Module-1:VCin Ordinary Differential Equations Expected outcome: To solve ODE and $\angle DE$ of higher order with constant coefficients and apply then in engineer problems

Res: Shastry sis - eng moth Erwin Kreyszig-Ad eng math Vecrarajon - Eng math. Grewel S.S - Higher Eng north * Core Concepts ?) Introduction 2) First Ordo DiS Eq 3) Exact Diff Eq 4) Berroullis $E\eta$ 6) Method of Sold and Simple Applteration c) Linear Diff Eq of Higher order with constant corf. 7) Method of sols of $\angle OE$ e) Cauchy's Linear Diff Eq? 9) Simultaneous $\angle DE$ re) Some Application - Electrical circuit - Mechanical system Introduction Now I'm a budding eclentiat so whenever I feel z vibration I flow in nature I feel I can write its abstract using differential eqn and that flow will be some 4roooth curve. A curve can be represented as a fund: in m-therpatics. In $y=f(2)$, ex: $y=x^2+2$, the variable x is independent variable and the value of y depends on variable $x, 00$ y is dependant variable. In $z=f(x,y)$, there are two independent vorithe x and y and one dependent variable = Drferential Equations An en involving derivatives of the dependent vamable wort independent variables is called differeshal eph. ex: $x\frac{dy}{dx}+y=0$ E.dioary Dis. Eq n - Diff Eon involving derivatives of dependent variables wow. 1 only one independent variable. ex. $3\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^5=0$ Order of diff ep order of highest order dalvative of dependent variable wort bodependent variable involved in the ep. ex, $3\frac{d^2y}{dx^2}+\left(\frac{dy}{dz}\right)^5=0$ order $=2$

Degree of diff eqn- If diff eq is in the form of polynomial eqn. in derivatives, the highest power C the positive integral Index) of the highest order derivative involved in the eqn is its degree.

$$\begin{aligned}&\text{ex: } \frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^5=0\\&\text{order } =2\\&\text{degree } =1\end{aligned}$$

degree not defined.

degree = 2 not linear

First order $\angle D\varepsilon$

Standard form: $y'+P(x)y=Q(x)$

$$\begin{aligned}&y'+P(x)y=Q(x)\\&\text{order } =1 \qquad \Rightarrow \text{ Pinery}\\&\text{degree } =1\end{aligned}$$

P and Q are continuous fund in $\frac{I}{Cd}$ (domain)

Integrating factor, $I_F=\int e^{Rdx}e^{\int pdx}$

Soln $\underline{y(F)=\int Q(IF)dx+C}$

1. Solve $y'+2xy=x$

fLD

$$P = 2x$$
$$Q = x$$
$$If = e^{\int Pdx} = e^{\int 2xdx}$$
$$= e^{x^2}$$

Sols: $\quad y \cdot e^{x^2} = \int x \cdot e^{x^2} dx + c$

$$x^2 = t$$
$$2xdx = dt$$
$$xdx = 1/2dt$$

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2. Solve $\frac{dy}{dx} + 2y \tan x = \sin x$

given eqn is $fLD\varepsilon$

$$P(x) = 2 \tan x$$
$$Q(x) = \sin x$$
$$1f = e^{\int 2 \tan x dx}$$
$$= e^{2 \ln |\sec x|} = \sec^2 x$$
$$y \left(\sec^2 x\right) = \int \sin x \cdot \sec^2 x + dx + c$$
$$y \sec^2 x = \int \tan x \sec x + c$$
$$y \sec^2 x = \sec x + c$$
$$y = \sec^{-1} x + c \sec^{-2} x$$
$$y = \cos x + \cos x$$

π

3. Find the sol. of the initial value problem

$$x^2 y' - xy = x^4 \cos 2x, \quad y(\pi) = 2\pi$$

$\div$ by x^2 we get,
$y \to y^2 - \frac{y}{x} = x^2 \cos 2x$

$$P(x) = -1/x$$
$$Q(x) = x^2 \cos 2x$$
$$IF = e^{SPdx}$$
$$= e^{S-1/xdx} = e^{-\ln x} = e^{\ln x^{-1}}$$
$$= x^{-1}$$

Soln.

$$\begin{aligned} y(\text{ IF }) &= \int Q(\text{ IF}) \text{ dx } + c \\ yx^{-1} &= \int \frac{x^2}{x} \cos 2x dx + c \\ \frac{y}{x} &= \int x \cos 2x dx + c \end{aligned} \tag{1}$$

$$\begin{aligned} \int I_1 I_2 &= I_1 \int I_2 - \int \left(\frac{d}{dx} I_1 \int I_2 \right) \\ &= x \int \cos 2x dx - \int 1 - \int \cos 2x dx \\ &= \frac{x}{2} \sin 2x - \int \frac{1}{2} \sin 2x dx \\ \frac{y}{x} &= \frac{x}{2} \sin 2x + \frac{1}{4} \cos 2x + c \end{aligned}$$

general sols.

$$\frac{y}{x} = \frac{x}{2} \sin 2x + \frac{1}{4} \cos 2x + c$$

given $y(\pi) = 2\pi$

$$\begin{aligned} 2\pi &= \frac{\pi^2}{2} \sin 2\pi + \frac{\pi}{4} \cos 2\pi + c\pi \\ 2\pi &= 0 + \frac{\pi}{4} + c\pi \\ 2\pi &= \pi \left(\frac{1}{4} + c \right) \\ 2 - \frac{1}{4} &= c \\ c &= 7/4 \end{aligned}$$

Substituting in (2),

$$y = \frac{x^2}{2} \sin 2x + \frac{x}{4} \cos 2x + \frac{7}{4} x$$

A) Solve $y' - 2xy = 2x$

5. Solve $xy' - 2y = -x$

6. Solve $xy' + 2y = \frac{\cos x}{x}$

7. Solve $\frac{dy}{dx} + \frac{2y}{x} = \frac{4}{x}, y(1) = 6$

$$
\begin{aligned}
&\text{A) } P(x) = -2x \\
&Q(x) = 2x \\
&\text{ if } = e^{-\int 2x dx} \\
&= e^{-x^2} \\
&y\left(e^{-x^2}\right) = \int 2x\left(e^{-x^2}\right) dx + c \\
&ye^{-x^2} = 2\int xe^{-x^2} dx + c \\
&ye^{-x^2} = -e^{-x^2} + c \\
&y = -1 + ce^{x^2} \\
&y + 1 = ce^{x^2}
\end{aligned}
$$

$$
\begin{aligned}
-\int (2x)e^{-x^2} \\
= -\frac{e^{-x^2}}{2}
\end{aligned}
$$

5.

$$
\begin{aligned}
xy' - 2y &= -x \\
y' - \frac{2y}{x} &= -1 \\
P(x) &= -2/x \\
Q(x) &= -1 \\
&= e^{\int -2/x dx} \\
&= e^{-2\int 1/x dx} \\
&= e^{-2\log x} \\
&= e^{\log x^{-2}} = x^{-2} \\
y\left(x^{-2}\right) &= \int -1\left(x^{-2}\right) dx + c \\
y\left(x^{-2}\right) &= -\int x^{-2} dx + c \\
\frac{y}{x^2} &= -\int \frac{x^{-2+1}}{-2+1}\Bigg] + c \\
\frac{y}{x^2} &= -\frac{x^{-1}}{-1} + c \\
\frac{y}{x^2} &= \frac{1}{x} + c
\end{aligned}
$$

6.

$$
\begin{aligned}
& x y^{\prime}+2 y=\frac{\cos x}{x} \\
& y^{\prime}+\frac{2 y}{x}=\frac{\cos x}{x^2} \\
& P(x)=2 / x \\
& Q(x)=\frac{\cos x}{x^2} \\
& \text { If }=e^{\int P(x) d x} \\
& =e^{2 \int 1 / x d z} \\
& =e^{2 \log x}=e^{\log x^2}=x^2
\end{aligned}
$$

$$
\begin{gathered}
y\left(x^{\prime \prime}\right)=\int a(\text { If }) d x+c \\
y\left(x^2\right)=\int \frac{\cos x}{x^2} \cdot x^2+c \\
y\left(x^2\right)=\sin x+c
\end{gathered}
$$

7. Solve

$$
\begin{gathered}
\frac{d y}{d x}+\frac{2 y}{x}=\frac{4}{x}, y(1)=6 \\
y^{\prime}+\frac{2 y}{x}=\frac{4}{x} \\
P(x)=2 / x \\
Q(x)=\frac{4}{x} \\
\text { If }=e^{\int 2 / x d x} \\
=e^{2 \log ^x}=x^2 \\
y\left(x^2\right)=\int \frac{4}{x}\left(x^2\right) d x+c \\
y\left(x^2\right)=4 \int x d x+c \\
y\left(x^2\right)=\frac{x^2}{2}+c \\
y\left(x^2\right)=2 x^2+c \\
y=2+c x^{-2} \\
6=2+c \\
y=x^2=2 x^2+4 \\
y=2+4 x^{-2}
\end{gathered}
$$

1. Solve $(x+1) \frac{d y}{d x}+2 y=(x+1)^{5 / 2}$

2. Solve $xy' - 2y = x^4 e^x$

3. Solve $xy' - y = 2x \ln x$

A) Solve $\frac{dy}{dx} + y \tan x = \cos^2 x$

5. Solve $\frac{dy}{dx} + y \cot x = \operatorname{cosec}^2 \cdot x$

c) Solve $xy' + y = (1+x)e^x$

7. Solve $y' + 2xy = xe^{-x^2}$

8. Solve $xy' - 2y = x^3 e^x, y(1) = 0$

Variable seperable Eq?

A differential eq of the form $M(\dot{x}, y)dx + N(x, y)dy = 0$ is a variable separable ep. if it can be expressed in the form: $f(x)dx + g(y)dy = 0$

i) Solve $\frac{dy}{dx} = y/x$

$$
\begin{gathered}
\frac{dy}{y} = \frac{dx}{x} \\
\int \frac{1}{y} dy = \int \frac{1}{x} dx \\
\text{fog } y = \log x + \log c \\
y = xc \rightarrow \text{ general sols:}
\end{gathered}
$$

2. Solve $(y+2)dx + y(x+4)dy = 0$

$$
\begin{aligned}
&(y+2)dx = -y(x+4)dy \\
&\int \frac{dx}{x+4} + \int \frac{y}{(y+2)} dy = \int 0 + \ln c \\
&\ln(x+4) + \int \frac{y+2-2}{y+2} dy = \ln c \\
&\ln(x+4) + \int 1 dy - \int z/y + 2 dy = \ln c \\
&\ln(x+2) + y - 2\ln(y+2) = \ln c \\
&y = 2\ln(y+2) + \ln c - \ln(x+2) \\
&y = \ln(y+2)^2 + \ln c - \ln(x+2) \\
&y = \ln\left[\frac{c(y+2)^2}{(x+4)}\right]
\end{aligned}
$$

3. $x \sin y dx + \left(x^2 + 1\right) \cos y dy = 0$

$$x \sin y dx = -\left(x^2+1\right) \cos y dy$$

$\int \cot x dx = \ln|\sin x| + c$
$\div$ by $\sin y \left(x^2+1\right)$

$$\frac{x}{(x^2+1)} dx + \cot y dy = 0$$
$$\frac{1}{2} \int \frac{2x}{(x^2+1)} dx + \int \frac{\cos y}{\sin y} dy = \ln c$$
$$\frac{1}{2} \ln\left|x^2+1\right| + \ln|\sin y| = \ln c$$
$$\ln\left|x^2+1\right|^{1/2} + \ln|\sin y| = \ln c$$
$$\left(x^2+1\right)^{1/2} \sin y = c$$
$$\left(x^2+1\right) \sin 2y = c$$

A) Solve $\tan \theta dr + 2r d\theta = 0$.

$$\begin{array}{rl} \tan \theta dr = -2r d\theta & \\ -\frac{dr}{2r} = \frac{d\theta}{\operatorname{tnn} \theta} & \frac{1}{\sqrt{r}} = \sin \theta c \\ -\frac{1}{2} \int \frac{dr}{r} = \int \cot \theta d\theta + r & \frac{1}{r} = \sin^2 \theta \cdot c \\ -\frac{1}{2} \ln|r| = \ln|\sin \theta| + \ln c & \sin^2 \theta r \\ \ln|r|^{-1/2} = \ln|\sin \theta| c & r - 1/2 = \sin \theta \cdot c \end{array}$$

Solve $4xy dx + \left(x^2+1\right) dy = 0$

$$\div \left(x^2+1\right) y$$
$$= \int \frac{4x}{(x^2+1)} dx + \int \frac{1}{y} dy = \int 0$$
$$= 2 \cdot \int \frac{2x}{x^2+1} + \ln|y| = \ln c$$
$$= \ln\left|x^2+1\right|^2 + \ln|y| = \ln c$$
$$\left|x^2+1\right|^2 y = c$$

Homogeneous diff ego

A diff eqn that can be reduced to the form $\mu d\ \frac{dy}{dx} = f(y/x)$ is called homogeneous differs. This can be solved by putting $y = vx$ and hence reducing to variable separable form.

Q.1) Solve $2xy \frac{dy}{dx} - y^2 + x_1^2 = 0$.

$$
\begin{aligned}
&2xy\frac{dy}{dx} = y^2 - x^2 \\
&\frac{dy}{dx} = \frac{x^2 - y^2 - x^2}{2xy} = \frac{x^2\left(y^2/x^2 - 1\right)}{x^2 + 2(y/x)} = f(y/x) \\
&\frac{y = vx}{dx} = v + x\frac{dv}{dx} \\
&v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{x^2} \\
&v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v} \\
&x\frac{dv}{dx} = \frac{v^2 - 1}{2v} - v
\end{aligned}
$$

$$
\begin{aligned}
&x\frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v} \\
&x\frac{dv}{dx} = -\frac{\left(v^2 + 1\right)}{2v} \\
&2v - \frac{2v}{v^2 + 1}dv = \frac{dx}{x} \\
&-\ln\left|v^2 + 1\right| = \ln|x| + \ln|c| \\
&\left\{\left(v^2 + 1\right)^{-1} = xc\right. \\
&\left(y^2/y^2 + 1\right)^{-1} = xc \\
&\left(\frac{y^2 + x^2}{x^2}\right)^{-1} = xc
\end{aligned}
$$

2. Solve $xy' = x + y$
3. Solve $\frac{dy}{dx} = \frac{y+x}{y-x}$
4. Solve $\left(y + \sqrt{x^2 + y^2}\right)dx - xdy = 0$
5.

$$
\begin{aligned}
&xy' = x + y \\
&\frac{dy}{dx} = \frac{x+y}{x} \\
&v + x\frac{dv}{dx} = \frac{x+vx}{x} \\
&y + x\frac{dv}{dx} = 1 + y \\
&x\frac{dv}{dx} = 1 \\
&\int dv = \int \frac{dx}{x} \\
&v = \ln|x| + d \\
&y/x = \ln|x| + c
\end{aligned}
$$

3. $\frac{dy}{dx} = \frac{y+x}{y-x}$

$$v + x\frac{dv}{dx} = \frac{vx+x}{\sqrt{x-x}}$$

$$
\begin{aligned}
v + x\frac{dv}{dx} &= \frac{v+1}{v-1} \\
\times\frac{dv}{dx} &= \frac{v+1}{v-1} - v \\
x\frac{dv}{dx} &= \frac{v+1-v(v-1)}{v-1} \\
\times\frac{dv}{dx} &= \frac{v+1-v^2+v}{v-1}
\end{aligned}
$$

$$\frac{xdv}{dx} = \frac{2v+1-v^2}{v-1}$$
$$x\frac{dv}{dx} = \frac{-\left(v^2-2v-1\right)}{v-1}$$
$$-\frac{(2v-2)}{(x^2-2v-1)}dv = \frac{dx}{x}$$
$$2\left(v^2-2v-1\right)$$
$$\frac{-1}{2}\ln\left|v^2-2v-1\right| = \ln|x| + \ln|c|$$
$$\ln\left|v^2-2v-1\right|^{-1/2} = \ln|x||c|$$
$$\left|v^2-2v-1\right|^{-1/2} = xc$$
$$\left|(y/x)^2-2(y/x)-1\right|^{-1/2} = x_c$$
$$\frac{1}{x^2c^2} = \left((y/x)^2+2(y/x)^2-1\right)^{1/3}$$
$$\frac{1}{c^2} = y^2-2y^2-x^2$$
$$y_c^2 = y^2-2yx-x^2$$

4. Solve

$$\left(y+\sqrt{x^2+y^2}\right)dx - \int xdy = 0$$
$$\frac{\left(y+\sqrt{x^2+y^2}\right)}{x} = \frac{dy}{dx}$$
$$v + x\frac{dv}{dx} = \frac{vx+\sqrt{x^2+v^2x^2}}{x}$$
$$v + x\frac{dv}{dx} = \frac{vx+\sqrt{x^2\left(1+v^2\right)}}{x}$$
$$v + x\frac{dv}{dx} = \frac{vx+x\sqrt{1+v^2}}{x}$$
$$x\frac{dv}{dx} = y+\sqrt{1+v^2}-x$$
$$\int\frac{1}{\sqrt{1+v^2}}dv = \frac{dx}{x}$$

$$\int \frac{dx}{\sqrt{a^2+x^2}} = \ln\left|x+\sqrt{a^2+x^2}\right| + \ln|c| \tag{cs}$$

$$\ln\left|v+\sqrt{1+v^2}\right| = \ln|x| + \ln|c|$$

$$v+\sqrt{1+v^2} = xc$$

$$y/x+\sqrt{1+(y/x)^2} = xc$$

$$y/x+\sqrt{x^2+y_1^2} = xc$$

$$y+\sqrt{x^2+y^2} = cx^2$$

Bernoullis Diff eat

A diff eqn. of the form: $\frac{dy}{dx}+p(x)y = a(1)y^2, n \neq 0, 1$ where n is a real no. except 0 and 1 is called a bernoullis diff e eqn.

Method to solve:

Divide by y^n

$$y^{-n}\frac{dy}{dx}+p(x)y^{1-n} = Q(x)$$

$$\text{put } z = y^{1-n}$$

$$\frac{dy}{dx} = (1-n)y^{1-n-1}\frac{dy}{dx}$$

$$= (1-n)y^{-n}\frac{dy}{dx}$$

$$y^{-n}\frac{dy}{dx} = \frac{1}{(1-n)}\frac{dyz}{dx}$$

Substituting in egg. (1)

$$(1) \Rightarrow \frac{1}{1-n}\frac{dz}{dx}+P(x)z = Q(x)$$

x lying by $(1-n)$

$$\frac{dz}{dx}+(1-n)P(x)z = (1-n)a(x)$$

This is a FLDE in dependent variable is z Sold is.

$$z(1F) = \int q(T)(1F)dx + c$$

1. Solve $\frac{dy}{dx}+2y = y^2$

$\div$ by y^2

$$y^{-2}\frac{dy}{dx}+2y^{-1} = 1 \tag{1}$$

put $z = y^{-1}$

$$\frac{dz}{dx} = -1y^{-2}\frac{dy}{dx}$$
$$y^{-2}\frac{dy}{dx} = -\frac{dz}{dx}$$

Sub in eq. (1)

$$-\frac{dz}{dx} + 2z = 1$$
$$\frac{dz}{dx} - 2z = -1$$
$$P = -2$$
$$Q = -1$$
$$1f = e^{-2\int dx} = e^{-2x}$$
$$z \cdot e^{-2x} = \int -1\left(e^{-2x}\right)dx$$
$$ze^{-2x} = \frac{-1}{-2}e^{-2x} + c$$
$$ze^{-2x} = \frac{1}{2}e^{-2x} + c$$
$$2ze^{-2x} = e^{-2x} + c$$
$$\frac{2}{y}e^{-2x} = e^{-2x} + c$$
$$\frac{z}{y} = 1 + ce^{2x}$$

2. Solve

$$x\frac{dy}{dx} + y = xy^3$$
$$\frac{dy}{dx} + y/x = y^3$$
$$y^{-3}\frac{dy}{dx} + \frac{y^{-2}}{-x} = 1 \tag{1}$$
$$z = y^{-2}$$
$$\frac{dz}{dx} = -2y^{-3}\frac{dy}{dx}$$

$$y^{-2} = \frac{2x^{-1}}{x-2} + cx^2$$
$$y^{-2} = 2x + cx^2$$
$$\frac{1}{y^2} = 2x + ccx^2$$
$$\frac{y}{2x + cx^2} = y^2$$
$$y = \frac{1}{\sqrt{2x + cx^2}}$$

Sub in (1)

$$\frac{dy}{dx} = -\frac{1}{2} y^3 \frac{dz}{dx}$$

$$\begin{aligned}
&-\frac{1}{z} D^{\$} \frac{dz}{dx} + y^{-3} + \frac{z}{x} = 1 \\
&-\frac{1}{2}\frac{dz}{dx} + \frac{1}{x} z = 1 \\
&-\frac{dz}{dx} + \frac{z}{x} z = 2 \\
&\frac{dz}{dx} - \frac{z}{x} z = -2 \\
&P(x) = e^{-2\int yx dx} = e^{\log|x|^{-2}} = x^{-21} \\
&zx^{-2} = \int -2 \cdot x^{-2} dx + c \\
&zx^{-2} = -2 \int x^{-2} dx \\
&zx^{-2} = -2 \cdot \frac{x^{-2+1}}{-2+1} \\
&zx^{-2} = -z \frac{x^{-1}}{-1} + c \\
&zx^{-2} = 2x^{-1} + c \\
&y^{-2} x^{-2} = 2x^{-1} + c
\end{aligned}$$

$$x^{-2} = t$$

$$\int \sec^3 x \sec x dx$$
$$= \int \sec^2 x \cdot \sec^2 x dx$$
$$= \int \sec^2 x \left(1 + \tan^2 \cdot x dx\right.$$
$$= \int \sec^2 x + \int \sec^2 x \tan^2 x$$
$$= \tan x + \int (\sec x + \operatorname{san} x)^2$$
$$y^{-4} \frac{dy}{dx} = \frac{-1}{3} \frac{dz}{dx}$$
$$-\frac{1}{3} \frac{dz}{dx} - 3z \tan x = 3 \sec x$$
$$\frac{dz}{dx} + 3z \tan x = -3 \sec x$$

$$IF = e^{\int 3 \tan x} = e^{\log|\sec x|^3} \quad \sec x^3$$

$\tan x = t$
$t = \tan x$
$dt = \sec^2 x d$

$\frac{1}{2} \int dt \, (1 + t^2)$

$$2 = 1 - 2 \cdot \sec x^3 = -3 \int \sec^4 x dx + c$$
$$z \sec x^3 = -\frac{1}{3} \operatorname{san} x + \frac{\tan^3 x}{x^5} + c$$
$$y^{-3} \sec x^3 = -3 \tan x - \frac{\tan^3 x}{x} + c$$
$$-y^{-3} \sec x^3 = 3 \tan x + \tan^3 x - c$$

$$\left\{\begin{array}{l} 1 \\ t = \sec x \left(18 \tan^2 x\right)^2 dx \\ dt = \sec^2 x dx \\ \int dt \left(1 + t^2\right) \\ \int dt + t^2 dt \\ = t + \frac{t^3}{3} \end{array}\right.$$

1.

4. Solve $\frac{dy}{dx} + \tan x \tan y = \cos x \sec y$

$B \cdot D \cdot \varepsilon$

$$\therefore \sec y$$

$\cos y \frac{dy}{dx} + \frac{\tan x \tan y}{\sec y} = \cos x$
$\cos y \frac{dy}{dx} + \tan x \sin y = \cos x$

$$
\begin{aligned}
z &= \sin y \\
\frac{dz}{dx} &= \cos y \frac{dy}{dx} \\
\frac{dy}{dx} &= \frac{1}{\cos y}\frac{dz}{dx}
\end{aligned}
$$

$\cos y \cdot \frac{1}{\cos y}\frac{dz}{dx} + \tan x \cdot z = \cos x$

$$
\begin{aligned}
1f = e^{\int \tan x} - e^{\log|\sec x|} = \sec x \\
z \cdot \sec x = \int \cos x \cdot \sec x + c \\
z \cdot \sec x = \int 1dx + c \\
z \cdot \sec x = x + c \\
\sin y \sec x^3 = c^2 x + c
\end{aligned}
$$

5. $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$

$$
\sec^2 y \frac{dy}{dx} + x \sin 2y \cdot \sec^2 y = x^3
$$

$\sec^2 y \frac{dy}{dx} + x \cdot 2 \sin y \cos y \cdot \frac{1}{\cos^x y} = x^3$

$$
\begin{aligned}
&\sec^2 y \frac{dy}{dx} + x \cdot 2 \tan y = x^3 \\
&z = \tan y \\
&\frac{dz}{dx} = \sec^2 y \frac{dy}{dz} \\
&\frac{dy}{dz} = \frac{1}{\sec^2 y}\frac{dz}{dx}
\end{aligned}
$$

$\sec^2 /y \cdot \frac{1}{\sec^2 y}\frac{dz}{dx} + 2x \cdot z = x^3$

$$
\begin{aligned}
\text{IF } &= e^{\int 2xdx} = e^{2\int xdx} \\
&= e^{2x^2/2} = e^{x^2}
\end{aligned}
$$

$$z \cdot e^{x^2} = \int x^3 \cdot e^{x^2} + c$$
$$z \cdot e^{x^2} = \frac{1}{2} \int \frac{2x}{} \cdot x^2 \cdot e^{x^2} + c$$
$$z \cdot e^{x^2} = \frac{1}{2} \int dt \cdot t \cdot e^t + c$$
$$z \cdot e^{x^2} = \frac{1}{2} e^t (t-1) + c$$
$$z \cdot e^{x^2} = \frac{1}{2} e^{x^2} \left(x^2 - 1\right) + c$$
$$\tan y e^{x^2} = \frac{1}{2} e^{x^2} \left(x^2 - 1\right) + c$$
$$\tan y = \frac{1}{2} \left(x^2 - 1\right) + \frac{c}{e^{x^2}}$$
$$x^3 \int e^{x^2} - \int \frac{d}{dx} x^3 \int e^x dx$$
$$x^2 = t^6$$
$$t = x^2$$
$$dt = 2x dx$$
$$\int t - e^t dt$$
$$t \int e^t - \int \frac{d}{dt} t \int e^t$$
$$te^{t-1} = \int e^t$$
$$te^t - e^t = e^t (t-1)$$

Solve:

i) $\frac{dy}{dx} + y = xy^3$
2) $\frac{dy}{dx} - y \tan x = y^2 \sec x$
3) $\frac{dy}{dx} + y/x = \frac{y^2}{x} V_n x$

$$4) \ \frac{dy}{dx} + xy = x^3 y^3$$

$$y = f(x)$$
$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$z = f(x, y)$$
$$\frac{dz}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$$
$$\frac{\partial z}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y}$$
$$z = x^2 + y^2 + 2xy$$
$$\frac{\partial z}{\partial x} = 2x + 0 + 2y$$
$$\frac{\partial z}{\partial y} = 0 + 2y + 2x$$

Exact diff-eqn.

$$M dx + N dy = 0$$
$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Partial diff ego

If z is a func of x and $y : z = f(x, y)$, it can be differentiated partially w.x.t or partially o.r.t. y

$$\frac{\partial z}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$$

here, y is treated as constant.

eg: =

$$z = x^2 + y^2 + 2xy$$
$$\frac{\partial z}{\partial x} = 2x + 0 + 2y$$
$$\frac{\partial z}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x; y + \Delta y) - f(x, y)}{\Delta y}$$
$$\frac{\partial z}{\partial y} = 2x + 2y$$

here, $x \to$ constant

1. find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ of the following
   i) $z = x^3 + 3x^2 y + xy^3$
2. $z = \tan^{-1}(y/x)$
3. $z = 2x \cos y + 3x^2 y$
4. $z = x^3 - x^2 \sin y - y$

$$\text{i)}\ \frac{\partial z}{\partial x} = 3x^2 + 6xy + y^3$$

$$3y = \frac{\partial z}{\partial y} = 3x^2 + 3y^2 x$$

$$\frac{\partial z}{\partial x} = \frac{1}{x^1}\frac{\partial}{\partial x}\tan^{-1}(y/x)$$

$$= \frac{1}{1+(y/x)^2} - \frac{\partial}{\partial x}(y/x) = \frac{1}{1+\frac{y^2}{x^2}} \quad y\left(-1/x^2\right)$$

$$= \frac{x}{x^2+y^2}\frac{y}{x^2} = \frac{-y}{x^2+y^2\|}$$