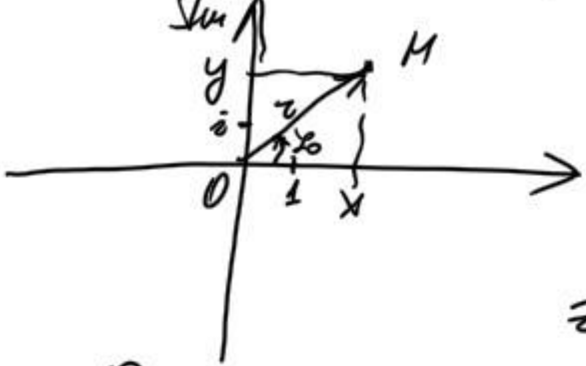
$$\mathbb{C} = \{(x,y) \mid x,y \in \mathbb{R}\} = \left\{ z = x + yi \;\middle|\; \begin{array}{l} x,y \in \mathbb{R} \\ i^2 = -1 \end{array} \right\} =$$

$$= \left\{ z = r(\cos\varphi_0 + i\sin\varphi_0) \;\middle|\; \begin{array}{l} r = |z| = \sqrt{x^2+y^2} \\ \varphi_0 \in [0, 2\pi) \end{array} \right\}$$



Гаусова р-на

$$\varphi = \varphi_0 + 2k\pi, \; k \in \mathbb{Z}$$

$$z_1 = r_1(\cos\alpha + i\sin\alpha)$$

$$z_2 = r_2(\cos\beta + i\sin\beta)$$

$$z_1 = z_2 \iff \begin{array}{l} r_1 = r_2 \\ \alpha_0 = \beta_0 \end{array}$$

$$z_1 + z_2 := (r_1\cos\alpha + r_2\cos\beta) + (r_1\sin\alpha + r_2\sin\beta)i$$

$$z_1 z_2 := r_1(\cos\alpha + i\sin\alpha) \cdot r_2(\cos\beta + i\sin\beta) =$$

$$= r_1 r_2 \left((\cos\alpha\cos\beta - \sin\alpha\sin\beta) + i(\cos\alpha\sin\beta + \cos\beta\sin\alpha) \right)$$

$$= r_1 r_2 \left(\cos(\alpha+\beta) + i\sin(\alpha+\beta) \right)$$

A1 ÷ A9 $\Rightarrow$ $\mathbb{C}$ е поле

$$\frac{z_1}{z_2} = \frac{z_1 \overline{z_2}}{r_2^2} = \frac{r_1 r_2\left(\cos(\alpha-\beta) + i\sin(\alpha-\beta)\right)}{r_2^2} =$$

$$= \frac{r_1}{r_2}\left(\cos(\alpha-\beta) + i\sin(\alpha-\beta)\right)$$

$$z^n = r^n(\cos n\varphi + i\sin n\varphi)$$

формула за степенуване / I ф-ла на Моавър

$$(1+i)^{2021} = \left(\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right)^{2021} =$$

$$3 = 3(\cos 0 + i\sin 0)$$

$$-7 = 7(\cos\pi + i\sin\pi)$$

$$2i = 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

$$-4i = 4\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) = 4\left(\cos\frac{-\pi}{2} + i\sin\frac{-\pi}{2}\right)$$

$? \; \varphi_0 \in (-\pi, 0) \cup [0, \pi] \,! \quad \cos\varphi_0 = \frac{1}{\sqrt{2}}$

$\sin\varphi_0 = \frac{1}{\sqrt{2}}$

$$= (\sqrt{2})^{2021}\left(\cos\frac{2021\pi}{4} + i\sin\frac{2021\pi}{4}\right) =$$

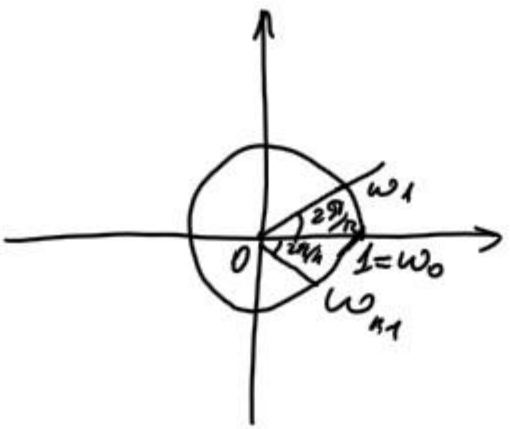
$$= (\sqrt{2})^{2021}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right) =$$

$z^n = 1$, $z \in \mathbb{C}$ $\boxed{\sqrt[n]{1} \in \mathbb{R}}$

корените се наричат n-ти корени на единицата, които са означени

$$\omega_k = \left(\cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}\right),$$

$k = 0, 1, \dots, n-1.$



и ω_k са върховете на правилния n-ъгълник, вписан в единичната окръжност.

$\omega_0 = 1$

$$\omega_k = \omega_1^k$$

$$\left\{\sqrt[n]{r(\cos\varphi + i\sin\varphi)}\right\} = \sqrt[n]{r}\left(\cos\frac{\varphi+2k\pi}{n} + i\sin\frac{\varphi+2k\pi}{n}\right)$$

$k = 0, 1, \dots, n-1$

формула за коренуване или II-ра формула на Моавър.

Пр.: $\sqrt[17]{\dfrac{(1+i\sqrt{3})^{34}}{(-1+i)^{13}}} = ?$ С.р.

последното действие е $\sqrt{\ }$-та

F – поле (числово) ; $\mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$

$\mathbb{Q}(\sqrt{7}) = \{a + b\sqrt{7} \mid a, b \in \mathbb{Q}\}$

$(a+b\sqrt{7}) + (c+d\sqrt{7}) := (a+c) + (b+d)\sqrt{7}$

$(a+b\sqrt{7}) \cdot (c+d\sqrt{7}) := (ac+7bd) + (ad+bc)\sqrt{7}$

A1 ÷ A9 $\Rightarrow$ поле $\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{7}) \subseteq \mathbb{R}$

$\mathbb{Q}(\sqrt{7}, \sqrt{13}) = \{a + b\sqrt{7} + c\sqrt{13} + d\sqrt{7 \cdot 13} \mid a, b, c, d \in \mathbb{Q}\}$

$\mathbb{Z}_7 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}\} =$

$= \{\overline{-3}, \overline{-2}, \overline{-1}, \overline{0}, \overline{1}, \overline{2}, \overline{3}\}$

$\overline{a} = \overline{b}$, $a = b + 7k$

$\overline{6} = \overline{-1}$

$\mathbb{Z}_p = \{\overline{0}, \overline{1}, \ldots, \overline{p-1}\}$ – класове остатъци по модул $\underline{\underline{p}}$

p – просто число

$\mathbb{Z}_7 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}\}$ $(-\overline{4}) = \overline{3}$ $\overline{4}^{-1} = \overline{2}$

противоположен обратен

$\overline{a} + \overline{b} := \overline{a+b}$; $\overline{a}\overline{b} := \overline{ab}$

$\overline{3} + \overline{6} = \overline{9} = \overline{2}$ $\overline{3} \cdot \overline{6} = \overline{4}$

$\overline{10} + \overline{13} = \overline{2}$ $\overline{10} \cdot \overline{13} = \overline{4}$

$\mathbb{Z}_p$ – поле A1 + A9 са в сила

$\mathbb{Z}_2 = \{\overline{0}, \overline{1}\}$
