ФН: 0MI0600041

Филип Красимиров Филчев
Софтуерно инженерство, I курс, I група

Домашна работа №1

1) а) $z^3 = -7 \quad \approx \quad z^3 = -7 + i.0$

$a = -7 \quad b = 0$

$r = \sqrt{49 + 0} = 7$

$\sin\varphi = \frac{b}{r} = 0 \quad \cos\varphi = \frac{a}{r} = -1$

$\Rightarrow \measuredangle \varphi = \pi$

$\Rightarrow z = 7.(\cos\pi + i.\sin\pi)$

$\sqrt[3]{z} = \sqrt[3]{7}\left(\cos\frac{\pi + 2k\pi}{3} + i.\sin\frac{\pi + 2k\pi}{3}\right) \quad k = 0, 1, 2$

$\underset{k=0}{W_0} = \sqrt[3]{7}\left(\cos\frac{\pi}{3} + i.\sin\frac{\pi}{3}\right)$

$r = \sqrt[3]{7} \qquad \cos\frac{\pi}{3} = \frac{a}{r} = \frac{1}{2} \Rightarrow a = \frac{\sqrt[3]{7}}{2}$

$\sin\frac{\pi}{3} = \frac{b}{r} = \frac{\sqrt{3}}{2} \Rightarrow b = \frac{\sqrt{3}.\sqrt[3]{7}}{2}$

алг. вид на $W_0 = \frac{\sqrt[3]{7}}{2} + \frac{i.\sqrt{3}.\sqrt[3]{7}}{2} = \frac{\sqrt[3]{7}}{2}(1 + i\sqrt{3})$

$\underset{k=1}{W_1} = \sqrt[3]{7}.\left(\cos\frac{3\pi}{3} + i.\sin\frac{3\pi}{3}\right) \Rightarrow$

$\Rightarrow r = \sqrt[3]{7} \quad \sin\pi = 0 = \frac{b}{r} \Rightarrow b = 0$

$\cos\pi = -1 = \frac{a}{r} \Rightarrow a = -\sqrt[3]{7}$

алг. вид на $W_1 = -\sqrt[3]{7} + i.0$

$$W_{2,\,k=2} = \sqrt[3]{7}\left(\cos\frac{5\pi}{3} + i\cdot\sin\frac{5\pi}{3}\right)$$

$r = \sqrt[3]{7}$ $\quad \sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2} = \frac{b}{r} \Rightarrow b = -\frac{\sqrt{3}\cdot\sqrt[3]{7}}{2}$

$\cos\frac{5\pi}{3} = \frac{1}{2} = \frac{a}{r} \Rightarrow a = \frac{\sqrt[3]{7}}{2}$

алг. вид на $W_2 = \frac{\sqrt[3]{7}}{2}\left(1 - i\sqrt{3}\right)$

① б) $x^{63} + x^{42} - 2 = 0$

пол. $x^{21} = y$

$y^3 + y^2 - 2 = 0$

	1	1	0	-2
1	1	2	2	0

$\Rightarrow (y-1)(y^2+2y+2) = 0$

$D = -4 = 4i^2$

$y_{1,2} = \frac{-2 \pm 2i}{2} = -1 \pm i$

$y_1 = 1 \quad y_2 = -1+i \quad y_3 = -1-i$

I $y = 1 \quad x^{21} = 1$

$a = 1 \quad b = 0 \quad r = 1$

$\sqrt[21]{z} = 1\cdot\left(\cos\frac{2k\pi}{21} + i\cdot\sin\frac{2k\pi}{21}\right) \quad k = \overline{0,20}$

II $y = -1+i \quad x^{21} = -1+i$

$a = -1 \quad b = 1 \quad r = \sqrt{2}$

$$\left.\begin{aligned}\sin\varphi &= \frac{b}{r} = \frac{\sqrt{2}}{2}\\ \cos\varphi &= \frac{a}{r} = -\frac{\sqrt{2}}{2}\end{aligned}\right\} \Rightarrow \varphi = \frac{3\pi}{4}$$

$z = \sqrt{2}\cdot\left(\cos\frac{3\pi}{4} + i\cdot\sin\frac{3\pi}{4}\right)$

$= 2 =$

$$w_k = \sqrt[21]{z} = \sqrt[42]{2}\left(\cos\frac{\frac{3\pi}{4}+2k\pi}{21} + i\cdot\sin\frac{\frac{3\pi}{4}+2k\pi}{21}\right) =$$

$k=\overline{0,20}$

$$= \sqrt[42]{2}\left(\cos\frac{3\pi+8k\pi}{84} + i\cdot\sin\frac{3\pi+8k\pi}{84}\right)$$

$k=\overline{0,20}$

$\underline{\text{III}}$ $y=-1-i$ $\quad x^{21}=-1-i$

$a=-1 \quad b=-1 \quad r=\sqrt{2}$

$\sin\varphi = -\frac{\sqrt{2}}{2} \quad \cos\varphi = -\frac{\sqrt{2}}{2} \Rightarrow \varphi = \frac{5\pi}{4}$

Аналогично на $\underline{\text{II}}$:

$$w_k = \sqrt[42]{2}\left(\cos\frac{5\pi+8k\pi}{84} + i\sin\frac{5\pi+8k\pi}{84}\right)$$

$k=\overline{0,20}$

① b) $A = \frac{(-11-i\sqrt{3})^{161}}{(-92+48i\sqrt{3})^{80}} = (-11-i\sqrt{3})\left(\frac{(-11-i\sqrt{3})^2}{-92+48i\sqrt{3}}\right)^{80}$

$z = \frac{(-11-i\sqrt{3})^2}{-92+48i\sqrt{3}} = \frac{-121-22i\sqrt{3}+3}{-92+48i\sqrt{3}} = \Big| \cdot(-1)$

$= \frac{118+22i\sqrt{3}}{92-48i\sqrt{3}} = \frac{59+11i\sqrt{3}}{\cancel{92-48i\sqrt{3}}\ 46-24i\sqrt{3}} = \Big| \cdot \frac{46+24i\sqrt{3}}{46+24i\sqrt{3}}$

$= \frac{1922+1922i\sqrt{3}}{3884} = \frac{1}{2} + \frac{i\sqrt{3}}{2}$

$z = \frac{1}{2} + \frac{i\sqrt{3}}{2}$

$a = \frac{1}{2} \quad b = \frac{\sqrt{3}}{2} \qquad r = 1 \quad \sin\varphi = \frac{b}{r} = \frac{\sqrt{3}}{2} \quad \cos\varphi = \frac{1}{2} = \frac{a}{r}$

$\varphi = \frac{\pi}{3}$

$z^{80} = 1^{80} \cdot \left(\cos\frac{\pi}{3} + i\cdot\sin\frac{\pi}{3}\right)^{80} = \cos\frac{80\pi}{3} + i\cdot\sin\frac{80\pi}{3} =$

$= \cos 26\pi + \frac{2\pi}{3} + i\cdot\sin 26\frac{\pi}{3} + \frac{2\pi}{3} =$

$= \left(\cos\frac{2\pi}{3} + i\cdot\sin\frac{2\pi}{3}\right) \Rightarrow$

$\Rightarrow r = 1 \qquad \sin\frac{2\pi}{3} = \frac{b}{r} = \frac{\sqrt{3}}{2} = b$

$\cos\frac{2\pi}{3} = \frac{a}{r} = a = -\frac{1}{2}$

$=4=$

$$z^{80} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$A = (-11 - i\sqrt{3}) \cdot \left(\frac{-1 + i\sqrt{3}}{2}\right) =$$

$$= \frac{1}{2} \cdot \left(11 - 11i\sqrt{3} + i\sqrt{3} + 3\right) = \frac{1}{2}\left(14 - 10i\sqrt{3}\right) =$$

$$= 7 - 5i\sqrt{3}$$

② $-3x_1 - 8x_2 - 6x_3 - x_4 = \lambda$

$-x_1 - 3x_2 - 3x_3 - 3x_4 = -3$

$2x_1 \qquad -9x_3 + (7+\mu)x_4 = 4\lambda + 17$

$-3x_1 - 6x_2 - x_3 - x_4 = -3$

$$\left(\begin{array}{cccc|c} -3 & -8 & -6 & -1 & \lambda \\ -1 & -3 & -3 & -3 & -3 \\ 2 & 0 & -9 & 7+\mu & 4\lambda+17 \\ -3 & -6 & -1 & -1 & -3 \end{array}\right) \quad .(-3) \quad .(2)$$

$$\left(\begin{array}{cccc|c} 0 & 1 & 3 & 8 & 9+\lambda \\ -1 & -3 & -3 & -3 & -3 \\ 0 & 6 & -15 & 1+\mu & 4\lambda+11 \\ 0 & 3 & 8 & 8 & 6 \end{array}\right) \sim \quad .(-1) \quad .(1)$$

$$.(-3) \left(\begin{array}{cccc|c} 0 & -2 & -5 & 0 & 3+\lambda \\ -1 & 0 & 5 & 5 & 3 \\ 0 & -6 & -15 & 1+\mu & 4\lambda+11 \\ 0 & 3 & 8 & 8 & 6 \end{array}\right) \sim \left(\begin{array}{cccc|c} 0 & -2 & -5 & 0 & 3+\lambda \\ -1 & 0 & 5 & 5 & 3 \\ 0 & 0 & 0 & 1+\mu & \lambda+2 \\ 0 & 3 & 8 & 8 & 6 \end{array}\right)$$

От 3ти ред $(1+\mu)x_4 = \lambda+2 \Rightarrow x_4 = \dfrac{\lambda+2}{1+\mu}$

От 1ви ред $x_3 = p \Rightarrow -2x_2 - 5p = 3+\lambda$

$x_2 = -\dfrac{5p+3+\lambda}{2}$

От 2ри ред $-x_1 + 5p + 5\left(\dfrac{\lambda+2}{1+\mu}\right) = 3$

$x_1 = 5.\left(p + \dfrac{\lambda+2}{1+\mu}\right) - 3$

$=6=$

Системата е съвместима неопределена с решения от вида:

$$\left(5\cdot\left(p+\frac{\lambda+2}{1+\mu}\right)3,\ -\frac{5p+3+\lambda}{2},\ p,\ \frac{\lambda+2}{1+\mu}\right)$$

(3) б)

$$\begin{array}{c} \\ \frac{8}{9} \\ \frac{8}{8} \\ \frac{8}{7} \\ \vdots \\ \frac{8}{2} \\ (-1)\cdot \end{array}
\begin{array}{c} \left(\frac{1}{2}\right) \quad \ldots \quad \frac{1}{7}\ \ \frac{1}{8}\ \ \frac{1}{9} \\ \left|\begin{matrix} 8 & 8 & 8 & \ldots & 8 & 8 & -27 \\ 8 & 8 & 8 & \ldots & 8 & -24 & 8 \\ 8 & 8 & 8 & \ldots & -21 & 8 & 8 \\ \ldots & & & & & & \ldots \\ 8 & 6 & 8 & \ldots & 8 & 8 & 8 \\ -3 & 8 & 8 & \ldots & 8 & 8 & 8 \end{matrix}\right| \end{array} \quad \det = ?$$

$$\begin{array}{c} \frac{35}{9} \\ \frac{32}{8} \\ \frac{29}{7} \\ \vdots \\ \frac{14}{2} \\ \ \end{array}
\left|\begin{matrix} 0 & 0 & \ldots & 0 & 0 & -35 \\ 0 & 0 & & 0 & -32 & 0 \\ 0 & 0 & \ldots & -29 & 0 & 0 \\ \ldots & & & & & \ldots \\ -14 & 0 & & 0 & 0 & 0 \\ 3 & 8 & 8 \ldots & 8 & 8 & 8 \end{matrix}\right| \sim \underbrace{\left|\begin{matrix} 0 & 0 & 0 & \ldots & 0 & 0 & -35 \\ 0 & 0 & 0 & \ldots & 0 & -32 & 0 \\ 0 & 0 & 0 & \ldots & -29 & 0 & 0 \\ 0 & \ldots & 17 & \ldots & & & \\ 0 & -14 & 0 & \ldots & 0 & 0 & 0 \\ * & 8 & 8 & \ldots & 8 & 8 & 8 \end{matrix}\right|}_{=A}$$

$$\det = *\cdot\underbrace{(-14)\cdot(-17)\cdot(-20)\cdot(-23)\cdot(-26)\cdot(-29)\cdot(-32)\cdot(-35)}_{8 \text{ на брой (махат се минусите)}}$$

$$* = -3 + 8\cdot\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\ \ldots\ +\frac{1}{9}\right) = $$

$$= \frac{3664}{315}.$$

$$\Rightarrow \det = \frac{3664}{315}\cdot 35\cdot A = \frac{3664}{9}\cdot A \quad \Big| = 7 =$$

det A=?

(3) a)

$$A=\begin{pmatrix} 3 & 5 & 4 & 2 & -5 \\ -9 & 15 & 24 & 18 & -5 \\ 27 & 45 & 144 & 98 & -5 \\ -81 & 135 & 864 & 686 & -5 \\ 243 & 405 & 5184 & 4802 & -5 \end{pmatrix} \cdot(3)\cdot(-9)\cdot(27)\cdot(-81)$$

$$\sim\begin{pmatrix} 3 & 5 & 4 & 2 & -5 \\ 0 & 30 & 36 & 20 & -20 \\ 0 & 0 & 108 & 80 & 40 \\ 0 & 270 & 972 & 740 & -140 \\ 0 & 0 & 4860 & 4640 & 400 \end{pmatrix} \cdot(-9)$$

$$\sim\begin{pmatrix} 3 & 5 & 4 & 2 & -5 \\ 0 & 30 & 36 & 20 & -20 \\ 0 & 0 & 108 & 80 & 40 \\ 0 & 0 & 648 & 560 & 40 \\ 0 & 0 & 4860 & 4640 & 400 \end{pmatrix} \cdot(-6)\cdot(-45)$$

$$\sim\begin{pmatrix} 3 & 5 & 4 & 2 & -5 \\ 0 & 30 & 36 & 20 & -20 \\ 0 & 0 & 108 & 80 & 40 \\ 0 & 0 & 0 & 80 & -200 \\ 0 & 0 & 0 & 1040 & -1400 \end{pmatrix} \cdot(-13)$$

$$\sim\begin{pmatrix} 3 & 5 & 4 & 2 & -5 \\ 0 & 30 & 36 & 20 & -20 \\ 0 & 0 & 108 & 80 & 40 \\ 0 & 0 & 0 & 80 & -200 \\ 0 & 0 & 0 & 0 & 1200 \end{pmatrix}$$

$\det A = 3 \cdot 30 \cdot 80 \cdot 1200 =$
$= 933\ 120\ 000$

(4

=)

1) 4

$$X \cdot \underbrace{\begin{pmatrix} -1 & -3 & -3 \\ 1 & 4 & 6 \\ 1 & 2 & 1 \end{pmatrix}}_{=A} = \underbrace{\begin{pmatrix} -3 & -9 & -11 \\ -6 & -17 & -19 \\ -5 & -15 & -18 \\ 0 & -4 & -9 \end{pmatrix}}_{=B}$$

$X \cdot A = B$

$A^t \cdot X^t = B^t \Rightarrow$

$$\Rightarrow \left(\begin{array}{ccc|cccc} -1 & 1 & 1 & -3 & -6 & -5 & 0 \\ -3 & 4 & 2 & -9 & -17 & -15 & -4 \\ -3 & 6 & 1 & -11 & -19 & -18 & -9 \end{array}\right) \cdot (-3)$$

$$\left(\begin{array}{ccc|cccc} -1 & 1 & 1 & -3 & -6 & -5 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 & -4 \\ 0 & 3 & -2 & -2 & -1 & -3 & -9 \end{array}\right) \cdot (-3) \cdot (-1)$$

$$\left(\begin{array}{ccc|cccc} -1 & 0 & 2 & -3 & -7 & -5 & -4 \\ 0 & 1 & -1 & 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -2 & -4 & -3 & 3 \end{array}\right) \cdot (-2) \cdot (1)$$

$$\left(\begin{array}{ccc|cccc} -1 & 0 & 0 & 1 & 1 & 1 & -2 \\ 0 & 1 & 0 & -2 & -3 & -3 & -1 \\ 0 & 0 & 1 & -2 & -4 & -3 & 3 \end{array}\right) \cdot (-1)$$

$= X^t \Rightarrow$

$$\Rightarrow X = \begin{pmatrix} -1 & -2 & -2 \\ -1 & -3 & 4 \\ -1 & -3 & -3 \\ 2 & -1 & 3 \end{pmatrix} \quad V$$

=9=