Answers for Homework 2 - STAT451

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1 Exercise 17:

Let A denote the event that the next request for assistance from a statistical software consultant relates to the SPSS package, and let B be the event that the next request is for help with SAS. Suppose that $P(A) = .30$ and $P(B) = .50$.

a. Why is it not the case that $P(A) + P(B) = 1$?

b. Calculate $P(A')$.

c. Calculate $P(A \cup B)$.

d. Calculate $P(A' \cap B')$.

1.1 a. Why is it not the case that $P(A) + P(B) = 1$?

The events A and B are not mutually exclusive (disjointed), because, apart from the 2 package above, there are probably other packages that the consultant can use. Therefore, the sum of the probabilities of A and B is not equal to 1.

1.2 b. Calculate $P(A')$.

$P(A') = 1 - P(A) = 1 - 0.30 = 0.70$

1.3 c. Calculate $P(A \cup B)$.

Because A and B are disjointed, the intersection of A and B is 0:

$$
\begin{aligned}
P(A \cup B) &= P(A) + P(B) - P(A \cap B) = 0.30 + 0.50 - P(A \cap B) \\
&= 0.80 - P(A \cap B) \\
&= 0.80 - 0 \\
&= 0.80
\end{aligned} \tag{1}
$$

1.4 d. Calculate $P(A' \cap B')$.

Use De Morgan's Law:

$$P(A' \cap B') = P[(A \cup B)'] = 1 - P(A \cup B) = 1 - 0.80 = 0.20 \tag{2}$$

2 Exercise 26:

A certain system can experience three different types of defects. Let $A_i (i = 1, 2, 3)$ denote the event that the system experiences a defect of type i. Suppose that:

$P(A_1) = 0.12$, $P(A_2) = 0.07$, $P(A_3) = 0.50$
$P(A_1 \cup A_2) = 0.13$, $P(A_1 \cup A_3) = 0.14$
$P(A_2 \cup A_3) = 0.10$, and $P(A_1 \cap A_2 \cap A_3) = 0.10$.

2.1 What is the probability that the system does not have a type 1 defect?

Does not have a type 1 defect $\Leftrightarrow \bar{A}_1$.

$$P(\bar{A}_1) = 1 - P(A_1) = 1 - 0.12 = 0.88 \tag{3}$$

2.2 What is the probability that the system has both type 1 and type 2 defects?

Both type 1 and type 2 defects $\Leftrightarrow A_1 \cap A_2$.

$$\begin{aligned} P(A_1 \cap A_2) &= P(A_1) + P(A_2) - P(A_1 \cup A_2) \\ &= 0.12 + 0.07 - 0.13 \\ &= 0.06 \end{aligned} \tag{4}$$

2.3 What is the probability that the system has both type 1 and type 2 defects but not a type 3 defect?

Both type 1 and type 2 defects but not a type 3 defect $\Leftrightarrow (A_1 \cap A_2) \cap \bar{A}_3$.

$$\begin{aligned} P[(A_1 \cap A_2) \cap \bar{A}_3] &= P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3) \\ &= 0.06 - 0.10 \\ &= -0.04 \end{aligned} \tag{5}$$

2.4 What is the probability that the system has at least two types of defect?

At least two types of defect $\Leftrightarrow$ all cases where we don't have all 3 types of defect.

$$\begin{aligned} P &= 1 - P(A_1 \cap A_2 \cap A_3) \\ &= 1 - 0.10 = 0.90 \end{aligned} \tag{6}$$

3 Exercise 29:

As of April 2006, roughly 50 million .com web domain names were registered (e.g., yahoo.com).

3.1 How many domain names consisting of just two letters in sequence can be formed? How many domain names of length two are there if digits as well as letters are permitted as characters? [Note: A character length of three or more is now mandated.]

- There are 26 letters in the English alphabet. Therefore, the number of domain names consisting of just two letters in sequence is $26^2 = 676$.
- If digits are permitted as characters, then there are 26 letters and 10 digits. Therefore, the number of domain names of length two is $36^2 = 1296$.

3.2 How many domain names are there consisting of three letters in sequence? How many of this length are there if either letters or digits are permitted? [Note: All are currently taken.]

- $26^3 = 17576$.
- $36^3 = 46656$.

3.3 Answer the questions posed in 3.2 for four-character sequences.

- $26^4 = 456976$.
- $36^4 = 1679616$.

3.4 As of April 2006, 97,786 of the four-character se quences using either letters or digits had not yet been claimed. If a four-character name is randomly selected, what is the probability that it is already owned?

Let P be the probability that a four-character name is not claimed.

$$\begin{aligned} P &= \frac{97786}{1679616} \\ &\approx 0.058 \end{aligned} \tag{7}$$

Therefore, the probability that a four-character name is already owned is $1 - 0.058 = 0.942$.

4 Exercise 35:

A production facility employs 10 workers on the day shift, 8 workers on the swing shift, and 6 workers on the graveyard shift. A quality control consultant is to select 5 of these workers for in-depth interviews. Suppose the selection is made in such a way that any particular group of 5 workers has the same chance of being selected as does any other group (drawing 5 slips without replacement from among 24).

4.1 How many selections result in all 5 workers coming from the day shift? What is the probability that all 5 selected workers will be from the day shift?

- Based on the question, we pick 5 out of the 10 day workers.
  - Number of selections: $\binom{10}{5} = 252$
  - Probability: $P(all\ 5\ from\ day\ shift) = \frac{\binom{10}{5}}{\binom{24}{6}} \approx 0.00592$

4.2 What is the probability that all 5 selected workers will be from the same shift?

Let A, B, C are the events that all 5 workers come from the day, swing, and graveyard shifts, respectively. Then, we have:

$$\begin{aligned} P(6 from same shifts) &= P(A) + P(B) + P(C) \\ &= \frac{\binom{10}{5}}{\binom{24}{5}} + \frac{\binom{8}{5}}{\binom{24}{5}} + \frac{\binom{6}{5}}{\binom{24}{5}} \\ &\approx 0.00592 + 0.00131 + 0.00014 \\ &\approx 0.00737 \end{aligned} \tag{8}$$

4.3 What is the probability that at least two different shifts will be represented among the selected workers?

At least 2 different shifts will be represented $\Leftrightarrow$ all cases where we don't have all 5 workers from the same shift.

$$\begin{aligned} P &= 1 - P(6\ from\ same\ shifts) \\ &= 1 - 0.00737 = 0.99263 \end{aligned} \tag{9}$$

5 Exercise 41:

An ATM personal identification number (PIN) consists of four digits, each a 0, 1, 2,... 8, or 9, in succession.

5.1 How many different possible PINs are there if there are no restrictions on the choice of digits?

The number of different possible PINs is $10^4 = 10000$.

5.2 According to a representative at the author's local branch of Chase Bank, there are in fact restrictions on the choice of digits. The following choices are prohibited: (i) all four digits identical (ii) sequences of consecutive ascending or descending digits, such as 6543 (iii) any sequence start ing with 19 (birth years are too easy to guess). So if one of the PINs in (a) is randomly selected, what is the probability that it will be a legitimate PIN (that is, not be one of the prohibited sequences)?

To calculate the probability that a PIN is legitimate, we need to calculate the number of prohibited PINs and then subtract that number from the total number of PINs. Let:

- A be the event that all four digits are identical.
- B be the event that the PIN is a sequence of consecutive ascending or descending digits.
- C be the event that the PIN starts with 19.

Then, we have:

- $P(A) = \frac{10}{10000} = 0.001$.
- $P(B)$:
  - For ascending sequences, we have 7 sequences: 0123, 1234, 2345, 3456, 4567, 5678, 6789.
  - For descending sequences, we have 7 sequences: 9876, 8765, 7654, 6543, 5432, 4321, 3210.
  - So, we have 14 prohibited sequences. Therefore, $P(B) = \frac{14}{10000} = 0.0014$.
- $P(C)$:
  - First and 2nd slot is 19, while 3rd and 4th slot have 10 choices each.
  - Therefore, there are $1 \times 1 \times 10 \times 10 = 100$ prohibited sequences. Therefore, $P(C) = \frac{100}{10000} = 0.01$.
- $P(A \cup B \cup C) = P(A) + P(B) + P(C) = 0.001 + 0.0014 + 0.01 = 0.0124$.
- The probability that a PIN is legitimate is $1 - 0.0124 = 0.9876$.