Control of Adjustable Speed Drives

ECE 730

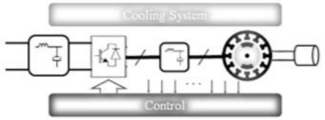
Babak Nahid
PhD, HDR, Fellow IEEE
ECE Department
Faculty of Engineering
McMaster University

Contact Info :

Office: **ITB A109**
Email: **babak.nahid@mcmaster.ca**

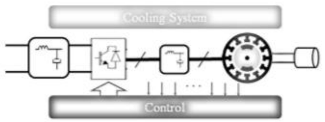
Office Hours:
By appointment

Topic 1



Course Outline

1. Introduction to Adjustable Speed Drives (ASD)
2. Topic 1: Modeling of PMSM for Control Purposes
3. Topic 2: Average Modeling of Voltage-Source Inverters
4. Topic 3: Torque Control of PMSM
5. Topic 4: Torque Control of Other Electric Motors
6. Topic 5: Speed Control of Electric Motors
7. Topic 6: Common Failures in ASD
8. Topic 7: Modeling of ASD Under Fault Conditions
9. Topic 8: Fault-Tolerant Capability of ASD
10. Topic 9: Fault-Tolerant Control of ASD
11. Future Trends and Conclusion



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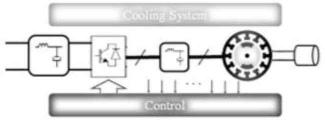

Modeling of PMSM for Control Purposes

1. **Circuit-based modeling of PMSM**

2. **αβ transformation**

3. **Park transformation**

4. **Motor torque**

5. **Mechanical model**

6. **Electric motor losses**

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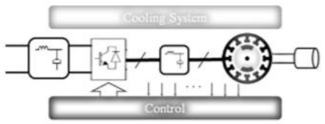
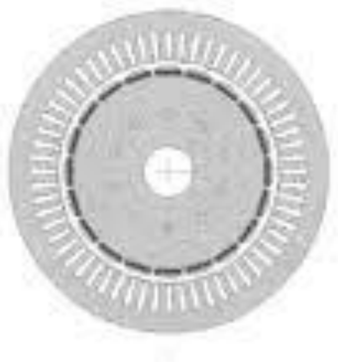


Permanent-Magnet Synchronous Motors (PMSM)

Three-Phase PM Motors:



Surface-Mounted PMSM
(SPMSM or PMSM)

PM-assisted Synchronous
Reluctance Motor (PMa-SynRM)

Interior PMSM
(IPMSM or IPM)

Permanent-Magnet Synchronous Motors (PMSM)

Three-Phase PM Motors:

Non-Salient Pole PMSM

Surface-Mounted PMSM
(SPMSM or PMSM)

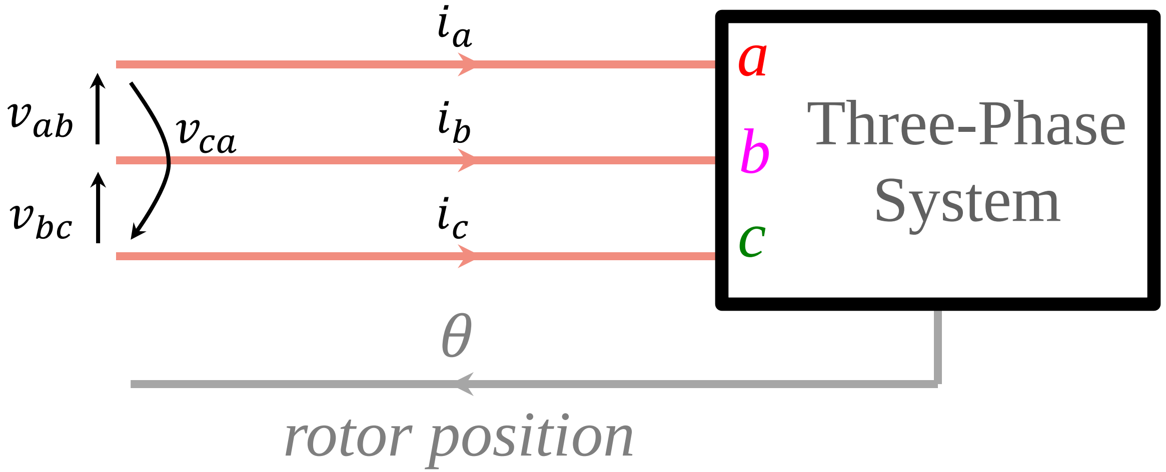
Salient Pole PMSM

Interior PMSM
(IPMSM or IPM)

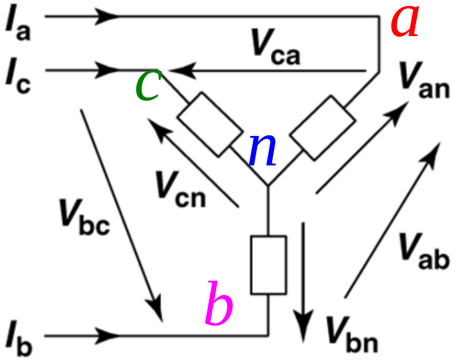
PM-assisted Synchronous
Reluctance Motor (PMa-SynRM)

Permanent-Magnet Synchronous Motors (PMSM)

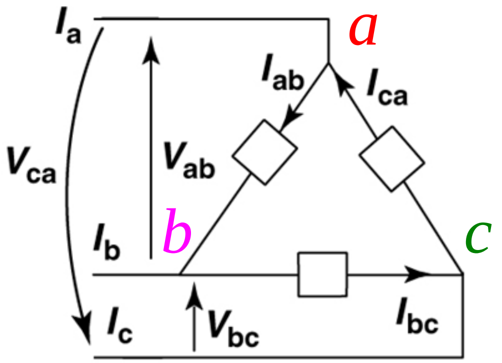
Three-Phase PM Motors:



Y-connection

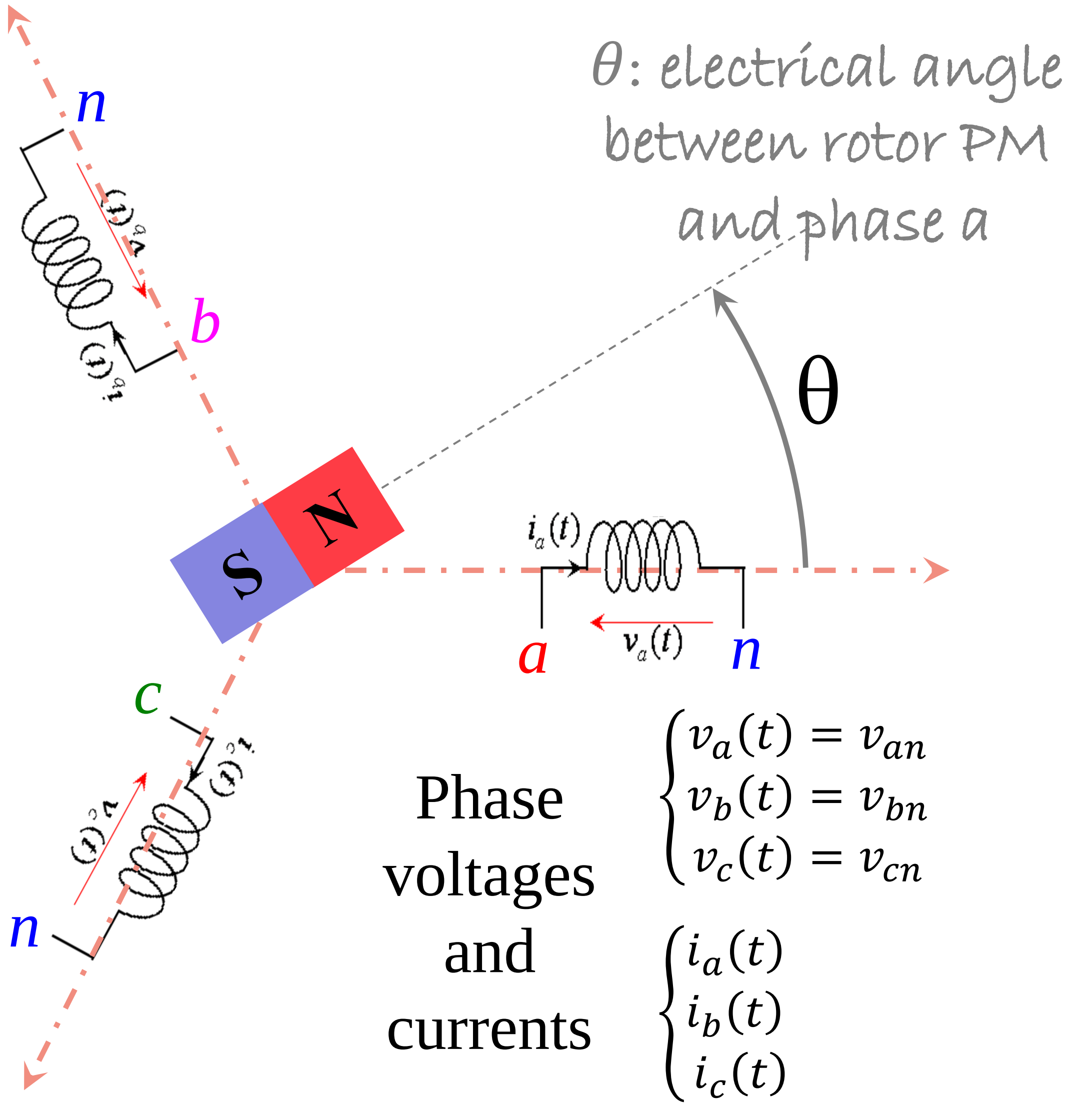


Δ-connection



Circuit Modeling of PM Synchronous Motors

Three-Phase PM Motors:



Modeling for control:

- Stator: Y-connection
- Rotor: Permanent-Magnet

Phase voltages and currents

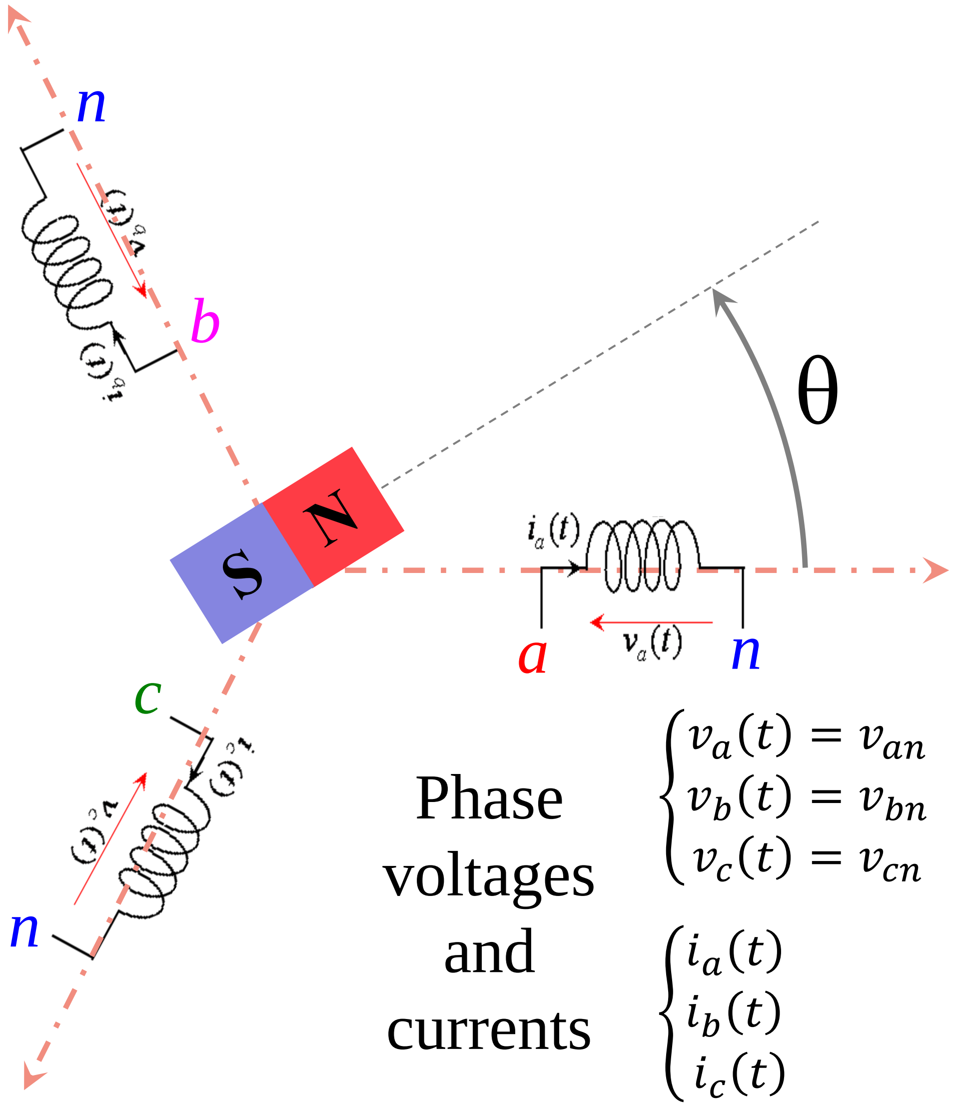
$$\begin{cases} v_a(t) = v_{an} \\ v_b(t) = v_{bn} \\ v_c(t) = v_{cn} \end{cases}$$

$$\begin{cases} i_a(t) \\ i_b(t) \\ i_c(t) \end{cases}$$

Circuit Modeling of PM Synchronous Motors

Modeling assumptions:

- Stator windings are balanced.
- Hysteresis phenomena and eddy currents are ignored.
- High frequency dynamics (beyond a few kHz) are not considered.
- Capacitive couplings between stator windings are neglected.



Phase voltages and currents

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$$\begin{cases} i_a(t) \\ i_b(t) \\ i_c(t) \end{cases}$$

Circuit Modeling of PM Synchronous Motors

<u>Circuit model:</u>

Using basic circuit laws
and Faraday's law,
we can write:

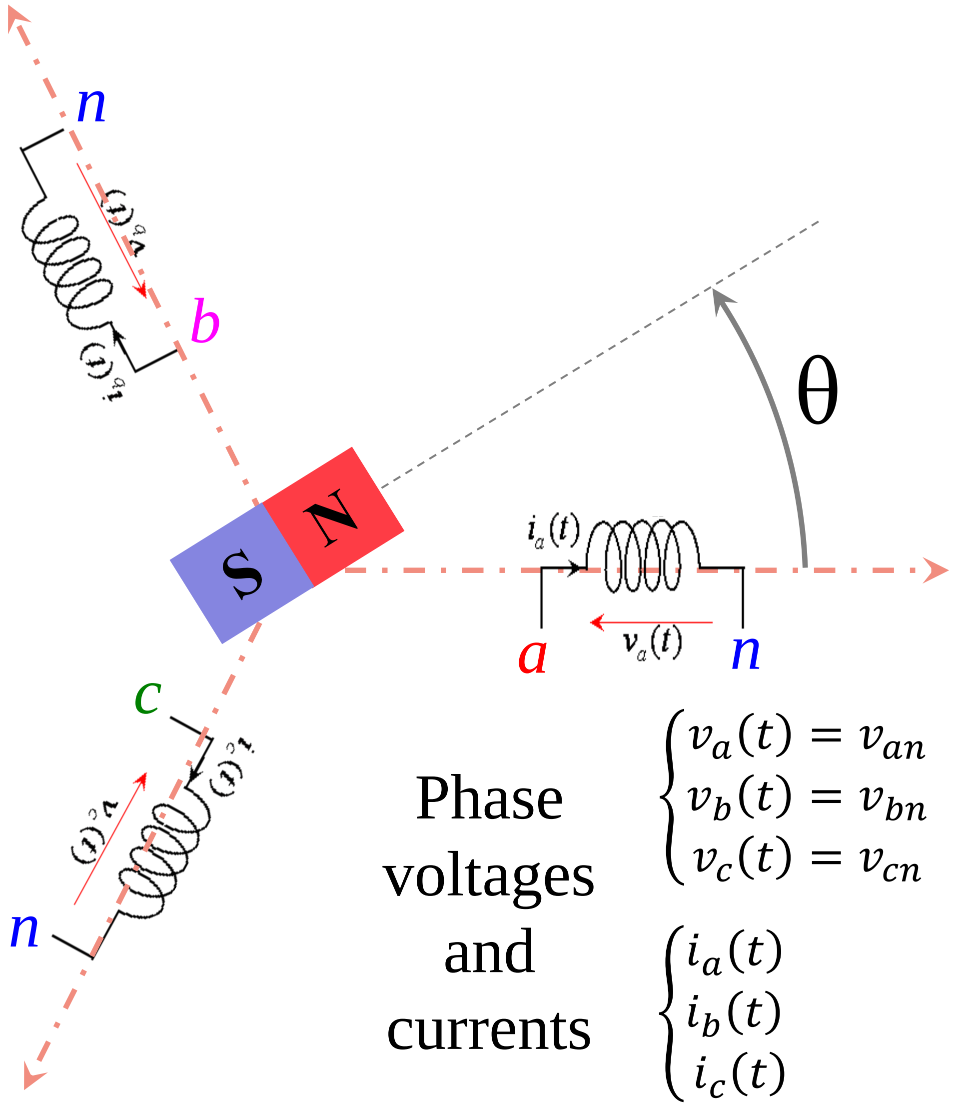
$$v_a = R_s \cdot i_a + \frac{d}{dt}\psi_a$$

where:

v_a: voltage across winding a

$R_s \cdot i_a$: ohmic voltage drop in winding a

ψ_a : total magnetic flux through winding a



Phase voltages and currents

$$\begin{cases} v_a(t) = v_{an} \\ v_b(t) = v_{bn} \\ v_c(t) = v_{cn} \end{cases}$$

$$\begin{cases} i_a(t) \\ i_b(t) \\ i_c(t) \end{cases}$$

Circuit Modeling of PM Synchronous Motors

Circuit model:

Put together three phases,
it yields:

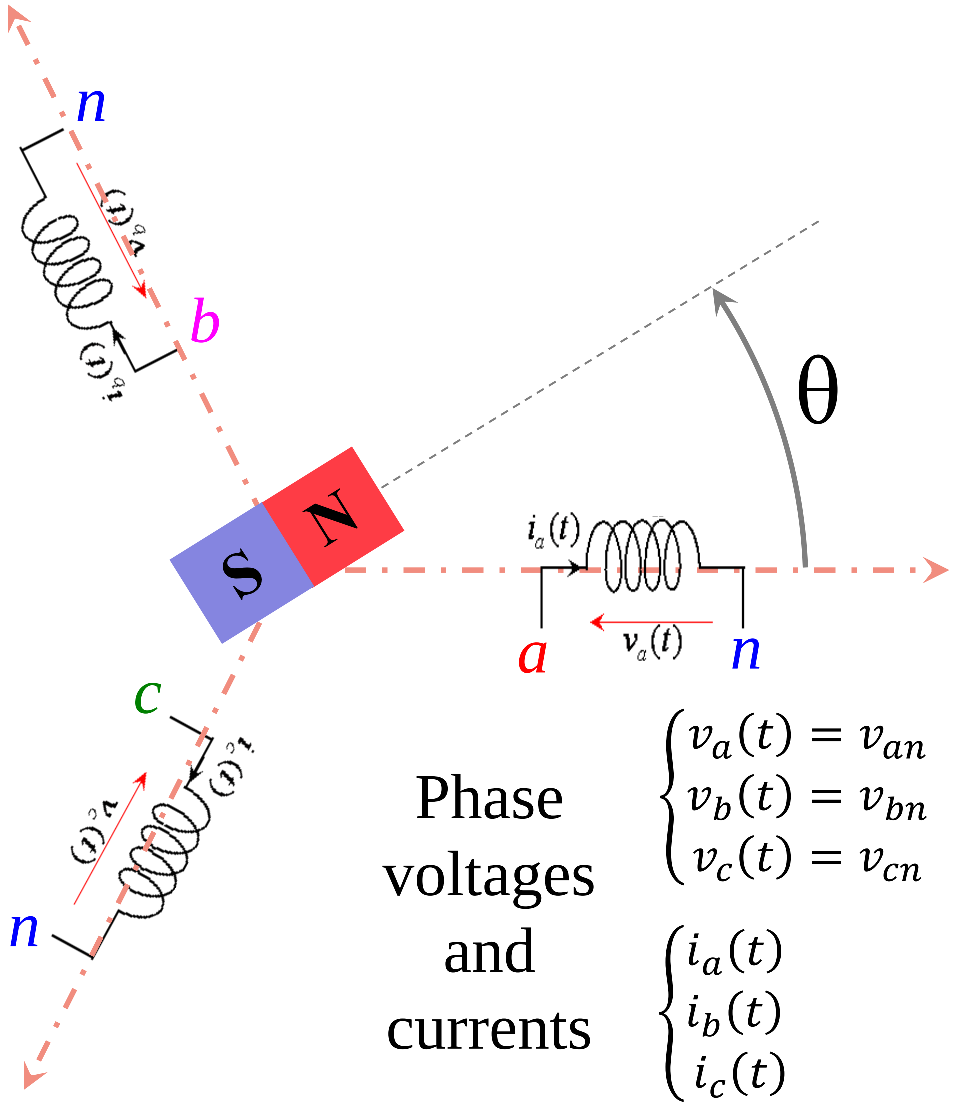
$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = R_s \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \end{bmatrix}$$

with magnetic flux:

PM flux linkage

$$\begin{aligned} \psi_a &= \psi_a(i_a, i_b, i_c, \theta, \Psi_{PM}) \\ \psi_b &= \psi_b(i_a, i_b, i_c, \theta, \Psi_{PM}) \\ \psi_c &= \psi_c(i_a, i_b, i_c, \theta, \Psi_{PM}) \end{aligned}$$

where ψ_{abc} are
nonlinear memoryless functions!



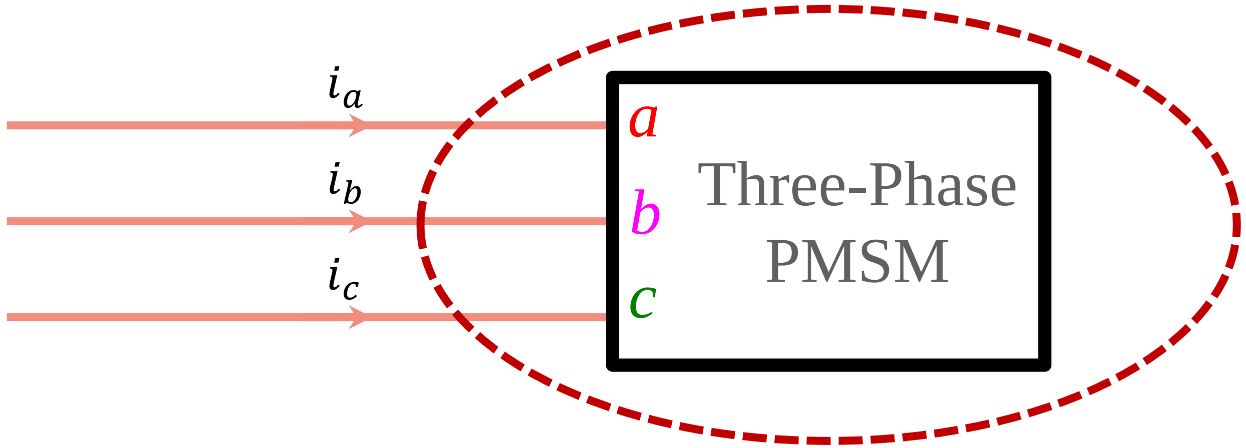
Phase voltages and currents

$$\begin{cases} v_a(t) = v_{an} \\ v_b(t) = v_{bn} \\ v_c(t) = v_{cn} \end{cases}$$

$$\begin{cases} i_a(t) \\ i_b(t) \\ i_c(t) \end{cases}$$

Zero-Sequence Current

Three-phase PMSM with isolated neutral point:



Δ-connected stator or Y-connected stator with isolated neutral point

$$i_a + i_b + i_c = 0$$

Important note: above equation holds whatever the phase currents shape (sinusoidal or any other waveform).

$\Longrightarrow$ **Only two independent currents!**

Electrical Variables Transformation

<u>Three-phase PMSM with isolated neutral point:</u>

$$i_a + i_b + i_c = 0$$

Only two independent currents:

three-phase $\rightarrow$ two-phase transformation

<u>Alpha-beta (αβ) transformation:</u>

- Clarke transformation (current/voltage invariant):
  - Preserves current and voltage magnitudes
  - Does not preserve power magnitude
- Concordia transformation (power invariant):
  - Does not preserve current and voltage magnitudes
  - Preserves power magnitude

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Modeling of PMSM for Control Purposes

1. Circuit-based modeling of PMSM

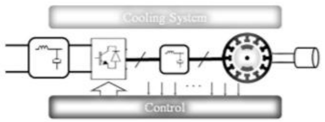
2. αβ transformation

3. Park transformation

4. Motor torque

5. Mechanical model

6. Electric motor losses

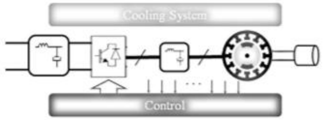


Alpha-Beta (αβ) Transformation

Concordia transformation: three-phase → two-phase transformation

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \triangleq \underbrace{\sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & 0 \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-1}{2} & \frac{-\sqrt{3}}{2} \end{bmatrix}}_{= T_{32}} \cdot \begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix}$$

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \underbrace{\sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \end{bmatrix}}_{= T_{32}^{-1} = T_{32}^{T}} \cdot \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}$$



Alpha-Beta (αβ) Transformation

Clarke transformation: three-phase → two-phase transformation

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \triangleq \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-1}{2} & \frac{-\sqrt{3}}{2} \end{bmatrix}}_{= C_{32}} \cdot \begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix}$$

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \underbrace{\frac{2}{3} \cdot \begin{bmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \end{bmatrix}}_{= C_{32}^{-1} = \frac{2}{3} C_{32}^{T}} \cdot \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}$$

Alpha-Beta (αβ) Transformation

Application to three-phase systems:

Clarke:

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = C_{32} \cdot \begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix}$$

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = \begin{bmatrix} X_m \cdot sin(\theta) \\ X_m \cdot sin(\theta - 2\pi/3) \\ X_m \cdot sin(\theta + 2\pi/3) \end{bmatrix}$$

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \begin{bmatrix} +X_m \cdot sin(\theta) \\ -X_m \cdot cos(\theta) \end{bmatrix}$$

Application to three-phase systems:

Concordia:

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = T_{32} \cdot \begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix}$$

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = \begin{bmatrix} X_m \cdot sin(\theta) \\ X_m \cdot sin(\theta - 2\pi/3) \\ X_m \cdot sin(\theta + 2\pi/3) \end{bmatrix}$$

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \sqrt{\frac{3}{2}} \begin{bmatrix} +X_m \cdot sin(\theta) \\ -X_m \cdot cos(\theta) \end{bmatrix}$$

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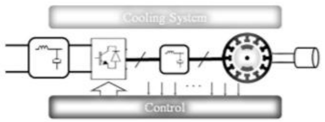
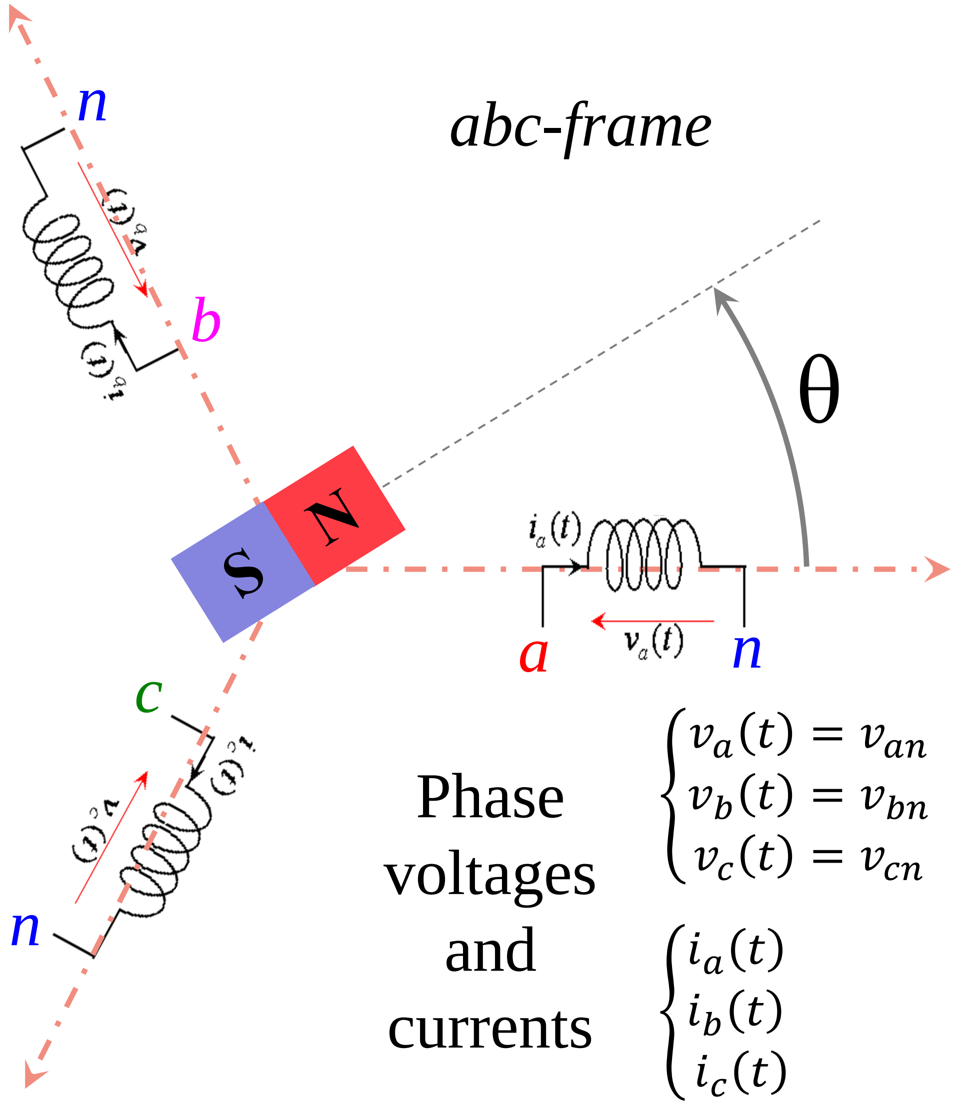


Alpha-Beta (αβ) Transformation

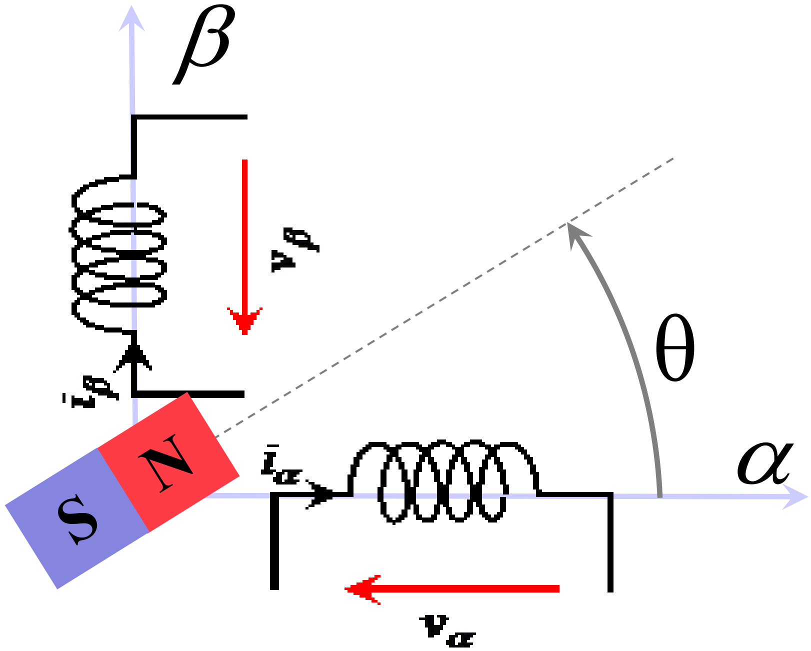
abc-frame



T_{32}^{-1}

or C_{32}^{-1}

αβ-frame



Phase voltages and currents

$$\begin{cases} v_a(t) = v_{an} \\ v_b(t) = v_{bn} \\ v_c(t) = v_{cn} \end{cases}$$

$$\begin{cases} i_a(t) \\ i_b(t) \\ i_c(t) \end{cases}$$

αβ voltages and currents

$$\begin{cases} v_\alpha(t) \\ v_\beta(t) \end{cases}$$

$$\begin{cases} i_\alpha(t) \\ i_\beta(t) \end{cases}$$

Alpha-Beta (αβ) Transformation

Application to three-phase systems:

Clarke:

$$\|x_{abc}\| = \sqrt{x_a^2 + x_b^2 + x_c^2} = \sqrt{\frac{3}{2}}\, X_m$$

$$\left\|x_{\alpha\beta}\right\| = \sqrt{x_\alpha^2 + x_\beta^2} = X_m = \sqrt{\frac{2}{3}}\, \|x_{abc}\|$$

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = \begin{bmatrix} X_m \cdot sin(\theta) \\ X_m \cdot sin(\theta - 2\pi/3) \\ X_m \cdot sin(\theta + 2\pi/3) \end{bmatrix}$$

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \begin{bmatrix} +X_m \cdot sin(\theta) \\ -X_m \cdot cos(\theta) \end{bmatrix}$$

Application to three-phase systems:

Concordia:

$$\|x_{abc}\| = \sqrt{x_a^2 + x_b^2 + x_c^2} = \sqrt{\frac{3}{2}}\, X_m$$

$$\left\|x_{\alpha\beta}\right\| = \sqrt{x_\alpha^2 + x_\beta^2} = \sqrt{\frac{3}{2}}\, X_m = \|x_{abc}\|$$

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = \begin{bmatrix} X_m \cdot sin(\theta) \\ X_m \cdot sin(\theta - 2\pi/3) \\ X_m \cdot sin(\theta + 2\pi/3) \end{bmatrix}$$

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \sqrt{\frac{3}{2}} \begin{bmatrix} +X_m \cdot sin(\theta) \\ -X_m \cdot cos(\theta) \end{bmatrix}$$

Alpha-Beta (αβ) Transformation

αβ transformation properties:

$$C_{32}^{-1}\cdot C_{32}=\frac{2}{3}C_{32}^{T}\cdot C_{32}=\begin{bmatrix}1&0\\0&1\end{bmatrix}\qquad T_{32}^{-1}\cdot T_{32}=T_{32}^{T}\cdot T_{32}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$C_{32}\cdot C_{32}^{-1}=\frac{1}{3}\begin{bmatrix}2&-1&-1\\-1&2&-1\\-1&-1&2\end{bmatrix}\qquad T_{32}\cdot T_{32}^{-1}=\frac{1}{3}\begin{bmatrix}2&-1&-1\\-1&2&-1\\-1&-1&2\end{bmatrix}$$

Application to PMSM model:

$$\begin{bmatrix}v_a\\v_b\\v_c\end{bmatrix}=T_{32}\cdot\begin{bmatrix}v_\alpha\\v_\beta\end{bmatrix},\quad \begin{bmatrix}i_a\\i_b\\i_c\end{bmatrix}=T_{32}\cdot\begin{bmatrix}i_\alpha\\i_\beta\end{bmatrix},\quad \begin{bmatrix}\Psi_a\\\Psi_b\\\Psi_c\end{bmatrix}=T_{32}\cdot\begin{bmatrix}\Psi_\alpha\\\Psi_\beta\end{bmatrix}$$

electrical power:
$$\begin{cases}\text{abc:} & p_e=[v]^T\cdot[i]\\ \boldsymbol{\alpha\beta}\text{-Concordia:} & p_e=[v]^T\cdot[i] \quad \text{power invariant}\\ \boldsymbol{\alpha\beta}\text{-Clarke:} & p_e=\frac{3}{2}\cdot[v]^T\cdot[i] \quad \text{vi invariant}\end{cases}$$

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Alpha-Beta (αβ) Transformation

Application to PMSM model:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = R_s \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \end{bmatrix}$$

$$\Rightarrow \quad T_{32} \cdot \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = R_s \cdot T_{32} \cdot \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \frac{d}{dt}\left\{T_{32} \cdot \begin{bmatrix} \Psi_\alpha \\ \Psi_\beta \end{bmatrix}\right\}$$

$$\Rightarrow \quad T_{32} \cdot \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = T_{32} \cdot R_s \cdot \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + T_{32} \cdot \frac{d}{dt}\begin{bmatrix} \Psi_\alpha \\ \Psi_\beta \end{bmatrix}$$

Alpha-Beta (αβ) Transformation

Application to PMSM model:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = R_s \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \end{bmatrix}$$

$$\Rightarrow \quad T_{32} \cdot \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = R_s \cdot T_{32} \cdot \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \frac{d}{dt}\left\{ T_{32} \cdot \begin{bmatrix} \Psi_\alpha \\ \Psi_\beta \end{bmatrix} \right\}$$

$$\Rightarrow \quad T_{32} \cdot \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = T_{32} \cdot R_s \cdot \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + T_{32} \cdot \frac{d}{dt}\begin{bmatrix} \Psi_\alpha \\ \Psi_\beta \end{bmatrix}$$

Multiplying both sides by T_{32}^{-1}, it yields:

$$\Rightarrow \quad \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = R_s \cdot \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_\alpha \\ \Psi_\beta \end{bmatrix}$$

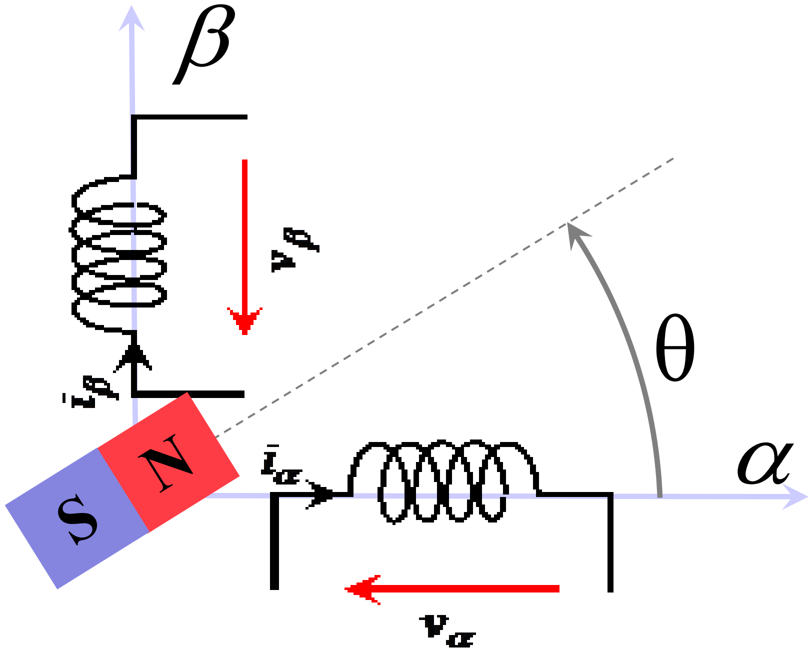
Alpha-Beta (αβ) Transformation

New circuit model:

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = R_s \cdot \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_\alpha \\ \Psi_\beta \end{bmatrix}$$

with:

$$\Psi_\alpha = ?$$
$$\Psi_\beta = ?$$



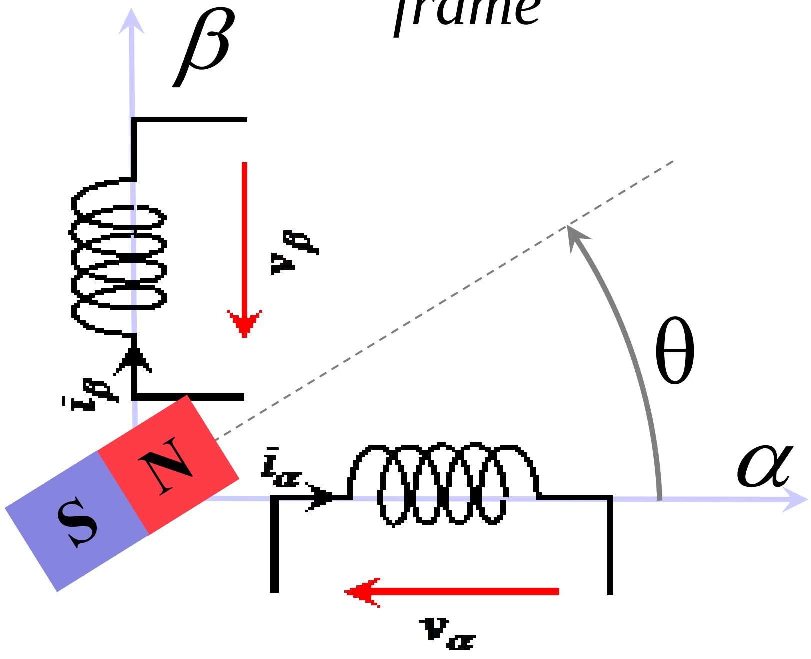
Alpha-Beta (αβ) Transformation

New circuit model:

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = R_s \cdot \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_\alpha \\ \Psi_\beta \end{bmatrix}$$

with:

$$\psi_\alpha = \psi_\alpha(i_\alpha, i_\beta, \theta, \Psi_{PM})$$
$$\psi_\beta = \psi_\beta(i_\alpha, i_\beta, \theta, \Psi_{PM})$$



where ψ_α and ψ_β depend on θ!

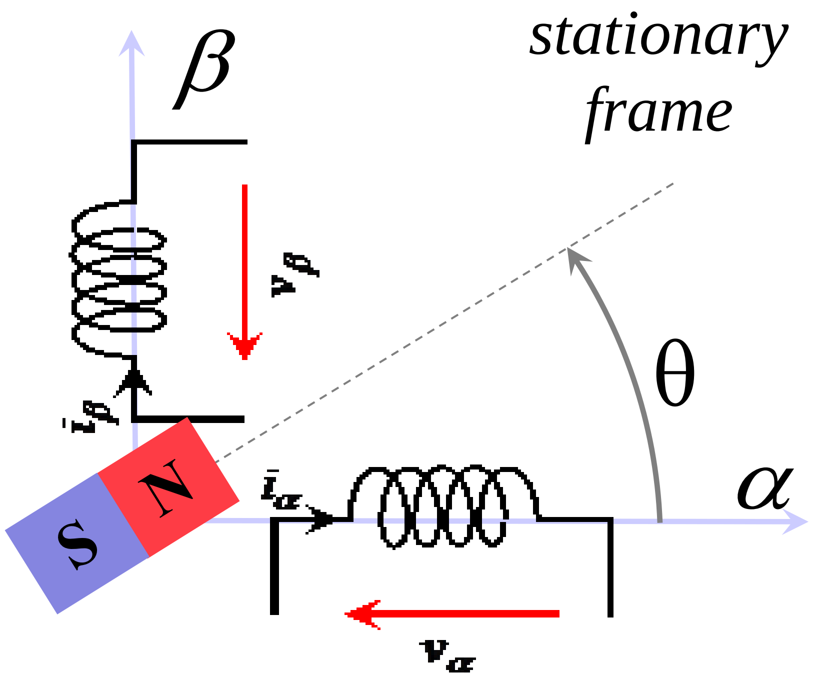
→ trigonometric functions of θ because **αβ** windings and permanent-magnets (rotor) move relative to each other.

Modeling of PMSM for Control Purposes

1. Circuit-based modeling of PMSM
2. αβ transformation
3. **Park transformation**
4. Motor torque
5. Mechanical model
6. Electric motor losses

Park Transformation

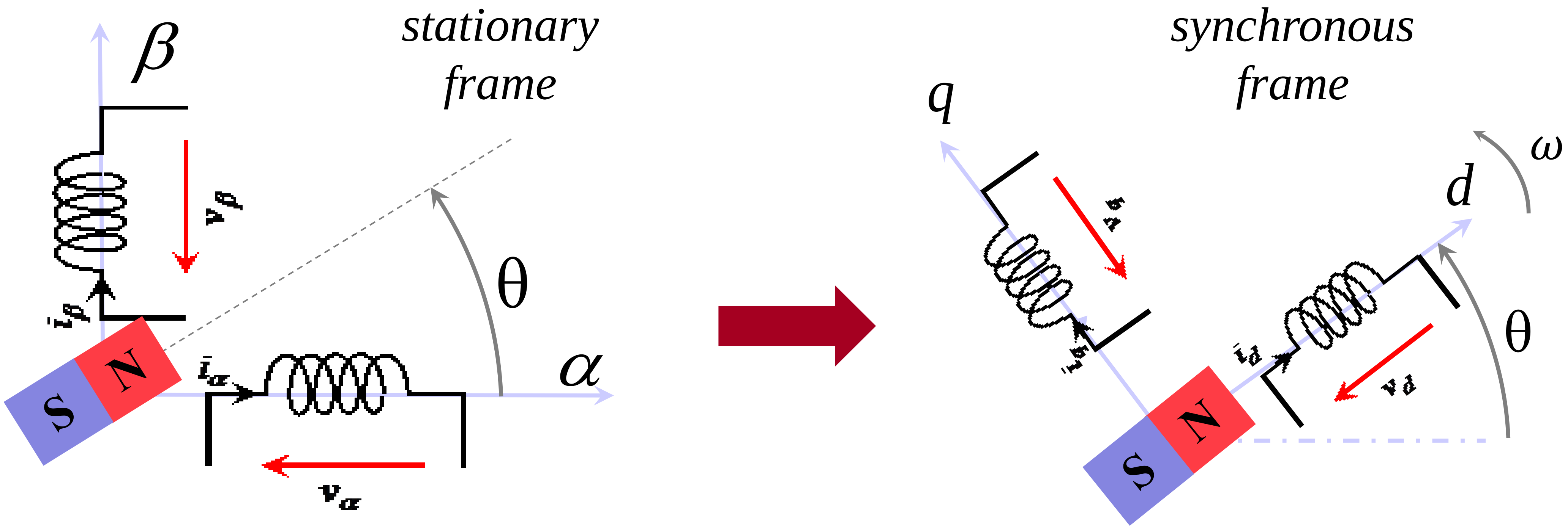
Question: is it possible to simplify the model in such a way that flux functions are (almost) independent from θ?



ψ_α and ψ_β include trigonometric functions of θ because **αβ** windings and permanent-magnets (rotor) move relative to each other.

Park Transformation

<u>Question:</u> is it possible to simplify the model in such a way that flux functions are (almost) independent from θ?



Introduce new windings not moving relative to permanent-magnets (synchronous rotation with rotor).

Park Transformation

<u>Note:</u> rotating **αβ** windings by θ (counter-clockwise)
→ $\boldsymbol{dq}$ windings

<u>Park transformation:</u>

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} \triangleq \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}_{= P(\theta)} \cdot \begin{bmatrix} x_d \\ x_q \end{bmatrix}$$

$$\begin{bmatrix} x_d \\ x_q \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}}_{= P^{-1}(\theta)} \cdot \begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix}$$

Park Transformation

Park transformation properties: $P(\theta) = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$

$$P^{-1}(\theta) = P(-\theta)$$

$$P(\theta_1) \cdot P(\theta_2) = P(\theta_2) \cdot P(\theta_1) = P(\theta_1 + \theta_2)$$

$$P(\theta) \cdot P^{-1}(\theta) = P^{-1}(\theta) \cdot P(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\frac{d}{dt}\{P(\theta)\} = \frac{d}{dt}\theta \cdot \begin{bmatrix} -sin\theta & -cos\theta \\ cos\theta & -sin\theta \end{bmatrix} = \omega \cdot P\left(\theta + \frac{\pi}{2}\right)$$

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Park Transformation

Application to PMSM **αβ** variables:

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = P(\theta) \cdot \begin{bmatrix} v_d \\ v_q \end{bmatrix}, \quad \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = P(\theta) \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix}, \quad \begin{bmatrix} \Psi_\alpha \\ \Psi_\beta \end{bmatrix} = P(\theta) \cdot \begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix}$$

Application to PMSM **αβ** circuit model:

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = R_s \cdot \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \Psi_\alpha \\ \Psi_\beta \end{bmatrix}$$

Park Transformation

<u>Application to PMSM **αβ** variables:</u>

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = P(\theta) \cdot \begin{bmatrix} v_d \\ v_q \end{bmatrix}, \qquad \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = P(\theta) \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix}, \qquad \begin{bmatrix} \Psi_\alpha \\ \Psi_\beta \end{bmatrix} = P(\theta) \cdot \begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix}$$

<u>Application to PMSM **αβ** circuit model:</u>

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = R_s \cdot \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \Psi_\alpha \\ \Psi_\beta \end{bmatrix}$$

$$\Rightarrow \quad P(\theta) \cdot \begin{bmatrix} v_d \\ v_q \end{bmatrix} = R_s \cdot P(\theta) \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{d}{dt} \left\{ P(\theta) \cdot \begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix} \right\}$$

$$\Rightarrow \quad P(\theta) \cdot \begin{bmatrix} v_d \\ v_q \end{bmatrix} = P(\theta) \cdot R_s \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{d}{dt} \{P(\theta)\} \cdot \begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix} + P(\theta) \cdot \frac{d}{dt} \begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix}$$

Park Transformation

$$P(\theta) \cdot \begin{bmatrix} v_d \\ v_q \end{bmatrix} = P(\theta) \cdot R_s \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{d}{dt}\{P(\theta)\} \cdot \begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix} + P(\theta) \cdot \frac{d}{dt}\begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix}$$

Multiplying both sides by $P^{-1}(\theta) \ = P(-\theta)$, it yields:

$$\Rightarrow \ \begin{bmatrix} v_d \\ v_q \end{bmatrix} = R_s \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + P(-\theta) \cdot \frac{d}{dt}\{P(\theta)\} \cdot \begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix}$$

Park Transformation

Multiplying both sides by $P^{-1}(\theta) \;= P(-\theta)$, it yields:

$$\Rightarrow \begin{bmatrix} v_d \\ v_q \end{bmatrix} = R_s \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + P(-\theta) \cdot \frac{d}{dt}\{P(\theta)\} \cdot \begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} v_d \\ v_q \end{bmatrix} = R_s \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + P(-\theta) \cdot \omega \cdot P\left(\theta + \frac{\pi}{2}\right) \cdot \begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} v_d \\ v_q \end{bmatrix} = R_s \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \omega \cdot P\left(-\theta + \theta + \frac{\pi}{2}\right) \cdot \begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} v_d \\ v_q \end{bmatrix} = R_s \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \omega \cdot P\left(\frac{\pi}{2}\right) \cdot \begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} v_d \\ v_q \end{bmatrix} = R_s \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \omega \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix}$$

Park Transformation

<u>New circuit model in Park *dq*-frame:</u>

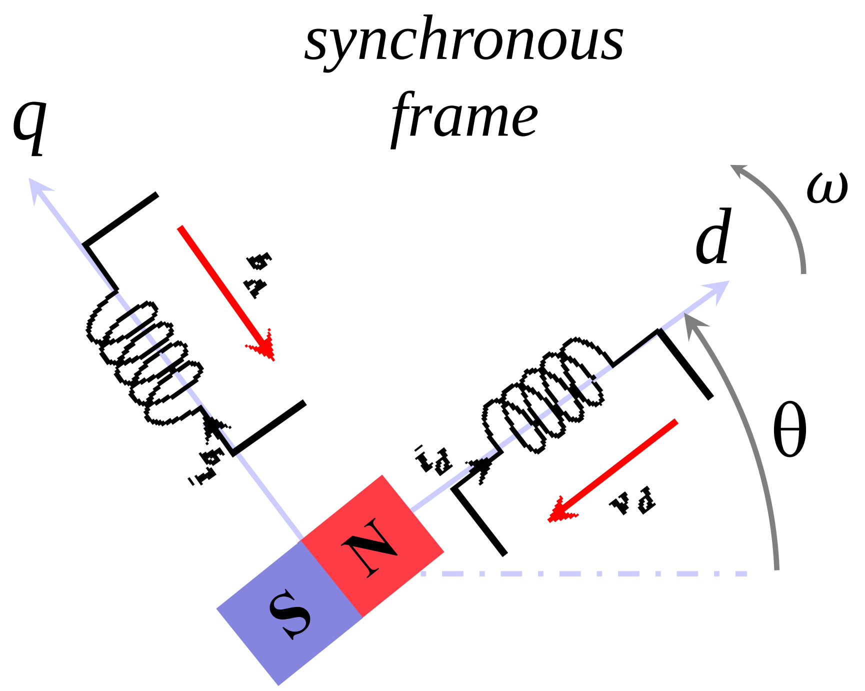
$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = R_s \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix} + \omega \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix}$$

where: $\omega = \frac{d}{dt}\theta = \dot{\theta} = P_p \cdot \Omega$

electrical speed of the rotor

no. of pole pairs

mechanical speed of the rotor

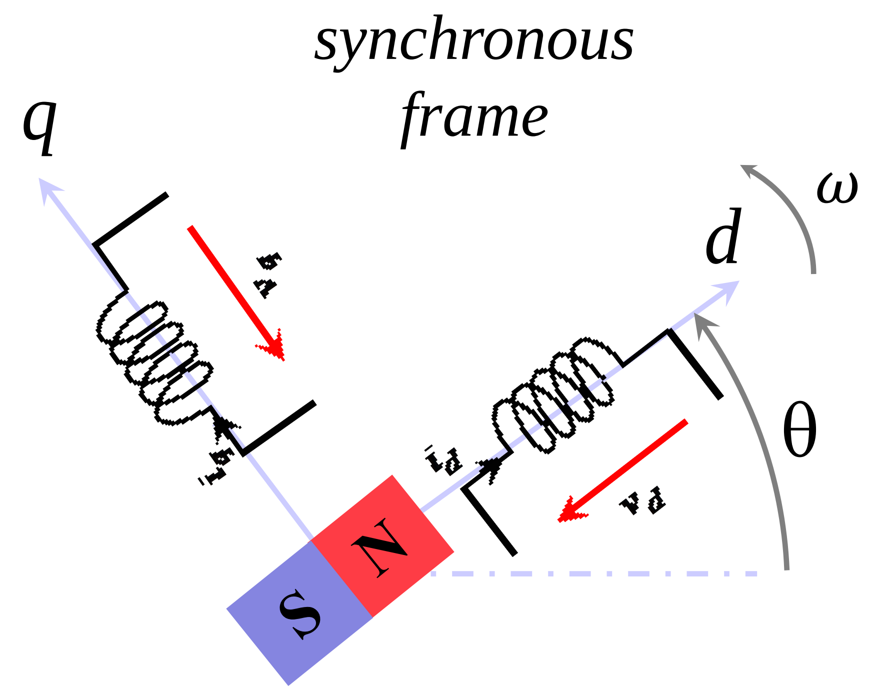


Park Transformation

New circuit model in Park *dq*-frame:

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = R_s \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix}$$
$$+\omega \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix}$$

where: $\omega = \frac{d}{dt}\theta = \dot{\theta} = P_p \cdot \Omega$



Reminder: modeling assumptions

- Stator windings are balanced.
- High frequency dynamics are not considered.
- Hysteresis phenomena and eddy currents are ignored.
- Capacitive couplings between stator windings are neglected.

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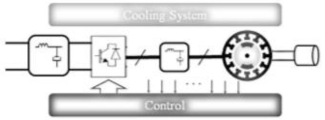


Circuit Modeling of PM Synchronous Motors

Additional modeling assumptions:

- Distribution of magnetomotive forces is sinusoidal.
- Magnetic circuit of the machine is not saturated.
- Damping effect at the rotor is neglected.
- Air gap irregularities due to stator slots are ignored.
- Permanent-magnet flux linkage magnitude is constant.

Then, ψ_d and ψ_q are linear functions of stator *dq*-currents:

$$\begin{cases} \psi_d = L_d \cdot i_d + M_{dq} \cdot i_q + \Psi_f \\ \psi_q = L_q \cdot i_q + M_{qd} \cdot i_d \end{cases}$$

$= \Psi_{PM}$ Clarke

$= \sqrt{\frac{3}{2}} \cdot \Psi_{PM}$ Concordia

negligible if no saturation

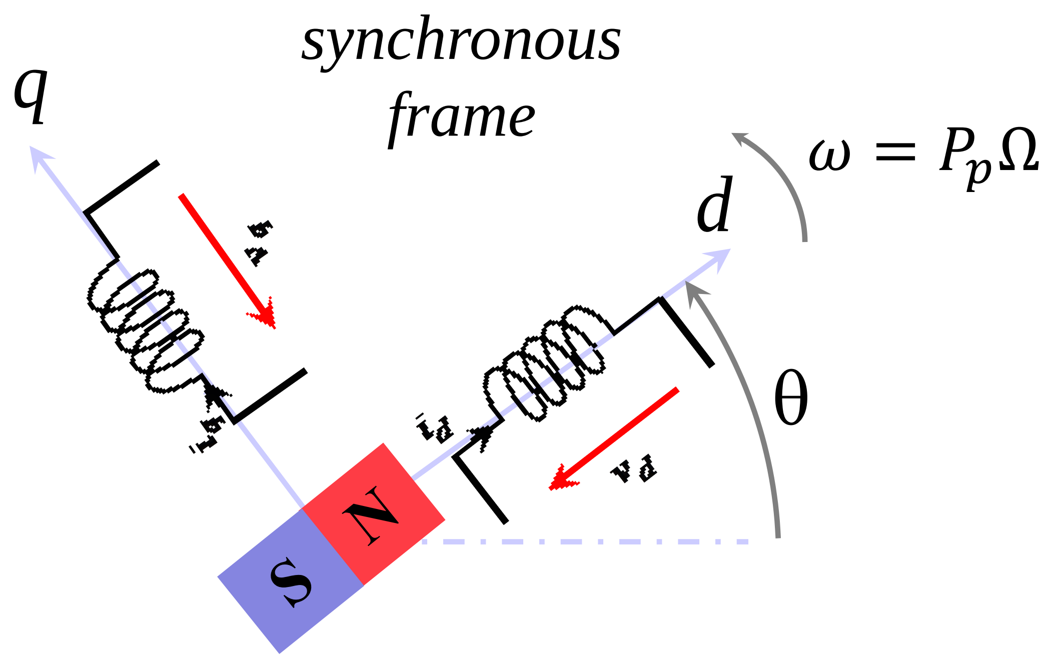
Modeling of PMSM for Control Purposes

1. Circuit-based modeling of PMSM
2. αβ transformation
3. Park transformation
4. **Motor torque**
5. Mechanical model
6. Electric motor losses

Electromechanical Energy Conversion

<u>Park model of PMSM:</u>

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = R_s \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \psi_d \\ \psi_q \end{bmatrix} + \omega \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \psi_d \\ \psi_q \end{bmatrix}$$



with: $$\begin{cases} \psi_d = L_d \cdot i_d + \Psi_f \\ \psi_q = L_q \cdot i_q \end{cases}$$

Motoring mode:

electrical power = losses + mechanical power + electromagnetic power

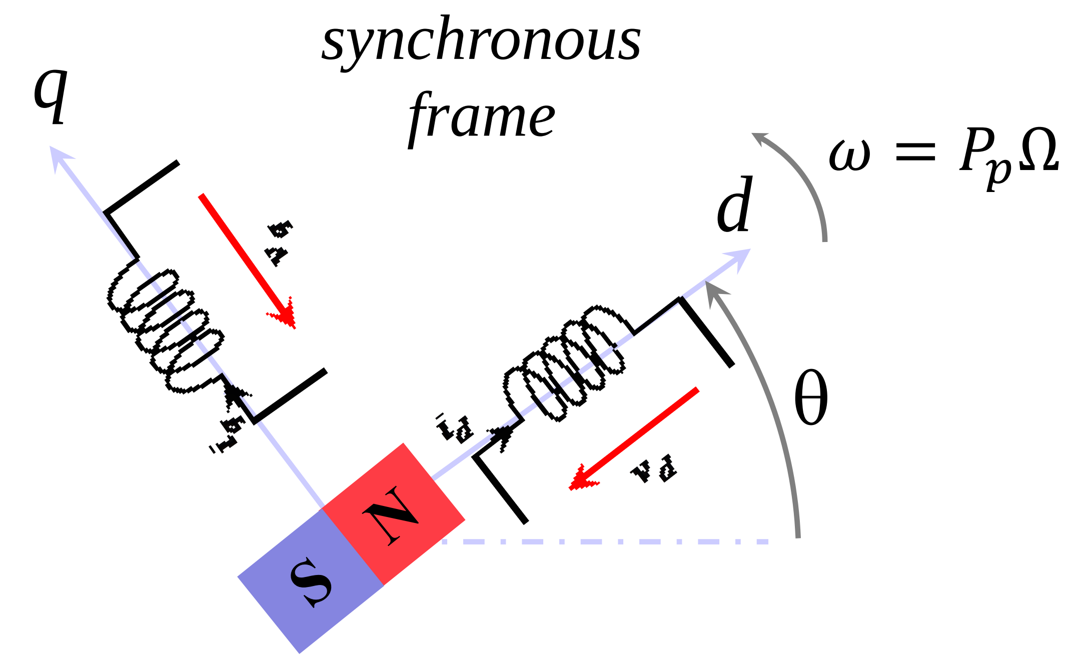
Concordia: $p_e = [v]^T \cdot [i] = R_s \cdot \left(i_d^2 + i_q^2\right) + P_p\Omega \cdot \left(\psi_d i_q - \psi_q i_d\right) + \cdots$

Clarke: $p_e = \frac{3}{2} \cdot [v]^T \cdot [i] = \cdots$

Electromechanical Energy Conversion

<u>Park model of PMSM:</u>

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = R_s \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \psi_d \\ \psi_q \end{bmatrix} + \omega \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \psi_d \\ \psi_q \end{bmatrix}$$



with: $$\begin{cases} \psi_d = L_d \cdot i_d + \Psi_f \\ \psi_q = L_q \cdot i_q \end{cases}$$

Motoring mode:

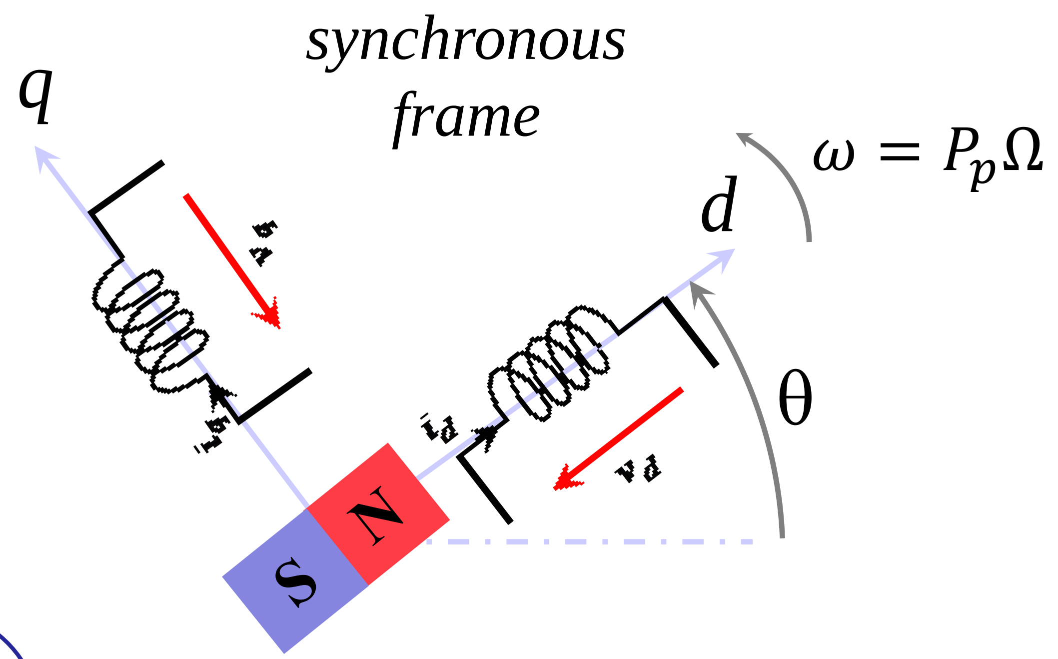
mechanical power: $p_m = P_p\Omega \cdot (\psi_d i_q - \psi_q i_d)$ (Concordia)

From Dynamics, we know: $p_m = T_m \cdot \Omega$

motor torque — mechanical speed

Park model of PMSM:

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = R_s \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \psi_d \\ \psi_q \end{bmatrix} + \omega \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \psi_d \\ \psi_q \end{bmatrix}$$



with: $$\begin{cases} \psi_d = L_d \cdot i_d + \Psi_f \\ \psi_q = L_q \cdot i_q \end{cases}$$

$$\sqrt{\frac{3}{2}} \cdot \Psi_{PM}$$

Motor torque (Concordia Transformation):

$$T_m = P_p \cdot \left(\psi_d \cdot i_q - \psi_q \cdot i_d\right) = P_p \cdot \left[\Psi_f + (L_d - L_q) \cdot i_d\right] \cdot i_q$$

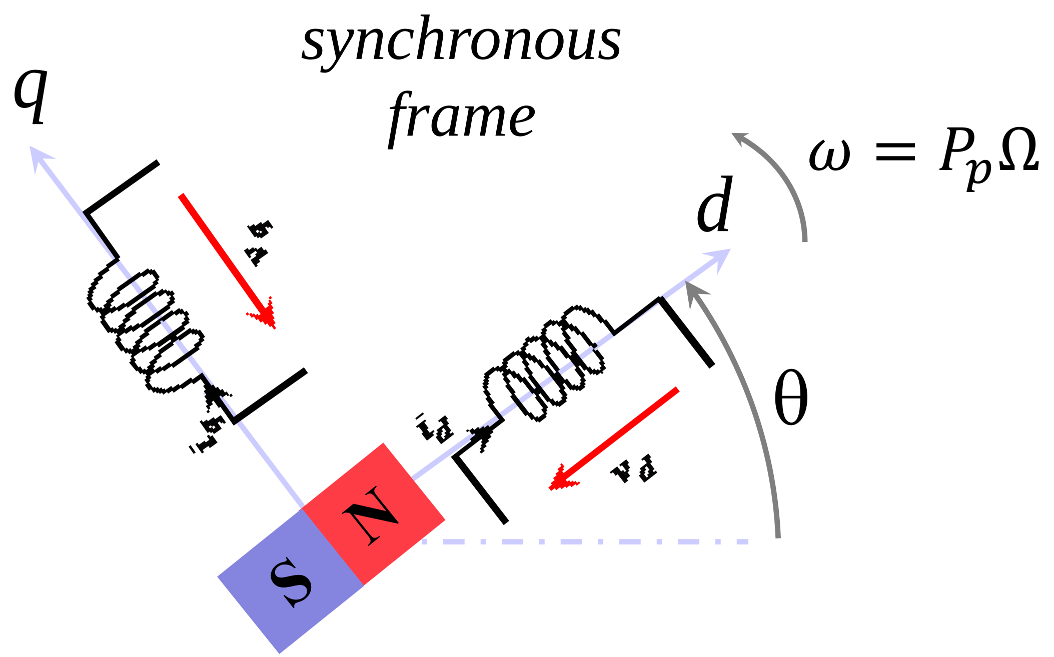
Interaction Torque

Reluctance Torque

Electromechanical Energy Conversion

Park model of PMSM:

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = R_s \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \psi_d \\ \psi_q \end{bmatrix} + \omega \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \psi_d \\ \psi_q \end{bmatrix}$$



with: $$\begin{cases} \psi_d = L_d \cdot i_d + \Psi_f \\ \psi_q = L_q \cdot i_q \end{cases}$$

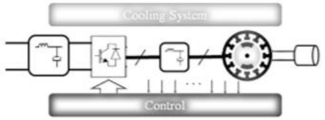
Ψ_{PM}

Motor torque (Clarke Transformation):

$$T_m = \frac{3}{2}P_p \cdot \left(\psi_d \cdot i_q - \psi_q \cdot i_d\right) = \frac{3}{2}P_p \cdot \left[\Psi_f + (L_d - L_q) \cdot i_d\right] \cdot i_q$$

Interaction Torque

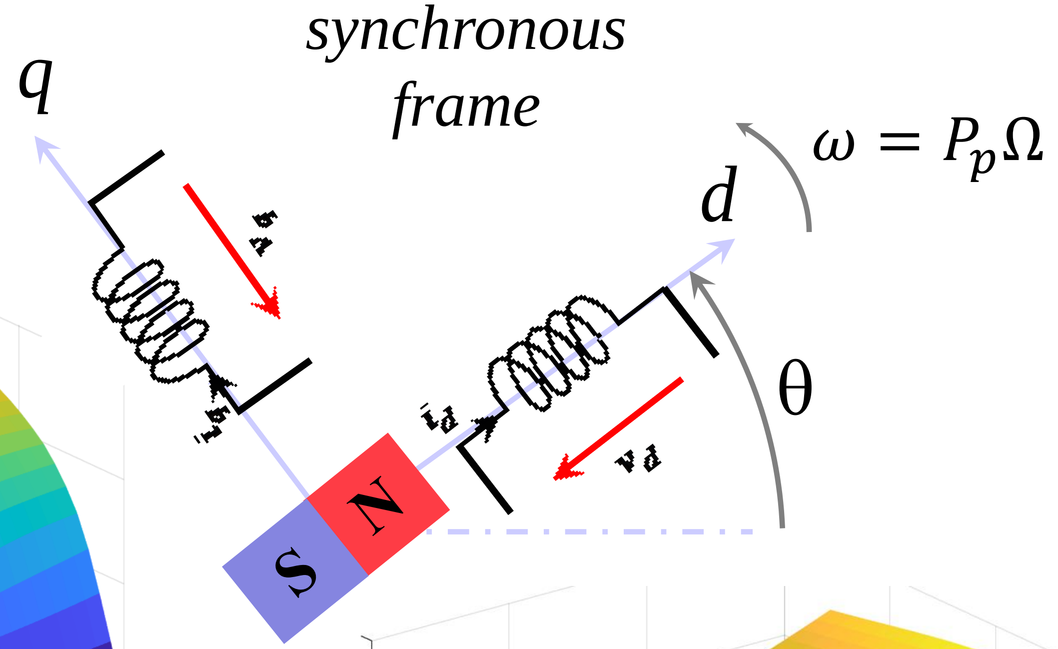
Reluctance Torque



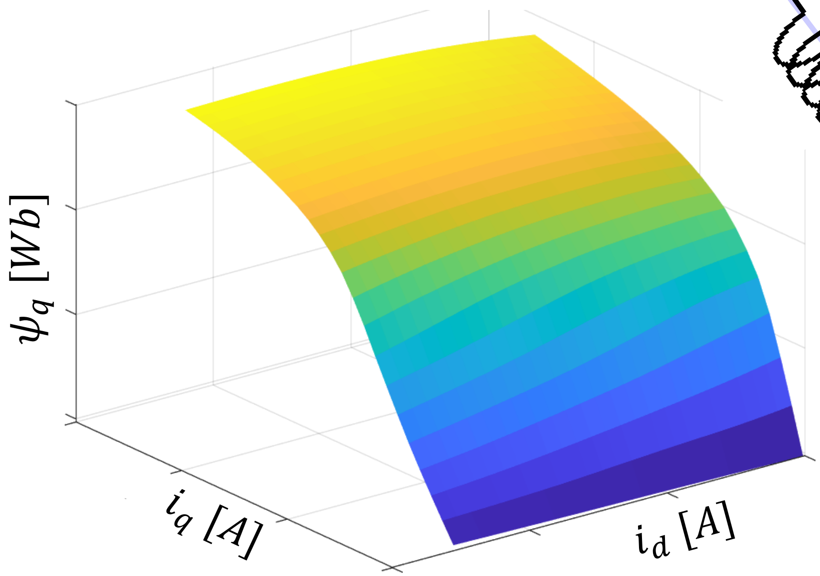
PMSM Model with Magnetic Circuit Saturation

<u>Park model of PMSM:</u>

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = R_s \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \psi_d \\ \psi_q \end{bmatrix} + \omega \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \psi_d \\ \psi_q \end{bmatrix}$$

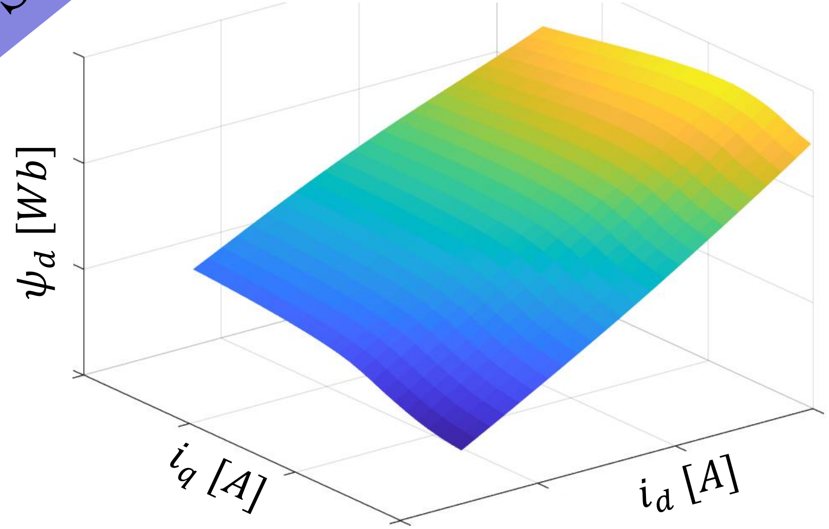


with: $\begin{cases} \psi_d = \psi_d(i_d, i_q) \\ \psi_q = \psi_q(i_d, i_q) \end{cases}$



<u>Note 1:</u> In general:

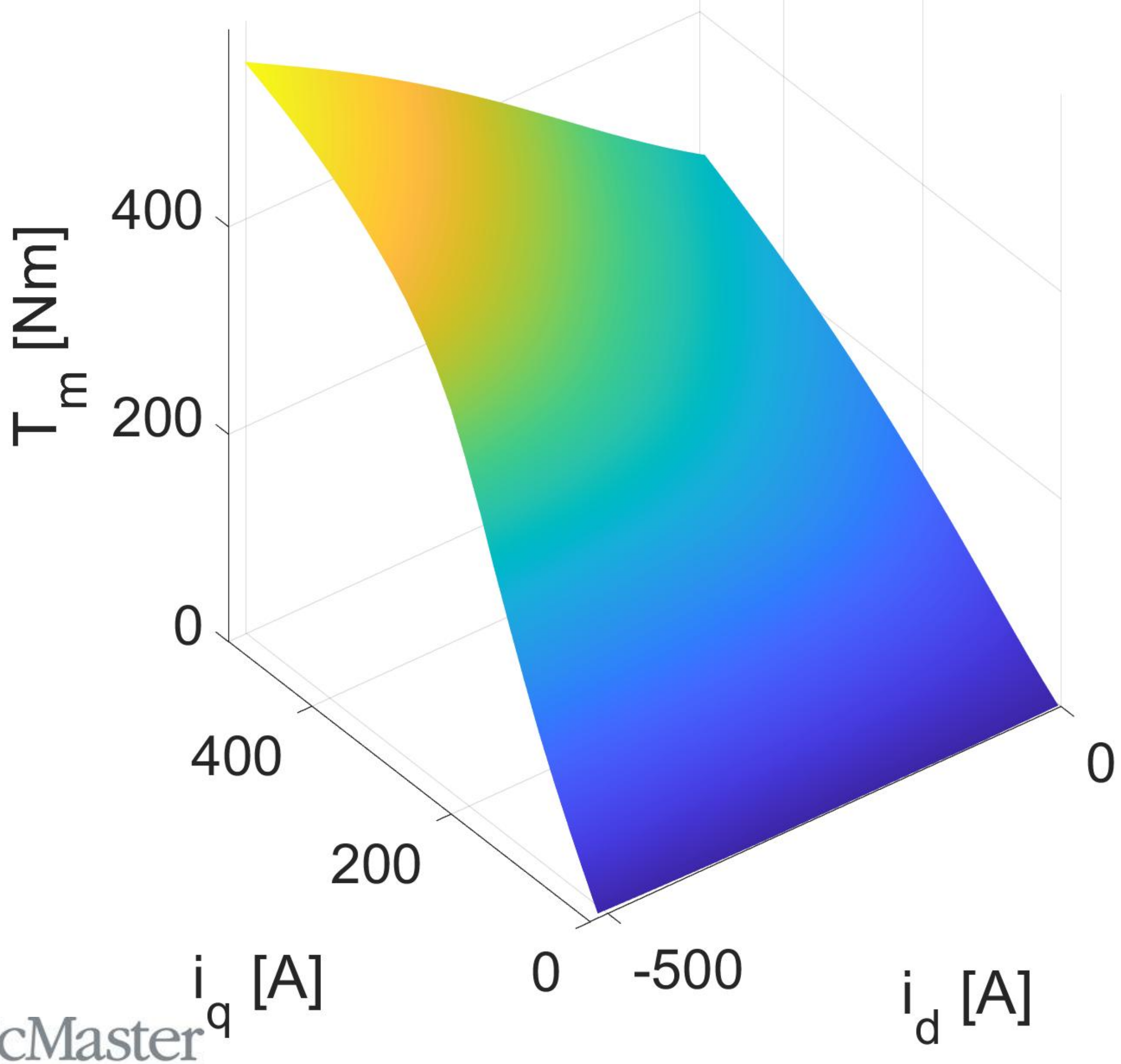
$$\begin{cases} \psi_d = \psi_d(i_d, i_q, \theta, T°) \\ \psi_q = \psi_q(i_d, i_q, \theta, T°) \end{cases}$$



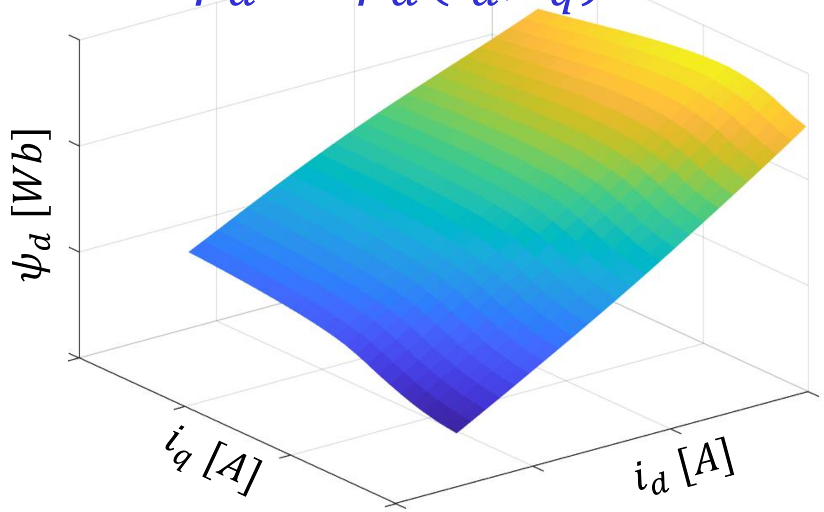
<u>Note 2:</u> Some authors consider $\psi_d = L_d i_d + \Psi_f$ and $\psi_q = L_q i_q$ with:
$L_d = L_d(i_d, i_q, \theta, T°)$, $L_q = L_q(i_d, i_q, \theta, T°)$ and $\Psi_f = \Psi_f(i_d, i_q, \theta, T°)$

PMSM Model with Magnetic Circuit Saturation

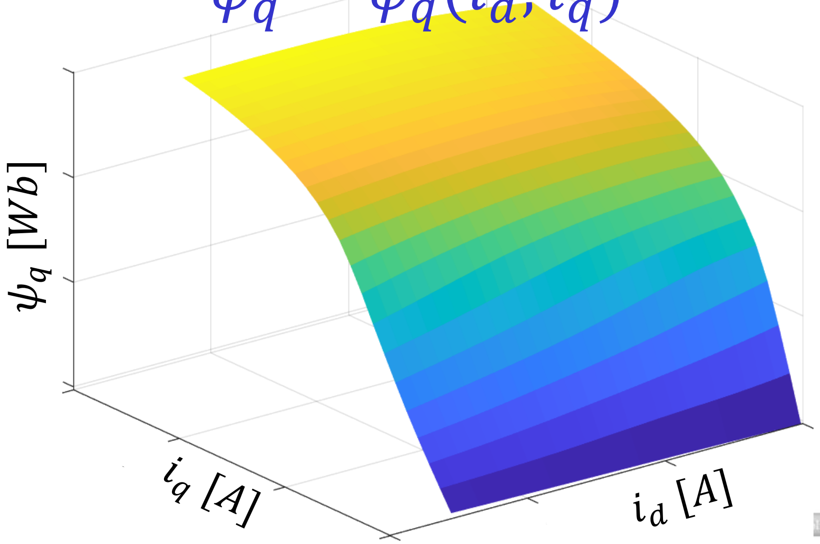
Motor torque (Clarke Transformation):



$\psi_d = \psi_d(i_d, i_q)$



$\psi_q = \psi_q(i_d, i_q)$



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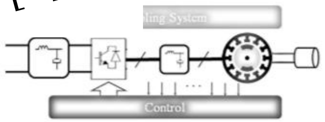
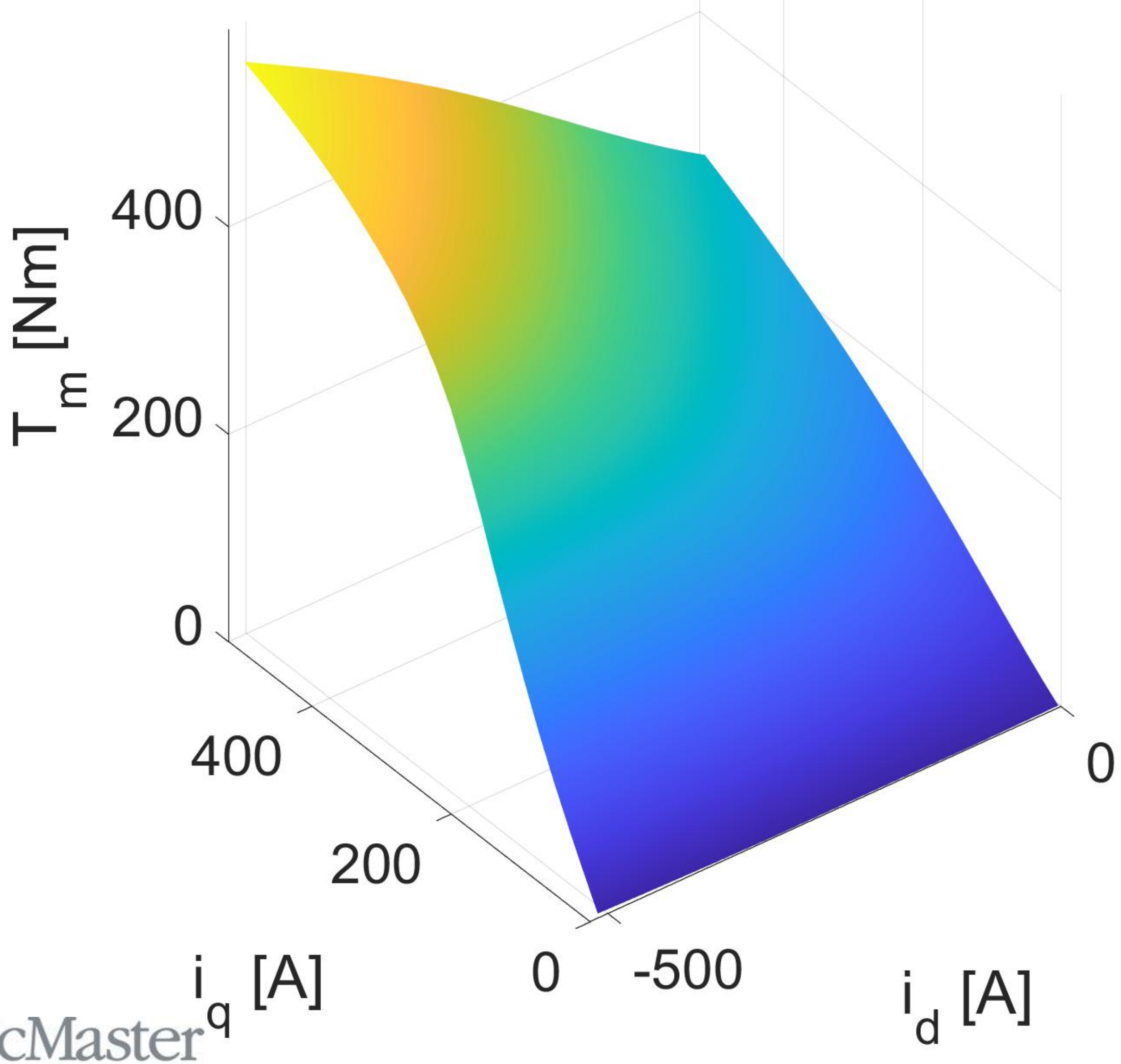
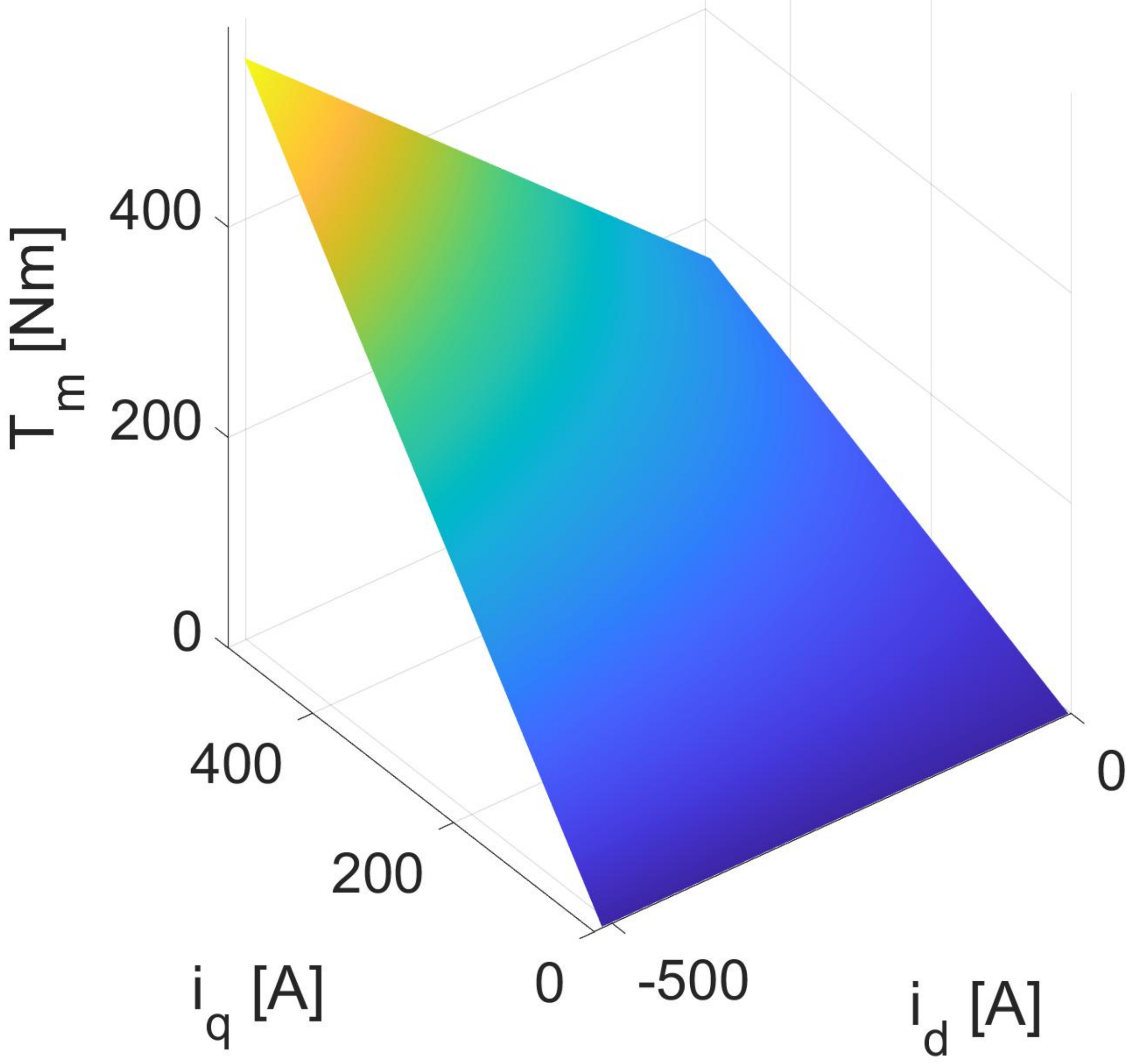


Effect of Magnetic Circuit Saturation

Motor torque (T_m) with saturation

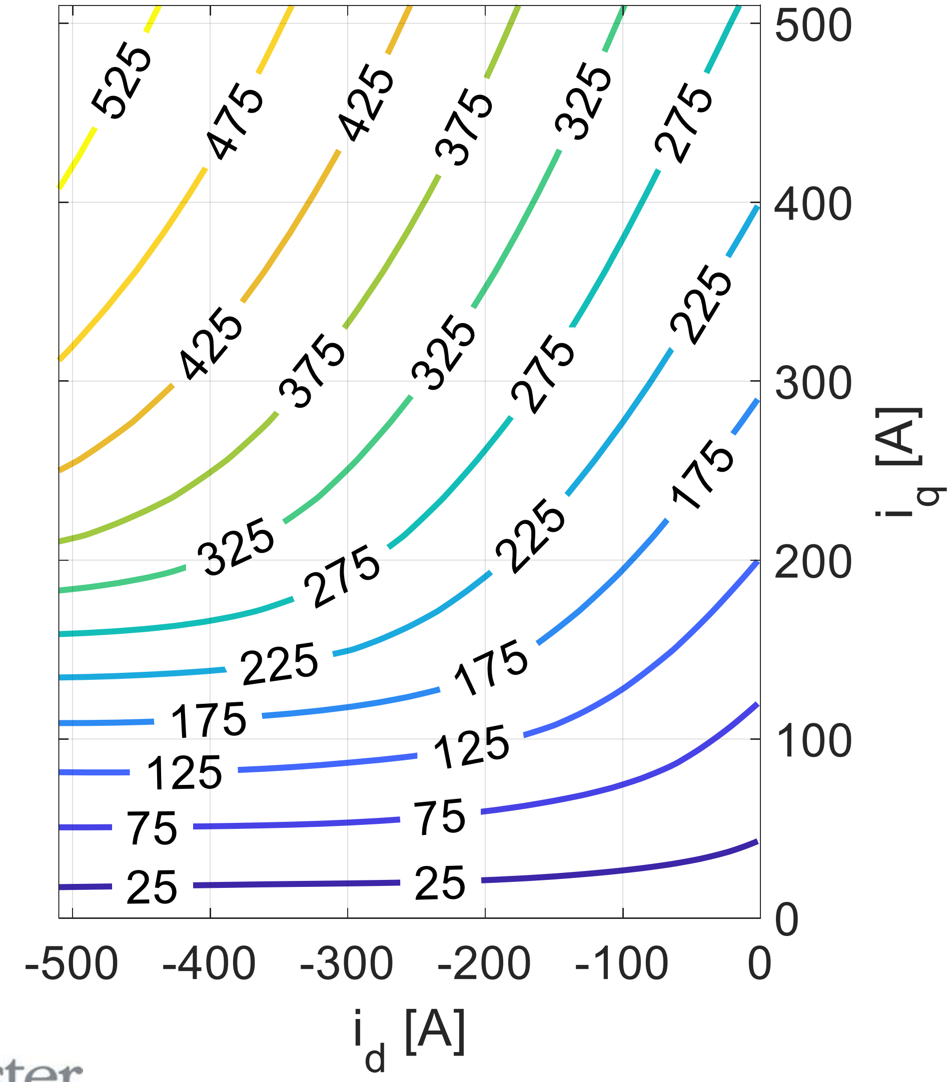


Motor torque (T_m) without saturation

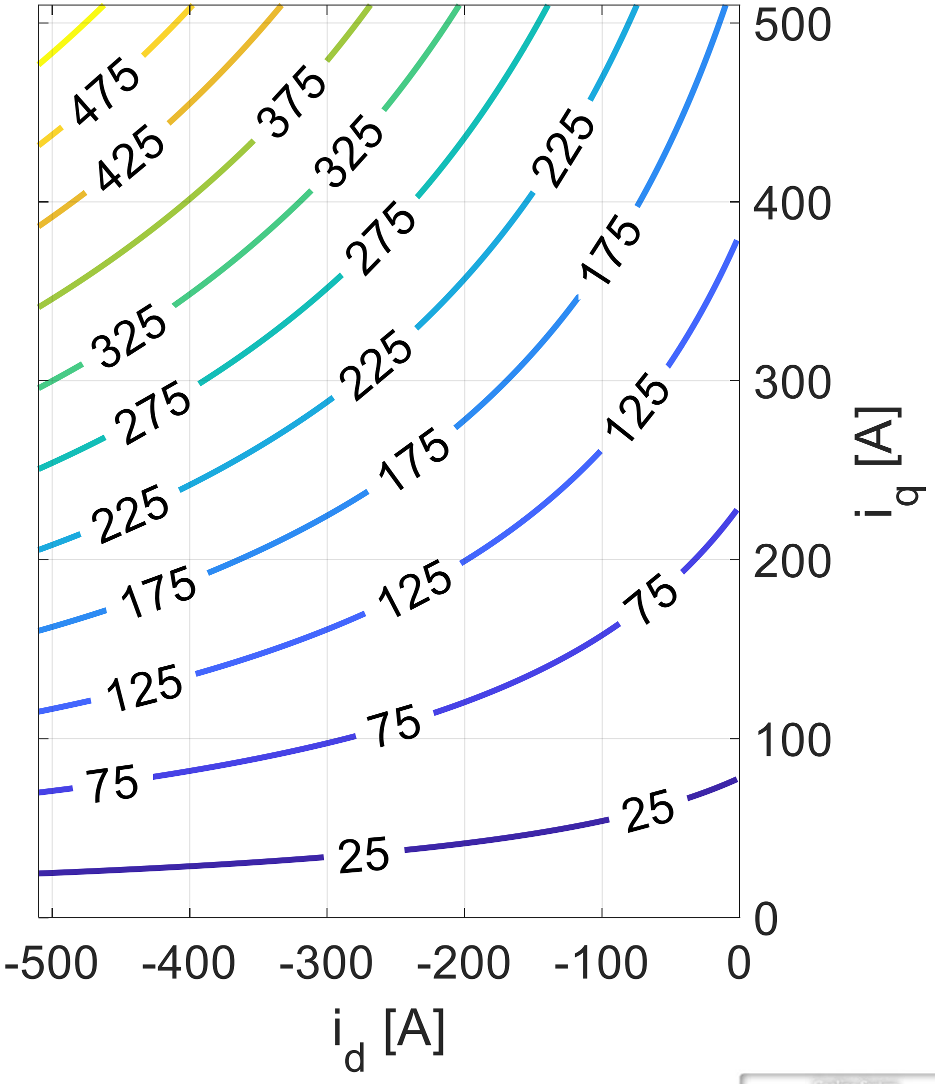


Effect of Magnetic Circuit Saturation

Motor torque (T_m) with saturation

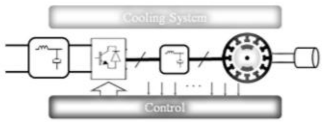


Motor torque (T_m) without saturation



Modeling of PMSM for Control Purposes

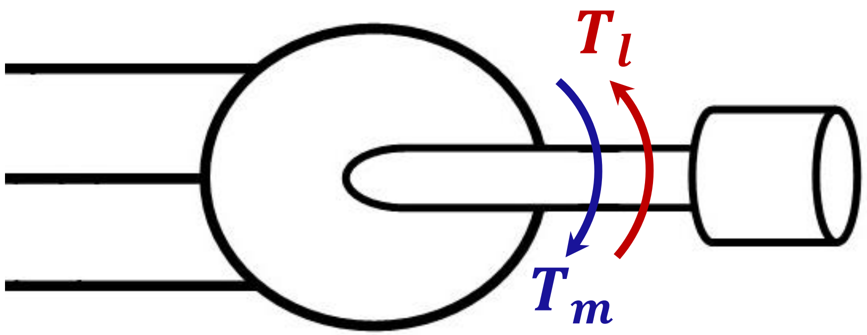
1. Circuit-based modeling of PMSM
2. αβ transformation
3. Park transformation
4. Motor torque
5. **Mechanical model**
6. Electric motor losses



Mechanical Model

Mechanical model for all rotating machines:

Dynamic of rotating systems: $$J\frac{d}{dt}\Omega = \sum Torques$$



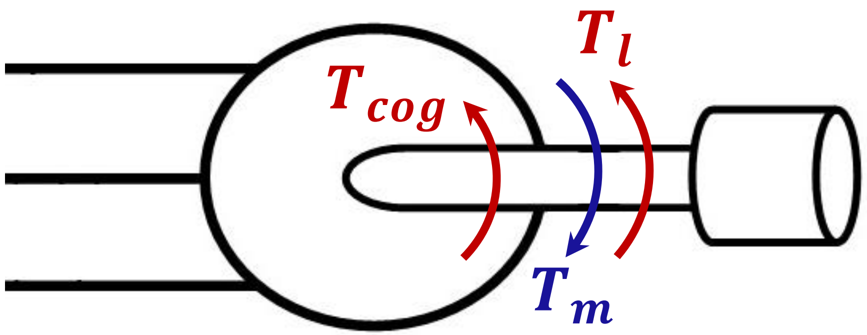
Mechanical model: $$J\frac{d}{dt}\Omega = T_m - T_l - \underbrace{friction\quad torque}_{T_f = f_0 \cdot sign(\Omega) + f_1 \cdot \Omega + f_2 \cdot |\Omega| \cdot \Omega + \cdots}$$

Mechanical Model

Mechanical model: $$J\frac{d}{dt}\Omega = T_m - T_l - \underbrace{friction\quad torque}_{T_f = f_0 \cdot sign(\Omega) + f_1 \cdot \Omega + f_2 \cdot |\Omega| \cdot \Omega + \cdots}$$

J: inertia constant (moment of inertia) of all rotating parts $[kg/m^2]$
T_f: friction torque of all rotating parts $[Nm]$

Permanent-magnet motors:



Load torque (T_l): external torque applied to the rotor shaft $[Nm]$
in general: $T_l = T_l\left(T_{l0}, \theta, \Omega, \dot{\Omega}, \ddot{\Omega}, \ldots\right)$

Cogging torque: an additional torque due to the interaction force between permanent magnets and stator teeth ($T_{cog} = T_{cog}\left(\theta, \Psi_f\right)$)

Modeling of PMSM for Control Purposes

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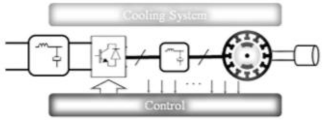


Main Losses in Electric Motors

<u>Mechanical losses:</u> $Loss_{mec} = T_f \cdot \Omega$

T_f includes dry friction, viscous friction, and aerodynamic friction (drag)

Windage losses due to relative motion of the fluid between rotor and stator

<u>Copper (ohmic) losses:</u> $Loss_{Cu} = \frac{3}{2} R_s \cdot \left(i_d^2 + i_q^2\right)$ (Clarke)

$$Loss_{Cu} = \frac{3}{2} R_s \cdot \left(i_\alpha^2 + i_\beta^2\right) = R_s \cdot \left(i_a^2 + i_b^2 + i_c^2\right)$$

<u>Core losses:</u> $Loss_{core} \cong K_H\, B_m^n \cdot f_1 + K_E\, B_m^2 \cdot f_1^2 + K_O\, B_m^{3/2} \cdot f_1^{3/2}$

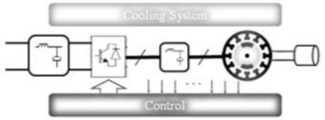
fundamental frequency

peak flux density

- Hysteresis losses (Steinmetz equation, $1.8 < n < 2.2$)
- Eddy current losses
- Other (or excess Eddy current) losses

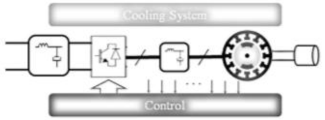
<u>Magnet losses:</u>

Due to eddy currents flowing in magnets; proportional to f_1^2



Course Outline

1. Introduction to Adjustable Speed Drives (ASD)
2. Topic 1: Modeling of PMSM for Control Purposes
3. Topic 2: Average Modeling of Voltage-Source Inverters
4. Topic 3: Torque Control of PMSM
5. Topic 4: Torque Control of Other Electric Motors
6. Topic 5: Speed Control of Electric Motors
7. Topic 6: Common Failures in ASD
8. Topic 7: Modeling of ASD Under Fault Conditions
9. Topic 8: Fault-Tolerant Capability of ASD
10. Topic 9: Fault-Tolerant Control of ASD
11. Future Trends and Conclusion



ECE 730: Control of Adjustable Speed Drives

- Course Website:

  http://avenue.mcmaster.ca/

- Please send your questions/appointment requests to:

  babak.nahid@mcmaster.ca

  with subject:

  ECE730 question/appointment