MXB103 S2 2020

Project: Bungee!

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Introduction

As part of Brisbane's "New World City" transformation, the Brisbane City Council is proposing to allow bungee jumping off the Story Bridge. A commercial bungee jump company has expressed interest and has provided the council with facts and figures of their potential bungee plan. Shu Du, James Kosiol, Joshua Wakeling & Jai McGarrigal are a part of a professional consultation company called "Project Team 38" who have been hired by the Council to verify certain aspects of the provided information.

Project Team 38 has been requested to create a numerical solution which will provide answers to the questions proposed by the Council. The questions asked by the Council involve the behaviour of a standard bungee jump (10 bounces and distance travelled in 60 seconds, the distance the jumper falls from the platform, if the jumper touches the water at first bounce) and the associated forces (max acceleration and speed the jumper will experience). This numerical solution will be made using modified Euler's method and a mathematical model of a bungee jump with the provided data and MATLAB software. The next section of this report will explain this model and its derivation.

Mathematical Model

The mathematical model associated with this bungee jump contains the following parameters:

Table 1: Parameters and their meanings with unit values

Symbol	**Meaning**	**Value**	**Units**
H	Height from water level	74	m
D	Height of bridge deck from water level	31	m
m	Mass of the jumper	80	kg
g	Gravitational Acceleration	9.8	m/s^2
c	Air drag coefficient	0.9	kg/m
L	Length of unstretched bungee rope	25	m
k	Elasticity of bungee rope (spring constant)	90	N/m

This model was derived by considering the forces acting on the jumper at all times. These forces are outlined below in Table 2.

Table 2: Forces acting on Jumper with definitions

Force	Definition
Gravitational Acceleration	mg
Drag (air resistance)	$-c\,\lvert v \rvert\, v$
Rope Tension	-max(0, k(y-L))

For drag force, $\lvert v \rvert$ represents the jumpers speed as a magnitude of the velocity. This allows speed to always be measured as a positive number.

For rope tension, y represents the position of the jumper measured from the jumping point on the bridge.

Newton’s Second Law

Summing all the forces acting upon the jumper throughout the entire jump and setting it equal to the force of the jumpers mass multiplied by the jumpers acceleration, will result in the following Ordinary Differential Equation (ODE):

$$m \frac{dv}{dt} = mg - c|v|v - \max(0, k(y - L))$$

Where $\frac{dv}{dt}$ is the acceleration of the jumper. This ODE can be further simplified by dividing through m. This results in the following ODE:

$$\frac{dv}{dt} = g - C|v|v - \max(0, K(y - L))$$

Where C = c/m and K = k/m.

Assumptions and limitations of the model

The model in this report assumes that the jumper is experiencing constant drag for the duration of the jump. Weather conditions are also assumed. An assumption made is the bounces are only assumed vertical, limiting the possibility of horizontal movement. A limitation for the model assumes the jumpers are all of constant mass of 80 kilograms and height of 1.75 metres.

Conducted study limitations

Limitations to the mathematical design include catastrophic cancellation errors and round off errors. Catastrophic cancellation occurs when the floating point subtraction of two numbers that appear to be exact, however one number is not represented exactly and hence resulting in error. Round off error occurs when rounding numbers with decimals to whole numbers.

Numerical Solution

Two ordinary differential equations were used to solve numerical solution, as the initial ordinary differential equation was too complex to solve analytically. The modified Euler's method was used to solve the two coupled ordinary differential equations.

Table 3: Parameters for modified Euler's method with definition and values

Parameter	Definition	Value
T	Final time	60 seconds
n	Max number of intervals	10000
h	Timestep	$\frac{T}{n}$

$$y_{i+1} = y_i + hv_i\,, \quad i = 0, 1, \ldots (n - 1)$$

$$v_{i+1} = v_i + \tfrac{1}{2}(k_1 + k_2)\,, \quad i = 0, 1, \ldots (n - 1)$$

Where:

$$k_1 = h(g - C|v_i|v_i - max(0, K(y_i - L)))$$

$$k_2 = h(g - C|v_i + k_1|(v_i + k_1) - max(0, K(y_i - L)))$$

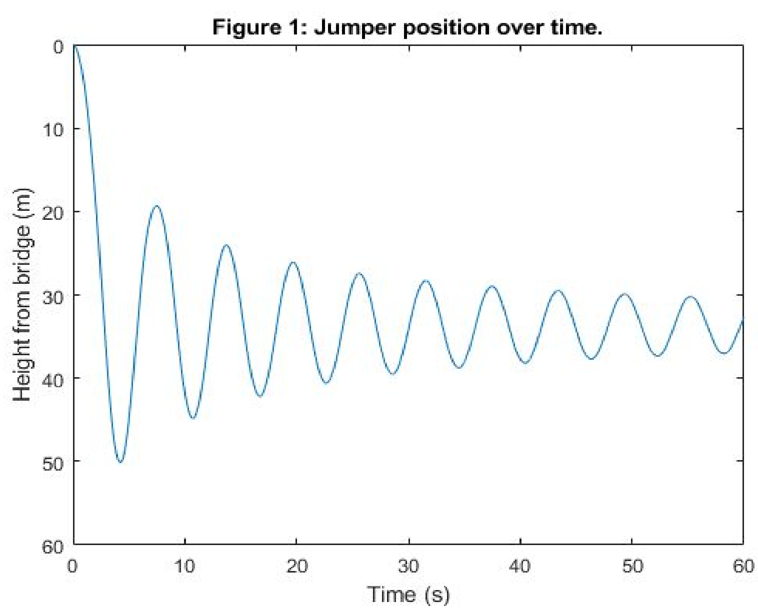


Figure 1 shows the Jumper's position over the 60 second jump. On the first bounce the jumper reaches a max height of roughly 50 meters away from the bridge. The y axis of the graph has 0 at the top to emphasize that the distance is measured away from the bridge.

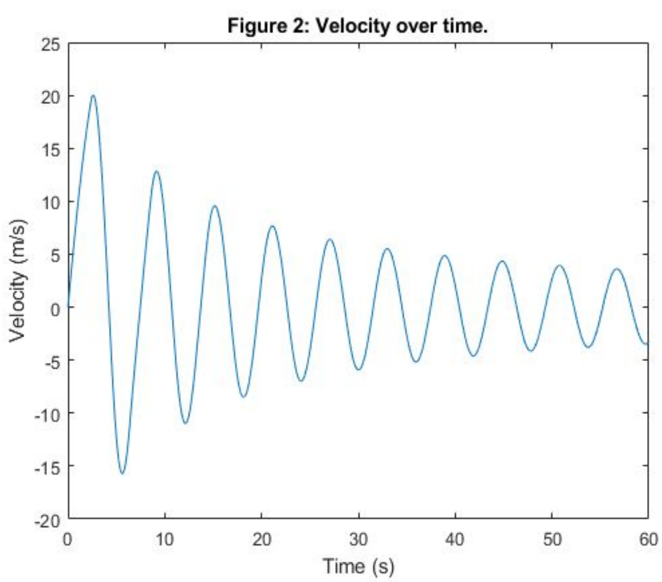
Analysis

The following section of this report will focus on answering the questions posed by the council. The answers provided will explain the method used to achieve the results as well as visual representation of the answers with graphs.

Timing and bounces

The commercial bungee jump company states that their bungee model will achieve ten "bounces" within the 60 second jump. A bounce is defined as the upward movement of the jumper caused by the elasticity of the bungee rope. After the ten bounces, the jumper will be at rest. The council has questioned if the model will achieve this statement. As shown in Figure 1, each trough of the wave is counted as a bounce. The jumper makes ten bounces just before the 60 second mark.

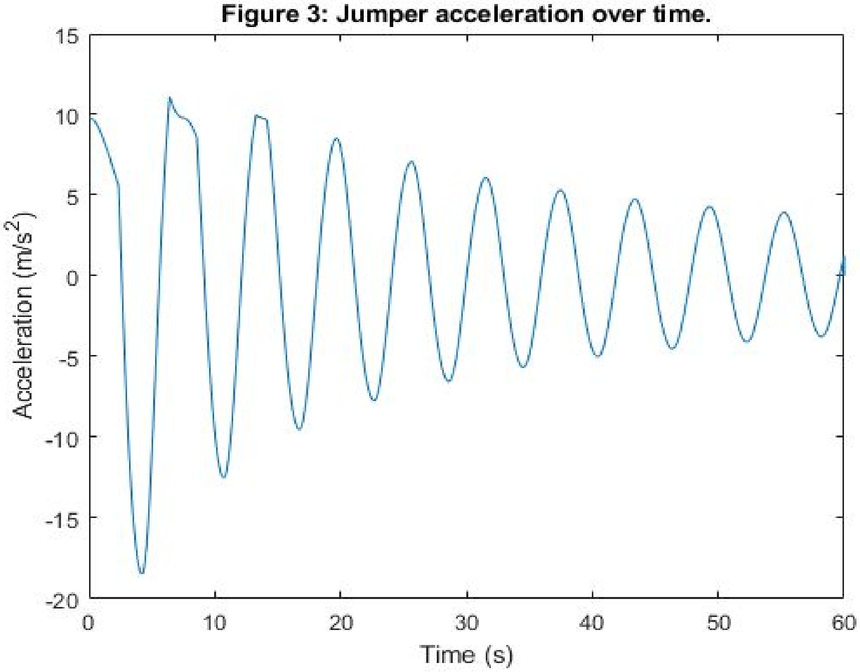
Maximum speed experienced by the jumper



It is suggested that the amount of “thrill factor” is partially determined by the maximum velocity experienced by the jumper. The council has asked for the model’s maximum velocity experienced by the jumper and at what time it will occur. Figure 2 shows the jumpers' maximum speed of 20.04m/s will occur 2.61 seconds into the jump.

Maximum acceleration experienced by the jumper

Figure 3: Jumper acceleration over time.



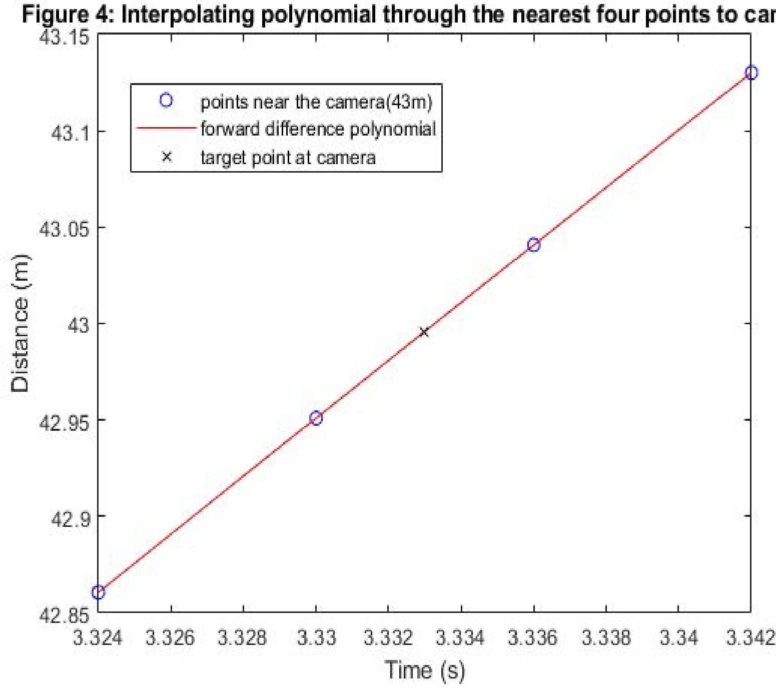
The “thrill factor” of the bungee jump can also be attributed to the maximum acceleration experienced by the jumper. It is stated that the more acceleration the jumper experiences, the higher the “thrill factor”. However, too much acceleration is dangerous for the jumper. The commercial bungee jump company has stated that the jumper will experience up to 2g of acceleration. This stated max acceleration is defined as 2 multiplied by gravitational acceleration: $2 \times 9.8m/s^2 = 19.6m/s^2$. The council has asked for the value of the maximum acceleration and when it will occur. To calculate the maximum acceleration, the derivative of the velocity function was calculated using second order central method. Figure 3 shows that at 4.26 seconds the jumper will experience a maximum acceleration of 18.461m/s^2 (downwards) which is less than the stated value.

Distance travelled by the jumper

The council wishes to use promotional material for the bungee jump which requires the result of the total distance travelled over the duration of the jump. This is achieved by using numerical integration of the speed of the jumper from 0 to 60 seconds. Using the trapezium rule, it was found that the jumper will travel 292.10 metres in 60 seconds.

Automated camera system

Figure 4: Interpolating polynomial through the nearest four points to camera.



Part of the proposal by the commercial bungee jump company states that there is to be a camera installed on the deck of the bridge at height D. The council has asked for the time said camera will trigger in order to take an accurate photo of the jumper descending. To answer this question, four positions closest to the camera were taken and using Newton's forward difference, created an interpolating polynomial. Using the bisection method to find the roots of this polynomial showed that the camera should trigger at 3.33 seconds into the jump. This is visually represented in Figure 4.

Water touch option

Figure 5: Distance change over time (Water Touch).

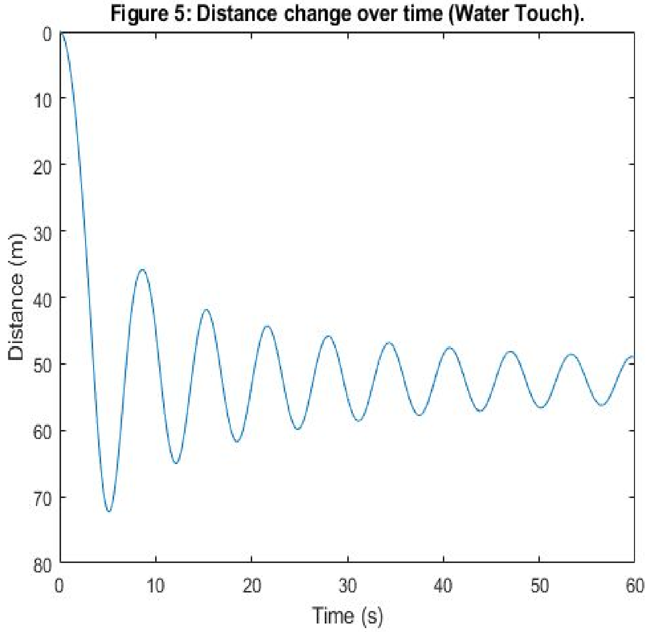
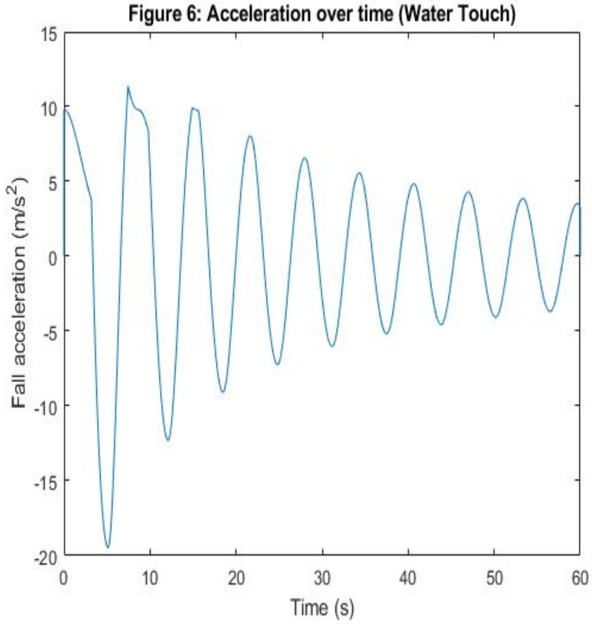


Figure 6: Acceleration over time (Water Touch)



As part of the proposal by the commercial bungee jump company, a "water touch" option could be considered, where the jumper just touches the water at first bounce. The council asks if the jumper does touch the water upon first bounce and if not, what needs to be altered to allow a "water touch". In order for the jumper to experience a "water touch" a distance of 74 metres total needs to be reached. As shown in Figure 1, the jumper barely makes it past the 50 metre mark. To achieve a "water touch" adjustments to the rope length and spring constant of the rope need to be made, whilst making sure the maximum acceleration does not exceed 2g ($19.6m/s^2$). As shown in Figure 5, if the spring constant is changed to 78.8N/m and the rope length increased to 42.55m, then on the first bounce the jumper will reach a distance of 72.25m. This distance plus the jumpers height will allow for a "water touch" whilst having a safe maximum acceleration of 19.466m/s^2 as shown in Figure 6.

Conclusion

To summarise, the bungee jumping attraction that the Brisbane City Council has proposed is a very plausible option of amusement for the 'New World City' transformation. The mathematical model and numerical solution using modified Euler's method, generated through MATLAB programming, assisted in meeting all of the questions asked by the council.

The jumper will experience 10 bounces within a 60-second timeframe. To validate the questions posed by the council, the maximum velocity reached by the jumper will occur about 2.61 seconds into freefall peaking at 20.04m/s which satisfies the 'thrill factor' aspect of the jump. The maximum acceleration of 18.461m/s^2 will be experienced by the jumper at 4.26 seconds into the jump. This meets the safety requirement of the attraction as the bungee jump acceleration is not to exceed a max acceleration of 2g which is approximately 19.61m/s^2 . The council requested figures for total distance covered over the jump for promotional purposes. This was met, with the jumper covering a total of 292.10 metres over the course of the 60-second jump.

The council had also raised questions on the possibility of future implementations into the design. A question raised by the council was at what time during the jump would a camera that is installed on the bridge deck be needed in order to take a perfect image of the person bungee jumping. The time taken was calculated to be 3.33 seconds into the jump. The council was also interested in whether the bungee jump had a "water touch" moment upon first bounce and if it there wasn't, what changes would need to occur to ensure it does. Originally the bungee jump did not have a "water touch" as it only reached a total distance of roughly 50 metres upon first bounce, when a minimum distance of 74 metres (including jumpers height) was required to touch the water. The changes required were modifying the spring constant to 78.8N/m and the bungee length to 42.55 metres. This allowed for the bungee to reach a maximum distance, without jumper height included, of 72.25 metres while also maintaining a safe maximum acceleration of 19.466m/s^2. Therefore, with the jumper's height taken into consideration, a “water touch” can be accomplished.

The numerical solution to the mathematical model is sufficient enough to model the design as proposed by the commercial bungee jump company as it meets all concerns of safety, thrill and practicality as raised by the council.

Future recommendations to modify the design would include having a safety factor of 3 for varying weights and heights so that the bungee jump is safe for everyone not just an assumed constant weight and height.