Color depth	Bytes of storage per pixel	Common name
4bit	0.5	standard
8bit	1	256 color mode
16bit	2	High color
24 bit	3	True color

Q. A monitor with a horizontal scanning frequency of 96 KHz at a resolution of 1280 X 1024. Determine refresh rate based on calculation.

Soln;

96 KHz

$\Rightarrow 96 \times 10^3 Hz$

$$f_r = \frac{f_H}{\text{no. of } \cancel{\text{vertical}} \text{ horizontal lines}} * 0.95$$

$$= \frac{96 \times 10^3}{\cancel{1280} \times 1024} \times 0.95$$

$= 89.06$

$\approx 89 Hz$.

Q. If pixel are accessed from the frame buffer with an average access the 300ns. Then will this rate produce the flickering effect? screen resolution (640 X 480)

Soln;

Access time for 1 pixel = 300ns

" " " (640 x 480) pixel = 640 x 480 x 300 ns

we know,

$$\text{frequency} = 1/t = \frac{1}{640 \times 480 \times 300 \times 10^{-9}}$$

= 10.85 fps less than 50 fps.

∴ Flickering.

Date
Page

Q. If the total intensity available for a pixel is 256 and screen resolution is 640 x 480. What will be size of the frame buffer.

Soln,

Size in frame buffer for 1 pixel = 8 bit.

1 byte = 8 bits

for 640 x 480 = 640 x 480 x 8 bits

= 300Kb

Q. Consider 256 pixel x 256 scan lines image with 24 bit true color. If 10 min video is required to capture. Calculate the total memory required?

Soln:

for 1 sec = (256 x 256 x 3 x 50) bytes

for 10 min = (256 x 256 x 3 x 50 x 10 x 60) bytes

$$= \frac{5898240000}{1024 \times 1024 \times 1024}$$

= 5.49 GB

Hardware Concept :-

1. Tablet → Electrical
   → Sonic
   → Resistive

3. Light pen
4. Keyboard & mouse
5. Barcode Scanner / reader
6. Data glove.

2. Touch panels → optical
   → sonic
   → electric

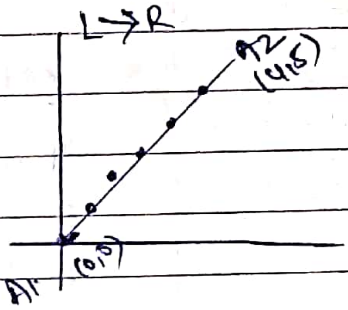
Advantage :

- faster method than simple line drawing algorithm for calculating pixel position as it eliminates multiplication.
- Avoids directly deal of eqn. of st. line $y = mx + c$.

Disadvantages :-

- 'm' is usually stored in floating point number.
- There could be round off error.
- The line will move away from the true line path especially when it is long due to sucessive round off error.
- Accumulation of round off error in sucessive addition of floating point increament.

Q.No.1 Plot a straight line between (0,0) & (4, 5)



Solution;

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-0}{4-0} = 1.25 > 1$$

$$\text{or, } 1/m = 1/1.25 = 0.8$$

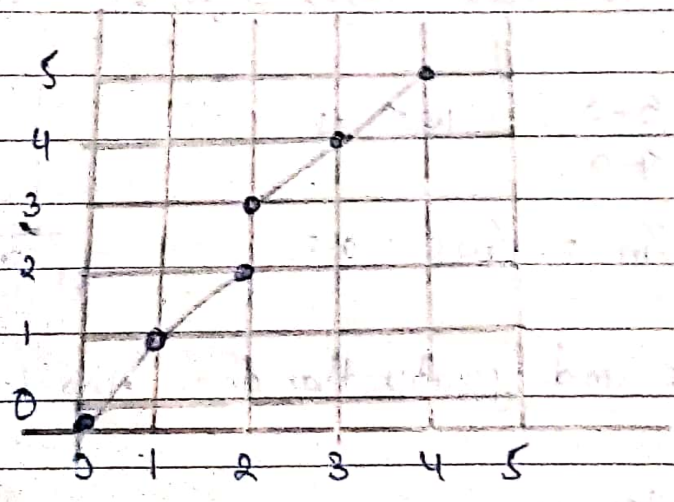
Since m is positive and greater than one. And taking from left to Right.

$y \rightarrow$ unit intervals

$x \rightarrow x_k + 1/m$

$$\therefore (x_{k+1}, y_{k+1}) = \left(x_k + \frac{1}{m}, y_k + 1\right)$$

x_k	y_k	x plot	y plot	(x_{k+1}, y_{k+1})	
0	0	0	0	(0,0)	→ start
0.8+0=0.8	0+1=1	0.8 ≈ 1	1	(1,1)	
0.8+0.8=1.6	1+1=2	1.6 ≈ 2	2	(2,2)	
1.6+0.8=2.4	2+1=3	2.4 ≈ 2	3	(2,3)	
2.4+0.8=3.2	3+1=4	3.2 ≈ 3	4	(3,4)	
3.2+0.8=4	4+1=5	4	5	(4,5)	stop



Q.No 2 Plot the straight line between $(0,0)$ & $(5,4)$.
Solution.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(4-0)}{(5-0)} = 4/5 = 0.8 < 1$$

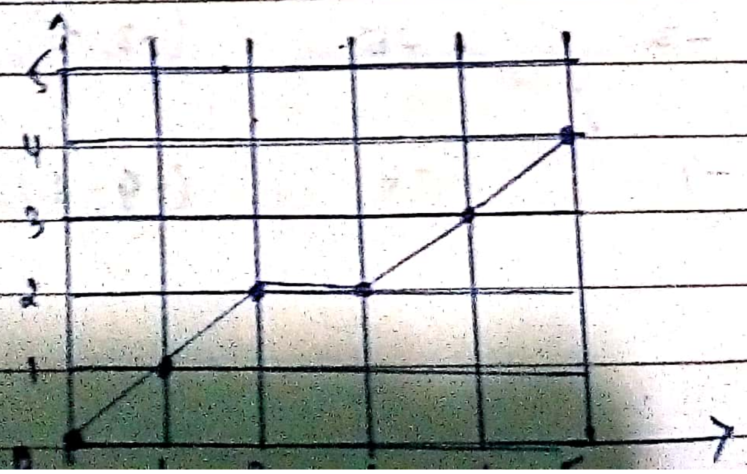
since m is +ve and ~~greater~~ less than one.
And taking from left to right.

$x \rightarrow$ unit intervals

$y \rightarrow y_k + m$

$\therefore (x_{k+1}, y_{k+1}) = (x_k + 1, y_k + m)$

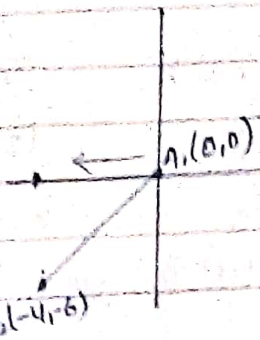
x_k	y_k	x plot	y plot	(x_{k+1}, y_{k+1})
0	0	0	0	$(0,0)$ → start
0+1=1	0+0.8=0.8	1	$0.8 \approx 1$	$(1,1)$
1+1=2	0.8+0.8=1.6	2	$1.6 \approx 2$	$(2,2)$
2+1=3	1.6+0.8=2.4	3	$2.4 \approx 2$	$(3,2)$
3+1=4	2.4+0.8=3.2	4	$3.2 \approx 3$	$(4,3)$
4+1=5	3.2+0.8=4.0	5	$4 \approx 4$	$(5,4)$ stop



Q.No.2) Plot the st. line betn (0,0) & (-4,-6).

Solution

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-6-0)}{(-4-0)} = \frac{-6}{-4} = 1.5$



$m = 1.5 > 1$; $1/m = 1/1.5 = 0.66$

since m is positive and greater than one.

And taking from right to left

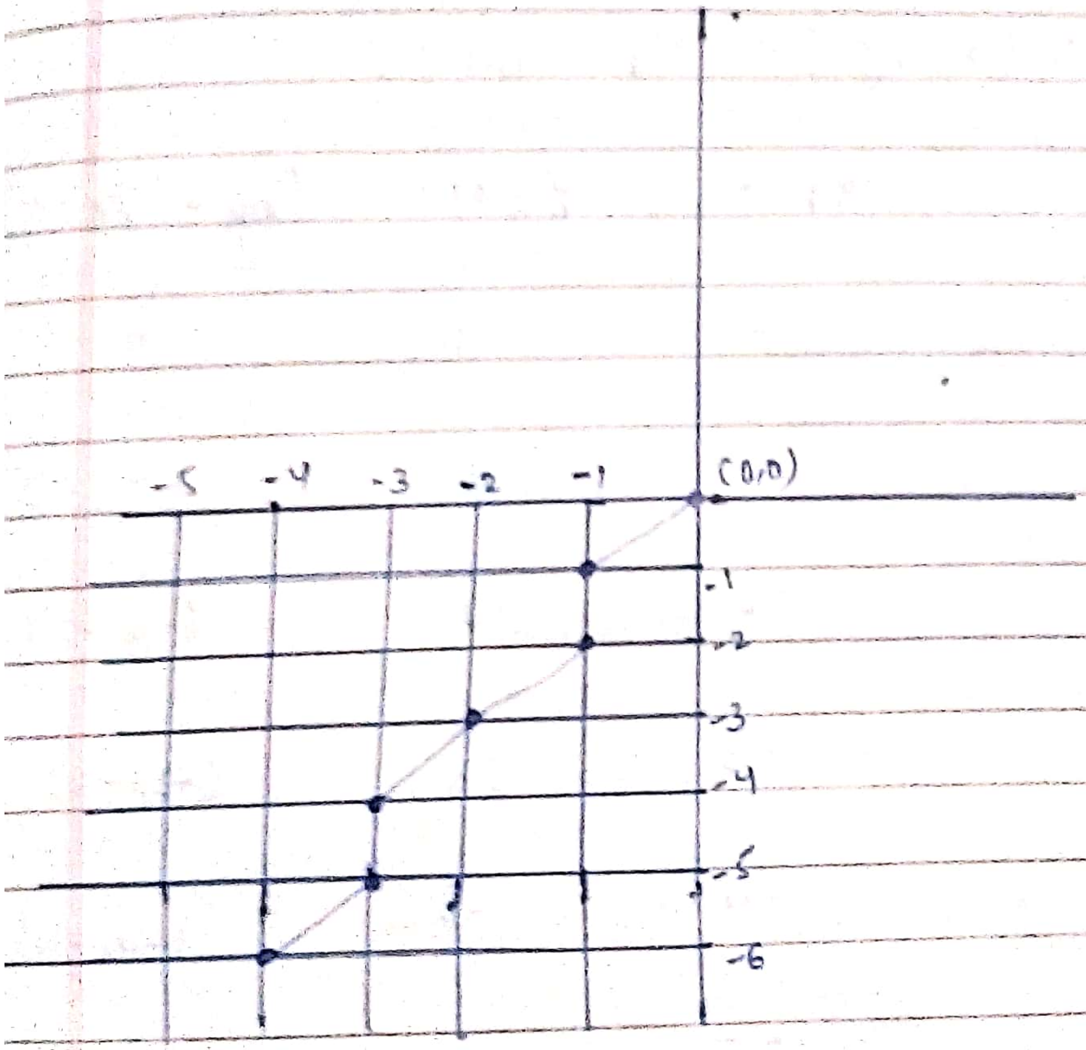
$y = y_k - 1$

$x = x_k - 1/m$

$\therefore (x_k, y_k) = (x_k - 1/m, y_k - 1)$

x_k	y_k	x plot	y plot	$(x_k - 1/m, y_k - 1)$
0	0	0	0	(0,0)
-0.66	-1	$-0.66 \approx -1$	-1	(-1,-1)
-1.32	-2	-1	-2	(-1,-2)
-1.98	-3	-2	-3	(-2,-3)
-2.64	-4	-3	-4	(-3,-4)
-3.3	-5	-3	-5	(-3,-5)
-3.96	-6	$-3.96 \approx -4$	-6	(-4,-6)

~~-4.62~~


-5
-4
-3
-2
-1
(0,0)
-1
-2
-3
-4
-5
-6

Q.No.4 plot the straight line between (0,0) & (-6,-6).

solution,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-6-0)}{(-6-0)} = -6/-6 = 1$$

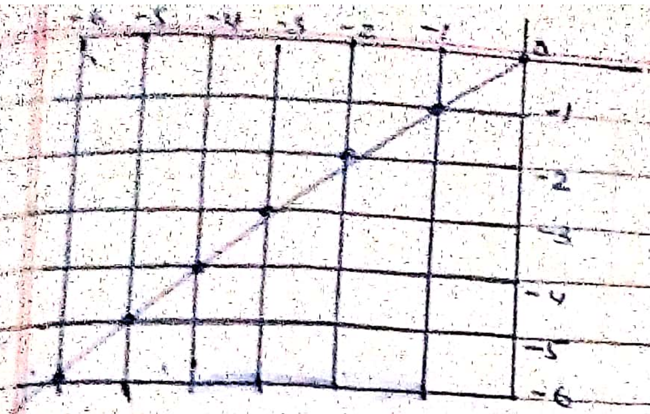
$m = 1$ (since m is equals to 1 and positive, taking right to left

$x = x_K - 1$

$y = y_K - m$

$\therefore (x_K, y_K) = (x_K - 1, y_K - m)$

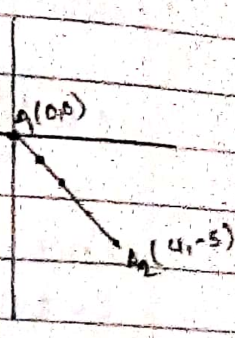
x_K	y_K	x plot	y plot	$(x_K - 1, y_K - m)$
0	0	0	0	0
-1	-1	-1	-1	(-1, -1)
-2	-2	-2	-2	(-2, -2)
-3	-3	-3	-3	(-3, -3)
-4	-4	-4	-4	(-4, -4)
-5	-5	-5	-5	(-5, -5)
-6	-6	-6	-6	(-6, -6)



Q.no.5. Plot the straight line between (0,0) & (4,-5)

Solution.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-0}{4-0} = -5/4 = -1.25 < 1$



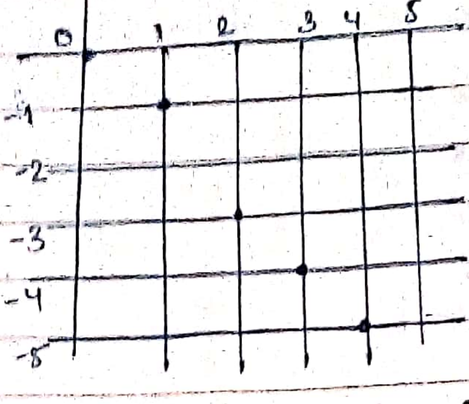
Since m is negative and less than one. so taking left to right.

$x_k = x_k + 1$

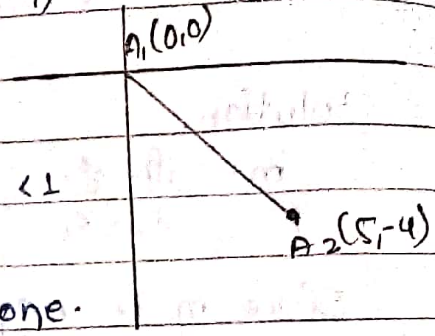
$y_k = y_k + m$

$\therefore (x_k, y_k) = (x_k + 1, y_k + m)$

x_k	y_k	xplot	yplot	(x_k+1, y_k+m)
0	0	0	0	(0,0)
1	-1.25	1	$-1.25 \approx -1$	(1,-1)
2	-2.5	2	$-2.5 \approx -3$	(2,-3)
3	-3.75	3	$-3.75 \approx -4$	(3,-4)
4	-5	4	-5	(4,-5)



Q.no 6. Plot the st. line between (0,0) & (5,-4).



Solution,

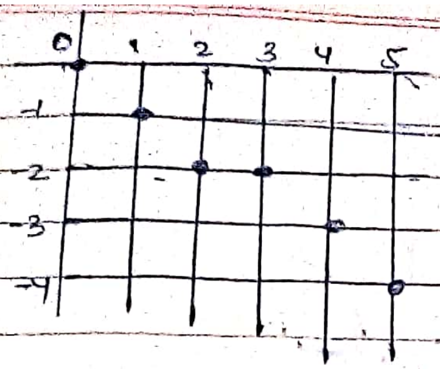
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-0}{5-0} = \frac{-4}{5} = -0.8 < 1$$

Since m is negative and less than one.
So taking left to right.

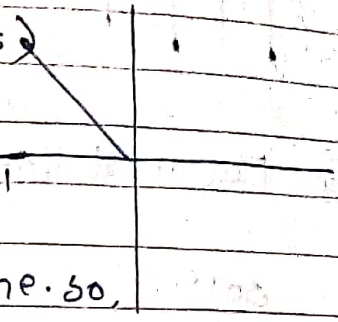
$x_k = x_k + 1$ $\therefore (x_k, y_k) = (x_k+1, y_k+m)$

$y_k = y_k + m$

x_k	y_k	xplot	yplot	(x_k+1, y_k+m)
0	0	0	0	(0,0)
1	-0.8	1	$-0.8 \approx -1$	(1,-1)
2	-1.6	2	$-1.6 \approx -2$	(2,-2)
3	-2.4	3	$-2.4 \approx -2$	(3,-2)
4	-3.2	4	$-3.2 \approx -3$	(4,-3)
5	-4.0	5	$-4 \approx -4$	(5,-4)



Q.no.7 Plot the st. line between (0,0) & (-4,5)



Soln

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-0}{-4-0} = \frac{5}{-4} = -1.25 < 1$

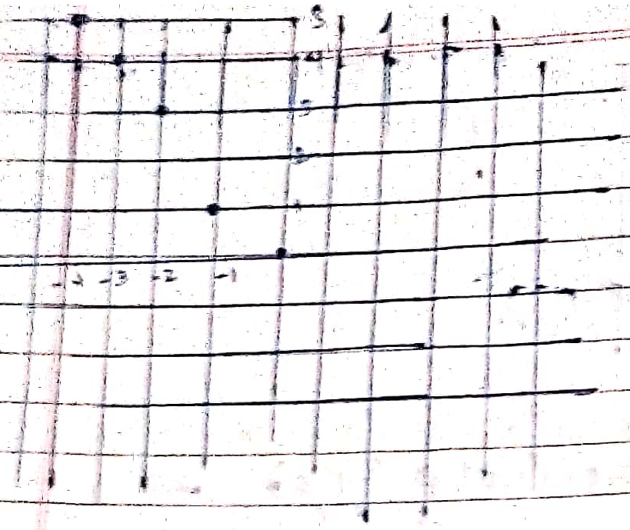
Since m is negative and less than one. so, taking right to left.

$x_K = x_K - 1$

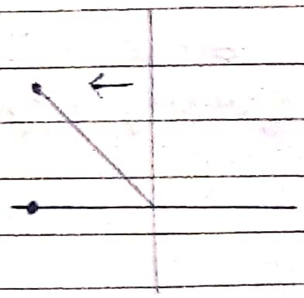
$y_K = y_K - m$

$\therefore (x_K, y_K) = (x_K - 1, y_K - m)$

x_K	y_K	xplot	yplot	(x_K-1, y_K-m)
0	0	0	0	(0,0)
-1	1.25	-1	1.25 ≃ 1	(-1,1)
-2	2.5	-2	2.5 ≃ 3	(-2,3)
-3	3.75	-3	3.75 ≃ 4	(-3,4)
-4	5	-4	5	(-4,5)



Q.8. Plot a st line between $(0,0)$ & $(-5,4)$



Soln

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{4-0}{-5-0}$

$= 4/_{-5} = -0.8 < 1$

Since, m is -ve and is less than 1 and taking right to left.

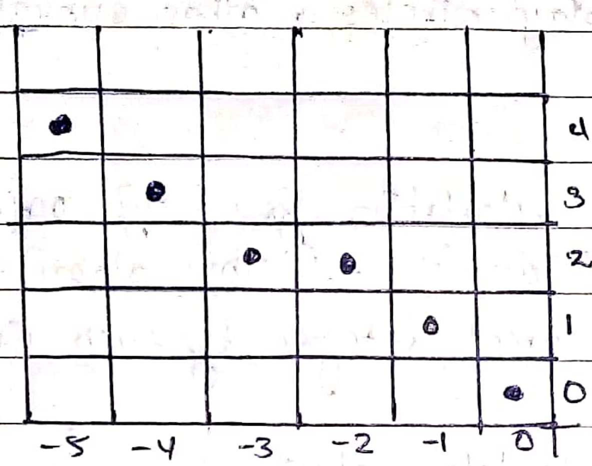
$x_{k+1} \rightarrow$ unit interval

$y_{k+1} \rightarrow y_k - m$

$(x_{k+1}, y_{k+1}) = (x_k - 1, y_k - m)$

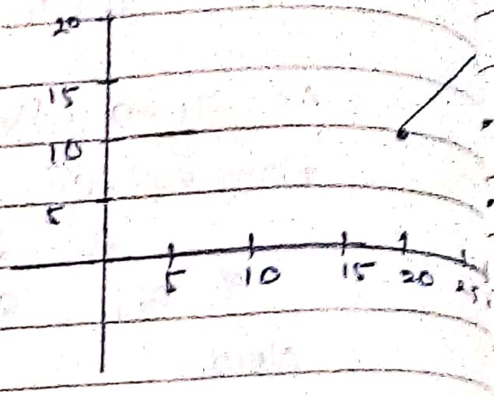
x_k	y_k	xplot	yplot	(x_{k+1}, y_{k+1})
0	0	0	0	(0, 0)
-1	0.8	-1	$0.8 \approx 1$	(-1, 1)
-2	1.6	-2	$1.6 \approx 2$	(-2, 2)

-3	2.4	-3	24 ≈ 2	(-3, 2)
-4	3.2	-4	3.2 ≈ 3	(-4, 3)
-5	4	-5	~~5~~ 4	(-5, 4)



Example:
Digitize the line with endpoints $(\overset{x_1}{20}, \overset{y_1}{10})$ & $(\overset{x_2}{30}, \overset{y_2}{18})$

Soln:



$\Delta x = |x_2 - x_1| = |30-20| = 10$

$\Delta y = |y_2 - y_1| = |18-10| = 8$

$m = \frac{\Delta y}{\Delta x} = 8/10 = 0.8 < 1$

Initial parameter $P_0 = 2\Delta y - \Delta x = 2 \times 8 - 10$
$= 16 - 10 = 6$

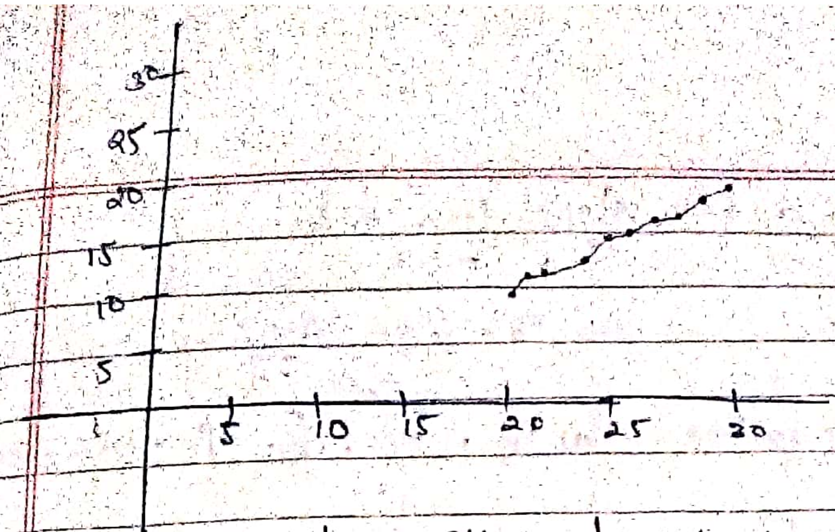
$2\Delta y - 2\Delta x = 2\times 8 - 2\times 10 = -4$

$2\Delta y = 2\times 8 = 16$

Now, plot $(x_0, y_0) = (20, 10)$

K	P_K	(x_{k+1}, y_{k+1})
0	$P_0 = 6$	(21, 11)
1	$P_1 = 2$	(22, 12)
2	$P_2 = -2$	(23, 12)
3	$P_3 = 14$	(24, 13)
4	$P_4 = 10$	(25, 14)
5	$P_5 = 6$	(26, 15)
6	$P_6 = 2$	(27, 16)
7	$P_7 = -2$	(28, 16)
8	$P_8 = 14$	(29, 17)
9	$P_9 = 10$	(30, 18)

$P_1 = P_0 + 2\times 8 - 2\times 10$
$= 6 + 16 - 20$
$= 2$



Q Digitize the line with end points (15,8) & (20,18)

Soln

$\Delta y = |18-8| = 10$

$\Delta x = |x_2 - x_1| = |20-15| = 5$

$m = \frac{\Delta y}{\Delta x} = 10/5 = 2 > 1$

Initial parameter $(P_0) = 2\Delta x - \Delta y = 2\times5 - 10 = 0$

$2\Delta x - 2\Delta y = 2\times5 - 2\times10 = 10 - 20 = -10$

$2\Delta x = 2\times5 = 10$

Now plot $(x_0, y_0) = (15, 8)$

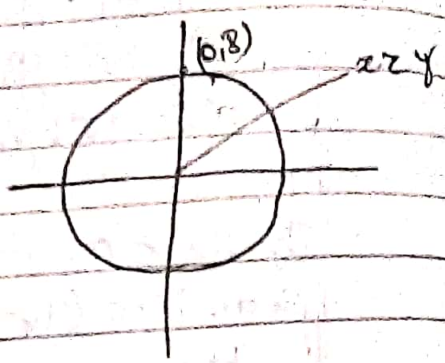
K	P_K	x_{K+1}, y_{K+1}
0	$P_0 = 0$	(16, 9)
1	$P_1 = -10$	(16, 10)
2	$P_2 = 0$	(17, 11)
3	$P_3 = -10$	(17, 12)
4	$P_4 = 0$	(18, 13)
5	$P_5 = -10$	(18, 14)
6	$P_6 = 0$	(19, 15)
7	$P_7 = -10$	(19, 16)
8	$P_8 = 0$	(20, 17)
9	$P_9 = -10$	(20, 18)

Formula

$\boxed{\begin{array}{l} (a)\ P_K < 0 \to NC \to (x_K+1, y_K), \quad P_{K+1} = P_K + 2x_{K+1} + 1 \\ (b)\ P_K > 0 \to NC \to (x_{K+1}, y_K - 1), \quad P_{K+1} = P_K + 2x_{K+1} - 2y_{K+1} + 1 \end{array}}$

Examples

1. Draw a circle with radius 8



Solution

Initial decision parameter (P_0):

$= 1 - r$

$= 1 - 8$

$= -7$

K	(x_K, y_K)	P_K	x_{K+1}, y_{K+1}	Remarks
0	(0, 8)	-7	(1, 8)	$-7 + 2*1+1 = -4$
1	(1, 8)	-4	(2, 8)	$-4+2*2+1 = 1$
2	(2, 8)	1	(3, ~~8~~7)	$1+2*3-2*\cancel{8}7+1 = -6$
3	(3, 7)	-6	(4, 7)	$-6+2*4+1 = 3$
4	(4, 7)	3	(5, 6)	$3+2*5-2*6+1 = 2$
5	(5, 6)	2	(6, 5)	
			$x \geq y$	
			stop	

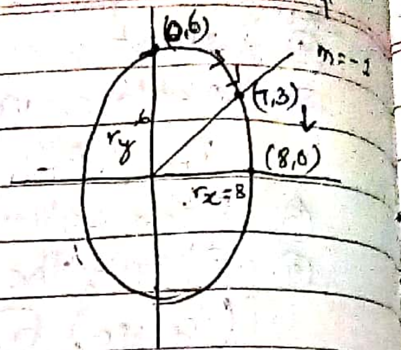
(x, y) $Q_1(x_k, y_k)$	$(-x, y)$ $Q_2(x_k, y_k)$	$(-y, -x)$ $Q_3(x_k, y_k)$	$(y, -x)$ $Q_4(x_k, y_k)$
(0,8)	(0,8)	(-8,0)	(8,0)
(1,8)	(-1,8)	(-8,-1)	(8,-1)
(2,8)	(-2,8)	(-8,-2)	(8,-2)
(3,~~8~~7)	(-3,7)	(-7,-3)	(7,-3)
(4,7)	(-4,7)	(-7,-4)	(7,-4)
(5,6)	(-5,6)	(-6,-5)	(6,-5)

$Q_5(y, x)$	$Q_6(-y, x)$	$Q_7(-x, -y)$	$Q_8(x, -y)$
(6,5)	(-6,5)	(-5,-6)	(0,-8)
(7,4)	(-7,4)	(-4,-7)	(1,-8)
(7,3)	(-7,3)	(-3,-7)	
(8,2)	(-8,2)	(-2,-8)	
(8,1)	(-8,1)	(-1,-8)	
(8,0)	(-8,0)	(-8,0)	

$2x_{k+1} r_y^2 \geq 2y_{k+1} r_x^2$

Example;

Draw ellipse with $r_x = 8$ & $r_y = 6$.



solution.

Here, for octave 1

$(x_0, y_0) = (0, 6)$

$$P_{10} = r_y^2 + \frac{r_x^2}{4} - r_y r_x^2$$

$$= 6^2 + \frac{8^2}{4} - 6 \times 8^2$$

$$= -332 < 0$$

condition,

$P_{1K} < 0$	$P_{1K} > 0$
(x_{k+1}, y_{k+1}) NC $= (x_k+1, y_k)$	NC $= (x_k+1, y_k-1)$
$P_{1k+1} = P_{1k} + 2r_y^2 x_{k+1} + r_y^2$	$P_{1k+1} \rightarrow P_{1K} + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_y^2$

K	x_k, y_k	P_{1K}	$2x_{k+1} r_y^2$		$2y_{k+1} r_x^2$	x_{k+1}, y_{k+1}
0	(0,6)	-332<0	~~(1,6)~~ 72	<	736	(1,6)
1	(1,6)	-224<0	144	<	768	(2,6)
2	(2,6)	-44<0	216	<	640	(3,6)
3	(3,6)	208	288	<	640	(4,5)
4	(4,~~6~~5)	-108	360	<	640	(5,5)
5	(5,5)	288	432	<	512	(6,4)
6	(6,4)	244	504	>	384	(7,3)

True (m=-1)

since $2x_{k+1} r_y^2 \geq 2y_{k+1} r_x^2$. We enter an region 2 and coordinate (7,3) will be initial decision parameter for region 2. $[(x_k, y_k) = (7,3)]$

so,

$$P_{20} = (x_k + 1/2)^2 r_y^2 + (y_k - 1)^2 r_x^2 - r_x^2 r_y^2$$
$$= -23 < 0$$

conditions,

$P_{2k} \leq 0$	$P_{2k} \geq 0$
$NC \to (x_k+1, y_k-1)$	$NC \to (x_k, y_k-1)$
$P_{2k+1} = P_{2k} + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_x^2$	$P_{2k+1} = P_{2k} - 2r_x^2 y_{k+1} + r_x^2$

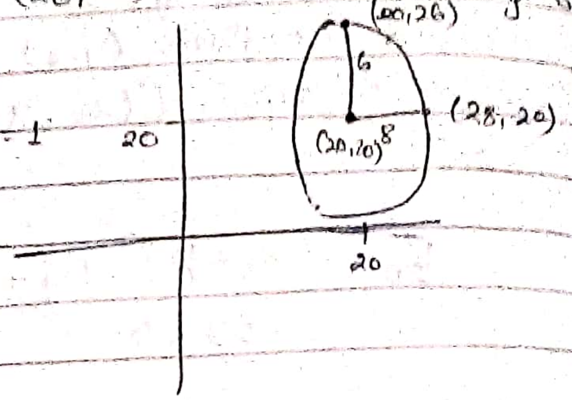
k	x_k, y_k	P_{2k}	(x_{k+1}, y_{k+1})	~~$2x_{k+1} r_y^2$~~	$2r_x^2 y_{k+1}$
7	(7,3)	-23	(8, 2)		
8	(8,2)	261	(8, 1)		
9	(8,1)	297	(8,0)		

stop

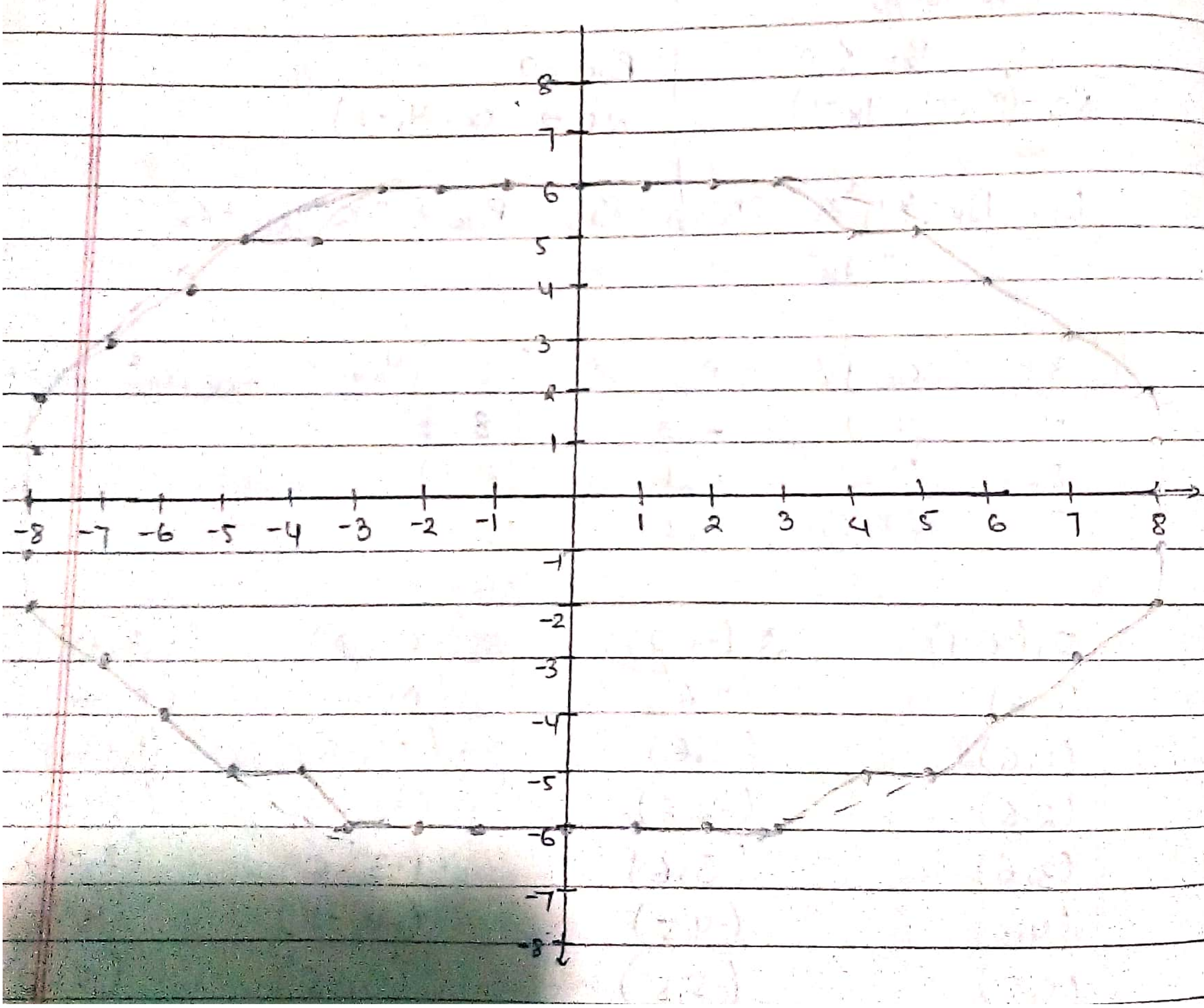
$Q_1(x,y)$	$Q_2(-x,y)$	$Q_3(-x,-y)$	$Q_4(x,-y)$
(0,6)	(0,6)	(0,-6)	(0,-6)
(1,6)	(-1,6)	(-1,-6)	(1,-6)
(2,6)	(-2,6)	(-2,-6)	(2,-6)
(3,6)	(-3,6)	(-3,-6)	(3,-6)
(4,5)	(-4,5)	(-4,-5)	(4,-5)
(5,5)	(-5,5)	(-5,-5)	(5,-5)
(6,4)	(-6,4)	(-6,-4)	(6,-4)
(7,3)	(-7,3)	(-7,-3)	(7,-3)
(8,2)	(-8,2)	(-8,-2)	(8,-2)
(8,1)	(-8,1)	(-8,-1)	(8,-1)
(8,0)	(-8,0)	(-8,0)	(8,0)

Q. Draw an ellipse with center (20,20) and $r_x = 8$ & $r_y = 6$

Q. Draw an ellipse $\frac{(x-2)^2}{25} + \frac{y+5}{36} = 1$



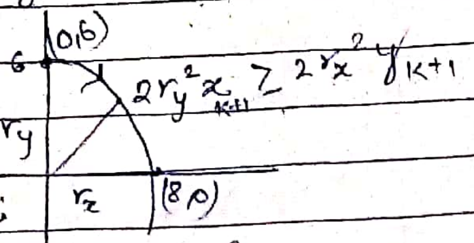
Center = (2,-5) $r_x = 5$, $r_y = 6$



Here, Given that,

$r_y = 6, \ r_x = 8$

center $(x, y) = (20, 20)$



In octane 0_1;

$$P_{1K} = \frac{r_x^2}{4} + r_y^2 - r_y \cdot r_x^2$$

$$= \frac{64}{4} + 36 - 6 \cdot 64$$

$$= -332 < 0$$

Conditions;

if $P_{1K} > 0$	$P_{1K} < 0$
$NC \to (x_{K}+1, y_K - 1)$	$NC \to (x_K+1, y_K)$
$P_{1K+1} = P_{1K} + 2r_y^2 x_{K+1} - 2r_x^2 y_{K+1} + r_y^2$	$P_{1K+1} = P_{1K} + 2r_y^2 x_{K+1} + r_y^2$

K	(x_K, y_K)	P_{1K}	(x_{K+1}, y_{K+1})	$2r_y^2 x_{K+1}$	$2r_x^2 y_{K+1}$
0	(0, 6)	-332	(1, 6)	72	< 768
1	(1, 6)	-224	(2, 6)	144	< 768
2	(2, 6)	-44	(3, 6)	216	< 768
3	(3, 6)	208	(4, 5)	288	< 640
4	(4, 5)	-108	(5, 5)	360	< 640
5	(5, 5)	288	(6, 4)	432	< 512
6	(6, 4)	244	(7, 3)	504	> 384

True

since $2x_{k+1} r_y^2 \geq 2y_{k+1} r_x^2$. we enter into an region 2 and coordinate (7,3) will be initial decision parameter for O_2

$$[x_k, y_k] = [(7,3)]$$

$$~~(x_k + 1/2, y_k - 1)~~$$

So,

$$P_{20} = (x_k + 1/2)^2 r_y^2 + (y_k - 1)^2 \cdot r_x^2 - r_x^2 \cdot r_y^2$$

$$= (7+0.5)^2 \cdot 36 + (3-1)^2 \cdot 64 - 36 \cdot 64$$

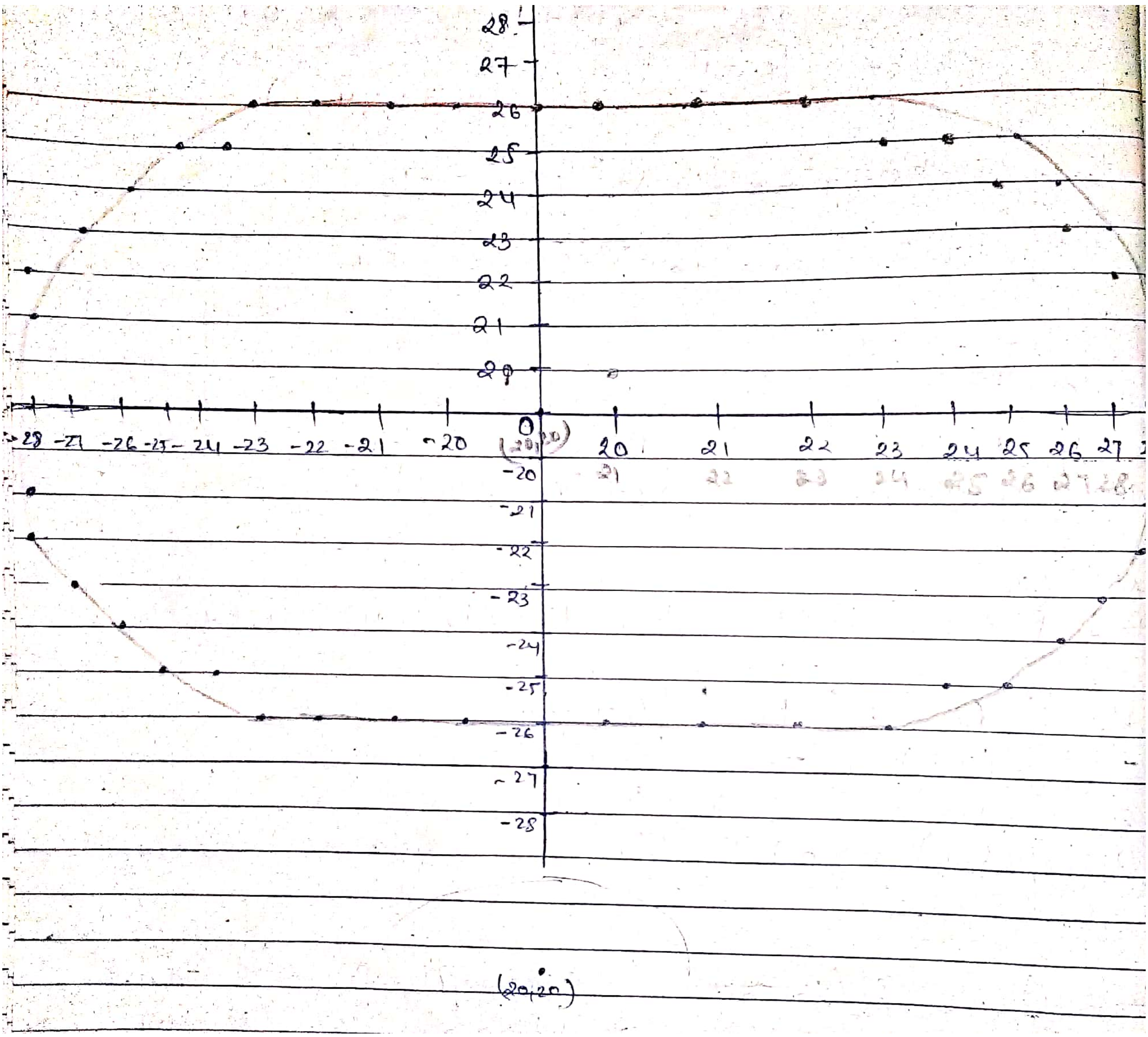
$$= -23 < 0$$

Condition;

If $P_{2k} > 0$	$P_{2k} < 0$
$NC \rightarrow (x_k, y_k - 1)$	$NC \rightarrow (x_k + 1, y_k - 1)$
$P_{2k+1} = P_{2k} - 2r_x^2 y_{k+1} + r_x^2$	$P_{2k+1} = P_{2k} + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_x^2$

K	(x_k, y_k)	P_{2k}	(x_{k+1}, y_{k+1})
7	(7,3)	$-23 < 0$	(8, 2)
8	(8,2)	$361 > 0$	(8, 1)
9	(8,1)	$297 > 0$	(8, 0)
		Stop.	

$Q_1(x, y)$	$Q_2(-x, y)$	$Q_3(-x, -y)$	$Q_4(x, -y)$
(20,26)	(-20,26)	(-20,-26)	(20,-26)
(21,26)	(-21,26)	(-21,-26)	(21,-26)
(22,26)	(-22,26)	(-22,-26)	(22,-26)
(23,26)	(-23,26)	(-23,-26)	(23,-26)
(24,25)	(-24,25)	(-24,-25)	(24,-25)
(25,25)	(-25,25)	(-25,-25)	(25,-25)
(26,24)	(-26,24)	(-26,-24)	(26,-24)
(27,23)	(-27,23)	(-27,-23)	(27,-23)
(28,22)	(-28,22)	(-28,-22)	(28,-22)
(28,21)	(-28,21)	(-28,-21)	(28,-21)


28
27
26
25
24
23
22
21
20
0
-28 -27 -26 -25 -24 -23 -22 -21 -20
20 21 22 23 24 25 26 27
-20
-21
-22
-23
-24
-25
-26
-27
-28
(20,20)

Q. Draw a circle of radius 10.

Solution.

Given r = 10

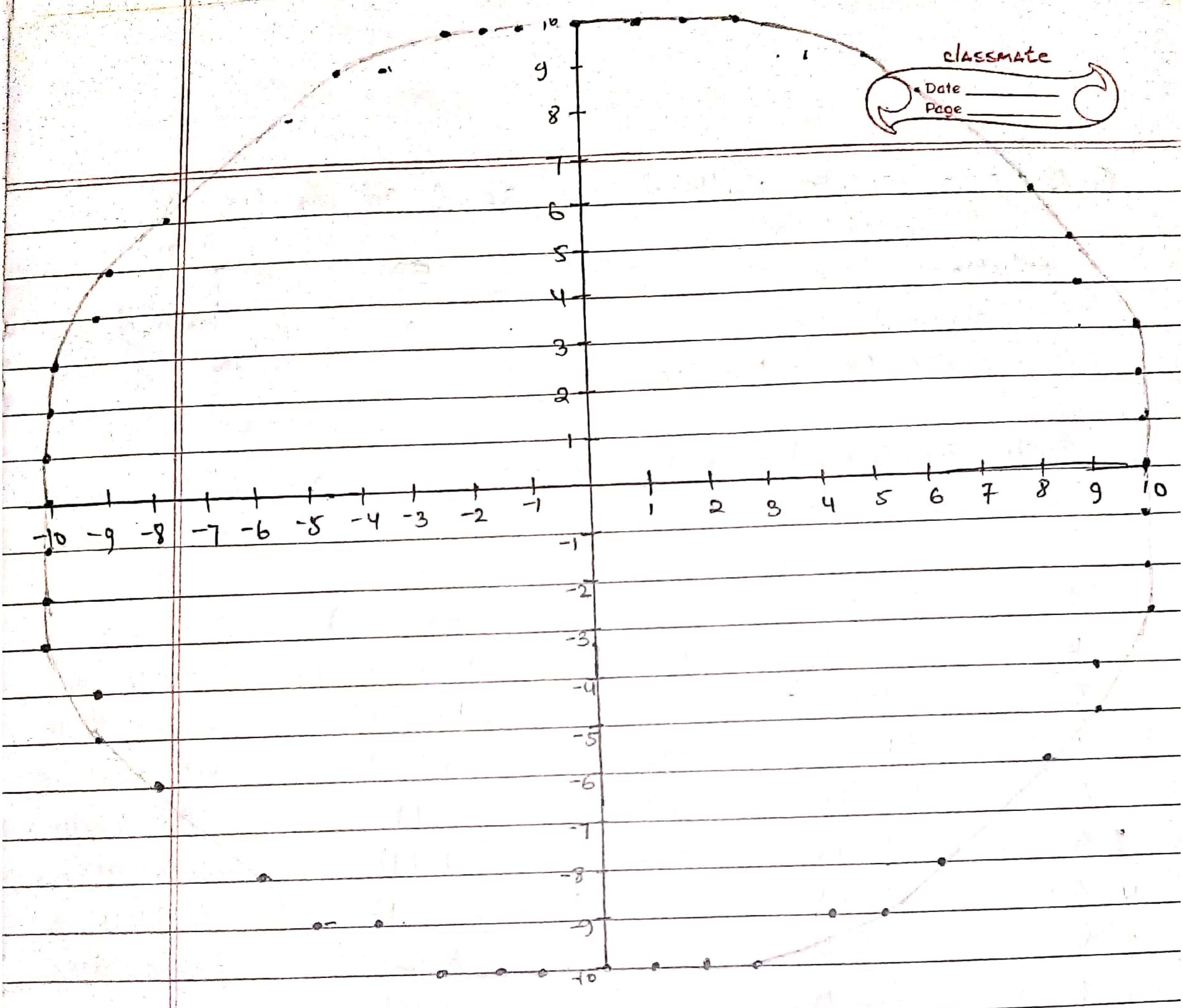
Initial decision parameter (P_0) = $1 - r$

= $1 - 10$

= -9

K	(x_K, y_K)	P_K	(x_{K+1}, y_{K+1})	Remarks
0	(0,10)	-9	(1,10)	$-9+2\times1+1=-6$
1	(1,10)	-6	(2,10)	$-6+2\times2+1=-1$
2	(2,10)	-1	(3,10)	$-1+2\times3+1=6$
3	(3,10)	6	(4,9)	$6+8-18+1=-3$
4	(4,9)	-3	(5,9)	$-3+2\times5+1=8$
5	(5,9)	8	(6,8)	$8+2\times6-2\times6+1=5$
6	(6,8)	5	(7,7)	

$Q_1 (x_K, y_K)$ (x,y)	$Q_2 (-x, y)$	$Q_3 (-x, -y)$	$Q_4 (x, -y)$
(0,10)	(0,10)	(0,-10)	(0,-10)
(1,10)	(-1,10)	(-1,-10)	(1,-10)
(2,10)	(-2,10)	(-2,-10)	(2,-10)
(3,10)	(-3,10)	(-3,-10)	(3,-10)
(4,9)	(-4,9)	(-4,-9)	(4,-9)
(5,9)	(-5,9)	(-5,-9)	(5,-9)
(6,8)	(-6,8)	(-6,-8)	(6,-8)
$Q_1 (y, x)$	(-y, x)	(-y, -x)	(y, -x)
(10,0)	(-10,0)	(-10,0)	(0,0)
(10,1)	(-10,1)	(-10,-1)	(10,-1)
(10,2)	(-10,2)	(-10,-2)	(10,-2)
(10,3)	(-10,3)	(-10,-3)	(10,-3)
(9,4)	(-9,4)	(-9,-4)	(9,-4)
(9,5)	(-9,5)	(-9,-5)	(9,-5)
(8,6)	(-8,6)	(-8,-6)	(8,-6)

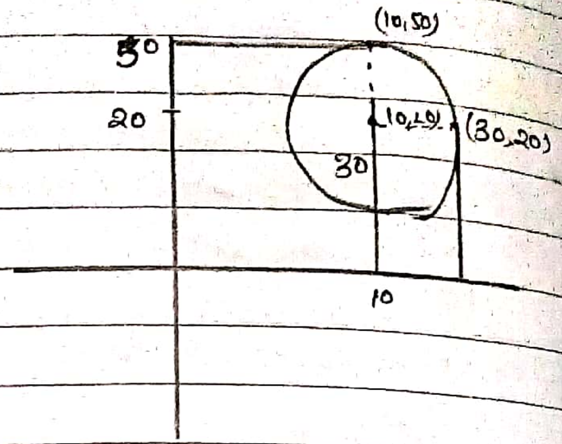

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1
1 2 3 4 5 6 7 8 9 10
10 9 8 7 6 5 4 3 2 1
-1 -2 -3 -4 -5 -6 -7 -8 -9 -10

Q. Draw a circle with radius 30 & center (10,20)

Solution

Given, r = 30

center $(x_k, y_k) = (10,20)$



Initial Decision parameter (P_0);

$= 1 - r$

$= 1 - 30$

$= -29$

K	(x_k, y_k)	P_k	(x_{k+1}, y_{k+1})	Remarks
0	(10,50)	-29	(11, 50)	-29+2×11+1=-6
1	(11,50)	-6	(12,50)	-6+2×12+1=19
2	(12,50)	19	(13,49)	19+2×13-2×49+1=-52
3 ~~4~~	(13,49)	-52	(14,49)	-52+2×14+1=-23
4 ~~5~~	(14,49)	-23	(15,49)	-23+2×15+1=8
5 ~~6~~	(15,49)	8	(16,48)	8+2×16-2×48+1=-55
6 ~~7~~	(16,48)	-55	(17,48)	-55+2×17+1=-20
7 ~~8~~	(17,48)	-20	(18,48)	-20+2×18+1=17
8 ~~9~~	(18,48)	17	(19,47)	17+2×19-2×47+1=-38
9 ~~10~~	(19,47)	-38	(20,47)	-38+2×20+1=3
10 ~~11~~	(20,47)	3	(21,46)	3+2×21-2×46+1=-46
11 ~~12~~	(21, 46)	-46	(22,46)	-46+2×22+1=-1
12 ~~13~~	(22,46)	-1	(23,46)	-1+2×23+1=46
13 ~~14~~	(23, 46)	46	(24,45)	46+2×24-2×45+1=5
14 ~~15~~	(24, 45)	5	(25,44)	5+2×25-2×44+1=-32
15 ~~16~~	(25,44)	-32	(26,44)	-32+2×26+1=21
16 ~~17~~	(26,44)	21	(27,43)	-21+2×27-2×43+1=-10
17 ~~18~~	(27,43)	-10	(28,43)	-10+2×28+1=47
18 ~~19~~	(28,43)	47	(29,42)	47+2×29-2×42+1=22
19 ~~20~~	(29,42)	22	(30,41)	22+2×30-2×41+1=-1

20	(30,41)	1	(31,40)	1+2×31−2×40+1 = −16
21	(31,40)	−16	(32,40)	−16+2×32+1 = 49
22	(32,40)	49	(33,39)	49+2×33−2×39+1 = 38
23	(33,39)	38	(34,38)	38+2×33−2×38+1 = 27
24	(34,38)	27	(35,37)	27+2×35−2×37+1 = 24
25	(35,37)	24	(36,36)	
~~26~~				

$Q_1(x,y)$	$Q_2(-x,y)$	$Q_3(-x,-y)$	$Q_4(x,-y)$
(10,50)	(−10,50)	(−10,−50)	(10,−50)
(11,50)	(−11,50)	(−11,−50)	(11,−50)
(12,50)	(−12,50)	(−12,−50)	(12,−50)
(13,49)	(−13,49)	(−13,−49)	(13,−49)
(14,49)	(−14,49)	(−14,−49)	(14,−49)
(15,49)	(−15,49)	(−15,−49)	(15,−49)
(16,48)	(−16,48)	(−16,−48)	(16,−48)
(17,48)	(−17,48)	(−17,−48)	(17,−48)
(18,48)	(−18,48)	(−18,−48)	(18,−48)
(19,47)	(−19,47)	(−19,−47)	(19,−47)
(20,47)	(−20,47)	(−20,−47)	(20,−47)
(21,46)	(−21,46)	(−21,−46)	(21,−46)
(22,46)	(−22,46)	(−22,−46)	(22,−46)
(23,46)	(−23,46)	(−23,−46)	(23,−46)
(24,45)	(−24,45)	(−24,−45)	(24,−45)
(25,44)	(−25,44)	(−25,−44)	(25,−44)
(26,44)	(−26,44)	(−26,−44)	(26,−44)
(27,43)	(−27,43)	(−27,−43)	(27,−43)
(28,43)	(−28,43)	(−28,−43)	(28,−43)
(29,42)	(−29,42)	(−29,−42)	(29,−42)
(30,41)	(−30,41)	(−30,−41)	(30,−41)
(31,40)	(−31,40)	(−31,−40)	(31,−40)
(32,40)	(−32,40)	(−32,−40)	(32,−40)

(33,39)	(-33,39)	(-33,-39)	(33,-39)
(34,38)	(-34,38)	(-34,-38)	(34,-38)
(35,37)	(-35,37)	(-35,-37)	(35,-37)

(y,x)	(-y,x)	(-y,-x)	(y,-x)
~~(10,50)~~ (50,10)	(-50,10)	(-50,-10)	(50,-10)
(50,11)	(-50,11)	(-50,-11)	(50,-11)
(50,12)	(-50,12)	(-50,-12)	(50,-12)
(49,13)	(-49,13)	(-49,-13)	(49,-13)
(49,14)	(-49,14)	(-49,-14)	(49,-14)
(49,15)	(-49,15)	(-49,-15)	(49,-15)
(48,16)	(-48,16)	(-48,-16)	(48,-16)
(48,17)	(-48,17)	(-48,-17)	(48,-17)
(48,18)	(-48,18)	(-48,-18)	(48,-18)
(47,19)	(-47,19)	(-47,-19)	(47,-19)
(47,20)	(-47,20)	(-47,-20)	(47,-20)
(46,21)	(-46,21)	(-46,-21)	(46,-21)
(46,22)	(-46,22)	(-46,22)	(46,-22)
(46,23)	(-46,23)	(-46,-23)	(46,-23)
(45,24)	(-45,24)	(-45,-24)	(45,-24)
(44,25)	(-44,25)	(-44,-25)	(44,-25)
(44,26)	(-44,26)	(-44,-26)	(44,-26)
(43,27)	(-43,27)	(-43,-27)	(43,-27)
(43,28)	(-43,28)	(-43,-28)	(43,-28)
(42,29)	(-42,29)	(-42,-29)	(42,-29)
(41,30)	(-41,30)	(-41,-30)	(41,-30)
(40,31)	(-40,31)	(-40,-31)	(40,-31)
(40,32)	(-40,32)	(-40,-32)	(40,-32)
(39,33)	(-39,33)	(-39,-33)	(39,-33)
(38,34)	(-38,34)	(-38,-34)	(38,-34)
(37,35)	(-37,35)	(-37,-35)	(37,-35)

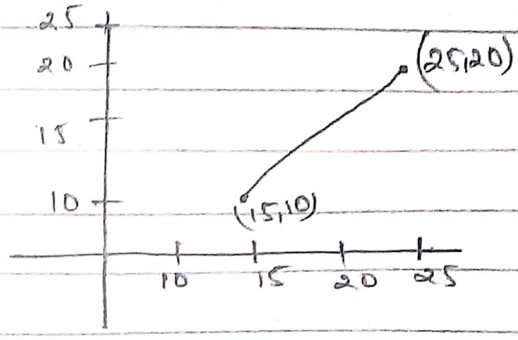
Draw the circle

Q. Use BLA to draw a line having endpoints (25,20) & (15,10).

Soln;

$\Delta x = |x_2 - x_1| = |15-25| = |-10| = 10$

$\Delta y = |y_2 - y_1| = |10-20| = |-10| = 10$



$m = \frac{\Delta y}{\Delta x} = \frac{10}{10} = 1$

Initial parameter $(P_0) = 2\Delta x - \Delta y$

$= 2\times10 - 10 = 10$

$2\Delta x - 2\Delta y = 2\times10 - 2\times10 = 20-20 = 0$

$2\Delta x = 2\times10 = 20$

Now plot $(x_0, y_0) = (15, 10)$

K	P_K	x_{K+1}, y_{K+1}
0	$P_0 = 10$	(16, 11)
1	$P_1 = 10$	(17, 12)
2	$P_2 = 10$	(18, 13)
3	$P_3 = 10$	(19, 14)
4	$P_4 = 10$	(20, 15)
5	$P_5 = 10$	(21, 16)
6	$P_6 = 10$	(22, 17)
7	$P_7 = 10$	(23, 18)
8	$P_8 = 10$	(24, 19)
9	$P_9 = 10$	(25, 20)

E.g. : Translate square with vertices $(0,0), (2,0), (0,2)$, & $(2,2)$ with $t_x = 2$ & $t_y = 3$.

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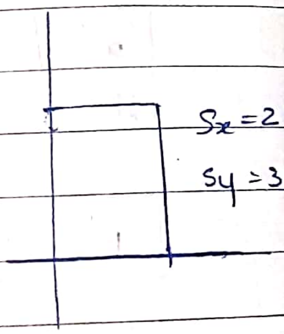
Eg ⓐ ~~Sac~~ Scale Square (0,0), (2,0), (0,2) & (2,2) with $S_x = 2$
$S_y = 3$.
ⓑ $S_x = 0.5$, $S_y = 0.5$

Solution.

ⓐ Let the given vertices of square be given as,
A(0,0), B(2,0), C(0,2) & D(2,2).
also, the given scaling points are $S_x = 2$ & $S_y = 3$.



Transformation eqn. are

$x' = x \cdot S_x$

$y' = y \cdot S_y$.

Now, using scaling formula, we have

$$A(0,0) \Rightarrow A' \Rightarrow \begin{bmatrix} 0 \cdot 2 \\ 0 \cdot 3 \end{bmatrix} \Rightarrow [0, 0]$$

$B(2,0) \Rightarrow$

Q. example;

scale an object (4,4), (3,2), (5,2) about a fixed point (4,3) by 2.

solution;

$(x_f, y_f) = (4,3)$

$(S_x, S_y) = (2,2)$

Now, we have fixed point scaling formula as,

$$CT = \begin{bmatrix} S_x & 0 & x_f(1-S_x) \\ 0 & S_y & y_f(1-S_y) \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 4(1-2) \\ 0 & 2 & 3(1-2) \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & -4 \\ 0 & 2 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

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Now, Scaling the given object $P = \begin{bmatrix} 4 & 3 & 5 \\ 4 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$

$$P' = CT * P = \begin{bmatrix} 2 & 0 & -4 \\ 0 & 2 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 & 5 \\ 4 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}.$$

$$= \begin{bmatrix} 4 & 2 & 6 \\ 5 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

∴ Scaled point are (4,5), (2,1), & (6,1)

Example:

Q. Rotate Δ^{le} (0,0), (1,0), (1,1) with $\theta = -90^\circ$ (clockwise) &

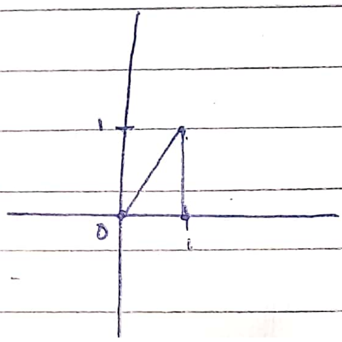
$\theta = 90^\circ$ (Anticlockwise)

Solution

(a) Rotation with $\theta = -90^\circ$ (clockwise):-

Given,

Let Vertices of a Δ^{le} ABC be A(0,0), B(1,0), C(1,1) respectively.



Example:

Q Rotate a Δ (5,5), (7,3), (3,3) about a fixed point (5,4) in counter clockwise by 90°.

Solution,

Here,

$$P = \begin{bmatrix} 5 & 7 & 3 \\ 5 & 3 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$CM = T_{(5,4)} R_{90°} T_{(-5,-4)}$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 90° & -\sin 90° & 0 \\ \sin 90° & \cos 90° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 9 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Now,

$$P' = CM * P$$

$$= \begin{bmatrix} 0 & -1 & 9 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 7 & 3 \\ 5 & 3 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 & 6 \\ 4 & 6 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$\therefore$ Co-ordinates are (4,4), (6,6) & (6,2).

Q. Rotate the ΔABC by $45°$ clockwise about the origin & scale it by (2,3) about origin.

Soln:

Step 1 : Rotation by $45°$ clockwise

2 : Scaling by (2,3)

Here,

$$P = \begin{bmatrix} 7 & 5 & 10 \\ 15 & 8 & 10 \\ 1 & 1 & 1 \end{bmatrix}$$

Now,

net transformation $= S(2,3) \cdot R_{45°}$

$$T = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45° & \sin 45° & 0 \\ -\sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2} & \sqrt{2} & 0 \\ -3/\sqrt{2} & 3/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now,

$P' = T * P$

$$= \begin{bmatrix} \sqrt{2} & \sqrt{2} & 0 \\ -3/\sqrt{2} & 3/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 & 5 & 10 \\ 15 & 8 & 10 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 31.113 & 18.38 & 28.28 \\ 16.97 & 6.363 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

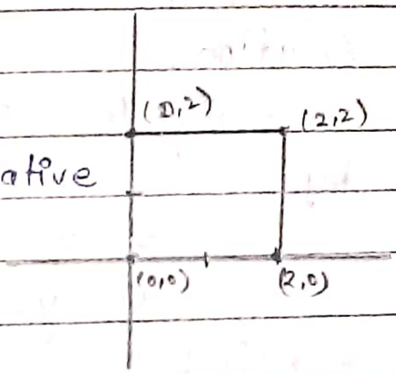
$\therefore$ Coordinates are (31.11, 16.97), (18.38, 6.36), (28.28, 0) example;

Q. A square with vertices (0,0), (2,0), (2,2) & (0,2) is shear with 2 units in x-direction. Find the new shear vertices.

Solution.

The given vertices of square are (0,0), (2,0), (2,2) & (0,2) & $sh_x = 2$ units

we have,



Shearing towards x-direction relative to x-axis is given by

$x' = x + sh_x \cdot y$

$y' = y$

now using formula, we have

For (0,0);

$x_1' = 0 + 2 \times 0 = 0$; $y_1' = 0$

$\therefore (x_1', y_1') = (0,0)$

For (2,0);

$x_2' = 2 + 2 * 0 = 2$; $y_2' = 0$

$\therefore (x_2', y_2') = (2,0)$

For (2,2);

$x_3' = 2 + 2 * 2 = 2+4 = 6$; $y_3' = 2$

$\therefore (x_3', y_3') = (6,2)$

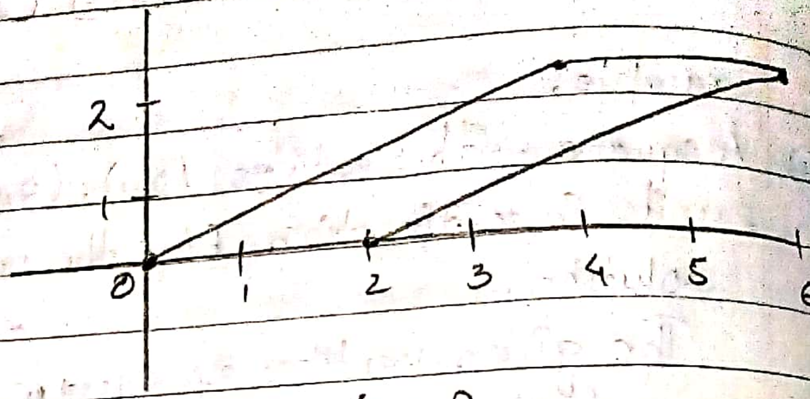
For (0,2);

$x_4' = 0 + 2 * 2 = 4$; $y_4' = 2$

$\therefore (x_4', y_4') = (4,2)$

∴ The sheared vertices of a square are:
(0,0), (2,0), (6,2), (4,2)

Now plotting above vertices on graph as below,



Q. Also, with 2 units in y-direction, find?

Solution,

Shearing towards y-direction relative to y-axis is given by,

$$x' = x$$
$$y' = y + sh_y \cdot x$$

For (0,0);

$$x_1' = 0 \; ; \; y_1' = 0 + 2*0 = 0$$
$$\therefore (x_1', y_1') = (0,0)$$

for (2,0);

$$x_2' = 2 \; ; \; y_2' = 2*2+0 = 4$$
$$\therefore (x_2', y_2') = (2,4)$$

For (2,2);

$$x_3' = 2 \; ; \; y_3' = 2*2+2 = 6$$
$$\therefore (x_3', y_3') = (2,6)$$

for

For $(0,2)$;

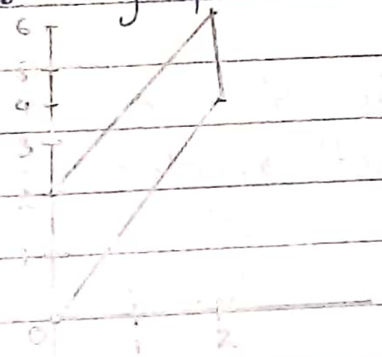
$x_4' = 0$; $y_4' = 0 * 2 + 2 = 2$

$\therefore (x_4', y_4') = (0,2)$

$\therefore$ The new sheared vertices of a square are,

$(0,0), (2,4), (2,6)$ & $(0,2)$

now, plotting above vertices in graph as below,



11 Q Rotate a triangle A(5,6), B(6,2) & C(4,1) by 45° about an arbitary pivot point (3,3).

Soln

The given vertices of ΔABC are A(5,6), B(6,2), C(4,1).

$\theta = 45°$

Here,

$$P = \begin{bmatrix} 5 & 6 & 4 \\ 6 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$CM = T_{(3,3)} \, R_{45°} \, T_{(-3,-3)}$$

$$= \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45° & -\sin 45° & 0 \\ \sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now,

$$P' = CM * P$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 6 & 4 \\ 6 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.70 & 2.82 & 2.12 \\ 7.77 & 5.65 & 3.53 \\ 1 & 1 & 1 \end{bmatrix}$$

$\therefore$ Co-ordinates after rotation are $(-0.70, 7.77)$, $(2.82, 5.65)$, & $(2.12, 3.53)$

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$\boxed{P' = CM * P}$

Q. Reflect an object (2,3), (4,3), (4,5) about line $y = x+1$. ($y = mx + c$, $c = 1$.)

$\tan\theta = m$

$\theta = \tan^{-1}(m)$

$$P = \begin{bmatrix} 2 & 4 & 4 \\ 3 & 3 & 5 \\ 1 & 1 & 1 \end{bmatrix}$$

$$CM = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$P' = CM * P$

$= [\ ]\ [\ ]$

$$P' = \begin{bmatrix} 2 & 2 & 4 \\ 3 & 5 & 5 \\ 1 & 1 & 1 \end{bmatrix}$$

Required points are (2,3), (2,5) & (4,5)

Here,

The given coordinates of an object are (2,3), (4,3), (4,5).

i.e. $P = \begin{bmatrix} 2 & 4 & 4 \\ 3 & 3 & 5 \\ 1 & 1 & 1 \end{bmatrix}$

Given st. line is $y = x + 1$. Comparing given line with $y = mx + c$, we have

$c = 1$, $m = 1$.

then.

$\tan\theta = m$

$\theta = \tan^{-1}(m) = \tan^{-1}(1) = 45^\circ$

Also, we know the composite matrix is given by

$$CM = \begin{bmatrix} \frac{1-m^2}{1+m^2} & \frac{2m}{1+m^2} & -\frac{2cm}{1+m^2} \\ \frac{2m}{1+m^2} & \frac{m^2-1}{1+m^2} & \frac{2cm}{1+m^2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1-1}{1+1} & \frac{2.1}{1+1} & \frac{-2.1.1}{1+1} \\ \frac{2.1}{1+1} & \frac{1-1}{1+1} & \frac{2.1.1}{1+1} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Again,

$P' = CM * P$

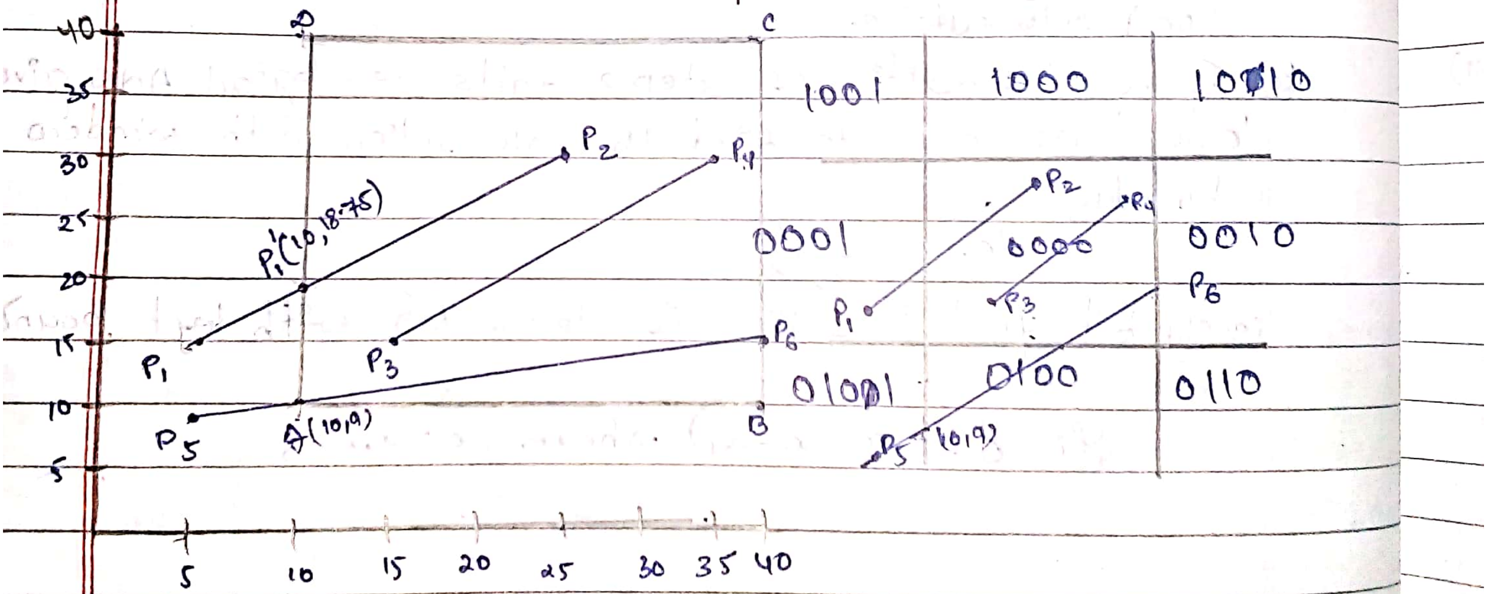
$$= \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 4 \\ 3 & 3 & 5 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 4 \\ 3 & 5 & 5 \\ 1 & 1 & 1 \end{bmatrix}$$

$\therefore$ The required coordinates are $(2,3)$, $(2,5)$ & $(4,5)$.

Example:-

Q. Given a clipped window A(10,10), B(40,10), C(40,40) & D(10,40) using cohen-sutherland line clipping. Find region code for each end points of lines P_1P_2, P_3P_4, P_5P_6 where $P_1(5,15)$, $P_2(25,30)$, $P_3(15,15)$, $P_4(35,30)$, $P_5(5,8)$ & $P_6(40,15)$. Also find the clipped line using above parameters.



Here,

For $P_1 P_2$

$P_1 = 0001$

$P_2 = 0000$

AND 0000

⇓

clipping required.

we have to find coordinate of P_1'.

$x = 10$

$y = ?$

we have two point formula for slope $P_1(5,15)$, $P_2(25,30)$

$$m = \frac{30-15}{25-5} = 0.75$$

$y = y_1 + m(x - x_1)$ passes through $P_1(5,15)$

$= 15 + 0.75(10-5)$

$= 18.75$

$\therefore P_1' = (10, 18.75)$

For $P_3 P_4$ [completely inside]

For $P_5 P_6$

$P_5 = 0101$

$P_6 = 0000$

0000

⇓ clipping required

we have to find coordinate of P_5'

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$P_5'(x,y) = (10, ?)$ Passes through $(5,8)$

$$m = \frac{15-8}{40-5} = 0.2$$

$$y = 8 + (10-5) * 0.2 = 9$$

$$P_5' = (10, 9)$$

Again,

$P_5' = 0100$

$P_6 = 0000$

$\overline{\quad\quad\quad\quad}$

0000

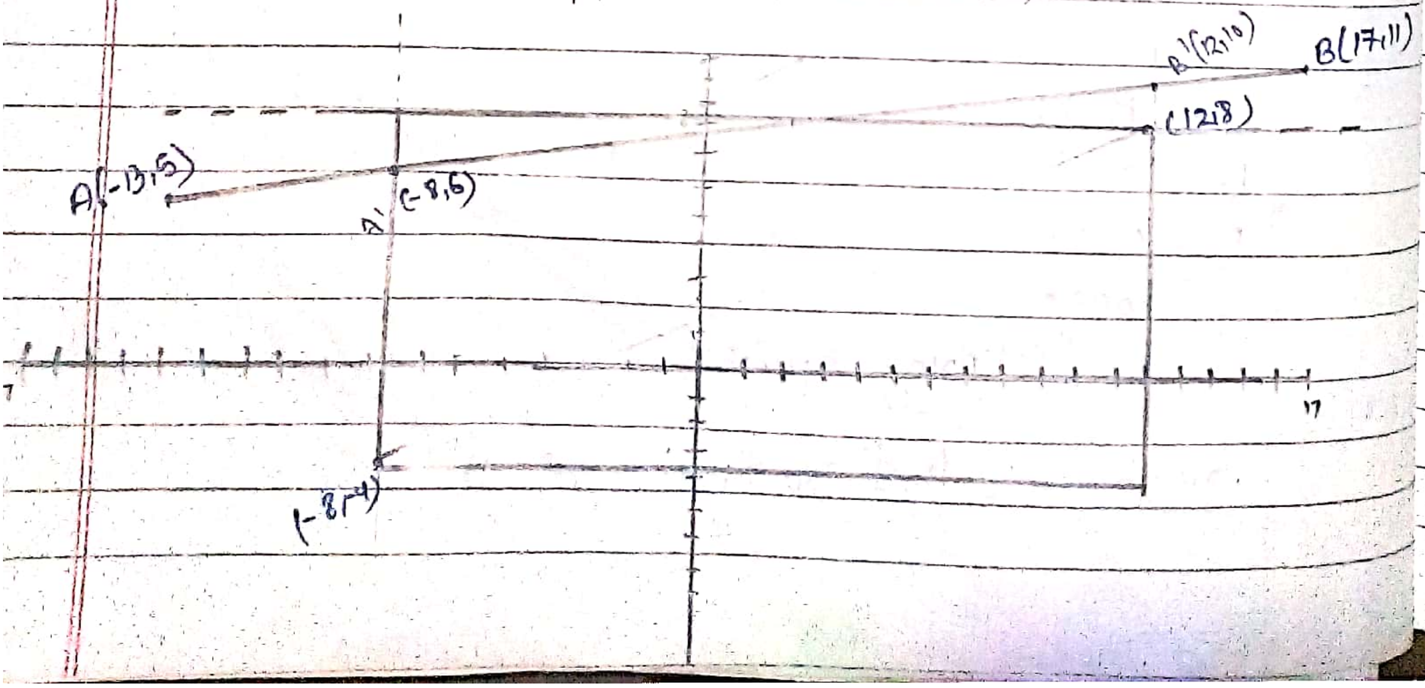
$\Downarrow$ clipping required

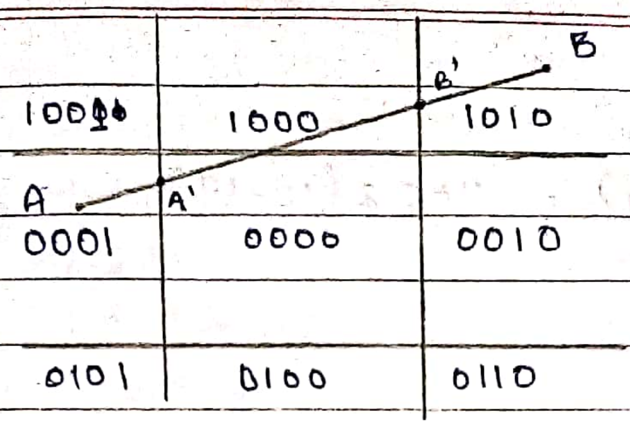
Coordinate of P_5'' at line $(10,9)$ & $m = 0.2$

$(x,y) = (15,10)$; $x' = x_1 + 1/m(y - y_1) = 15$

$y = y_{min} = 10$

Q.2. Use clipping method to clip a line starting from A(-13, 5) and ending at B(17,11) against the window having its lower corner at (-8,-4) & upper corner at (12,8).





Here

For AB

A = 0001

B = 1010

AND 0000

⇓ clipping required

we have to find the coordinate of A'

$x = -8$

$y = ?$

we have two point formula for slope A (-13, 5) and B(17, 11)

$$m = \frac{11-5}{17+13} = \frac{6}{30} = \frac{\cancel{6}^1}{5} = 0.2$$

$y = y_1 + m(x - x_1)$ passes through A (-13, 5)

$= 5 + 0.2(-8+13)$

$= 5 + 0.2(5)$

$= 6$

$\therefore A' = (-8, 6)$

For A'B

A' = 0000

B = 1010

AND 0000 ⇓

clipping is required

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$B' = (12, ?)$

Here,

$m = 0.2$

$y = y_1 + m(x - x_1) = 11 + 0.2(12 - 17) = 10$

$B' = (12, 10)$

Also,

For $A'B''$

$A' = 0000$

$B'' = 1000$

$\overline{\quad 0000}$

$\Downarrow$ clipping is required.

$m = 0.2$

$B''(?, 8)$

$x = x_1 + 1/m (y - y_1)$

$= 12 + 1/0.2 (8 - 10)$

$= 2$

$B'' = (2, 8)$

For $A'B''$

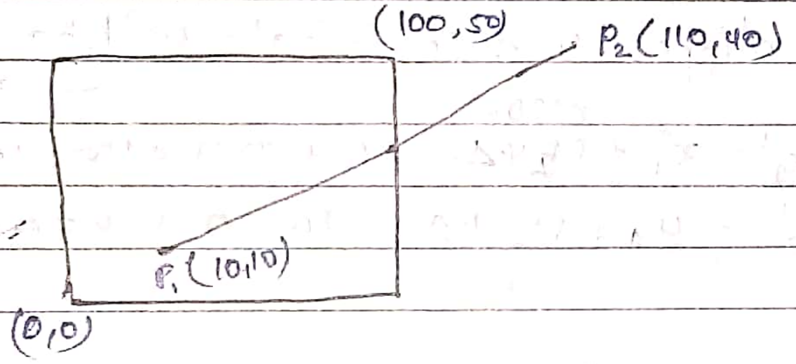
$A' = 0000$

$B'' = 0000$

$\overline{\quad 0000}$

~~clipping~~ completely inside.

Example :-



K	P_K	q_K	R_K
1	$-\Delta x$	$x_1 - x_{wmin}$	$r_1 = -10/100$
	$= -(110-10)$	$= 10 - 0$	$= -1/10$
	$= -100$	$= 10$	= candidate of U_1
	ie, $P_K < 0$		
2.	Δx	$x_{wmax} - x_1$	$r_2 = 90/100$
	$= 100$	$= 100 - 10$	$= 0.9 \Rightarrow$
	ie, $P_K > 0$	$= 90$	candidate of U_2
3.	$-\Delta y$	$y_1 - y_{wmin}$	$r_3 = 10/-30$
	$= -(40-10)$	$= 10 - 0$	$= -1/3$
	$= -30$	$= 10$	candidate of U_1.
	$P_K < 0$		

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4 Δy $= 30$ i.e. $P_k > 0$ | $y_{wmax} - y_1$ $= 50 - 10$ $= 40$ | $r_4 = 40/30$ $= 4/3 \Rightarrow$ candidate for u_2

Here,

For $P_k < 0$

Selection of $u_1 = \max(0, -1/10, -1/3)$
$= 0$

For $P_k > 0$

selection of $u_2 = \min(1, 0.9, 4/3)$
$= 0.9$

Now, clipped line coordinate will be →

$x_1' = x_1 + u_1 * \Delta x$
$= 10 + 0 \times 100 = 10$

$y_1' = y_1 + u_1 * \Delta y$
$= 10$

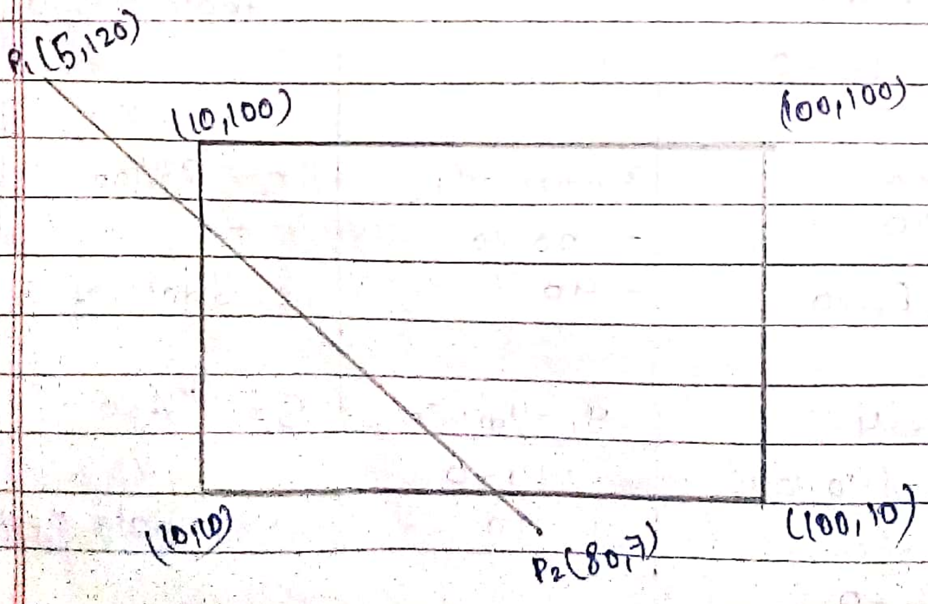
also,

$x_2' = x_1 + u_2 * \Delta x = 10 + 0.9 * 100 = 100$

$y_2' = y_1 + u_2 * \Delta y = 10 + 0.9 * 30 = 37$

Q. Find the clipped region in window of diagonal vertex (10,10) & (100,100) for line $P_1(5,120)$ & $P_2(80,7)$ using Liang Barsky line clipping.

Ans. $(x_1', y_1') = (18, 100)$
$(x_2', y_2') = (78, 10)$



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K	P_K	q_K	R_K
1	$-\Delta x$ $= -(80-5)$ $= -75$ $P_K < 0$	$x_1 - x_{wmin}$ $= 5-10$ $= -5$	$r_1 = -5/-75$ $= 1/15$ candidate of u_1
2.	Δx $= (80-5)$ $= 75$ $P_K > 0$	$x_{wmax} - x_1$ $= 100-5$ $= 95$	$r_2 = 95/75$ $= 19/15$ candidate of U_2
3.	$-\Delta y$ $= -(7-120)$ $= 113$ $P_K > 0$	$y_1 - y_{wmin}$ $= \cancel{100-120}$ $= 120-10$. $= 110$	$r_3 = 110/113$ Candidate of U_2
4.	Δy $= (7-120)$ $= -113$ $P_K < 0$	$y_{wmax} - y_1$ $= 100-120$ $= -20$	$r_4 = -20/-113$ $= 20/113$ candidate of u_1

for $P_K < 0$

selection of $u_1 = (0, 1/15, 20/113)$

$= 20/113$

For $P_K > 0$

selection of $u_2 = (1, 19/15, 110/113)$

$= 110/113$

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Now, clipped line coordinates will be

$x_1' = x_1 + U_1 * \Delta x$

$= 5 + 20/113 \times (80 - 5)$

$= 18.27$

$y_1' = y_1 + U_1 * \Delta y$

$= 120 + 20/113 (7 - 120)$

$= 120 - 20$

$= 100$

$\therefore (x_1', y_1') = (18.27, 100)$

Again,

$x_2' = x_1 + U_2 * \Delta x$

$= 5 + \frac{110}{113} * (80 - 5)$

$= 78$

$y_2' = y_1 + U_2 * \Delta y$

$= 120 + \frac{110}{113} * (7 - 120)$

$= 120 - 110$

$= 10$

$\therefore (x_2', y_2') = (78, 10)$

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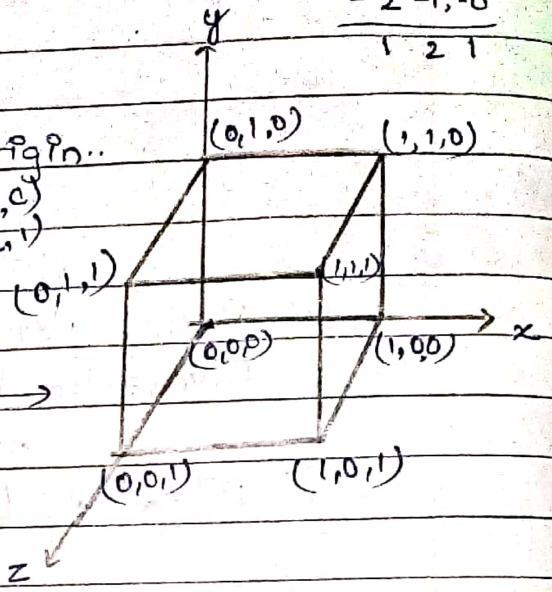
Example :-

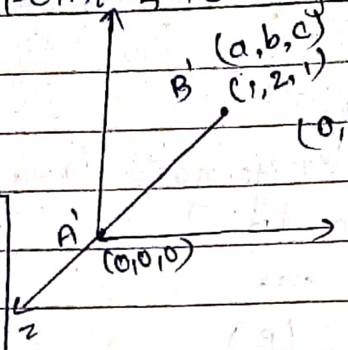
Q. Find the new coordinate of unit cube 90° rotate about an axis defined by its endpoints $A(2,1,0)$ & $B(3,3,1)$.

$$\begin{array}{r} 3,\ 3,\ 1 \\ -2\ -1,\ -0 \\ \hline 1\ \ 2\ \ 1 \end{array}$$

Here,



Step 1: Translate point A to origin..

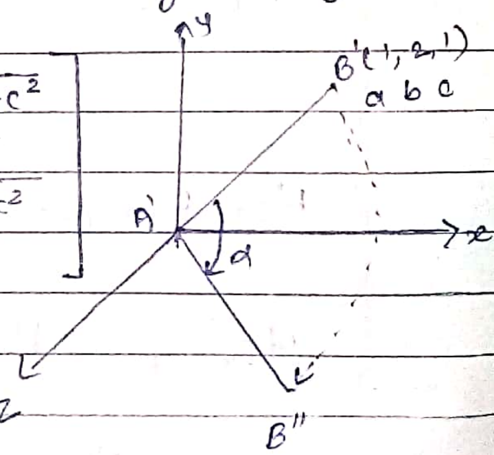


$$T = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 2 :- Rotate A'B' about the x-axis by an angle 'α' unit it lies on XZ plane

$$\left[\begin{array}{l} \therefore d = \sqrt{b^2+c^2} \\ \quad = \sqrt{5} \\ l = \sqrt{a^2+b^2+c^2} \\ \quad = \sqrt{6} \end{array}\right]$$

$\sin\alpha = b/d = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$



$\cos\alpha = \frac{c}{d}$

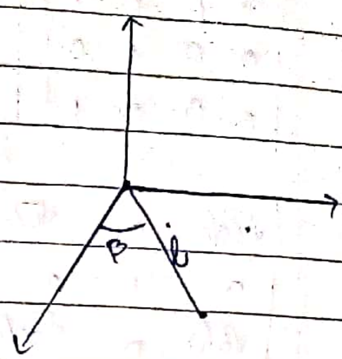
$= \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{5}/5 & -2\sqrt{5}/5 & 0 \\ 0 & 2\sqrt{5}/5 & \sqrt{5}/5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

step 3 : Rotate axis A'B'' about y-axis by and angle β unit it coincides with z-axis .

$\cos\beta = d/L = \sqrt{5}/\sqrt{6} = \sqrt{30}/6$

$\sin\beta = a/L = 1/\sqrt{6} = \sqrt{6}/6$



$$R_y(\beta) = \begin{bmatrix} \sqrt{30}/6 & 0 & -\sqrt{6}/6 & 0 \\ 0 & 1 & 0 & 0 \\ \sqrt{6}/6 & 0 & \sqrt{30}/6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

step 4 : Rotate the cube 90° about the z-axis .

$$R_z(90°) = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

step 5 :- Reverse transformation of above

$= T^{-1} R_x^{-1}(\alpha)\ R_y^{-1}(\beta)$

Finally , the composite rotational matrix about the arbitrary axis AB be comes .

$N T_m = T^{-1} R_x^{-1}(\alpha)\ R_y^{-1}(\beta) \cdot R_z(90=\theta)\ R_y(\beta)\ R_x(\alpha) \cdot T$

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$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{5}/5 & 2\sqrt{5}/5 & 0 \\ 0 & -2\sqrt{5}/5 & \sqrt{5}/5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{30}/6 & 0 & \sqrt{6}/6 & 0 \\ 0 & 1 & 0 & 0 \\ -\sqrt{6}/6 & 0 & \sqrt{30}/6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{30}/6 & 0 & -\sqrt{6}/6 & 0 \\ 0 & 1 & 0 & 0 \\ \sqrt{6}/6 & 0 & \sqrt{30}/6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{5}/5 & -\frac{2\sqrt{5}}{5} & 0 \\ 0 & 2\sqrt{5}/5 & \sqrt{5}/5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.166 & -0.075 & 0.983 & 1.742 \\ 0.742 & 0.667 & 0.875 & -1.151 \\ -0.650 & 0.741 & 0.167 & 0.560 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now,

$$P' = \begin{bmatrix} N_{Tm} \end{bmatrix} * \begin{bmatrix} P \end{bmatrix}$$

$$= \begin{bmatrix} 0.166 & -0.075 & 0.983 & 1.742 \\ 0.742 & 0.667 & 0.075 & -1.151 \\ -0.650 & 0.741 & 0.167 & 0.560 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.650 & 1.667 & 1.834 & 2.816 & 2.725 & 1.742 & 1.909 & 2.891 \\ -0.558 & -0.484 & 0.258 & 0.184 & -1.225 & -1.151 & -0.409 & 0.483 \\ 1.467 & 1.301 & 0.650 & 0.817 & 0.726 & 0.560 & -0.091 & 0.076 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

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Q. Perform Rotation of a line (10,10,10) & (20,20,15) about arbitrary y-axis in clockwise direction by 90°.

Numerical of perspective projection :-

Obtain perspective projection coordinate for the pyramid with vertices of base (15, 15, 10), (20, 20, 10), (25, 25, 10), (20, 10, 10) & apex (20, 15, 20) given that $Z_{prp} = 20$ & $Z_{vp} = 0$

Solution:

Given,

$$Z_{prp} = 20 \quad , \quad Z_{vp} = 0$$

we have,

$$x' = x - x \cdot u$$

$$y' = y - y \cdot u$$

$$z' = z - (z - Z_{prp})$$

on the view plane

$$z' = Z_{vp} \text{ so,}$$

$$u = \frac{Z_{vp} - Z}{Z_{prp} - Z}$$

Here, for vertex (15, 15, 10) - P_1

$$x' = x_p = x - x\left(\frac{Z_{vp} - Z}{Z_{prp} - Z}\right)$$

$$= x \cdot \left(\frac{Z_{prp} - Z_{vp}}{Z_{prp} - z}\right)$$

$$= 15 \times \frac{20 - 0}{20 - 10} = 30$$

Similarly,

$$y' = y_p = y \times \left(\frac{Z_{prp} - Z_{vp}}{Z_{prp} - Z}\right)$$

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$= 15 * \frac{20-0}{20-10} = 30$

$z' = Z_p = Z_{vp} = 0$

Projected point is $(x', y', z') = (30, 30, 0)$

For $P_2 (20, 20, 10)$

$x' = x_p = x \cdot \left(\frac{Z_{prp} - Z_{vp}}{Z_{prp} - z} \right) = 20 * \frac{20-0}{20-10} = 40$

$y' = y_p = y \cdot \left(\frac{Z_{prp} - Z_{vp}}{Z_{prp} - z} \right) = 20 * \frac{20-0}{20-10} = 40$

$z' = Z_p = Z_{vp} = 0$

Projected point is $(x', y', z') = (40, 40, 0)$

Again for $P_3 (25, 15, 10)$, we get

$x' = x_p = 50$, $y' = y_p = 30$, $z' = Z_p = 0$

Projected point is $(x', y', z') = (50, 30, 0)$

For $P_4 (20, 10, 10)$, we get

$x' = x_p = 40$

$y' = y_p = 20$

$z' = 0$

Projected point is $(40, 20, 0)$.

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control point = n-1

Bazier curve :-

Q. Derive the equation for quadratic Bezier curves.

Solution.

we have,

$$P(u) = \sum_{K=0}^{n} P_K \; BEZ_{K,n}(U) \qquad 0 \le U \le 1 \;\; \text{--- (a)}$$

where,

$$BEZ_{K,n}(U) = C(n,K) \; U^K(1-U)^{n-K} \;\; \text{.... (b)}$$

And,

$$x(U) = \sum_{K=0}^{n} x_K \cdot BEZ_{K,n}(U) \;\; \text{--- (I)}$$

$$y(U) = \sum_{K=0}^{n} y_K \cdot BEZ_{K,n}(U) \;\; \text{..... (II)}$$

$$z(U) = \sum_{K=0}^{n} z_K \cdot BEZ_{K,n}(U) \;\; \text{---- (III)}$$

Now, for quadratic eq^n. n = 2,

So from eq^n (a)

$$P(U) = \sum_{K=0}^{2} P_K \; BEZ_{K,2}(U)$$

$$= P_0 \, BEZ_{0,2}(U) + P_1 \, BEZ_{1,2}(U) + P_2 \, BEZ_{2,2}(U) \;\; \text{--- (c)}$$

from eq^n (b)

$$BEZ_{0,2}(U) = C(2,0) \, U^0 (1-U)^{2-0} = 1 \times (1-U)^2 = (1-U)^2$$

$$BEZ_{1,2}(U) = C(2,1) \, U^1 (1-U)^{2-1} = 2U(1-U)$$

$$BEZ_{2,2}(U) = C(2,2) \, U^2 (1-U)^{2-2} = 1 \times U^2 = U^2$$

Now from eq^n (c)

$$P(u) = (1-u)^2 P_0 + 2u(1-u)P_1 + u^2 P_2$$

is the required eqⁿ. of quadratic Bezier Curves.

similarly, for Cubic Bezier Curve $n=3$ do at home.

$$\boxed{P(u) = (1-u)^3 P_0 + 3(1-u)^2 u P_1 + 3(1-u)u^2 P_2 + u^3 P_3}$$

Q. Construct Bezier Curve for control points (4,2), (8,8) & (16,4).

Solution,

$P_0 = (4,2)$, $P_1 = (8,8)$, $P_2 = (16,4)$

control point = 3

$\therefore n = 3-1 = 2$

It is quadratic bezier curve so, we have

$$P(u) = \sum_{k=0}^{n} P_k \cdot BEZ_{k,2}(u) \quad 0 \le u \le 1$$

on putting $n=2$ we get

$$P(u) = (1-u)^2 P_0 + 2u(1-u)P_1 + u^2 P_2$$

{ so whole process of derivation.

Now, considering for x-axis, (4, 8, 16)

$$x(u) = (1-u)^2 4 + 2u(1-u)8 + u^2 16$$
$$= 4u^2 + 8u + 4$$

Now, considering for y-axis (2, 8, 4)

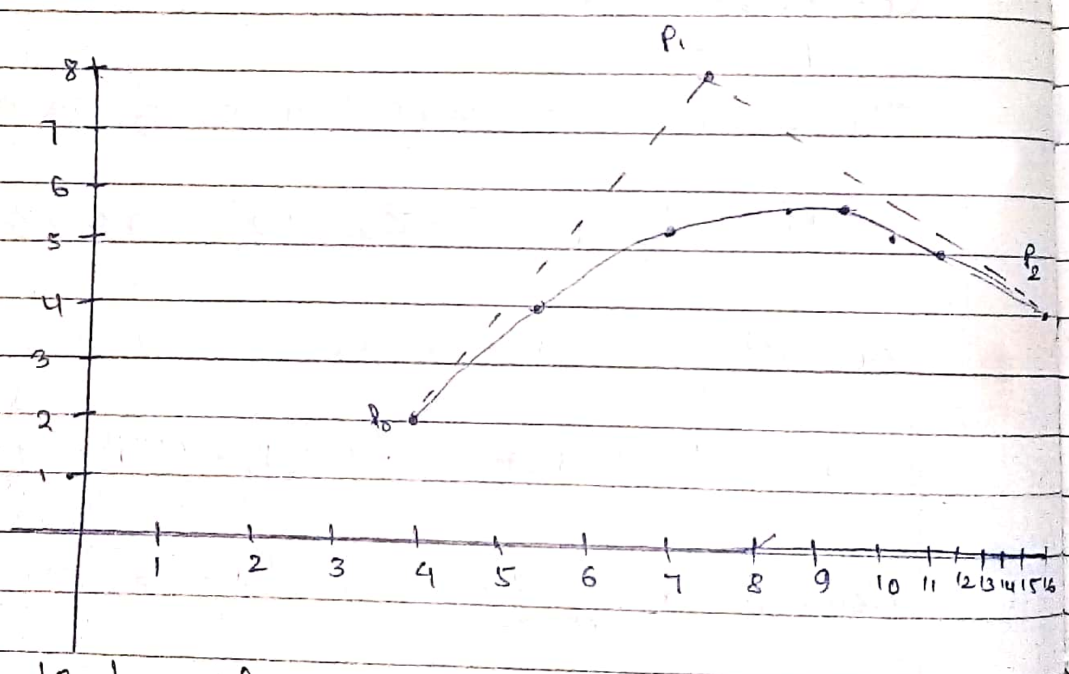
$$y(u) = (1-u)^2 2 + 2u(1-u)8 + u^2 4$$

$= -10u^2 + 12u + 2$

Now, we know, $0 \le u \le 1$

SN.	(u)	x(u)	y(u)	
1	0	4	2	P_0
2	0.2	~~4~~ 5.76	4	
3	0.4	~~5.76~~ 7.84	5.2	
4	0.6	~~7.84~~ 10.24	5.6	
5	0.8	~~10.24~~ 12.96	5.2	
6	1	16	4	P_2

Plotting point of x(u) & y(u) in graph. we get.



Q. The coordinate of four control points relative to curve are given by $P_1(2,2)$, $P_2(2,3)$, $P_3(3,3)$, $P_4(2,2)$. write eqn. of Bezier Curve. Also find coordinate pixels of curve for $u=0$, $u=1/4$, $u=1/2$, $u=3/4$ & $u=1$. And also plot graph.

Q. Construct the Bezier Curve of order 3 with 4 polygon vertices A (1,1), B(2,3), C(4,3) & D(6,4)

Solid Modeling

Q. A cubic Bezier Curve is described by the four control points (0,0), (2,1), (5,2) & (6,1). Find the tangent to the curve at t = 0.5.

Solution

We know, Bezier Cubic polynomial equation,

$$p(t) = (1-t)^3 V_0 + 3t(1-t)^2 V_1 + 3t^2(1-t) V_2 + t^3 V_3$$

$$= (1-3t+3t^2-t^3) V_0 + (3t-6t^2+3t^3) V_1 + (3t^2-3t^3) V_2 + t^3 V_3$$

$$= (-t^3+3t^2-3t+1) V_0 + (3t^3-6t^2+3t) V_1 + (-3t^3+3t^2) V_2 + t^3 V_3$$

In matrix form, we have

$$p(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ \cancel{0}1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

where,

$V_0 = (0,0)$, $V_1 = (2,1)$, $V_2 = (5,2)$, $V_3 = (6,1)$

The tangent is given by the derivative of the general eq.ⁿ above,

$$p'(t) = \begin{bmatrix} 3t^2 & 2t & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 1 \\ 5 & 2 \\ 6 & 1 \end{bmatrix}$$

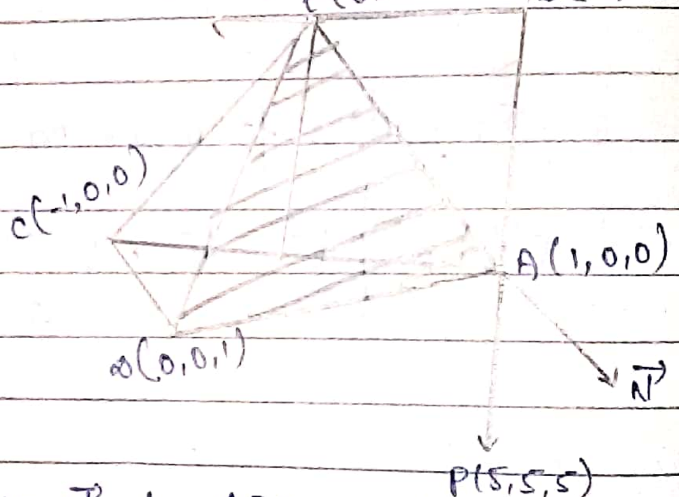
At t = 0.5.

$$P'(0.5) = \begin{bmatrix} 3(0.5)^2 & 2(0.5) & 1 & 0 \end{bmatrix} \begin{bmatrix} -3 & 4 \\ 3 & -6 \\ 6 & 3 \\ 0 & 0 \end{bmatrix}$$

$$\begin{array}{cc} x(t) & y(t) \end{array}$$
$$= \begin{bmatrix} -15.75 & 19 \end{bmatrix}$$

$$\text{Slope} = \frac{y(t)}{x(t)} = \frac{19}{-15.75} = 1.21$$

Q. Find the visibility for the surface AED where observer at p(5,5,5)



Solution:-

Step 1: find the normal vector $\vec{N}$ for AED surface.
(Always take anticlockwise direction convention)
(ie, AE * AD not AD * AE)

$AE = E - A = (0-1)\hat{i} + (1-0)\hat{j} + (0-0)k = -\hat{i} + \hat{j}$

$AD = D - A = (0-1)\hat{i} + (0-0)\hat{j} + (1-0)k = -\hat{i} + k$

Now,

$$\vec{N} = AE * AD = \begin{vmatrix} \hat{i} & \hat{j} & k \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = \hat{i} + \hat{j} + k$$

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Step2:

The observer is at P(5,5,5) so we can construct the view vector $\vec{V}$ from surface to view point A(1,0,0) as

$$V = PA = A - P = (1-5)i + (0-5)j + (0-5)k$$
$$= -4i - 5j - 5k$$

Step3:

To find the visibility of the object, we use dot product of view vector $\vec{V}$ & normal vector $\vec{N}$ as:

$$\vec{V} \cdot \vec{N} = (-4i - 5j - 5k) \cdot (i + j + k)$$
$$= -4 - 5 - 5$$
$$= -14 < 0$$

This shows that surface is visible for observer.