2024-January Session-01-29-2024-shift-1

AI24BTECH11006

I. SECTION - A

1) Let a die be rolled until a 2 is obtained. The probability that a 2 is obtained on an even-numbered toss is equal to: $[January - 2024]$
   a) $\frac{5}{11}$
   b) $\frac{5}{6}$
   c) $\frac{1}{11}$
   d) $\frac{6}{11}$

2) $\lim_{x\to\frac{\pi}{2}^-} \frac{\int_{x^3}^{\left(\frac{\pi}{2}\right)^2} \cos t^{\frac{1}{3}}\, dt}{\left(x-\frac{\pi}{2}\right)^2}$ $[January - 2024]$
   a) $\frac{3\pi^2}{4}$
   b) $\frac{3\pi}{4}$
   c) $\frac{3\pi^2}{8}$
   d) $\frac{3\pi}{8}$

3) Consider the equation $4\sqrt{2}x^3 - 3\sqrt{2}x - 1 = 0$.
   Statement 1: The solution of this equation is $\cos\frac{\pi}{12}$.
   Statement 2: This equation has only one real solution. $[January - 2024]$
   a) Both statements are true.
   b) Statement 1 is true but Statement 2 is false.
   c) Statement 1 is false but Statement 2 is true.
   d) Both statements are false.

4) If $\mod 2A^3 = 2^{21}$ and $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$, then α is: $[January - 2024]$
   a) 5
   b) 3
   c) 9
   d) 17

5) In a GP with 64 terms, if the sum of all terms is seven times the sum of the odd terms, the common ratio is: $[January - 2024]$
   a) 3
   b) 4
   c) 5
   d) 6

6) Given $\frac{dy}{dx} - \left(\frac{\sin 2x}{1+\cos^2 x}\right) y = \left(\frac{\sin x}{1+\cos^2 x}\right)$ and $y(0) = 0$, find $y\left(\frac{\pi}{2}\right)$. $[January - 2024]$
   a) -1
   b) 1
   c) 0
   d) 2

7) $4\cos\theta + 5\sin\theta = 1$. Then find $\tan\theta$, where $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. $[January - 2024]$
   a) $\frac{10-\sqrt{10}}{6}$
   b) $\frac{10-\sqrt{10}}{12}$
   c) $\frac{\sqrt{10}-10}{6}$
   d) $\frac{\sqrt{10}-10}{12}$

8) In an increasing arithmetic progression $a_1, a_2, \ldots, a_n$, if $a_6 = 2$ and the product of a_1, a_5, a_4 is the greatest, then the common difference d is: $[January - 2024]$
   a) 1.6
   b) 1.8
   c) 0.6
   d) 2.0

9) If the relation $R : (a, b) R (c, d)$ holds only if $ad - bc$ is divisible by 5 where $a, b, c, d \in \mathbb{Z}$, then R is: $[January - 2024]$
   a) Reflexive
   b) Symmetric, Reflexive but not Transitive
   c) Reflexive, Transitive but not Symmetric
   d) An Equivalence Relation

10) Let $f(x)$ and $g(x)$ be defined as follows:
$$f(x) = \begin{cases} 2x + 2 & \text{if } x \in (-1, 0) \\ 1 - \frac{x}{3} & \text{if } x \in [0, 3] \end{cases}$$
$$g(x) = \begin{cases} x & \text{if } x \in [0, 1] \\ -x & \text{if } x \in (-3, 0) \end{cases}$$
   The range of $fog(x)$ is: $[January - 2024]$
   a) $[0, 1]$
   b) $[-1, 1]$
   c) $[0, 1]$
   d) $(-1, 1)$

11) If $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x^2 \cos x}{1+\pi^x} + \frac{1+\sin^2 x}{1+e^{(\sin x)^{2023}}}\right) dx = \frac{\pi}{4}(\pi + \alpha) - 2$, then α is equal to: $[January - 2024]$
   a) 1
   b) 2

c) 3
d) 4

12) The area under the curve $x^2 + y^2 = 169$ and below the line $5x - y = 13$ is: $[January - 2024]$

a) $\frac{169\pi}{4} - \frac{65}{2} + \frac{169}{2}\sin^{-1}\frac{12}{13}$
b) $\frac{169\pi}{4} + \frac{65}{2} - \frac{169}{2}\sin^{-1}\frac{12}{13}$
c) $\frac{169\pi}{4} - \frac{65}{2} + \frac{169}{2}\sin^{-1}\frac{13}{14}$
d) $\frac{169\pi}{4} + \frac{65}{2} + \frac{169}{2}\sin^{-1}\frac{13}{14}$

13) If $f(x) = \frac{\left(2^x + 2^{-x}\right)(\tan x)\tan^{-1}\left(2x^2 - 3x + 1\right)}{\left(7x^2 - 3x + 1\right)^3}$, then $f'(0)$ is: $[January - 2024]$

a) $\sqrt{\pi}$
b) $\sqrt{\frac{\pi}{4}}$
c) π
d) $2 \cdot \pi^{\frac{3}{2}}$

14) Evaluate $\int \frac{(\sin x - \cos x)\sin^2 x}{\sin x \cos^2 x + \tan x \sin^3 x}\,dx$ is equal to $[January - 2024]$

a) $\frac{1}{3}\ln\left|\sin^3 x - \cos^3 x\right| + C$
b) $\frac{1}{3}\ln\left|\sin^3 x + \cos^3 x\right| + C$
c) $\frac{1}{2}\ln\left|\sin^3 x - \cos^3 x\right| + C$
d) $\frac{1}{4}\ln\left|\sin^3 x + \cos^3 x\right| + C$