

# Supplementary materials: Are Bayesian Regularization Methods a Must for Multilevel Dynamic Latent Variables Models?

We provide here supplementary materials containing results that are not shown in the manuscript for simulation study 1 and simulation study 2. We present here several graphics showing the small sample properties of the estimates. In particular, we illustrated the convergence rates and the sampling precision rates across data conditions. We also computed the absolute bias, the relative bias as well as the root mean squared error (RMSE). Their behavior across the data conditions are shown here. Furthermore, in the manuscript we only presented the type-I-error rates and the NDR when 50% of the components of the vector of slopes  $\beta$  are zeros. In this document, we presented these metrics when 75% of  $\beta$ 's components are zeros.

## 1 Simulation study 1

### 1.1 Convergence rates and sampling precision rates

**Convergence** Here, we describes the convergence rates across replications and the data conditions. We say that a model converge if we have  $\hat{R} > 1.1$  for each parameter of the model.

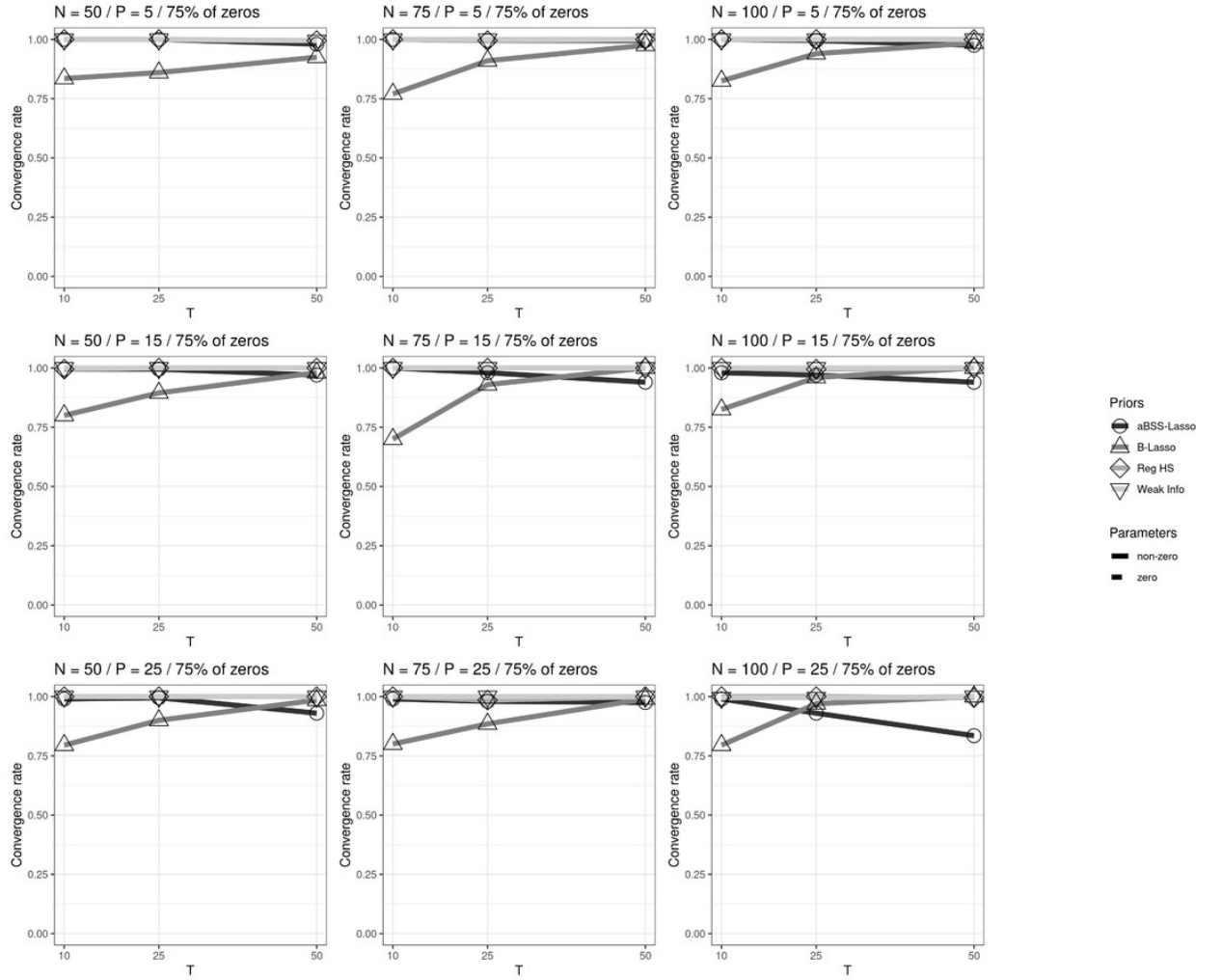


Figure 1: Convergence rates (based on  $\hat{R} > 1.1$ ) across  $N, T$  and  $P$  when 75% of  $\beta$  components are zeros

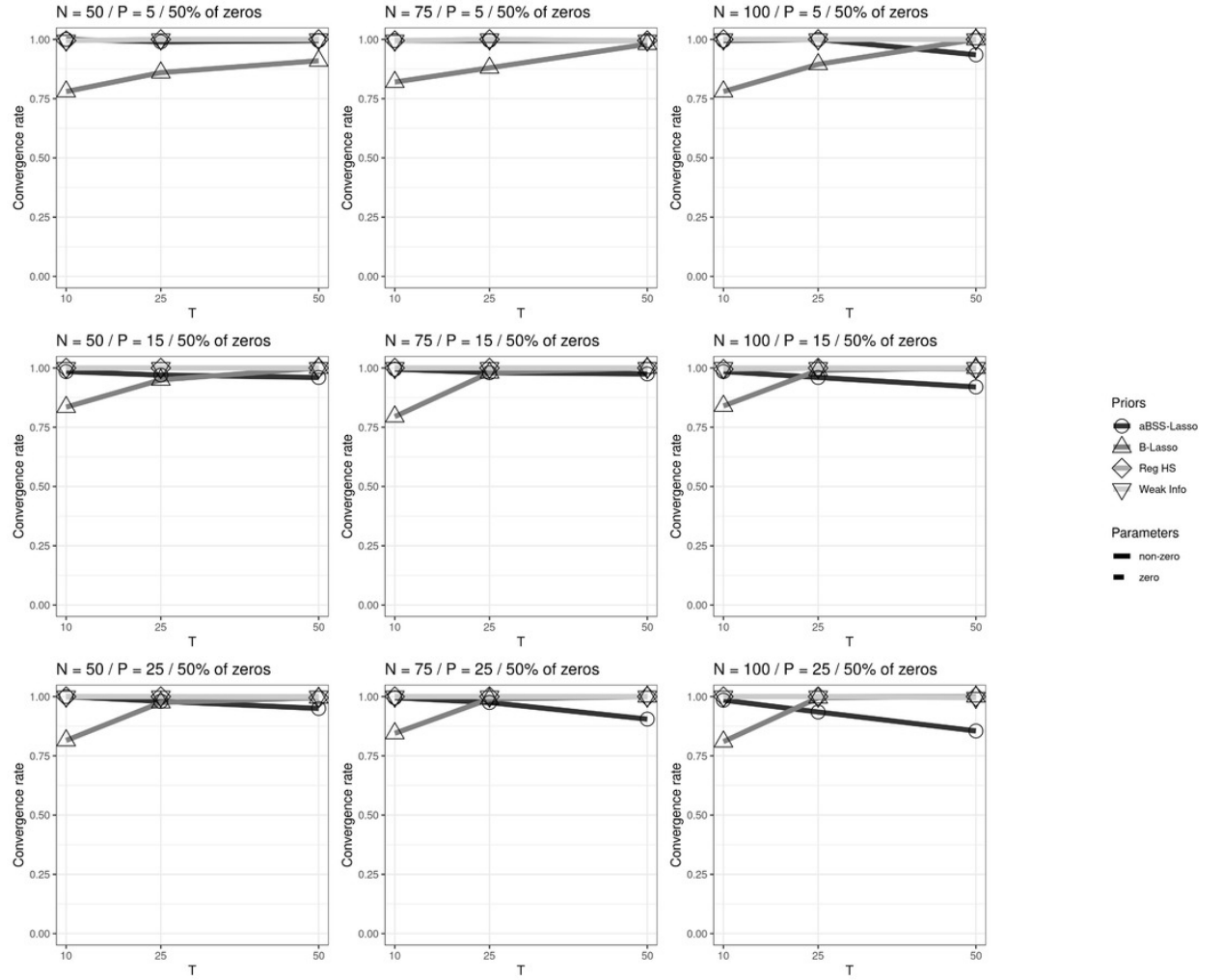


Figure 2: Convergence rates (based on  $\hat{R} > 1.1$ ) across  $N, T$  and  $P$  when 50% of  $\beta$  components are zeros

**Sampling Precision** We also calculated the sampling precision based on the threshold over the effective sample size (ESS). These thresholds are  $x = 100, 400, 1000$ . The precision rates are calculated across replications, so that for a given replication, if all parameters had  $ESS > x$ , then we say that the HMC sampling applied to that replication provided the expected sampling precision.

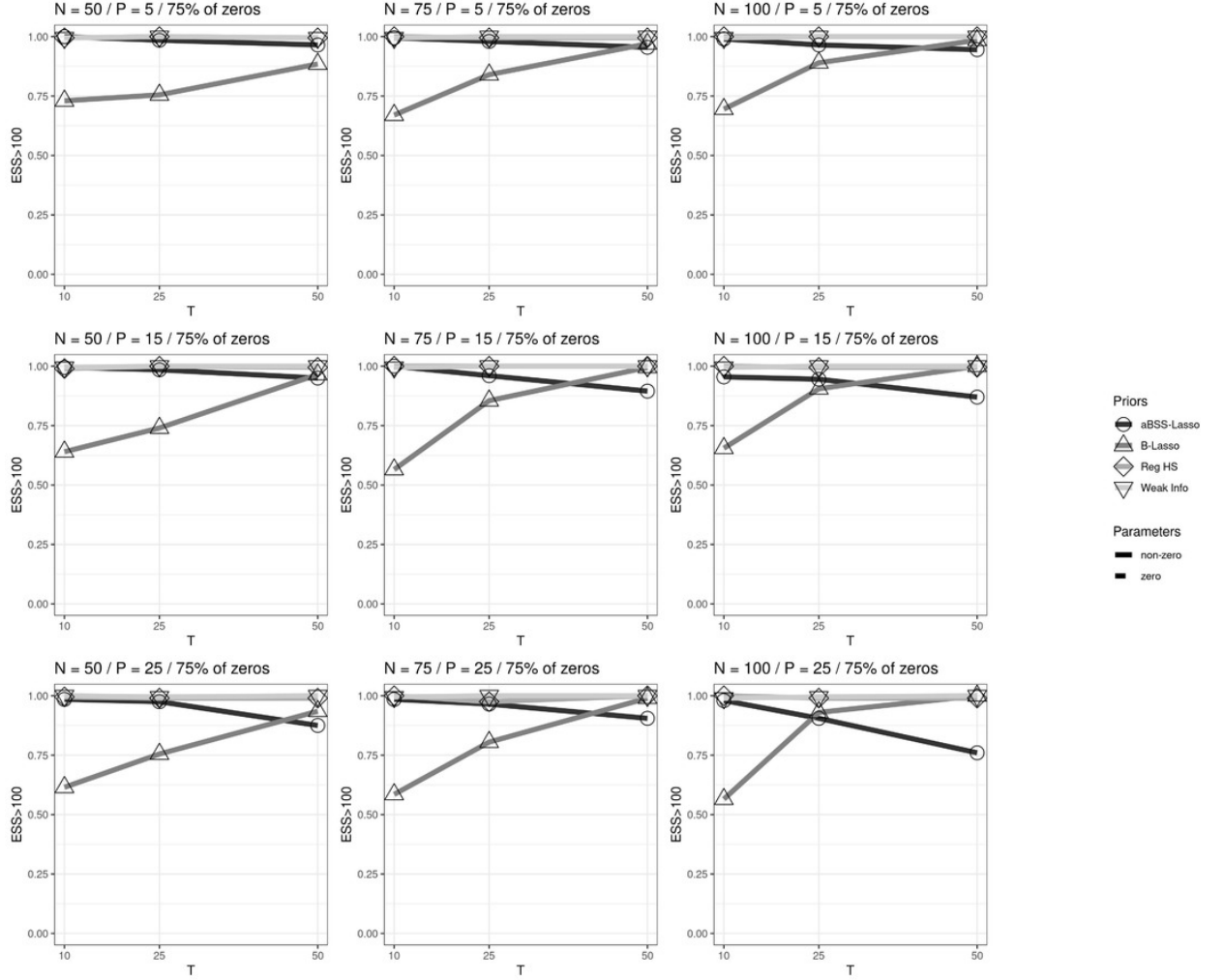


Figure 3: Sampling precision rates (based on  $ESS > 100$ ) across  $N, T$  and  $P$  when 75% of  $\beta$  components are zeros

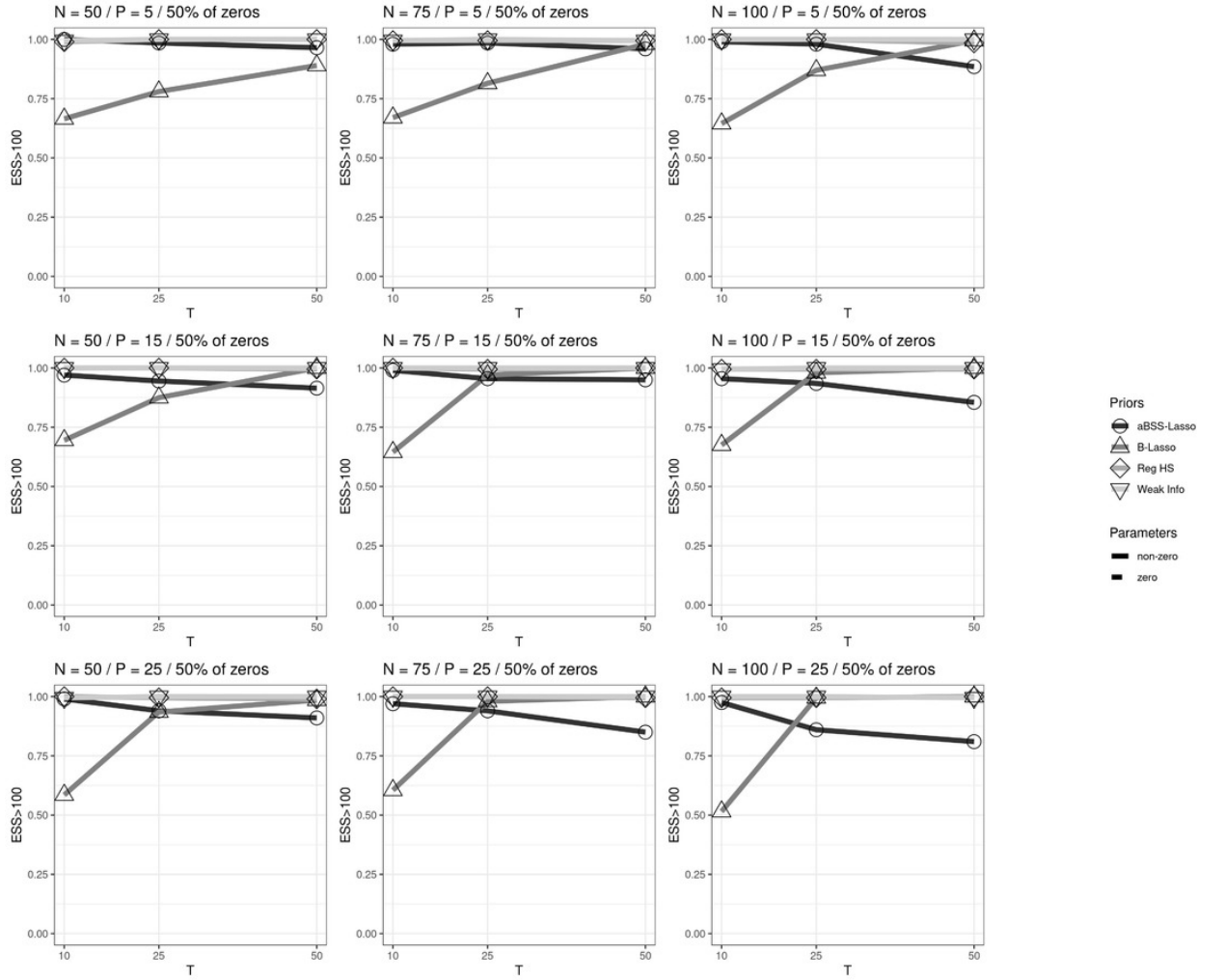


Figure 4: Sampling precision rates (based on  $ESS > 100$ ) across  $N, T$  and  $P$  when 50% of  $\beta$  components are zeros

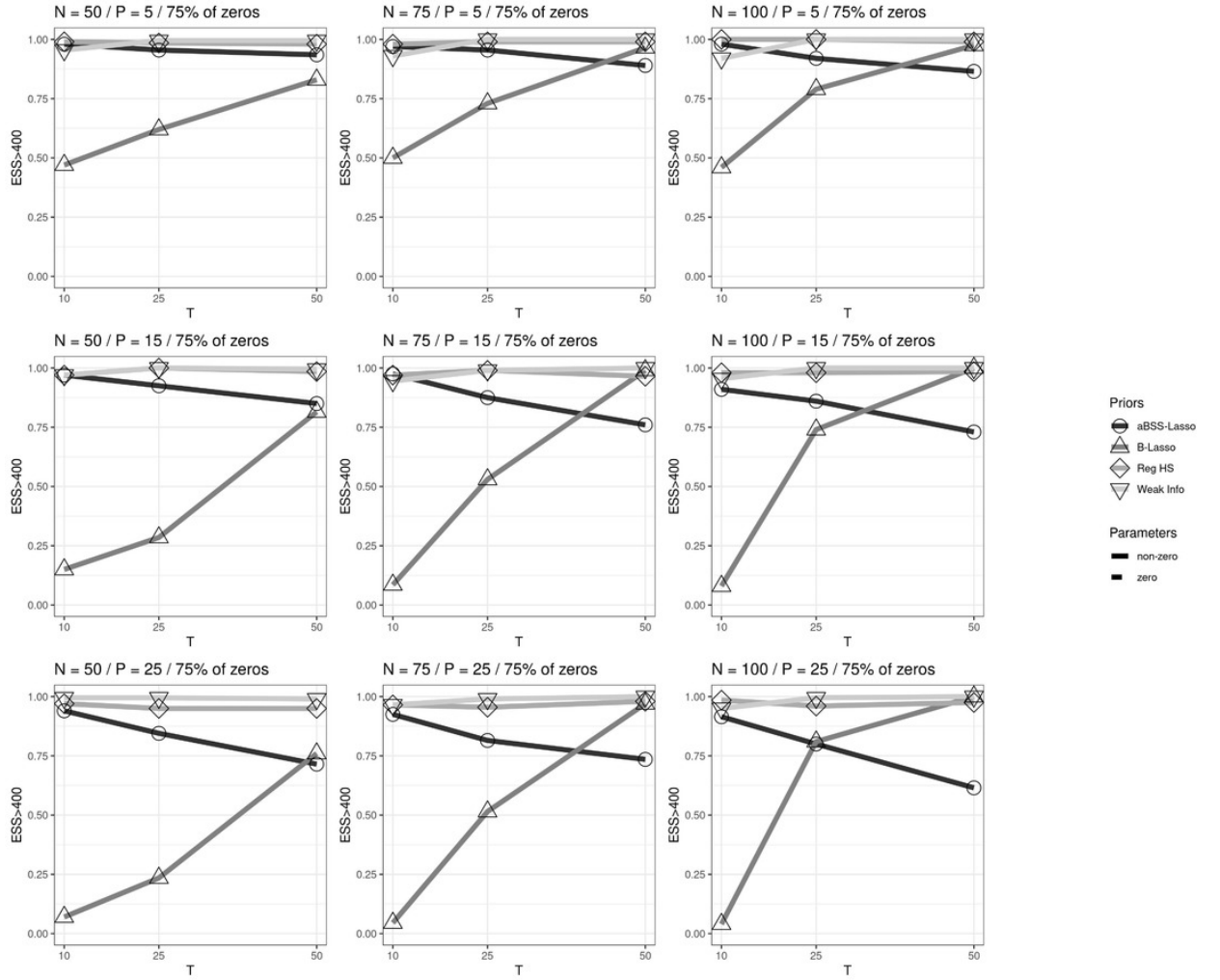


Figure 5: Sampling precision rates (based on  $ESS > 400$ ) across  $N, T$  and  $P$  when 75% of  $\beta$  components are zeros

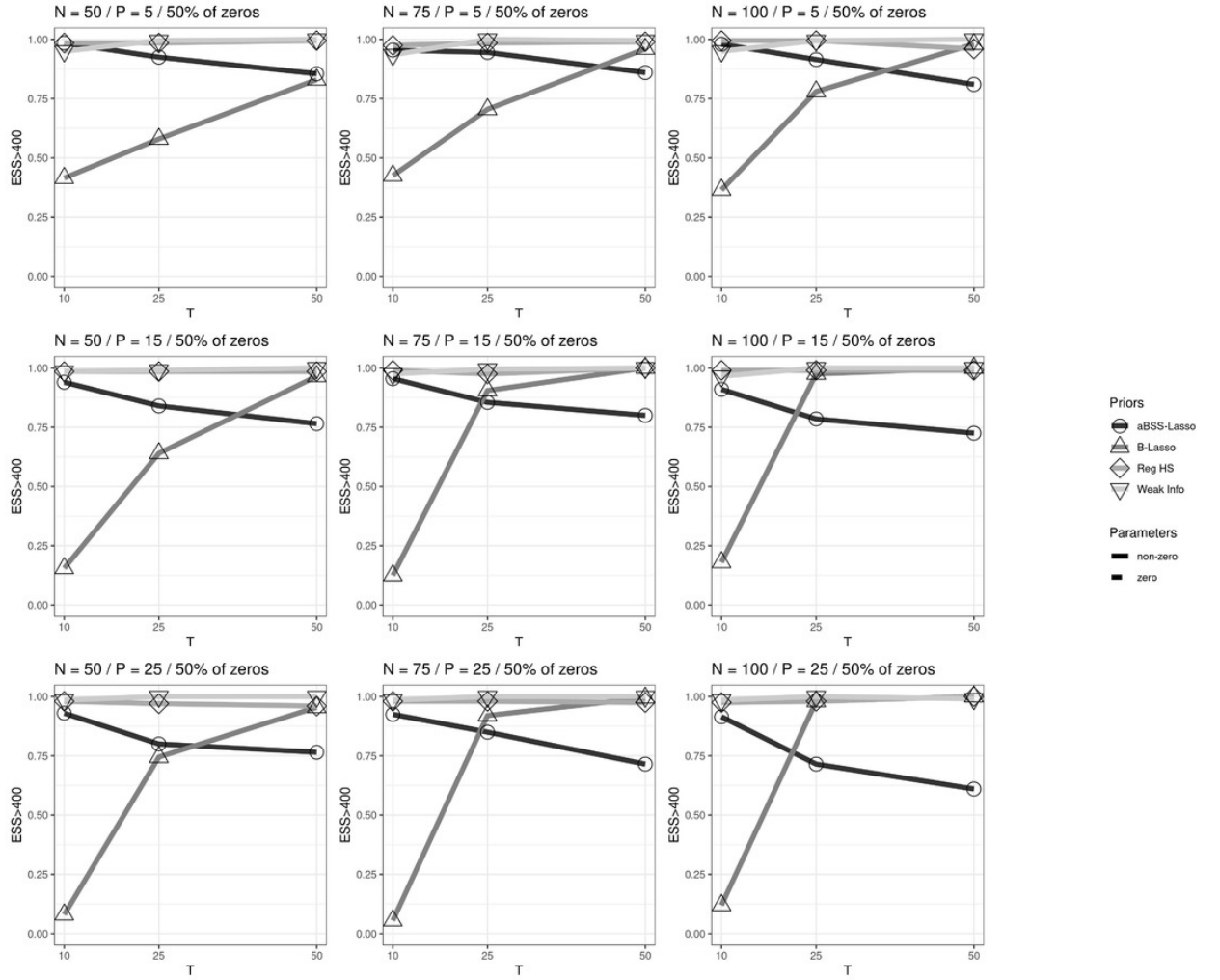


Figure 6: Sampling precision rates (based on  $ESS > 400$ ) across  $N, T$  and  $P$  when 50% of  $\beta$  components are zeros

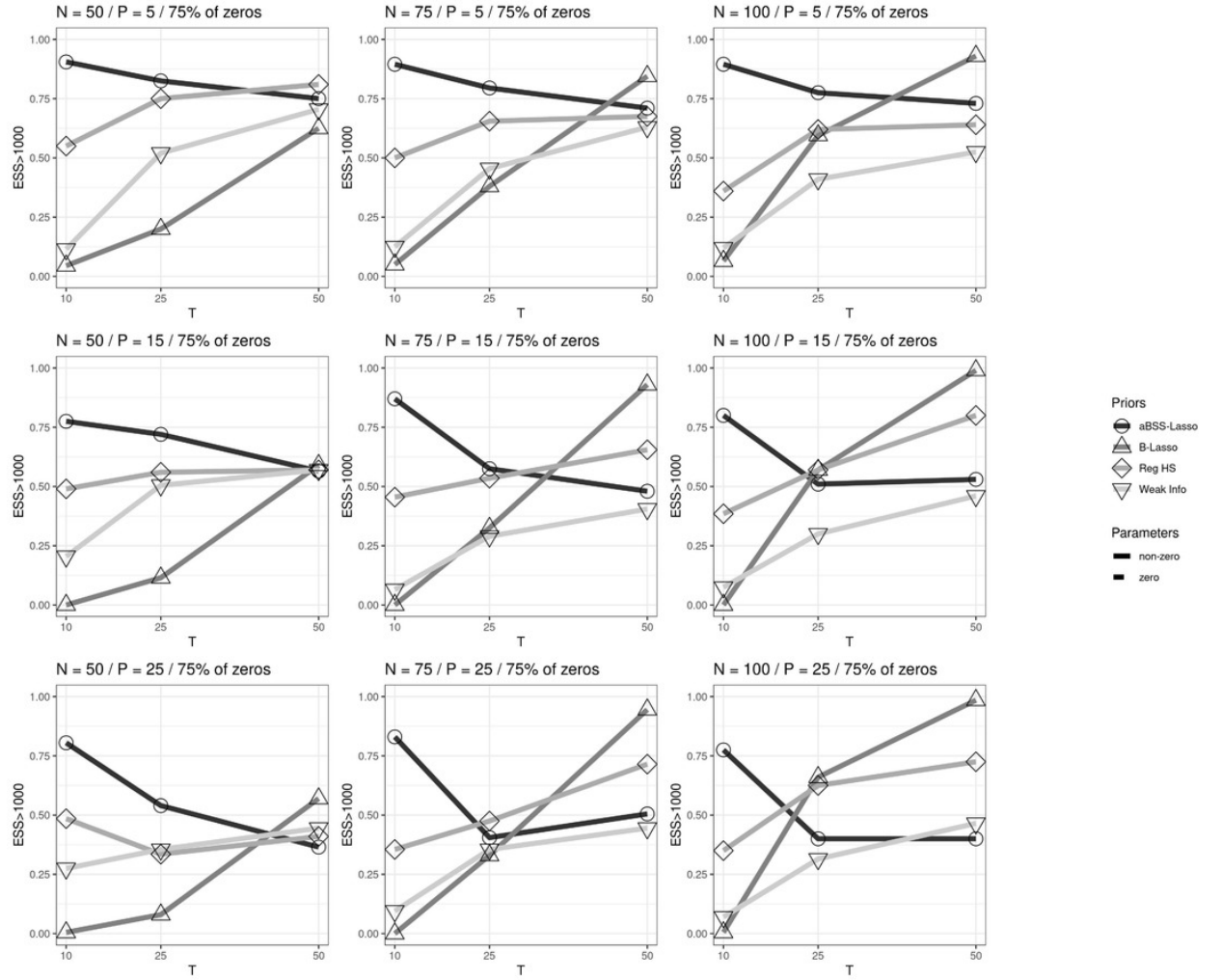


Figure 7: Sampling precision rates (based on  $ESS > 1000$ ) across  $N, T$  and  $P$  when 75% of  $\beta$  components are zeros



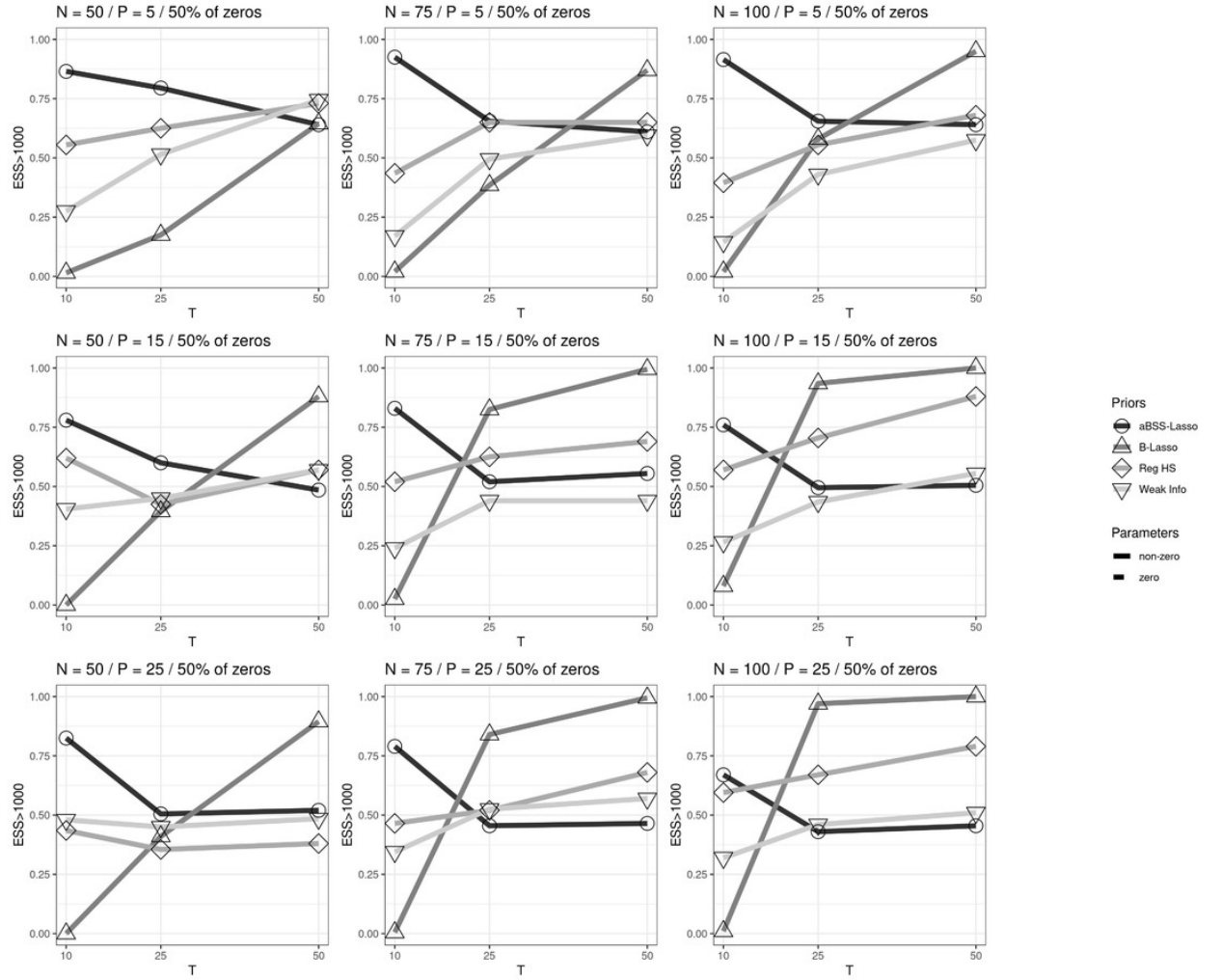


Figure 8: Sampling precision rates (based on  $ESS > 1000$ ) across  $N, T$  and  $P$  when 50% of  $\beta$  components are zeros

## 1.2 Finite sample properties

**Absolute Bias** The absolute bias of an estimator  $\hat{\theta}_m$  of a parameter  $\theta$  is computed by the following quantity:

$$AB = \frac{1}{M} \sum_{m=1}^M |\theta - \hat{\theta}_m|$$

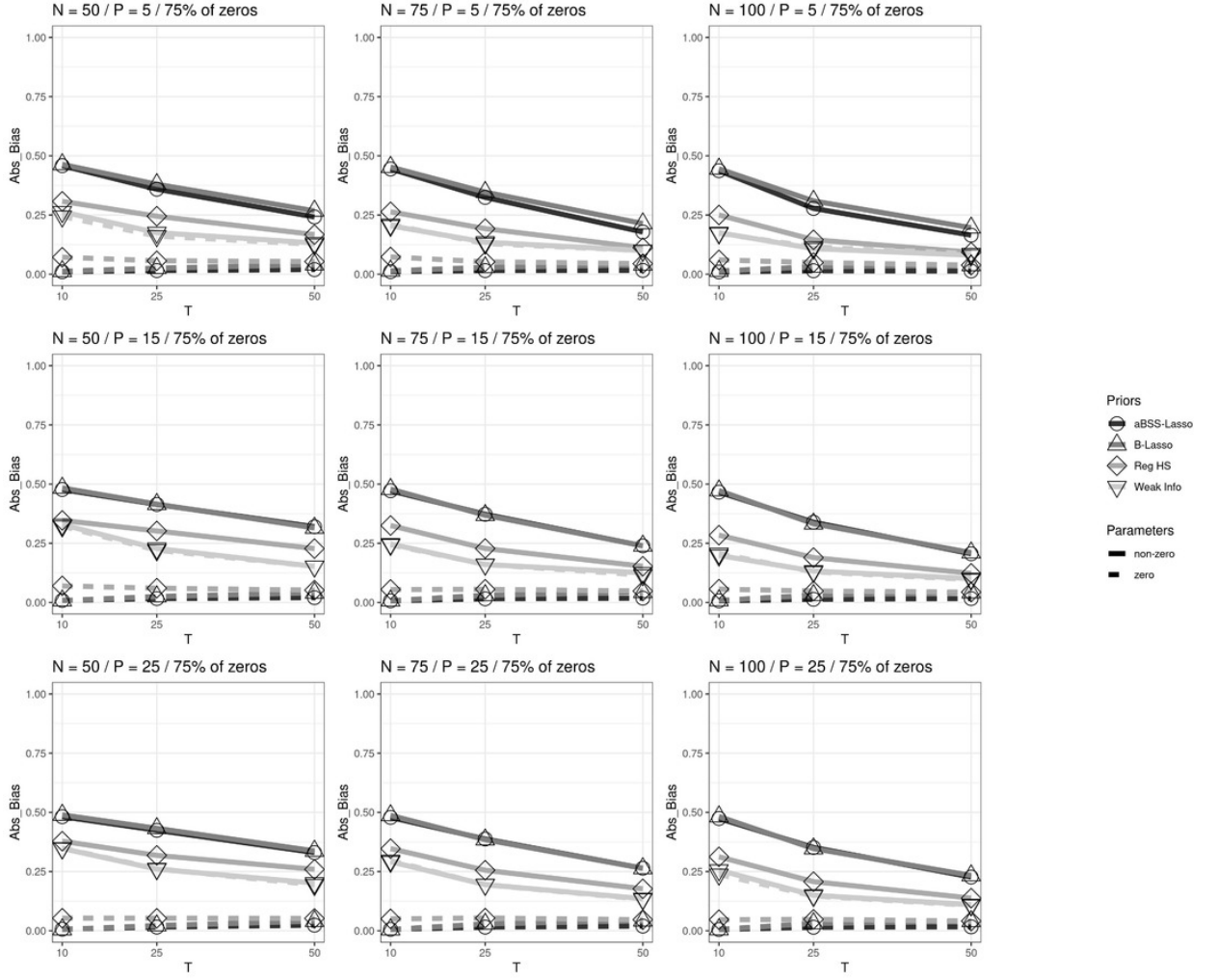


Figure 9: Absolute Bias across  $N, T$  and  $P$  slope when 75% of  $\beta$  components are zeros

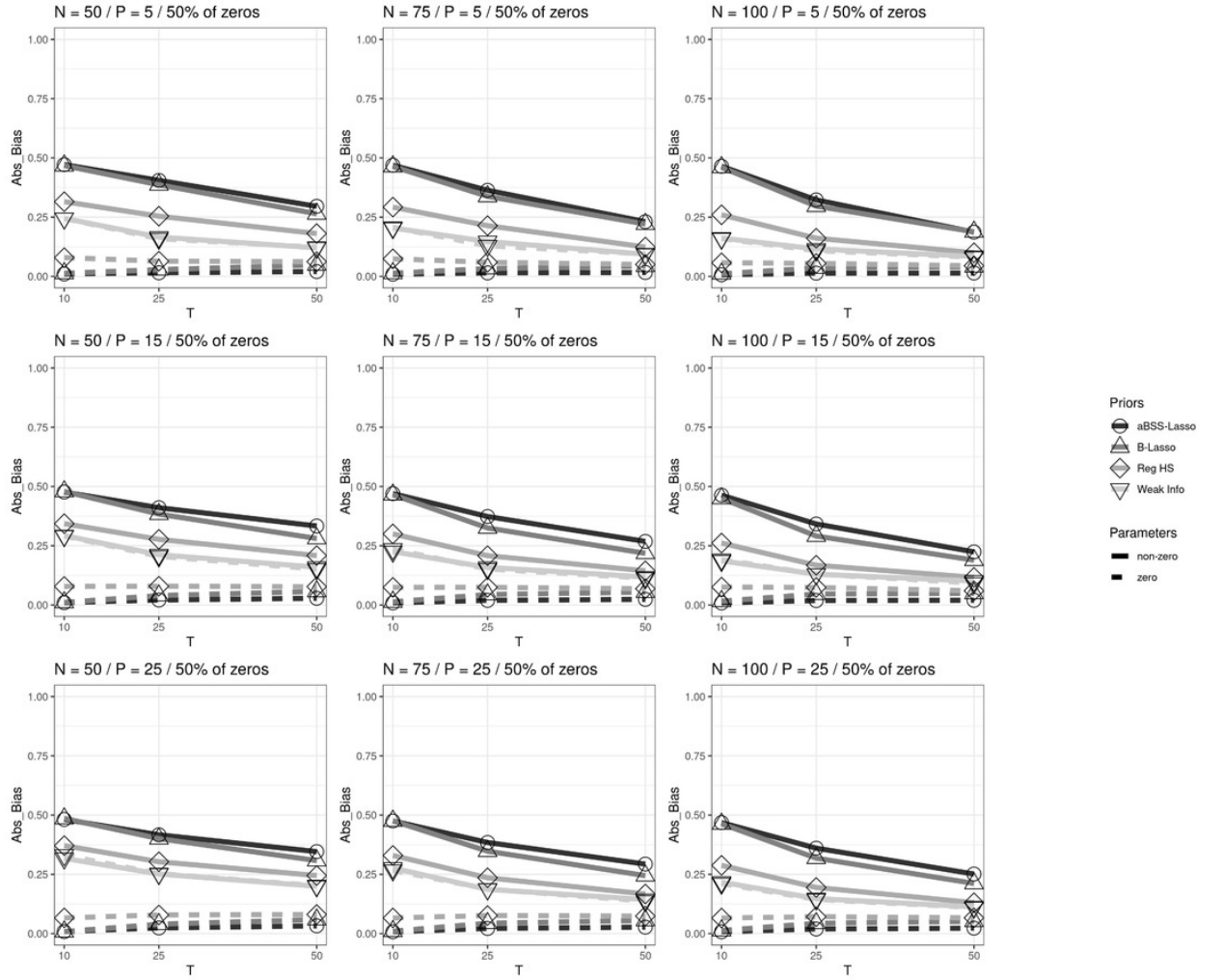


Figure 10: Absolute Bias across  $N, T$  and  $P$  slope when 50% of  $\beta$  components are zeros

**Relative Bias** The relative bias of an estimator  $\hat{\theta}_m$  of a parameter  $\theta$  is computed by the following quantity:

$$RB = \frac{1}{M} \sum_{m=1}^M \frac{\hat{\theta}_m}{\theta}$$

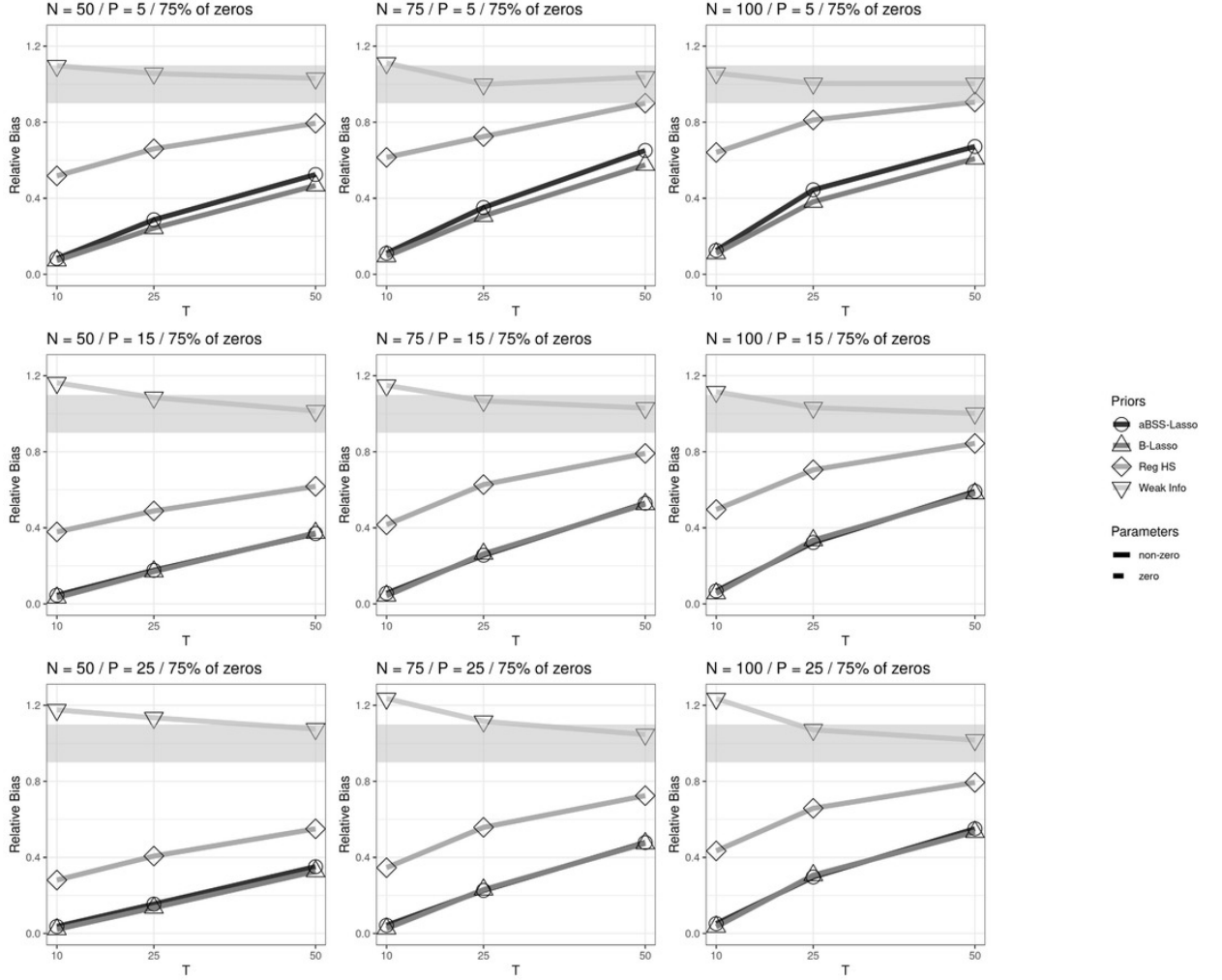


Figure 11: Absolute Bias across  $N, T$  and  $P$  slope when 75% of  $\beta$  components are zeros

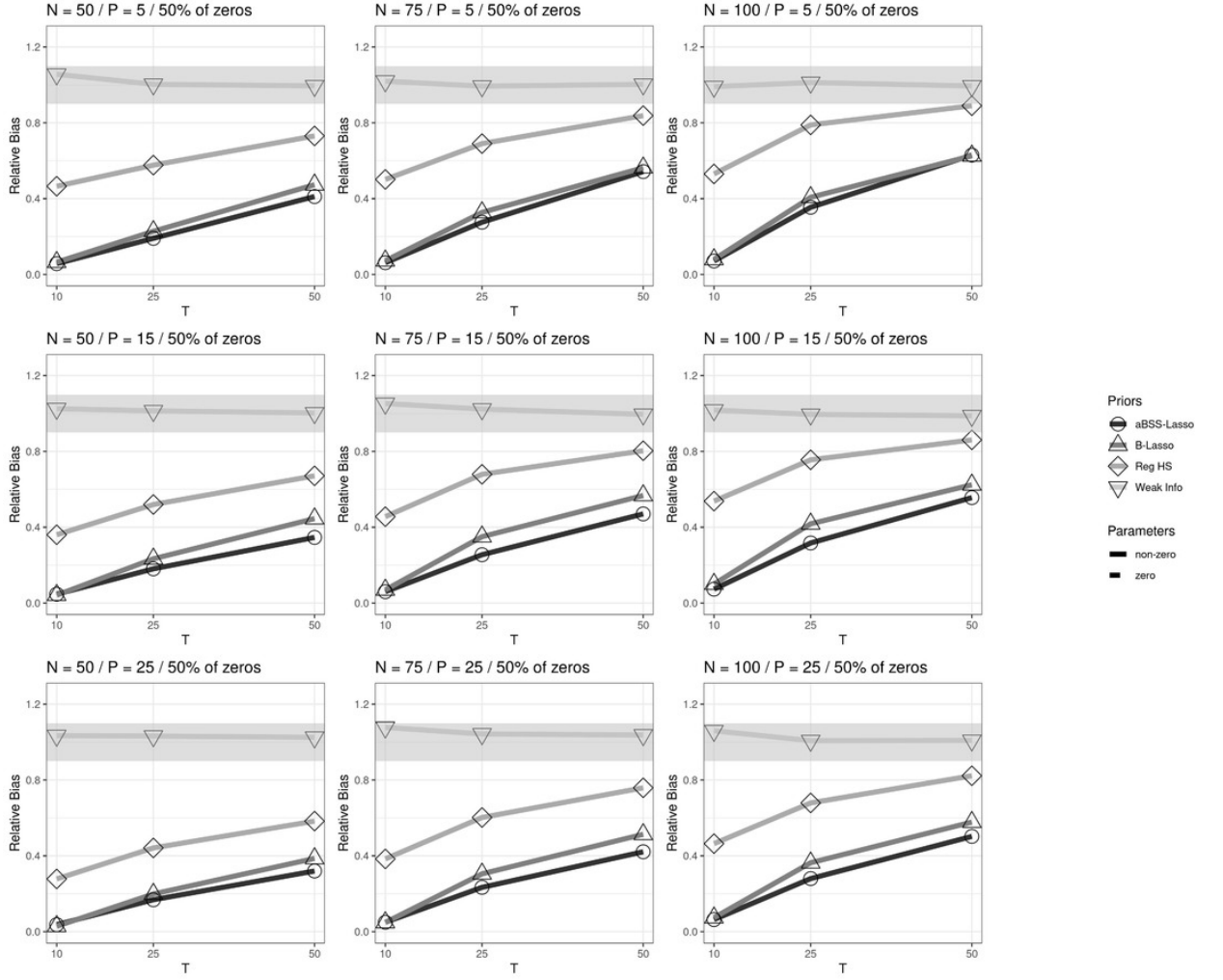


Figure 12: Absolute Bias across  $N, T$  and  $P$  slope when 50% of  $\beta$  components are zeros

**RMSE** The RMSE of an estimator  $\hat{\theta}_m$  of a parameter  $\theta$  is computed by the following quantity:

$$RMSE = \sqrt{\frac{1}{M} \sum_{m=1}^M (\theta - \hat{\theta}_m)^2}$$

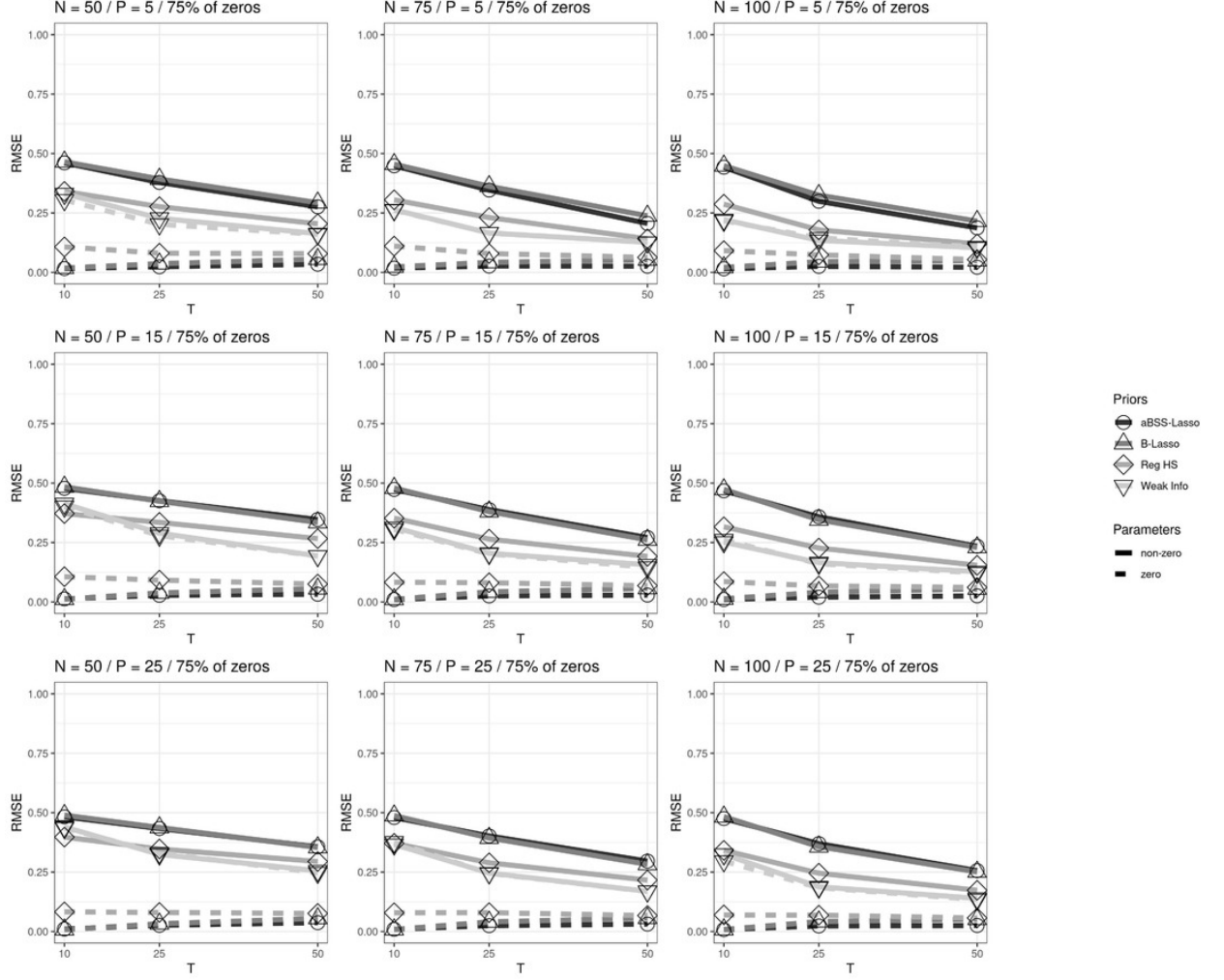


Figure 13: RMSE across  $N, T$  and  $P$  slope when 75% of  $\beta$  components are zeros

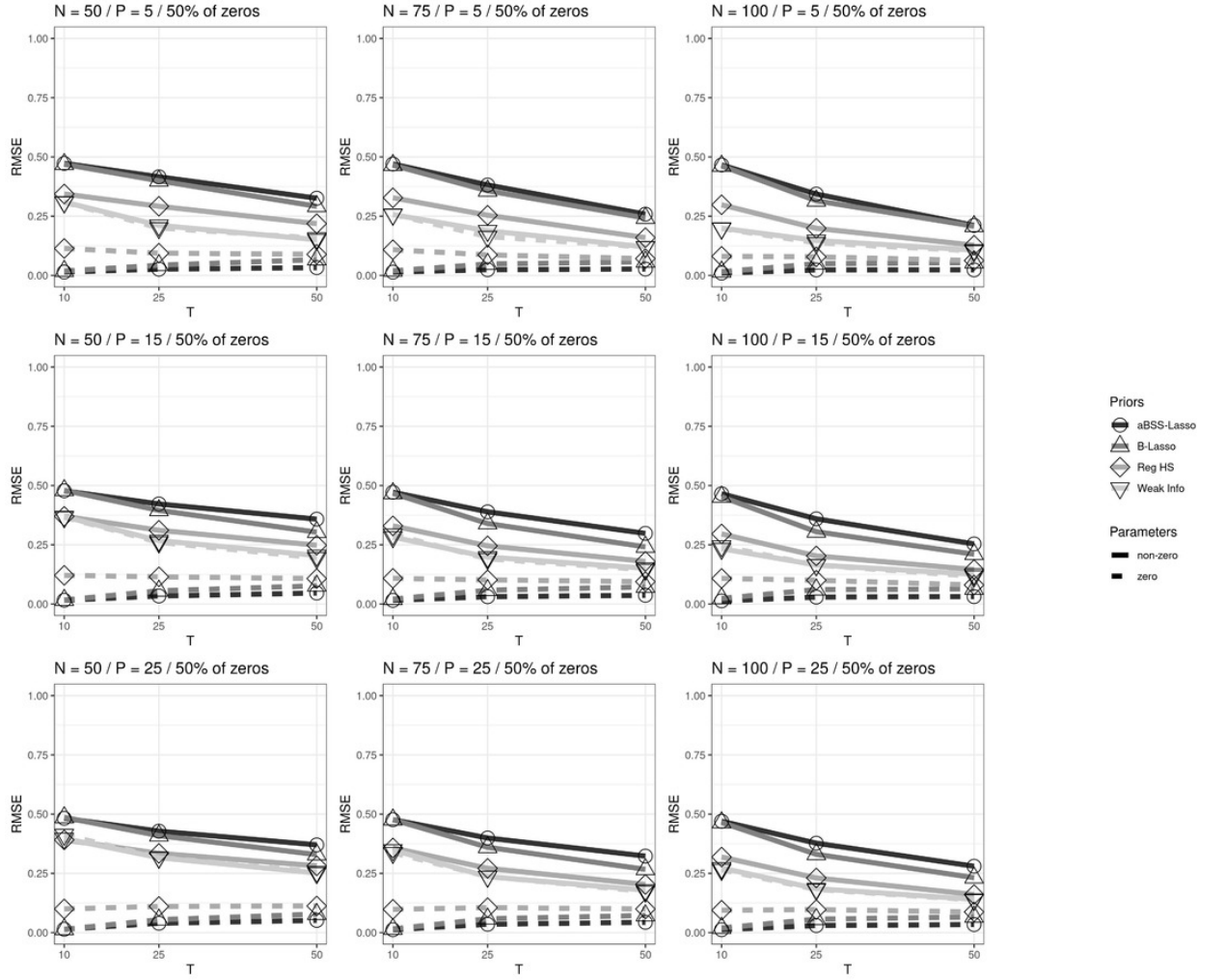


Figure 14: RMSE across  $N, T$  and  $P$  slope when 50% of  $\beta$  components are zeros

**Coverage rates and Type-I-error rates** The following figures show the Coverage rates and Type-I-error rates when 75% of the  $\beta$  components are zeros.

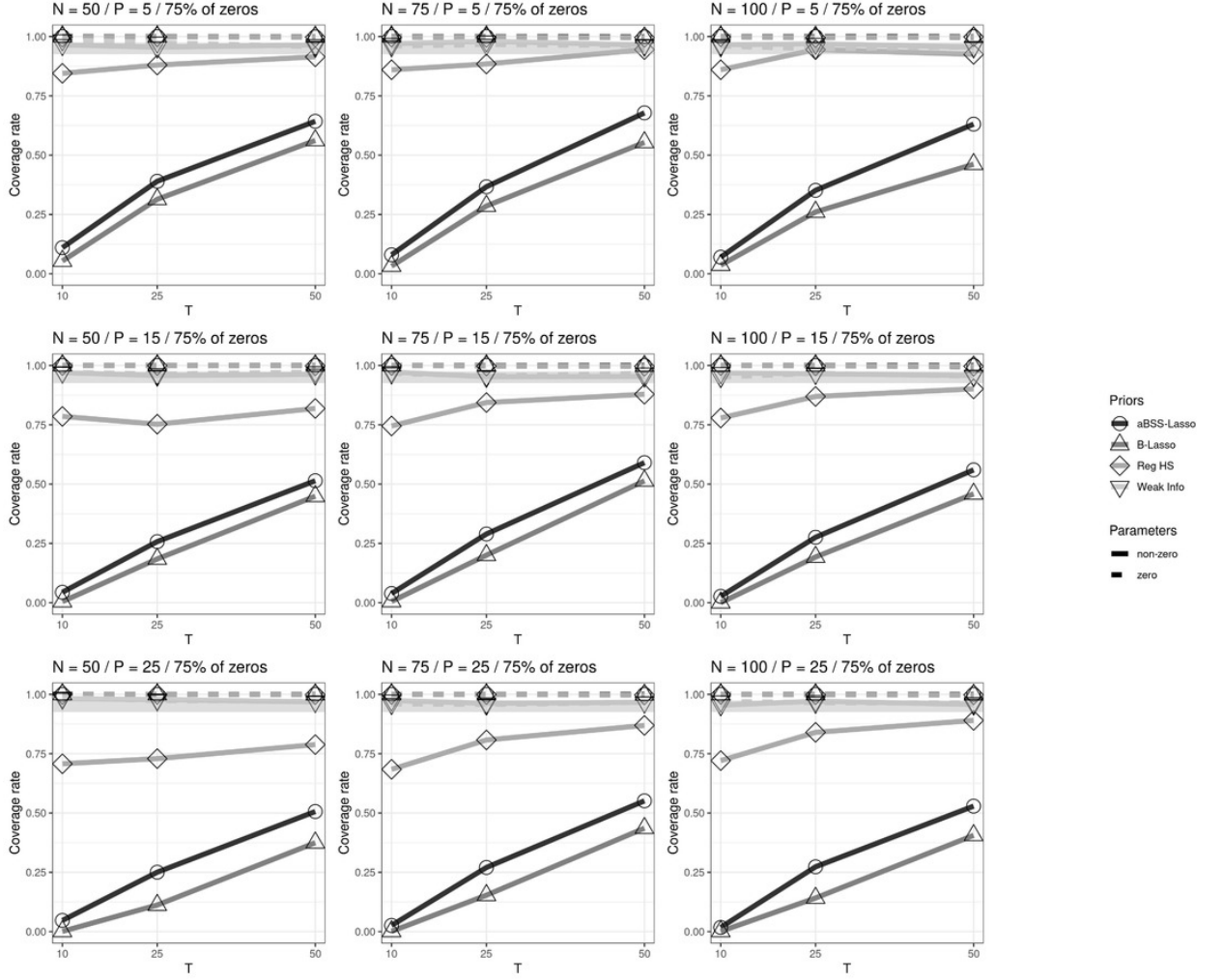


Figure 15: Coverage rates across  $N, T$  and  $P$  slope when 75% of  $\beta$  components are zeros



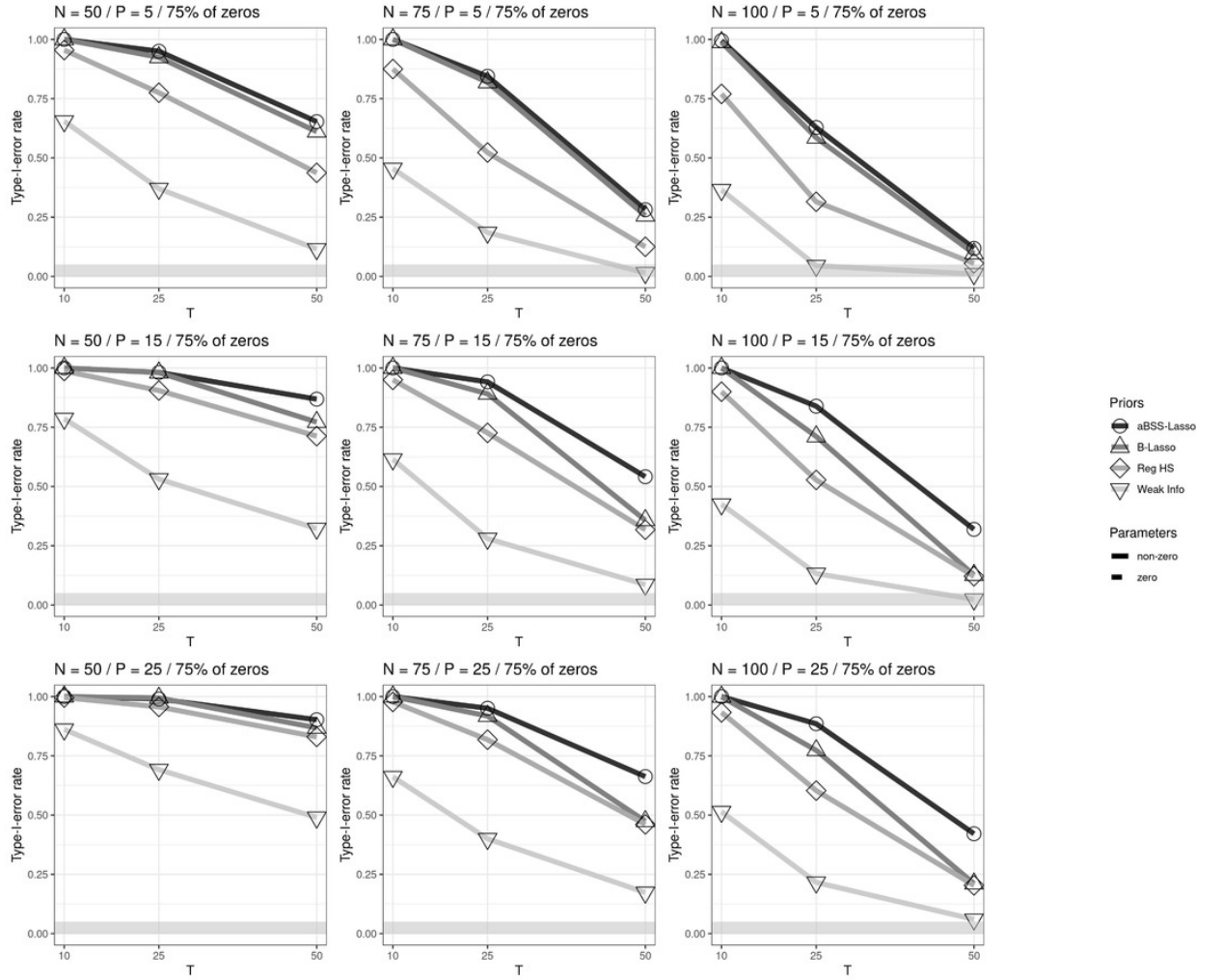


Figure 16: Type-I-error rates across  $N, T$  and  $P$  slope when 75% of  $\beta$  components are zeros

## 2 Simulation study 2

Here, we present the sampling precision over the data conditions of Simulation Study 2. To compute this quantity we applied the same thresholds to ESS as in simulation study 1, i.e.  $x = 100, 400, 1000$ . Figure 17 shows the corresponding results.

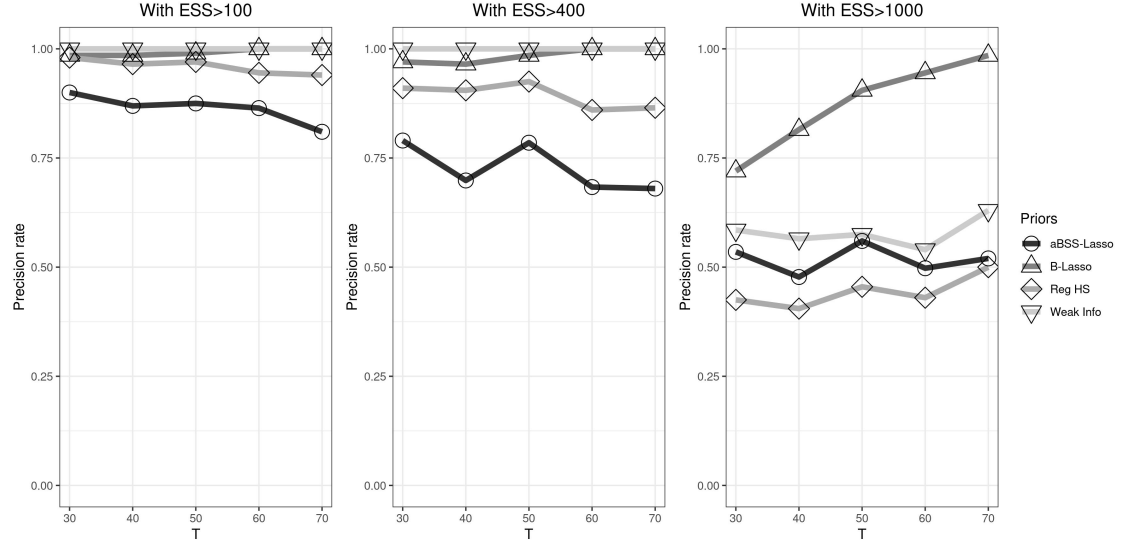


Figure 17: Comparison of the sampling precision rates for each prior distribution across different numbers of time points  $T$ .