Question

A student in materials sciences wants to determine the refraction index n_2 of a certain material using Snell's law of refraction. He put a sample of this material in a container filled with air at a certain pressure, with a goniometer.

Snell's law.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Assume the refraction index of the pressurized air, n_1, is knows with an uncertainty σ_{n_1}.

The student measures both θ_1 and θ_2 with the same goniometer. Both angles have a systematic uncertainty σ_θ.

How do these uncertainties propagate to the measurement of n_2? Are the uncertainties on θ_1 and θ_2 likely to be correlated?

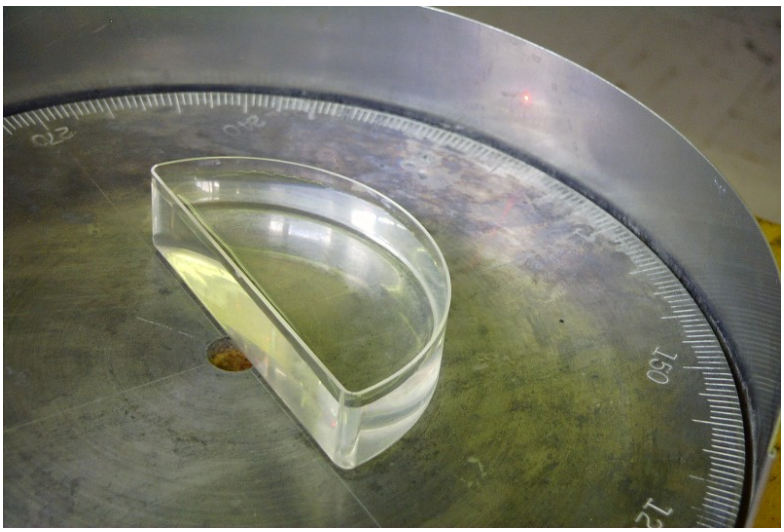

Figure 1: A refraction experiment [photo credits `clemson.edu`].

Exercise

1. Write a function that computes n_2 given the measurements of n_1, θ_1 and θ_2.

2. Write a function that computes the uncertainty on n_2 given the measurements of n_1, θ_1 and θ_2 and their respective uncertainties, and the correlation coefficient ρ of θ_1 and θ_2.

- you can "factorize" this problem and keep your code simpler by developing separate functions to compute all the needed partial derivatives.

3. What is the likeliest value of ρ? Call it $\bar{\rho}$. Compute n_2 and its uncertainty given the following data:

   - $n_1 = 1.0010 \pm 0.0005$
   - $\theta_1 = (46 \pm 1)^\circ$
   - $\theta_2 = (38 \pm 1)^\circ$

   Pay attention to the units and perform the appropriate conversions (degrees versus radians)...

   Store $\bar{\rho}$, n_2 and σ_{n_2} in the `results` dictionary in `correlated.py`.