2022-July Session-07-27-2022-shift-1

AI24BTECH11006 - Bugada Roopansha

I. SECTION - A

1) Let R_1 and R_2 be two relations defined on $\mathbb{R}$, defined as follows:$aR_1b \iff ab \geq 0$ and $aR_2b \iff a \geq b$. Then,
   a) R_1 is an equivalence relation but not R_2
   b) R_2 is an equivalence relation but not R_1
   c) Both R_1 and R_2 are equivalence relation
   d) Neither R_1 and R_2 are equivalence relation
2) Let $f, g : \mathbb{N} = \{1\} \to \mathbb{N}$ be functions defined by f$(a) = a$, where a is the maximum of the powers of those primes p such that p^a divides a, and $g(a) = a + 1$, for all $a \in N - \{1\}$. Then the function f+g is
   a) One-one but not onto
   b) Onto but not one-one
   c) Both one-one and onto
   d) Neither one-one nor onto
3) Let the minimum value v_0 of $v = |z|^2 + |z-3|^2 + |z-6i|^2, z \in \mathbb{C}$ is attained at z $=z_0$. Then $\left|2z_0^2 - \bar{z}_0^3 + 3\right|^2 + v_0^2$ is equal to
   a) 1000
   b) 1024
   c) 1105
   d) 1196
4) Let $A = \begin{pmatrix} 1 & 2 \\ -2 & -5 \end{pmatrix}$. Let$\alpha, \beta \in \mathbb{R}$ be such that $\alpha A^2 + \beta A = 2I$.Then $\alpha + \beta$ is equal to
   a) -10
   b) -6
   c) 6
   d) 10
5) The remainder when$(2021)^{2022} + (2022)^{2021}$ is divided by 7 is
   a) 0
   b) 1
   c) 2
   d) 6
6) Suppose $a_1, a_2, \ldots, a_n, \ldots$ be an arithmetic progression of natural numbers. If the ration of the sum of first five terms to the sum of first nine terms of the progression is 5 : 17 and 110 $<a_{15}$ $<$120, then the sum of the first ten terms of the progression is equal to
   a) 290
   b) 380
   c) 460
   d) 510
7) Let $\mathbb{R} \to \mathbb{R}$ be function defined as $f(x) = a\sin\left(\frac{\pi[x]}{2}\right) + [2-x], a \in \mathbb{R}$ where $[t]$ is the greatest integer less than or equal to t. If $\lim_{x\to 1} f(x)$ exists, then the value of$\int_0^4 f(x)\,dx$ is equal to
   a) -1
   b) -2
   c) 1
   d) 2
8) Let $I = \int_{\pi/4}^{\pi/3} \left(\frac{8\sin x - \sin 2x}{x}\right) dx$ Then
   a) $\frac{\pi}{2} < I < \frac{3\pi}{4}$
   b) $\frac{\pi}{5} < I < \frac{5\pi}{12}$
   c) $\frac{5\pi}{12} < I < \frac{\sqrt{2}\pi}{3}$
   d) $\frac{3\pi}{4} < I < \pi$
9) The area of the smaller region enclosed by the curves $y^2 = 8x + 4$ and $x^2 + y^2 + 4\sqrt{3}x - 4 = 0$ is equal to
   a) $\frac{1}{3}\left(2 - 12\sqrt{3} + 8\pi\right)$
   b) $\frac{1}{3}\left(2 - 12\sqrt{3} + 6\pi\right)$
   c) $\frac{1}{3}\left(4 - 12\sqrt{3} + 8\pi\right)$
   d) $\frac{1}{3}\left(4 - 12\sqrt{3} + 6\pi\right)$
10) Let $y = y_1(x)$ and $y = y_2(x)$ be two distinct solution of the differential equation $\frac{dy}{dx} = x + y$, with $y_1(0) = 0$ and $y_2(0) = 1$ respectively. Then, the number of points of intersection of $y = y_1(x)$ and $y = y_2(x)$ is
    a) 0
    b) 1
    c) 2
    d) 3
11) Let $P(a, b)$ be a point on the parabola $y^2 = 8x$ such that the tangent at P passes through the centre of the circle $x^2 + y^2 - 10x - 14y + 65 = 0$. Let A be the product of all possible values of

a and B be the product of all possible values of b. Then the value of A + B is equal to

a) 0
b) 25
c) 40
d) 65

12) Let $\mathbf{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$ and $\mathbf{b} = 3\hat{i} + 5\hat{j} + 4\hat{k}$ be two vectors, such that $\mathbf{a} \times \mathbf{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$. Then the projection of $\mathbf{b} - 2\mathbf{a}$ on $\mathbf{b} + \mathbf{a}$ is equal to

a) 2
b) $\frac{39}{5}$
c) 9
d) $\frac{46}{5}$

13) Let $\mathbf{a} = 2\hat{i} - \hat{j} + 5\hat{k}$ and $\mathbf{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$. If $\left((\mathbf{a} \times \mathbf{b}) \times \hat{i}\right) \cdot \hat{k} = \frac{23}{2}$, then $\left|\mathbf{b} \times 2\hat{j}\right|$ is equal to

a) 4
b) 5
c) $\sqrt{21}$
d) $\sqrt{17}$

14) Let S be the sample space of all five digit numbers. It p is the probability that a randomly selected number from S, is multiple of 7 but not divisible by 5, then 9p is equal to

a) 1.0146
b) 1.2085
c) 1.0285
d) 1.1521

15) Let a vertical tower AB of height 2h stands on a horizontal ground. Let from a point P on the ground a man can see upto height h of the tower with an angle of elevation 2α. When from P, he moves a distance d in the direction of $\overrightarrow{AP}$ he can see the top B of the tower with an angle of elevation α. if d = $\sqrt{7}$ h, then $\tan\alpha$ is equal to

a) $\sqrt{5} - 2$
b) $\sqrt{3} - 1$
c) $\sqrt{7} - 2$
d) $\sqrt{7} - \sqrt{3}$