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I. SECTION - A

16) If the solution of the equation $\log(\cos x)\cot x + 4\log(\sin x)\tan x = 1,\ x \in \left[0, \frac{\pi}{2}\right]$ is $\sin^{-1}\left(\frac{\alpha+\beta}{2}\right)$, where α, β are integers, then $\alpha+\beta$ is equal to
   a) 6
   b) 5
   c) 4
   d) 3
17) A straight line cuts off the intercepts $OA = a$ and $OB = b$ on the positive direction of the x-axis and y-axis respectively. If the perpendicular from the origin O to this line makes an angle of $\frac{\pi}{6}$ with the positive direction of the y-axis and the area of $\triangle OAB$ is $\frac{98}{3\sqrt{3}}$, then $a^2 - b^2$ is equal to
   a) 196
   b) $\frac{196}{3}$
   c) $\frac{392}{3}$
   d) 98
18) If $a_n = \frac{-2}{4n^2-16n+5}$, then $a_1 + a_2 + \cdots + a_{25}$ is equal to
   a) $\frac{49}{138}$
   b) $\frac{52}{147}$
   c) $\frac{51}{144}$
   d) $\frac{50}{141}$
19) Let the solution curve $y = y(x)$ of the differential equation $\frac{dy}{dx} - \frac{3x^5\tan^{-1}\left(x^3\right)}{\left(1+x^6\right)^{\frac{3}{2}}}y = 2x\exp\frac{x^3 - tan^{-1}x^3}{\sqrt{1+x^6}}$ pass through the origin. Then $y(1)$ is equal to
   a) $\exp\left(\frac{1-\pi}{4\sqrt{2}}\right)$
   b) $\exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$
   c) $\exp\left(\frac{4+\pi}{4}\right)$
   d) $\exp\left(\frac{\pi-4}{2\sqrt{2}}\right)$
20) If an unbiased die, marked with $-2, -1, 0, 1, 2, 3$ on its faces, is thrown five times, then the probability that the product of the outcomes is positive is:
   a) $\frac{881}{2592}$
   b) $\frac{440}{2592}$
   c) $\frac{27}{288}$
   d) $\frac{521}{2592}$

II. SECTION-B

21) Let $S = [1, 2, 3, 4, 5, 6]$. The number of one-to-one functions $f : S \to P(S)$, such that $f(n) \subset f(m)$ where $n<m$, is equal to $\cdots$
22) The number of four-digit numbers $(repetitionofdigitsallowed)$ made using the digits $1, 2, 3, and 5$ and divisible by 15, is $\cdots$
23) If $\lambda_1 < \lambda_2$ are two values of λ such that the angle between the planes $P_1 : \bar{r}\left(3\hat{i} - 5\hat{j} + \hat{k}\right) = 7$ and $P_2 : \bar{r}\left(\lambda\hat{i} + \hat{j} - 3\hat{k}\right) = 9$ is $\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$, then the square of the length of the perpendicular from the point $(38\lambda_1, 10\lambda_2, 2)$ to the plane P_1 is $\cdots$
24) Let $\sum_{n=0}^{\infty} \frac{n^3((2n)!)+(2n-1)(n!)}{(n!)((2n)!)} = ae + \frac{b}{e} + c$, where $a, b, c \in \mathbb{Z}$ and $e = \sum_{n=0}^{\infty} \frac{1}{n!}$.Then $a^2 - b + c$ is equal to $\cdots$
25) Let $z = 1 + i$ and $z_1 = \frac{1+i\bar{z}}{\bar{z}(1-z)+\frac{1}{z}}$. Then $\frac{12}{\pi}\arg(z_1)$ is equal to $\cdots$
26) Let $f^1(x) = \frac{3x+2}{2x+3}, x \in \mathbb{R} - \{\frac{-3}{2}\}$.For $n \geq 2$, define $f^n(x) = f^1 o f^{n-1}(x)$.If$f^5(x) = \frac{ax+b}{bx+a}$,gcd$(a, b) = 1$, then a + b is equal to $\cdots$
27) $\lim_{x\to 0} \frac{48}{x^4}\int_0^x \frac{t^3}{t^6+1}dt$ is equal to $\cdots$
28) The mean and variance of 7 observations are 8 and 16respectively. If one observation 14 is omitted and a and b are respectively mean and variance of remaining 6 observation, then $a + 3b - 5$ is equal to $\cdots$
29) If the equation of the plane passing through the point $(1, 1, 2)$ and perpendicular to the line$(x - 3y + 2z - 1 = 0 = 4x - y + z)$ is Ax + By + Cz = 1, then $140(C - B + A)$ is equal to $\cdots$
30) Let α be the area of the larger region bounded by the curve $y^2 = 8x$, the line $y = x$, and $x = 2$, which lies in the first quadrant. Then the value of 3α is equal to $\cdots$