Response for Project 5

Problem 1

For a range of implied volatilities between 10% and 80%, plot the value of the call and the put

I use BSM formula to find options' values. Let inputs be:

```
strike = 165
underlying = 165
current = pd.to_datetime("2022-02-25")
expire = pd.to_datetime("2022-03-18")
rf = 0.0025
coupon = 0.0053
b = rf - coupon
days = (expire - current).days
ttm = days / 365
iVol = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8]
```

and pass them into the BSM formula:

$$d_1 = \frac{\ln\left(\frac{S}{X}\right)+\left(b+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

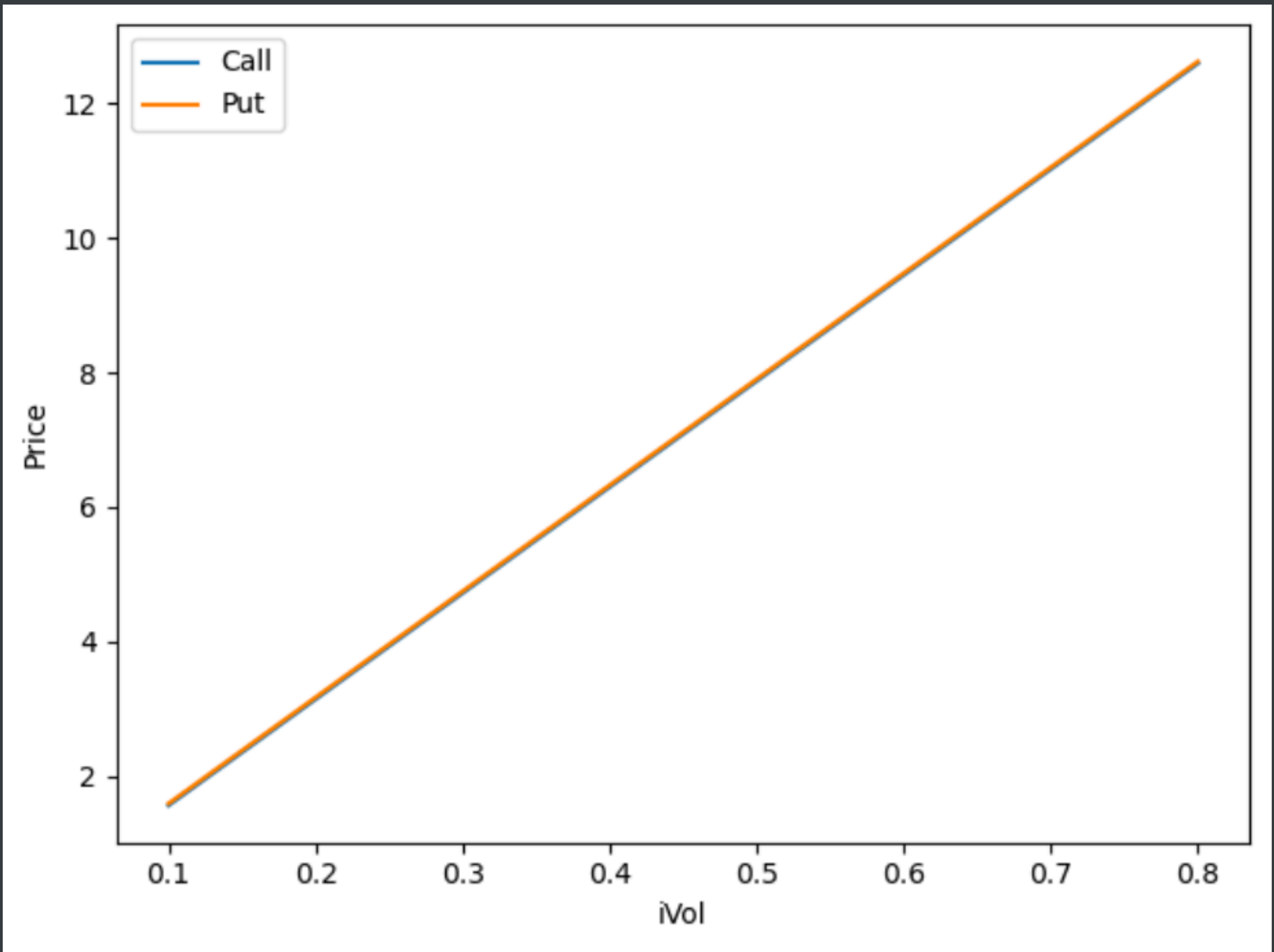
$$Call = Se^{(b-r)T}\Phi(d_1) - Xe^{-rT}\Phi(d_2)$$

$$Put = Xe^{-rT}\Phi(-d_2) - Se^{(b-r)T}\Phi(-d_1)$$

I get option values under different implied volatilities.

	iVol	Call	Put
0	0.1	1.565266	1.591840
1	0.2	3.143538	3.170113
2	0.3	4.721369	4.747943
3	0.4	6.298521	6.325096
4	0.5	7.874768	7.901343
5	0.6	9.449882	9.476457
6	0.7	11.023638	11.050213
7	0.8	12.595810	12.622385

Plot the value of the call and the put:



Discuss these graphs. How does the supply and demand affect the implied volatility?

Observations:

1. When the option price is at-the-money, implied volatility seems to have a linear relationship with the option price. The larger the implied volatility, the larger the option price.
2. The graph for call overlaps with the graph for put.

Implied volatility is directly influenced by the supply and demand of the underlying options and by the market's expectation of the share price's direction. As expectations rise, or as the demand for an option increases, option price will rise, implied volatility will rise, and option values will rise.

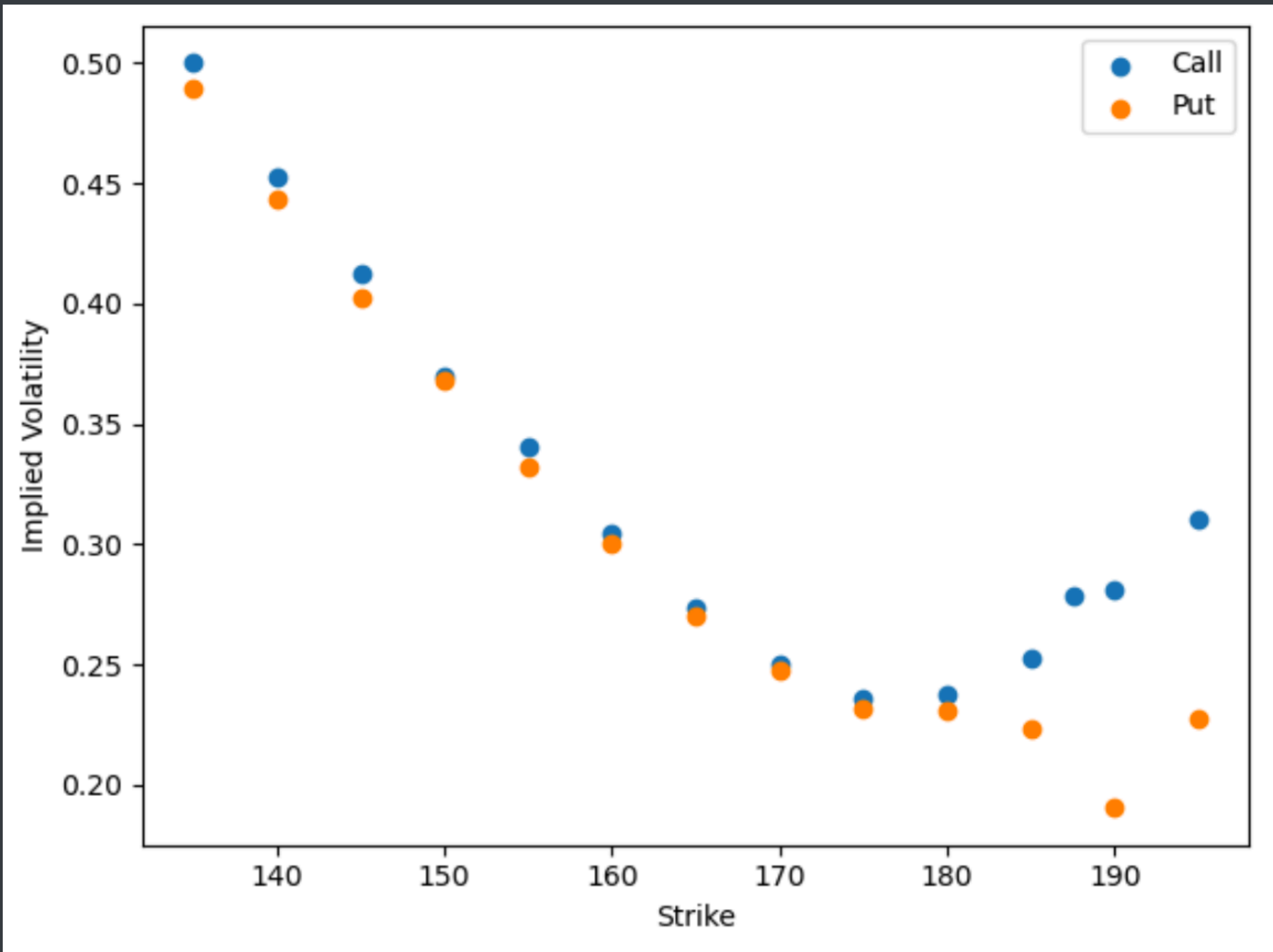
Problem 2

Calculate the implied volatility for each option

I use the **fsolve** function in Python to find implied volitility given an option price. For specific declarations, see the **cal_ivol** function in BS class in Q1.py.

	Stock	Expiration	Type	Strike	Last Price	isCall	iVol
0	AAPL	2022-03-18	Call	135.0	30.175	True	0.499898
1	AAPL	2022-03-18	Call	140.0	25.300	True	0.452886
2	AAPL	2022-03-18	Call	145.0	20.525	True	0.412720
3	AAPL	2022-03-18	Call	150.0	15.850	True	0.369706
4	AAPL	2022-03-18	Call	155.0	11.525	True	0.340394
5	AAPL	2022-03-18	Call	160.0	7.525	True	0.304316
6	AAPL	2022-03-18	Call	165.0	4.225	True	0.273378
7	AAPL	2022-03-18	Call	170.0	1.935	True	0.249889
8	AAPL	2022-03-18	Call	175.0	0.715	True	0.235938
9	AAPL	2022-03-18	Call	180.0	0.260	True	0.237799
10	AAPL	2022-03-18	Call	185.0	0.115	True	0.252563
11	AAPL	2022-03-18	Call	187.5	0.120	True	0.278469
12	AAPL	2022-03-18	Call	190.0	0.075	True	0.280884
13	AAPL	2022-03-18	Call	195.0	0.055	True	0.310271
14	AAPL	2022-03-18	Put	135.0	0.320	False	0.489545
15	AAPL	2022-03-18	Put	140.0	0.435	False	0.443027
16	AAPL	2022-03-18	Put	145.0	0.640	False	0.402427
17	AAPL	2022-03-18	Put	150.0	1.015	False	0.368100
18	AAPL	2022-03-18	Put	155.0	1.610	False	0.332262
19	AAPL	2022-03-18	Put	160.0	2.640	False	0.299955
20	AAPL	2022-03-18	Put	165.0	4.350	False	0.270110
21	AAPL	2022-03-18	Put	170.0	7.075	False	0.247330
22	AAPL	2022-03-18	Put	175.0	10.850	False	0.231569
23	AAPL	2022-03-18	Put	180.0	15.400	False	0.230668
24	AAPL	2022-03-18	Put	185.0	20.225	False	0.223064
25	AAPL	2022-03-18	Put	190.0	25.175	False	0.190425
26	AAPL	2022-03-18	Put	195.0	30.175	False	0.227408

Plot the implied volatility vs the strike price for Puts and Calls



Discuss the shape of these graphs. What market dynamics could make these graphs?

The shape of these graphs looks like a skewed smile. Academically, this is called an implied volatility smirk.

Theoretically, implied volatility measures the perceived volatility of the underlying asset. Regardless of the strike price, all options with the same underlying asset should have the same actual volatility. Therefore, the actual volatility should be a horizontal line in the gragh above. When the strike price equals to the underlying asset price, implied volatility should be the most accurate measure of the actual volatility. therefore, the actual volatility line should pass across (164.85, f(164.85)).

However, we observe a skewed smile shape of the implied volatility. The IV for options at the lower strikes are higher than the IV at higher strikes. This pattern suggests that in-the-money calls and out-of-the-money puts are more expensive compared to out-of-the-money calls and in-the-money puts.

The popular explanation for the manifestation of the reverse volatility skew is that investors are generally worried about market crashes and buy puts for protection.

Another possible explanation is that in-the-money calls have become popular alternatives to outright stock purchases as they offer leverage. This leads to greater demands for in-the-money calls and therefore increased IV at the lower strikes.

Problem 3

For each of the portfolios, graph the portfolio value over a range of underlying values. Plot the portfolio values.

Steps:

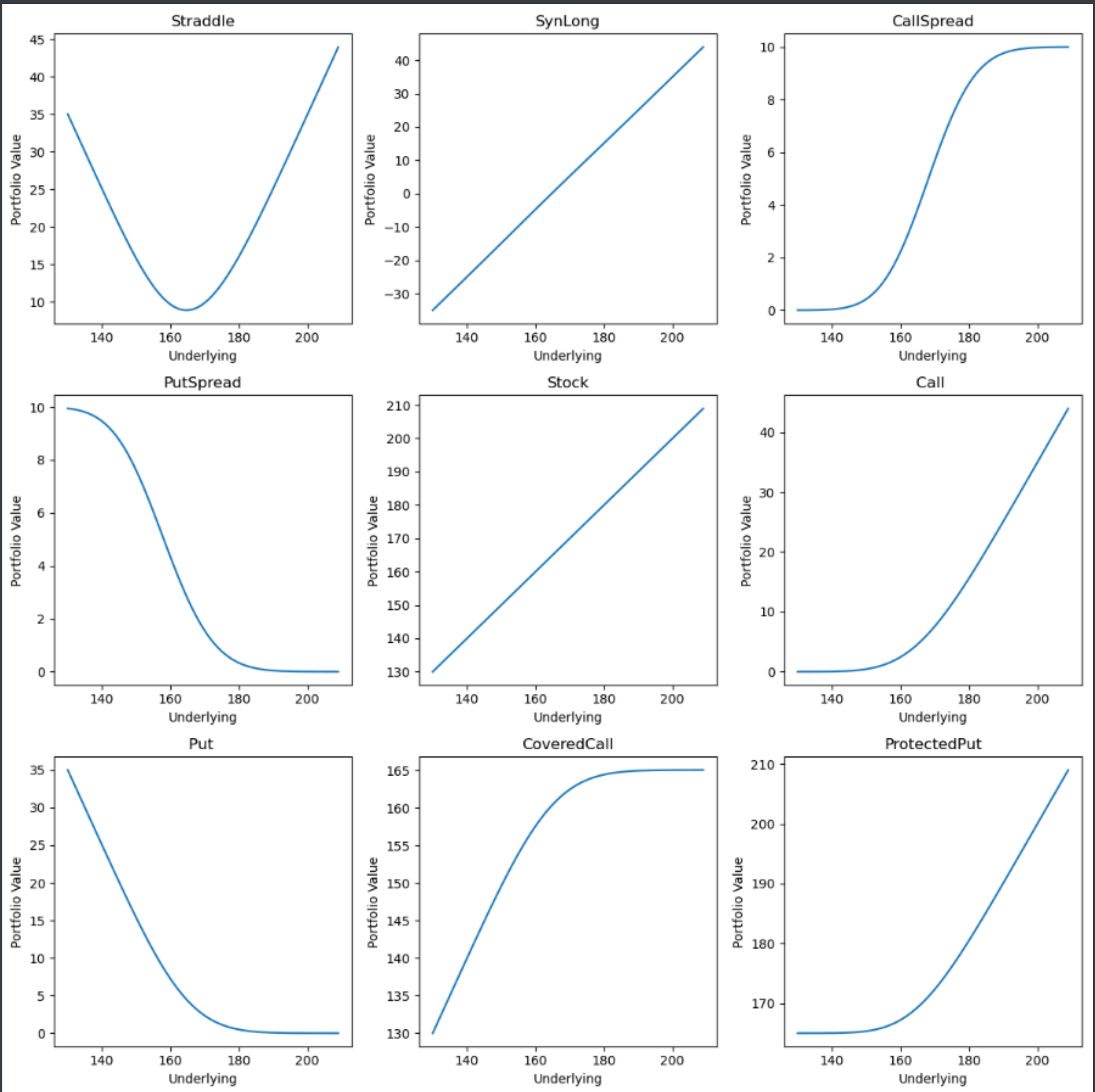
1. For each option, calculate the implied volatility given the underlying inputs.

   ```
   currUnderlying = 164.85
   current = pd.to_datetime("2022-02-25")
   rf = 0.0025
   coupon = 0.0053
   b = rf - coupon
   initVol = 0.5
   daysInYear = 365
   ```

2. Simulate a range of underlying prices and calculate the corresponding option prices using the implied volatilities calculated above.

   ```
   underlyings = list(range(130, 210))
   ```

3. Multiply options prices with holdings to calculate the portfolio values.
4. Plot.



Discuss the shapes.

1. Straddle has a smile shape. The goal of this strategy is to profit from a very strong move, usually triggered by a newsworthy event, in either direction by the underlying asset.
2. SynLong has the same shape with Stock. It can be used to simulate a stock payoff. This is related to the Put Call Parity.

$$C - P = S - Xe^{-rT}$$

3. CallSpread and PutSpread benefit from a stock's limited increase in price.

4. ProtectedCall has the same shape with the reversed Put. It can be used to simulate a put payoff. This is related to the Put Call Parity.

$$S - C = Xe^{-rT} - P$$

5. ProtectedPut has the same shape with Call. It can be used to simulate a call payoff. This is related to the Put Call Parity.

$$S + P = C + Xe^{-rT}$$

Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above). Calculate Mean, VaR and ES. Discuss.

Steps:

1. Calculate the standard deviation of AAPL returns in DailyReturn.csv.
2. Simulate 10 normally distributed returns according to my fitted standard deviation. Assume loc=0.
3. Use my simulated returns to get 10 day predicted prices ahead, starting from the current AAPL price.
4. For each option, calculate the tenth day's option price.
5. Multiply options prices with holdings to calculate the portfolio values.
6. Iterate 2-4 10000 times. Calculate the mean, VaR, and ES for the portfolio values.

Results:

	Mean	VaR	ES
Straddle	9.238795	2.444581	2.452591
SynLong	-0.053501	13.477768	16.689049
CallSpread	3.529414	3.627343	3.710826
PutSpread	3.184107	2.666481	2.740758
Stock	164.820570	13.258781	16.449877
Call	4.592647	4.346952	4.430713
Put	4.646148	4.245900	4.331782
CoveredCall	160.227924	8.911828	12.019164
ProtectedPut	169.466718	4.127965	4.191541

(VaR and ES measure loss rather than absolute portfolio values)

1. Straddle has the smallest VaR and ES, since it is a strategy that benefits from extreme cases.
2. SynLong has similar VaR and ES with Stock, since it simulates Stock.
3. CallSpread and PutSpread have relatively small VaR and ES, since they benefit from a stock's limited increase in price.
4. ProtectedPut has the same VaR and ES with Call, since it simulates Call.