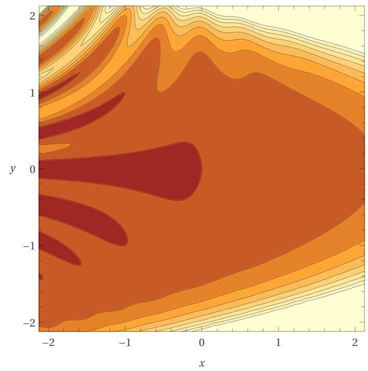
Monte Carlo integration

An important application of the Monte Carlo method is the numerical integration of complicated ed functions, especially in functions of several variables.

Consider, for example, the two-dimensional function

$$f(x, y) = e^{y-x} \sin^2(\pi x y) + \sinh(x + y^2)$$

in the range $(x, y) \in I \equiv ([-1, 1], [-1, 1])$. Its graph is shown on the right.



Computed by Wolfram|Alpha

1. Compute the integral of f numerically. You can approximate the volume of f by dividing the domain of each variable in n bins, and by evaluating f in the center of each bin and multiplying for the bin size:
$$\int_I f(x, y)\mathrm{d}x\mathrm{d}y \approx \sum_{i,j} f(x_i, y_j)\Delta x \Delta y$$

2. Compute the integral of f using the "hit and miss" method. Pay attention to the following points:
   - generate random points in a volume that fully contains the range of f in its domain I.
   - f is negative in some regions. Carefully handle the sign of f.

3. Which one of the two approaches is more convenient? In what conditions? If you compute the integral numerically using 1000×1000 bins, how many random points do you have to generate to get the same result with the Monte Carlo method up to the second decimal digit? We will discuss your findings during the next exercise session.

Attention: this exercise's "submit.pyc" is a bit slower than what you are used to. Please be patient.