Enhancing Logical Reasoning of Large Language Models through Logic-Driven Data Augmentation

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Abstract

Combining large language models with logical reasoning enhance their capacity to address problems in a robust and reliable manner. Nevertheless, the intricate nature of logical reasoning poses challenges to gathering reliable data from web for building comprehensive training datasets, subsequently affecting the performance on downstream tasks. To address this, we introduce a novel logic-driven data augmentation approach, AMR-LDA. AMR-LDA converts the original text into an Abstract Meaning Representation (AMR) graph, a structured semantic representation that encapsulates the logic structure of the sentence, upon which operations are performed to generate logically modified AMR graphs. The modified AMR graphs are subsequently converted back into texts to create augmented data. Notably, our methodology is architecture-agnostic and enhances generative large language models, such as GPT-3.5 and GPT-4, through prompt augmentation, and fine-tuning discriminative large language models through contrastive learning with logic-driven data augmentation. Empirical evidence underscores the efficacy of our proposed method with improvement in performance across seven downstream tasks, such as logical reasoning reading comprehension, textual entailment, and natural language inference. Furthermore, our method ranked first on the ReClor leaderboard ¹. The source code and data are publicly available ².

Introduction

Enabling pre-trained large language models (LLMs) to reliably perform logical reasoning is an important step towards strong artificial intelligence (Chollet 2019). However, data annotation for logical reasoning tasks is a difficult, time-consuming and costly process that has led to the scarcity of large-scale logical reasoning datasets derived from natural

language on the web. Therefore, LLMs are usually trained on generic corpora or smaller datasets on logical reasoning that lead to poor generalisation (Wang et al. 2022). The automatic augmentation of logical reasoning data has the potential to enhance the generalisation and performance of LLMs on logical reasoning tasks.

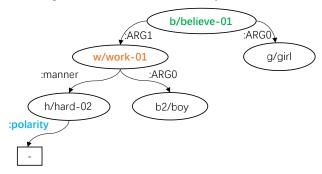
To address this challenge, we propose a logic-driven data augmentation method based on Abstract Meaning Representation (AMR). AMR is a structural representation of the semantic meaning and logical structure of a text via a rooted directed acyclic graph (DAG) (Shou, Jiang, and Lin 2022). Figure 1 shows an example of AMR graph. The AMR graph can be easily modified by changing the nodes or arguments to create logically equivalent or nonequivalent graphs. Publicly available text-to-AMR and AMR-totext models can perform end-to-end conversion between natural language and AMR graph. By taking advantage of the ease of logical manipulation of AMR graph and the end-to-end conversion between natural language and AMR graph, our proposed data augmentation is not task-specific or template-dependent, and can generate logically equivalent and nonequivalent sentences that are diverse in language.

In order to improve the performance of LLMs on downstream tasks that require logical reasoning, we investigate two different applications of the proposed logic-driven data augmentation for two different types of language models. In this paper, we call models such as RoBERTa (Liu et al. 2019) and DeBERTa (He et al. 2021) as discriminative large language models, and models like GPT-3.5 (OpenAI 2023a) as generative LLMs. Discriminative large language models typically cast a question-answering task into a classification task and require fine-tuning to perform downstream tasks. Generative LLMs are autoregressive language models that answer questions by generating textual outputs conditioned on the input prompts. We improve the reasoning ability of discriminative large language models by applying contrastive learning to identify logically equivalent and nonequivalent sentence pairs generated using the proposed data augmentation, before fine-tuning the model further on downstream tasks. In order to improve the performance of

¹https://eval.ai/web/challenges/challenge-page/503/leaderboard/1347

²https://github.com/Strong-AI-Lab/Logical-Equivalence-driven-AMR-Data-Augmentation-for-Representation-Learning Preprint. Under review.

S1: The girl believes that the boy doesn't work hard. S2: The girl doesn't believe that the boy works hard.



S3: If Alan is kind, then Bob is **not** clever.

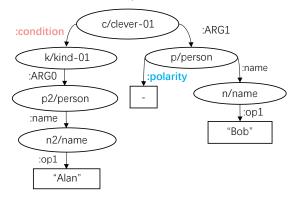


Figure 1: An example of AMR. Two sentences with the same semantic meaning can be represented as the same AMR graph. "b", "g", and "w" are variables. "w/work-01" refers to the variable "w" has an instance relation with the AMR concept "work-01". "work" is the frame from Propbank (Kingsbury and Palmer 2002) and "-01" is the sense of frame. ":ARG0", ":ARG1", ":condition", ":polarity" are frame arguments, following PropBank instructions. ":condition" and ":polarity -" are used to represent conditional and negative relationships.

generative LLMs on logical reasoning tasks without finetuning, we augment the input prompt by extending the question context and options using data augmentation.

We summarize key contributions as follows:

- 1. We propose an AMR-based logic-driven data augmentation method to automatically construct logically equivalent/nonequivalent sentences.
- 2. We enhance the logical reasoning of large language models through logical-equivalence-identification contrastive learning and prompt augmentation.
- 3. The experimental results show that our method can improve large language models' performance on downstream tasks including logical reasoning, textual entailment and natural language inference. Furthermore, our method can improve generative large language models' performance without fine-tuning.

Related Work

Data Augmentation Data augmentation is a widely applied method in natural language processing to improve a model's performance. There are typically two types of data augmentation (Feng et al. 2021): rule-based and modelbased data augmentation. Rule-based data augmentation relies on predefined templates or rules. This approach has been used to automatically generate multi-step deductive reasoning datasets (Clark, Tafjord, and Richardson 2021), to ensure that the generated examples are logically correct. However, rule-based augmentation methods suffer from a lack of diversity in generated texts. To increase the diversity of augmented texts, model-based data augmentation has been applied to paraphrase original sentences (Li, Hou, and Che 2022). Model-based data augmentation typically uses a language model to encode the original sentence with a latent representation, then decode it back into natural language (Ng, Cho, and Ghassemi 2020; Qi et al. 2023).

Counterfactual Data Augmentation MERIt is a state-of-the-art model on the ReClor leaderboard that automatically synthesises context-option pairs from unlabelled corpus (e.g., Wikipedia), with the help of relation extraction (Jiao et al. 2022). The system operates under the assumption that the extracted sentences are factually correct, using them as positive examples. For the creation of negative examples, MERIt applies counterfactual data augmentation by randomly replacing the entities within a example with entities extracted from other documents. However, its assumptions heavily depend on the transitive relationships between sentences, which are established through relation extraction. This approach does not perform logic manipulation to the sentences.

Logic-Driven Data Augmentation To address the data sparsity associated with deeper reasoning depths, PARARULE-Plus (Bao et al. 2022) introduces an expanded synthetic multi-step logical reasoning dataset. This dataset aims to enhance the performance of large language models on tasks that require deeper reasoning depths. Moving toward a more realistic logical reasoning data augmentation, LReasoner (Wang et al. 2022) improves logical reasoning performance by logically augmenting sentences on a reading comprehension dataset such as ReClor (Yu et al. 2020) and adding the augmented contextual information to the options. Specifically, it uses a constituency parser to capture the syntactical structure of a sentence, and uses manually created templates to logically augment the parsed tree. However, the logical laws considered are limited and the augmentation is also constrained by the templates (Yu et al. 2020), weakening the generalisability of the method. Furthermore, the constituency parser cannot capture rich logical information about the sentences.

AMR-Based Data Augmentation AMR-DA (Shou, Jiang, and Lin 2022) uses an AMR-based semantic parser, instead of a constituency parser, to extract the semantic information from a sentence and applies four different modifications – random swap, random deletion, random insertion, and synonym replacement – on the AMR graph.

It takes advantage of text-to-AMR and AMR-to-text models to convert sentences into AMR graphs and convert them back into natural sentences after modifying the AMR graphs. However, AMR-DA is only designed to increase the diversity of the sentences in a dataset, and does not aim to logically manipulate original sentences. In contrast, our AMR-LDA is designed to construct more logical reasoning data. It aims to address the existing gap due to the scarcity of logical reasoning datasets in the real world web and to enhance the performance of logical reasoning tasks on large language models.

Logical Equivalence Logical equivalence is a fundamental concept in formal logic (Mendelson 2009). It can be formally defined as: Two propositions or statement forms P and Q are logically equivalent if they have the same truth value in every possible circumstance, or in every possible model. This can be denoted as $P \equiv Q$. This condition can also be described by the statement: P and Q are logically equivalent if and only if the statement "P if and only if Q" is a tautology. A tautology is a statement that is always true, regardless of the truth values of its components. In terms of truth tables, P and Q are logically equivalent if their truth tables are identical, i.e., P and Q have the same truth value for each possible assignment of truth values to their components.

Method

System Architecture

Our system, shown in Figure 2, features an AMR-Based Logic-Driven Data Augmentation Module that parses sentences into AMR graphs, modifies the graphs to generate corresponding logically equivalent and nonequivalent graphs, then converts these back into natural language. The Logical-Equivalence-Identification Contrastive Learning Module aims to improve the logical reasoning ability of discriminative large language models by conducting contrastive learning to identify equivalent and nonequivalent sentence pairs, before further fine-tuning the model on downstream tasks. The Prompt Augmentation Module is intended to improve the performance of generative autoregressive LLMs on logical reasoning tasks by applying the data augmentation module to the input fed into the models at inference time, without performing any fine-tuning.

AMR-Based Logic-Driven Data Augmentation

We propose Abstract Meaning Representation-based Logic-driven Data Augmentation (AMR-LDA) to construct logically equivalent and nonequivalent sentences automatically. For simplicity, we consider only individual sentences, and propositional logic statements expressed in natual language. AMR-LDA involves the following steps: 1): Convert a sentence into AMR graph. 2): Logically augment the AMR graph. 3): Convert the logically augmented AMR graph back into natural language.

Text-To-AMR Parsing A text-to-AMR model is used to parse a natural language sentence into an AMR graph. In this step, the input is a natural language sentence written in

English. The output is a rooted, labeled, directed, and acyclic AMR graph that captures the main semantic information of the sentence.

AMR Graph Modification The AMR graph is modified to construct logically equivalent and nonequivalent graphs. To create logically equivalent graphs, we consider four different logical equivalence laws: double negation, commutative, implication, and contraposition laws. These laws of logical equivalence are defined below using propositional statements \mathcal{A} and \mathcal{B} , followed by examples in natural language (e.g. \mathcal{A} is "Alan is kind" and \mathcal{B} is "Bob is clever").

Definition 1: Contraposition Law

$$(\mathcal{A} \to \mathcal{B}) \Leftrightarrow (\neg \mathcal{B} \to \neg \mathcal{A})$$

If Alan is kind, then Bob is clever. \Leftrightarrow If Bob is not clever, then Alan is not kind.

To implement the contraposition law, we first swap the first half of the sentence with the second half if the AMR parser detects that the sentence is a conditional statement (e.g. "if-then", as marked by the blue background in Table 1). In the second step, we construct logically equivalent sentences for the four potential scenarios in which the negation may appear. Here, we use one such scenario as an example. If the first half of the sentence has no negation and the second half of the sentence has no negation either, then we will add the negative polarity argument, ":polarity -", to the first half and the second half of the sentence to construct logically equivalent sentences (marked with the yellow background in Table 1). AMR uses ":polarity -" to represent negation (e.g. "not").

Definition 2: Implication Law

$$(A \to B) \Leftrightarrow (\neg A \lor B)$$

If Alan is kind, then Bob is clever. \Leftrightarrow Alan is not kind or Bob is clever.

We consider two scenarios. If the sentence is detected by the AMR parser as a conditional statement, then we replace the conditional connective with a disjunction connective (marked with yellow background in Table 1). In the second scenario, if the sentence contains disjunction connectives, we replace the disjunction connective with conditional connective and remove the negative polarity from the AMR graph if it exits. Otherwise, a negative polarity will be added.

Definition 3: Commutative Law

$$(A \wedge B) \Leftrightarrow (B \wedge A)$$

Alan is kind and Bob is clever. \Leftrightarrow Bob is clever and Alan is kind.

If the AMR graph has a conjunction connective, we swap the order of the first half of the graph with the second half.

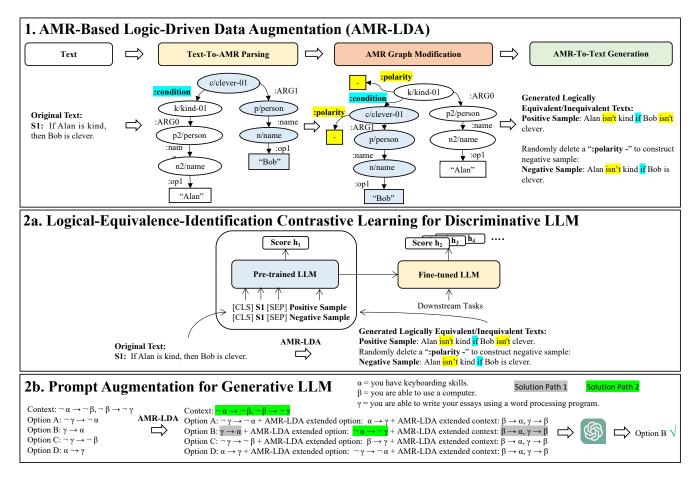


Figure 2: Architecture of AMR-LDA (1) and its applications to improve the reasoning performance of discriminative LLMs with contrastive learning (2a) and autoregressive generative LLMs by augmenting input prompts without fine-tuning (2b).

An example can be found in the Table 1. The sub-sentence "The wolf is fierce" and "the bald eagle is clever" have been marked as blue and swapped because our method detects the conjunction connective word "and" in the original sentence "The bald eagle is clever and the wolf is fierce".

Definition 4: Double Negation Law

$$\mathcal{A} \Leftrightarrow \neg \neg \mathcal{A}$$

Alan is kind. \Leftrightarrow Alan is not unkind.

We apply the double negation law only to those sentences and their AMR graphs that do not contain the ":polarity -" argument which represents negative polarity. There are several words that can be represented as ":polarity -", such as "not", "no", "never", "none", and "nothing". A representative example can be seen in Table 1. The original sentence is "The bald eagle is strong". The logically equivalent sentence we construct using double negation law is "The bald eagle is not weak", while the logically nonequivalent sentence is "The bald eagle is weak". Note that the generated sentences do not contain the word "not" twice. We avoid generating sentences with "not" appearing multiple times

consecutively because they are uncommon and unnatural. The process of applying double negation law is as follows: convert the sentence into an AMR graph; augment the AMR graph by adding a negative polarity argument ": polarity -"; convert the modified AMR graph back into a natural language sentence; lastly, replace the adjective word with its antonym by using WordNet (Miller 1992).

To create logically nonequivalent sentences, we randomly delete or add a negative polarity argument ":polarity -" in the AMR graph. Additionally, we randomly sample a sentence from the corpus and consider it as logically nonequivalent to the original sentence.

AMR-To-Text Generation Lastly, an AMR-to-text model is used to convert the modified AMR graph back into natural language, to generate a sentence that is logically equivalent or nonequivalent to the original sentence.

Logical-Equivalence-Identification Contrastive Learning

Inspired by SimCSE (Gao, Yao, and Chen 2021) and Sim-CLR (Chen et al. 2020), we propose to improve dicriminative language models' logical reasoning ability by perform-

Original sentence	Positive sample	Negative sample		
If Alan is kind,	Alan isn't kind if Bob isn't clever.	Alan isn't kind if Bob is clever.		
then Bob is clever.	Alan is not kind or Bob is clever.	Alan is kind or Bob is clever.		
The bald eagle is strong.	The bald eagle is not weak.	The bald eagle is weak.		
The bald eagle is clever and the wolf is fierce.	The wolf is fierce and the bald eagle is clever.	The wolf is not fierce and the bald eagle is not clever.		

Table 1: Examples of generated logically equivalent (positive) and nonequivalent sentences(negative). The blue background highlights the parts of the original sentence that have been moved from their original positions. The yellow background highlights the change in polarity from the original sentence.

ing contrastive learning to identify logically equivalent and nonequivalent sentence pairs that are generated using AMR-LDA (Figure 2, 2a).

Contrastive Learning The goal of contrastive learning is to minimise the distance of the hidden representations of two similar inputs while maximising the distance between two representations of dissimilar inputs. In our case, the goal is to optimise the model to map logically equivalent sentences to hidden representations that are close to each other.

$$h\left(s,s^{+}\right) \gg h\left(s,s^{-}\right). \tag{1}$$

h is a score function used to measure the distance between two representations. s is an original sentence, s^+ is a positive sample logically equivalent to the original sentence s, s^- is a negative sample logically nonequivalent to the original sentence s. The expected semantic representation distance between s and s^+ should be much closer than that of s and s^- . The training loss can be written with the following formula:

$$\mathcal{L} = -\sum \log \frac{\exp(h(+))}{\exp(h(+)) + \exp(h(-))}, \quad (2)$$

where $h\left(+\right)$ and $h\left(-\right)$ are short for $h\left(s,s^{+}\right)$ and $h\left(s,s^{-}\right)$. After the contrastive learning step, we further fine-tune the model on downstream tasks, including logical reasoning reading comprehension, natural language inference, and textual entailment.

Prompt Augmentation

To improve the performance of generative LLMs (e.g. GPT-3.5 (OpenAI 2023a) or GPT-4 (OpenAI 2023b)) on logical reasoning tasks, we propose to augment the input prompt using AMR-LDA, before feeding it to the model (Figure 2, 2b). If AMR-LDA can be applied to an option, we generate a logically equivalent sentence using AMR-LDA to extend the option. The original option is "If you are able to write your essays using a word processing program, then you have keyboarding skills.", it can be extended as "If you have no keyboarding skills, then you are not able to write your essays using a word processing program." using contraposition law. If an option can be augmented, we further augment the context and append the augmented context into

that option as well. In this example, we can generate the augmented context "If you are able to use a computer, then you have keyboarding skills. If you are able to write your essays using a word processing program, then you are able to use a computer.". Then, we extend the original option with the augmented option and augmented context. Based on the extended information, we can find **two solution paths** marked as grey and green background under the **Module 2b** in the Figure 2. **Solution Path 1**: use the sentence from extended context marked as grey background to support the Option B is correct. **Solution Path 2**: use the sentence from the original context marked as green background to support the extended Option B is correct.

Experiments

Datasets

ReClor (Yu et al. 2020) and LogiQA (Liu et al. 2021) are two challenging logical reasoning datasets. ReClor is summarised from Graduate Management Admission Test (GMAT) and Law School Admission Test (LSAT). LogiQA is collected by national civil service examination (Liu et al. 2021). We use five additional datasets for natural language inference and textual entailment tasks: MNLI (Williams, Nangia, and Bowman 2018), RTE (Wang et al. 2018), MRPC (Dolan and Brockett 2005), QNLI (Rajpurkar et al. 2016), and QQP (Wang et al. 2018). The primary distinction lies in the nature of the relationships they evaluate. The main difference is that MNLI, RTE, and MRPC assess the relationship between two sentences, while QNLI focuses on the relationship between a question and a sentence, and QQP evaluates the relationship between two questions.

Synthetic Data for Contrastive Learning In this paper, we perform contrastive learning for discriminative large language models on sentences augmented from a synthetic dataset. This dataset contains 14,962 sentences with different combinations of 23 entities, 2 relations and 40 attributes. Synthetic data is used to generate more controllable sentences given logical equivalence laws, which are less noisy than real-world datasets, and are more conducive to validate our experimental design and ideas. More details about the synthetic dataset can be found in the .

Models/ Datasets		ReC	Clor		Log	iQA	MNLI	MRPC	RTE	QNLI	QQP
Tributis, Dutagets	Dev	Test	Test-E	Test-H	Dev	Test			Eval		
RoBERTa	0.5973	0.5320	0.7257	0.3797	0.3543	0.3450	0.8895	0.9044	0.8339	0.9473	0.9089
RoBERTa AMR-LDA	0.6526	0.5686	0.7734	0.4077	0.4029	0.3814	0.8978	0.9093	0.8664	0.9449	0.9314
RoBERTa LReasoner-LDA	0.5946	0.5366	0.7219	0.3910	0.3481	0.3481	0.8941	0.8946	0.8628	0.9425	0.9001
RoBERTa AMR-DA	0.5866	0.5393	0.6681	0.4380	0.3645	0.3722	0.8974	0.9044	0.8628	0.9442	0.9206
DeBERTaV2	0.7393	0.7046	0.8082	0.6231	0.3972	0.3962	0.8945	0.8971	0.8448	0.9500	0.9254
DeBERTaV2 AMR-LDA	0.7940	0.7763	0.8575	0.7124	0.4234	0.3988	0.8967	0.9020	0.8809	0.9524	0.9247
DeBERTaV2 LReasoner-LDA	0.7573	0.7070	0.8408	0.6017	0.3087	0.2851	0.8923	0.8995	0.8700	0.9515	0.9250
DeBERTaV2 AMR-DA	0.7906	0.7590	0.8462	0.6904	0.2995	0.3010	0.8992	0.8971	0.8339	0.9502	0.9242

Table 2: Comparison between our proposed AMR-LDA and baseline models. We use RoBERTa-Large, DeBERTaV2-XXLarge as the pre-trained backbone models. Our fine-tuned LLMs perform equally well or better than baseline methods.

Settings

We conduct all the experiments on 8 NVIDIA A100 GPUs, each with 80G of VRAM. We conduct the primary experiments on three different random seeds, and report the average values. We adopted the parse_xfm_bart_large and T5Wtense models from AMRLib ⁴ to perform text-to-AMR and AMR-to-text conversions when generating logically augmented sentence pairs. The reason why we select those two models is explained in subsection . In our experiments, we use RoBERTa (Liu et al. 2019) and DeBERTa (He et al. 2021) as the discriminative large language models. For comparative purposes, we adopt MERIt (Jiao et al. 2022), the leading model on the ReClor leaderboard, as a baseline in both our primary experiment and the subsequent ablation studies. As for generative large language models, we utilise GPT-3.5 (gpt-3.5-turbo) (OpenAI 2023a) and GPT-4 (OpenAI 2023b).

We follow the training script from Huggingface and the default hyperparameters ⁵ to conduct the logical-equivalence-identification contrastive learning and fine-tuning. For the contrastive learning, we fine-tune RoBERTa-Large, DeBERTa-Large, and DeBERTaV2-XXLarge using the constructed logical equivalence sentence pair from our AMR-LDA data augmentation method, LReasoner's logic-driven data augmentation method (LReasoner-LDA) and AMR-DA data augmentation method. We use DeBERTaV2-XXLarge for ReClor and LogiQA tasks because DeBERTaV2 supports multiple-choice question tasks with a DeBERTaV2ForMultipleChoice head. More details about hyperparameters used during the experiment can be found in .

Conversion Between Texts and AMR

In order to decide which models to use to perform text and AMR conversions, we experiment with different combinations of text-to-AMR and AMR-to-text models. In the experiment, a sentence is converted to AMR, and then is converted back to text without any modification to the AMR. We pick the combination that can recover the original sentence the most, as measured in BLEU score. The results are re-

Text-To-AMR Parser	AMR-To-Text Generator	BLEU
Spring	Spring T5wtense	25.08 30.86
Spring	T5	24.76
	T5wtense	29.33
13	T5	30.82
parse_xfm_bart_large	T5wtense T5	38.45 30.10

Table 3: Comparison of different combinations of text-to-AMR and AMR-to-text models in recovering original texts after the conversions without any augmentation to the AMR. We adopt the combination with the highest BLEU score in the rest of the experiments.

ported in Table 3. We find that using parse_xfm_bart large as the AMR parser and T5Wtense as the AMR generator produces the highest BLEU score. Therefore, we select them as the text-to-AMR parser and AMR-to-text generator in all the remaining experiments. Parse_xfm_bart_large is an AMR parser that uses BART-Large as the backbone model (Lewis et al. 2020). T5Wtense is an AMR generator that uses T5 as the backbone model (Raffel et al. 2020).

Logical-Equivalence-Identification Contrastive Learning for Discriminative LLM

To increase the performance of discriminative large language models on downstream tasks that require logical reasoning, we apply contrastive learning for logical equivalence identification on sentences augmented from the synthetic dataset using AMR-LDA, as described in Section, and two baseline augmentation methods: AMR-DA and LReasoner-LDA. It is important to note that we do not use the whole system or pipeline from LReasoner, we only use the data augmentation method from LReasoner in our experiment. For each augmentation method, 14,962 pairs of logically equivalent and logically nonequivalent sentences are constructed with a positive to negative sample ratio of 1:1. Twenty percent of the augmented data are used as the validation set during contrastive learning. All the models are further fine-tuned and compared on downstream tasks requiring logical reasoning and natural language inference. The results

⁴https://amrlib.readthedocs.io/en/latest/models/

⁵https://github.com/huggingface/transformers/tree/main/examples/pytorch/text-classification

Models/Datasets	RoBERTa AMR-LDA	RoBERTa LReasoner-LDA
Depth=1	1	1
Depth=1 (with altered rules)	1	0.9987
Depth=2	1	1
Depth=2 (with altered rules)	0.9973	0.7400

Table 4: Comparison between AMR-LDA and LReasoner-LDA with RoBERTa-Large on PARARULE-Plus and PARARULE-Plus (with altered rules). Depth=1 means that only one rule was used to infer the answer. Depth=1 (with altered rules) means one of the rules has been augmented using logical equivalence laws.

as shown in Table 2, suggest that the models trained using AMR-LDA perform better in most cases compared with the other augmentation methods.

We perform additional experiments to evaluate the robustness of the models trained using AMR-LDA and LReasoner-LDA on a synthetic multi-step logical reasoning dataset, PARARULE-Plus (Bao et al. 2022). This dataset is a set of logic problems expressed in natural language and require multiple reasoning steps. Examples from this dataset can be found in Figures 7 and 8. To evaluate the robustness of the models, we alter the original test set using contraposition law to randomly rewrite a rule in each test sample to create a distribution shift between the training set and the test set. Our experiments were conducted using RoBERTa-Large. Like before, we fine-tune the models obtained from the contrastive learning on PARARULE-Plus. The results on the original test sets and the altered test sets are shown in Table 4. AMR-LDA performs as well as LReasoner-LDA on the original test sets with Depth=1 and Depth=2. However, AMR-LDA performs better than LReasoner-LDA on test sets with altered rules, demonstrating enhanced robustness of the models.

Prompt Augmentation for Generative LLM

To increase the performance of generative large language models on the tasks that require logical reasoning, we apply AMR-LDA for prompt augmentation on GPT-3.5 (gpt-3.5-turbo) (OpenAI 2023a) and GPT-4 (OpenAI 2023b), on the ReClor and LogiQA datasets. A precise example showcasing the input and output of augmented prompts through AMR-LDA for GPT-4 is available in Figure 3. The data segments highlighted in bold italics and colored blue were generated using the contraposition law from each option. Sentences highlighted in cyan were derived through the application of the contraposition law via AMR-LDA from the context sentences, while those in brown were similarly derived using the implication law through AMR-LDA from the context sentences. Furthermore, Figure 3 illustrates two solution paths for option B: one marked in grey and the other in green. Both paths validate that option B is correct. We compare the models with augmented prompt against the models without it. The results are shown in Table 5. The models with prompt augmentation achieved better performance in

Models/Datasets		ReClor				LogiQA	
	Dev	Test	Test-E	Test-H	Dev	Test	
GPT-3.5	0.5702	0.5620	0.5931	0.5375	0.3763	0.3732	
GPT-3.5 AMR-LDA	0.5862	0.5669	0.6090	0.5339	0.3974	0.3947	
GPT-4	0.8735	0.8960	0.9090	0.8857	0.4324	0.5388	
GPT-4 AMR-LDA	0.8773	0.9020	0.9159	0.8911	0.4751	0.5806	

Table 5: Comparison between GPT-3.5 AMR-LDA, GPT-4 AMR-LDA with GPT-3.5 and GPT-4 alone for evaluating on ReClor and LogiQA test sets.

Models/Dataset	s Con	Con Con-dou		Con-dou imp-com
Re	oBERTa-Larg	ge as backbo	ne model	
ReClor	0.6040	0.6080	0.6180	0.5980
LogiQA	0.3778	0.3317	0.3394	0.3870
MNLI	0.8955	0.9015	0.8968	0.8978
MRPC	0.9069	0.8922	0.9044	0.9093
RTE	0.8123	0.8520	0.8484	0.8664
QNLI	0.9416	0.9405	0.9451	0.9449
QQP	0.9212	0.8988	0.9206	0.9314
DeBl	ERTaV2-XXI	arge as baci	kbone model	!
ReClor	0.8180	0.7220	0.7940	0.7880
LogiQA	0.3225	0.4546	0.3824	0.4055
De	eBERTa-Lar	ge as backbo	ne model	
MNLI	0.9080	0.9059	0.9068	0.8967
MRPC	0.9020	0.8848	0.8995	0.9020
RTE	0.8484	0.8736	0.8556	0.8809
QNLI	0.9528	0.9504	0.9497	0.9524
QQP	0.9233	0.9240	0.9229	0.9247

Table 6: An experiment to assess the influence of different logical equivalence laws on downstream logical reasoning and natural language inference tasks. "Con", "dou", "imp" and "com" are the abbreviation for contraposition law, double negation law, implication law and commutative law. "Con-dou" denotes data constructed using both the contraposition law and the double negation law. Other terms are derived in a similar manner.

all cases except for the "hard" test set for ReClor.

Ablation Studies

We perform experiments using only a subset of the logical equivalence laws for fine-tuning. We use RoBERTa-Large, DeBERTaV2-XXLarge and DeBERTa-Large as ablated models. We present the results in Table 6. This ablation study serves as the basis for our selection of four logical equivalence rules in the main experiment as Table 2 shown. Since the test sets are private and used to rank models on the leaderboard, we evaluated directly using the validation sets instead of the test sets. To make a fair comparison, we ensure the sizes of the training sets are the same for con, con-dou, con-dou-imp and com-dou-imp-com. For this ablation study, we constructed training sets of size 1,000.

We conduct another ablation study where we modify the positive and negative sample ratios. We select DeBERTaV2-XXLarge as the backbone model. We compare the generated

AMR-LDA Prompt Augmentation Case Study

GPT-4 Input: "context": "If you have no keyboarding skills at all, you will not be able to use a computer. And if you are not able to use a computer, you will not be able to write your essays using a word processing program.", "question": "If the statements above are true, which one of the following must be true?", "answers":

A. "If you are not able to write your essays using a word processing program, you have no keyboarding skills. If you have the skill of a keyboard, you can write your essay using a word processing program. If you can use a computer, you have keyboarding skills. If you can write your essay with a word processing program, you can use a computer. Whether you have keyboard skills at all or can't use a computer. Whether you can use a computer or you can't write your own essay with a word processing program.",

B. "If you are able to write your essays using a word processing program, you have at least some keyboarding skills. If you don't have at least some keyboard skills, you can't write your essay with a word processing program. If you can use a computer, you have keyboarding skills. If you can write your essay with a word processing program, you can use a computer. Whether you have keyboard skills at all or can't use a computer. Whether you can use a computer or you can't write your own essay with a word processing program.",

C. "If you are not able to write your essays using a word processing program, you are not able to use a computer. If you can use a computer, you can write your essay using word processing programs. If you can use a computer, you have keyboarding skills. If you can write your essay with a word processing program, you can use a computer. Whether you have keyboard skills at all or can't use a computer. Whether you can use a computer or you can't write your own essay with a word processing program.",

D. "If you have some keyboarding skills, you will be able to write your essays using a word processing program. If you can't write your essay with a word processing program, you don't have some keyboard skills. If you can use a computer, you have keyboarding skills. If you can write your essay with a word processing program, you can use a computer. Whether you have keyboard skills at all or can't use a computer. Whether you can use a computer or you can't write your own essay with a word processing program."

GPT-4 output: B

Figure 3: Example for using AMR-LDA to augment the prompt from ReClor dataset and their subsequent utilisation as input for GPT-4. Data segments that are marked in bold italics and appear in blue were generated using the contraposition law, while those in brown were generated using the implication law.

Models/Datasets		ReClor				LogiQA	
Wodels/ Datasets	Dev	Test	Test-E	Test-H	Dev	Test	
DeBI	ERTaV2-X	XXLarge a	s backbor	e model			
AMR-LDA-1:1	0.7880	0.7610	0.8477	0.6928	0.4055	0.4147	
AMR-LDA-1:2	0.8020	0.7640	0.8477	0.6982	0.4700	0.4393	
AMR-LDA-1:3	0.8120	0.7570	0.8409	0.6910	0.4270	0.4101	
MERIt-1:3	0.8020	0.7580	0.8500	0.6857	0.3732	0.4239	
MERIt-Del	BERTaV2	-XXLarge	-1:3 as ba	ckbone m	odel		
AMR-LDA-Con-1:3	0.8260	0.7660	0.8613	0.6910	0.4500	0.4301	
AMR-LDA-Merged-1:3	0.8180	0.7690	0.8750	0.6857	0.4454	0.4562	

Table 7: An experiment to validate how ratios of positive and negative samples influence downstream tasks. AMR-LDA 1:1 means the ratio of positive and negative samples is 1:1.

data against our AMR-LDA and MERIt. Table 7 shows that a higher proportion of negative samples may help increase the performance on logical reasoning tasks. Furthermore, we chose MERIt-DeBERTaV2-XXLarge-1:3 as the backbone model. We then performed logical equivalence identification contrastive learning, using data constructed solely from the AMR-LDA contraposition law and subsequently merging all four logical equivalence laws. Subsequent finetuning on downstream tasks demonstrated that incorporating more logical equivalence laws can enhance the performance of language models on logical reasoning tasks.

Human Evaluation

Human evaluation is conducted to evaluate the correctness and fluency of the logically manipulated sentences generated using AMR-LDA and LReasoner-LDA. We constructed the survey with 20 questions, and each question consists of two randomly selected sentences: one from those generated by our AMR-LDA and the other by LReasoner-LDA, respectively. 45 participants participate to the survey anonymously. We ask them to evaluate the sentences on two aspects: which sentence is logically equivalent to the original sentence or both of them are logically equivalent to the original sentence. The other one is which sentence is more fluent. 63.92% and 76.44% of people prefer AMR-LDA's logically equivalent and fluent sentences over those generated by LReasoner-LDA, respectively.

Conclusion

The sparsity of web data related to logical reasoning constrains the advancement of large language models in their performance on logical reasoning tasks. Existing methods for constructing logically equivalent sentences are restricted to templates and specific datasets. Our AMR-based logic-driven data augmentation method considers more logical equivalence laws than existing methods: double negation, contraposition, commutative, and implication laws. Further-

more, our approach allows the construction of a logically equivalent sentence from the semantic meaning of a sentence, independent of any templates. We use our method to build a set of logical equivalence and inequivalence sentences to fine-tune different discriminative LLMs and augment the prompt of generative LLMs (GPT-3.5 and GPT-4), which yield better results than baseline methods on logical reasoning tasks.

Limitations

In this paper, we only applied logic-driven data augmentation to synthetically generated sentences for contrastive learning using discrimative language models. We have yet to apply it to real-world sentences. Note that we did apply AMR-LDA to real-world datasets for augmenting input prompt when using autoregressive generative LLMs. We decided to use synthetic sentences for contrastive learning because we wanted to control the types of logical operations and relationships contained in the generated sentences. Additionally, we wanted to be certain that we could robustly apply our data augmentation to the input sentences, in order to perform contrastive learning reliably. These conditions are very difficult to guarantee using real-world sentences. Extracting sentences from real-world corpora so that one can reliably apply logic-driven data augmentation is itself a very challenging research question. This research problem is out of the scope of this paper and we intend to address this challenging and important problem in our future work.

Ethics Statement

All the data used in this paper are either synthetically generated or open-source datasets. All the code used to run the experiments is written using open-source libraries or adapted from published code from other papers. We will also release our code and any synthetically generated data to ensure that the work can be reproduced. The human evaluation has been approved by the Ethics Committee of the main authors' employer.

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Appendices

Case Studies

We present several case studies comparing our AMR-LDA method with LReasoner-LDA in terms of constructing logically equivalent sentences. These constructions leverage four logical equivalence laws. LReasoner-LDA, however, does not design for the implication law, double negation law, or the commutative law, leading to its inability to handle scenarios that require these laws. Additionally, LReasoner-LDA struggles to construct logically equivalent sentences using the contraposition law when encountering new sentences not found in the ReClor and LogiQA datasets

	Contraposition law
Original Sentence	If the bald eagle is small, then the mouse is not small.
AMR-LDA	The bald eagle isn't small, unless the mouse is small.
LReasoner-LDA	If it is not small, then it will be not the bald eagle.

Table 8: Logically equivalent sentences constructed by contraposition law.

	Contraposition law
Original Sentence AMR-LDA	If the bald eagle is kind, then Dave is not short. If Dave is short, the bald eagle is not kind.
LReasoner-LDA	If it is not kind, then it will be not the bald eagle.

Table 9: Logically equivalent sentences constructed by contraposition law.

	Implication law
Original Sentence	The bear is not sleepy or Bob is not cute.
AMR-LDA	If the bear is not sleepy, then Bob is not cute.
LReasoner-LDA	-

Table 10: Logically equivalent sentences constructed by implication law.

	Double negation law
Original Sentence AMR-LDA LReasoner-LDA	The bald eagle is beautiful. The bald eagle isn't ugly.

Table 11: Logically equivalent sentences constructed by double negation law.

	Implication law
Original Sentence	If the lion is not funny, then the tiger is beautiful.
AMR-LDA	Lions aren't funny or Tigers beautiful.
LReasoner-LDA	-

Table 12: Logically equivalent sentences constructed by implication law.

	Double negation law
Original Sentence AMR-LDA LReasoner-LDA	The bald eagle is strong. The bald eagle is not weak.

Table 13: Logically equivalent sentences constructed by double negation law.

	Commutative law
Original Sentence	The bald eagle is kind and the wolf is not dull.
AMR-LDA	The wolf is not dull and the bald eagle is kind.
LReasoner-LDA	-

Table 14: Logically equivalent sentences constructed by commutative law.

	Commutative law
Original Sentence AMR-LDA	The lion is thin and the dinosaur is not angry. The dinosaur was not angry
LReasoner-LDA	and the lion was thin.

Table 15: Logically equivalent sentences constructed by commutative law.

Synthetic Dataset Construction

Here are the entities, relationships, and attributes we used to construct our synthetic dataset. We used the synthetic dataset to conduct the AMR-based logic-driven data augmentation and logical-equivalence-identification contrastive learning. For the subject, we used "the bald eagle", "the tiger", "the bear", "the lion", "the wolf", "the crocodile", "the dinosaur", "the snake", "the leopard", "the cat", "the dog", "the mouse", "the rabbit", "the squirrel", "Anne", "Alan", "Bob", "Charlie", "Dave", "Erin", "Harry", "Gary", and "Fiona". For the relationships, we used "is" and "is not". For the attributes, we used "kind", "quiet", "round", "nice", "smart", "clever", "dull", "rough", "lazy", "slow", "sleepy", "boring", "tired", "reckless", "furry", "small", "cute", "lovely", "beautiful", "funny", "big", "strong", "awful", "fierce", "heavy", "horrible", "powerful", "angry", "tall", "huge", "short", "thin", "little", "tiny", "wealthy", "poor", "dull", "rough", "bad", and "sad".

Here are the entities, relationships, and attributes we used to fine-tune T5-Large. After T5-Large had been fine-tuned, we used the fine-tuned model to generate logical equivalence sentences as the label for the above synthetic sentences and then conducted the logical-equivalence-identification contrastive learning and downstream task. For the subject, based on the above subject name entities, we add "the duck", "the goat", "the goose", "the donkey", "the cow", "James", "Robert", "John", "Michael", "David", "William", "Richard", "Anthony", "Paul", "Andrew". For the attributes, we add "cautious", "careful", "brainy", "bored", "adorable", "aggressive", "anxious", "dizzy", "depressed", "disturbed", and "awful".

The entity names used for the "change name" experiment in Table 20. For the new entity names that we used "the sheep", "the kitten", "the Garfield", "the lion", "the goat", "the bull", "the cow", "the elephant", "the butterfly", "the fish", "Peter", "Bill", "Tom", "Amy", "Charles", "Tim", "Lucy", and "John".

Table 16, 17, 18, and 19 are the logic pattern and changed logic pattern that we consider to replace the original logic pattern for the experiment on Table 20.

To validate whether the current pre-trained language model can distinguish logically equivalent sentences. We design a preliminary experiment as Table 20 shown. We use RoBERTa-Large to conduct the experiment. We first generate a synthetic test set 1, which includes 1312 test samples with 23 entities, 2 relationships, 40 attributes, and 4 logical equivalence laws (double negation, contraposition, implication, and commutative laws). Model's performance can improve if we fine-tune language model on the logical equivalence training set, which is constructed by our AMR-LDA data augmentation method. Also, The result shows that the model's performance will not drop if we change the entity name or logic pattern, this indicates that the fine-tuned discriminative large language model can handle scenarios requiring greater robustness more effectively.

Logic pattern for double negation l	
Original sentence	subject + verb + adj
Positive sample	subject + verb + "not" + the antonym of the adj
Negative sample	subject + verb + "not" + adj

Table 16: We used the logic pattern for double negation law for constructing the test set for the experiment in Table 20.

	Original logic pattern for commutative law	Changed logic pattern
s1	sub1 + verb1 + adj1	sub1 + verb1 + "not" + adj1
s2	sub2 + verb2 + adj2	sub2 + verb2 + "not" + adj2
s3	sub1 + verb1 + "not" + adj1	sub2 + verb2 + "not" + adj2
Original sentence	s1 + "and" + s2	-
Positive sample	s2 + "and" + s1	
Negative sample	s1 + "and" + s3	

Table 17: We used the logic pattern for commutative law for constructing the test set for the experiment in Table 20.

	Logic pattern for contraposition law
Original sentence1 Positive sentence1 Negative sentence1	"If" + sub1 + verb + adj1 + ", then" + sub2 + verb + adj2 "If" + sub2 + verb + "not" + adj2 + ", then" + sub1 + verb + "not" + adj1 "If" + sub1 + verb + adj1 + ", then" + sub2 + verb + "not" + adj2
Original sentence2 Positive sentence2 Negative sentence2	"If" + sub1 + verb + adj1 + ", then" + sub2 + verb + "not" + adj2 "If" + sub2 + verb + adj2 + ", then" + sub1 + verb + "not" + adj1 "If" + sub1 + verb + adj1 + ", then" + sub2 + verb + adj2
Original sentence3 Positive sentence3 Negative sentence3	"If" + sub1 + verb + "not" + adj1 + ", then" + sub2 + verb + adj2 "If" + sub2 + verb + "not" + adj2 + ", then" + sub1 + verb + adj1 "If" + sub1 + verb + "not" + adj1 + ", then" + sub2 + verb + "not" + adj2
Original sentence4 Positive sentence4 Negative sentence4	"If" + sub1 + verb + "not" + adj1 + ", then" + sub2 + verb + "not" + adj2 "If" + sub2 + verb + "not" + adj2 + ", then" + sub1 + verb + "not" + adj1 "If" + sub1 + verb + "not" + adj1 + ", then" + sub2 + verb + adj2

Table 18: We used the logic pattern for contraposition law for constructing the test set for the experiment in Table 20.

	Original logic pattern for implication law
Original sentence Positive sample Negative sample	"If" + sub1 + verb + adj1 + ", then" + sub2 + verb + adj2 sub1 + verb + "not" + adj1 + "or" + sub2 + verb + adj2 sub1 + verb + "not" + adj1 + "or" + sub2 + verb + "not" + adj2
Original sentence Positive sample Negative sample	Changed logic pattern sub1 + verb + "not" + adj1 + "or" + sub2 + verb + adj2 "If" + sub1 + verb + adj1 + ", then" + sub2 + verb + adj2 sub1 + verb + "not" + adj1 + "or" + sub2 + verb + "not" + adj2

Table 19: We used the logic pattern for implication law for constructing the test set for the experiment in Table 20.

$\overline{\text{Test sets }\downarrow;\text{Models}\rightarrow}$	RoBERTa	Fine-tuned RoBERTa
Test set 1	0.5335	0.8513
Test set 2 (change name)	0.5347	0.8510
Test set 3 (change logic)	0.4672	0.9482

Table 20: Compared fine-tuned RoBERTa-Large and RoBERTa-Large on three different synthetic test sets.

	Stage-1	Stage-2
	Fine-tuning	Fine-tuning
Seed	2021	0/21/42
Batch Size	32	16/32
Initial Learning Rate	2e-5	2e-5/3e-6
Learning Rate Scheduler Type	Linear	
Epoch	10	
Num Warmup Steps	0	
Weight Decay	0	
Max Sequence Length	256	
Gradient Accumulation Steps	1	

Table 21: Hyperparameter details for stage-1 fine-tuning and stage-2 fine-tuning except ReClor and LogiQA. Stage-1 fine-tuning means logical-equivalence-identification contrastive learning, and stage-2 fine-tuning means fine-tuning on the downstream tasks. 32 for batch size for all tasks (except we used 16 for batch size on RTE and QNLI tasks), 2e-05 for learning rate (except we used 3e-6 for QNLI and QQP).

	Stage-2 Fine-tuning ReClor LogiQA	
-		
Seed	42	
Batch Size	4	
Gradient Accumulation Steps	2	
Initial Learning Rate	1e-05	
Epoch	10	
Adam Betas	(0.9, 0.98)	
Adam Epsilon	1e-6	
No Clip Grad Norm	True	
Warmup Proportion	0.1	
weight_decay	0.01	

Table 22: Model hyperparameter tuning details for stage-2 fine-tuning on ReClor and LogiQA.

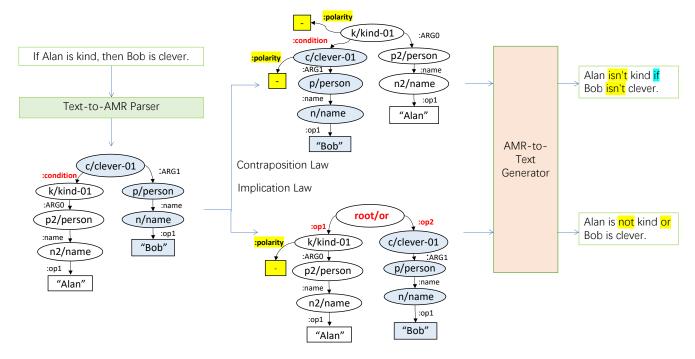


Figure 4: An example of our AMR-based logic-driven data augmentation method using contraposition law and implication law

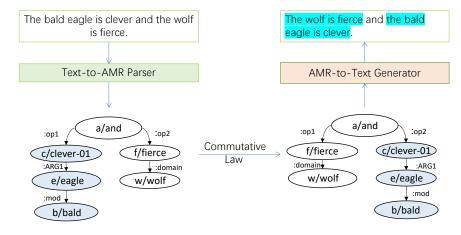


Figure 5: An example of our AMR-based logic-driven data augmentation method using commutative law

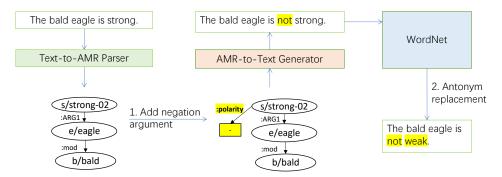


Figure 6: An example for our AMR-based logic-driven data augmentation method using double negation law

Context (Facts+Rules):

Facts: Alan is tall. Alan is big. Alan is huge. Fiona is thin. Fiona is small. Charlie is quiet. Charlie is smart. Charlie is wealthy. Anne is dull. Anne is sad. Anne is poor.

Rules for Depth=1: If someone is tall then they are quiet. If someone is thin then they are little. If someone is dull and sad then they are bad. If someone is quiet and smart then they are kind.

Rules for Depth=1 (with altered rules: If someone is not quiet then they are not tall. If someone is not little then they are

not thin. If someone is sad and dull then they are bad. If someone is smart and quiet then they are kind.

Question 1: Alan is quiet? Label: True.

Question 2: Alan is not smart? Label: False.

Question 3: Fiona is little? Label: True.

Question 4: Fiona is not little? Label: False.

Question 5: Charlie is kind? Label: True.

Question 6: Charlie is not kind? Label: False.

Question 7: Anne is bad? Label: True.

Question 8: Anne is not bad? Label: False.

Figure 7: An example for PARARULE-Plus Depth=1 and Depth=1 (with altered rules). The input includes context (facts + rules) and questions. The output is either true or false. In this example, we use logical equivalence laws (contraposition and commutative laws to extend the sentence in the rule sets to logical equivalence sentences. The highlighted words are the logical equivalence laws that we used. The green and lime green background mean the sentences are constructed by contraposition law, and the cyan background means the sentences are constructed by commutative law.)

Context (Facts+Rules):

Facts: Erin is strong. Erin is tall. Erin is huge. Dave is thin. Dave is short. Fiona is kind. Fiona is wealthy. Fiona is quiet. Bob is sad. Bob is poor. Bob is bad.

Rules for Depth=2: Strong people are kind. If someone is thin and short then they are little. If someone is sad and poor then they are dull. If someone is kind and wealthy then they are nice. All little people are small. All kind people are wealthy. All nice people are smart. All dull people are rough.

Rules for Depth=2 (with altered rules): If someone is not kind then they are not strong. If someone is thin and short then they are little. If someone is sad and poor then they are dull. If someone is not nice then they are not both kind and wealthy.

There are no little people who are not small. All kind people are wealthy. All nice people are smart. There are no dull people who are not rough.

Question 1: Erin is wealthy? Label: True.

Question 2: Erin is not wealthy? Label: False.

Question 3: Dave is small? Label: True.

Question 4: Dave is not small? Label: False.

Question 5: Fiona is smart? Label: True.

Question 6: Fiona is not smart? Label: False.

Question 7: Bob is rough? Label: True.

Question 8: Bob is not rough? Label: False.

Figure 8: An example for PARARULE-Plus Depth=2 and Depth=2 (with altered rules). The input includes context (facts + rules) and questions; the output is either "True" or "False". In this example, we use the contraposition law and De Morgan's law to convert sentences in the rule set to logically equivalent sentences. We highlighted the keywords that were changed when the alternative rules were constructed. Green and lime green backgrounds indicate sentences constructed using the contraposition law, while pink and magenta indicate sentences constructed with De Morgan's law.)

Algorithm 1: AMR-Based Logic-Driven Data Augmentation

return total_list

```
Require: Synthetic sentence lists (list1, list2, list3, and list4) generated following the patterns from
  Table 16, 17, 18, and 19 respectively. total_list = []
  for sent in synthetic_sentence_lists do
     amr_graph = Text-To-AMR-Parser(sent)
    if sent in list1 then
       ##double negation law
       if ":polarity -" in amr_graph then
         Remove ":polarity -" from the amr_graph
         Add ":polarity -" into the amr_graph
       aug_text = AMR-To-Text-Generator(amr_graph)
       Use WordNet to replace an adjective word to antonym word from aug_text.
     else if sent in list2 then
       ##commutative law
       Switch the order of two arguments.
       aug_text = AMR-To-Text-Generator(amr_graph)
     else if sent in list3 then
       ##implication law
       Change the root node as "or".
       if ":polarity -" in a condition argument then
          Remove the ":polarity -".
       else
          Add ":polarity -" into the argument.
       end if
       aug_text = AMR-To-Text-Generator(amr_graph)
     else if sent in list4 then
       ##contraposition law
       Switch the order of two arguments.
       if ":polarity -" in the argument of the amr_graph then
         Remove the ":polarity -".
       else
         Add ":polarity -" into the argument.
       aug_text = AMR-To-Text-Generator(amr_graph)
     end if
     total_list = total_list.append((sent, aug_text, 1))
  end for
```

Algorithm 2: Negative samples construction

```
Require: Synthetic sentence lists (list1, list2, list3, and list4) generated following the patterns from
  Table 16, 17, 18, and 19 respectively. total_list = [], total_list2 = []
  for sent in synthetic_sentence_lists do
     amr_graph = Text-To-AMR-Parser(sent)
     if ":polarity -" in amr_graph then
       Remove ":polarity -"
     else
       Add ":polarity -" into the amr_graph
     aug_text = AMR-To-Text-Generator(amr_graph)
     total_list = total_list.append((sent, aug_text, 0))
     for sent in total_list do
       random select an index i from total_list
       total_list2 = total_list2.append((sent, total_list[i], 0))
     end for
  end for
  total_list = total_list.extend(total_list2)
  return total_list
```

Algorithm 3: Logical-Equivalence-Identification Contrastive Learning

```
Require: positive_list and negative_list from Algorithm 1 and 2, pre-trained large language model (LLM), stage-2 downstream task datasets (ReClor, LogiQA, MNLI, RTE, QNLI, QQP), batch_size bs, learning_rate lr Stage-1 fine-tuning
for sents, pos_sents, neg_sents from zip(positive_list, negative_list, bs) do
    LLM, Loss = Contrastive loss(LLM,
    sents, pos_sents, neg_sents, label, lr)
end for
Stage-2 fine-tuning
for sent1, sent2 from zip(downstream_tasks, bs) do
    LLM, Loss = Cross_entropy_loss(LLM, sent1, sent2, label, lr)
end for
```