Module - 1

Lasers & Fibre optics

Laser

LASER stands for Light Amplification by Stimulated Emission of Radiation. Differing from conventional sources, lasers produce highly directional, monochromatic, coherent and stimulated radiation. Lasing been extended from microwave(Maser) to γ-rays (Graser)

Characteristics

1. **Monuchromaticity**: The line width of laser beams are extremely narrow. (1 in 10^{15}, in conventional light source, it is 1 in 10^6). Degree of non-monochromaticly: $\xi = \Delta\lambda/\lambda = \Delta\nu/\nu (\Delta\lambda or \Delta\nu$ variation in wavelength or frequency of light)

2. **Directionality**: Laser can travel very long distances without divergence.

   Divergence: $\Delta\theta = \dfrac{r_2 - r_1}{D_2 - D_1}$ or $\dfrac{D^2}{\lambda}$ (Reliegh's range)

   For laser: 0.01 milliradian
   for search light 0.5 radian $>\Rightarrow$ divergence

3. **Coherence**
   Spatial coherence: If a wave maintains a constant phase difference or in phase at two different points on the wave over a time t, then the wave is said to have spatial coherence.

   Temporal coherence: If there is no change in phase over a time t at a point on the wave, then it is said to be coherent temporally during that time. (these reference points are taken on electric field)

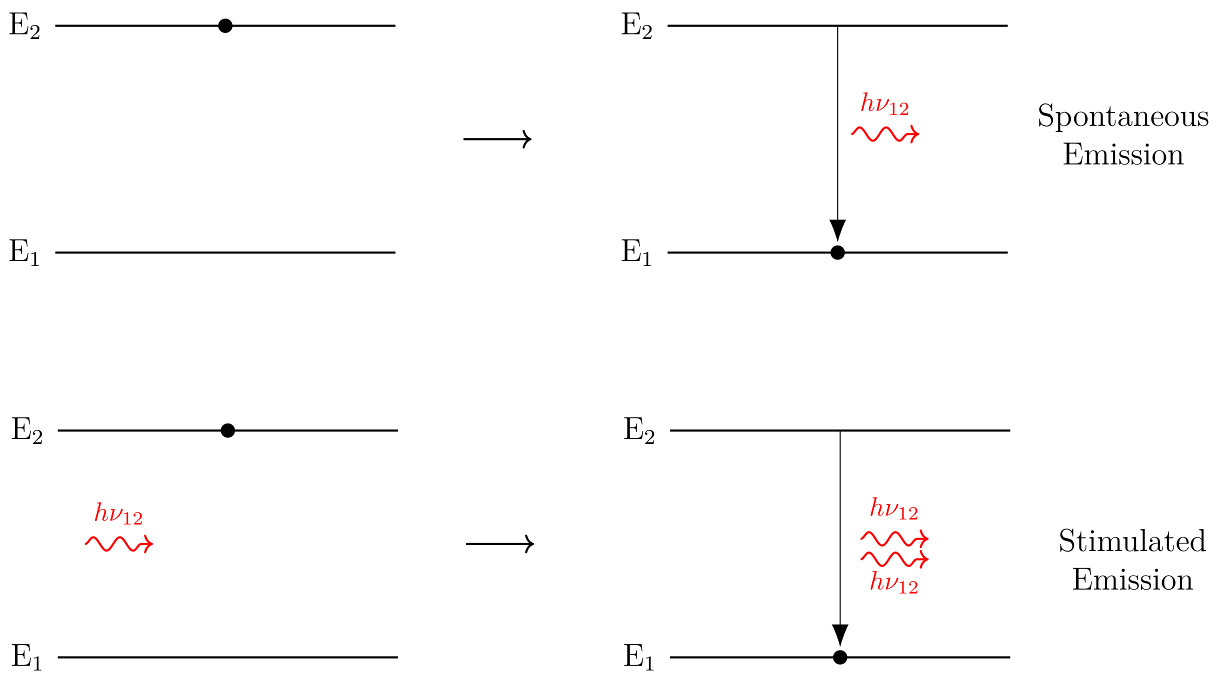
4. **Brightness/intensity**
   Laser produce highly intense beams, because more light energy is concentrated in a small region. Also laser light is coherent, so at a time many photons are in phase. They superimpose to produce a wave of larger amplitude. Hence resultant intensity ($\propto$amplitude^2) is very high.

Coherence length (L_c):Propagation distance over which a coherent wave maintains a specified degree of coherence or phase difference
for He-Ne: 600km

Non-chromaticity $\propto \dfrac{1}{L_c}$

Coherence time(τ_c): time over which a propagating wave remains coherent or the maximum time interval for which the wave have definite phase relation.
For He-Ne: $\tau_c = 2 \times 10^{-3}$ s
Sodium lamp: 10^{-10} s

Spontaneous & stimulated emission



Spontaneous emission

Atom initially at excited state makes transition voluntarily on its own, without aid of any external agency, to ground state, and emits photon of energy

$$h\nu_{12} = E_2 - E_1$$

Different atoms of medium emits light at different times and different directions. Hence emitted photons are incoherent. The rate of emission is independent of incident energy (as no energy is supplied to the system.)

Stimulated emission

Here photon having energy $h\nu_{12}(= E_2 - E_1)$ impinges on (or passes in the vicinity of) an atom present in its excited state and atom is stimulated to make transition to ground state and gives off a photon of energy $h\nu_{12}$. Emitted photon in is in phase with the incident photon. These two travel in same direction and posses same frequency. They're coherent.

Note:

⋆ At thermal equilibrium, all energy transitions are equally probable

Spontaneous emission	**Stimulated emission**
Polychromatic radiation	Monochromatic radiation
Low intensity	High intensity
Low Directionality (so more angular spread during Propagation)	High Directionality (so less angular spread during Propagation)
Spacially and Temporally incoherent radiation	Spacially and Temporally coherent radiation
Takes place when atom transition from higher to lower energy level	Takes place when photon of suitable frequency Stimulates an excited atom to make transition to lower energy level

Einstein's coefficients

For a system in steady state containing atoms and radiation, ratio of atoms in ground state(E_1) to atoms in excited state(E_2) is:

$$\frac{n_2}{n_1} = \exp\left(-\frac{E_2 - E_1}{k_B T}\right) = \exp\left(-\frac{h\nu}{k_B T}\right)$$

- When atom in ground state gets excited to higher state, rate or absorption of radiation is : $B_{12} n_1 \rho_v$ (ρ_v = energy density of radiation incident on ground state)
- Rate of spontaneous emission: $R_{21} = n_2 A_{21}$
- Rate of absorption : $R_{ab} = n_1 \rho_v B_{12}$
- Rate of stimulated emission: $R_{12} = n_2 \rho_v B_{21}$

Where

n_1	=	number of availoble atoms per unit volume
n_2	=	number of atoms per unit volume
ν	=	frequency of radiation
T	=	absolute temperature of the atoms
k_{B}	=	Boltzmann constant
A_{21}	=	probability that atom will spontaneously jump to E_1 (ground state) per unit time (spontaneous emission proportionality constant)

B_{12} = probability of absorpion per unit time (absorption proportionality constant)

B_{21} = probability that atom will transition from E_2 to E_1 via stimulated emission per unit time (stimulated emission proportionality constant)

Finding ρ_v

Inside the container, radiation is present so, the number of photons per unit volume having frequencies around ν in unit range (i.e., radiation density) ,represented as ρ_v is given by Planck's black body radiation law as:

$$\rho_v = \frac{8\pi h\nu^3}{c^3\left[\exp\left(\frac{h\nu}{k_\mathrm{B}T}\right)-1\right]}$$

In steady state:

$$\begin{aligned} R_{ab} &= R_{12} + R_{21} \\ B_{12}n_1\rho_v &= A_{21}n_2 + B_{21}n_2\rho_v \\ &\text{(or)} \\ \rho_v\left[B_{12}n_1 - B_{21}n_2\right] &= A_{21}n_2 \end{aligned}$$

$$\begin{aligned} \rho_v = \frac{A_{21}n_2}{B_{12}n_1 - B_{21}n_2} &= \frac{A_{21}n_2}{n_2 B_{21}\left[\frac{B_{12}n_1}{B_{21}n_2}-1\right]} \\ &= \frac{A_{21}/B_{21}}{\frac{B_{12}n_1}{B_{21}n_2}-1} \end{aligned}$$

Substituting Equation (6.1) in (6.8) for n_1/n_2, we have:

$$\rho_v = \frac{A_{21}/B_{21}}{\frac{B_{12}}{B_{21}}\exp\left(\frac{h\nu}{K_\mathrm{B}T}\right)-1}$$

In thermal equilibrium state, Equations (6.2) and (6.9) are equal. so,

$$\frac{8\pi h\nu^3}{c^3\left[\exp\left(\frac{h\nu}{K_\mathrm{B}T}\right)-1\right]} = \frac{A_{21}/B_{21}}{\frac{B_{12}}{B_{21}}\exp\left(\frac{h\nu}{K_\mathrm{B}T}\right)-1}$$

Under stimulated emission, the probability of upward transitions and probability of downward transitions are equal, so:

$$B_{12} = B_{21} = B \text{ and } A_{21} = A \text{ (say)}.$$

Then, Equation (6.10) becomes:

$$\frac{A_{21}}{B_{21}} = \frac{A}{B} = \frac{8\pi h\nu^3}{c^3}$$

The proportionality constants A_{21}, B_{12} and B_{21} are called Einstein's A and B coefficients. From Equations (6.5) and (6.6), the ratio of spontaneous emission rate to stimulated emission rate is:

$$\frac{A_{21} n_2}{B_{21} n_2 \rho_v} = \frac{A_{21}}{B_{21} \rho_v}$$

Substituting Equations (6.2) and (6.11) in (6.12) gives:

$$= \frac{8\pi h\nu^3}{c^3} \bigg/ \frac{8\pi h\nu^3}{c^3\left[\exp\left(\frac{h\nu}{K_{\mathrm{B}}T}\right) - 1\right]} = \exp\left(\frac{h\nu}{K_{\mathrm{B}}T}\right) - 1$$

This ratio works out to be 10^{10}, thus at optical frequencies, the emission is predominantly spontaneous. So, the conventional light sources emit incoherent radiation. 6.5 Population inversion

Usually in a system the number of atoms (N_1) present in the ground state (E_1) is larger than the number of atoms (N_2) present in the higher energy state. The process of making $N_2 > N_1$ is called population inversion. Population inversion can be explained with three energy levels E_1, E_2 and E_3 of a system. Let E_1, E_2 and E_3 be ground state, metastable state and excited states of energies of the system respectively such that $E_1 < E_2 < E_3$. In a system, the population of atoms (N) in an energy level E, at absolute temperature T has been expressed in terms of the population (N_1) in the ground state using Boltzmann's distribution law

$$N = N_1 \exp\left(-E/K_R T\right) \quad \text{where } K_B = \text{ Boltzmann's constant}$$

Graphically this has been shown in Fig. 6.2(b). As shown in Fig. 6.2(a), let the atoms in the system be excited from E_1 state to E_3 state by supplying energy equal to $E_3 - E_1 (= h\nu)$ from an external source. The atoms in E_3 state are unstable, they make downward transition in a time approximately 10^{-8} seconds to E_2 state. In E_2 state, the atoms stay over a very long duration of the order of milliseconds. So, the population of E_2 state increases steadily. As atoms in E_1 state are continuously excited to E_3 so, the population in E_1 energy level goes on decreasing. A stage will reach at which the population in E_2 state exceeds as that present in E_1 state (i.e., $N_2 > N_1$). This situation is known as population inversion. Graphically the population inversion has been shown in Fig. 6.2(c). Conditions for population inversion are: (a) The system should possess at least a pair of energy levels ($E_2 > E_1$), separated by an energy equal to the energy of a photon (bv). (b) There should be a continuous supply of energy to the system such that the atoms must be raised continuously to the excited state.

$$\begin{aligned}
\frac{A_{21}}{B_{21}} &= \frac{8\pi h\nu^3}{c^3} \\
&= \frac{8\pi h c^3/\lambda^3}{c^3} \\
&= \frac{8\pi h}{\lambda^3} \\
\frac{R_{21}}{R_{21}} &= e^{h\nu/(kT)} - 1
\end{aligned}$$

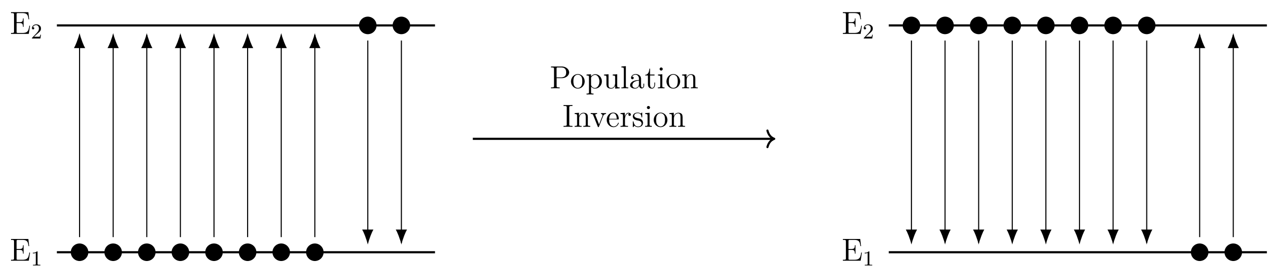
Note:

- Spontaneous emission produce incoherent light
- Stimulated emission produce coherent light

Population Inversion

Population inversion: a state of system in which more members of system are in high energy level than in lower level.

- When photon of Energy ΔE (equal to difference in energy level) enters atom probability of absorption = probability of spontaneous emission
- At Room temperature: $\Delta E = E_2 - E_1 \approxeq 0.025$ eV (Corresponding to IR radiation of 6×10^{12} Hz)



$N(E_1) >> N(E_2)$

$\Rightarrow R_{12} > R_{21}$

$N(E_1) << N(E_2)$

$\Rightarrow R_{12} < R_{21}$

At thermal equilibrium: $N\left(E_2\right)/N\left(E_1\right) = e^{-(E_2-E_1)/(kT)}$

pumping: Supplying energy to excite atoms to upper levels.

Metastable state

The atoms excited will undergo spontaneous emission and attain popalation inversion. These atoms reach a intermediate state between the transitioning states which has more life time than higher energy state. This state is known as **metastable state**. From here, atoms undergo stimulated emission and lasing action takes place. (In two level systems, lasing action doesn't takes place because of lack of metastable state)

Representing transitions

- Absorption: $A + h\nu \to A^*$
- Spontaneous Emission: $A^* \to A + h\nu$
- Stimulated Emission : $A^* + h\nu \to A + 2h\nu$

$\star$ Emitted photon is identical to incident one. (have same frea, phase, polarisation)
$\star$ Since excited states are highly unstable (atom will returns to ground state in less than 1ns) an intermedioute energy level (which has life time of 10^{-3} to 10^{-4}s) is required to create stimulated emission. This is achieved by using certain dopants.

Purity of spectral line

Finite purity or sharpress $= \frac{\lambda}{\Delta\lambda}$
$\Delta\nu/\Delta\lambda =$ Spread of frequency.
Fading of amplitude is explained by setting $\Delta\nu$ in time τ_c by unity:

$$\tau_c \Delta\nu = 1 \Rightarrow \Delta\lambda = \frac{\lambda^2}{\tau_c c}$$
$$L = c\tau_c = \frac{\lambda^2}{\Delta\lambda} = \lambda Q$$

For Sodium line: $\Delta\lambda = 0.12\text{Å}$ For He-Ne laser $\Delta\lambda = 0.00050.12\text{Å}$

1 For a source of monochromatic radiation with wavelength 5000Å, and frequency 6×10^{14} Hz, what will be the sharpress for conventional light with $\Delta\nu = 10^{10}$ Hz and les with $\Delta\nu = 500$ Hz ?

<u>**Answer**</u>
For conventional radiation: $\dfrac{\Delta\nu}{\nu} = \dfrac{10^{10}}{6 \times 10^{14}} = 1.67 \times 10^{-5}$
For laser: $\dfrac{\Delta\nu}{\nu} = \dfrac{500}{6 \times 10^{14}} = 6 \times 10^{-12}$

2 For a He-Ne laser, Output diameters are 4 nm and 5 nm at distances 1m and 2m. Calculate beam divergence

<u>**Answer**</u>
Beam divergence $= \dfrac{R_2 - R_1}{D_2 - D_1} = \dfrac{.5 \text{ mm}}{1 \text{ m}} = \dfrac{1}{2}10^{-3}$ rad

3 For ordinary source of light, $\tau_c = 10^{-10}$ s. Obtain its sharpness if wavelength is 5400Å

<u>**Answer**</u>

$$Q = \frac{\Delta\nu}{\nu}$$
$$\nu = \frac{c}{\lambda} = 5.56 \times 10^{14} \text{ Hz}$$
$$\tau_c = \frac{1}{\Delta\nu} \Rightarrow \Delta\nu = 1/\tau_c = 10^{10} \text{ Hz}$$
$$\therefore \text{Sharpness} = \frac{10^{10}}{5.56 \times 10^{14}} = 1.8 \times 10^{-5}$$

4 A monochromatic source emits light of wavelength 5461Å. Its bandwidth $\Delta\nu = 10^9$Hz. Find τ_c, L_c and frequency stability (sharpness)

Answer

$\tau_c = \dfrac{1}{\Delta v} = 10^{-9}s$

$L_c = \tau_c c = 30$ cm

$\nu = \dfrac{c}{\lambda} = 5.49 \times 10^{44}$ Hz

$\text{Sharpress} = \dfrac{\Delta \tau}{\nu} = \dfrac{10^9}{5.49 \times 10^{14}} = 1.82 \times 10^{-6}$

5 At what temperature is the ratio of spontaneous emission to stimulated emission equal to 1 for a source emitting light of wavelength 5000 Å ?

Answer

$$\frac{\text{Spontaneous emission}}{\text{Stimulated emission}} = e^{h\nu/(kT)} - 1 = 1$$

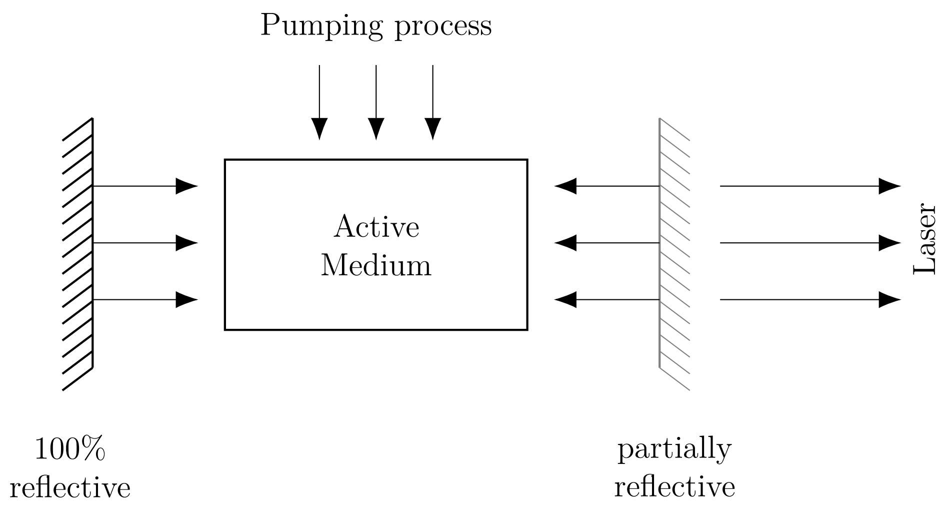
$$\Rightarrow T = \frac{h\nu}{\ln(2)k} = 41543K$$

6 Find relative popalation of 2 states in a ruby laser that produces a light beam of wave 6943 Å at 3000 K

Answer

$$\begin{aligned}\text{relative population} &= \frac{N_2}{N_1} e^{\frac{E_1 - E_2}{kT}} \\ &= e \wedge \left(\frac{-2.86 \times 10^{-19}}{1.83 \times 10^{-25} \times 3000} \right) \\ &= 1.0002 \times 10^{-3}\end{aligned}$$

Fabrication of laser



Active medium: material in which popalation inversion takes place. (eg. ruby, Ga-In, He-Ne)
Active center: Atoms which can produce more stimulated emission than spontaneous emission: (e.g: Cr in ruby, Ne in He-Ne medium).
Pumping mechanism: Adding energy to system. This can be done by:

- Optical pumping (used in ruby laser, xe-flash lamp)
- Direct electron excitation.
- Inelastic atom-atom collisions: electric discharge is employed to cause collision and excitation of atoms. (used in He-Ne)
- Chemical reactions.

Optical resonance & Resonance cavity

Two mirrors are used to reflect light back into active medium one will be 100 % reflective.(Usually made of dielectic material) and other one is partially reflective.
Two arrangements of mirrors are used to reflect light:

- **plane-parallel**: needs to be strictly parallel (with deviation less than 1" ($1/3600^{\text{deg}}$)) and must have smooth surface(irregularities $<1/100\lambda$).
- **confocal**: need to be parallel (with deviation less than 0.025^{deg})

Types of Laser

- **Pulsed mode**: Train of pulses are produced. It can have produce power in the order of 1MW. (e.g: Ruby laser)
- **Continuous mode**: Light is produced continuously. It has less powerful output (less than 1W) (eg: He-Ne laser)

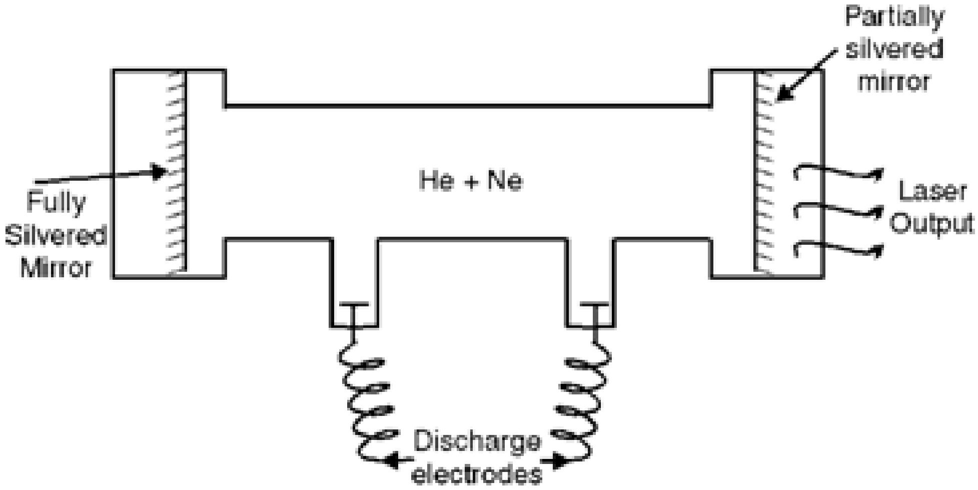
Uses of Laser

- Used in holograms
- To study the internal structure of molecules.
- To detect earthquakes
- For cutting gilling and welding
- Used to perform surgery
- Used to treat cancer, kidncy stone, tumers etc.
- Used to detect and destroy enemy missiles and also to control rockets and satellites.

Different Lasers

Helium - Neon Laser

- It is a 4 level Gaseous system.
- Created by Ali Jawan, W. Bennet, D. Harrul in 1962
- It produces continuous output.
- Pumping mechanism: 10kV electron dischange
- Active medium: mixture of He and Ne in 10:1 ratio. (Active center is Ne)
- wavelength of light emitted: 632.8 nm (red color)

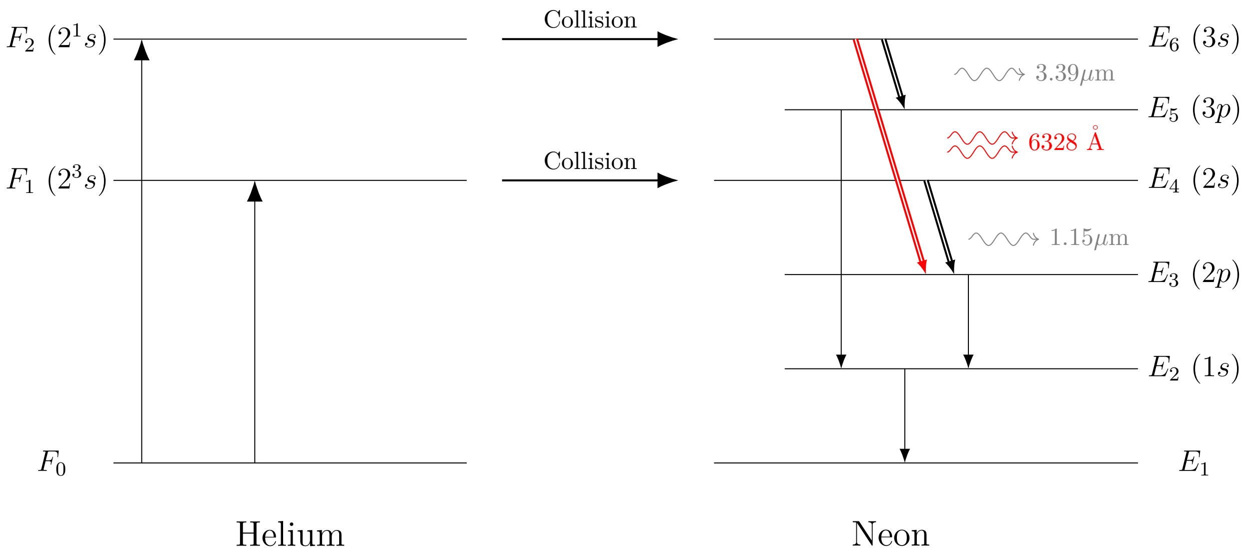


Working

Due to electric dischange in gas, energetic electron interacts with ground state helium atoms. The impact of e^- results in exchange of some of its energy to the atom. As a result. Helium atoms gets excited to higher energy levels: F_1&F_2. These two energy levals are close to E_4&E_6 levels of Neon atoms and collision of second kind takes place between them, hence we goes to excited state. E_5 of E_3. E_2 level of we forms metastable state.
3 types of transition are possible :

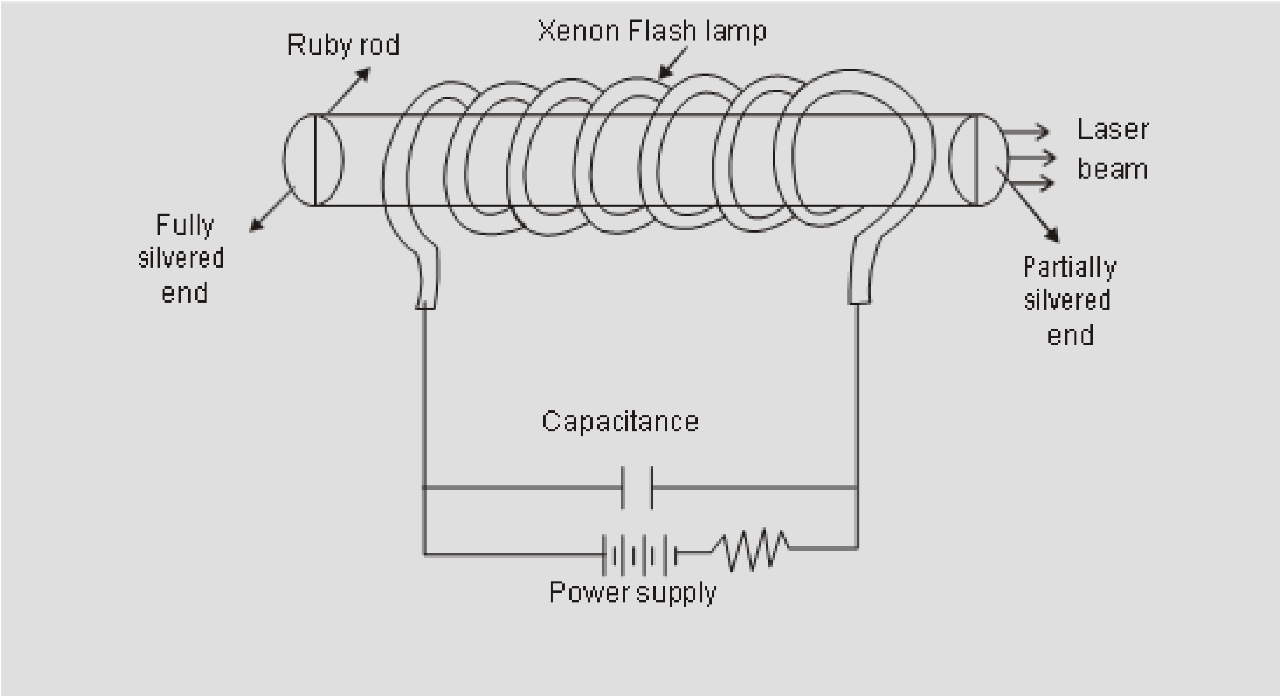
- $E_4 \rightarrow E_3$, emitting infrared light of $\lambda = 1.15\mu$m
- $E_6 \rightarrow E_5$, emitting FR of $\lambda = 3.39\mu$m
- $E_6 \rightarrow E_3$, emitting red light of $\lambda = 632.8$ nm

IR light emitted is absorbed by the mirrors. Neon atoms in terminal level E_3 decay very rapialy to E_2 metastable state, much faster than spontaneously decaying from $E_6 \rightarrow E_3$ level. Thus lower level E_3 is kept empty and population inversion is achieved between E_6&E_3. Intensity of light produced is inversely proportional to diameter of quartz tube.

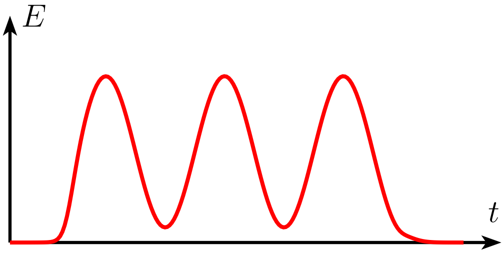


Ruby Laser

- first laser fabricated
- Created in 1960 by Maiman
- It is a 3 level system
- Produced light in pulsating maner (continuous mode not possible due to heating of active medium)
- Active medium: $Al_2O_3 \cdot Cr^{3+}$ (0.05-0.5 % of Al atoms are replaced with Cr atoms)
- Pumping source: Xenon lamp. (the lamp has a helical structure that wounds the ruby rod).
- Eavelength of light emitted: 6943 Å



⋆ Population inversion is achieved between E_m & E,



Intensity vs time graph

Working

Cr^{3+} ions absorb pumping light and go to excited states E_2 & E_3. From these a rapid non-radiation transition to the metastable level E_m takes place. Decay from E_m is very slow that with sufficient excitation, populations inversion between E_m & E_1 can occur. Those photons are allowed to pass many times in the active medium with the help of mirrors. When threshold conditions are. satisfied, an intense pulse of light of wavelength 6934Å is emitted. Continuous operation is not possible due to excessive heating of active medium.

1 For semiconductor, band gap is 0.9eV, what is wavelength of light emitted from it?

Answer

$$h\nu = 0.9 \text{ eV} = 1.442 \times 10^{-19} \quad \text{J}$$
$$\therefore \nu = \frac{1.442 \times 10^{-19}}{h}$$
$$= \frac{1.442 \times 10^{-19}}{6.626 \times 10^{-34}} = 2.176 \times 10^{14} \quad \text{Hz}$$
$$\therefore \lambda = c/\nu = 1.38 \quad \mu\text{m}$$

2 For a He-Ne laser with wavelength 632.8 nm and output power 3.14 mW how many photons are emitted in each minute in operation?

Answer

$$\text{Energy of each photon } = h\nu = \frac{hc}{\lambda}$$
$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{632.8^{-9}}$$
$$= 3.14 \times 10^{-19} \quad \text{J}$$

$$\therefore \text{ Energy emitted in 1 minute } = 3.14 \quad \text{mW} \times 60 = 0.1884 \text{ J}$$
$$\therefore \text{ Number of photons emitted } = \frac{0.1884}{3.14 \times 10^{-19}} = 6 \times 10^{17}$$

3 Calculate ratio of population of two states of ruby laser in which transition from one state to another is responsible for emission of photon of wavelength 6928 Å (Assuming that the transition temperature is 18°C)

Answer

Transition temperature (T) = 18°C = 291K

frequency of light emitted $\nu = \text{c}/\lambda = \dfrac{3 \times 10^{8}}{6928 \times 10^{-10}} = 4.3302 \times 10^{14}$ Hz

$$\frac{N_2}{N_1} = e \wedge (-h\nu/(kT))$$
$$= e \wedge (-6.626 \times 10^{-34} \times 4.3302 \times 10^{14}/(1.38 \times 10^{-23} \times 291))$$
$$= e^{-71.45} = 9.32 \times 10^{-32}$$

Fibre Optics

Fibre optics deals with transmission of information using light (which has minimal energy loss). Total internal reflection is the principle behind it.

Total internal reflection

Total internal reflection: It is a phenomenon which happens when light traveling from denser to lighter medium will reflect back to the same media if the incident angle is greater than a particular angle(known as critical angle of the media interface).

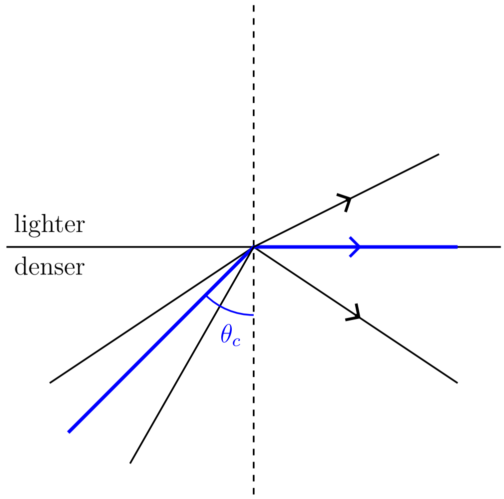
By snell's law:

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

At critical angle $(\theta_1 = \theta_c)$, $\theta_2 = 90$

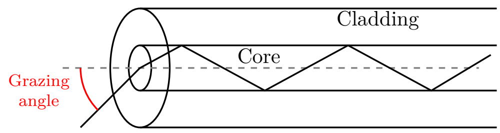
$$\therefore n_1 \sin(\theta_c) = n_2$$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

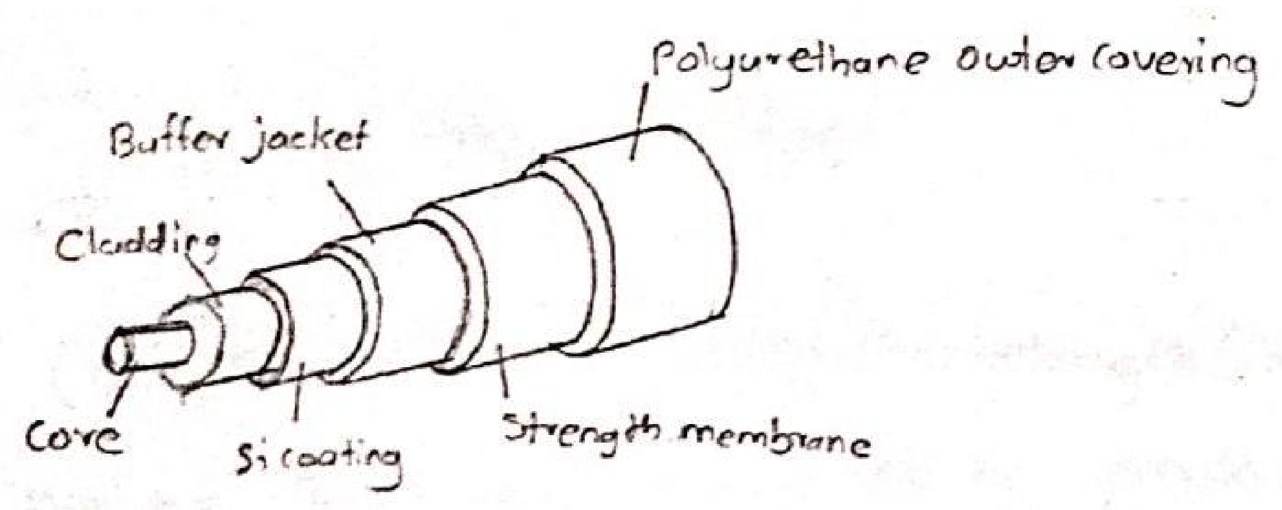


θ_c for various material interfaces:

- Water - air =49°
- Diamond - air 24°
- Gluss - air 42°



Structure of Fiber Optic Cable

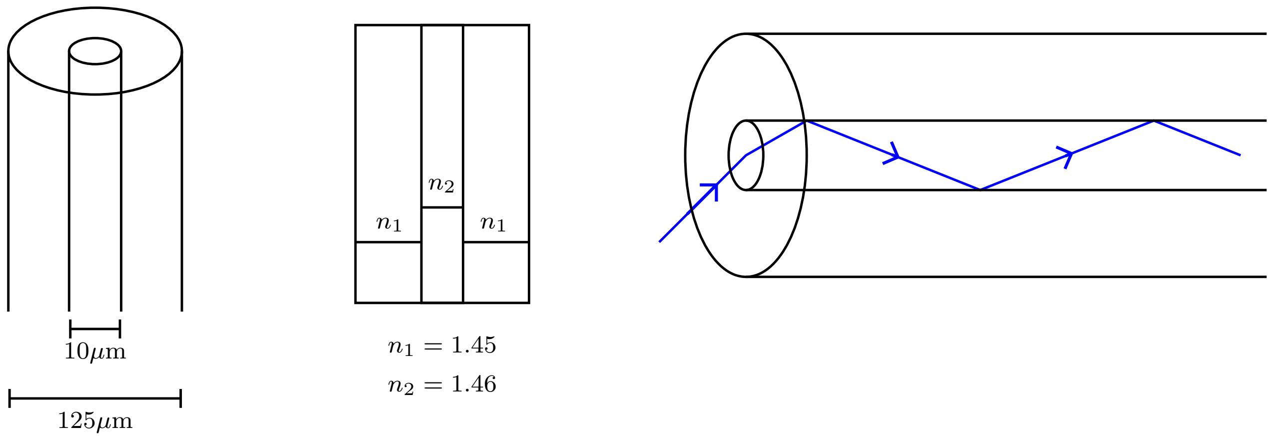


- Polyurethane: for mecharical crushing
- Butfor jacket : to prevent moistume
- Si-coarting : protect from dust & soratches

Classification of Fibre Optic Cables

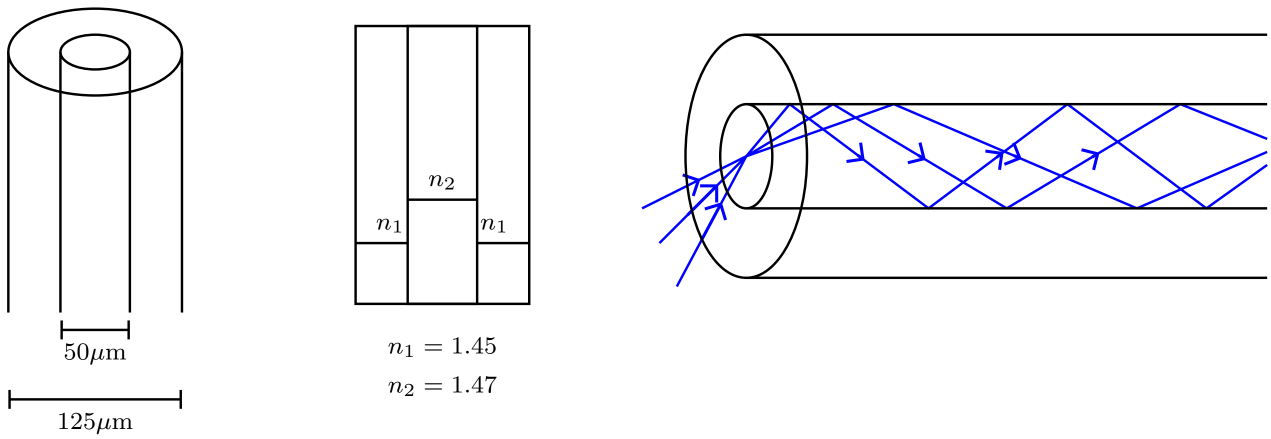
- Based on Index profile: step-index, graded index
- Based on made : single mode, multi mode.

Step Index Single Mode



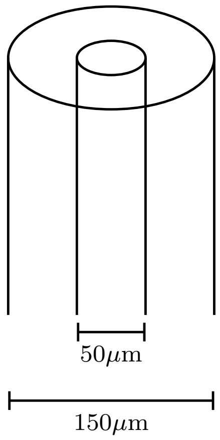
- Allows only one path for light
- Bandwidth distance product: > 3 GHzkm
- Splicing is difficut.

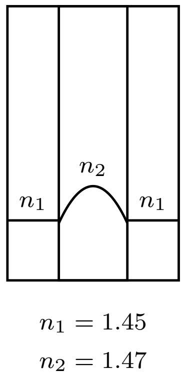
Step Index Multi Mode

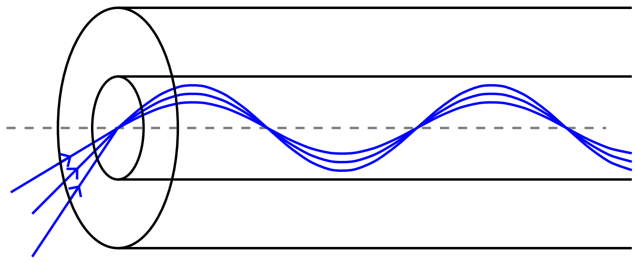


- Allows multiple path for light
- Easier to splice
- Bandwidth distance product (BwDP) ~ 300 MHzkm

Graded Index Multi Mode







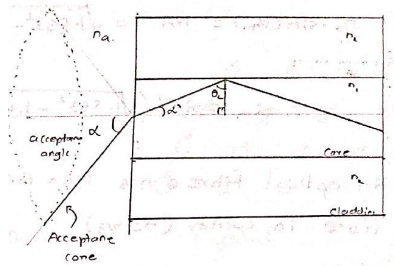
- Gradual decrease in refractive index.
- BWDP $\approx$ 3 GHzkm

Comparison between different Cables

Step Index	**Graded Index**
Refractive index changes sharply from core-cladding interface	Refractive index changes gradually from core-cladding interface
Refractive index profile looks like steps	Refractive index profile looks like parabola
Attenuation is more for multimode and less for single mode	Attenuation is usually less
Numerical Aperture is more for multimode and less for single mode	Numerical Aperture is usually less

Single Mode	**Multi Mode**
Only one mode can propagate through fibre	Allows a large number of modes
Smaller core diameter	Larger core diameter
Difference in refractive indices is very less	Difference in refractive indices is more compared to single mode
No multimodal dispersion	There is multimodal dispersion

Numerical Aperture (NA)



Numerical Aperture is defined as sine of largest angle that a incident ray should have for it to undergo TIR in core

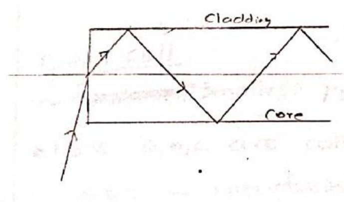
By snell's law: $n_a \sin(\alpha) = n_1 \sin(\alpha)$
For TIR, angle of incidence in the medium must be $> \theta_c$.
α at θ_c is called acceptance angle. (α_m)

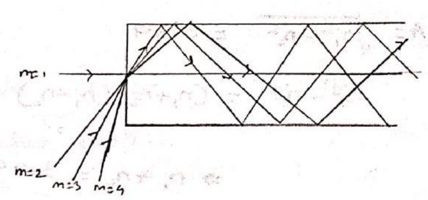
$$
\begin{aligned}
n_a \sin(\alpha_m) &= n_1 \sin(90 - \theta_c) \\
&= n_1 \cos(\theta) \\
&\equiv n_1 \sqrt{1 - \sin^2(\theta_c)} \\
&= n_1 \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} \\
&= \sqrt{n_1^2 - n_2^2}
\end{aligned}
$$

$$\boxed{\therefore NA = n_a \cdot \sin(\alpha_m) = \sqrt{n_1^2 - n_2^2}}$$

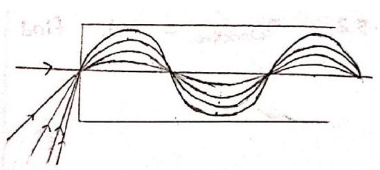
Propagation of light ray in Fibre Optic cable

Single mode & Multi mode





Graded index



⋆ Eventhough all waves have different velocities, they reach at same time, due to precise differences in distance which they travel.

1 An optical fibre has core refractive index of 1.55 & cladding refractive index of 1.50. Calculate Numerical Aperture

<u>**Answer**</u>

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.55^2 - 1.5^2} = 0.391$$

2 Calculate acceptance angle of given fibre, if core refractive index = 1.563 cladding refractive index =1.498

Answer
Assuming refractive index of air $(n_a) = 1$.

$$\begin{aligned} n_a \cdot \sin(\alpha_m) &= \sqrt{1.563^2 - 1.498^2} \\ \therefore \text{acceptance angle } (\alpha_m) &= \sin^{-1}\left(\sqrt{1.563^2 - 1.498^2}\right) = 26.49^\circ \end{aligned}$$

3 An optical fibre Numerical Aperture = 0.20, refractive index of cladding = 1.59. Find acceptance angle for fibre in water (n=4/3)

Answer Here $n_a = 4/3$

$$\begin{aligned} n_a \sin(\alpha_m) &= 0.20 \\ \therefore \alpha_m &= \sin^{-1}\left(\frac{3}{4} \times 0.20\right) = 8.63^\circ \end{aligned}$$

4 For a cable, Numerical Aperture = 0.39, and difference between refractive indices of core and cladding is 0.05. Find refractive index of core

Answer

$$\begin{aligned} NA = \sqrt{n_1^2 - n_2^2} &= 0.39 \\ n_1^2 - n_2^2 \equiv (n_1 + n_2)(n_1 - n_2) &= 0.1521 \end{aligned}$$

$$\Rightarrow n_1 + n_2 = \frac{0.1521}{n_1 - n_2} = 3.042$$

$$\begin{aligned} \therefore n_1 = n_{\text{core}} &= \frac{(n_1 + n_2) + (n_1 - n_2)}{2} \\ &= \frac{3.042 + 0.05}{2} = 1.546 \end{aligned}$$

5 For step index fibre, $n_{\text{core}} = 1.52, n_{\text{cladding}} = 1.41$. Find θ_c

Answer

$$\theta_c = \sin^{-1}\left(\frac{1.41}{1.52}\right) = 68.1^\circ$$