Interpolation

Introduction

Finite differences play an important role in numerical techniques, where tabulated values of the functions are available.

For instance, consider a function $y = f(x)$.

As x takes values $x_0, x_1, x_2, \ldots, x_n$,

Let the corresponding values of y be $y_0, y_1, y_2, \ldots, y_n$.

That is, for given a table of values, $(x_k, y_k), k = 0,1,2,\ldots,n;$

The process of estimating the value of y, for any intermediate value of x, is called interpolation. The method of computing the value of y, for a given value of x, lying outside the table of values of x is known as extrapolation. If the function $f(x)$ is known, the value of y corresponding to any x can be readily computed to the desired accuracy. For interpolation of a tabulated function, the concept of finite differences is important. The knowledge about various finite difference operators and their symbolic relations are very much needed to establish various interpolation formulae.

Finite Difference Operators

Forward Differences

For a given table of values $(x_k, y_k), k = 0,1,2,...,n$ with equally spaced abscissas of a function $y = f(x)$, we define the forward difference operator Δ as follows

$\Delta y_i = y_{i+1} - y_i, \qquad i = 0,1,...,(n-1)$

To be explicit, we write

$$\Delta y_0 = y_1 - y_0$$
$$\Delta y_1 = y_2 - y_1$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$\Delta y_{n-1} = y_n - y_{n-1}$$

These differences are called *first differences of the function y* and are denoted by the symbol Δy_i Here, Δ is called the first difference operator

Similarly, the differences of the first differences are called second differences, defined by

$$\Delta^2 y_0 = \Delta y_1 - \Delta y_0, \qquad \Delta^2 y_1 = \Delta y_2 - \Delta y_1$$

Thus, in general

$$\Delta^2 y_i = \Delta y_{i+1} - \Delta y_i$$

Here Δ^2 is called the *second difference* operator. Thus, continuing, we can define, *r-th* difference of *y*, as

$$\Delta^r y_i = \Delta^{r-1} y_{i+1} - \Delta^{r-1} y_i$$

By defining a difference table as a convenient device for displaying various differences, the above defined differences can be written down systematically by constructing a difference table for values

$(x_k, y_k), k = 0,1,...,6$

Forward Difference Table