18.Definite Integrals and Applications of Integrals

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Section-A JEE Advanced/IIT-JEE

C. MCQs with One Correct Answer

21) If $l(m,n)=\int_0^1 t^m(1+t)^n$dt, then the expression for $l(m,n)$ in terms of $l(m+1,n-1)$is (2003S)
   a) $\frac{2^n}{m+1}$-$\frac{n}{m+1}l(m+1,n-1)$
   b) $\frac{n}{m+1}l(m+1,n-1)$
   c) $\frac{2^n}{m+1}+\frac{n}{m+1}l(m+1,n-1)$
   d) $\frac{m}{n+1}l(m+1,n-1)$
22) If f(x)= $\int_{x^2}^{x^2+1} e^{-t^2}$dt ,then f($x$)increases in (2003S)
   a) $(-2,2)$
   b) no value of x
   c) $(0,\infty)$
   d) $(-\infty,0)$
23) The area bounded by the curves $y=\sqrt{x}$,2y+3=x and x-axis in the 1^{st}quadrant is (2003S)
   a) 9
   b) $\frac{27}{4}$
   c) 36
   d) 18
24) If f(x)is differentiable and$\int_0^{t^2} xf(x)\,dx = \frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals (2004S)
   a) $\frac{2}{5}$
   b) $\frac{-5}{2}$
   c) 1
   d) $\frac{5}{2}$
25) The value of the integral$\int_0^1 \sqrt{\frac{1-x}{1+x}}dx$ is (2004S)
   a) $\frac{\pi}{2}+1$
   b) $\frac{\pi}{2}-1$
   c) -1
   d) 1
26) The area enclosed between the curves $y=ax^2$and$x = ay^2(a>0)$is $1sq.unit$,then the value of a is (2004S)
   a) $\frac{1}{\sqrt{3}}$
   b) $\frac{1}{2}$
   c) 1
   d) $\frac{1}{3}$
27) $\int_{-2}^0 x^3+3x^2+3x+3+(x+1)\,cos\,(x+1)$dx is equal to (2005S)
   a) -4
   b) 0
   c) 4
   d) 6
28) The area bounded by the parabolas $y = (x+1)^2$and$y = (x-1)^2$and the line$y = \frac{1}{4}$is (2005S)
   a) $4sq.units$
   b) $\frac{1}{6}sq.units$
   c) $\frac{4}{3}$sq.units
   d) $\frac{1}{3}$sq.units
29) The area of the region between the curves $y = \sqrt{\frac{1+sinx}{cosx}}andy = \sqrt{\frac{1-sinx}{cosx}}$bounded by the lines $x = 0$and$x = \frac{\pi}{4}$is (2008)
   a) $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}}dt$
   b) $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}$dt
   c) $\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$
   d) $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}}dt$
30) Let $f$be a non-negative function defined on the interval $[0,1]$·If$\int_0^x \sqrt{1-(f'(t))^2} = \int_0^x f(t)\,dt$,$0 \le x \le 1$,and$f(0)=0$,then (2009)
   a) $f\left(\frac{1}{2}\right)< \frac{1}{2}andf\left(\frac{1}{3}\right) > \frac{1}{3}$
   b) $f\left(\frac{1}{2}\right) > \frac{1}{2}and f\left(\frac{1}{3}\right) > \frac{1}{3}$
   c) $f\left(\frac{1}{2}\right) < \frac{1}{2}and f\left(\frac{1}{3}\right) < \frac{1}{3}$
   d) $f\left(\frac{1}{2}\right) > \frac{1}{2}and f\left(\frac{1}{3}\right) < \frac{1}{3}$
31) The value of $\lim_{x\to 0}\frac{1}{x^3}\int_0^x \frac{t\ln(1+t)}{t^4+4}dt$ is (2010)
   a) 0
   b) $\frac{1}{12}$
   c) $\frac{1}{24}$
   d) $\frac{1}{64}$
32) Let f be a real valued function defined on the interval $(-1,1)$ such that $e^{-x}f(x) = 2+\int_0^x \sqrt{t^4+1}dt$, for all $x \in (-1,1)$,and let f^{-1}be the inverse function f.Then $\left(f^{-1}\right)'(2)$is equal

to (2010)

a) 1
b) $\frac{1}{3}$
c) $\frac{1}{2}$
d) $\frac{1}{e}$

33) The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x sin x^2}{sin x^2 + sin\left(\ln 6 - x^2\right)} dx$ is (2011)

a) $\frac{1}{4} \ln \frac{3}{2}$
b) $\frac{1}{2} \ln \frac{3}{2}$
c) $\ln \frac{3}{2}$
d) $\frac{1}{6} \ln \frac{3}{2}$

34) Let the straight line $x = b$ divide the area enclosed by $y = (1 - x)^2$, $y = 0$, and $x = 0$ into two parts $R_1\,(0 \le x \le b)$ and $R_2\,(b \le x \le 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals (2011)

a) $\frac{3}{4}$
b) $\frac{1}{2}$
c) $\frac{1}{3}$
d) $\frac{1}{4}$

35) Let $f : [-1, 2] \to [0, \infty)$ be a continuous function such that $f(x) = f(1 - x)$ for all $x \in (-1, 2)$. Let $R_1 = \int_{-1}^{2} x f(x)\, dx$, and R_2 be the area of the region bounded by $y = f(x)$, $x = -1$, $x = 2$, and the x-axis $\cdot$
Then (2011)

a) $R_1 = 2R_2$
b) $R_1 = 3R_2$
c) $2R_1 = R_2$
d) $3R_1 = R_2$