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РР №10 Рус 2

Пресметнете обема на тялото образувано
от неравенствата:

$$2 \leq x^2 + y^2, \quad 0 \leq x \leq y \leq 2, \quad 0 \leq z(1+x^2+y^2)^2 \leq 1$$

$$K: \begin{cases} x^2 + y^2 \geq 2 \\ 0 \leq x \leq y \leq 2 \\ 0 \leq z(1+x^2+y^2)^2 \leq 1 \Rightarrow 0 \leq z \frac{1}{(1+x^2+y^2)^2} \end{cases}$$

$$\Rightarrow V(x) = \iint_D \frac{1}{(1+x^2+y^2)^2} dx dy, \text{ където } D: \begin{cases} x^2 + y^2 \geq 2 \\ 0 \leq x \leq y \leq 2 \end{cases}$$

Преходим Полярна система $x = \rho \cos \varphi$
 $y = \rho \sin \varphi$

$$\text{като } 0 \leq \varphi \leq 2\pi \\ \rho \geq 0, \Delta = \rho$$

$$D': \begin{cases} \rho^2 \geq 2 \\ 0 \leq \rho \cos \varphi \leq \rho \sin \varphi \leq 2 \\ 0 \leq \varphi \leq 2\pi, \rho \geq 0 \end{cases} \Leftrightarrow$$

$$0 \leq \rho \leq \sqrt{2}$$

$$\sin \varphi \geq \cos \varphi, \cos \varphi \geq 0$$

$$\rho \sin \varphi \leq 2$$

$$0 \leq \varphi \leq 2\pi$$

$$\Rightarrow D' \quad \begin{cases} 0 \leq \rho \leq \sqrt{2} \\ \frac{\pi}{4} \leq \varphi \leq \frac{\pi}{2} \\ \rho \sin \varphi \leq 2 \end{cases}$$

$$\Rightarrow V(x) = \iint_D \frac{\rho}{(1+\rho^2)^2} d\rho d\varphi = \int_0^{\sqrt{2}} \left(\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\rho}{(1+\rho^2)^2} d\varphi \right) d\rho$$

$$\begin{aligned}
 \frac{\pi}{4} \int_0^{\sqrt{2}} \frac{y}{(1+y^2)^2} dy &= \frac{\pi}{8} \int_0^{\sqrt{2}} \frac{d(1+y^2)}{(1+y^2)^2} = \\
 &= \frac{\pi}{8} \left(-\frac{1}{1+y^2} \right) \bigg|_0^{\sqrt{2}} = \frac{\pi}{8} \cdot \frac{2}{3} = \frac{\pi}{12} \quad \square
 \end{aligned}$$