Trigonometry

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1 Basic Identities

- $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{1}{\cot(\theta)}$
- $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)} = \frac{1}{\tan(\theta)}$
- $\sec(\theta) = \frac{1}{\cos(\theta)}$
- $\csc(\theta) = \frac{1}{\sin(\theta)}$

2 Pythagorian Relations

- $\sin^2(\theta) + \cos^2(\theta) = 1$
- $\sec^2(\theta) - \tan^2(\theta) = 1$
- $\csc^2(\theta) - \cot^2(\theta) = 1$

3 Sign of trigonometric functions for negative angles

- $\sin(-x) = -\sin(x)$
- $\cos(-x) = \cos(x)$
- $\tan(-x) = -\tan(x)$
- $\csc(-x) = -\csc(x)$
- $\sec(-x) = \sec(x)$
- $\cot(-x) = -\cot(x)$

4 Expansion for trigonometric functions with two angles

- $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
- $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
- $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
- $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
- $\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$
- $\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
- $\cot(x+y) = \frac{\cot(x)\cot(y) - 1}{\cot(x) + \cot(y)}$
- $\cot(x-y) = \frac{\cot(x)\cot(y) + 1}{\cot(y) - \cot(x)}$

- **With π**
  1. $\sin(\dfrac{\pi}{2} + \theta) = cos(\theta)$
  2. $\sin(\pi - \theta) = sin(\theta)$
  3. $\cos(\dfrac{\pi}{2} + \theta) = -sin(\theta)$
  4. $\cos(\pi - \theta) = -cos(\theta)$

5 Product formula

- $\sin(x+y) + \sin(x-y) = 2\sin(x)\cos(y)$
- $\sin(x+y) - \sin(x-y) = 2\cos(x)\cos(y)$
- $\cos(x+y) + \cos(x-y) = 2\cos(x)\cos(y)$
- $\cos(x+y) - \cos(x-y) = -2\sin(x)\sin(y)$

6 Sum formula

- $\sin(a) + \sin(b) = 2\sin\left(\dfrac{a+b}{2}\right)\cos\left(\dfrac{a-b}{2}\right)$
- $\sin(a) - \sin(b) = 2\cos\left(\dfrac{a+b}{2}\right)\sin\left(\dfrac{a-b}{2}\right)$
- $\cos(a) + \cos(b) = 2\cos\left(\dfrac{a+b}{2}\right)\cos\left(\dfrac{a-b}{2}\right)$
- $\cos(a) - \cos(b) = -2\sin\left(\dfrac{a+b}{2}\right)\sin\left(\dfrac{a-b}{2}\right)$

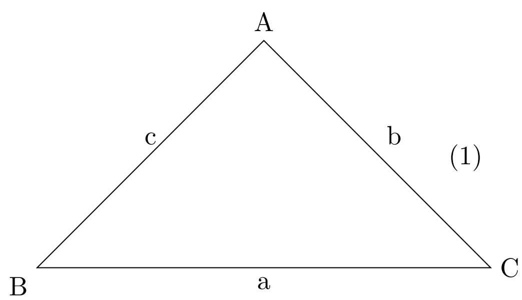
7 Expansion for multiple angles

- $\sin(2x) = 2sin(x)cos(x) = \dfrac{2\tan(x)}{1 + tan^2(x)}$
- $\sin(3x) = 3\sin(x) - 4\sin^3(x)$
- $\cos(2x) = \cos^2(x) - \sin^2(x)$
- $\cos(3x) = 4\cos^3(x) - 3\cos(x)$
- $\tan(2x) = \dfrac{2\tan(x)}{1 - \tan^2(x)}$
- $\tan(3x) = \dfrac{3\tan(x) - \tan^3(x)}{1 - 3\tan^2(x)}$
- $\sin^2(x) = \dfrac{1 - \cos(2x)}{2}$
- $\cos^2(x) = \dfrac{1 + \sin(2x)}{2}$

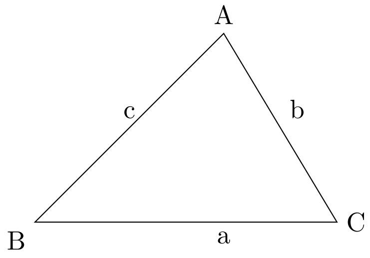
8 Law of sines

In any triangle, sides are proportional to to the sins of the angles

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c} \qquad or \qquad \frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \tag{1}$$



9 Law of cosines



For any ΔABC

$$\begin{aligned} a^2 &= b^2 + c^2 - bc\cos(A) \\ b^2 &= a^2 + c^2 - ac\cos(B) \\ c^2 &= a^2 + b^2 - ab\cos(C) \end{aligned}$$

also

$$\begin{aligned} \cos(A) &= \frac{b^2 + c^2 - a^2}{2bc} \\ \cos(B) &= \frac{a^2 + c^2 - b^2}{2ac} \\ \cos(C) &= \frac{a^2 + b^2 - c^2}{2ab} \end{aligned}$$

From above equations, we also get:

$$\frac{\cos(A)}{a} = \frac{\cos(B)}{b} = \frac{\cos(C)}{c} = \frac{a^2 + b^2 + c^2}{2abc}$$

10 Solutions of some trigonometric equations

- $\sin(x) = 0 \implies x = n\pi,\ n \in \mathbb{Z}$
- $\cos(x) = 0 \implies x = (2n+1)\dfrac{\pi}{2},\ n \in \mathbb{Z}$
- $\sin(x) = \sin(y) \implies x = n\pi + (-1)^n y,\ n \in \mathbb{Z}$
- $\cos(x) = \cos(y) \implies x = n\pi \pm y,\ n \in \mathbb{Z}$
- $\tan(x) = \tan(y) \implies x = n\pi + y,\ n \in \mathbb{Z}$
- $\tan(x) = 0 \implies x = n\pi,\ n \in \mathbb{Z}$

11 Other useful stuffs...

- $\tan\left(\dfrac{\pi}{4} - x\right) = \dfrac{1 - \tan(x)}{1 + \tan(x)}$
- $\tan\left(\dfrac{\pi}{4} + x\right) = \dfrac{1 + \tan(x)}{1 - \tan(x)}$
- $\dfrac{\tan\left(\dfrac{\pi}{4} + x\right)}{\tan\left(\dfrac{\pi}{4} - x\right)} = \left[\dfrac{1 + \tan(x)}{1 - \tan(x)}\right]^2$
- $\sin(x+y)\sin(x-y) = \sin^2(x) - \sin^2(y)$
- $\sin(x+y)\sin(x-y) = \cos^2(x) - \sin^2(y)$
- $\tan(3x)\tan(2x)\tan(x) = \tan(3x) - \tan(2x) - \tan(x)$

12 Some *Condensed* trigonometric identities

- $\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$
- $\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$
- $\tan(x \pm y) = \dfrac{\tan(x) \pm \tan(y)}{1 \mp \tan(x)\tan(y)}$
- $\cot(x \pm y) = \dfrac{\cot(x)\cot(y) \mp 1}{\cot(x) \pm \cot(y)}$
- $\tan\left(\dfrac{\pi}{4} \pm x\right) = \dfrac{1 \pm \tan(x)}{1 \mp \tan(x)}$