2021-March Session-03-18-2021-shift-2

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I. SECTION - A

16) If P and Q are two statements, then which of the following compound statements is a tautology?
    a) $((P \Rightarrow Q) \wedge \neg Q) \Rightarrow P$
    b) $((P \Rightarrow Q) \wedge \neg Q) \Rightarrow \neg P$
    c) $(P \Rightarrow Q) \wedge \neg Q$
    d) $((P \Rightarrow Q) \wedge \neg Q) \Rightarrow Q$
17) Consider a hyperbola H: $x^2 - 2y^2 = 4$. Let the tangent at a point P $\left(4, \sqrt{6}\right)$ meet the x-axis at Q and latus rectum at R (x_1, y_1), $x_1>0$. If F is a focus of H which is nearer to the point P, then the area of $\triangle QFR$ is equal to:
    a) $\sqrt{6} - 1$
    b) $4\sqrt{6} - 1$
    c) $4\sqrt{6}$
    d) $\frac{7}{\sqrt{6}} - 2$
18) Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined as

$$f(x) = \begin{cases} \frac{\sin((a+1)x)+\sin(2x)}{2x}, & \text{if } x<0 \\ b, & \text{if } x = 0 \\ \frac{\sqrt{x+bx^3}-\sqrt{x}}{bx^{5/2}}, & \text{if } x>0 \end{cases} \tag{1}$$

. If f is continuous at $x = 0$, then the value of a + b is equal to
    a) -2
    b) $\frac{-2}{5}$
    c) $\frac{-3}{2}$
    d) -3
19) Let y=y(x) be the solution of the differential equation $\frac{dy}{dx} = (y+1)\left[(y+1)e^{x^2/2} - x\right]$,0<x<2.1,with y(2) = 0. Then the value of $\frac{dy}{dx}$ at x=1 is equal to:
    a) $\frac{e^{5/2}}{\left(1+e^2\right)^2}$
    b) $\frac{5e^{1/2}}{\left(e^2+1\right)^2}$
    c) $-\frac{2e^2}{\left(1+e^2\right)^2}$
    d) $-\frac{e^{3/2}}{\left(e^2+1\right)^2}$
20) Let a tangent be drawn to the ellipse $\left(x^2/27\right) + y^2 = 1$ at $\left(3\sqrt{3}\cos\theta, \sin\theta\right)$ where $\theta \in \left(0, \frac{\pi}{2}\right)$. Then the value of θ such that the sum of intercepts on axes made by a tangent is minimum is equal to:
    a) $\frac{\pi}{8}$
    b) $\frac{\pi}{6}$
    c) $\frac{\pi}{3}$
    d) $\frac{\pi}{4}$

II. SECTION - B

21) Let P be a plane containing the line $\frac{[x-1]}{3} = \frac{[y+6]}{4} = \frac{[z+5]}{2}$ and parallel to the line $\frac{[x-3]}{4} = \frac{[y-2]}{-3} = \frac{[z+5]}{7}$. If the point $(1, -1, \alpha)$ lies on the plane P, then the value of $|5\alpha|$ is equal to ...
22) $\sum_{r=1}^{10} r!\left(r^3 + 6r^2 + 2r + 5\right) = \alpha(11!)$. Then the value of α is equal to ...
23) The term independent of x in the expansion of $\left[\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right]^{10}$, $x \neq 1$,is equal to...
24) Let $\binom{n}{r}$ denote the binomial coefficient of x^r in the expansion of $(1+x)^n$. If $\sum_{k=0}^{10}\left[2^2 + 3k\right]\binom{n}{k} = \alpha \cdot 3^{10} + \beta \cdot 2^{10}$then $\alpha + \beta$ is equal to...
25) Let P (x) be a real polynomial of degree 3 which vanishes at x =- 3. Let P(x) have local minima at x = 1, local maxima at x = -1 and $\int_{-1}^{1} P(x)\,dx = 18$,then the sum of all the coefficients of the polynomial P (x) is equal to...
26) Let the mirror image of the point $(1, 3, a)$ with respect to the plane r. $(2i - j + k) - b = 0$ be $(-3, 5, 2)$. Then, the value of$|a + b|$ is equal to...
27) If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x) = f\left(x^3\right) + xg\left(x^3\right)$ is divisible by $^2 + x + 1$, then $P(1)$ is equal to...
28) Let I be an identity matrix of order 2×2 and $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of $n \in N$ for which $P^n = 5I - 8P$ is equal to...

29) Let $f : \mathbb{R} \to \mathbb{R}$ satisfy the equation $f(x+y) = f(x) \cdot f(y)$for all $x, y \in \mathbb{R}$and $f(x) \neq 0$for any$x \in \mathbb{R}$. If the function f is differentiable at x = 0 and $f\prime(0) = 3$,then $\lim_{h\to 0} \frac{1}{h}[f(h) - 1]$ is equal to ...
30) Let $y = y(x)$ be the solution of the differential equation $x\,dy - y\,dx = \sqrt{x^2 - y^2}\,dx, x \geq 1$ with $y(1) = 0$. If the area bounded by the line $x = 1, x = e^{\pi}, y = 0$ and $y = y(x)$ is $\alpha e^{2\pi} + \beta$ then the value of $10(\alpha + \beta)$ is equal to ...