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СИ, I курс, I група

ДИС2, ДР 1.

$f(x) = y = x^2 - 5x$

$g(x) = y = x + 16$

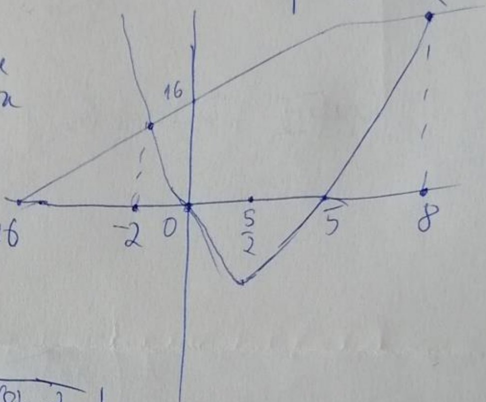
Дължина на кривата отсечена от правата = ?

$f(x) = g(x) \to$ пресечни точки

$x^2 - 5x = x + 16$

$x^2 - 6x - 16 = 0$

$x_1 = 8 \ ; \ x_2 = -2$



Дължина на крива: $\int^{a} \sqrt{1 + (f'(x))^2}\, dx$

Търсим $\int_{-2}^{8} \sqrt{1 + ((x^2 - 5x)')^2}\, dx$

$f'(x) = (x^2 - 5x)' = 2x - 5 \Rightarrow \int_{-2}^{8} \sqrt{4x^2 - 20x + 26}\, dx$

$I = \int_{-2}^{8} \sqrt{(2x-5)^2 + 1}\, dx \ ; \quad 2x - 5 = t$

$x = \frac{t+5}{2}$

$\frac{dx}{dt} = \frac{1}{2} \quad dx = \frac{dt}{2}$

граници: $2(-2) + 5 = -9$

$2.(8) - 5 = 11$

$I = \frac{1}{2} \int_{-9}^{11} \sqrt{t^2 + 1}\, dt =$ по части $\frac{1}{2} \left(t\sqrt{t^2+1} \Big|_{-9}^{11} - \int_{-9}^{11} t\, d\sqrt{t^2+1} \right) =$

$= 1 =$

$(\sqrt{t^2+1})' = \frac{1}{2\sqrt{t^2+1}}$

$$I \Rightarrow \frac{1}{2}\int_{-9}^{11}\sqrt{t^2+1}\,dt \underset{\text{по частям}}{=} \frac{1}{2}t\sqrt{t^2+1}\Big|_{-9}^{11} - \frac{1}{2}\int_{-9}^{11} t\,d\sqrt{t^2+1} =$$

$$I = \frac{11}{2}\cdot\sqrt{122} + \frac{9}{2}\sqrt{82} - \frac{1}{2}\int_{-9}^{11}\frac{t^2+1-1}{\sqrt{t^2+1}}\,dt =$$

$$I = \frac{1}{2}\left(11\sqrt{122} + 9\sqrt{82}\right) - \underbrace{\frac{1}{2}\int_{-9}^{11}\sqrt{t^2+1}\,dt}_{=I} - \frac{1}{2}\int_{-9}^{11}\frac{dt}{\sqrt{t^2+1}} \Rightarrow$$

$$I = \frac{1}{2}\left(\frac{1}{2}\left(11\sqrt{122} + 9\sqrt{82}\right) - \frac{1}{2}\ln\left(\frac{11+\sqrt{122}}{-9+\sqrt{82}}\right)\right)$$

отг. Ргиx $= \frac{1}{2}\left(\frac{11\sqrt{122} + 9\sqrt{82}}{2} - \frac{1}{2}\ln\left(\frac{11+\sqrt{122}}{-9+\sqrt{82}}\right)\right)$

$=2=$