1

$0 \to i_1 + i_2 - I_s = 0$ $R_1 i_1 = V_{b_1} = 0 - V_1$

$1 \to i_3 - i_1 = 0$ $R_2 i_2 = V_{b_2} = V_2 - 0$

$2 \to i_3 + i_2 - I_s = 0$ $R_3 i_3 = V_{b_3} = V_2 - V_1$

$$\Rightarrow \left.\begin{aligned} &\bullet\ i_3 - i_1 = 0 \\ &\bullet\ i_3 + i_2 = 0 \end{aligned}\right\} \underbrace{\begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}}_{A} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ I_s \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_{b_1} \\ V_{b_2} \\ V_{b_3} \end{bmatrix} = \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}}_{A^T} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\Rightarrow \left.\begin{aligned} i_1 &= \frac{V_{b_1}}{R_1} = \frac{-V_1}{R_1} \\ i_2 &= \frac{V_{b_2}}{R_2} = \frac{V_2}{R_2} \\ i_3 &= \frac{V_{b_3}}{R_3} = \frac{V_2 - V_1}{R_3} \end{aligned}\right\} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 1/R_1 & 0 & 0 \\ 0 & 1/R_2 & 0 \\ 0 & 0 & 1/R_3 \end{bmatrix}}_{\alpha} \begin{bmatrix} V_{b_1} \\ V_{b_2} \\ V_{b_3} \end{bmatrix}$$

a) $1 \rightarrow \dfrac{V_2 - V_1}{R_3} + \dfrac{V_1}{R_1} = 0$

$2 \rightarrow \dfrac{V_2 - V_1}{R_3} + \dfrac{V_2}{R_2} - I_s = 0$

$$\begin{bmatrix} 1/R_1 - 1/R_3 & 1/R_3 \\ -1/R_3 & 1/R_2 + 1/R_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ I_s \end{bmatrix}$$

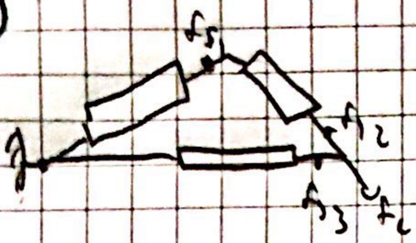
b)

$$\begin{bmatrix} A & \text{"}0\text{"} \\ \Sigma_N & -\alpha A^T \end{bmatrix} \begin{bmatrix} I_b \\ V_N \end{bmatrix} = \begin{bmatrix} I_s \\ \text{"}0\text{"} \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|cc} -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ \hline 1 & 0 & 0 & 1/R_1 & 0 \\ 0 & 1 & 0 & 0 & -1/R_2 \\ 0 & 0 & 1 & -1/R_3 & -1/R_3 \end{array}\right] \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ I_s \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-\alpha \cdot A = \begin{bmatrix} 1/R_1 & 0 \\ 0 & -1/R_2 \\ -1/R_3 & -1/R_3 \end{bmatrix}$$

2



$$\Rightarrow f_{s_1} - f_{s_2} = 0$$

$$f_{s_2} + f_{s_3} + f_{Load} = 0$$

$$\Rightarrow f_{s_1} \rightarrow f^*_{s_{1,x}} = \frac{\Delta x}{L_{s_1}} \xi (L_{s_{1,0}} - L_{s_1}) \quad , \quad f^*_{s_{1,y}} = \frac{\Delta x}{L_{s_1}} \xi (L_{s_{1,0}} - L_{s_1})$$

$$f_{s_2} \rightarrow f^*_{s_{2,x}} = \frac{\Delta x}{L_{s_2}} \xi (L_{s_{2,0}} - L_{s_2}) \quad , \quad f^*_{s_{2,y}} = \frac{\Delta y}{L_{s_2}} \xi (L_{s_{1,0}} - L_{s_2})$$

$$f_{s_3} \rightarrow f^*_{s_{3,x}} = \frac{\Delta x}{L_{s_1}} \xi (L_{s_{3,0}} - L_{s_3}) \quad , \quad f^*_{s_{3,y}} = \frac{\Delta y}{L_{s_3}} \xi (L_{s_{3,0}} - L_{s_3})$$

for $F_x(x,y) = \frac{x}{L} \xi (L_0 - L)$ disregard

$L = \sqrt{x^2 + y^2}$

$$\frac{dF_x}{dx} = -\frac{x^2 (L_0 + L)}{L^3} + \frac{L_0 + L}{L} + \frac{x^2}{L^2}$$

→ same method for y

for $F_y(x,y) = \frac{x}{L} \xi (L_0 - L)$ disregard

$L = \sqrt{x^2 + y^2}$

$$\frac{dF_x}{dy} = -\frac{L_0 x y}{L^3}$$

making calculations for $f_{s_1,x}, f_{s_1,y}, \dots f_{s_3,x}, f_{s_3,y}$ }

for $s_1 \to (x_u, y_u) = (1,1) \to L_{o_1} = \sqrt{2}$

for $s_2 \to (x_u, y_u) = (1,-1) \to L_{o_2} = \sqrt{2}$

for $s_3 \to (x_u, y_u) = (2,0) \to L_{o_3} = 2$

$$\alpha(1,1) = \begin{bmatrix} \frac{df_{s_1,x}}{dx} = (2^{-1}+2).0.5 & \frac{df_{s_1,x}}{dy} = -2^{-1} * 0.5 \\ \frac{df_{s_1,y}}{dy} = -2^{-1} * 0.5 & \frac{df_{s_1,y}}{dy} = (2^{-1}+1) * 0.5 \end{bmatrix}$$

$$\alpha(2,2) = \begin{bmatrix} \frac{df_{s_2,x}}{dx} = (2^{-1}+1) * 0.5 & \frac{df_{s_2,x}}{dy} = +2^{-1} * 0.5 \\ \frac{df_{s_2,y}}{dx} = +2^{-1} * 0.5 & \frac{df_{s_2,y}}{dy} = (2^{-1}+1) * 0.5 \end{bmatrix}$$

$$\alpha(3,3) = \begin{bmatrix} \frac{df_{s_3,x}}{dx} = 1 * 0.5 & \frac{df_{s_3,x}}{dy} = 0 \\ \frac{df_{s_3,y}}{dx} = 0 & \frac{df_{s_3,y}}{dy} = 2 * 0.5 \end{bmatrix}$$

c. Looking at to $-\alpha A^T$ matrix ;

=> joint at (1,1) -> movement $x \to -$

$y \to +y$

=> joint at (2,0) -> movement $x \to -x$

$y \to -y$

Answers are accurate and makes sense

d.
$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} * & * \\ * & * \end{bmatrix} \begin{bmatrix} x_1^* \\ y_1^* \end{bmatrix}$$
=> non-linear equations can not be solved like this

$$\to F_x^* = \frac{x_1 - 0}{L} \, (L_0 - L) \Rightarrow \text{non-linear}$$

$$(1) \quad f(x,y) \cong f(x_e, y_e) + \left.\frac{df(x,y)}{dx}\right|_{\substack{x=x_e \\ y=y_e}} \cdot (x - x_e) + \left.\frac{df(x,y)}{dy}\right|_{\substack{x=x_e \\ y=y_e}} (y - y_e)$$

ODS is given as matlab file.