Leisen-Reimer binomial model

We consider binomial model for pricing European and American options. Parameters of the binomial model are the following: r - risk-free continuous annual interest rate; σ - volatility (standard deviation) of the underlyuing asset;
T - expiration time (in years); $S(0)$ - price of the underlying asset at time $t = 0$; E - strike price; N - number of time periods in the binomial model.

To achieve the faster convergence in binomial model we can use **Leisen-Reimer method** for choosing the parameters of the binomial model. In case of Leisen-Reimer method the number of time periods must be odd; so, if $N = 2k$ then take $N = N + 1$.

The parameters in case of Leisen-Reimer method are the following

$$R = \exp(r\Delta t),$$
$$U = \frac{Rq'}{q},$$
$$D = \frac{R - qU}{1 - q},$$

where

$$q' = h(d1), \qquad q = h(d2),$$
$$d1 = \frac{log(S(0)/E) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}},$$
$$d2 = d1 - \sigma\sqrt{T},$$
$$h(z) = 0.5 + 0.5sign(z)\sqrt{1 - \exp\left(-a(z)\right)},$$
$$a(z) = \frac{z^2}{(N + 1/3 + 0.1/(N + 1))^2}(N + 1/6).$$

Here $sign(z) = 1$ if $z > 0$, $sign(z) = 0$ if $z = 0$, $sign(z) = -1$ if $z < 0.$ (In Python there is function numpy.sign(z) to find corresponding value). Note, that the function $h(z)$ is a discrete approximation to the cummulative distribution function for a normal distribution.

It is easy to check that q is the **risk-neutral probability**: $q = \frac{R-D}{U-D}$.