Separable and Entangled

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Theory

Consider the state $|\psi>=\frac{|00>-|11>+p(|01>|-10>)}{\sqrt{2+2p^2}}$
for p=0 this is the bell state $|\phi^->=\frac{|00>-|11>}{\sqrt{2}}$
for p=1 this is the separable state $|\psi>=\frac{|0>-|1>)}{\sqrt{2}}\otimes\frac{|0>+|1>)}{\sqrt{2}}$
Hence as we vary p from 0 to 1, we go from an entangled state to a separable state.

We know E_{ab} for $|\psi^->$ is -cos(θ).(Considering we have orthonormal measurement Basis for Alice and Bob each)

For, $|\phi^->$, Consider two measurements $\bar{a}$=(a_x,a_y,a_z) and $\bar{b}$=(b_x,b_y,b_z), where $|\bar{a}|$=1 and $|\bar{b}|$=1. So, $\bar{\sigma}$.a=

$$\begin{bmatrix} a_z & a_x - ia_y \\ a_x + ia_y & -a_z \end{bmatrix}$$

and $\bar{\sigma}$.b=

$$\begin{bmatrix} b_z & b_x - ib_y \\ b_x + ib_y & -b_z \end{bmatrix}$$

So,
The Expectation value of $\bar{\sigma_a}\otimes\bar{\sigma_b}=<\phi^-|\bar{\sigma_a}\otimes\bar{\sigma_b}|\phi^->$

where,$\bar{\sigma_a}\otimes\bar{\sigma_b}$=

$$\begin{bmatrix} a_z b_z & a_z(b_x - ib_y) & b_z(a_x - ia_y) & (a_x - ia_y)(b_x - ib_y) \\ a_z(b_x + ib_y) & -a_z b_z & (a_x - ia_y)(b_x + ib_y) & -b_z(a_x - ia_y) \\ b_z(a_x + ia_y) & (a_x + ia_y)(b_x - ib_y) & -a_z b_z & -a_z(b_x - ib_y) \\ (a_x + ia_y)(b_x + ib_y) & -b_z(a_x + ia_y) & -a_z(b_x + ib_y) & a_z b_z \end{bmatrix}$$

$\bar{\sigma_a} \otimes \bar{\sigma_b}|\phi^- >=$

$$\begin{bmatrix} a_z b_z - (a_x - ia_y)(b_x + ib_y) \\ a_z(b_x + ib_y) + b_z(a_x - ia_y) \\ b_z(a_x + ia_y) + a_z(b_x - ib_y) \\ (a_x + ia_y)(b_x + ib_y) - a_z b_z \end{bmatrix}$$

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$< \phi^-|\bar{\sigma_a} \otimes \bar{\sigma_b}|\phi^- >=$

$$a_x b_x - (a_x - ia_y)(b_x + ib_y) - (a_x + ia_y)(b_x + ib_y) + a_z b_z$$

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$$= a_y b_y + a_z b_z - a_x b_x$$

So,

$$< \phi^-|\bar{\sigma_a} \otimes \bar{\sigma_b}|\phi^- >= a_y b_y + a_z b_z - a_x b_x$$

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Therefore,

$$|\psi >= \frac{|00 > -|11 > +p(|01 > -|10 >)}{\sqrt{2 + 2p^2}}$$

$$|\psi >= \frac{\sqrt{2}|\phi^- > +\sqrt{2}p(|\psi^- >)}{\sqrt{2 + 2p^2}}$$

$$|\psi >= \frac{|\phi^- > +p(|\psi^- >)}{\sqrt{1 + p^2}}$$

$$< \psi|\bar{\sigma_a} \otimes \bar{\sigma_b}|\psi >= \frac{-p^2 cos(\theta) + a_z b_z + a_y b_y - a_x b_x}{1 + p^2}$$

Without loss of generality, we can assume, $\bar{b_1} = (b_1, 0, 0), \bar{b_2} = (0, b_2, 0), \bar{b_3} = (0, 0, b_3)$ and $\bar{a_1} = (a_{1x}, a_{1y}, a_{1z}), \bar{a_2} = (a_{2x}, a_{2y}, a_{2z}), \bar{a_3} = (a_{3x}, a_{3y}, a_{3z})$

$$cos(\theta_{11}) = \bar{a_1}.\bar{b_1} = a_{1x} b_1$$
$$cos(\theta_{12}) = a_{1x} b_2$$
$$cos(\theta_{13}) = a_{1x} b_3$$
$$cos(\theta_{21}) = a_{2x} b_1$$
$$cos(\theta_{22}) = a_{2x} b_2$$
$$cos(\theta_{23}) = a_{2x} b_3$$
$$cos(\theta_{31}) = a_{3x} b_1$$
$$cos(\theta_{32}) = a_{3x} b_2$$
$$cos(\theta_{33}) = a_{3x} b_3$$

$$E_{11} = \frac{cos(\theta_{11})(-1-p^2)}{1+p^2}$$
$$E_{12} = \frac{cos(\theta_{12})(1-p^2)}{1+p^2}$$
$$E_{13} = \frac{cos(\theta_{13})(1-p^2)}{1+p^2}$$
$$E_{21} = \frac{cos(\theta_{21})(-1-p^2)}{1+p^2}$$
$$E_{22} = \frac{cos(\theta_{22})(1-p^2)}{1+p^2}$$
$$E_{23} = \frac{cos(\theta_{23})(1-p^2)}{1+p^2}$$
$$E_{31} = \frac{cos(\theta_{31})(-1-p^2)}{1+p^2}$$
$$E_{32} = \frac{cos(\theta_{32})(1-p^2)}{1+p^2}$$
$$E_{32} = \frac{cos(\theta_{33})(1-p^2)}{1+p^2}$$

$$|E_{11} + E_{12} + E_{21} + E_{22}| = \frac{|-cos(\theta_{11})(1+p^2) + cos(\theta_{12})(1-p^2) - cos(\theta_{21})(1+p^2) + cos(\theta_{22})(1-p^2)|}{1+p^2}$$

When p=0(Entangled State), we get our usual CHSH values

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When p=1(Separable State), we get $|-\cos(\theta_{11}) - \cos(\theta_{21})|$ which has a maximum value 2.

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Therefore Bell inequality is not violated for a Separable State.