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I. SECTION - A

- 16) For $\lambda > 0$ let θ be the angle between the vectors $\mathbf{a} = \hat{i} + \lambda \hat{j} - 3\hat{k}$ and $\mathbf{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. If the vectors $\bar{\bf a} + {\bf b}$ and $\bar{\bf a} - {\bf b}$ are mutually perpendicular, then the value of $(14\cos\theta)^2$ is equal to:
 - a) 50
 - b) 25
 - c) 20
 - d) 40
- 17) If the value of the integral $\int_{-1}^{1} \frac{\cos \alpha}{1+3^{x}} dx i s \frac{2}{\pi}$, then a value of α is:

 - a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$
- 18) Let $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}$, $\mathbf{b} = 2\hat{i} + 4\hat{j} 5\hat{k}$, and $\mathbf{c} = \hat{k}$ $x\hat{i}+2\hat{j}+3\hat{k}$, where $x \in \mathbb{R}$. If $\bar{\mathbf{d}}$ is the unit vector in the direction of $\mathbf{\bar{b}} + \mathbf{\bar{c}}$ such that $\mathbf{\bar{a}} \cdot \mathbf{\bar{d}} = 1$, then $(\bar{\mathbf{a}} \times \mathbf{b}) \cdot \bar{\mathbf{c}}$ is equal to:
 - a) 3
 - b) 6
 - c) 11
 - d) 9
- 19) Let a relation R on $\mathbb{N} \times \mathbb{N}$ be defined as: $(x_1, y_1)R(x_2, y_2)$ if and only if $x_1 \le x_2$ or $y_1 \le y_2$. Consider the two statements:
 - a) R is reflexive but not symmetric
 - b) R is transitive

Which one of the following is true:

- a) Both (I) and (II) are correct
- b) Only (I) is correct
- c) Only (II) is correct
- d) Neither (I) nor (II) is correct
- 20) If the function:

$$f(x) = \begin{cases} \frac{72^{a} - 9^{a} - 8^{a} + 1}{\sqrt{2} - \sqrt{1 + \cos x}} & x \neq 0\\ a \ln 2 \ln 3, & x = 0 \end{cases}$$

is continuous at x = 0, then the value of a^2 is:

- a) 1152
- b) 746

- c) 968
- d) 1250
- 21) There are 4 men and 5 women in Group A, and 5 men and 4 women in Group B. If 4 persons are selected from each group, then the number of ways of selecting 4 men and 4 women is...
- 22) Consider a triangle ABC having the vertices A(1,2), $B(\alpha,\beta)$, and $C(\gamma,\delta)$ and angles $\angle ABC = \frac{\pi}{6}$ and $\angle BAC = \frac{2\pi}{3}$. If points *B* and *C* lie on the line y = x + 4, then $\alpha^2 + \gamma^2$ is equal to \cdots
- 23) Let y = y(x) be the solution of the differential equation $(x + y + 2)^2 dx = dy$, y(0) = -2. Let the maximum and minimum values of the function y(x) in $\left[0, \frac{\pi}{3}\right]$ be α and β , respectively. If $(3\alpha + \pi)^2 + \beta^2 = \gamma + \delta \sqrt{3}$, where $\gamma, \delta \in \mathbb{Z}$, then $\gamma + \delta$ equals \cdots
- $\int \csc^5 x dx$ 24) If $\alpha \cot x \csc x \left(\csc^2 x + \frac{3}{2} \right) + \beta \ln \left| \tan \frac{x}{2} \right| + C$ where $\alpha, \beta \in \mathbb{R}$, then the value of $8(\alpha + \beta)$ is:
- 25) Let $f: \mathbb{R} \to \mathbb{R}$ be a thrice differentiable function such that f(0) = 0, f(1) = 1, f(2) = -1, f(3) = 2, and f(4) = -2. Then, the minimum number of zeros of 3f'f'' + ff''' is...
- 26) Let A be a 2×2 symmetric matrix such that $A \begin{vmatrix} 1 & 1 \end{vmatrix} = \begin{vmatrix} 3 & 7 \end{vmatrix}$, and the determinant of A is 1. If $A^{-1} = \alpha A + \beta I$, where I is the identity matrix of order 2, then $\alpha + \beta$ equals:
- 27) Consider the function $f(x) = \frac{2x}{\sqrt{1+9x^2}}$. If the composition of $f \cdot \frac{(f \cdot f \cdot f \cdot f)(x)}{10 \text{ times}} = \frac{2^{10}x}{\sqrt{1+9ax^2}}$, then the value of $\sqrt{3}a + 1$ is equal to \cdots
- 28) Consider a line L passing through points P(1,2,1) and Q(2,1,-1). If the mirror image of point A(2,2,2) in the line L is (α,β,γ) , then $\alpha + \beta + 6\gamma$ is...
- 29) In a tournament, a team plays 10 matches with probabilities of winning and losing each match $\frac{1}{3}$ and $\frac{2}{3}$, respectively. Let x be the number of matches that the team wins, and y be the number of matches that the team loses. If the probability $P(|x-y| \le 2)$ is p, then 3^9p

equals...
30) Let $S = \{\sin^2 2\theta : (\sin^4 \theta + \cos^4 \theta) x^2 + (\sin 2\theta) x + (\sin^6 \theta + \cos^6 \theta) = 0\}$ has real root. If α and β are the smallest and largest elements of S, respectively, then $3((\alpha - 2)^2 + (\beta - 1)^2)$ equals...