

$$P(X_{t+1} = \text{colored} \mid X_t = \text{blank}, U_{t+1} = \text{paint}) = 0,9$$

$$P(z_t = \text{colored} \mid X_t = \text{blank}) = 0,2$$

$$P(z_t = \text{colored} \mid X_t = \text{colored}) = 0,7$$

$$P(X_{t+1} = \text{blank} \mid X_t = \text{blank}, U_{t+1} = \text{paint}) = 0,1 \quad \text{bel}(X_1 = \text{blank}) = ?$$

$$S = \{ \text{colored}, \text{blank} \} \quad A = \{ \text{paint}, \text{nothing} \}$$

$$\mathcal{U} = \{ \text{colored}, \text{blank} \}$$

$$P(X_0 = \text{blank}) = 0,5 = \text{bel}(X_0 = \text{blank})$$

$$P(X_0 = \text{colored}) = 0,5 = \text{bel}(X_0 = \text{colored})$$

$$\begin{aligned} \overline{\text{bel}}(X_1 = \text{blank}) &= \sum_{X_0 \in S} P(X_1 = \text{blank} \mid X_0, U_1 = \text{paint}) \cdot \text{bel}(X_0) \\ &= P(X_1 = \text{blank} \mid X_0 = \text{blank}, U_1 = \text{paint}) \cdot \text{bel}(X_0 = \text{blank}) \\ &= 0,5 \cdot P(X_1 = \text{blank} \mid X_0 = \text{blank}, U_1 = \text{paint}) \end{aligned}$$

$\hookrightarrow P(X_{t+1} = \text{blank} \mid X_t = \text{colored}, U_{t+1} = \text{paint}) = 0$ because there is no way to make a colored object blank again

$$P(X_{t+1} = \text{colored} \mid X_t = \text{colored}, U_{t+1} = \text{paint}) = 1$$

$$P(X_{t+1} = \text{blank} \mid X_t = \text{colored}, U_{t+1} = \text{paint}) = 0$$

$$P(z_t = \text{blank} \mid X_t = \text{blank}) = 1 - P(z_t = \text{colored} \mid X_t = \text{blank}) = 0,8$$

$$P(z_t = \text{blank} \mid X_t = \text{colored}) = 1 - P(z_t = \text{colored} \mid X_t = \text{colored}) = 0,3$$

$$\begin{aligned} \overline{\text{bel}}(X_1 = \text{colored}) &= P(X_1 = \text{colored} \mid X_0 = \text{blank}, U_1 = \text{paint}) \cdot \text{bel}(X_0 = \text{blank}) \\ &\quad + P(X_1 = \text{colored} \mid X_0 = \text{colored}, U_1 = \text{paint}) \cdot \text{bel}(X_0 = \text{colored}) \\ &= 0,9 \cdot 0,5 + 1 \cdot 0,5 \\ &= 0,95 \end{aligned}$$

$$P(X_{t+1} = \text{blank} \mid X_t = \text{blank}, U_t = \text{paint}) = 0,1$$

$$\overline{\text{bel}}(X_1 = \text{blank}) = 0,05$$

②

$$\text{bel}(x_1) = \eta p(z_1 = \text{blank} | x_1) \bar{\text{bel}}(x_1)$$

$$\text{bel}(x_1 = \text{blank}) = \eta 0,8 \cdot 0,05 = \eta 0,04$$

$$\text{bel}(x_1 = \text{colored}) = \eta 0,3 \cdot 0,95 = \eta 0,285$$

$$\eta = \frac{1}{0,04 + 0,285} = 3,0769$$

~~= irrelevant, after painting robot always senses colored!~~

$$\text{bel}(x_1) = \eta p(z_1 = \text{colored} | x_1) \bar{\text{bel}}(x_1)$$

$$\text{bel}(x_1 = \text{blank}) = \eta 0,2 \cdot 0,05 = \eta 0,01$$

$$\text{bel}(x_1 = \text{colored}) = \eta 0,7 \cdot 0,95 = \eta 0,665$$

$$\eta = \frac{1}{0,01 + 0,665} = 1,481$$

$$\text{bel}(x_1 = \text{blank}) = 0,01481$$

$$\text{bel}(x_1 = \text{colored}) = 0,9852$$

$$p(x_{t+1} = b | u_{t+1} = p, z_{t+1} = c) = \frac{p(z_{t+1} = c | u_{t+1} = p, x_{t+1} = b) p(x_{t+1} = b | u_{t+1})}{p(z_{t+1} = c | u_{t+1} = p)}$$

=> Bayes Theorem, Markov assumption

$$p(z_{t+1} = c | x_{t+1} = b) = 0,2$$

$$p(x_{t+1} = b | u_{t+1} = p) = p(x_{t+1} = b | u_{t+1} = p, x_t = b) p(x_t = b)$$

=> total prob.

$$+ p(x_{t+1} = b | u_{t+1} = p, x_t = c) p(x_t = c)$$

$$= p(x_{t+1} = b | u_{t+1} = p, x_t = b) p(x_t = b)$$

$$= 0,9 \cdot 0,5 = 0,05 = \text{bel}(x_1 = \text{blank})$$

$$p(z_{t+1} = c | u_{t+1} = p) = p(z_{t+1} = c | u_{t+1} = p, x_{t+1} = c) p(x_{t+1} = c | u_{t+1} = p)$$

=> total prob.

markov assumption,
total prob.

$$+ p(z_{t+1} = c | u_{t+1} = p, x_{t+1} = b) p(x_{t+1} = b | u_{t+1} = p)$$

$$= 0,7 (p(x_{t+1} = c | u_{t+1} = p, x_t = b) p(x_t = b | u_{t+1} = p)$$

$$+ p(x_{t+1} = c | u_{t+1} = p, x_t = c) p(x_t = c | u_{t+1} = p))$$

$$0,2 (p(x_{t+1} = b | u_{t+1} = p, x_t = b) p(x_t = b | u_{t+1} = p)$$

$$+ p(x_{t+1} = b | u_{t+1} = p, x_t = c) p(x_t = c | u_{t+1} = p))$$

$$= 0,7 (0,9 \cdot 0,5 + 1 \cdot 0,5) + 0,2 (0,1 \cdot 0,5 + 0 \cdot 0,5)$$

$$= 0,7 \cdot 0,95 + 0,2 \cdot 0,05 = 0,675$$