Problem Statement:

A nuclear power company is deciding whether to build a nuclear power plant at Diablo Canyon or at Roy Rogers City. The cost of building the power plant is $10million at Diablo and $20 million at Roy Rogers City. If the company builds at Diablo, however, and an earthquake occurs at Diablo during the next five years, construction will be terminated, and the company will lose $10 million (and will still have to build a power plant at Roy Rogers City). Without further expert information the company believes that there is a 20% chance that an earthquake will occur at Diablo during the next five years. For $1 million, a geologist can be hired to analyse the fault structure at Diablo Canyon. She will predict either that an earthquake will occur or that an earthquake will not occur. The geologist's past record indicates that she will predict an earthquake on 95% of the occasions for which an earthquake will occur and no earthquake on 90% of the occasions for which an earthquake will not occur. Should the power company hire the geologist?

SOLUTION:

Abbreviations:

Diablo Canyon = DC

Roy Rogers = RR

Cost in million dollars = $m

Given Information:

Building Nuclear power plant in city	Cost if earthquake in DC	Cost if no earthquake in DC
DC	$(10+20) m	$10m
RR	$20m	$20m

According to nuclear power company:

City	Probability of earthquake in DC	Probability of no earthquake in DC
DC	**P(E) = 0.20**	**P(NE) = 0.80**

WITHOUT GEOLOGIST, cost (of building nuclear power plant) to the company:

Building Nuclear power plant in city	If earthquake in DC	If no earthquake in DC	Total Cost $m
DC	30*0.20 = 6	10*0.80 = 8	14
RR	20*0.20 = 4	20*0.80 = 16	20

So, it is advisable for the company to go ahead and build nuclear power plant at **Diablo Canyon city** as it costs only **$14m**

WITH GEOLOGIST, cost (of building nuclear power plant) to the company:

Geologist consultation fees = $1m

Geologist will **predict an earthquake** on 95% of the occasions for which an **earthquake will occur** = **P(PE|E) = 0.95**, so **P(PNE|E) = 0.05**

Geologist will **predict no earthquake** on 90% of the occasions for which an **earthquake will not occur** = **P(PNE|NE) = 0.90**, so **P(PE|NE) = 0.10**

By applying Conditional Probability:

	Predict earthquake **(PE)**	Predict no earthquake **(PNE)**	Marginal probability
Earthquake **(E)**	P(PE|E) * P(E) = 0.95 * 0.20 = **0.19**	P(PNE|E) * P(E) = 0.05 * 0.20 = **0.01**	**0.20**
No earthquake **(NE)**	P(PE|NE) * P(NE) = 0.10 * 0.80 = **0.08**	P(PNE|NE) * P(NE) = 0.90 * 0.80 = **0.72**	**0.80**
Marginal probability	**0.27**	**0.73**	

If geologist predicts earthquake will occur:

What is the probability of earthquake occurring given geologist predicted earthquake will occur?

By applying Bayes theorem:

P(E|PE) = P(PE|E)*P(E) / [P(PE|E)*P(E) + P(PE|NE)*P(NE)]

= 0.19 / [0.19 + 0.08] = 0.19 / 0.27 = **0.7037**

So, **P(NE|PE)** = 1 – 0.7037 = **0.2963**

Building Nuclear power plant in city	If earthquake in DC	If no earthquake in DC	Total Cost $m
DC	30*0.7037 = 21.111	10*0.2963 = 2.963	24.074
RR	20*0.7037 = 14.074	20*0.2963 = 5.926	20

So, if geologist predicts earthquake will occur, it is advisable for the company to go ahead and build nuclear power plant at **Roy Rogers city** as it costs only **$20m**

If geologist predicts earthquake will not occur:

What is the probability of earthquake occurring given geologist predicted earthquake will not occur?

By applying Bayes theorem:

P(E|PNE) = P(PNE|E)*P(E) / [P(PNE|E)*P(E) + P(PNE|NE)*P(NE)]

= 0.01 / [0.01 + 0.72] = 0.01 / 0.73 = **0.0137**

So, **P(NE|PNE)** = 1 – 0.0137 = **0.9863**

Building Nuclear power plant in city	If earthquake in DC	If no earthquake in DC	Total Cost $m
DC	30*0.0137 = 0.411	10*0.9863 = 9.863	10.274
RR	20*0.0137 = 0.274	20*0.9863 = 19.726	20

So, if geologist predicts earthquake will not occur, it is advisable for the company to go ahead and build nuclear power plant at **Diablo Canyon city** as it costs only **$10.274m**

Should the power company hire the geologist? What is the value of expert information?

The probability of geologist predicting earthquake = 27%

And so, the probability of geologist predicting no earthquake = 73%

Total estimated cost with geologist = (Estimated cost when geologist predicts earthquake will occur) + (Estimated cost when geologist predicts earthquake will not occur)

Total estimated cost with geologist = 20*0.27 + 10.274*0.73 = 5.4 + 7.5 = $12.9m

Total estimated cost without geologist = $14m

Net value = (Total estimated cost without geologist) – (Total estimated cost with geologist + Geologist consultation fees)

= 14 – (12.9 + 1) = $0.1m

So, if the nuclear power company hires geologist, it can reduce the overall cost of building the nuclear power plant by $100,000.

Hence it is advisable for the nuclear power company to hire the geologist.