Филип Красимиров Филчев № 0МI0600041
Софтуерно инженерство I курс I група

Домашна работа № 4

a) $\lim\limits_{n\to\infty} \frac{1^4+2^4+3^4+\dots+n^4}{n^5}$

$1^4+2^4+3^4+\dots n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$

Док. чрез индукция

1) $n=1$ $\quad 1 = \frac{1.2.3.5}{30} = \frac{30}{30} = 1$ ✓ вярно

2) допускаме, че е вярно за $\forall n$

3) доказваме за $n+1$

$1^4+2^4+3^4+\dots+n^4+(n+1)^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)+30(n+1)^4}{30} =$

$= \frac{(n+1)\left((2n^2+n)(3n^2+3n-1)+30(n+1)^3\right)}{30} =$

$= \frac{(n+1)(6n^4+6n^3+2n^2+3n^3+3n^2-n+30n^3+90n^2+90n+30)}{30} =$

$= \frac{(n+1)(6n^4+39n^3+91n^2+89n+30)}{30} =$ Хорнер =>

=1= Фн: 0МI0600041

$$\Rightarrow \frac{(n+1)(n+2)(6n^3+27n^2+37n+15)}{30} = \text{Хорнер} \Rightarrow ?$$

$$\Rightarrow \frac{(n+1)(n+2)\left(n+\frac{3}{3}\right)(6n^2+18n+10)}{30} =$$

$$= \frac{(n+1)(n+2)(2n+3)(3n^2+9n+5)}{30} \checkmark$$

изпълнено за $n+1$

Вярно за $\forall n$

$$\Rightarrow \lim_{n\to\infty} \frac{1^4+2^4+3^4+\dots+n^4}{n^5} = \lim_{n\to\infty} \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30n^5} =$$

$$= \lim_{n\to\infty} \frac{6n^5+15n^4+10n^3-n}{30n^5} =$$

$$= \lim_{n\to\infty} \frac{\cancel{n^5}\left(6+\frac{15}{n}+\frac{10}{n^2}-\frac{1}{n^4}\right)}{30\cancel{n^5}} = \frac{6}{30} = \frac{1}{5}$$

 фн: 0MI0600041

б) $\lim\limits_{n\to\infty} \frac{1}{n} \sqrt[n]{\frac{(3n+1)!}{(2n)!}} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{(3n+1)!}{n^n(2n)!}}$

$a_n = \frac{(3n+1)!}{n^n \cdot (2n)!} \Rightarrow a_{n+1} = \frac{(3n+4)!}{(n+1)^{n+1} \cdot (2n+2)!}$

$$\ell = \frac{a_{n+1}}{a_n} = \frac{\frac{(3n+4)!}{(n+1)^{n+1} \cdot (2n+2)!}}{\frac{(3n+1)!}{n^n \cdot (2n)!}} =$$

$$= \frac{(3n+4)(3n+2)(3n+3) \cdot n^n}{(n+1)^n \cdot (n+1) \cdot (2n+2)(2n+1)} = \frac{\cancel{n^3}\left(3+\frac{4}{n}\right)\left(3+\frac{2}{n}\right)\left(3+\frac{3}{n}\right)}{\cancel{n^3} \cdot \left(1+\frac{1}{n}\right) \cdot \left(2+\frac{2}{n}\right)\left(2+\frac{1}{n}\right)} \cdot \frac{n^n}{(n+1)^n} =$$

$$= \frac{27}{4} \cdot \left(\frac{n}{n+1}\right)^n = \frac{27}{4} \cdot \left(\frac{n+1}{n}\right)^{-n} = \frac{27}{4} \cdot \left(1+\frac{1}{n}\right)^{-n} =$$

$$= \frac{27}{4} \cdot e^{-1} = \frac{27}{4 \cdot e}$$

 фн: 0MI0600041