18.Definite Integrals and Applications of Integrals

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Section-A JEE Advanced/IIT-JEE

C. MCQs with One Correct Answer

21) If $l(m,n)= \int_0^1 t^m(1+t)^n$dt, then the expression for $l(m,n)$ in terms of $l(m+1,n-1)$is (2003S)
(a) $\frac{2^n}{m+1}-\dfrac{n}{m+1}l(m+1,n-1)$
(b) $\frac{n}{m+1}l(m+1,n-1)$
(c) $\frac{2^n}{m+1}+\dfrac{n}{m+1}l(m+1,n-1)$
(d) $\frac{m}{n+1}l(m+1,n-1)$

22) If f(x)= $\int_{x^2}^{x^2+1} e^{-t^2}$dt ,then f(x)increases in (2003S)
(a) (-2,2) (b) no value of x
(c) $(0,\infty)$ (d) $(-\infty,0)$

23) The area bounded by the curves $y=\sqrt{x}$,$2y+3=x$ and x-axis in the 1^{st}quadrant is (2003S)
(a) 9 (b) $\frac{27}{4}$ (c) 36 (d) 18

24) If f(x)is differentiable and$\int_0^{t^2} xf(x)dx = \frac{2}{5}t^5, then f(\frac{4}{25})$ equals (2004S)
(a)$\frac{2}{5}$ $(b)\frac{-5}{2}$ $(c)1$ $(d)\frac{5}{2}$

25) The value of the integral$\int_0^1 \sqrt{\frac{1-x}{1+x}}dx$ is (2004S)
(a)$\frac{\pi}{2}+1$ (b)$\frac{\pi}{2}-1$ (c)-1 (d)1

26) The area enclosed between the curves $y=ax^2$and$x=ay^2(a>0)$is 1 sq.unit,then the value of a is (2004S)
(a)$\frac{1}{\sqrt{3}}$ (b)$\frac{1}{2}$ (c)1 (d)$\frac{1}{3}$

27) $\int_{-2}^{0}x^3+3x^2+3x+3+(x+1)cos(x+1)$dx is equal to (2005$S$)
(a)-4 (b)0 (c)4 (d)6

28) The area bounded by the parabolas $y=(x+1)^2$and$y=(x-1)^2$and the line$y=\frac{1}{4}$is (2005S)
(a) 4 sq.units (b)$\frac{1}{6}sq.units$
(c)$\frac{4}{3}$sq.units (d)$\frac{1}{3}$sq.units

29) The area of the region between the curves $y=\sqrt{\frac{1+sinx}{cosx}}andy=\sqrt{\frac{1-sinx}{cosx}}$bounded by the lines $x=0$and$x=\frac{\pi}{4}$is (2008)
(a)$\int_0^{\sqrt{2}-1}\frac{t}{(1+t^2)\sqrt{1-t^2}}dt$ (b)$\int_0^{\sqrt{2}-1}\frac{4t}{(1+t^2)\sqrt{1-t^2}}$dt
(c)$\int_0^{\sqrt{2}+1}\frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$ (d)$\int_0^{\sqrt{2}+1}\frac{t}{(1+t^2)\sqrt{1-t^2}}dt$

30) Let $f$be a non-negative function defined on the interval [0,1].If$\int_0^x\sqrt{1-(f'(t))^2} = \int_0^x f(t)dt$,$0\le x\le 1$,and$f(0)=0, then$ (2009)
$(a)f(\frac{1}{2})<\frac{1}{2}andf(\frac{1}{3})>\frac{1}{3}$
(b)$f(\frac{1}{2})>\frac{1}{2} and f(\frac{1}{3})>\frac{1}{3}$
(c)$f(\frac{1}{2})<\frac{1}{2} and f(\frac{1}{3})<\frac{1}{3}$
(d)$f(\frac{1}{2})>\frac{1}{2} and f(\frac{1}{3})<\frac{1}{3}$

31) The value of $\lim_{x\to 0}\frac{1}{x^3}\int_0^x\frac{t\ln(1+t)}{t^4+4}dt$ is (2010)
(a) 0 (b)$\frac{1}{12}$ $(c)\frac{1}{24}$ $(d)\frac{1}{64}$

32) Let f be a real valued function defined on the interval (-1,1) such that $e^{-x}f(x) = 2+\int_0^x\sqrt{t^4+1}dt$, for all $x\in(-1,1)$,and let f^{-1}be the inverse function f.Then $(f^{-1})'(2)$is equal to (2010)
(a) 1 (b)$\frac{1}{3}$ $(c)\frac{1}{2}$ $(d)\frac{1}{e}$

33) The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x sin x^2}{sin x^2 + sin(\ln 6 - x^2)} dx$ is (2011)
(a)$\frac{1}{4}\ln\frac{3}{2}$ $(b)\frac{1}{2}\ln\frac{3}{2}$ $(c)\ln\frac{3}{2}$ $(d)\frac{1}{6}\ln\frac{3}{2}$

34) Let the straight line $x = b$ divide the area enclosed by $y = (1-x)^2$, $y = 0$, and $x = 0$ into two parts $R_1(0 \leq x \leq b)$ and $R_2(b \leq x \leq 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals (2011)
(a)$\frac{3}{4}$ $(b)\frac{1}{2}$ $(c)\frac{1}{3}$ $(d)\frac{1}{4}$

35) Let $f : [-1, 2] \to [0, \infty)$ be a continuous function such that $f(x) = f(1-x)$ for all $x \in [-1, 2]$. Let $R_1 = \int_{-1}^{2} x f(x) dx$, and R_2 be the area of the region bounded by $y = f(x), x = -1, x = 2$, and the x-axis.
Then (2011)
(a)$R_1 = 2R_2$ (b)$R_1 = 3R_2$
(c)$2R_1 = R_2$ (d)$3R_1 = R_2$