Trigonometry

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1 Basic Identities

- $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)} = \dfrac{1}{\cot(\theta)}$
- $\cot(\theta) = \dfrac{\cos(\theta)}{\sin(\theta)} = \dfrac{1}{\tan(\theta)}$
- $\sec(\theta) = \dfrac{1}{\cos(\theta)}$
- $\csc(\theta) = \dfrac{1}{\sin(\theta)}$

2 Trigonometric table

Degree	$0°$	$30°$	$45°$	$60°$	$90°$	$180°$	$270°$	$360°$
Radian	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined	0	Not defined	0
cosec	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	Not defined	-1	Not defined
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined	-1	Not defined	1
cot	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Not defined	0	Not defined

3 Pythagorian Relations

- $\sin^2(\theta) + \cos^2(\theta) = 1$
- $\sec^2(\theta) - \tan^2(\theta) = 1$
- $\csc^2(\theta) - \cot^2(\theta) = 1$

4 Sign of trigonometric functions for negative angles

- $\sin(-x) = -\sin(x)$
- $\cos(-x) = \cos(x)$

- $\tan(-x) = -\tan(x)$
- $\csc(-x) = -\csc(x)$
- $\sec(-x) = \sec(x)$
- $\cot(-x) = -\cot(x)$

5 Expansion for trigonometric functions with two angles

- $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
- $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
- $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
- $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
- $\tan(x+y) = \dfrac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$
- $\tan(x-y) = \dfrac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
- $\cot(x+y) = \dfrac{\cot(x)\cot(y)-1}{\cot(x)+\cot(y)}$
- $\cot(x-y) = \dfrac{\cot(x)\cot(y)+1}{\cot(y)-\cot(x)}$
- **With** π
  1. $\sin(\frac{\pi}{2}+\theta) = cos(\theta)$
  2. $\sin(\pi-\theta) = sin(\theta)$
  3. $\cos(\frac{\pi}{2}+\theta) = -sin(\theta)$
  4. $\cos(\pi-\theta) = -cos(\theta)$

6 Product formula

- $\sin(x+y) + \sin(x-y) = 2\sin(x)\cos(y)$
- $\sin(x+y) - \sin(x-y) = 2\cos(x)\cos(y)$
- $\cos(x+y) + \cos(x-y) = 2\cos(x)\cos(y)$
- $\cos(x+y) - \cos(x-y) = -2\sin(x)\sin(y)$

7 Sum formula

- $\sin(a) + \sin(b) = 2\sin\left(\dfrac{a+b}{2}\right)\cos\left(\dfrac{a-b}{2}\right)$
- $\sin(a) - \sin(b) = 2\cos\left(\dfrac{a+b}{2}\right)\sin\left(\dfrac{a-b}{2}\right)$
- $\cos(a) + \cos(b) = 2\cos\left(\dfrac{a+b}{2}\right)\cos\left(\dfrac{a-b}{2}\right)$
- $\cos(a) - \cos(b) = -2\sin\left(\dfrac{a+b}{2}\right)\sin\left(\dfrac{a-b}{2}\right)$

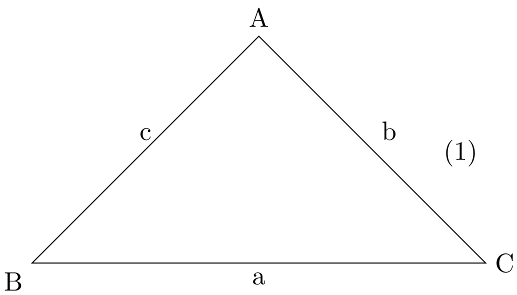
8 Expansion for multiple angles

- $\sin(2x) = 2sin(x)cos(x) = \dfrac{2\tan(x)}{1+tan^2(x)}$
- $\sin(3x) = 3\sin(x) - 4\sin^3(x)$
- $\cos(2x) = \cos^2(x) - \sin^2(x)$
- $\cos(3x) = 4\cos^3(x) - 3\cos(x)$
- $\tan(2x) = \dfrac{2\tan(x)}{1-\tan^2(x)}$
- $\tan(3x) = \dfrac{3\tan(x) - \tan^3(x)}{1-3\tan^2(x)}$
- $\sin^2(x) = \dfrac{1-\cos(2x)}{2}$
- $\cos^2(x) = \dfrac{1+\sin(2x)}{2}$

9 Law of sines

In any triangle, sides are proportional to to the sins of the angles

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c} \qquad or \qquad \frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$



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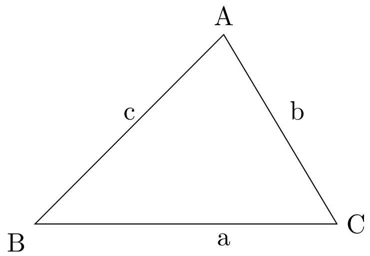
10 Law of cosines

For any ΔABC

$$a^2 = b^2 + c^2 - bc\cos(A)$$
$$b^2 = a^2 + c^2 - ac\cos(B)$$
$$c^2 = a^2 + b^2 - ab\cos(C)$$



also

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

From above equations, we also get:

$$\frac{\cos(A)}{a} = \frac{\cos(B)}{b} = \frac{\cos(C)}{c} = \frac{a^2 + b^2 + c^2}{2abc}$$

11 Solutions of some trigonometric equations

- $\sin(x) = 0 \implies x = n\pi,\ n \in \mathbb{Z}$
- $\cos(x) = 0 \implies x = (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$
- $\sin(x) = \sin(y) \implies x = n\pi + (-1)^n y,\ n \in \mathbb{Z}$
- $\cos(x) = \cos(y) \implies x = n\pi \pm y,\ n \in \mathbb{Z}$
- $\tan(x) = \tan(y) \implies x = n\pi + y,\ n \in \mathbb{Z}$
- $\tan(x) = 0 \implies x = n\pi,\ n \in \mathbb{Z}$

12 Other useful stuffs...

- $\tan\left(\frac{\pi}{4} - x\right) = \frac{1 - \tan(x)}{1 + \tan(x)}$
- $\tan\left(\frac{\pi}{4} + x\right) = \frac{1 + \tan(x)}{1 - \tan(x)}$
- $\dfrac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left[\frac{1 + \tan(x)}{1 - \tan(x)}\right]^2$
- $\sin(x+y)\sin(x-y) = \sin^2(x) - \sin^2(y)$
- $\sin(x+y)\sin(x-y) = \cos^2(x) - \sin^2(y)$
- $\tan(3x)\tan(2x)\tan(x) = \tan(3x) - \tan(2x) - \tan(x)$

13 Some *Condensed* trigonometric identities

- $\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$
- $\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$
- $\tan(x \pm y) = \frac{\tan(x) \pm \tan(y)}{1 \mp \tan(x)\tan(y)}$
- $\cot(x \pm y) = \frac{\cot(x)\cot(y) \mp 1}{\cot(x) \pm \cot(y)}$
- $\tan\left(\frac{\pi}{4} \pm x\right) = \frac{1 \pm \tan(x)}{1 \mp \tan(x)}$