

$$x_1 + x_2 = 2$$

$$10^{-5}x_1 + x_2 = 1$$

Using Gaussian elimination method, the solution is found to be $x_1 = 1$, $x_2 = 1$, which is a meaningful and perfect result.

In full pivoting (or complete pivoting), we interchange rows and columns, such that the largest element in the matrix of the variables also get changed. Full pivoting, in fact, is more complicated than the partial pivoting. Partial pivoting is preferred for hand computation.

The general form of a system of m linear equations in n unknowns $x_1, x_2, x_3, \dots, x_n$ can be represented in matrix form as under:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

Using matrix notation, the above system can be written in compact form as $[A](X) = (B)$

Note:

1. This method fails if any of the pivots become zero in such cases, by interchanging the rows we can get the non-zero pivots.

Example

Solve the system of equations by Gaussian elimination method with partial pivoting.

$$x + y + z = 7$$

$$3x + 3y + 4z = 24$$

$$2x + y + 3z = 16$$

Solution

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 24 \\ 16 \end{bmatrix}$$

To start with, we observe that the pivot element

$$a_{11} = 1 (\neq 0).$$

However, a glance at the first column reveals that the numerically largest element is 3 which is in second row. Hence $R/2$

Thus the given equation takes the form after partial pivoting

$$\begin{bmatrix} 3 & 3 & 4 \\ 1 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 24 \\ 7 \\ 16 \end{bmatrix}$$

Stage I (Reduction to upper triangular form):

$$\begin{bmatrix} 1 & 1 & \frac{4}{3} \\ 0 & -1 & \frac{1}{3} \\ 0 & 0 & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ -1 \end{bmatrix}$$

Stage II (Back substitution):

$$z = 3$$

$$-y + 1 = 0 \text{ or } y = 1$$

$$x + 1 + 4 = 8 \text{ or } x = 3$$

Example

Solve the following system of equations by Gaussian elimination method with partial pivoting

$$0x_1 + 4x_2 + 2x_3 + 8x_4 = 24$$

$$4x_1 + 10x_2 + 5x_3 + 4x_4 = 32$$

$$4x_1 + 5x_2 + 6.5x_3 + 2x_4 = 26$$

$$9x_1 + 4x_2 + 4x_3 + 0x_4 = 21$$

Solution

In matrix notation, the given system can be written as

$$\begin{bmatrix} 0 & 4 & 2 & 8 \\ 4 & 10 & 5 & 4 \\ 4 & 5 & 6.5 & 2 \\ 9 & 4 & 4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 24 \\ 32 \\ 26 \\ 21 \end{bmatrix}$$

To start with, we observe that the pivot row, that is, the first row has a zero pivot element ($a_{11} = 0$). This row should be interchanged with any row following it, which on becoming a pivot row should not have a zero pivot element.