CFRM 501 Investment Science

Homework 4

Total marks: 15

October 30, 2022, 11:59 PM

General instructions. Submit your answers as a single PDF file on Canvas. To ensure full marks, include working out and explanations. For some questions, you may want to use computer software. For these, your written explanation should be sufficient to explain your answer without having to read code. Please check that your submission on Canvas can be opened and the content of the file is complete and correct.

Submissions more than 1 day late will not be accepted.

Question 1. Consider the daily prices from October 2, 2017, to September 30, 2022, of the following equities: GNK, BBBY, HCCI, SKM, CS, AZN, NVDA, EPAC. The dataset is on Canvas as a CSV file.

(a) Compute the sample mean vector and sample covariance matrix for the daily arithmetic returns, and then convert these quantities to annualized means and covariances using the formulas

$$\mu_i^{(a)} = (\mu_i^{(d)} + 1)^N - 1,$$

$$\sigma_{ij}^{(a)} = \left(\sigma_{ij}^{(d)} + (\mu_i^{(d)} + 1)(\mu_j^{(d)} + 1) \right)^N - (\mu_i^{(d)} + 1)^N (\mu_j^{(d)} + 1)^N,$$

respectively, where the number of days in a year is $N = 252$ (the covariance formula comes from similar arguments as in the Week 2 Notes, pg 18). Using the annualized mean vector and covariance matrix, find the global minimum variance portfolio in terms of its weights, and also the expected return and standard deviation. [5 marks]

(b) Suppose an investor wants to achieve an annualized expected return of 0.25. Find the weights of the minimum variance portfolio that achieves this annualized expected return, and the standard deviation. [3 marks]

(c) Suppose instead the investor wants to achieve an annualized standard deviation of 0.23. What is the maximum annualized expected return the investor can obtain? [2 marks]

Question 2. There are 2 assets, and their volatilities are $\sigma_1, \sigma_2 > 0$.

(a) Assume $\sigma_1 \neq \sigma_2$. Determine all the conditions on the covariance matrix such that the global minimum variance portfolio involves fully investing in one asset. Also, include any further conditions that are needed for the covariance matrix to be valid. [3 marks]

(b) What happens if $\sigma_1 = \sigma_2$? [2 marks]