2024-April Session-04-04-2024-shift-2

AI24BTECH11006

I. SECTION - A

16) For λ >0 let θ be the angle between the vectors $\mathbf{a} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\mathbf{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. If the vectors $\bar{\mathbf{a}} + \bar{\mathbf{b}}$ and $\bar{\mathbf{a}} - \bar{\mathbf{b}}$ are mutually perpendicular, then the value of $(14\cos\theta)^2$ is equal to: $[April - 2024]$
    a) 50
    b) 25
    c) 20
    d) 40
17) If the value of the integral $\int_{-1}^{1} \frac{\cos\alpha}{1+3^x} dxis\frac{2}{\pi}$, then a value of α is: $[April - 2024]$
    a) $\frac{\pi}{3}$
    b) $\frac{\pi}{2}$
    c) $\frac{\pi}{4}$
    d) $\frac{\pi}{6}$
18) Let $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}$, $\mathbf{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$, and $\mathbf{c} = x\hat{i} + 2\hat{j} + 3\hat{k}$, where $x \in \mathbb{R}$. If $\bar{\mathbf{d}}$ is the unit vector in the direction of $\bar{\mathbf{b}} + \bar{\mathbf{c}}$ such that $\bar{\mathbf{a}} \cdot \bar{\mathbf{d}} = 1$, then $(\bar{\mathbf{a}} \times \bar{\mathbf{b}}) \cdot \bar{\mathbf{c}}$ is equal to: $[April - 2024]$
    a) 3
    b) 6
    c) 11
    d) 9
19) Let a relation R on $\mathbb{N} \times \mathbb{N}$ be defined as: $(x_1, y_1) R (x_2, y_2)$ if and only if $x_1 \leq x_2$ or $y_1 \leq y_2$. Consider the two statements: $[April - 2024]$
    a) R is reflexive but not symmetric
    b) R is transitive

    Which one of the following is true: $[April - 2024]$
    a) Both (I) and (II) are correct
    b) Only (I) is correct
    c) Only (II) is correct
    d) Neither (I) nor (II) is correct
20) If the function:

$$f(x) = \begin{cases} \frac{72^a - 9^a - 8^a + 1}{\sqrt{2} - \sqrt{1+\cos x}} & x \neq 0 \\ a \ln 2 \ln 3, & x = 0 \end{cases}$$

is continuous at $x = 0$, then the value of a^2 is: $[April - 2024]$
    a) 1152
    b) 746
    c) 968
    d) 1250
21) There are 4 men and 5 women in Group A, and 5 men and 4 women in Group B. If 4 persons are selected from each group, then the number of ways of selecting 4 men and 4 women is$\cdots$ $[April - 2024]$
22) Consider a triangle ABC having the vertices $A(1,2)$, $B(\alpha,\beta)$, and $C(\gamma,\delta)$ and angles $\angle ABC = \frac{\pi}{6}$ and $\angle BAC = \frac{2\pi}{3}$. If points B and C lie on the line $y = x + 4$, then $\alpha^2 + \gamma^2$ is equal to$\cdots$ $[April - 2024]$
23) Let $y = y(x)$ be the solution of the differential equation $(x + y + 2)^2 dx = dy$, $y(0) = -2$. Let the maximum and minimum values of the function $y(x)$ in $\left[0, \frac{\pi}{3}\right]$ be α and β, respectively. If $(3\alpha + \pi)^2 + \beta^2 = \gamma + \delta\sqrt{3}$, where $\gamma, \delta \in \mathbb{Z}$, then $\gamma + \delta$ equals $\cdots$ $[April - 2024]$
24) If $\int \operatorname{cosec}^5 x dx = \alpha \cot x \operatorname{cosec} x \left(\operatorname{cosec}^2 x + \frac{3}{2} \right) + \beta \ln \left| \tan \frac{x}{2} \right| + C$, where $\alpha, \beta \in \mathbb{R}$, then the value of $8(\alpha + \beta)$ is: $[April - 2024]$
25) Let $f : \mathbb{R} \to \mathbb{R}$ be a thrice differentiable function such that $f(0) = 0$, $f(1) = 1$, $f(2) = -1$, $f(3) = 2$, and $f(4) = -2$. Then, the minimum number of zeros of $3f\prime f\prime\prime + ff\prime\prime\prime$ is$\cdots$ $[April - 2024]$
26) Let A be a 2×2 symmetric matrix such that $A\begin{bmatrix} 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 7 \end{bmatrix}$, and the determinant of A is 1. If $A^{-1} = \alpha A + \beta I$, where I is the identity matrix of order 2, then $\alpha + \beta$ equals: $[April - 2024]$
27) Consider the function $f(x) = \frac{2x}{\sqrt{1+9x^2}}$. If the composition of $f \cdot \frac{(f \cdot f \cdot f \cdots f)(x)}{10 times} = \frac{2^{10} x}{\sqrt{1+9ax^2}}$, then the value of $\sqrt{3a+1}$ is equal to $\cdots$ $[April - 2024]$
28) Consider a line L passing through points $P(1,2,1)$ and $Q(2,1,-1)$. If the mirror image

of point $A(2,2,2)$ in the line L is (α, β, γ), then $\alpha + \beta + 6\gamma$ is$\cdots$ $[April - 2024]$

29) In a tournament, a team plays 10 matches with probabilities of winning and losing each match $\frac{1}{3}$ and $\frac{2}{3}$, respectively. Let x be the number of matches that the team wins, and y be the number of matches that the team loses. If the probability $P(|x-y| \leq 2)$ is p, then $3^9 p$ equals$\cdots$ $[April - 2024]$

30) Let $S = \{\sin^2 2\theta : \left(\sin^4\theta + \cos^4\theta\right)x^2 + (\sin 2\theta)x + \left(\sin^6\theta + \cos^6\theta\right) = 0\} has real root$. If α and β are the smallest and largest elements of S, respectively, then $3\left((\alpha-2)^2 + (\beta-1)^2\right)$ equals$\cdots$ $[April - 2024]$