Newton -Raphson Method

This method is one of the most powerful method and well known methods, used for finding a root of f(x)=0 the formula many be derived in many ways the simplest way to derive this formula is by using the first two terms in Taylor's series expansion of the form,

$$f(x_{n+1}) = f(x_n) + (x_{n+1} - x_n) f'(x_n)$$

$$setting\ f(x_{n+1}) = 0\ gives,$$

$$f(x_n) + (x_{n+1} - x_n) f'(x_n) = 0$$

$$thus\ on\ simplification, we\ get,$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad for\ n = 0,1,2...$$

Geometrical interpretation

Let the curve f(x)=0 meet the x-axis at x=α meet the x axis at x=α .it means that α is the original root of the f(x)=0.Let x_0 be the point near the root α of the equation f(x)=0 then the equation of the tangent $P_0[x_0, f(x_0)]$ is

$$y - f(x_0) = f'(x_0)(x - x_0)$$

This cuts the x-axis at $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

This is the first approximation to the root α .if $P_1[x_1, f(x_1)]$ is the point corresponding to x_1 on the curve then the tangent at P_1 is

$$y - f(x_1) = f'(x_1)(x - x_1)$$

This cuts the x-axis at $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

This is the second approximation to the root α .Repeating this process we will get the root α with better approximations quite rapidly.

Note:

1. When $f'(x)$ very large .i.e. is when the slope is large, then h will be small (as assumed) and hence, the root can be calculated in even less time.
2. If we choose the initial approximation x_0 close to the root then we get the root of the equation very quickly.
3. The process will evidently fail if $f'(x) = 0$ is in the neighborhood of the root. In such cases the regula falsi method should be used.

4. If the initial approximation to the root is not given, choose two say, a and b, such that f(a) and f(b) are of opposite signs. If |f(a)|<|f(b)| then take a as the initial approximation.
5. Newton's raphson method is also referred to as the method of tangents.

Example

Find a real root of the equation $x3 - x - 1 = 0$ using Newton - Raphson method, correct to four decimal places.

Solution

$f(x)=x^3 - x - 1$

$f(1)=1^3 - 1 - 1 =-1<0$

$f(2)=2^3 - 2 - 1 = 8-2-1=5>0$

$so\ the\ root\ lies\ between\ 1\ and\ 2$

$here\ f'(x)=3x^2-1 \quad and \quad f''(x)=6x$

$f'(1)=3*1^2-1=2$

$f'(x)=3*2^2-1=11$

$here$

$f''(1)=6$

$f''(2)=6(2)=12$

$here\ f(2)\ and\ f''(2)\ have\ the\ same\ signs\ so\ x_0=2$

The second approximation is computed using Newton-Raphson method as