

2021-March Session-03-18-2021-shift-2

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I. SECTION - A

16) If P and Q are two statements, then which of the following compound statements is a tautology?

- a)
- b) $((P \Rightarrow Q) \wedge \neg Q) \Rightarrow P$
- c) $((P \Rightarrow Q) \wedge \neg Q) \Rightarrow \neg P$
- d) $(P \Rightarrow Q) \wedge \neg Q$
- e) $((P \Rightarrow Q) \wedge \neg Q) \Rightarrow Q$

17) Consider a hyperbola H: $x^2 - 2y^2 = 4$. Let the tangent at a point P $(4, \sqrt{6})$ meet the x-axis at Q and latus rectum at R (x_1, y_1) , $x_1 > 0$. If F is a focus of H which is nearer to the point P, then the area of $\triangle QFR$ is equal to:

- a) $\sqrt{6} - 1$
- b) $4\sqrt{6} - 1$
- c) $4\sqrt{6}$
- d) $\frac{7}{\sqrt{6}} - 2$

18) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined as

$$f(x) = \begin{cases} \frac{\sin((a+1)x) + \sin(2x)}{2x}, & \text{if } x < 0 \\ b, & \text{if } x = 0 \\ \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}}, & \text{if } x > 0 \end{cases} \quad (1)$$

. If f is continuous at $x = 0$, then the value of $a + b$ is equal to

- a) -2
- b) $-\frac{2}{3}$
- c) $-\frac{5}{2}$
- d) -3

19) Let $y=y(x)$ be the solution of the differential equation $\frac{dy}{dx} = (y+1) \left[(y+1)e^{x^2/2} - x \right]$, $0 < x < 2.1$, with $y(2) = 0$. Then the value of $\frac{dy}{dx}$ at $x=1$ is equal to:

- a) $\frac{e^{5/2}}{(1+e^2)^2}$
- b) $\frac{5e^{1/2}}{(e^2+1)^2}$
- c) $-\frac{2e^2}{(1+e^2)^2}$
- d) $-\frac{e^{3/2}}{(e^2+1)^2}$

20) Let a tangent be drawn to the ellipse $(x^2/27) + y^2 = 1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ where $\theta \in (0, \frac{\pi}{2})$.

Then the value of θ such that the sum of intercepts on axes made by a tangent is minimum is equal to:

- a) $\frac{\pi}{8}$
- b) $\frac{\pi}{6}$
- c) $\frac{\pi}{3}$
- d) $\frac{\pi}{4}$

II. SECTION - B

21) Let P be a plane containing the line $\frac{[x-1]}{3} = \frac{[y+6]}{4} = \frac{[z+5]}{2}$ and parallel to the line $\frac{[x-3]}{4} = \frac{[y-2]}{-3} = \frac{[z+5]}{7}$. If the point $(1, -1, \alpha)$ lies on the plane P, then the value of $|5\alpha|$ is equal to ...

22) $\sum_{r=1}^{10} r!(r^3 + 6r^2 + 2r + 5) = \alpha(11!)$. Then the value of α is equal to ...

23) The term independent of x in the expansion of $\left[\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}} \right]^{10}$, $x \neq 1$, is equal to ...

24) Let $\binom{n}{r}$ denote the binomial coefficient of x^r in the expansion of $(1+x)^n$. If $\sum_{k=0}^{10} [2^2 + 3k] \binom{n}{k} = \alpha \cdot 3^{10} + \beta \cdot 2^{10}$ then $\alpha + \beta$ is equal to ...

25) Let P(x) be a real polynomial of degree 3 which vanishes at $x = -3$. Let P(x) have local minima at $x = 1$, local maxima at $x = -1$ and $\int_{-1}^1 P(x) dx = 18$, then the sum of all the coefficients of the polynomial P(x) is equal to ...

26) Let the mirror image of the point $(1, 3, a)$ with respect to the plane $r. (2i - j + k) \cdot b = 0$ be $(0, -3, 5, 2)$. Then, the value of $|a + b|$ is equal to ...

27) If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x) = f(x^3) + xg(x^3)$ is divisible by $x^2 + x + 1$, then $P(1)$ is equal to ...

28) Let I be an identity matrix of order 2×2 and $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of $n \in \mathbb{N}$ for which $P^n = 5I - 8P$ is equal to ...

29) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfy the equation $f(x+y) = f(x) \cdot f(y)$ for all $x, y \in \mathbb{R}$ and $f(x) \neq 0$ for any $x \in \mathbb{R}$. If the function f is differentiable at

$x = 0$ and $f'(0) = 3$, then $\lim_{h \rightarrow 0} \frac{1}{h} [f(h) - 1]$ is equal to ...

- 30) Let $y = y(x)$ be the solution of the differential equation $x dy - y dx = \sqrt{x^2 - y^2} dx$, $x \geq 1$ with $y(1) = 0$. If the area bounded by the line $x = 1$, $x = e^\pi$, $y = 0$ and $y = y(x)$ is $\alpha e^{2\pi} + \beta$ then the value of $10(\alpha + \beta)$ is equal to ...