

2022-July Session-07-27-2022-shift-1

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I. SECTION - A

- 1) Let a die be rolled until a 2 is obtained. The probability that a 2 is obtained on an even-numbered toss is equal to:
 - a) $\frac{5}{11}$
 - b) $\frac{5}{6}$
 - c) $\frac{1}{11}$
 - d) $\frac{6}{11}$
- 2) $\lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\int_{x^3}^{\left(\frac{\pi}{2}\right)^2} \cos t^{\frac{1}{3}} dt}{\left(x - \frac{\pi}{2}\right)^2}$
 - a) $\frac{3\pi^2}{4}$
 - b) $\frac{3\pi}{4}$
 - c) $\frac{3\pi^2}{8}$
 - d) $\frac{3\pi}{8}$
- 3) Consider the equation $4\sqrt{2}x^3 - 3\sqrt{2}x - 1 = 0$.
 Statement 1: The solution of this equation is $\cos \frac{\pi}{12}$.
 Statement 2: This equation has only one real solution.
 - a) Both statements are true.
 - b) Statement 1 is true but Statement 2 is false.
 - c) Statement 1 is false but Statement 2 is true.
 - d) Both statements are false.
- 4) If $\text{mod } 2A^3 = 2^{21}$ and $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$, then α is:
 - a) 5
 - b) 3
 - c) 9
 - d) 17
- 5) In a GP with 64 terms, if the sum of all terms is seven times the sum of the odd terms, the common ratio is:
 - a) 3
 - b) 4
 - c) 5
 - d) 6
- 6) Given $\frac{dy}{dx} - \left(\frac{\sin 2x}{1+\cos^2 x}\right)y = \left(\frac{\sin x}{1+\cos^2 x}\right)$ and $y(0) = 0$, find $y\left(\frac{\pi}{2}\right)$.
 - a) -1
 - b) 1
 - c) 0
 - d) 2
- 7) $4 \cos \theta + 5 \sin \theta = 1$. Then find $\tan \theta$, where $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.
 - a) $\frac{10-\sqrt{10}}{6}$
 - b) $\frac{10-\sqrt{10}}{12}$
 - c) $\frac{\sqrt{10}-10}{6}$
 - d) $\frac{\sqrt{10}-10}{12}$
- 8) In an increasing arithmetic progression a_1, a_2, \dots, a_n , if $a_6 = 2$ and the product of a_1, a_5, a_4 is the greatest, then the common difference d is:
 - a) 1.6
 - b) 1.8
 - c) 0.6
 - d) 2.0
- 9) If the relation $R : (a, b)R(c, d)$ holds only if $ad - bc$ is divisible by 5 where $a, b, c, d \in \mathbb{Z}$, then R is:
 - a) Reflexive
 - b) Symmetric, Reflexive but not Transitive
 - c) Reflexive, Transitive but not Symmetric
 - d) An Equivalence Relation
- 10) Let $f(x)$ and $g(x)$ be defined as follows:

$$f(x) = \begin{cases} 2x+2 & \text{if } x \in (-1, 0) \\ 1 - \frac{x}{3} & \text{if } x \in [0, 3] \end{cases}$$

$$g(x) = \begin{cases} x & \text{if } x \in [0, 1] \\ -x & \text{if } x \in (-3, 0) \end{cases}$$

The range of $f \circ g(x)$ is:

 - a) $[0, 1]$
 - b) $[-1, 1]$
 - c) $[0, 1]$
 - d) $(-1, 1)$
- 11) If $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x^2 \cos x}{1+\pi^x} + \frac{1+\sin^2 x}{1+e^{(\sin x)^{2023}}} \right) dx = \frac{\pi}{4}(\pi + \alpha) - 2$, then α is equal to:
 - a) 1
 - b) 2

- c) 3
- d) 4

12) The area under the curve $x^2 + y^2 = 169$ and below the line $5x - y = 13$ is:

- a) $\frac{169\pi}{4} - \frac{65}{2} + \frac{169}{2} \sin^{-1} \frac{12}{13}$
- b) $\frac{169\pi}{4} + \frac{65}{2} - \frac{169}{2} \sin^{-1} \frac{12}{13}$
- c) $\frac{169\pi}{4} - \frac{65}{2} + \frac{169}{2} \sin^{-1} \frac{13}{14}$
- d) $\frac{169\pi}{4} + \frac{65}{2} + \frac{169}{2} \sin^{-1} \frac{13}{14}$

13) If $f(x) = \frac{(2^x + 2^{-x})(\tan x) \tan^{-1}(2x^2 - 3x + 1)}{(7x^2 - 3x + 1)^3}$, then $f'(0)$ is:

- a) $\sqrt{\pi}$
- b) $\sqrt{\frac{\pi}{4}}$
- c) π
- d) $2 \cdot \pi^{\frac{3}{2}}$

14) Evaluate $\int \frac{(\sin x - \cos x) \sin^2 x}{\sin x \cos^2 x + \tan x \sin^3 x} dx$ is equal to

- a) $\frac{1}{3} \ln |\sin^3 x - \cos^3 x| + C$
- b) $\frac{1}{3} \ln |\sin^3 x + \cos^3 x| + C$
- c) $\frac{1}{2} \ln |\sin^3 x - \cos^3 x| + C$
- d) $\frac{1}{4} \ln |\sin^3 x + \cos^3 x| + C$