

# 2021-March Session-03-18-2021-shift-2

AI24BTECH11006

## I. SECTION - A

- 16) If P and Q are two statements, then which of the following compound statements is a tautology? [March – 2021]

- $((P \Rightarrow Q) \wedge \neg Q) \Rightarrow P$
- $((P \Rightarrow Q) \wedge \neg Q) \Rightarrow \neg P$
- $(P \Rightarrow Q) \wedge \neg Q$
- $((P \Rightarrow Q) \wedge \neg Q) \Rightarrow Q$

- 17) Consider a hyperbola H:  $x^2 - 2y^2 = 4$ . Let the tangent at a point P  $(4, \sqrt{6})$  meet the x-axis at Q and latus rectum at R  $(x_1, y_1)$ ,  $x_1 > 0$ . If F is a focus of H which is nearer to the point P, then the area of  $\triangle QFR$  is equal to: [March – 2021]

- $\sqrt{6} - 1$
- $4\sqrt{6} - 1$
- $4\sqrt{6}$
- $\frac{7}{\sqrt{6}} - 2$

- 18) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function defined as

$$f(x) = \begin{cases} \frac{\sin((a+1)x) + \sin(2x)}{2x}, & \text{if } x < 0 \\ b, & \text{if } x = 0 \\ \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}}, & \text{if } x > 0 \end{cases} \quad (1)$$

. If f is continuous at  $x = 0$ , then the value of  $a + b$  is equal to [March – 2021]

- 2
- $-\frac{2}{5}$
- $-\frac{3}{2}$
- 3

- 19) Let  $y=y(x)$  be the solution of the differential equation  $\frac{dy}{dx} = (y+1) \left[ (y+1)e^{x^2/2} - x \right]$ ,  $0 < x < 2.1$ , with  $y(2) = 0$ . Then the value of  $\frac{dy}{dx}$  at  $x=1$  is equal to: [March – 2021]

- $\frac{e^{5/2}}{(1+e^2)^2}$
- $\frac{5e^{1/2}}{(e^2+1)^2}$
- $-\frac{2e^2}{(1+e^2)^2}$
- $-\frac{e^{3/2}}{(e^2+1)^2}$

- 20) Let a tangent be drawn to the ellipse  $(x^2/27) + y^2 = 1$  at  $(3\sqrt{3}\cos\theta, \sin\theta)$  where  $\theta \in (0, \frac{\pi}{2})$ . Then the value of  $\theta$  such that the sum of intercepts on axes made by a tangent is minimum is equal to: [March – 2021]

- $\frac{\pi}{8}$
- $\frac{\pi}{6}$
- $\frac{\pi}{3}$
- $\frac{\pi}{4}$

## II. SECTION - B

- 21) Let P be a plane containing the line  $\frac{[x-1]}{3} = \frac{[y+6]}{4} = \frac{[z+5]}{2}$  and parallel to the line  $\frac{[x-3]}{4} = \frac{[y-2]}{-3} = \frac{[z+5]}{7}$ . If the point  $(1, -1, \alpha)$  lies on the plane P, then the value of  $|\alpha|$  is equal to ... [March – 2021]

- 22)  $\sum_{r=1}^{10} r! (r^3 + 6r^2 + 2r + 5) = \alpha (11!)$ . Then the value of  $\alpha$  is equal to ... [March – 2021]

- 23) The term independent of x in the expansion of  $\left[ \frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}} \right]^{10}$ ,  $x \neq 1$ , is equal to... [March – 2021]

- 24) Let  $\binom{n}{r}$  denote the binomial coefficient of  $x^r$  in the expansion of  $(1+x)^n$ . If  $\sum_{k=0}^{10} [2^2 + 3k] \binom{n}{k} = \alpha \cdot 3^{10} + \beta \cdot 2^{10}$  then  $\alpha + \beta$  is equal to... [March – 2021]

- 25) Let P(x) be a real polynomial of degree 3 which vanishes at  $x = -3$ . Let P(x) have local minima at  $x = 1$ , local maxima at  $x = -1$  and  $\int_{-1}^1 P(x) dx = 18$ , then the sum of all the coefficients of the polynomial P(x) is equal to... [March – 2021]

- 26) Let the mirror image of the point  $(1, 3, a)$  with respect to the plane  $r \cdot (2i - j + k) - b = 0$  be  $(-3, 5, 2)$ . Then, the value of  $|a + b|$  is equal to... [March – 2021]

- 27) If  $f(x)$  and  $g(x)$  are two polynomials such that the polynomial  $P(x) = f(x^3) + xg(x^3)$  is divisible by  $x^2 + x + 1$ , then  $P(1)$  is equal to... [March – 2021]

- 28) Let  $I$  be an identity matrix of order  $2 \times 2$  and  $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$ . Then the value of  $n \in \mathbb{N}$  for which  $P^n = 5I - 8P$  is equal to...  
[March – 2021]
- 29) Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  satisfy the equation  $f(x+y) = f(x) \cdot f(y)$  for all  $x, y \in \mathbb{R}$  and  $f(x) \neq 0$  for any  $x \in \mathbb{R}$ . If the function  $f$  is differentiable at  $x = 0$  and  $f'(0) = 3$ , then  $\lim_{h \rightarrow 0} \frac{1}{h} [f(h) - 1]$  is equal to ...  
[March – 2021]
- 30) Let  $y = y(x)$  be the solution of the differential equation  $x dy - y dx = \sqrt{x^2 - y^2} dx$ ,  $x \geq 1$  with  $y(1) = 0$ . If the area bounded by the line  $x = 1$ ,  $x = e^\pi$ ,  $y = 0$  and  $y = y(x)$  is  $\alpha e^{2\pi} + \beta$  then the value of  $10(\alpha + \beta)$  is equal to ...  
[March – 2021]