

18. Definite Integrals and Applications of Integrals

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Section-A JEE Advanced/IIT-JEE

C. MCQs with One Correct Answer

- 21) If $l(m, n) = \int_0^1 t^m (1+t)^n dt$, then the expression for $l(m, n)$ in terms of $l(m+1, n-1)$ is (2003S)
- $\frac{2^n}{m+1} - \frac{n}{m+1} l(m+1, n-1)$
 - $\frac{n}{m+1} l(m+1, n-1)$
 - $\frac{2^n}{m+1} + \frac{n}{m+1} l(m+1, n-1)$
 - $\frac{m}{n+1} l(m+1, n-1)$
- 22) If $f(x) = \int_{x^2}^{x^2+1} e^{-t^2} dt$, then $f(x)$ increases in (2003S)
- $(-2, 2)$
 - no value of x
 - $(0, \infty)$
 - $(-\infty, 0)$
- 23) The area bounded by the curves $y = \sqrt{x}$, $2y+3=x$ and x -axis in the 1st quadrant is (2003S)
- 9
 - $\frac{27}{4}$
 - 36
 - 18
- 24) If $f(x)$ is differentiable and $\int_0^{t^2} x f(x) dx = \frac{2}{5} t^5$, then $f\left(\frac{4}{25}\right)$ equals (2004S)
- $\frac{2}{5}$
 - $\frac{-5}{2}$
 - 1
 - $\frac{5}{2}$
- 25) The value of the integral $\int_0^1 \sqrt{\frac{1-x}{1+x}} dx$ is (2004S)
- $\frac{\pi}{2} + 1$
 - $\frac{\pi}{2} - 1$
 - 1
 - 1
- 26) The area enclosed between the curves $y = ax^2$ and $x = ay^2$ ($a > 0$) is 1 sq. unit, then the value of a is (2004S)
- $\frac{1}{\sqrt{3}}$
 - $\frac{1}{2}$
 - 1
 - $\frac{1}{3}$
- 27) $\int_{-2}^0 x^3 + 3x^2 + 3x + 3 + (x+1) \cos(x+1) dx$ is equal to (2005S)
- 4
 - 0
 - 4
 - 6
- 28) The area bounded by the parabolas $y = (x+1)^2$ and $y = (x-1)^2$ and the line $y = \frac{1}{4}$ is (2005S)
- 4 sq. units
 - $\frac{1}{6}$ sq. units
 - $\frac{4}{3}$ sq. units
 - $\frac{1}{3}$ sq. units
- 29) The area of the region between the curves $y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the lines $x = 0$ and $x = \frac{\pi}{4}$ is (2008)
- $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}} dt$
 - $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$
 - $\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$
 - $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}} dt$
- 30) Let f be a non-negative function defined on the interval $[0, 1]$. If $\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt$, $0 \leq x \leq 1$, and $f(0) = 0$, then (2009)
- $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$
 - $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$
 - $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$
 - $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$
- 31) The value of $\lim_{x \rightarrow 0} \frac{1}{x^3} \int_0^x \frac{t \ln(1+t)}{t^4 + 4} dt$ is (2010)
- 0
 - $\frac{1}{12}$
 - $\frac{1}{24}$
 - $\frac{1}{64}$
- 32) Let f be a real valued function defined on the interval $(-1, 1)$ such that $e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4 + 1} dt$, for all $x \in (-1, 1)$, and let f^{-1} be

the inverse function f . Then $(f^{-1})'(2)$ is equal to (2010)

- a) 1
- b) $\frac{1}{3}$
- c) $\frac{1}{2}$
- d) $\frac{1}{e}$

33) The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)} dx$ is (2011)

- a) $\frac{1}{4} \ln \frac{3}{2}$
- b) $\frac{1}{2} \ln \frac{3}{2}$
- c) $\ln \frac{3}{2}$
- d) $\frac{1}{6} \ln \frac{3}{2}$

34) Let the straight line $x = b$ divide the area enclosed by $y = (1 - x)^2$, $y = 0$, and $x = 0$ into two parts R_1 ($0 \leq x \leq b$) and R_2 ($b \leq x \leq 1$) such that $R_1 - R_2 = \frac{1}{4}$. Then b equals (2011)

- a) $\frac{3}{4}$
- b) $\frac{1}{2}$
- c) $\frac{1}{3}$
- d) $\frac{1}{4}$

35) Let $f : [-1, 2] \rightarrow [0, \infty)$ be a continuous function such that $f(x) = f(1 - x)$ for all $x \in (-1, 2)$. Let $R_1 = \int_{-1}^2 x f(x) dx$, and R_2 be the area of the region bounded by $y = f(x)$, $x = -1$, $x = 2$, and the x -axis.

Then (2011)

- a) $R_1 = 2R_2$
- b) $R_1 = 3R_2$
- c) $2R_1 = R_2$
- d) $3R_1 = R_2$