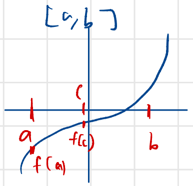
Bisection Method (closed) - for non linear function

[a, b]



$c = \frac{a+b}{2}$

f(a) or f(b) . f(c) = 0 ∴ root is on C

f(a) . f(c) > 0 ∴ root is on [c, b]

a → c

(c is between a and b) f(a) . f(c) < 0 ∴ root is on [a, c]

b → c

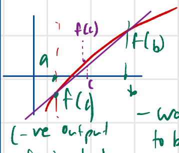
if f(a) and f(c) are both -ve or +ve = didn't cross x-axis

∴ Root is between b and

DrawBacks - linear convergence rate - uses fixed interval size
- require initial interval
(Bisection) - insufficient for oscillating functions

1. Closed methods will always converge to a solution, provided that the root or roots actually lie in the domain of x chosen for investigation.
2. Open methods, by contrast, may not converge towards a root (case b in Figure), but if they do (case c) they tend to be far more efficient in reaching a solution in fewer steps than bisection.

False Position Method (closed) non linear function



(-ve output for input a)

- want f(b) to be opposite of f(c)

- define interval

if f(a) . f(b) < 0 ∴ Function crossed x-axis

- find midpoint

Equation of Secant line:

$y = \frac{f(b) - f(a)}{b - a}(x - b) + f(b)$

y = 0 @ x-axis

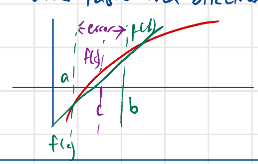
$$\therefore \boxed{C = x = \frac{a.f(b) - b.f(a)}{f(b) - f(a)}}$$

Apply to see if we will move left or right of c, which ever product is less than zero ∴ root lies in that direction.

f(a) . f(c) < 0 ∴ Must move to the left ∴ b = c's values

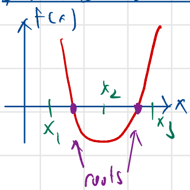
f(b) . f(c) < 0

- often faster than bisection



error = |a - b|

INCREMENTAL SEARCH METHODS



$f(x_1)$ and $f(x_2)$ opposite signs ∴ at least one root in interval $[x_1, x_2]$

if small enough ∴ likely a single root

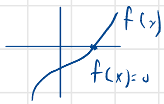
Drawbacks

- possible to miss closely spaced roots
- if search increment > root spacing
- identical roots will not be detected
- f(x) can be mistaken for roots

FIxed Point Iteration (Open)

eg root of $f(x) = x^3 + x - 0.5$

when $f(x) = 0$, $x = g(x)$



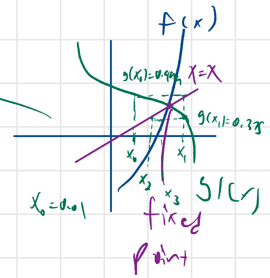
$0 = x^3 + x - 0.5$

$x = 0.5 - x^3$ $\qquad$ $x^3 = 0.5 - x$

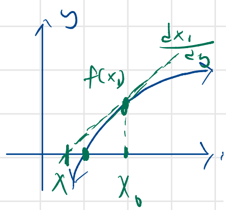
$X_{n+1} = 0.5 - X_n^3$ $\qquad$ $x_n = \sqrt[3]{0.5 - x_n}$

When $f(x) = 0$ $\qquad$ When $f(x) = 0$

Error = $|X_{i+1} - X_i|$ $\qquad$ $X_{n+1} = g_1(x)$ $\qquad$ $X_{n+1} = g_2(x)$



NEWTON'S RAPHSON (OPEN) - Non-linear function



(trying to find the x-intercept) (the root)

X_0 = initial guess

$$\boxed{X = X_0 - \frac{f(x_0)}{f'(x_0)}}$$

(faster than secant, but less likely to converge than secant)

Functions not solvable

1. Asymptotic functions
2. Functions with multiple minima: no solution will be found.
3. Cyclic/periodic functions
4. Functions with no real root in the domain.

Measures to mitigate

- Graph the function to see the function shape and establish whether a root exists in the domain (range of x) of interest.
- Check each solution to establish its closeness to zero.
- Include an upper limit in the number of iterations to prevent infinite cycling.

4) Repeat until absolute relative error < desired error

$|E_A|$ = Absolute relative error

$$|E_A| = \left|\frac{X_{i+1} - X_i}{X_{i+1}}\right| . 100\%$$

3) find where f(x) crosses x-axis

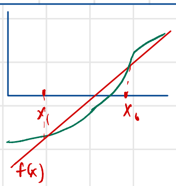
$0 \; \cancel{f(x)} = f(x_1) + f'(x_1)[x - x_1]$

$0 = f(x_1) + f'(x_1).x - f'(x_1).x_1$

$$\boxed{X = X_1 + \frac{f(x_1)}{f'(x_1)}}$$

- formula for next X iteration value

e.g. $X_2 = X_1 + \frac{f(x_1)}{\left[\frac{f(x_2) - f(x_1)}{x_2 - x_1}\right]}$



SECant - Open

- similar to Newton
- nonlinear equations

(1) Pick 2 root estimates

(2) Generate 1st order Taylor's Expansion

$f(x) = f(x_0) + \underbrace{f'(x_0)}_{\text{approximate by slope formula}}[x - x_0]$

$f'(x_0) = \frac{f(x_0) - f(x_1)}{x_0 - x_1}$