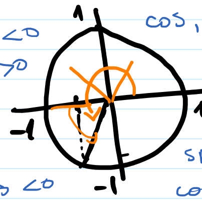
25 November 2021 14:02

① a) $(\underbrace{-1-\sqrt{3}i}_{a+b.i})^{25}$

$a=-1 \quad b=-\sqrt{3}$

$r=\sqrt{1+3}=2$ $\Rightarrow$ $\cos\varphi=-\frac{1}{2}$, $\sin\varphi=-\frac{\sqrt{3}}{2}$



$\varphi=\frac{4\pi}{3}$ $\qquad 100:3=33$, ост. 1

$$\left(2.\left(\cos\frac{4\pi}{3}+i.\sin\frac{4\pi}{3}\right)\right)^{25}=2^{25}.\left(\cos\frac{25.4\pi}{3}+i.\sin\frac{100\pi}{3}\right)$$

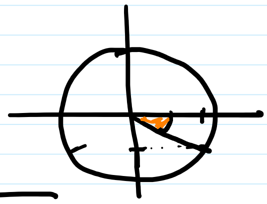
$$=2^{25}.\cos\left(32\pi+\frac{4\pi}{3}\right)+i.\sin\frac{4\pi}{3}=$$

Отг. $2^{25}.\left(\cos\frac{4\pi}{3}+i.\sin\frac{4\pi}{3}\right)$

б) $\sqrt[17]{5\sqrt{3}-5i}$ $\qquad a=5\sqrt{3}$, $b=-5$ $\Rightarrow$ $r=\sqrt{75+25}=10$

$\Rightarrow \cos\varphi=\frac{5\sqrt{3}}{10}=\frac{\sqrt{3}}{2}$ $\qquad \frac{\pi}{6}=\varphi$

$\varphi=\frac{11\pi}{6}$ $\qquad \sin\varphi=-\frac{5}{10}=-\frac{1}{2}$ $=-\frac{\pi}{6}$



$-\frac{\pi}{6}=\frac{11\pi}{6}$ $\qquad \varphi\in[0,2\pi)$

$$\sqrt[17]{10.\left(\cos\frac{11\pi}{6}+i.\sin\frac{11\pi}{6}\right)}=\sqrt[17]{10}.\left(\cos\frac{\frac{11\pi}{6}+2k\pi}{17}+i.\sin\frac{\frac{11}{6}\pi+2k\pi}{17}\right)$$

за $k\in[0,16]$

Задача 2. Решете системите спрямо стойностите на параметрите λ и μ.

$$\text{а)}\left|\begin{array}{l} 0x_1\ (2\lambda-1)x_2-x_3+(\lambda-1)x_4=\lambda \\ 2x_1+x_2-x_3+4x_4=3 \\ \lambda x_1+(\lambda^2+1)x_2-(\lambda-1)x_3+2\lambda x_4=\lambda+1 \\ x_1+\lambda x_2-x_3+2x_4=1 \end{array}\right.$$

$$\text{б)}\left|\begin{array}{l} (-12+2\lambda+4\lambda^2)x_1-6x_2-(17-4\mu)x_3=-2\lambda+4\mu^2+12 \\ (-4+2\mu^2)x_1-2x_2-(6-2\mu)x_3=2\mu^2+4 \\ -2x_1-x_2-3x_3=2 \end{array}\right.$$

1

5 6 7 4
0 1 2 3 $\quad R_2-R_1$

$R_{ij}(q)\to R_i+q.R_j$

$R_i(p)\to p.R_i,\ p\neq 0$

$R_{ij}\to R_i\leftrightarrow R_j$

а) $\left(\begin{array}{cccc|c} 0 & 2\lambda-1 & -1 & \lambda-1 & \lambda \\ 2 & 1 & -1 & 4 & 3 \\ \lambda & \lambda^2+1 & 1-\lambda & 2\lambda & \lambda+1 \\ 1 & \lambda & -1 & 2 & 1 \end{array}\right)$ $\begin{array}{c} R_3-\lambda R_4 \\ \sim \\ R_2-2R_4 \end{array}$ $\left(\begin{array}{cccc|c} 0 & 2\lambda-1 & -1 & \lambda-1 & \lambda \\ 0 & 1-2\lambda & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & \lambda & -1 & 2 & 1 \end{array}\right)$ $\overset{R_1+R_2}{\sim}$

$(0\quad 0\quad 0\quad \lambda-1 \mid \lambda+1)$, $\quad (\lambda-1)x_4=\lambda+1$

$$\sim\left(\begin{array}{cccc|c} 0 & 0 & 0 & \lambda-1 & \lambda+1 \\ 0 & 1-2\lambda & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & \lambda & -1 & 2 & 1 \end{array}\right)\sim$$

$(\lambda-1)x_4=\lambda+1$

I сл. $\lambda=1$ $\quad 0x_4=1+1 \quad 0x_4=2$ ↯

$\lambda-1=0 \Rightarrow$ СЛУ е несъвместима

II сл. $\lambda\neq1$ $\quad x_4=\frac{\lambda+1}{\lambda-1}$

$$\frac{1}{\lambda-1}R_1 \sim \left(\begin{array}{cccc|c} 0 & 0 & 0 & 1 & \frac{\lambda+1}{\lambda-1} \\ 0 & 1-2\lambda & 1 & 0 & 1 \\ 0 & \lambda & 1 & 0 & 1 \\ 1 & \lambda & -1 & 2 & 1 \end{array}\right) \underset{\sim}{R_3-R_2} \left(\begin{array}{cccc|c} 0 & 0 & 0 & 1 & \frac{\lambda+1}{\lambda-1} \\ 0 & 1-2\lambda & 1 & 0 & 1 \\ 0 & 2\lambda & 0 & 0 & 0 \\ 1 & \lambda & -1 & 2 & 1 \end{array}\right)$$

$2\lambda\cdot x_2=0 \qquad 1)\ \lambda\neq0 \qquad a.\ x_2=0 \Rightarrow x_2=0$

$$\left(\begin{array}{cccc|c} 0 & 0 & 0 & 1 & \frac{\lambda+1}{\lambda-1} \\ 0 & 1-2\lambda & 1 & 0 & 1 \\ 0 & 2\lambda & 0 & 0 & 0 \\ 1 & \lambda & -1 & 2 & 1 \end{array}\right)$$

$$\begin{array}{c} \left(\begin{array}{ccc|c} 0 & 0 & 1 & \frac{\lambda+1}{\lambda-1} \\ 0 & 1 & 0 & 1 \\ 1 & -1 & 2 & 1 \end{array}\right) \\ x_1 \quad x_3 \quad x_4 \end{array}$$

От R_2:

$(1-2\lambda)x_2+1\cdot x_3=1$

$0+1\cdot x_3=1$

$1x_3=1$

$\Rightarrow x_3=1$

От R_4: $1\cdot x_1+\lambda\cdot \underset{0}{x_2}-\underset{1}{x_3}+2\underset{\frac{\lambda+1}{\lambda-1}}{x_4}=1$

$x_1=1+1-2\cdot\frac{\lambda+1}{\lambda-1}=\frac{\cancel{2\lambda}-2-\cancel{2\lambda}-2}{\lambda-1}=\frac{-4}{\lambda-1}$

В случая $\lambda\neq0,\neq1$ $\quad\left(\frac{-4}{\lambda-1},0,1,\frac{\lambda+1}{\lambda-1}\right)$

$\frac{-4}{-1}=4$

2сл.) $\lambda=0$

$0x_2=0$

$=p$

$$\left(\begin{array}{cccc|c} 0 & 0 & 0 & 1 & \frac{\lambda+1}{\lambda-1} \\ 0 & 1-2\lambda & 1 & 0 & 1 \\ 0 & 2\lambda & 0 & 0 & 0 \\ 1 & \lambda & -1 & 2 & 1 \end{array}\right)\sim\left(\begin{array}{cccc|c} 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 1 & 0 & 1 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ 1 & 0 & -1 & 2 & 1 \end{array}\right)\sim$$

$x_4=-1$

$$\left(\begin{array}{cccc|c} 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 2 & 1 \end{array}\right)$$

3 ур, 4 неизв.

1 пар.

$x_2=p\ ;\ x_3=1-p$

$1x_1-x_3+2x_4=1$

$x_1=1-2x_4+x_3=1+2+1-p=4-p$

$\lambda=0$ $\quad(4-p,p,1-p,-1)$

Отговор: 1) $\lambda=1$ $\Rightarrow$ несъвместима

1) $\lambda \neq 1,\ \lambda = 0 \quad (4-p_1,\ p,\ 1-p_1\ -1)\ \forall p$

3) $\lambda \neq 1,\ \lambda \neq 0 \quad \left(\frac{-4}{\lambda-1},\ 0,\ 1,\ \frac{\lambda+1}{\lambda-1}\right)$

б) $\left|\begin{aligned} (-12+2\lambda+4\lambda^2)x_1 - 6x_2 - (17-4\mu)x_3 = -2\lambda+4\mu^2+12 \\ (-4+2\mu^2)x_1 - 2x_2 - (6-2\mu)x_3 = 2\mu^2+4 \\ -2x_1 - x_2 - 3x_3 = 2 \end{aligned}\right.$

$$\left(\begin{array}{ccc|c} 4\lambda^2+2\lambda-12 & -6 & 4\mu-17 & 4\mu^2-2\lambda+12 \\ 2\mu^2-4 & -2 & 2\mu-6 & 2\mu^2+4 \\ -2 & -1 & -3 & 2 \end{array}\right) \sim \quad \mu \text{ - мю}$$

$$-R_3 \left(\begin{array}{ccc|c} 4\lambda^2+2\lambda-12 & -6 & 4\mu-17 & 4\mu^2-2\lambda+12 \\ 2\mu^2-4 & -2 & 2\mu-6 & 2\mu^2+4 \\ 2 & 1 & 3 & -2 \end{array}\right) \underset{R_1+6R_3}{\overset{R_2+2R_3}{\sim}}$$

$$\left(\begin{array}{ccc|c} 4\lambda^2+2\lambda & 0 & 4\mu+1 & 4\mu^2-2\lambda \\ 2\mu^2 & 0 & 2\mu & 2\mu^2 \\ 2 & 1 & 3 & -2 \end{array}\right) \overset{R_1-2R_2}{\sim} \left(\begin{array}{ccc|c} 4\lambda^2+2\lambda-4\mu^2 & 0 & 1 & -2\lambda \\ 2\mu^2 & 0 & 2\mu & 2\mu^2 \\ 2 & 1 & 3 & -2 \end{array}\right)$$

$$2\mu^2 x_1 + 2\mu x_3 = 2\mu^2$$

$$2\mu(\mu x_1 + x_3) = 2\mu^2$$

I $\mu = 0 \qquad 0\underset{(x_1,x_2,x_3)}{X} = 0 \qquad \left(\begin{array}{ccc|c} 4\lambda^2+2\lambda & 0 & 1 & -2\lambda \\ 0 & 0 & 0 & 0 \\ 2 & 1 & 3 & -2 \end{array}\right) \sim$

$$\sim \begin{array}{ccc} x_1 & & x_3 \\ \end{array} \left(\begin{array}{ccc|c} 4\lambda^2+2\lambda & 0 & 1 & -2\lambda \\ 2 & 1 & 3 & -2 \end{array}\right)$$

2 ур., 3 неизв.

$x_1 = p \qquad x_3 = -2\lambda - p\cdot(4\lambda^2+2\lambda)$

$x_2 = -2 - 3x_3 - 2x_1$

$x_2 = -2 + 6\lambda + 3p(4\lambda^2+2\lambda) - 2p$

$= (12\lambda^2+6\lambda-2)p + 6\lambda - 2$

$$\left(p,\ (12\lambda^2+6\lambda-2)p+6\lambda-2,\ -2\lambda - p(4\lambda^2+2\lambda)\right)$$

II $\mu \neq 0$

$$\left(\begin{array}{ccc|c} 4\lambda^2+2\lambda-4\mu^2 & 0 & 1 & -2\lambda \\ 2\mu^2 & 0 & 2\mu & 2\mu^2 \\ 2 & 1 & 3 & -2 \end{array}\right) \overset{\frac{1}{2\mu}R_2}{\sim} \left(\begin{array}{ccc|c} 4\lambda^2+2\lambda-4\mu^2 & 0 & 1 & -2\lambda \\ \mu & 0 & 1 & \mu \\ 2 & 1 & 3 & -2 \end{array}\right) \sim$$

$$\underset{\sim}{R_1-R_2} \left(\begin{array}{ccc|c} 4\lambda^2-4\mu^2+2\lambda-\mu & 0 & 0 & -2\lambda-\mu \\ \mu & 0 & 1 & \mu \\ 2 & 1 & 3 & -2 \end{array}\right)$$

$(4\lambda^2-4\mu^2+2\lambda-\mu)x_1 = -2\lambda-\mu$

$x_1 = \frac{-2\lambda-\mu}{}$!

$\sim \Big(\quad \begin{matrix} \mu \\ 2 \end{matrix} \qquad \begin{matrix} 0 \\ 1 \end{matrix} \quad \begin{matrix} 1 \\ 3 \end{matrix} \Big| \begin{matrix} \mu \\ -2 \end{matrix} \Big)$

$$x_1 = \frac{-2\lambda-\mu}{4\lambda^2-4\mu^2+2\lambda-\mu} \quad !$$

Ако $\mu = \lambda$:

$0 + 2\lambda - \lambda = 0$

$\lambda = 0 \Downarrow$

$\cancel{\mu \neq \lambda}$

$\Rightarrow 4\lambda^2 - 4\mu^2 + 2\lambda - \mu \overset{?}{=} 0$

$4\lambda^2 - 4\mu^2 = \mu - 2\lambda$

$$\frac{4\lambda^2-4\mu^2}{\mu-2\lambda} = 1 \quad !$$

$\mu = 2\lambda$

$\mu \neq 2\lambda$

$\lambda + \mu = 0$

$\lambda - \mu = 0$

$\Rightarrow \mu = \pm\lambda$

II.1) $\mu \neq 0$, $\mu = 2\lambda$, но ако $\lambda = 0 \Downarrow$ – няма реш.

$(4\lambda^2 - 4\mu^2 + 2\lambda - \mu)\, x_1 = -2\lambda - \mu$

$\lambda \neq 0 \quad (4\lambda^2 - 4 \cdot 4\lambda^2)\, x_1 = -4\lambda \qquad -12\lambda^2 x_1 = -4\lambda$

$3\lambda x_1 = 1$

$\Rightarrow x_1 = \frac{1}{3\lambda}$

$\mu x_1 + 1 x_3 = \mu$

$x_3 = \mu - \mu \cdot \frac{1}{3\lambda} = \mu\left(\frac{3\lambda - 1}{3\lambda}\right) = \frac{6\lambda - 2}{3}$

$x_2 = -2 - 2x_1 - 3x_3 = \dots$

✓

II.2) $\mu \neq 2\lambda$

$$x_1 = \frac{-2\lambda-\mu}{4\lambda^2-4\mu^2+2\lambda-\mu}$$

$x_2 = \mu - \mu \cdot x_1 \Rightarrow$ сметайте

$f(x) \to a \qquad f: \mathbb{C} \to \mathbb{C}$

Задача 4. Да се докаже, че множеството $V := \{f : \mathbb{C} \to \mathbb{C}\}$ е линейно пространство над полето $\mathbb{C}$ относно операциите:

$$(f+g)(x) = f(x) + g(x) \text{ и } (\lambda.f)(x) = \lambda.f(x).$$

$\mathbb{C}[x]$ – ∀ полиноми с $\mathbb{C}$ коеф

$f(x)$

1) $\big((f+g)+h\big)(x) \overset{?}{=} \big(f+(g+h)\big)(x)$

LHS RHS

$\big((f+g)+h\big)(x) = (f+g)(x) + h(x) = \big(f(x) + g(x)\big) + h(x) =$

$= f(x) + \big(g(x) + h(x)\big) = f(x) + (g+h)(x) =$

$= \big(f + (g+h)\big)(x)$ ✓

2) $(f+g)(x) \overset{?}{=} (g+f)(x) \qquad (f+g)(x) = f(x) + g(x) = g(x) + f(x) =$

$2)\ (f+g)(x) \stackrel{?}{=} (g+f)(x) \qquad (f+g)(x) = f(x)+g(x) = g(x)+f(x) =$
$= (g+f)(x) \checkmark$

$3)\ (f+\vec{\theta}_v)(x) = f(x)$ Търсим $\vec{\theta}_v$.
$f(x) + \vec{\theta}_v(x) = f(x) \Rightarrow \vec{\theta}_v(x) = 0$ за $\forall x$
$\vec{\theta}_v$ е ~~нулевата~~ функ.
$0(x) = 0$ за $\forall x$

$4)\ (f+(-f))(x) = \vec{\theta}_v(x) = 0$
$f(x) + (-f)(x) = 0 \Rightarrow (-f)(x) = -f(x)$
Противоп. ел. относно $+$ на $f(x)$ е $-f(x)$

5) дистр. над скаларен мн-л $\qquad (\pi f)(x) = \pi . f(x)$
$((\alpha+\beta)f)(x) \stackrel{?}{=} (\alpha f + \beta f)(x) \qquad (f+g)(x) = f(x)+g(x)$
$((\underbrace{\alpha+\beta}_{\pi})f)(x) = (\alpha+\beta).f(x) = \alpha f(x) + \beta . f(x) =$
$= (\alpha f)(x) + (\beta f)(x) = (\alpha f + \beta f)(x) \checkmark$

6) дистр. над векторен мн-л $\quad (\alpha(f+g))(x) \stackrel{?}{=} (\alpha f + \alpha g)(x)$
$(\alpha(\overbrace{f+g}^{h}))(x) = \alpha . (f+g)(x) = \alpha.(f(x)+g(x)) = \alpha.f(x) + \alpha.g(x) =$
$= (\alpha f + \alpha g)(x) \checkmark$

$7)\ ((\alpha.\beta)f)(x) = (\alpha.(\beta.f))x$
$((\alpha\beta)f)(x) = (\alpha.\beta).f(x) = \alpha.(\beta.f(x)) = \alpha.(\overbrace{(\beta f)(x)}^{h(x)}) = (\alpha.(\beta.f))(x)$

$8)\ (1.f)(x) = f(x) \qquad (1.f)(x) = 1.f(x) = f(x) \checkmark$

$\Rightarrow V$ е ЛП над $\mathbb{C}$. $\square$

Задача 5. Да се докаже, че мн-вото $U = \{ f(x) \in \mathbb{R}[x]^{\leq 3} \mid f(4) + f(3) = 0 \}$ е линейно пространство над реалните числа $\mathbb{R}$.

$\boxed{U \subseteq \mathbb{R}[x]^{\leq 3}}$

Задача 6. Да се докаже, че следните множества са подпространства на линейното пространство $M_3(\mathbb{C})$:
а) горнотриъгълните матрици 3x3 б) диагоналните матрици 3x3
обоб. вид $\qquad$ обоб. вид

$\boxed{U \leq \mathbb{R}[x]^{\leq 3}}$

$\forall$ подпространство в частност е линейно пр-во.

зад. 5) Да докажем, че U е подпр-во на $\mathbb{R}[x]^{\leq 3}$.

Взимаме произволни $f \in U$, $g \in U$ $\lambda \in K$

Щом $f \in U$, то f изпълнява $f(4)+f(3)=0$ 1)

Аналогично $g \in U \Rightarrow g(4)+g(3)=0$ 2)

Сега да проверим дали $(f+g) \in U$ и $\lambda.f \in U$

$$(f+g)(x) = f(x)+g(x) \Rightarrow \underbrace{f(4)+f(3)}_{=0} + \underbrace{g(4)+g(3)}_{=0} =$$

Да проверим дали $(f+g)(4)+(f+g)(3)=0$

$$f(4)+g(4)+f(3)+g(3) = (f+g)(4)+(f+g)(3) = 0$$

$$\Rightarrow f+g \in U \; *$$

$\lambda.f \in U$ $\qquad (\lambda f)(4)+(\lambda.f)(3) \stackrel{?}{=} 0$

$(\lambda f)(x) = \lambda.f(x)$ $\qquad$ Тъй като $f(4)+f(3)=0 \;|.\lambda$

$$\lambda f(4)+\lambda.f(3)=0$$

$$(\lambda f)(4)+(\lambda f)(3)=0 \Rightarrow \lambda f \in U \; *$$

От $* \Rightarrow \boxed{U \le \mathbb{R}[x]^{\le 3}} \Rightarrow$ В частност U е линейно пр-во. □

$U \le \Pi$

Полином $f(x) = \sum_{i=0}^{n-1} a_i x^i$ $\qquad$ полиномът е с $\mathbb{R}$ коеф.

$x \in \mathbb{Q}$, $x \in \mathbb{C}$

Функ. $f: \mathbb{C} \to \mathbb{C}$

$f(x) = z$, $x \in \mathbb{C}$

(3)

$$\Delta_5 = \begin{vmatrix} -5i & 5i & 5i & 5i & -7+5i \\ -5i & 5i & 5i & 8+5i & 5i \\ -5i & 5i & 2+5i & 5i & 5i \\ -5i & 7+5i & 5i & 5i & 5i \\ 5i+2 & 5i & 5i & 5i & 5i \end{vmatrix} \underset{R_5 - R_1}{\overset{R_2 - R_1}{\vdots}} \begin{vmatrix} -5i & 5i & 5i & 5i & -7+5i \\ 0 & 0 & 0 & 8 & 7 \\ 0 & 0 & 2 & 0 & 7 \\ 0 & 7 & 0 & 0 & 7 \\ 10i+2 & 0 & 0 & 0 & 7 \end{vmatrix}$$

$R_1 - \frac{5i}{8} R_2$

$R_1 - \frac{5i}{2} R_3$

$R_1 - \frac{5i}{7} R_4$

$R_1 + \frac{5i}{10i+2} R_5$

$$= \begin{vmatrix} 0 & 0 & 0 & 0 & * \\ 0 & 0 & 0 & 8 & 7 \\ 0 & 0 & 2 & 0 & 7 \\ 0 & 7 & 0 & 0 & 7 \\ 10i+2 & 0 & 0 & 0 & 7 \end{vmatrix} = (-1)^{\frac{5.4}{2}} \cdot * \cdot 8.2.7.(10i+2)$$

$$\frac{5i}{2+10i} \cdot \frac{2-10i}{2-10i} = \frac{10i+50}{104}$$

$$* = -7+5i - \frac{5i}{8} \cdot 7 - \frac{5i}{2} \cdot 7 - \frac{5i}{7} \cdot 7 + \frac{5i}{10i+2} \cdot 7 =$$

$$= -7+5i\left(K - \frac{7}{\ } - \frac{7}{\ } - K + \frac{7}{\ }\right)$$

$$= -7 + 5i\left(x - \frac{7}{8} - \frac{7}{2} - x + \frac{7}{10i+2}\right)$$

$$z^3 = -11 \quad \Rightarrow z = \sqrt[3]{-11}$$

I в. Моавър $\quad 11(-1 + i\cdot 0) = 11(\cos\pi + i\cdot\sin\pi)$

$$\sqrt[3]{11(\cos\pi + i\cdot\sin\pi)} = \sqrt[3]{11}\cdot\left(\cos\frac{\pi+2k\pi}{3} + i\cdot\sin\frac{\pi+2k\pi}{3}\right)$$

за $k = 0, 1, 2$

при $k=0 \Rightarrow \sqrt[3]{11}\cdot\left(\cos\frac{\pi+0}{3} + i\cdot\sin\frac{\pi}{3}\right) = \sqrt[3]{11}\cdot\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) =$

$$= \frac{\sqrt[3]{11}}{2} + \frac{\sqrt[3]{11}\cdot\sqrt{3}}{2}\cdot i$$

$k=1 \Rightarrow \sqrt[3]{11}\cdot\left(\cos\frac{3\pi}{3} + i\sin\frac{3\pi}{3}\right) = -\sqrt[3]{11}$

$k=2 \quad \sqrt[3]{11}\cdot\cos\frac{5\pi}{3} + i\cdot\sin\frac{5\pi}{3} = \ldots$

II $\quad z^3 = -11 \qquad z^3 + 11 = 0 \qquad z^3 + \left(\sqrt[3]{11}\right)^3 = 0$

$$\left(z + \sqrt[3]{11}\right)\left(z^2 - \sqrt[3]{11}\cdot z + \sqrt[3]{11^2}\right) = 0$$

$z_1 = -\sqrt[3]{11}$ $\qquad z_2 \quad z_3$

$$D = \left(\sqrt[3]{11}\right)^2 - 4\cdot\sqrt[3]{11^2} =$$
$$= -3\cdot\left(\sqrt[3]{11}\right)^2 =$$
$$= \left(\sqrt{3}\cdot i\cdot\sqrt[3]{11}\right)^2$$

$$z_2 = \frac{\sqrt[3]{11} + \sqrt{3}i\cdot\sqrt[3]{11}}{2}$$

$$z_3 = \frac{\sqrt[3]{11} - \sqrt{3}i\cdot\sqrt[3]{11}}{2}$$