

# 2022-July Session-07-27-2022-shift-1

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## I. SECTION - A

- 1) Let  $R_1$  and  $R_2$  be two relations defined on  $\mathbb{R}$ , defined as follows:  $aR_1b \iff ab \geq 0$  and  $aR_2b \iff a \geq b$ . Then,
  - a)  $R_1$  is an equivalence relation but not  $R_2$
  - b)  $R_2$  is an equivalence relation but not  $R_1$
  - c) Both  $R_1$  and  $R_2$  are equivalence relation
  - d) Neither  $R_1$  and  $R_2$  are equivalence relation
- 2) Let  $f, g : \mathbb{N} = \{1\} \rightarrow \mathbb{N}$  be functions defined by  $f(a) = a$ , where  $a$  is the maximum of the powers of those primes  $p$  such that  $p^a$  divides  $a$ , and  $g(a) = a + 1$ , for all  $a \in \mathbb{N} - \{1\}$ . Then the function  $f+g$  is
  - a) One-one but not onto
  - b) Onto but not one-one
  - c) Both one-one and onto
  - d) Neither one-one nor onto
- 3) Let the minimum value  $v_0$  of  $v = |z|^2 + |z - 3|^2 + |z - 6i|^2, z \in \mathbb{C}$  is attained at  $z = z_0$ . Then  $|2z_0^2 - \bar{z}_0^3 + 3|^2 + v_0^2$  is equal to
  - a) 1000
  - b) 1024
  - c) 1105
  - d) 1196
- 4) Let  $A = \begin{pmatrix} 1 & 2 \\ -2 & -5 \end{pmatrix}$ . Let  $\alpha, \beta \in \mathbb{R}$  be such that  $\alpha A^2 + \beta A = 2I$ . Then  $\alpha + \beta$  is equal to
  - a) -10
  - b) -6
  - c) 6
  - d) 10
- 5) The remainder when  $(2021)^{2022} + (2022)^{2021}$  is divided by 7 is
  - a) 0
  - b) 1
  - c) 2
  - d) 6
- 6) Suppose  $a_1, a_2, \dots, a_n, \dots$  be an arithmetic progression of natural numbers. If the ratio of the sum of first five terms to the sum of first nine terms of the progression is  $5 : 17$  and  $110 < a_{15}$ 
  - a) 290
  - b) 380
  - c) 460
  - d) 510
- 7) Let  $\mathbb{R} \rightarrow \mathbb{R}$  be function defined as  $f(x) = a \sin\left(\frac{\pi[x]}{2}\right) + [2 - x], a \in \mathbb{R}$  where  $[t]$  is the greatest integer less than or equal to  $t$ . If  $\lim_{x \rightarrow 1} f(x)$  exists, then the value of  $\int_0^4 f(x) dx$  is equal to
  - a) -1
  - b) -2
  - c) 1
  - d) 2
- 8) Let  $I = \int_{\pi/4}^{\pi/3} \left( \frac{8 \sin x - \sin 2x}{x} \right) dx$  Then
  - a)  $\frac{\pi}{2} < I < \frac{3\pi}{4}$
  - b)  $\frac{\pi}{5} < I < \frac{5\pi}{12}$
  - c)  $\frac{5\pi}{12} < I < \frac{\sqrt{2}\pi}{3}$
  - d)  $\frac{3\pi}{4} < I < \pi$
- 9) The area of the smaller region enclosed by the curves  $y^2 = 8x + 4$  and  $x^2 + y^2 + 4\sqrt{3}x - 4 = 0$  is equal to
  - a)  $\frac{1}{3} (2 - 12\sqrt{3} + 8\pi)$
  - b)  $\frac{1}{3} (2 - 12\sqrt{3} + 6\pi)$
  - c)  $\frac{1}{3} (4 - 12\sqrt{3} + 8\pi)$
  - d)  $\frac{1}{3} (4 - 12\sqrt{3} + 6\pi)$
- 10) Let  $y = y_1(x)$  and  $y = y_2(x)$  be two distinct solution of the differential equation  $\frac{dy}{dx} = x + y$ , with  $y_1(0) = 0$  and  $y_2(0) = 1$  respectively. Then, the number of points of intersection of  $y = y_1(x)$  and  $y = y_2(x)$  is
  - a) 0
  - b) 1
  - c) 2
  - d) 3
- 11) Let  $P(a, b)$  be a point on the parabola  $y^2 = 8x$  such that the tangent at P passes through the centre of the circle  $x^2 + y^2 - 10x - 14y + 65 = 0$ . Let A be the product of all possible values of
  - a) 290
  - b) 380
  - c) 460
  - d) 510

a and B be the product of all possible values of b. Then the value of A + B is equal to

- a) 0
- b) 25
- c) 40
- d) 65

12) Let  $\mathbf{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$  and  $\mathbf{b} = 3\hat{i} + 5\hat{j} + 4\hat{k}$  be two vectors, such that  $\mathbf{a} \times \mathbf{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$ . Then the projection of  $\mathbf{b} - 2\mathbf{a}$  on  $\mathbf{b} + \mathbf{a}$  is equal to

- a) 2
- b)  $\frac{39}{5}$
- c) 9
- d)  $\frac{46}{5}$

13) Let  $\mathbf{a} = 2\hat{i} - \hat{j} + 5\hat{k}$  and  $\mathbf{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$ . If  $((\mathbf{a} \times \mathbf{b}) \times \hat{i}) \cdot \hat{k} = \frac{23}{2}$ , then  $|\mathbf{b} \times 2\hat{j}|$  is equal to

- a) 4
- b) 5
- c)  $\sqrt{21}$
- d)  $\sqrt{17}$

14) Let S be the sample space of all five digit numbers. It p is the probability that a randomly selected number from S, is multiple of 7 but not divisible by 5, then  $9p$  is equal to

- a) 1.0146
- b) 1.2085
- c) 1.0285
- d) 1.1521

15) Let a vertical tower AB of height 2h stands on a horizontal ground. Let from a point P on the ground a man can see upto height h of the tower with an angle of elevation  $2\alpha$ . When from P, he moves a distance d in the direction of  $\overrightarrow{AP}$  he can see the top B of the tower with an angle of elevation  $\alpha$ . if  $d = \sqrt{7}h$ , then  $\tan \alpha$  is equal to

- a)  $\sqrt{5} - 2$
- b)  $\sqrt{3} - 1$
- c)  $\sqrt{7} - 2$
- d)  $\sqrt{7} - \sqrt{3}$