Домашна работа 4

*зад.*1

$$а)\int_{-\infty}^{\infty}\frac{dx}{(x^2-2x+5)^2}=\int_{-\infty}^{\infty}\frac{dx}{((x-1)^2+4)^2}, y=x-1$$

$$\int_{-\infty}^{\infty}\frac{dx}{((x-1)^2+4)^2}=\frac{1}{16}\int_{-\infty}^{\infty}\frac{dy}{\left(\left(\frac{y}{2}\right)^2+1\right)^2}, \frac{y}{2}=tgu,$$

$$\frac{1}{16}\int\frac{dy}{\left(\left(\frac{y}{2}\right)^2+1\right)^2}=\frac{1}{8}\int\frac{du}{\cos^2 u\left(tgu^2+1\right)^2}=\frac{1}{8}\int\cos^2 udu=\frac{1}{8}\int\frac{1+\cos 2u}{2}du=\frac{1}{8}.u+\frac{1}{16}\sin 2u\Rightarrow$$

$$\int_{-\infty}^{\infty}\frac{dx}{((x-1)^2+4)^2}=\frac{\pi}{16}$$

$$б)\int_{-\infty}^{\infty}\frac{x^2+2}{x^4+4}dx=\frac{1}{2}\int_{-\infty}^{\infty}\frac{x^2+2x+2+x^2-2x+2}{(x^2-2x+2)(x^2+2x+2)}dx=\frac{1}{2}\int_{-\infty}^{\infty}\frac{1}{x^2-2x+2}dx+\frac{1}{2}\int_{-\infty}^{\infty}\frac{1}{x^2+2x+2}dx=$$

$$=\frac{1}{2}\int_{-\infty}^{\infty}\frac{1}{(x-1)^2+1}dx+\frac{1}{2}\int_{-\infty}^{\infty}\frac{1}{(x+1)^2+1}dx=\frac{1}{2}\left(arctg(x-1)+arctg(x+1)\right)\Big|_{-\infty}^{\infty}=\pi$$

$$в)\int_{0}^{\infty}\frac{x|\ln x|dx}{(x^2+1)^2}=-\int_{0}^{e}\frac{x\ln xdx}{(x^2+1)^2}+\int_{e}^{\infty}\frac{x\ln xdx}{(x^2+1)^2},$$

$$\int\frac{x\ln xdx}{(x^2+1)^2}=\frac{1}{2}\int\frac{dx}{x(x^2+1)}-\frac{\ln x}{2(x^2+1)}, u=x^2$$

$$\frac{1}{4}\int\frac{dx}{u(u+1)}-\frac{\ln x}{2(x^2+1)}=\frac{1}{4}\int\frac{1}{u}-\frac{1}{u+1}du-\frac{\ln x}{2(x^2+1)}=$$

$$=\frac{1}{4}\int\frac{1}{u}du-\frac{1}{4}\int\frac{1}{u+1}du-\frac{\ln x}{2(x^2+1)}=-\frac{1}{4}\int\frac{1}{u+1}du+\frac{\ln x^2}{4}-\frac{\ln x}{2(x^2+1)}=$$

$$=-\frac{\ln(x^2+1)}{4}+\frac{\ln x^2}{4}-\frac{\ln x}{2(x^2+1)},$$

$$\int_{0}^{e}\frac{x\ln xdx}{(x^2+1)^2}=\int_{e}^{\infty}\frac{x\ln xdx}{(x^2+1)^2}$$

$$=>\int_{0}^{\infty}\frac{x|\ln x|dx}{(x^2+1)^2}=2\left(-\frac{\ln(x^2+1)}{4}+\frac{\ln x^2}{4}-\frac{\ln x}{2(x^2+1)}\right)\Bigg|_{e}^{\infty}=\frac{\ln 2}{2}$$

зад.2

$$a)\int_0^1 \frac{(\sin x - arctgx)^3}{x^p}\,dx \sim \int_0^1 \frac{x^9}{x^p}\,dx \sim \int_0^1 \frac{dx}{x^{p-9}} \Rightarrow p<10\,\textit{интегралът е сходящ}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

$$arctgx = x - \frac{x^3}{3} + \frac{x^5}{5}$$

$$б)\int_0^1 \frac{\ln(1+\sqrt[4]{x})}{x+x^p}\arcsin\sqrt[3]{\frac{x}{x+1}}dx$$

$$p>1, x\to 0, \ln(1+\sqrt[4]{x}) \sim \sqrt[4]{x}, x+x^p \sim x^p, \arcsin\sqrt[3]{\frac{x}{x+1}} \sim \sqrt[3]{\frac{x}{x+1}}$$

$$\int_0^1 \frac{\ln(1+\sqrt[4]{x})}{x+x^p}\arcsin\sqrt[3]{\frac{x}{x+1}}dx \sim \int_0^1 \frac{x^{\frac{7}{12}}}{x^{p+\frac{1}{3}}}\,dx \sim \int_0^1 \frac{dx}{x^{p-\frac{1}{4}}} \Rightarrow p<\frac{5}{4} \Rightarrow p\in\left(1;\frac{5}{4}\right)\textit{интегралът е сходящ}$$

$$p\le 1, x\to 0, \ln(1+\sqrt[4]{x}) \sim \sqrt[4]{x}, x+x^p \sim x, \arcsin\sqrt[3]{\frac{x}{x+1}} \sim \sqrt[3]{\frac{x}{x+1}}$$

$$\int_0^1 \frac{\ln(1+\sqrt[4]{x})}{x+x^p}\arcsin\sqrt[3]{\frac{x}{x+1}}dx \sim \int_0^1 \frac{x^{\frac{7}{12}}}{x^{\frac{1}{3}}}\,dx \sim \int_0^1 \frac{dx}{x^{-\frac{1}{4}}} \Rightarrow p\le 1\,\textit{интегралът е сходящ}$$

$$\Rightarrow p\in\left(-\infty;\frac{5}{4}\right)$$

зад.3

$$a)\int_0^\infty \frac{\ln(1+x^{3p})}{(x+x^2)^{4p}\,arctg\sqrt{x}}\,dx=\int_0^a \xi\xi\xi\,dx+\int_a^\infty \xi\xi\xi\,dx$$

$$x\to 0,\ln(1+x^{3p})\sim 1+x^{3p},(x+x^2)^{4p}\sim x^{4p},arctg\sqrt{x}\sim\sqrt{x}$$

$$x\to\infty,\ln(1+x^{3p})\sim\ln x^{3p}=\ln x,(x+x^2)^{4p}\sim x^{8p},arctg\sqrt{x}\sim\frac{\pi}{2}$$

$$\Rightarrow\int_0^\infty \frac{\ln(1+x^{3p})}{(x+x^2)^{4p}\,arctg\sqrt{x}}dx\sim\int_0^a \frac{x^{3p}}{x^{4p}\sqrt{x}}\,dx+\int_a^\infty \frac{\ln x}{x^{8p}}\,dx$$

$$\int_0^a \frac{x^{3p}}{x^{4p}\sqrt{x}}dx=\int_0^a \frac{dx}{x^{p+\frac{1}{2}}},\ p+\frac{1}{2}<1\Rightarrow p<\frac{1}{2}$$

$$\int_a^\infty \frac{\ln x}{x^{8p}}\,dx<\int_a^\infty \frac{1}{x^{8p-\varepsilon}}\,dx,8p-\varepsilon>1\Rightarrow p>\frac{1}{8}$$

$$\Rightarrow p\in\left(\frac{1}{8};\frac{1}{2}\right)$$

$$б)\int_0^\infty \frac{\ln(1+\sqrt[3]{x}+2x^{3p})}{x^3+x^{4p}}\arcsin\sqrt{\frac{x}{x+1}}dx=\int_0^a \xi\xi\xi\,dx+\int_a^\infty \xi\xi\xi\,dx$$

$$\Rightarrow\int_0^\infty \frac{\ln(1+\sqrt[3]{x}+2x^{3p})}{x^3+x^{4p}}\arcsin\sqrt{\frac{x}{x+1}}dx\sim\int_a^\infty \frac{\ln 2x}{x^{8p}}.\frac{\pi}{2}\,dx$$

$$\int_a^\infty \frac{\ln 2x}{x^{8p}}.\frac{\pi}{2}\,dx<\int_a^\infty \frac{1}{x^{4p-2\varepsilon}}\,dx,8p-2\varepsilon>1\Rightarrow p>\frac{1}{4}$$

$$=>p\in\left(\frac{1}{4};\infty\right)$$

$$в)\int_0^\infty \frac{arctg\sqrt[3]{x}}{(x+\sqrt{x})\ln^2(1+x^{2p})}\,dx=\int_0^a \dots dx+\int_a^\infty \dots dx$$

$$\int_0^\infty \frac{arctg\sqrt[3]{x}}{(x+\sqrt{x})\ln^2(1+x^{2p})}\,dx\sim\int_0^a \frac{\sqrt[3]{x}}{\sqrt{x}.x^{4p}}\,dx+\int_a^\infty \frac{dx}{x\ln^2 x}$$

$$\int_0^a \frac{\sqrt[3]{x}}{\sqrt{x}.x^{4p}}\,dx=\int_0^a \frac{1}{x^{4p+\frac{1}{2}-\frac{1}{3}}}\,dx,\ 4p+\frac{1}{2}-\frac{1}{3}<1\Rightarrow p<\frac{5}{24}$$

$$\int_a^\infty \frac{dx}{x\ln^2 x},\ln x=t,\frac{dx}{x}=dt=>\int_{\ln a}^\infty \frac{dt}{t^2}\Rightarrow сходящ$$

$$p>0\Rightarrow p\in\left(0;\frac{5}{24}\right)$$

$$г)\int_0^{\infty}\frac{\ln(1+2x^3)}{(x+x^2)^p\left(arctg\sqrt{x}\right)^{4p}}\,dx=\int_0^a ξξξ\,dx+\int_a^{\infty}ξξξ\,dx$$

$$\int_0^{\infty}\frac{\ln(1+2x^3)}{(x+x^2)^p\left(arctg\sqrt{x}\right)^{4p}}\sim\int_0^a\frac{x^3}{x^p\left(\sqrt{x}\right)^{4p}}\,dx+\int_a^{\infty}\frac{\ln(2x)}{x^{2p}\cdot\frac{\pi}{2}}\,dx$$

$$\int_0^a\frac{x^3}{x^p\left(\sqrt{x}\right)^{4p}}\,dx=\int_0^a\frac{dx}{x^{p+2p-3}},\ 3p-3<1\Rightarrow p<\frac{4}{3}$$

$$\int_a^{\infty}\frac{\ln(2x)}{x^{2p}}\,dx<\int_a^{\infty}\frac{1}{x^{2p-2\varepsilon}}\,dx, 2p-2\varepsilon>1\Rightarrow p>\frac{1}{2}$$

$$\Rightarrow p\in\left(\frac{1}{2};\frac{4}{3}\right)$$