## 2016-MA-'1-13'

## AI24BTECH11006 - Bugada Roopansha

- 1) Let  $\{X, Y, Z\}$  be a basis of  $\mathbb{R}^3$ . Consider the following statements P and Q:
  - a)  $P: \{X+Y, Y+Z, X-Z\}$  is a basis of  $\mathbb{R}^3$ .
  - b)  $Q: \{X + Y + Z, X + 2Y Z, X 3Z\}$  is a basis of  $\mathbb{R}^3$ .

Which of the above statements hold TRUE?

- a) both P and Q
- b) only Q
- c) only P
- d) Neither P nor Q
- 2) Consider the following statements P and Q:

$$P: \text{ If } M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}, \text{ then } M \text{ is singular.}$$

Q: Let S be a diagonalizable matrix. If T is a matrix such that S+5T=I, then T is diagonalizable.

Which of the above statements hold TRUE?

- a) both P and Q
- b) only Q
- c) only P
- d) Neither P nor Q
- 3) Consider the following statements P and Q:

P: If M is an  $n \times n$  complex matrix, then  $R(M) = (N(M^*))^{\perp} +$ .

Q: There exists a unitary matrix with an eigenvalue  $\lambda$  such that  $|\lambda| < 1$ .

Which of the above statements hold TRUE?

- a) both P and Q
- b) only Q
- c) only P
- d) Neither P nor Q
- 4) Consider a real vector space V of dimension n and a non-zero linear transformation  $T:V\to V$ . If dimension (T(V))< n and  $T^2=\lambda T$ , for some  $\lambda\in\mathbb{R}\setminus\{0\}$ , then which of the following statements is TRUE?
  - a) determinant (T) = |2|
  - b) There exists a non-trivial subspace  $V_1$  of V such that  $T\left(X\right)=0$  for all  $X\in V$

- c) T is invertible
- d) 2 is the only eigenvalue of T
- 5) Let  $S = (0,1) \cup [2,3]$  and  $f: S \to \mathbb{R}$  be a strictly increasing function such that f(S) is connected. Which of the following statements is TRUE?

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- a) f has exactly one discontinuity
- b) f has exactly two discontinuities
- c) f has infinitely many discontinuities
- d) f is continuous
- 6) Let  $a_1 = 1$  and  $a_n = a_{n-1} + 4$ ,  $n \ge 2$ . Then,

$$\lim_{n \to \infty} \frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \dots + \frac{1}{a_{n-1} a_n}$$

is equal to · · ·

- 7)  $\max\{x + y : (x, y) \in B(0, 1)\}$  is equal to ...
- 8) Let  $a, b, c, d \in \mathbb{R}$  such that  $c^2 + d^2 \neq 0$ . Then, the Cauchy problem

$$au_x + bu_y = e^{x+y}, x, y \in \mathbb{R},$$

$$u(x,y) = 0$$
 on  $cx + dy = 0$ 

has a unique solution if

- a)  $ac + bd \neq 0$
- b)  $ad bc \neq 0$
- c)  $ac bd \neq 0$
- $d) \ ad + bc \neq 0$
- 9) Let u(x,t) be the d'Alembert's solution of the initial value problem for the wave equation

$$u_{tt} - c^2 u_{xx} = 0,$$

$$u(x,0) = f(x), u_t(x,0) = g(x),$$

where c is a positive real number and f, g are smooth odd functions. Then,  $u\left(0,1\right)$  is equal to  $\cdots$ 

10) Let the probability density function of a random variable X be

$$f(x) = \begin{cases} c(2x-1) & 0 < x \le 1, \\ \frac{1}{x} & 1 < x \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

Then, the value of c is equal to  $\cdots$ 

- 11) Let V be the set of all solutions of the equation y'' + ay' + by = 0 satisfying y(0) = y(1), where a, b are positive real numbers. Then, dimension (V) is equal to
- 12) Let y'' + p(x)y' + q(x)y = 0,  $x \in (-\infty, \infty)$ , where p(x) and q(x) are continuous functions. If  $y_1(x) = \sin(x) 2\cos(x)$  and  $y_2(x) = 2\sin(x) + \cos(x)$  are two linearly independent solutions of the above equation, then |4p(0) + 2q(1)| is equal to
- 13) Let P(x) be the Legendre polynomial of degree n and  $I = \int_{-1}^{1} x^k P(x) dx$ , where k is a non-negative integer. Consider the following statements P and Q:
  - P: I = 0 if k < n.
  - Q: I = 0 if n + k is an odd integer.

Which of the above statements hold TRUE?

- a) both P and Q
- b) only Q
- c) only P
- d) Neither P nor Q