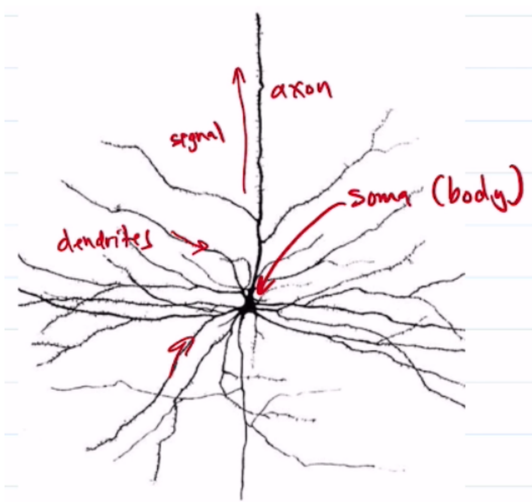
1 Topic 1.

1.1 The Hodgkin-Huxley Neuron Model.

1.1.1 Neurons

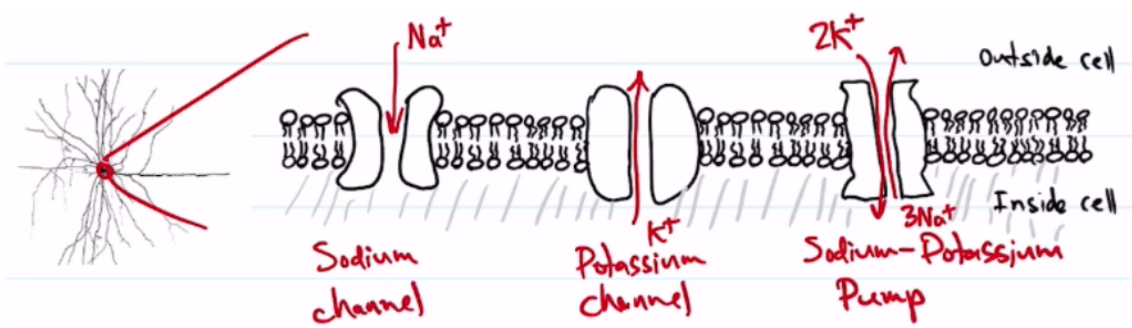
A **neuron** is a special cell that can send and receive signals from other neurons.



- **Soma**: generate electrical signals.
- **Axon**: transmit electrical signals.
- **Dendrites**: receive electrical signals.
- **Synapses**: send electrical signals.

1.1.2 Neuron Membrane Potential

Ions are molecules or atoms in which the number of electrons (-) does not match the number of protons (+), resulting in a net charge. Many ions float around your cells. The cell's **membrane**, a lipid bi-layer, stops most ions from crossing. However, ion channels embedded in the cell membrane allow ions to pass. There exist **sodium** and **potassium channels** which permits Na^+ and K^+ ions to move across the cell membrane, respectively.



The Na^+ channel moves Na^+ ions into the cell while the K^+ channel moves K^+ ions out of the cell. The **sodium-potassium pump** exchanges 3 Na^+ inside the cell for 2 K^+ ions outside the cell. This causes a higher concentration of Na^+ outside the cell and a higher concentration of K^+ inside the cell. It also creates a net positive charge outside and a net negative charge inside the cell. This difference in charge across the membrane induces a voltage difference and is called the **membrane potential**.

1.1.3 Action Potential

Neurons have a peculiar behavior: they can produce a **spike** of electrical activity called an **action potential**. This electrical burst travels along the neuron's **axon** to its **synapses**, where it passes signals to other neurons.

1.1.4 The Hodgkin-Huxley Model

The **Hodgkin-Huxley models** describes how action potentials in neurons are initiated and propagated. Their model is based on the non-linear interaction between membrane potential (aka **voltage**) and the opening/closing of Na^+ and K^+ ion channels. Both Na^+ and K^+ ion channels are voltage-dependent, so their opening and closing changes with the membrane potential.

Let v denote the membrane potential. A neuron usually keeps a membrane potential of around -70mV. We now wish to model the opening/closing of the channels.

Potassium Channels

The fraction of K^+ channels that are open is $n^4(t)$,[1] where

$$\frac{dn}{dt} = \frac{1}{\tau_n(v)}(n_\infty(v) - n).$$

n here is the dynamic variable. Both $\tau_n(v)$ and $n_\infty(v)$ depend on voltage. Thus, the dynamics of the K^+ channel depends on the voltage and varies over time.

As a remark, the DE converges to level n_∞; the rate of convergence is inversely proportional to τ, i.e., it converges faster if τ is smaller.

Sodium Channels

The fraction of Na^+ ion channels open is $(m(t))^3 h(t)$, [2] where

$$\frac{dm}{dt} = \frac{1}{\tau_m(v)}(m_\infty(v) - m)$$
$$\frac{dh}{dt} = \frac{1}{\tau_h(v)}(h_\infty(v) - h)$$

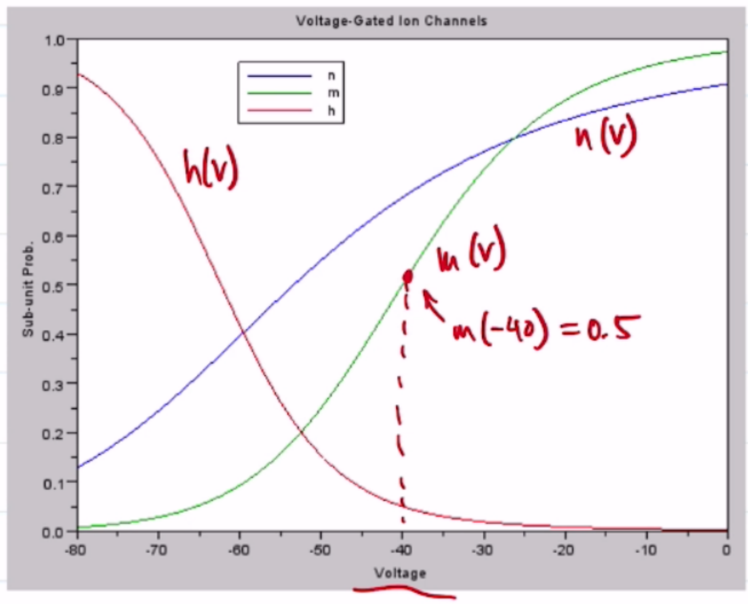
All quantities like τ_m, τ_h, τ_n, etc., are measured empirically.

[1] The intuition is that each K^+ channel is controlled by four gates wherein the probability of one gate being open is n, hence the probability of all gates being open is n^4.

[2] Similar to above, we can interpret this as the Na^+ channel is controlled by three gates with probability m being open and one gate with probability h being open.

Making Sense of DEs

Below is a graph showing how $h(v), m(v), n(v)$ change as functions of voltage. As we can see, as voltage increases (move rightward) the n-gates and m-gates tend to open while the h-gate tend to close. To see how the DEs work, fix membrane potential at $v = -40$. Then we have $m(-40) \approx 0.5$ and $h(-40) \approx 0.05$. With this, you can compute the number (fraction) of sodium channels that are open as $(m(t))^3 h(t)$.



Channels and Membrane Potential

Now these two types of channels allow ions to flow into and out of the cell, inducing a current, which affects the membrane potential V. We can thus describe the membrane potential as a DE in terms of the fraction of K^+ and Na^+ channels that are open:

$$C\frac{dV}{dt} = J_{in} - g_L(V - V_L) - g_{\mathrm{Na}}m^3h(V - V_{\mathrm{Na}}) - g_{\mathrm{K}}n^3(V - V_{\mathrm{K}}).$$

- C: **capacitance**.
- $\frac{dV}{dt}$: time rate of change in voltage, or **current**.
- J_{in}: **input current**, usually from other neurons.
- $V_L, V_{\mathrm{Na}}, V_{\mathrm{K}}$: **zero-current potentials**.
- $g_L, g_{\mathrm{Na}}, g_{\mathrm{K}}$: **maximum conductance**.
- $g_L(V - V_L)$: **leak current**.
- $g_{\mathrm{Na}}m^3h(V - V_{\mathrm{Na}})$: **sodium current**.
- $g_{\mathrm{Na}}m^3h(V - V_{\mathrm{Na}})$: **potassium current**.

This system of four DEs governs the dynamics of the membrane potential.