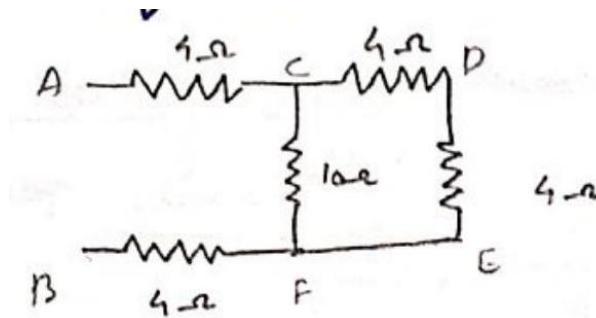


Elementary Concepts of Electric Circuits

1. Calculate equivalent resistance Across A & B



Answer

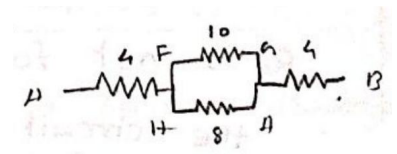
CDE contains resistors in series
 \therefore CDE combines to 8Ω .

FG & HA are in parallel.

$$\therefore \text{equivalent resistance} = \left(\frac{1}{10} + \frac{1}{8} \right)^{-1} = \frac{80}{18} = \frac{40}{9}\Omega$$

Remaining resistors are in series:

$$\therefore \text{equivalent resistance} = 4 + 4 + \frac{40}{9} = \frac{102}{9} = 11.3\Omega$$



Series Connection

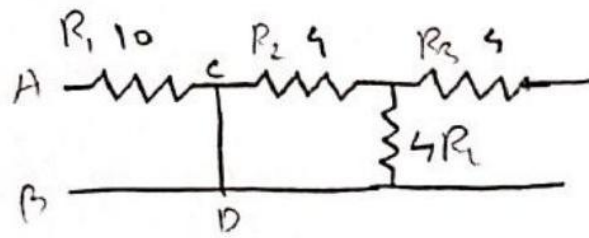
When two components are in series

- Current (I) flowing through them is same
- Have one common point & there are no intermediate element connected to common point
- (may) have potential drop

Parallel connection

- Have same potential across the component
- Two ends are joined by two common points with no element in between

2. Find equivalent resistance across A & B



Answer

R_3 is not fully connected. Hence it has no effect in the circuit and can be ignored.

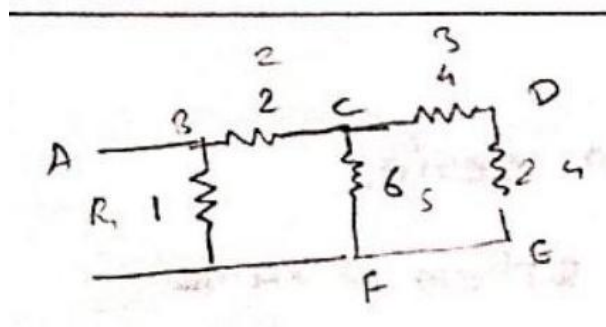
R_2 & $4R_1$ are in series.

\therefore Their effective resistance = 8Ω

There is no resistance in CD and whole current flow through CD (this condition is called short circuit).

\therefore net resistance is $R_1 = 10\Omega$

2. Find equivalent resistance across A & B



Answer

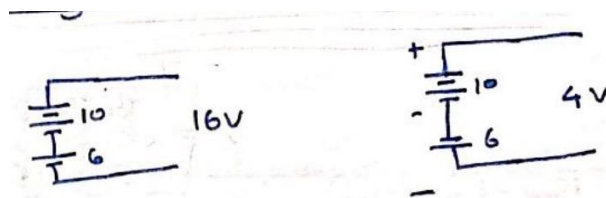
R_3 & R_4 are in series, \therefore their effective resistance is 6Ω

Net resistance in CDEF = $1 / \left(\frac{1}{6} + \frac{1}{6} \right) = 3\Omega = R'$

R_E & $R' = 2 + 3 = 5\Omega = R''$

\therefore Voltage across A & B = $R_1 \parallel R'' = \left(1 + \frac{1}{5} \right)^{-1} = \frac{5}{6}\Omega$

Voltage Source in series

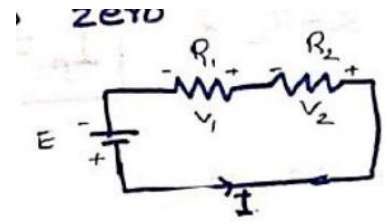


Kirchoff's voltage law: Algebraic sum of potential drop across a loop is zero

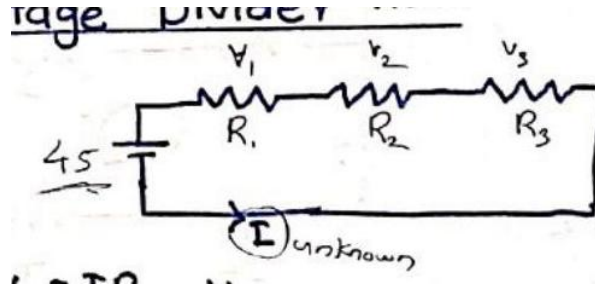
$$E = V_1 + V_2$$

$$= IR_1 + IR_2$$

Kirchoff's Current law: Algebraic sum of current flowing through junction is zero ($I_{in} = I_{out}$)



Voltage Divider Rule (VDR)



$$R_1 = 2K$$

$$R_2 = 5K$$

$$R_3 = 8K$$

$$V_n = IR_{n0} \quad V$$

$$\text{or } V_n = \frac{R_n \times V}{R_T}$$

R_T = total resistance

V = total voltage

$$\therefore V_1 = \underbrace{\frac{2k \times 45}{15k}}_{\text{less}} = 6V, V_2 = \frac{5k \times 45}{15k} = 15V, V_3 = \underbrace{\frac{8k \times 45}{15k}}_{\text{more}} = 24V$$

$$V_1 < V_2 \because R_1 < R_2 \Rightarrow V_{\text{drop}} \propto R_{\text{tm}}$$

*VDR is used in analysis of series circuit

Current divider rule

$$R_{\text{net}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots \right)^{-1}$$

$$\left. \begin{aligned} I_1 &= I \cdot R_1^{-1} \cdot \left(\frac{R_1 + R_2}{R_1 R_2} \right)^{-1} = I \cdot \frac{R_2}{R_1 + R_2} \\ I_2 &= I \cdot R_2^{-1} \cdot \left(\frac{R_1 + R_2}{R_1 R_2} \right)^{-1} = I \cdot \frac{R_1}{R_1 + R_2} \end{aligned} \right\} \quad \text{for 2 - branch circuits}$$

$$I_n = I \cdot \frac{R_{\text{net}}}{R_n}$$

Maxwell's Loop Current Method

1. Determine current in 4Ω branch.

Answer

In loop I

$$1 \times (I_1 - I_2) + 3 \times (I_1 + I_3) + 4 \times I_1 = 24$$

$$8I_1 - I_2 + 3I_3 = 24$$

In loop II

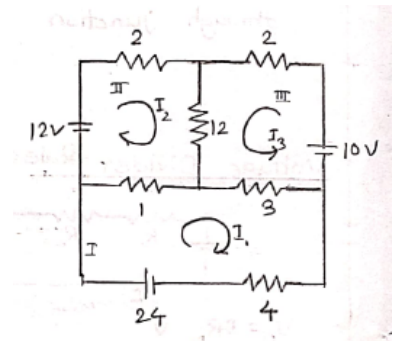
$$2 \cdot I_2 + (I_2 + I_3) 12 + 1 \times (I_2 - I_1) = 12$$

$$-I_1 + 15I_2 + 12I_3 = 12$$

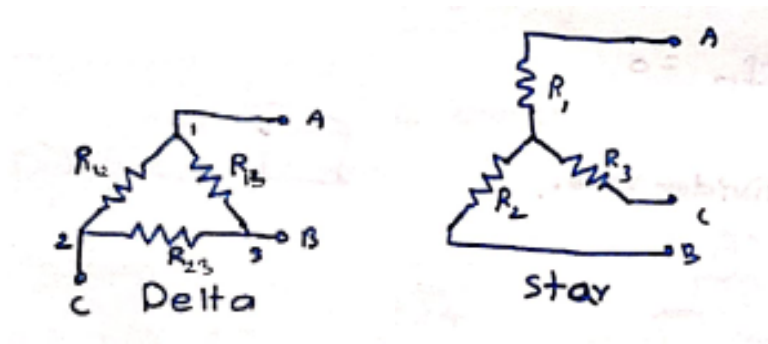
In loop III

$$2I_3 + 12(I_2 + I_3) + 3 \cdot (I_1 + I_3) = 10$$

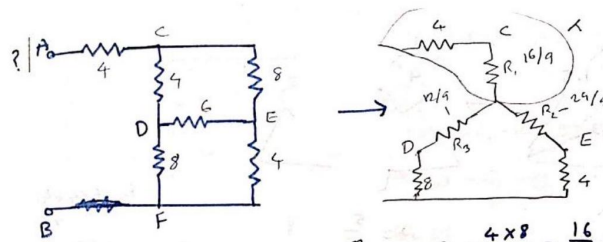
$$3I_1 + 12I_2 + 17I_3 = 10$$



Star Delta transformation



$$R_{AB} = R_{13} \parallel (R_{12} + R_{23}) = \frac{R_{13} (R_{12} + R_{23})}{R_{12} + R_{23} + R_{13}}$$



$$R_1 = \frac{4 \times 8}{4 + 8 + 6} = \frac{16}{9} \Omega$$

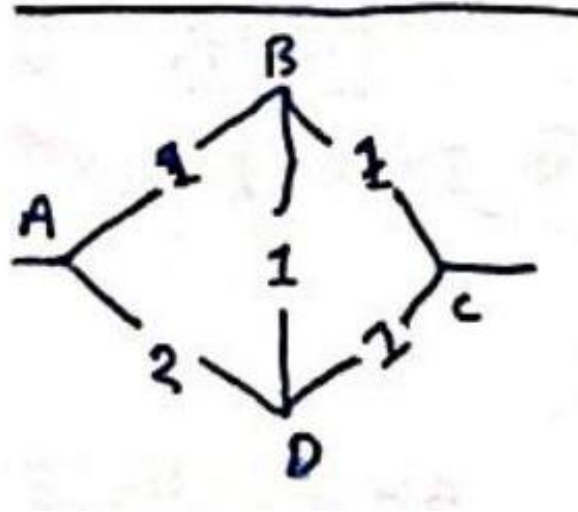
$$R_2 = \frac{6 \times 8}{18} = \frac{8}{3} \Omega = \frac{24}{9} \Omega$$

$$R_3 = \frac{6 \times 4}{18} = \frac{12}{9} \Omega = \frac{4}{3} \Omega$$

$$-4 - 16/9 - \left[\frac{\frac{24}{9} - 4}{\frac{12}{9} - 8} \right]$$

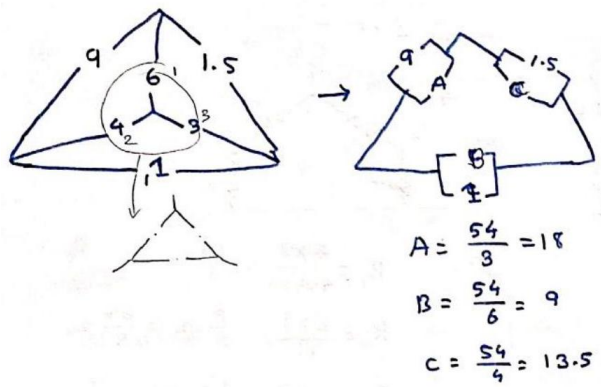
$$\begin{aligned} &\rightarrow 4 + \frac{16}{9} + \left(\frac{23}{20} + \frac{3}{28} \right)^{-1} = 4 + \frac{16}{9} + \frac{560}{144} \\ &= 47 + \frac{29}{3} \Omega \end{aligned}$$

$$\begin{aligned} R_1 + R_2 &= \frac{K(L+m)}{T}, R_1 + R_3 = \frac{M(K+K)}{T}, R_2 + R_3 = \frac{L(K+M)}{T} \\ R_1 &= \frac{C + (2) - (3)}{2} = \frac{KL + KM + KM + ML - KL - ML}{2T} = \frac{KM}{T} = \frac{R_{12} \times R_{13}}{R_{12} + R_{13} + R_{23}} \\ R_2 &= \frac{(3) - (2) + (0)}{2} = \frac{KL + ML - KM - ML + KM + KL}{2T} = \frac{KL}{T} = \frac{R_{12} \times R_{23}}{R_{12} + R_{13} + R_{23}} \\ R_3 &= \frac{(2) - (1)}{2} = \frac{KM + ML - KL - KM + KL + ML}{2T} = \frac{ML}{T} = \frac{R_{13} \times R_{23}}{R_{12} + R_{13} + R_{23}} \end{aligned}$$

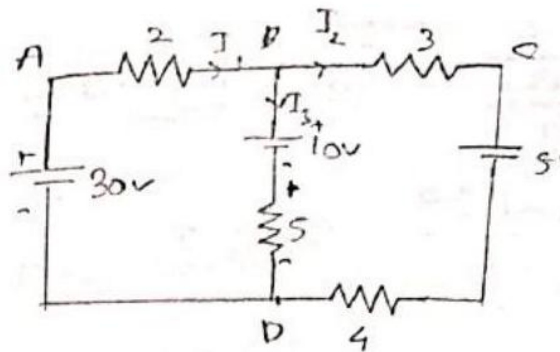


$$\begin{aligned} &\rightarrow \underbrace{1/2}_2 \rightarrow \underbrace{1/2}_{1/2} \\ A &= \frac{1 \times 2}{4} = \frac{1}{2} \\ B &= \frac{1}{2} \\ c &= \frac{1}{4} \\ &\rightarrow \frac{1}{2} + \left(\frac{2}{3} + \frac{5}{5} \right)^{-1} = \frac{1}{2} + \frac{15}{22} = \frac{52}{44} = \frac{26}{22} \\ &= \frac{13}{11} - 3 \end{aligned}$$

$$1 \rightarrow \rightarrow: R_{12} = \frac{R_\varphi}{R_3} \quad R_{23} = \frac{R_\psi}{R_p} \quad R_{13} = \frac{R_\psi}{R_2} \quad R_4 = R_1 R_2 + R_2 R_3 + R_1 R_3$$

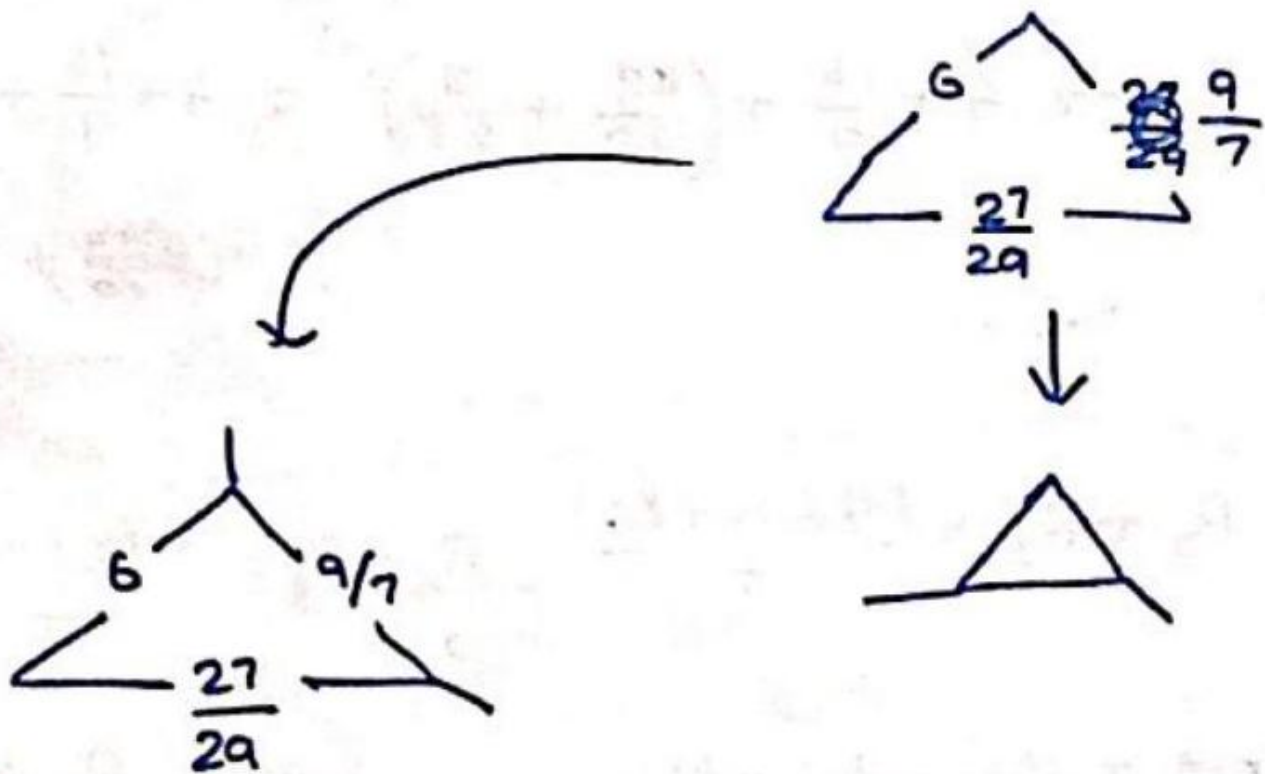


1.



$$R_y = 6 \times 4 + 4 \times 3 +$$

$$= 24 + 12 + 18 : 54 \therefore \text{reducable: } \left(\frac{1}{9} + \frac{1}{18}\right)^{-1} = 6, \left(\frac{1}{1.5} + \frac{1}{9}\right)^{-1} = \frac{9}{7}, \frac{1}{1} + \frac{1}{13.5} = \frac{27}{29}$$



$$= \frac{603}{556}$$

$$= \frac{225}{139}$$

mesh $\rightarrow \square$

Cromeristule

$$\begin{aligned} \therefore I_1 &= \frac{\Delta_1}{\Delta} = \frac{265}{59} \\ I_2 &= \frac{\Delta_2}{\Delta} = \frac{135}{59} \end{aligned}$$

By KCL : $I_3 = I_1 - I_2$

ABDA:

$$+30 - 2I_1 - 10 - 5I_3 = 0$$

$$\Rightarrow 20 - 7I_1 + 5I_2 + 20 = 0 \Rightarrow 7I_1 - 5I_2 = 20$$

BCDB

$$-3I_2 - 5 - 4I_2 + 8I_3 + 10 = 0$$

$$\Rightarrow 5I_1 - 12I_2 = -5$$

$$\therefore I_1 = \frac{265}{59}, \quad I_2 = \frac{135}{59}$$

$$\begin{bmatrix} 7 & -5 \\ 5 & -12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 20 \\ -5 \end{bmatrix}$$

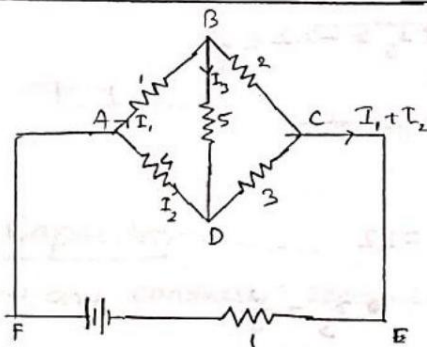
$$\Delta = \begin{vmatrix} 7 & -5 \\ 5 & -12 \end{vmatrix} = -59$$

Replac C_n by R

$$n^m Uv = \frac{U_n}{\Delta}$$

$$\Delta_1 = \begin{vmatrix} 20 & -5 \\ -5 & -12 \end{vmatrix} = -265$$

$$\Delta_2 = \begin{vmatrix} 7 & 20 \\ 5 & -5 \end{vmatrix} = -135$$



- 1) Calculate magnitude & direction of I through 5Ω resistor
- 2) Resistance b/w A & C.

a) ΔBDA : b) $BCDB$

$$I_1 + 5I_3 - 4I_2 = 0 \quad 2(I_1 - I_3) - 3(I_2 + I_3) - 5I_3 = 0$$

$$\Rightarrow 2I_1 - 3I_2 - 10I_3 = 0$$

8. *ABCEFA*

$$I_1 + 2(I_1 - I_3) + I_1 + I_2 - 4 = 0$$

$$4I_1 - I_2 - 2I_3 - 4 = 0$$

$$\Rightarrow I_1 = \frac{31}{28} \quad I_2 = -\frac{1}{28} - 0.34$$

$$I_3 = \frac{13}{56} \approx 0.232 \text{ A} - 0.087 \therefore 1) \text{ from } B \rightarrow D$$

$$2) R_{\text{net}} = \frac{V}{I} =$$

$$\frac{4}{\frac{15}{14} + 1} = \frac{56}{29} \approx 1.931 \text{ A}\Omega$$

$$\begin{bmatrix} 1 & 5 & 0 \\ 2 & -3 & -10 \\ 4 & -1 & -2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ 5 \end{bmatrix}$$

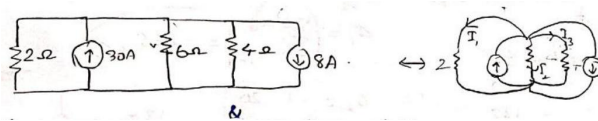
Handwritten notes: $4 - 1 \times (I_1 + I_2) = 4 - 1 \cdot 29 \approx -2.87$

$$\begin{bmatrix} 1 & 5 & 0 \\ 2 & -3 & 0 \\ 4 & -1 & 4 \end{bmatrix} \quad \Delta_3 = -52 \quad \begin{vmatrix} 0 & 5 & -4 \\ 0 & -3 & -10 \\ 4 & -1 & -2 \end{vmatrix} \quad \Delta_1 = \pm 248$$

$$\begin{vmatrix} 1 & 0 & -4 \\ 2 & 0 & -10 \\ 4 & 4 & -2 \end{vmatrix} \quad 0_2 = 8$$

$$I_1 = \frac{\Delta_1}{\Delta_4} = +\frac{31}{28}, \quad I_2 = -\frac{1}{28}$$

$$I_3 = \frac{13}{56}$$



1. Find polarity & magnitude of v

$$I_1 + I_2 + I_3 + 8 = 0 \Rightarrow I_1 + \frac{V}{2} + I_3 = 22$$

$$I_2 = \frac{V}{6}, \quad I_3 = \frac{V}{4}$$

$$\frac{V}{2} + \frac{U}{6} + \frac{U}{4} = 22$$

$$24V + 8V + 12V = 22 \times 48$$

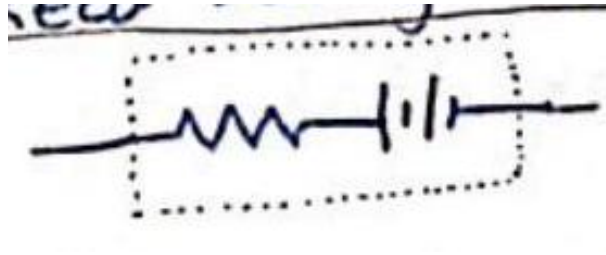
$$\Rightarrow U = 24V$$

Ideal voltage source

→ zero internal resistance

→ Supply constr. voltage at all currents

Real voltage source



- voltage source with very low internal resistance. can be treated as constant voltage source

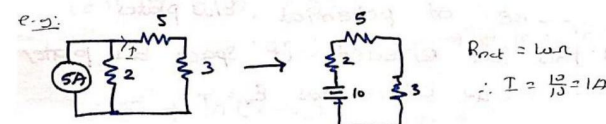
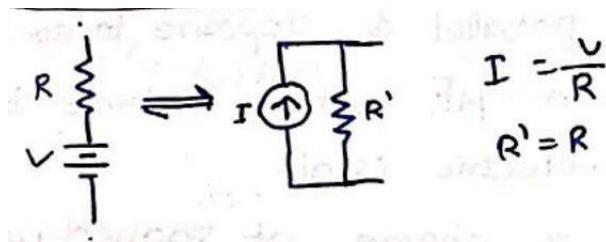
Ideal Current source (—⊕—)

Real current Source.

→ ∞ internal resistance winy? M Θ

→ supply constr. Current at all loads


Source conversion



Capacitor

☒ any conductor separated by insulator (dielectric)

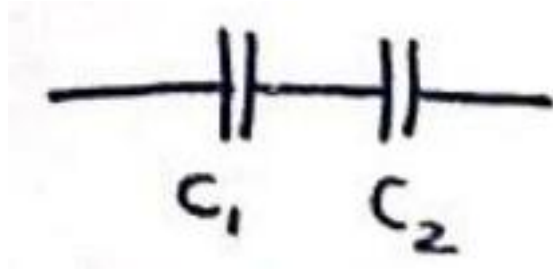
Capacitance: Ability of capacitor to store energy. (change → Ublwiwn) Dielectrics: air, mica, waxed paper, ceramic, electrolyte


$$C = \frac{\epsilon A}{d}$$

ϵ = dielectric const
= permittivity

$$\epsilon = \text{permittivity} = \epsilon_0 \cdot \epsilon_r.$$

$$\text{Energy stored: } \frac{1}{2} C v^2 = \frac{1}{2} Q v = \frac{1}{2} Q^2 / C$$



$$C = \left(\frac{1}{C_1} + \frac{1}{C_2} \right)^{-1}$$

$$V_1 = \text{pd}(C_1)$$

$$V_2 = \text{pd}(C_2)$$

$$Q_1 = Q_2 = Q$$

$$\therefore V = V_1 + V_2$$

$$\frac{Q}{C} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$\Rightarrow C = \left(\frac{1}{C_1} + \frac{1}{C_2} \right)^{-1}$$

1. A capacitor consists of two similar square Al. plates each $10 \text{ cm} \times 10 \text{ cm}$ bounded parallel & opposite to each other. what is their capacitance in MF when distance between them is 1 cm & dielectric is air

ii) If capacitor is given a charge of $500 \mu\text{C}$ what will be difference of potential between plates

iii) How will this be effected if space between plates is filled with wax which has $\epsilon_r = 4$
A.

$$C = \frac{\epsilon_0 \epsilon_r A}{d} = \frac{\epsilon_0 \times \epsilon_r \times (10 \times 10) \times 10^{-4}}{10^{-2}} = 8.85 \times 10^{-12} \text{ F}$$

$$= 8.85 \times 10^{-1} \mu\text{F}$$

$$\text{iii) } V = \frac{Q}{C} = \frac{500 \mu\text{C}}{8.85 \times 10^{-1} \mu\text{F}} = 56.47 \text{ V}$$

$$\text{iii) } v = \frac{Q}{C'} = \frac{CQ}{4C} = 19.12 \text{ V At } t = RC :$$

$$v_c = v (1 - e^{-Rt/R_c})$$

$$= 0.632.v$$

$$I_0 = I_m \cdot e^{-Rt/R_c}$$

$$= 0.368 I_m$$

RC : time constant:: Time at which the voltage across $G(\tau, \lambda)$ the capacitor reaches 63.2% of steady state T . voltage / current reaches 36.8% of initial value.

If $t = 2RC$:

$$v_c = v (1 - e^{-2}) = 0.865.V$$

$$t = 2RC, v_c = 0.993 \text{ V}$$

Rate of Rise of voltage.

$$v = RC \cdot \frac{dv_c}{dt} + v_c$$

at $t = 0, v_c = 0$

$$= 0, v_c = 0$$

$$\therefore v = RC \cdot \frac{dv_c}{dt} \Rightarrow \frac{dv_c}{dt} = \frac{v}{RC} \quad \begin{array}{l} \text{initial rate of-rise in} \\ \text{voltage (across capacitor)} \end{array}$$

1.

A $2\mu\text{f}$ capacitor is connected by closing a switch to a supply of 100 V through a $1\text{M}\Omega$ series resistance. Calculate.

i) Time constant ii) Initial charging current iii) $\frac{dv_c}{dt}|_{t=0}$ iv) Voltage across capacitor & s after the switch has been closed

v) Time taken for capacitor to be fully charged

$$\text{ii) } I_m = \frac{V}{R} = \frac{100}{10^6} = 10^{-4} \text{ A} = 109 \text{ A} \quad \text{iii) } \frac{dv_c}{dt}|_{t=0} = \frac{v}{RC} = \frac{100}{2} = 50 \text{ V/s}$$

$$\text{iv) } v_c = v(1 - e^{-t/\tau})$$

$$= 100(1 - e^{-6/2}) = 95.02 \text{ V}$$

$$\text{v) } \infty \text{ Q? } I = cv \Rightarrow cv(1 - e^{-t/\tau}) = c \cdot v \text{ (to fully charge)}$$

$$\Rightarrow 1 - e^{-t/\tau} = 1$$

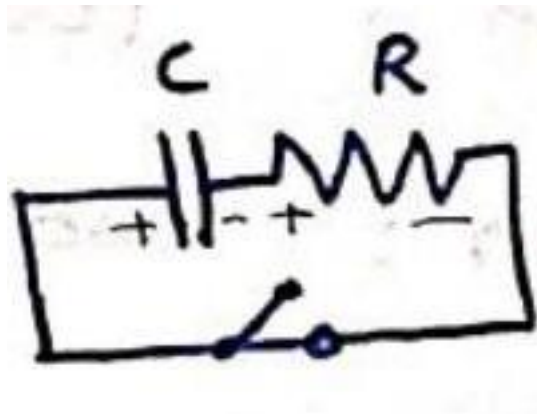
$$-e^{-t/\tau} = 0 \Rightarrow t = \infty$$

llor take 5 T = 105

Discharging of Capacitor

At $t = 0 : V_c = V$

$$I = 0$$



$t > 0 :$

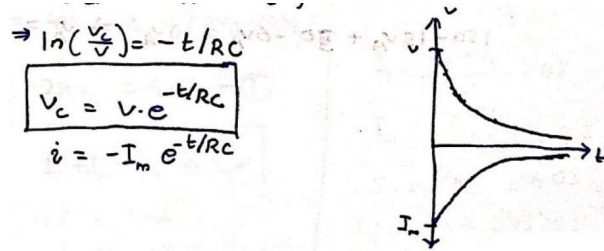
Applying KvL:

$$\begin{aligned}
 v_c + IR &= 0 \Rightarrow v_c + Rc \cdot \frac{dv_c}{dt} = 0 \\
 \Rightarrow \frac{dv_c}{v_c} &= -\frac{dt}{Rc} \\
 \Rightarrow \int \frac{dv_c}{v_c} &= -\int \frac{dt}{Rc} \\
 &= \ln(v_c) = -t/Rc + k
 \end{aligned}$$

to find $k, t = 0, v_c = v$

$$\ln(x_0) = k$$

$$\therefore (1) \equiv \ln(v_c) = -t/Rc + \ln(v)$$



2. A cable 10 km long and of capacitance $2.5\mu\text{F}$ discharges through its insulation resistance of $50\text{M}\Omega$. By what percentage the voltage would have fallen 1, 2, & 5 mins. respectively. After disconnection from bus-bars A) $V_c = V(1 - e^{-t/\tau})$ $T = RC = 50 \times 10^6 \times 10^{-6} \times 2.5 = 1255$ at 1min: $\frac{v - v_2}{v} = 100(1 - e^{-60/125}) = 37.12\%$ at 2 min: $100(1 - e^{-120/125}) = 61.71\%$ at 5 min: $100(1 - e^{-300/125}) = 90.93\%$ Nodal Analysis Potential at A: By $K \subset L: I_3 = I_1 + I_2$

$$\begin{aligned}
 I_1 &= \frac{V_{2A} - 0}{2} = \frac{V_B - V_A}{2Q} = \frac{10 - V_A}{2} \\
 I_2 &= \frac{5 - V_A}{5}, I_3 = \frac{V_A - 0}{3} \\
 \therefore 5 - \frac{V_A}{2} + 1 - \frac{V_A}{5} &= \frac{V_A}{3} \\
 150 - 15V_A + 30 - 6V_A &= 10V_A \Rightarrow V_A = \frac{-280}{31} = 2\frac{180}{31}.V
 \end{aligned}$$

at A:

$$\begin{aligned}
 10 &= I_1 + I_2 + I_3 \\
 10 &= \frac{V_A - V_C}{5} + \frac{V_A - V_B}{3} + \frac{V_A}{2} \\
 300 &= 6V_A - 6V_C + 10V_A - 10V_B + 15V_A
 \end{aligned}$$

10A

$$31v_A - 10v_B - 6v_C = 300$$

At B:

$$I_2 + I_4 + I_5 = 0$$

$$\frac{V_A - V_B}{3} \pm \frac{V_B}{5} + \frac{V_B - V_C}{1} = 0$$

$$5V_A - 5V_B \pm 3V_B + 15V_B - 15V_C = 0$$

$$-5V_A + 83V_B + 15V_C = 0 \quad (2)$$

At c:

$$2 + I_6 = I_1 + I_5$$

$$(2. \quad 1$$

$$5. \quad -1$$

$$3.8$$

$$2 + \frac{v_C}{4} = \frac{v_A - v_C}{5} + \frac{v_B - v_C}{1}$$

$$\Rightarrow 40 + 5v_c = 4v_A - 4v_C + 20v_B - 20v_c$$

$$\Rightarrow 4v_A + 20v_B - 29v_C = 40$$

(also true, but ∞ into. About ign)

Magnetic field lines

- * never intersect
- * always form closed path.

Magnetic flux (Φ) [Wb]

1 Wb = 10^8 lines of force

L flux density (\vec{B})

$$\vec{B} = \frac{\Phi}{A}$$

Permeability (μ) ~ measure of magnetization produced in a material in response to an applied field.

$\mu = \mu_0 \cdot \gamma$, $\gamma = 1$ (air/vacuum), $\gamma = 2000$ for Fe

$\mu_0 = 4\pi \times 10^{-7}$

flux density [Wb/m²]

1T = 1Wb/m²

$\approx 10^4$ Gauss

Diamagnetism

- * Cu, Bi, ...
- * arranges magnetic dipoles in opposite direction. of applied field.

Paramagnetism

- * Al, Sb
- * Arranges magnetic dipoles in applied field

Ferrimagnetism

- * Fe, ...
- * Arranges strongly in applied direction.

Magnetic circuits

Electric circuits

Here, there's resistor:

$$R = p_2$$

~ Reluctance

eryg's of anducto

$$S = \frac{2}{\mu a} \text{ y area of cross section of mag circuit}$$

$$\rightarrow \phi = \frac{MMF}{s} = \frac{NI}{s1} \cdot \mu \cdot a \cdot \mu.$$

$$= \frac{M \cdot NI a}{2}$$

→ Turns deroun n ? material:

→ No drop in power

due to flux

→ Permeance

→ Permeability

→ doesn't flow actually

→ No magnetic insulator ?

→ Permeability depends on the max. flux density (B_{max}) and it isn't constant

- Magnetic field intensity (H)

$$H = \frac{B}{\mu} = \frac{NI}{2}$$

$$\text{then } \phi = \frac{\mu \cdot NI a}{2} \Rightarrow B = \frac{\mu NI}{2}$$

$$\text{or } 1t = \frac{\omega f}{y}$$

$\therefore MMF = H \cdot 2 \rightarrow$ opposition to I

$$\rightarrow I = \frac{V}{R}$$

→ factors affecting the

conductors

→ Potential drop is

present. into resistance.

→ conductance.

→ conductivity

→ Power flow

→ electric insulators.

→ Resistivity of a material is almost constant except for slight change due to change in temp.

1. An Iron ring of circular cross sectional area: of 3 cm^2 and mean diameter of 20 cm is wound with a 500 turns of wire and carries a current of 2.09 A. to produce the magnetic flux of 0.5 mWb in the ring. Determine the permeability of the material

A) Given:

$$a = 3 \text{ cm}^2 \equiv 3 \times 10^{-9} \text{ m}^2$$

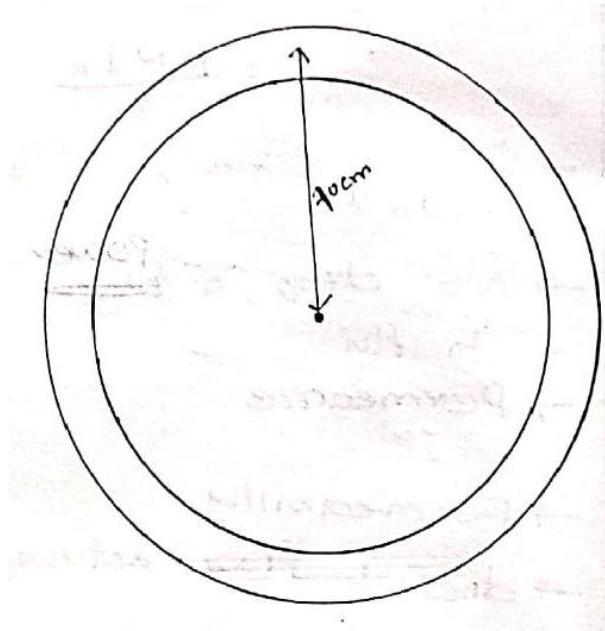
$$d = 20 \text{ cm}, N = 500$$

$$I = 2.09 \text{ A}, \phi = 0.5 \times 10^{-3} \text{ Wb}$$

$$\text{reluctance } s = \frac{2}{\mu a}$$

$$Z = \text{mean length} = \pi \cdot d$$

$$= 0.2 \pi \text{ m}$$



$$\therefore s = \frac{l}{\mu \cdot a}$$

$$\phi = \frac{mmf}{s} = \frac{NI}{s} \Rightarrow 0.5 \times 10^{-3} = \frac{500 \times 2.09}{s}$$

$$\Rightarrow s = 500 \times 2.09 \times 2 \times 10^3$$

$$= 2090000$$

$$\therefore \mu = \frac{l}{s \cdot a} = \frac{0.2\pi}{2090000 \times 63 \times 10^{-4}} = 0.0010021$$

$$\mu_r = \frac{\mu}{4\pi \times 10^{-7}} = 797$$

Linear circuit: linear rel. between voltage & Current output ($V \& I$) are linear function of input ($V \& I$), $\sim n, -A$

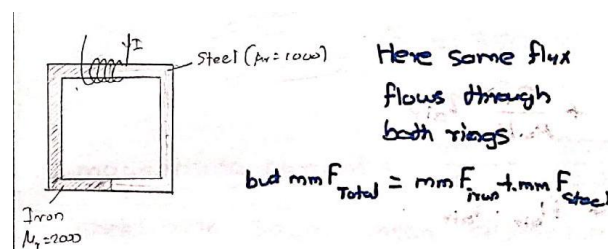
non-linear: rel between V & I is a non-linear function diodes, transistors, transformer whose core is saturated

Unilateral: allows flow of current in one direction only bilateral: can flow in both directions

Active element: require external power to operate produce energy, semiconductor,

- passive element: doesn't generate but dissipates, stores/releases it. R, C, I

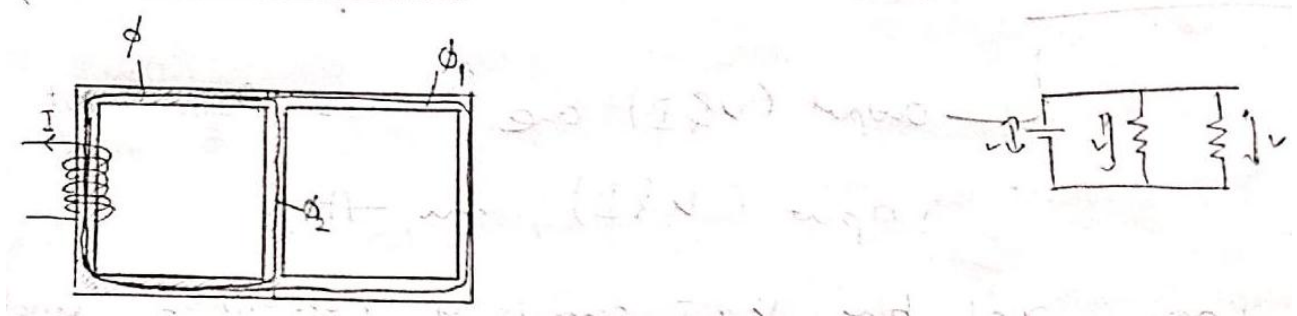
Series magnetic circuits



$$\text{but } mmF_{\text{Total}} = mmF_{\text{item}} + mmF_{\text{steel}}$$

Just like in series e^c circuits. $v = v_1 + v_2$

Parallel circuits



$$\begin{aligned} mmf_T &= mmf_\phi + mmf_\phi \\ &= mmf_\phi + mmf_{\phi_2} \end{aligned}$$

$$mmf_{\phi_1} = mmf_{\phi_2}$$

$$\phi = \phi_1 + \phi_2$$

$$\begin{aligned} mmF_T &= mmf_{air} + mmf_{iron} = \phi \cdot \text{reductome} \\ &= \phi \cdot s_{iron} + \phi \cdot s_{air} \quad s = \frac{2}{\mu a} \end{aligned}$$

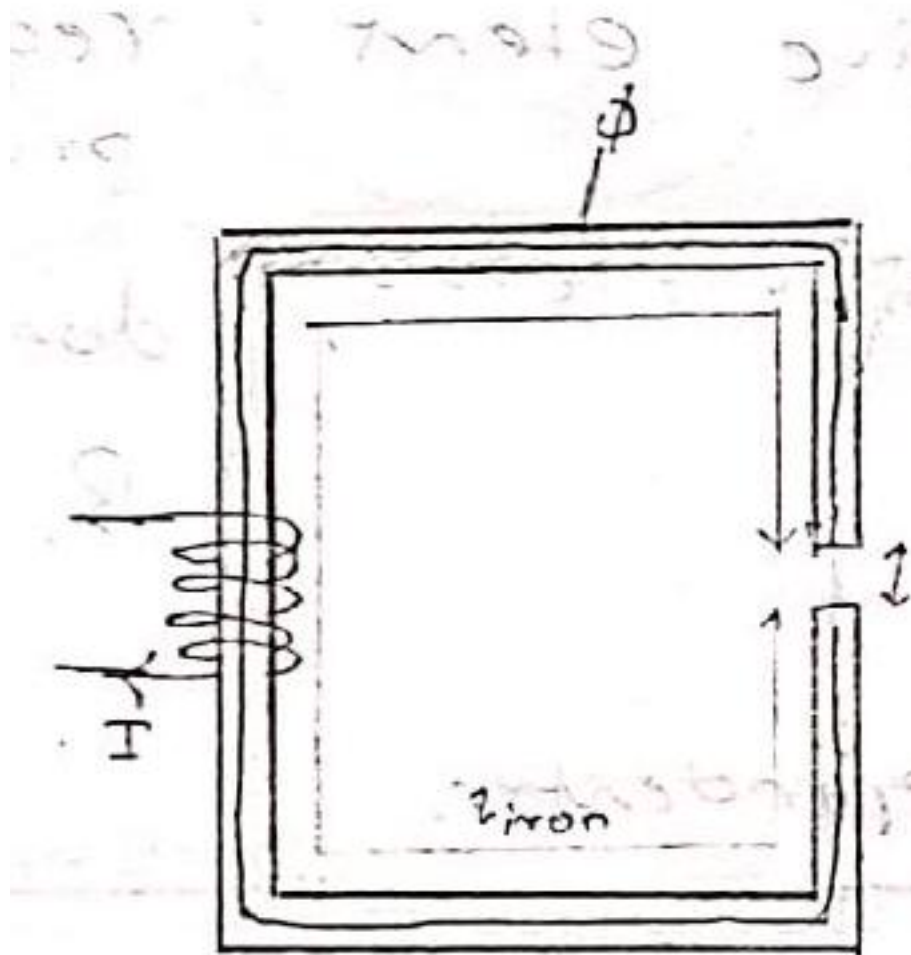
$$= \frac{\phi}{\mu_0 a} \left(\frac{\tau_{iron}}{\mu_{iron}} + \frac{\tau_{air}}{\mu_{air}} \right)$$

$$= \frac{B}{\mu_0} (\dots)$$

$$= \frac{B}{\mu_0} \cdot \tau_{iron} + \frac{B}{\mu_0 \cdot \mu_{air}} \tau_{air}$$

$$(A \cdot \text{tummy mm} = H_{iron} \cdot \tau_{iron} + H_{air} \cdot \tau_{air})$$

leakage flux: flow not flowing through desired



magnetic path

Useful flux: out through air gap/airflut.

- leakage factor (λ) = $\frac{\mu_{\text{setal}}}{\mu_{\text{total}}} H_{\text{us}}$
- Fringing: Bulging of flux cot air gap

I111). A cast steel a electromagnet a hos an air gap length of 3 mm and an iron path length of 40 cm. find number of ampere turns necessary to produce a flux density of 0.7 Wb/m^2 in the gap. neglect leakage A & fringing. Assume A-T for airgap to be 70% of tome AI

$$A) AT_{\text{air}} = \phi \cdot S_{\text{air}} = \phi \cdot \frac{2}{\mu_0 \cdot \mu_r a} = B_{\mu}^2 = \frac{0.7 \times 3 \times 10^{-3}}{4 \times 3.14 \times 10^{-7}} = 1072 A.$$

$$A.T T_{\text{total}} = \frac{10}{7} \times 1672 = 2388 \text{ ATT}$$

ϕ - same here

- An iron ring of crosse sectional area 6 cm^2 is wound with a wire of 100 turns. and has a saw-cut of 2 mm, calculate the magnetising current required to produce a flux of 0.1 mW if mean length of magnetic path is 30 cm. and $\mu_{0, \text{non}} \mu_0 \cdot 470$
A)

$$\begin{aligned}
 mmf &= \phi \cdot S_{\text{iron}} + \phi \cdot s_{\text{air}} = \phi \cdot \left(\frac{\tau_{\text{air}}}{\mu_0 \cdot \mu_r \cdot a} + \frac{\eta_{\text{iron}}}{\mu_0 \mu_{\text{inn}} a} \right) \\
 &= 0.1 \times 10^{-3} \times \left(\frac{2 \times 10^{-3}}{4\pi \times 10^{-1} \times 6 \times 10^{-4}} + \frac{0.3}{4\pi \times 10^{-7} \cdot 470 \times 10_0^4} \right) \\
 &\approx 350 \\
 &= NI
 \end{aligned}$$

$$\therefore \text{magnetising current} = \frac{mmf}{N} = \frac{350}{100} = 3.5 \text{ A}$$

- A steel ring 30 cm mean diameter and of circular cross section 2 cm in diameter, and an sp ind long, it is s wound uniformly with 600 turns of wire carrying current of 2.5 A, find

a) total mmf

b) total reluctance

c) flux

A neglect magnetic leakage & mon path takes 40% of total mme.

Handwritten calculations for a magnetic circuit problem. The left page shows calculations for total mmf, flux density, and flux. The right page shows calculations for temperature coefficient of resistivity and resistance of two coils in series.

Left page calculations:

- a) total mmf = $NI = 1500 \text{ AT}$
- b) $S = \frac{\pi d^2}{4}$
- area of cross section (π) = $\left(\frac{2 \times 10^{-2}}{2}\right)^2 \cdot \pi = 3.14 \times 10^{-4} \text{ m}^2$
- $mmf_{\text{iron}} = 4\pi \cdot 10^{-7} \cdot N I = 600 \text{ AT}$
- $mmf_{\text{air}} = 1500 - 600 = 900 \text{ AT}$
- $S_{\text{air}} = \frac{1}{\mu_0 \mu_r} = \frac{1}{4\pi \times 10^{-7} \times 1} = 7.96 \times 10^5$
- $\therefore S_{\text{iron}} = \frac{600}{7.96 \times 10^5} = 7.54 \times 10^{-5}$
- $\therefore S_{\text{total}} = 7.54 \times 10^{-5} + 900 \times 7.96 \times 10^5 = 7.18 \times 10^{-4}$
- b) $S_{\text{total}} = 4.22 \times 10^4$

Right page calculations:

- a) Total flux = $\frac{mmf}{S} = 0.36 \text{ mWb}$
- Temperature Coefficient of resistivity: $R = R_0 (1 + \alpha \Delta T)$
- 1) A coil has resistance 18Ω when mean temperature is 20°C , and 20Ω when $T_{\text{mean}} = 50^\circ\text{C}$. Find mean of rise in temperature when resistance is 21Ω and surrounding temperature is 15°C .
- A) $18 = R_0 (1 + \alpha_0 \times 20)$
- $20 = R_0 (1 + \alpha_0 \times 50)$
- $21 = R_0 (1 + \alpha_0 \times \Delta T)$
- $\therefore \text{Temperature rise} = \Delta T - T_{\text{surround}} = 55 - 15 = 40^\circ\text{C}$
- 2) Two coils connected in series have resistances 600Ω and 300Ω and $\alpha = 0.0045 / ^\circ\text{C}$ @ 20°C respectively. Find resistance of combination when temperature is 50°C . Also find effective coefficient.
- A) $R_{20} = R_0 (1 + \alpha_0 \Delta T)$
- $R_{50} = R_0 (1 + \alpha_0 \times 30)$
- $R_{20} = 600 \Omega$
- $R_{50} = 600 (1 + 0.0045 \times 30) = 681 \Omega$
- $R_{30} = 300 (1 + 0.0045 \times 30) = 336 \Omega$
- $\therefore R_{\text{total}} = R_{20} + R_{30} = 1017 \Omega$
- $R_{\text{total}} = R_0 (1 + \alpha_0 \times 30)$
- $1017 = 900 (1 + \alpha_0 \times 30) \Rightarrow \alpha_0 = 1/500 \approx 0.002$

Kirchhoff's laws

Voltage loop rule: algebraic sum of potential differences 1 directed (voltage) around any closed loop is zero.

$$\sum_{k=1}^n v_k = 0$$

Current law/junction rule: Algebraic sum of currents in a network of conductors meeting at a point is zero.

$$\sum_{k=1}^n I_k = 0 \quad \text{or} \quad I_{\text{entering}} = I_{\text{leaving}}$$

n = no. of branches

8 III

A cast steel magnetic structure made of a bar of (rose) section, 2×2 cm is shown in fig. Determine Current That the 500 tum magnetising coil. on the left limb should carry so that a flux of 2mWb is produced in the right limb. lake $\mu_{r_{\text{steel}}} = 600$, neglect leakage

A) mmf(AT) = NI

$$\begin{aligned}
 AT_{BD} &= AT_{BCD}. \\
 \therefore AT_{\text{total}} &= AT_{ABDA} + AT_{BD} = AT_{DEAB} + AT_{BCD} \\
 &= \phi S_{BEAB} + \phi_2 \cdot S_{BCD} \\
 \phi_2 &= 2 \text{ mW (given)} \\
 \phi_1 S_{BD} &= \phi_2 S_{BCD} \\
 \therefore \phi_1 \cdot \frac{l_{\infty}}{\mu_0 \mu_7 a_0} &= \phi_2 \cdot \frac{l_{BCP}}{\mu_0 H_2 a} \\
 \eta_{BD} &= 15 \text{ cm} \\
 \therefore \phi_1 \cdot \phi_2 \cdot \frac{e_{BCD}}{e_{BD}} \\
 \tau_{BCD} &= 25 \text{ cm} \\
 a &= 2 \times 2 \times 10^{-4} \text{ m}^2 = 4 \times 10^{-4} \text{ m}^2 \\
 &= \phi_2 \cdot \frac{RS}{15} \\
 &= \phi_2 \cdot \frac{(0)}{3} = \frac{10}{3} \times 10^{-3} \text{ Wm} \\
 \therefore \phi &= \phi_1 + \phi_2 = 2 + \frac{10}{3} = \frac{16}{3} \text{ mW} \\
 \therefore AT_{\text{total}} &= \phi \cdot S_{D_{GAB}} + \phi_2 S_{BCD} \\
 &= \frac{16}{3} \cdot \frac{2_{DEAB}}{\mu_0 \mu_4 \cdot a} + \frac{10}{3} \cdot \frac{2R_{B\infty}}{\mu_0 \mu_2 a} \\
 \left(\frac{16}{2} \cdot 25 + \frac{10}{3} \times .15 \right) &\frac{10^{-3}}{600 \times 4 \times 10^{-4} \times 4\pi \times 10^{-7}} = 6078 \text{ A} \cdot T \\
 AT_{\text{total}} &= N \cdot I \\
 \therefore I &= \frac{AT_{\text{tots}}}{N} = \frac{6078.8}{500} = 12.16 \text{ A}
 \end{aligned}$$

Electromagnetic Induction

The phenomenon of producing an emf in a conductor or coil whenever there's a magnetic flux linked with the coil or conductor is known as electromagnetic induction.

Faraday's laws of electromagnetic induction

1. Whenever there is a change in the flux linked with the coil or conductor, an emf is induced (known as induced emf).

if the coil is closed, a current will flow

2. magnitude of induced emf \propto rate of change is of flux linkage

flux linkage = $N\phi$.

change in flux linkage = $N(\phi_2 - \phi_1)$

According to Faraday's 2nd law:

$$e \propto N (\phi_2 - \phi_1) / t$$

$$e = k \cdot (\dots) \quad k = 1 \quad \text{in SI}$$

$$e = \frac{N\phi_2 - N\phi_1}{t}$$

or $e = N \cdot \frac{d\phi}{dt}$ in diff. form

Len 2's law

Induced current will oppose its cause I it will produce a magnetic flux which is opposite to the flux producing a induced current

$$e = -N \frac{d\phi}{dt}$$

Self Inductance

property of a coil which opposes the change in current flowing through it

list method: $e = n \cdot \frac{d\phi}{dt} \cdot \frac{d\phi}{dt} \propto \frac{d}{dt}$

change in I . induces change in flux producing self induced emf

$$e = L \frac{dI}{dt}, L = e / \frac{dI}{dt} \quad L = \text{self inductance}$$

2nd method:

$$e = \frac{d}{dt}(N\phi) = \frac{d}{dt}(LI)$$

$$\Rightarrow L = \frac{N\phi}{I} \quad [\text{wb.turns / A}]$$

III method:

$$L = \frac{N\phi}{\Delta I} \Rightarrow \phi = \frac{mmf}{s} = \frac{\phi I}{2t} \frac{NI}{\mu a} = \frac{NI_1}{\tau}$$

$$\therefore L = \frac{N^2 \mu_a}{2} \therefore L = \frac{N^2 / 4a}{2}$$

1. A coil wound on an iron core of permeability 400 has 150 turns & a cross sectional area of 5 cm² calculate inductance of coil given a steady current of 3 mA produces a magnetic field of 10 lines /cm² where air is present as the medium

A)

$$1 \text{wb} = 10^8 \text{ lines}$$

$$\phi = 10^3 \times 5 \times 10^{-4}$$

$$B = 10 \times 10^{-8} = 10^{-7} \text{ lines /cm}^2$$

$$\therefore B (\text{wb/cm}^2) = 400 \times 10^{-7} = 4 \times 10^{-5} \text{wb/cm}^2$$

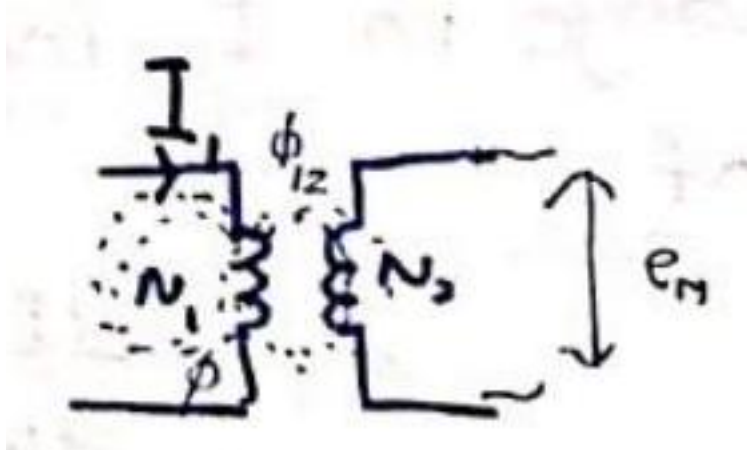
$$\therefore \phi = BA = 4 \times 10^{-5} \times 5 = 2 \times 10^{-4} \text{wb}$$

$$L = \frac{N\phi}{I} = \frac{150 \times 2 \times 10^{-4}}{3 \times 10^{-3}} = 10 \text{H}$$

$$a = 5 \times 10^{-4} \quad \mu_r = 400$$

Mutual Inductance

production of in one coil due to flux change in



other coil.

$$e_M = N_2 \cdot \frac{d\phi_{12}}{dt} \equiv M \cdot \frac{dI_1}{dt}$$

or mutual inductance $M = e_m / \frac{dI}{dt}$
or

$$\begin{aligned} e_m &= N_2 \frac{d\phi_{12}}{dt} = \frac{d}{dt} (N_2 \phi_{12}) \\ &= m \frac{dI_1}{dt} = \frac{d}{dt} (mI) \\ \therefore N_2 \phi_{12} &= mI_1 \Rightarrow m = \frac{N_2 \phi_{12}}{I_1} \\ \phi &= \frac{mmf}{s} \quad \therefore \phi_{12} = \frac{N_1 I_1}{2/\mu a} \end{aligned}$$

$$\phi_{12} = \frac{C1}{4\mu a}$$

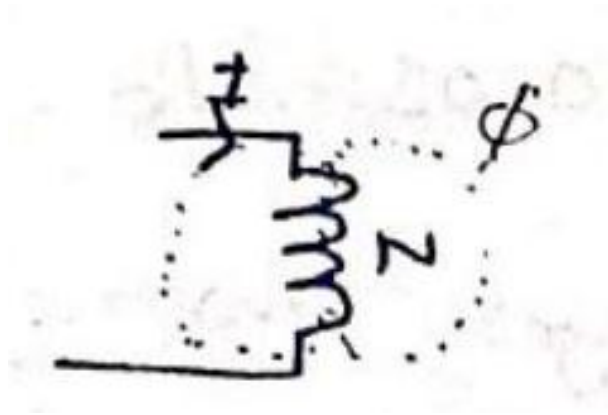
$$\begin{aligned} \text{since } M &= \frac{N_2 \phi_{12}}{I_1}; M = \frac{N_1 N_2 \mu a}{2} = \frac{N_1 N_2}{S} \\ &= \frac{N_2 N_1 I_1 \mu a}{2I_1}; \end{aligned}$$

\therefore Inductance

self (L)

$$\frac{e / \frac{dI}{dt}}{e_n / \frac{dI}{dt}} \text{ mutual } (M)$$

$$\frac{N_2 \phi_{12}}{I} \quad \therefore I_2$$



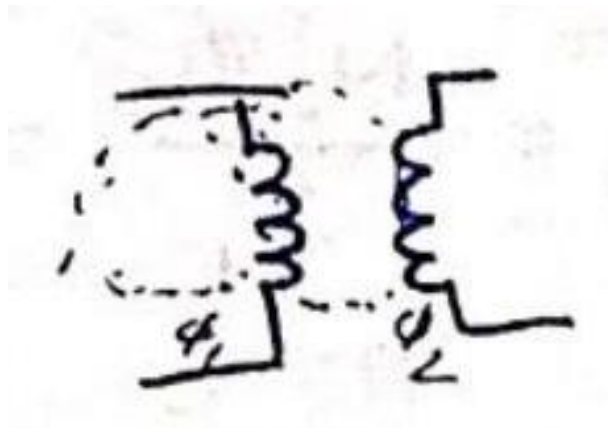
$$\frac{N^2 \mu a}{w^2/5}$$

$$\frac{N_1 N_2 \mu a}{2}$$

$$N_1 N_2 / s$$

Coefficient of Coupling (k)

$$k = \frac{\text{flux linked with 2nd coil}}{\text{flux produced by 1st coil.}} \quad \frac{\phi_{12}}{\phi_1}$$



$\phi_{12} = k\phi_1$ = Amount of flux produced in first coil linking with 2nd coil.

$$M_{12} = \frac{N_2 k \phi_{12}}{I_1}, M_{21} = \frac{N_1 k \phi_{21}}{I_2}, L_1 = \frac{N_1 \phi_1}{I_1} \quad L_2 = \frac{N_2 \phi_2}{I_2}$$

$$\text{so } M_{12} = M_{21} = m$$

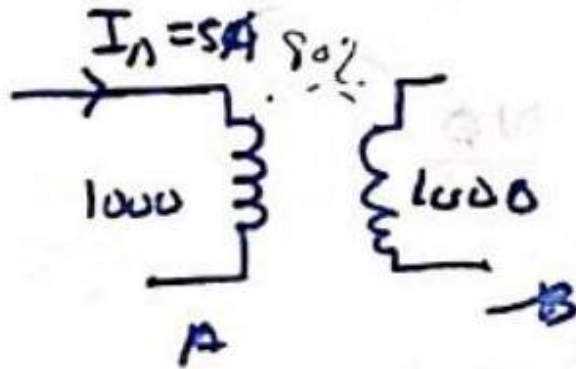
$$\therefore M_{12} \cdot M_{21} = \frac{k^2 N_1 N_2 \cdot \phi_1 \phi_2}{I_1 I_2} = k^2 L_1 L_2 = m^2$$

$$\therefore M = K \sqrt{L_1 L_2}$$

$$\text{or } K = \frac{M}{\sqrt{L_1 L_2}}$$

1. Two identical coils A, B of 1000 turns are lying parallel planes such that 80% of flux produced by one coil links with the other. A current of 5 A flowing in coil A produces a flux of 0.05 mWb in it. If the current in the coil A changes from 412 A to -12 A in 0.023 s. Calculate i) mutual inductance M ii) emf induced in coil B .

A) $I_A = 5A, \quad \phi_A = 0.05mW, \quad N = 1000$



$$\therefore L_A = \frac{N_1 \phi_1}{I_1} = \frac{1000 \times 0.05 \times 10^{-3}}{5}$$

Coils A & B in a magnetic circuit have 600 & 500 turns respectively, A current of 8 A in coil A produces a flux of 0.04 Wb. If $k = 0.2$, calculate

- i) Self inductance of A iii) Avg. emf induced in B ii) flux linking with coil B & flux with it changes from

iv) mutual inductance

v) Avg emf in B when I_A changes from 0 to 8 A in 0.05 s

Aji) Self inductance = $\frac{M \phi_A}{I_A}$

ii) flux linked $\phi_B = k \cdot \phi_A = 0.2 \times 0.04 = 0.008 \text{ WB} = 8 \text{ mWb}$

i) $M_{AB} = \frac{N_2 \phi_{12}}{I_1} = \frac{1000 \times 0.008 \times 10^{-3}}{5} =$

$$- \varepsilon_B = N \frac{\Delta \phi}{\Delta t} = 500 \times \frac{8 \text{ mWb} - 0}{0.025} = 500 \times \frac{0.008}{0.02} = 200$$

iv) $M = \frac{N_2 \phi_{12}}{I_1} = \frac{N_B \phi_B}{I_A} = \frac{500 \times 8 \times 10^{-3}}{8} = 0.51 \text{ H}$

v) When $I_A \in [0, 8]$

$$\phi_A \in [0, 40 \text{ mWb}], \phi_B \in [0, 8 \text{ mWb}]$$

$$\therefore \varepsilon_B = N \frac{d\phi_B}{dt} = 500 \cdot \frac{8 \text{ mWb} - 0}{0.05} = 500 \frac{0.008}{0.05}$$

ii) emf induced = $m \cdot \frac{dI}{dt} = m \frac{(12+12)}{0.02}$

$$\rightarrow M \cdot \frac{dI}{dt} = 0.5 \times \frac{8}{0.05} = 8$$

Inductance in series

$$L_{eq} = \begin{cases} \sum_{i=1}^n L_i, & \text{no mutual inductance} \\ L_1 + L_2 + 2m, & \text{w. mut. } E(\omega \cdot 2L). \end{cases}$$

fluxes are

"aiding"

$$L_1 + L_2 + 2m$$

Recons +855 Inductance in parallel

$$L_{eq} = \sum_{v_1} L_{U(2)} \omega / 0$$

$$L_{eq} = \frac{4L_2 - M^2}{L_1 + L_2 - \omega 2M}$$

$$L_{eq.} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

$$w/0 \cdot M \cdot I$$

fluxes are aiding

Energy stored: $\frac{1}{2}LI^2$ AC-fundamentals

$$v = V_m \sin(\omega t), i = I_m \sin(\omega t)$$

Advantages of sing. 1) Sinusoidal signals produce less disturbance to elect. network compared with other generating signals and the waveform is smooth and efficient 2) Sinusoidal signals applied to appropriately designed coils can produce a revolving field that can do the work. 3) mathematical calculations associated with a sinusoidal signals are much simpler 9). with the help of Fourier series, any alternating signal can be represented as the sum of different sinusoidal signals.

Parameters to represent alternating signal 1) Peak value \boxtimes Amplitude 2) peak to peak $\sim V_{pp}$? 3) Average value. /DC value.

$$= \frac{1}{T_0} \int_0^T f(t) dt$$

for symmetric, take 1/2 cycle. $\equiv \frac{2}{T} \int_0^{T/2} f(t) dt$ e.g.: $\frac{2}{2\pi} \int_0^\pi v_m \sin(ut) dt = \frac{a}{\pi} (2v_m)^0 = \frac{2}{\pi} v_m$

1.

$$= 25 \text{ V}$$

2 Area under curve

$$= \frac{10 \times 0.2}{2} + 0.4 \times 10 + \frac{0.2 \times 10}{2}$$

$$- \frac{0.2 \times 5}{2}$$

$$= 6.5$$

$$\therefore \text{Avg val} = \frac{6.5}{1} = \underline{6.5V}$$

3. FWR:

$$\text{Avg. val} = \frac{1}{\pi} \int_0^\pi \sin(\theta) d\theta$$

$$= \frac{\pi}{\pi} [-\cos \theta]_0^\pi$$

$$= \frac{\pi}{\pi} (n) = 0. \pi 1$$

4. HWR

$$\text{Avg val} = \frac{1}{2\pi} \int_0^\pi \sin(\theta) d\theta$$

$$= \frac{1}{\pi}$$

FWR, HWR wave. RMS value. \sim Rms value of an alternating current is that steady current (dc current) flowing through a given resistor for a given time which produces same heating effect as & by the a AC for the same resistance for the same time. Here total heating effect

$$I \simeq \underbrace{\sqrt{\frac{1}{N} \sum_{k=1}^N i_{15}^{m=m^2}}}_{\text{root mean square}}$$

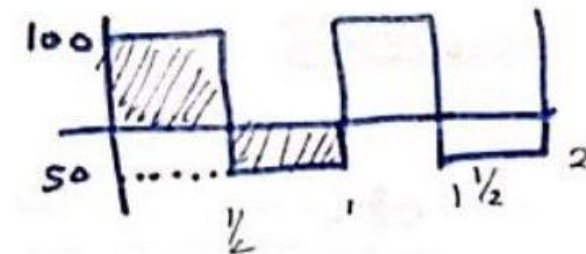
$$I = \sqrt{\frac{1}{T} \int_0^T i(t)^2 dt}$$

for symmetric wave: $R_{ms} = \sqrt{\frac{\text{sq. avg of } 1/2c_y}{1/2T}}$. for sin.

$$\begin{aligned} &= \frac{v_{rms}}{v_{avg}} \\ \text{- form factor for } \sin H\omega &= \frac{v_m}{\sqrt{2}} \cdot \frac{\pi}{2v_m} = \frac{\pi}{\sqrt{8}} = 1.111 \\ \text{- Peak factor} &= \frac{v_m}{v_{rms}} = v_m / \frac{v_m}{\sqrt{2}} = \sqrt{2} = 1.414 \end{aligned}$$

1.

$$\begin{aligned} A_{\text{vg. Val}} &= \frac{100 \times .5 \Phi 50 \times .5}{1} \\ &= 25 \text{ V} \end{aligned}$$



2 Area under curve

$$\begin{aligned} &= \frac{10 \times 0.2}{2} + 0.4 \times 10 + \frac{0.2 \times 10}{2} \\ &\quad - \frac{0.2 \times 5}{2} \\ &= 6.5 \\ \therefore \text{Avg val} &= \frac{6.5}{1} = 6.5V \end{aligned}$$

3. FWR:

$$\text{Avg. val} = \frac{1}{\pi} \int_0^{\pi} \sin(\theta) d\theta$$

$$= \frac{\pi}{\pi} [-\cos \theta]_0^{\pi}$$

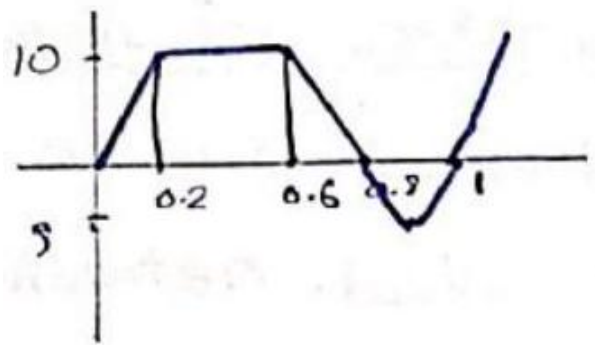
$$= 2/\pi$$

$$\pi(2) = 0.64$$

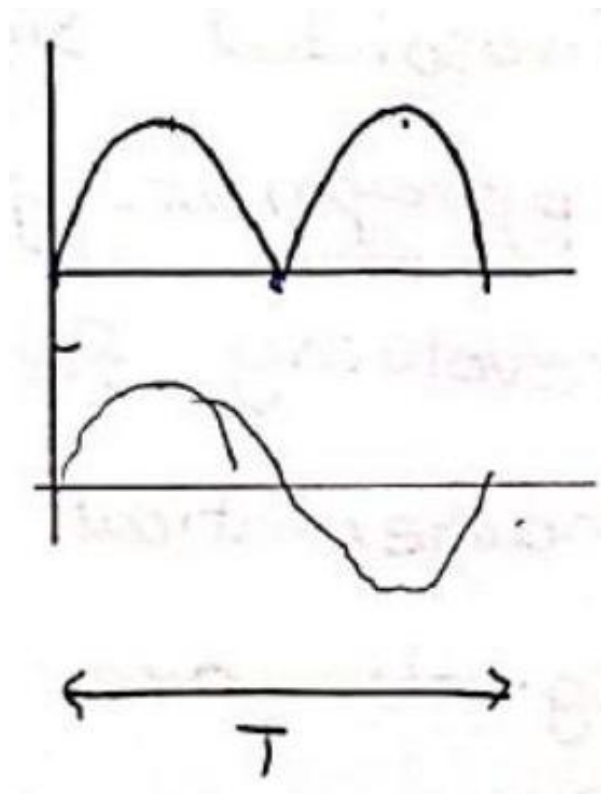
4. H'wR

$$\frac{HwR}{Avg} \text{ val} = \frac{1}{2\pi} \int_0^{\pi} \sin(\theta) d\theta$$

$$= \frac{1}{\pi} = 0.318$$



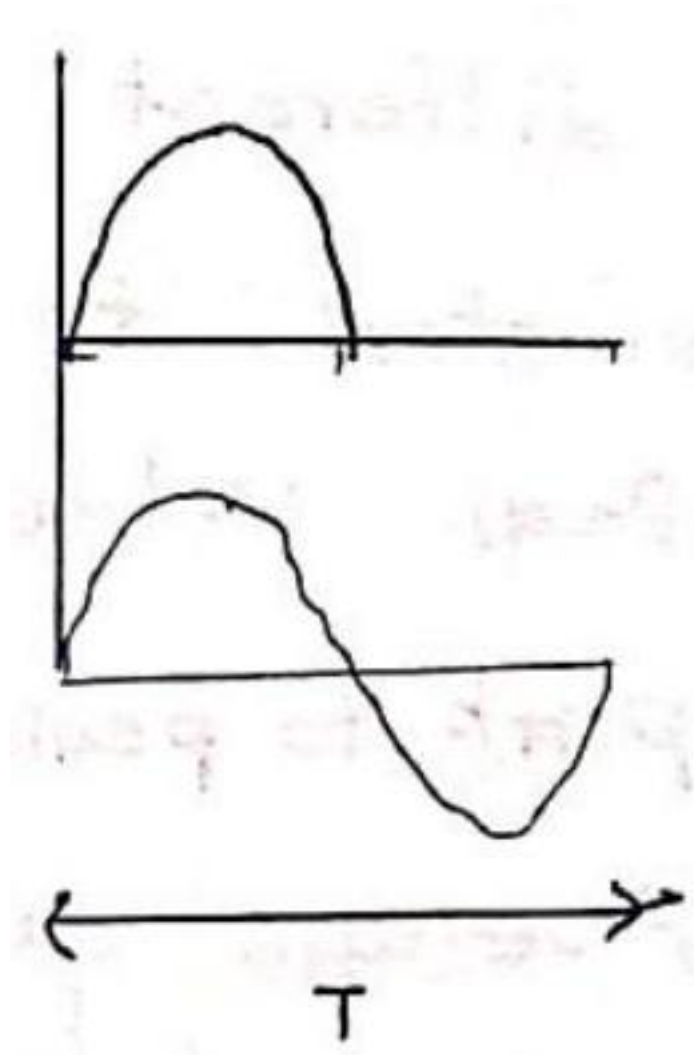
FWR, HWR wave.



$$I = \underbrace{\sqrt{\frac{1}{N} \sum_{k=1}^N i_{15}^2}}_{\text{root mew square}}$$

$$I = \sqrt{\frac{1}{T} \int_0^T i(t)^2 dt}$$

for symmetric wave: $R_{ms} = \sqrt{\frac{sq \cdot a00011/2cy}{1/2T}}$.
 for sin.

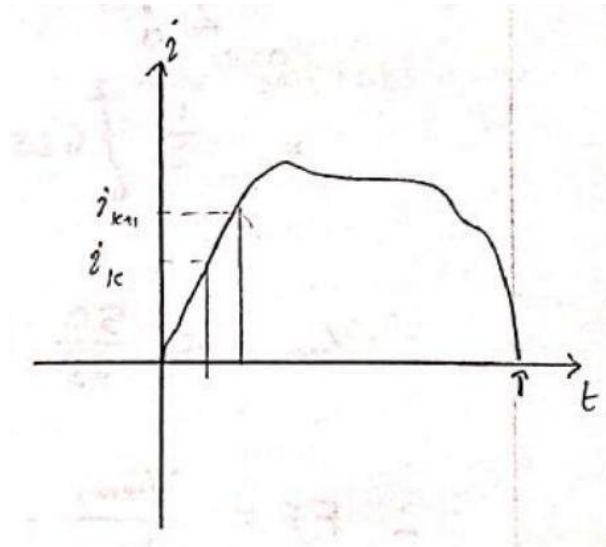


the same tire time
 Here total heating effect

$$\underbrace{I^2 R T}_{\text{dc eqiv heating}} \approx \frac{1}{N} \sum_{k=1}^N \left(\frac{i_{k+1} - i_k}{2} \right)^2 R \cdot T$$

RMS value.

~ Rms value of an alternating current is that steady current (dc current) flowing through a given resistance for a given time which produces same heating effect as by the AC for the same resistance for



$$\begin{aligned}
 J_{rms} &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} v_m^2 \sin^2(\theta) d\theta} = \\
 &= \sqrt{\frac{V_m^2}{2\pi} \int_0^{2\pi} \left(\frac{1}{2} - \frac{\cos(2\theta)}{2} \right) d\theta} = \sqrt{\frac{V_m^2}{4\pi} \left[\theta - \frac{\sin(2\theta)}{2} \right]_0^{2\pi}} \\
 &= \sqrt{\frac{v_m^2}{4\pi} (2\pi)} = \frac{v_m}{\sqrt{2}}
 \end{aligned}$$

- form factor = $\frac{V_{rms}}{V_{avg}}$

for $\sin H\omega = \frac{v_m}{\sqrt{2}} \cdot \frac{\pi}{2v_m} = \frac{\pi}{\sqrt{8}} = 1.111$

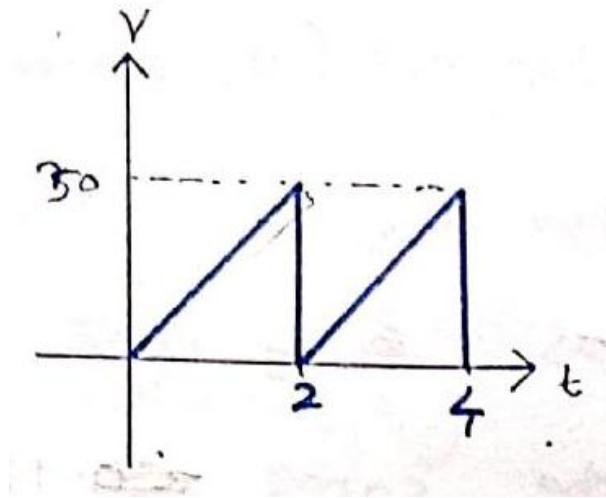
- Peak factor = $\frac{v_m}{v_{rms}} = v_m / \frac{v_m}{\sqrt{2}} = \sqrt{2} = 1.414$
- RmS of $FWR, T = \pi$ HWR $T = 2\pi$

RMS of a complex wave

1. find FF of given waveform

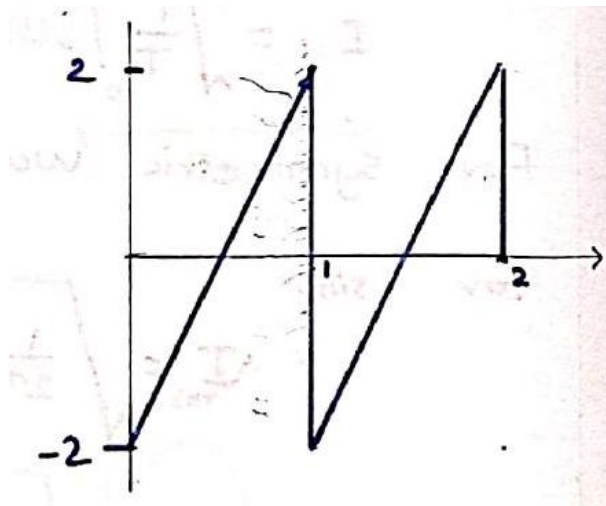
A)

$$\begin{aligned}
 V_{\text{Avg}} &= \frac{50 \times 2}{4} = 25V \\
 V_{rms}^2 &= \frac{1}{T} \int_0^T v(t) db \\
 &= \frac{1}{2} \int_0^2 (25t)^2 dt \\
 &= \frac{1}{2} \int_0^2 625t^2 dt = \frac{625}{2} \left[\frac{t^3}{3} \right]_0^2 = \frac{625}{6} \times 8 = \frac{4 \times 25^2}{3} \\
 \therefore V_{rms} &= \frac{50}{\sqrt{3}} = 28.87V \\
 \therefore FF &= \frac{V_{rms}}{V_{vuv}} = \frac{28.87}{25} = 1.155
 \end{aligned}$$



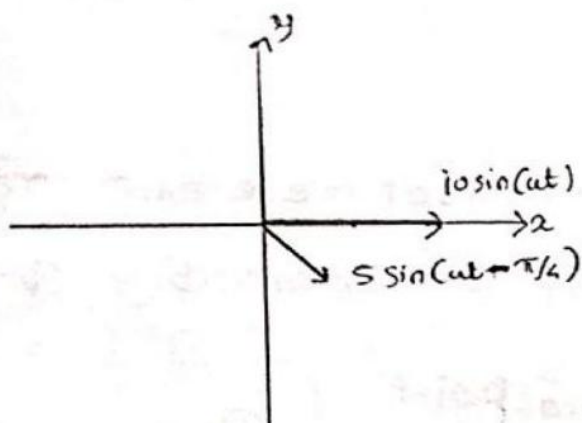
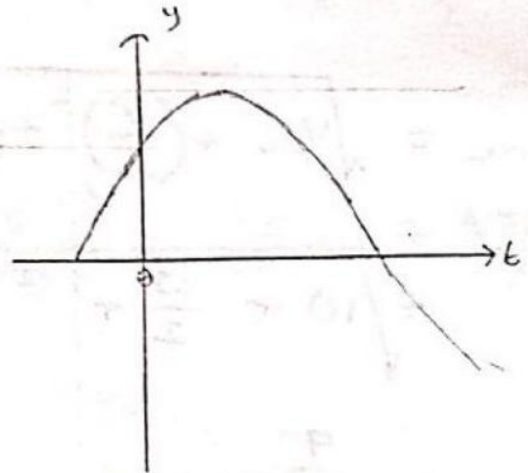
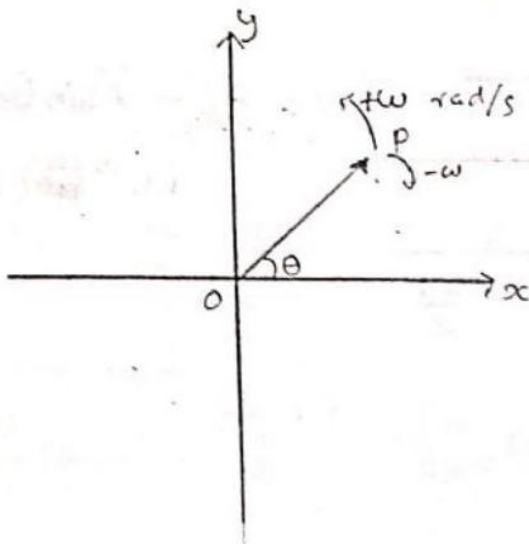
2. $V_{\text{Avg}} = 0$? 1

$$\begin{aligned}
 V_{\text{Rms}}^2 &= \frac{1}{1} \int_0^1 (4t - 2)^2 dt = \int_0^1 (16t^2 + 4 - 18t) dt \\
 &= \left[8\frac{16t^3}{3} + 4t - 8t^2 \right]_0^1 = \frac{16}{3} + 4 - 8 = \frac{4}{3} \\
 \therefore V_{\text{Taos}} &= \frac{2}{\sqrt{3}} = 1.155
 \end{aligned}$$



- Every alternating sign waveform can be represented completely if we have.

i) Amplitude ii) frequency iii) phase difference phasor representation
 VAF are not scalar nor vector but phasor-



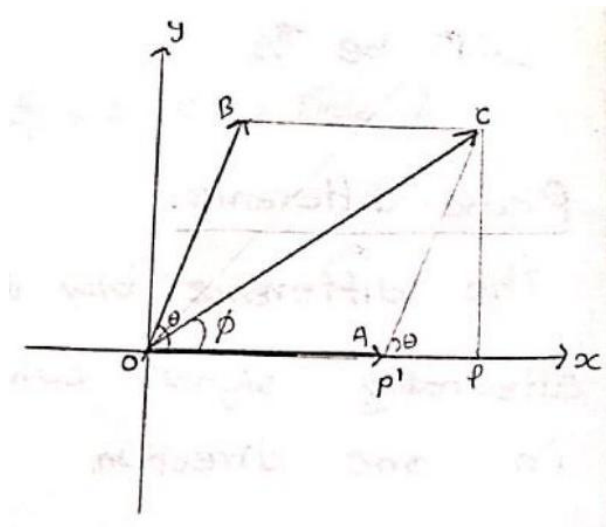
Addition of phasors

$$OC = \sqrt{OP^2 + PC^2}$$

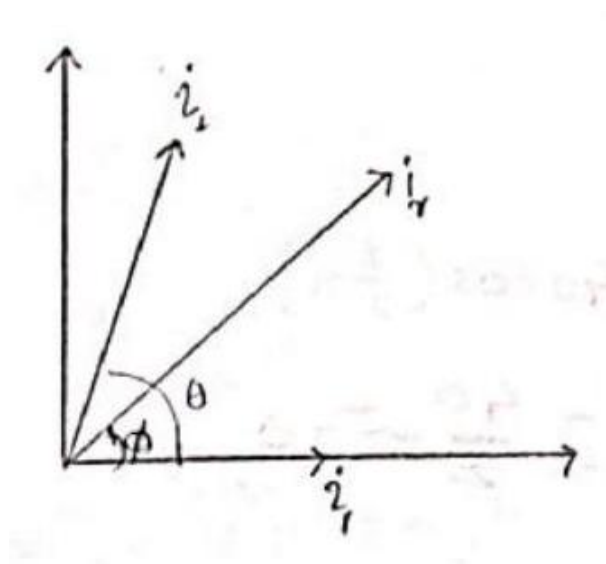
$$= \sqrt{(OP' + P'P)^2 + PC^2}$$

$$= \sqrt{(|A| + |B| \cdot \cos(\theta))^2 + (|B| \cdot \sin(\theta))^2}$$

$$= \sqrt{|A|^2 + |B|^2 + 2|A||B| \cdot \cos(\phi)}$$



$$\phi = \tan^{-1} \frac{|B| \sin(\theta)}{|A| + |B| \cos(\theta)} \text{ method of components.}$$



Split phasor to x, y camponents.

$$\therefore \text{Resultant} = \sqrt{x^2 + y^2}$$

$$i_1(x) = i_1, i_1(y) = 0$$

$$i_2(x) = i_2 \cos(\theta), i_2(y) = i_2 \sin(\theta)$$

$$\phi = \tan^{-1} \left(\frac{x}{y} \right) = \tan^{-1} \left(\frac{i_2 \sin(\theta)}{i_1 + i_2 \cos(\theta)} \right) i_r = \sqrt{(i_1 + i_2 \cos(\theta))^2 + (0 + i_2 \sin(\theta))^2}$$

$$= \sqrt{i_1^2 + i_2^2 + 2i_1 i_2 \cos(\theta)}$$

\Leftrightarrow

Three circuits in parallel take the follouing currents

$$i_1 = 20 \sin(314t), i_2 = 30 \sin(314t - \pi/4), i_3 = 40 \cos(314t + \frac{\pi}{6})$$

find

i) expression for resultant current

ii) its rms value and frequency.

iii) If the circuit has a resistance of 2Ω what is the energy loss in 1 hour

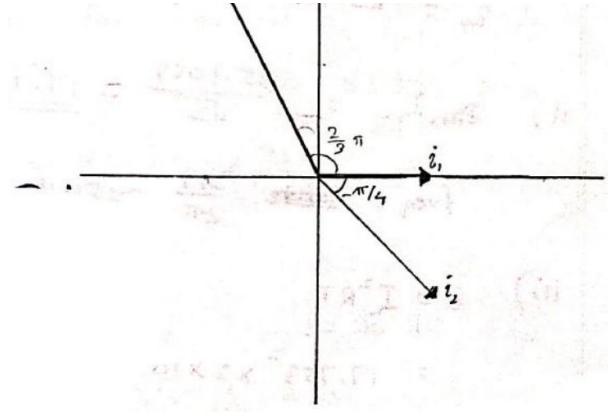
A) $i_2 = 40 \sin(314t + \frac{\pi}{2} + \frac{\pi}{6})$

$= 40 \sin(314t + \frac{2}{3}\pi)$

\dot{z}

i

i



Phasor algebra (\boxtimes)

$$\odot \vec{B} = j^2 \vec{A}$$

$$\vec{C} = j \vec{A}$$

$$i_1(x) = 20, \quad i_2(x) = 30 \cos(-\pi/4) \quad i_3(x) = 40 \cos\left(\frac{2}{3}\pi\right)$$

$$= \frac{30}{\sqrt{2}} = -\frac{40}{2} = -20$$

$$\therefore \dot{z}_r(x) = 20 + \frac{30}{\sqrt{2}} - 20 = \frac{30}{\sqrt{2}}$$

$$i_2(y) = 0 \quad i_2(y) = 30 \sin(-\pi/4), \quad i_3(y) = 40 \sin\left(\frac{2}{3}\pi\right)$$

$$= -\frac{30}{\sqrt{2}} = 40 \cdot \boxtimes \frac{\sqrt{3}}{2}$$

$$\therefore \dot{i}_r(y) = 40 \frac{\sqrt{3}}{2} - \frac{30}{\sqrt{2}} = \frac{40\sqrt{6} - 30\sqrt{2}}{2}$$

$$\therefore |\dot{i}_r| = \sqrt{\dot{i}_r(x)^2 + \dot{i}_r(y)^2} = \sqrt{\frac{900}{2} + (\dots)^2} = 25.1059$$

$$\theta = \frac{\eta_r(y)}{\dot{i}_r(x)} \tan^{-1} \theta = 32.3^\circ$$

$$\therefore i_r = 25.1059 \sin(314t + 32.3^\circ) \text{ A}$$

ii) $\mathcal{I}_{\text{reos}} I_{rms} = \frac{25.1059}{\sqrt{2}} = 17.753 \text{ A}$

$$f_{\text{req}} = \frac{314}{2\pi} \simeq 50 \text{ Hz}$$

iii).

$$\begin{aligned}
E &= I_m^2 RT \\
&= 17.753^2 \times 2 \times 10 \times 60 \times 60 = 22.69 \text{ MJ} \\
&= 17.753^2 \times 2 \times 10^{710^{-9}} = 6.303 \text{ kwh} \\
&= 4000 \left(\cos(\pi/4 - \pi/6) + \cos\left(2\omega t + \frac{\pi}{4} + \frac{\pi}{6}\right) \right) \\
P &= 4000 \left(\cos\left(\frac{\pi}{12}\right) + \cos\left(2\omega t + \frac{5}{12}\pi\right) \right) \\
\therefore P &= \int_0^{\pi/50} 4000 \left(\cos \frac{\pi}{12} \% + \cos\left(2\omega t + \frac{5}{12}\pi\right) \right) dt \\
&= 4000 \times \frac{\pi}{50} \cos \frac{\pi}{12} + \left[\frac{\sin\left(2\omega t + \frac{5}{12}\pi\right)}{2\omega} \times 4000 \right]_0^{\pi/50} \\
&= 80\pi \cos \pi/12 \\
&= \frac{242.76}{=} \\
&=
\end{aligned}$$

2 phosors are given in the rectangle form as: $\eta_1 = (15 + 10j)A$ and $\eta_2 = (12 + 6j)A$, perform $\eta_1 + \eta_2$ and $\eta_1 - \eta_2$

A) $z_1 + \eta_2 = 26 + 16j, \quad \eta_1 - \eta_2 = 3 + 4j$

3. Determine the resultant voltage of two sinusoidal generators in series whose voltages are, $v_1 = 25\angle 15^\circ \text{V}$, $v_2 = 15\angle 60^\circ \text{V}$

A)

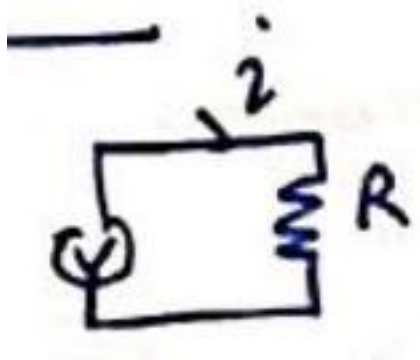
$$\begin{aligned}
v_1(x) &= 25 \cos(15^\circ), \quad v_2(x) = 15 \cos^3(60^\circ) \\
v_1(y) &= 25 \sin(15^\circ) \quad u_2(y) = 15 \sin(0^\circ) \\
R(x) &= \\
|R| &= 85) = 19.46
\end{aligned}$$

$$\therefore R = 37.15 < 31.59^\circ$$

AC circuits

Pure resistive circuit

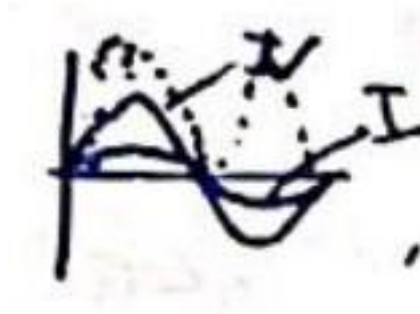
$$V = V_m \sin(\omega t)$$



Q inst. caners. valise
 . ins .t. current:

$$i = \frac{V_m}{R} \sin(\omega t)$$

$$:= N/R$$



Here both voltage & current case in phase.
 Instantaneous power = $v \cdot i$

$$= \frac{v_m^2}{R} \sin^2(\omega t)$$

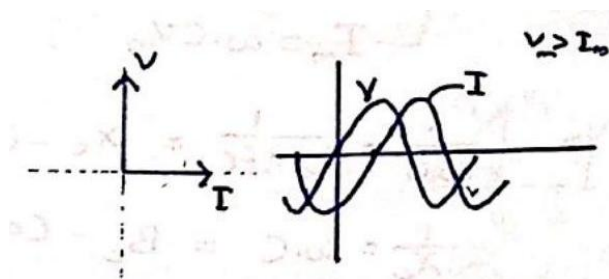
$$= \frac{V_m I_m}{2} - \frac{V_m I_m \cos(2\omega t)}{2}$$

$$\text{Avg. power} = \frac{1}{T} \int_0^T p dt = \frac{1}{2\pi} \int_0^{2\pi} \left[\frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos(2\omega t) \right] dt$$

$\underbrace{\qquad\qquad\qquad}_{=0} \quad \underbrace{\qquad\qquad\qquad}_{=V_m I_m}$

$$= \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = \underline{\underline{VI}} \text{ Rms values.}$$

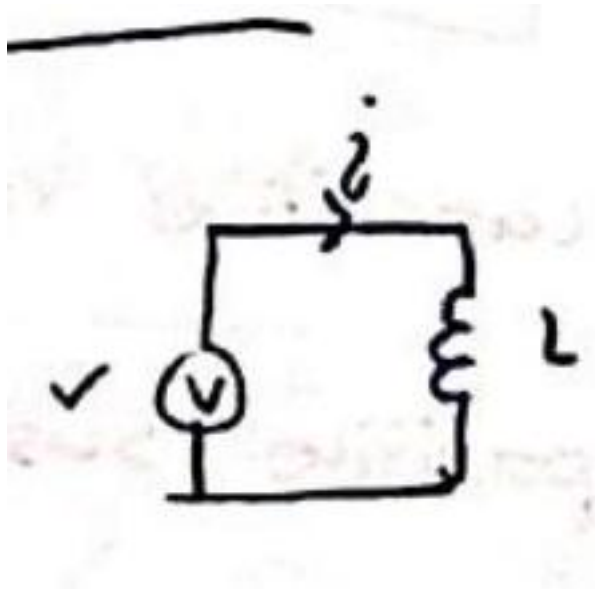
Purely inductive circuit



$$v = v_m \sin(\omega t)$$

$$= v_i = L \frac{di}{dt}$$

$$\therefore i = \int \frac{v_L}{L} dt = \frac{V_m}{\omega L} \cos^2(\omega t) = \frac{v_m}{L\omega} \cdot \sin\left(\omega t - \frac{\pi}{2}\right) = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$



$\therefore i$ is lagging behind V

$$I_m = \frac{v_m}{\omega L} \therefore \frac{\dot{V}_m}{I_m} = \omega L = 2\pi f L = X_c$$

$$\therefore G = \frac{1}{x_c} = \frac{1}{\omega L} = \frac{1}{2\pi f L} = B_L \text{ susceptance}$$

Instantaneous power : $p = V_i$

$$\begin{aligned} P &= V_i \\ &= V_m J_m \sin(\omega t) \sin\left(\omega t - \frac{\pi}{2}\right) \\ &= \frac{U_m I_n}{2} \cdot 2 \sin(\omega t) \sin\left(\omega t - \frac{\pi}{2}\right) \\ &= \frac{V_m I_m}{2} [-2 \sin(\omega t) \cos(\omega t)] \\ &= -\frac{V_m I_m}{2} \sin(2\omega t) \end{aligned}$$

$$\begin{aligned} \therefore \text{Avg. Power} &= \frac{1}{2\pi} \int_0^{2\pi} -\frac{V_m I_m}{2} \sin(2\omega t) dt \\ &= \frac{1}{2\pi} \left[\frac{y_m I_m}{2 \times 2\omega} \cdot \cos(2\omega t) \right]_0^{2\pi} = 0 \end{aligned}$$

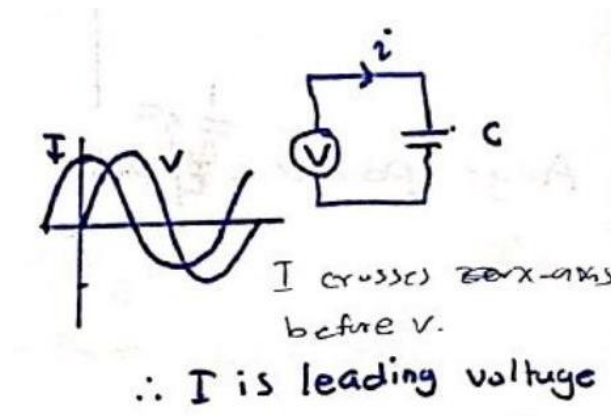
\therefore no power is consumed in purely inductive resistance

Purely capacitive circuit

$$v = v_m \sin(\omega t)$$

Instantaneous charge stored:

$$\begin{aligned} q &= C v \\ &= C \cdot v_m \sin(\omega t) \end{aligned}$$



∴ inst .t. current: $i = \frac{da}{dt} = C\omega v_m \cos(\omega t) = C\omega \cdot v_m \sin\left(\omega t + \frac{\pi}{2}\right)$

$$\frac{V_m}{I_m} = \frac{1}{\omega C} = \frac{1}{2\pi f C} = x_c - \text{(capacitive reactances)}$$

$$\frac{1}{x_c} = \omega \cdot C = B_c - \text{(capacitive susceptance)}$$

Instantaneous power: $p = vi = v_m I_m \sin(\omega t) \cos(\omega t)$

$$= \frac{V_m I_m}{2} \sin(2\omega t)$$

Average power = $\frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} \sin(2\omega t) dt = 0$

∴ Here no power drawn

1. A pure inductive coil allows a current of 10A to flow from a 230V 50Hz supply. rms find x_L , L , power absorbed, $i(t) = ?$ $v(t) = ?$

A))

$$v(t) = \frac{230\sqrt{2}}{\sqrt{2}} \sin(100\pi t) = 325.3 \sin(100\pi t)$$

$$i(t) = 10 \sin\left(\omega t - \frac{\pi}{2}\right) = 10 \sin\left(100\pi t - \frac{\pi}{2}\right)$$

$$x_L = \frac{v_m}{I_m} = \frac{230\sqrt{2}}{10\sqrt{2}} = 23 \quad 2\pi f L$$

$$\therefore L = \frac{23}{2\pi f} = \frac{23}{100\pi} = 0.037 = 37\text{mH}$$

power absorbed = 0.

Series AC circuit

RL circuit

- all real coils, literally * Choke coil

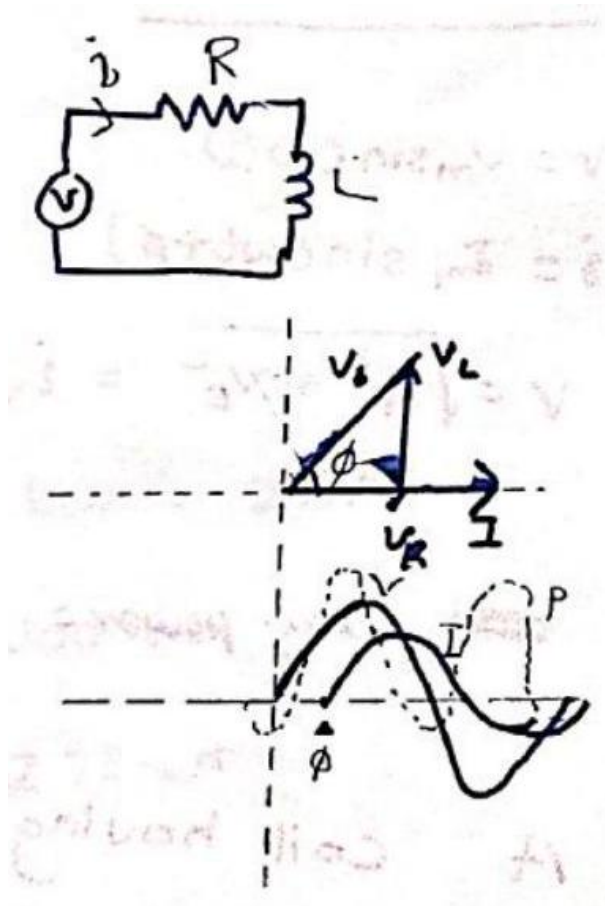
$$\vec{v} = \vec{v}_R + \vec{v}_L$$

$$v = \sqrt{u_R^2 + v_i^2} = \sqrt{(iR)^2 + (ix_L)^2}$$

$$= i\sqrt{R^2 + x_L^2}$$

Z Impedance

$$i = I_m \sin(\omega t - \phi), \quad \phi = \tan^{-1} \left(\frac{x_L}{R} \right)$$

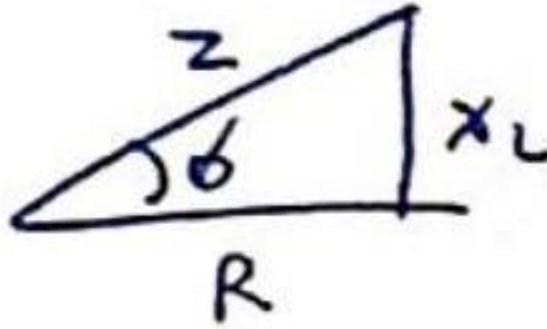


Instantaneous power:

$$\begin{aligned} p &= V_i = \frac{v_m I_m}{2} \cdot 2 \sin(\omega t) \cdot \sin(\omega t - \phi) \\ &= \frac{V_m I_m}{2} [\cos(\phi) - \cos(2\omega t - \phi)] \end{aligned}$$

Avg. Power: $\frac{1}{2\pi} \int_0^{2\pi} \frac{v_m I_m}{2} (\dots) dt = \frac{v_m I_m}{2} \cos(\phi) = V \cdot I \cos(\phi)$ as. angle between applied voltage and resultant current $= \frac{v_R}{v} = \frac{iR}{i^2} = R/2$

Impedance triangle Power triangle



S = apparent power

= power that is assumed to be drawn from source to circuit load

$$= V_m \cdot I_{rms} [\text{volt-ampere}]$$

P = active power true power

= power actively consumed / done useful work

$$= V_{rms} \cdot I_m \cdot \cos(\phi)$$

Q = reactive power

[watt]

= power not consumed / not done useful work

$$= v_{rms} \cdot I_{r-s} \cdot \sin \phi$$

~ consumed by capacitor/inductor

[volt-ampere-reactance (VAR)]

$$P = vI \cos(\phi)$$

$$\therefore I = \frac{230}{12.21} = 18.8 \text{ A}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = 58^\circ$$

$$\cos(\phi) = 0.57$$

$$P_{\text{C.V.}} \cos(\phi) \simeq 2.5 \text{ kW}$$

$$V_R = I \cdot R = 131.6$$

$$v_L = 188$$

$$\therefore \text{power factor} = \frac{\text{Active power}}{\text{Apparent power}} = \frac{P}{S} = I^2 R \text{ no } Z \therefore \text{con desist}$$

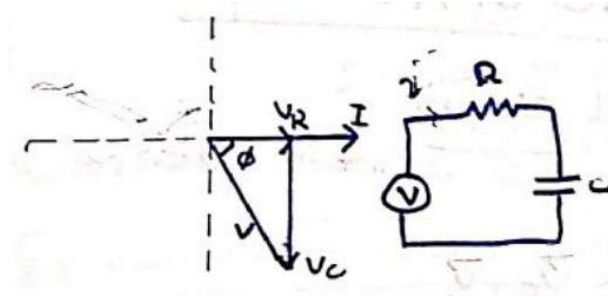
RC – circuit

$$v = v_m \sin(\omega t)$$

$$i = I_m \sin(\omega t + \phi)$$

$$v = \sqrt{v_R^2 + v_E^2} = i \underbrace{\sqrt{x_c^2 + R^2}}_2$$

$$\text{arg. power} = \frac{1}{2\pi} \int_a^{2\pi} v_i dt = \frac{1}{2\pi} \dots = VI \cos(\phi)$$



2. A coil having resistance of 7Ω and a inductance of 31.8mH is connected to $230\text{ V } 50\text{ Hz}$ supply. Calculate, $Q\dot{Q}I, \phi, \cos(\phi)$, power consumed, $V_R dV_L$

A). =

$$\begin{aligned} x_L &= L\omega = 2\pi fL = 100\pi \times 0.0318 \\ &= 9.989. = 10. \\ \therefore z &= \sqrt{7^2 + 10^2} = 12.21\Omega \end{aligned}$$

3. A choke coil takes a current of 2A lagging 60° behind

A the applied voltage of $200\text{ V } 50\text{Hz}$, Calculate Z, R, L of coil. Also find power consumed when coil is connected across $100\text{ V}, 25\text{ Hz}$ supply.

$$\begin{aligned} z &= \frac{V}{I} = \frac{200}{2} = 100\Omega \\ &= \frac{1}{2} \Rightarrow = 50 \end{aligned}$$

$$\cos(\phi) = R/z$$

$$\cos(60) = \text{loot } z, = \frac{1}{2} \Rightarrow = 50$$

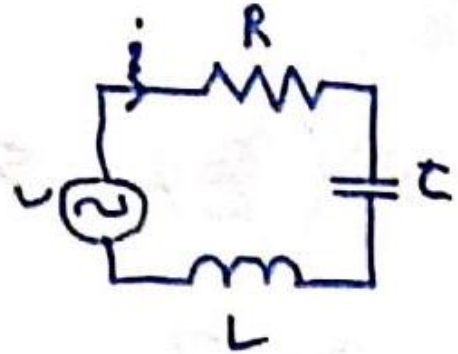
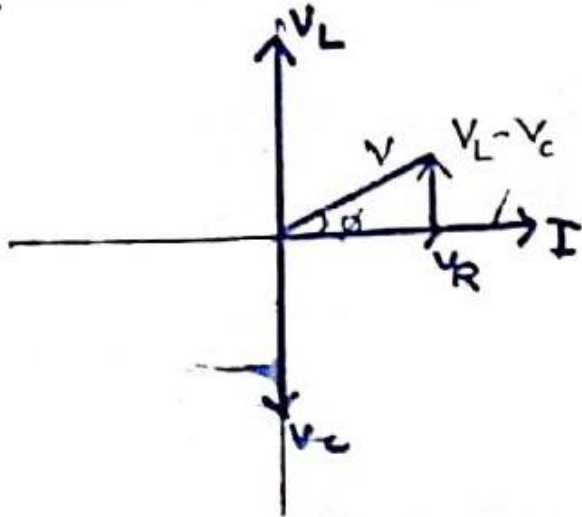
$$\therefore z = \sqrt{R^2 + X_L^2} \Rightarrow x_k = \frac{Z^2 - R^2}{173.2} = 100\pi L$$

$$X_L \text{ at } 100\text{ V } 25\text{Hz} : X_L = 2\pi fL = 43.3\Omega$$

$$\therefore z = \sqrt{R^2 + X_L^2} = \sqrt{50^2 + 93.3^2} = 66.1\Omega$$

$$\therefore I = \frac{100}{66.1} = 1.51, \Rightarrow \therefore P = I^2 \times R = 112.5$$

Series LCR



- if $x_L > x_C$. inductive in nature (I lags v)

$x_C > x_L$, capacitive " (I leads v)

$x_C = x_L$. resistive, (I in phase V)

$$v = \sqrt{v_R^2 + (v_L - v_C)^2} \text{ or } z = \sqrt{R^2 + (x_L - x_C)^2}$$

$$\cos(\beta) = \frac{R}{\sqrt{R^2 + (x_L - x_C)^2}}$$

1. A 230 V, 50 Hz AC supply is applied to a coil of 0.06H inductance, and 2.5Ω , resistance connected in series with a $6.8\mu\text{F}$, calculate Impedance, current, phase angle (ϕ), power factor; power consumed.

A)

$$x_L = 2\pi fL = 18.85\Omega$$

$$z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$x_C = \frac{1}{2\pi fC} = 468.1\Omega$$

$$= \sqrt{2.5^2 + (18.85 - 468.1)^2} = 468.1\Omega$$

$$I = \frac{V}{z} = \frac{230}{468.1} = 0.491\text{A}$$

$$\cos(\phi) = \frac{R}{z} = \frac{2.5}{468.1} \rightarrow \phi = 89.68^\circ \rightarrow \text{power factor} = 0.56 \times 10^{-3}$$

$$\phi = \cos^{-1}\left(\frac{R}{z}\right) = 89.68^\circ \rightarrow \text{leading}$$

power factor = 0.56×10^{-3} , \rightarrow leading

Resonance

A circuit consisting of LQC is said to be in resonance when circuit power factor is unity.

Resonance condition: $x_L = x_C \Rightarrow v$ in phase with I & $pf = 1$

Here impedance is minimum, I is max.

$$L\omega = 2\pi fL = \frac{1}{2\pi fC} \Rightarrow f_r = \frac{1}{2\pi\sqrt{LC}}$$

Quality / voltage umplification factor

$$Q = \frac{V_L}{V} = \frac{I_r x_L}{I_r R} (@Resnone) = \frac{x_L}{R} = \frac{V_C}{V_L} = \frac{x_L}{R}$$

$$Q \frac{I_r x_L}{I_r z} \leftarrow \text{fircoil}$$

$$Q = \frac{\omega L}{R} = \frac{1}{\omega C R}$$

$$Q = \frac{L}{R} \cdot \frac{1}{\sqrt{LC}} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

Band width

$$\beta = f_2 - f_1$$

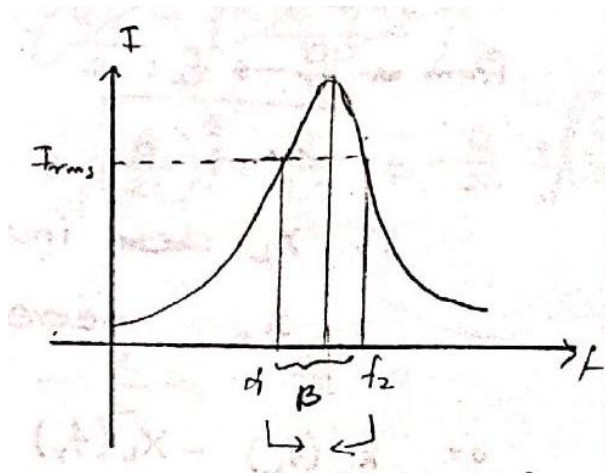
H_1 - Lower cutest freq

f. - uppa

f_1 ; E = half pour frow / - $3dR$ freq

$$P = J^2 R s, \quad P_{f_1} = I_h^2 R = \frac{P_{\max}}{2}$$

$$\Rightarrow \left(\frac{I_n}{s_2} \right)^2,$$



$$x_L - x_c = R$$

$$1z = R\sqrt{2}$$

Power in decibels: $P_r = 10 \log P_{\max}$ $P_{\gamma_{1,22}} = 10 \log \frac{P_{\max}}{2}$

\therefore Recharge in row from fo to tiu:

$$= 10 \log P_{mm} / (P_{mma}/2) = 10 \log 2 \approx 3dB \text{ from ream}$$

2 A coil of resistance 100Ω , & $1+ = 100\mu H$ is connected in series with $100pF$ capacitor, circuit is connected to a low variable frequency source, calculate, resonant freq.

I at resonance, voltage across LQC at resonance, Q factor, $Q = 104$

A) $f = \frac{1}{2\pi\sqrt{LC}} = 1.59\text{MHz}$

\therefore at resonance; $z = R$

$$\begin{aligned}\therefore I &= \frac{V}{Z} = \frac{10}{100} = 0.1 \text{ A} \\ V_R &= Ix_L = 0.1 \times 2\pi fL = 1000 \\ V_C &= Ix_C = \frac{0.1}{2\pi fC} = 1000 \\ Q &= \frac{V_R}{V} = \frac{1000}{100} = 10\end{aligned}$$

Expression for f_1 & f_2

At upper cut off :

$$x_L - x_C = R$$

from $\sim f_r \rightarrow f_2$:

$$0 \rightarrow R$$

$\therefore X_L$ der. increases by $R/2$ x_C decreases by $R/2$

or $x_L(f_2) - x_L(f_r) = \frac{R}{2}$

or $2\pi L f_2 - 2\pi L f_r = R/2$

or $f_2 - f_r = \frac{R}{4\pi L} \Rightarrow f_2 = f_r + \frac{R}{4\pi L}$ for lower cut off.

$$x_L f_r - x_C(f_r) = R/2$$

es.

$$f_2 - f_1 = \frac{R}{4\pi L}$$

$$f_1 = f_r - \frac{R}{4\pi L}$$

$\therefore \beta = f_2 - f_1 = \frac{R}{2\pi L} \leftarrow \text{bandwidth}$

$$\text{or } \frac{f_r}{r_r} \cdot (-) \quad \frac{R}{2\pi L f_v} \cdot f_v = \frac{f_v}{Q} = \beta \quad \frac{2\pi f_r L}{R} = Q$$

$$f = \sqrt{f_1 f_2}$$

$$x_L - x_C = R \quad (\text{eatsff})$$

for $f_1 =$

$$\begin{aligned}
x_L - x_C &= -R \\
\omega_1 L - \frac{1}{\omega_1 C} &= -R \Rightarrow \omega_1 / L \quad \omega_1^2 + \frac{R}{L} \omega_1 - \frac{L}{LC} = 0 \\
R &\rightarrow \Gamma / \\
\Rightarrow \omega_1 &= \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4L}{LC}}}{2} \\
&= \\
&= -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{C_2^2}{r}}
\end{aligned}$$

In the sarre lug for f_2 :

$$\begin{aligned}
\omega_2 L - \frac{1}{\omega_2 C} &= R \Rightarrow \omega_2^2 - \frac{R}{L} \omega_2 - \frac{1}{LC} = 0 \\
\Rightarrow \omega_2 &= \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \\
&= \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \omega_r^2} \\
\omega_1 \omega_2 &= \left(\frac{R}{2L}\right)^2 + \omega_r^2 - \left(\frac{R}{2L}\right)^2 = \omega_r^2 \Rightarrow \omega_r = \sqrt{\omega_1 \omega_2} \\
\therefore f_r &= \sqrt{f_1 f_2}
\end{aligned}$$

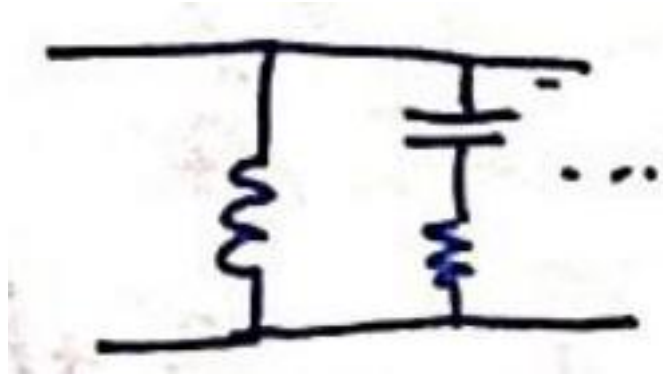
1. A series RLC circuit bos $R = 5\Omega, L = 0.2H, C = 50\mu F$. The for unity powr-factor. $I_1(y) = I_2(y)$ applied voltage is 200 V. find resonant frequency, Q -facter. $\beta, f_1 \& f_2, I_r, I_{f_1, f_2}, V_L(f_r)$.

⊗)

$$\begin{aligned}
f_r &= \frac{1}{2\sqrt{LC}} = \\
x_C &= \frac{1}{2 - \pi f C} \\
Q &= \frac{x_c}{R} = \frac{63.3}{5} = 12.65 \\
&= 63.3\Omega \\
\beta &= \frac{f_r}{Q} = \frac{3.97 \text{ Hz}}{48.3 \text{ Hz}} \\
f_1 &= f_r - B_2 = \\
I_{f_2} = I_{f_1} &= \frac{V}{2} = \frac{V}{R\sqrt{2}} = \frac{200}{5\sqrt{2}} = 28.28 = \frac{I_r}{\sqrt{2}} \\
V_L(f_r) &= I_{f_r} \cdot X_L = \frac{V}{R} \times 2\pi f_r \times L = 2528 \text{ V} \\
&= 6 \times 2 \\
I_r &= \frac{v}{R} = \frac{200}{5} = 40\theta
\end{aligned}$$

Parallel AC circuit

- equivalent impedance



+ admittance

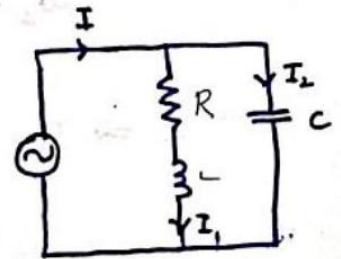
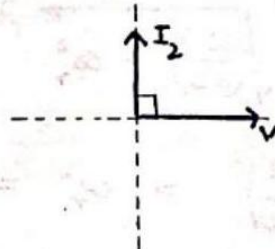
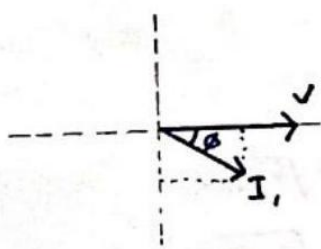
$$Z_{nd} = Z_1 \parallel Z_2 \parallel \dots \quad Z_n = \sqrt{\dots}$$

Resonance in parallel circuit

- take voltage as ref. as it's same in all branches.

Resonance in parallel circuit

take voltage as ref. as it's same in all branches.



$$\therefore I_1(y) = I_1 \sin(\phi), I_2(y) = I_2$$

$$\therefore I_1 \sin(\phi) = I_2$$

$$\frac{V}{Z_1} \frac{x_L}{Z_1} = \frac{V}{x_C} \Rightarrow \frac{x_L}{Z_1^2} = \frac{1}{x_C}$$

$$= \frac{x_L}{R^2 + x_L^2} = \frac{1}{x_C} \Rightarrow x_C x_2 = R^2 + x_L^2$$

$$= \frac{1}{\omega C} \cdot \omega L = \frac{L}{C} = R^2 + \omega^2 L^2 \Rightarrow 2\pi f_V = \sqrt{\frac{L}{LC} - \left(\frac{R}{L}\right)^2}$$

$$f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

if $R \gg L$, $f_r = \frac{1}{2\pi} \sqrt{\frac{1}{L}}$

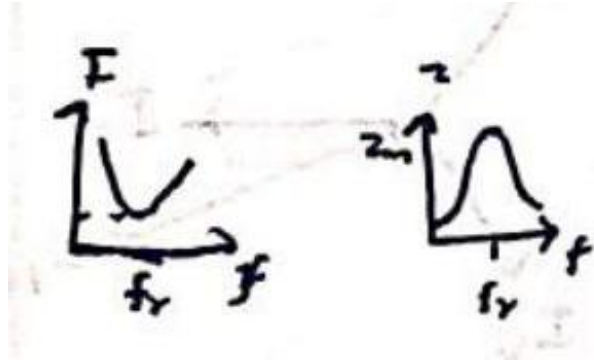
Current

at resonance: $I_L \sin(p) = I_L$

$$\therefore J_r = I_K \cos(\phi)$$

$$\frac{v}{z_r} = \frac{v}{z_L} \cdot \frac{R}{z_Z} \Rightarrow z_r = \frac{z_L^2}{R} \quad z_L = \sqrt{L}/c$$

$$Z_r = \frac{L}{CR} < \text{purely resistive}$$



at resonance, I is min. z is max.

Q-factor

$Q = \frac{I_c}{I_r}$ Current multiplication factor

$$= \frac{v}{x_c} / \frac{v}{Z_r} = \frac{Z_r}{x_c} = \frac{L}{CR} \cdot C\omega = \omega \cdot \frac{L}{R}$$

Bandwidth

$$\beta = f_2 - f_1$$

