Кристияна Атанасова, II курс, ф.н. 61311

Домашна работа №4

зад. а) $\int_{-\infty}^{+\infty}\frac{dx}{(x^2-2x+5)^2} = \int_{-\infty}^{+\infty}\frac{dx}{((x-1)^2+4)^2} = \left(\frac{1}{4}\right)^2\int_{-\infty}^{+\infty}\frac{dx}{\left(\left(\frac{x}{2}-\frac{1}{2}\right)^2+1\right)^2} = \frac{1}{\cancel{16}\,8}\cdot\cancel{2}\int_{-\infty}^{+\infty}\frac{d\left(\frac{x}{2}-\frac{1}{2}\right)}{\left(\left(\frac{x}{2}-\frac{1}{2}\right)^2+1\right)^2} =$

$= \frac{1}{8}\int_{-\infty}^{+\infty}\frac{du}{(u^2+1)^2} = \frac{1}{8}\int_{-\infty}^{+\infty}\frac{\cancel{1+u^2}}{(u^2+1)^{\cancel{2}}}du - \frac{1}{8}\int_{-\infty}^{+\infty}\frac{u^2du}{(u^2+1)^2} = \frac{1}{8}\operatorname{arctg} u\Big|_{-\infty}^{+\infty} -$

$\frac{x}{2}-\frac{1}{2}=u$

$-\left(-\frac{1}{8}\cdot\frac{1}{2}\int_{-\infty}^{+\infty}\frac{u\,d(u^2+1)}{(u^2+1)^2}\right) = \frac{\pi}{8} - \left(\frac{1}{16}\int_{-\infty}^{+\infty}u\,d\left(-\frac{1}{u^2+1}\right)\right) = \frac{\pi}{8}-\left(\frac{1}{16}\left(-\frac{u}{u^2+1}\right)\Big|_{-\infty}^{+\infty}+\right.$

$\left.+\frac{1}{16}\int_{-\infty}^{+\infty}\frac{du}{u^2+1}\right) = \frac{\pi}{8}-\left(+\frac{1}{16}\left(-\frac{u}{u^2+1}\right)\Big|_{-\infty}^{+\infty}+\frac{1}{16}\operatorname{arctg} u\Big|_{-\infty}^{+\infty}\right)=$

$=\frac{\pi}{8}-\frac{\pi}{16}=\frac{\pi}{16}$

б) $\int_{-\infty}^{+\infty}\frac{x^2+2}{x^4+4}dx$ $\qquad (x^2+2)^2 = x^4+4x^2+4$

$\int_{-\infty}^{+\infty}\frac{x^2+2}{(x^2+2)-4x^2}dx = \int_{-\infty}^{+\infty}\frac{x^2+2}{(x^2-2x+2)(x^2+2x+2)}dx$

$\frac{1}{x^2-2x+2}+\frac{1}{x^2+2x+2} = \frac{x^2+2+\cancel{2x}+x^2+2-\cancel{2x}}{(\dots)(\dots)} = \frac{2x^2+4}{(x^2-2x+2)(x^2+2x+2)}\ \Big|\cdot\frac{1}{2}$

$\frac{1}{2}\int_{-\infty}^{+\infty}\frac{1}{x^2+2x+2}dx+\frac{1}{2}\int_{-\infty}^{+\infty}\frac{1}{x^2-2x+2}dx = \frac{1}{2}\left(\int_{-\infty}^{+\infty}\frac{d(x+1)}{(x+1)^2+1}+\int_{-\infty}^{+\infty}\frac{1}{(x-1)^2+1}d(x+1)\right)=$

$=\frac{1}{2}\left(\operatorname{arctg}(x+1)\Big|_{-\infty}^{+\infty}+\operatorname{arctg}(x-1)\Big|_{-\infty}^{+\infty}\right) = \frac{1}{2}\left(\frac{\pi}{2}+\frac{\pi}{2}+\frac{\pi}{2}+\frac{\pi}{2}\right) = \frac{1}{\cancel{2}}\cdot\frac{\cancel{4}\pi}{\cancel{2}} = \pi$

в) $\int_{0}^{+\infty}\frac{x\ln|x|}{(x^2+1)^2}dx = \int_0^1\dots dx+\int_1^{+\infty}\dots dx$

$I_1 = \int_0^1 -\frac{x\ln x}{(x^2+1)^2}dx = -\frac{1}{2}\int_0^1\frac{2x\ln x}{(x^2+1)^2}dx = \frac{1}{2}\int_0^1\ln x\,d\left(\frac{1}{x^2+1}\right)=$

$=\frac{1}{2}\left(\frac{\ln x}{x^2+1}\Big|_0^1-\int_0^1\left(\frac{1}{x^2+1}\right)d\ln x\right)$

$I_2 = \int_1^{+\infty}\frac{x\ln x}{(x^2+1)^2}dx = \frac{1}{2}\int_1^{+\infty}\frac{2x\ln x}{(x^2+1)^2}dx = -\frac{1}{2}\int_1^{+\infty}\ln x\left(\frac{1}{x^2+1}\right)=$

$=-\frac{1}{2}\left(\frac{\ln x}{x^2+1}\Big|_1^{+\infty}-\int_1^{+\infty}\left(\frac{1}{x^2+1}\right)d\ln x\right)$

$I = I_1+I_2$

Зад. а) $\int\limits_0 \frac{(\sin x - \operatorname{arctg} x)^p}{x^p}\,dx = \int\limits_0 f(x)\,dx$

$(\sin x - \operatorname{arctg} x)^3 \sim x^3$

$x^p \sim x^p$

$f(x) \sim \frac{x^3}{x^p} = \frac{1}{x^{p-3}}$ $\qquad p-3<1$, $\quad p<4 \Rightarrow$ сх.

б) $\int\limits_0^{+\infty} \frac{\ln(1+\sqrt[4]{x})}{x+x^p} \arcsin\sqrt[3]{\frac{x}{x+1}}\,dx = \int\limits_0^a \dots dx + \int\limits_a^{+\infty} \dots dx$

$\arcsin\sqrt[3]{\frac{x}{x+1}} \sim \left(\frac{x}{x+1}\right)^3 \to 0$

$\ln(1+\sqrt[4]{x}) \sim x^{1/4}$

I сл. При $p>1$

$x+x^p \sim x$

$\int\limits_0^a \sim \frac{x^{1/4}}{x} = \frac{1}{x^{3/4}}$ $\qquad \frac{3}{4}<1 \Rightarrow$ сх.

II сл. При $p \geq 1$

$x+x^p \sim x^p$

$\int\limits_0^a \sim \frac{x^{1/4}}{x^p} = \frac{1}{x^{p-1/4}}$ $\qquad p-\frac{1}{4}<1$, $\quad p<\frac{5}{4} \Rightarrow$ сх.

I сл. $p>1$ $\quad \int\limits_a^{+\infty} \sim \frac{\ln x}{x^p}$

$\ln(1+\sqrt[4]{x}) \sim \ln(x^{1/4}) = \frac{1}{4}\ln x$

$\arcsin\sqrt[3]{\frac{x}{x+1}} \sim \arcsin\sqrt[3]{1} = \frac{\pi}{2}$

$x+x^p \sim x^p$

$p>1 \Rightarrow p = 1+2\varepsilon$

$\underbrace{\int\limits_a^{+\infty} \frac{\ln x\,dx}{x^{1+2\varepsilon}}}_{\text{сх.}} < \underbrace{\int\limits_a^{+\infty} \frac{x^{\varepsilon}}{x^{1+2\varepsilon}}\,dx}_{\text{сх.}}$ $\quad \Leftarrow$

II сл. $p \leq 1$

$\int\limits_a^{+\infty} \sim \frac{\ln x}{x}$

$\int\limits_a^{+\infty} \frac{\ln x}{x}\,dx$

$\int\limits_a^{+\infty} \frac{1}{x}\,dx$ − разх. $\Rightarrow \int\limits_a^{+\infty} \frac{\ln x}{x}\,dx$ − разх.

в) $\int_0^{+\infty} \frac{\operatorname{arctg}\sqrt[3]{x}}{(x+\sqrt{x})\ln^2(1+x^{2p})}dx = \int_0^a \dots dx + \int_a^{+\infty} \dots dx$ $\boxed{p>0}$

$\int_0^a \sim \frac{x^{1/3}}{x^{1/2}x^{4p}} \sim \int_0^a \frac{1}{x^{4p+\frac{1}{6}}}$ $4p+\frac{1}{6}<1$

$\operatorname{arctg}\sqrt[3]{x} \sim \pi$ $p<\frac{5}{24} \Rightarrow$ сх.

$\int_a^{+\infty} \sim \frac{1}{x\ln^2(x^{2p})} \sim \int_a^{+\infty}\frac{1}{x\ln^2 x}dx = \int_a^{+\infty}\frac{d\ln x}{\ln^2 x} = \int_{\ln a}^{+\infty}\frac{du}{u^2} \Rightarrow$ сх.

сх. за $\forall \left| \begin{array}{l} p<\frac{5}{24} \\ p>0 \end{array}\right.$

г) $\int_0^{+\infty}\frac{\ln(1+2x^3)}{(x+x^2)^p(\operatorname{arctg}\sqrt{x})^{4p}}dx = \underbrace{\int_0^a \dots dx}_{I_1} + \underbrace{\int_a^{+\infty}\dots dx}_{I_2}$ $\boxed{p>0}$

$I_1 \sim \frac{x^3}{x^p x^{2p}} = \frac{1}{x^{3p-3}}$ $3p-3>1$

$p>4/3 \Rightarrow$ сх.

$\ln(1+2x^3)\sim 2x^3 \sim x^3$

$(x+x^2)^p \sim x^p$

$(\operatorname{arctg}\sqrt{x})^{4p} \sim x^{\frac{1}{2}\cdot 4p} = x^{2p}$

$2p>1 \Rightarrow 2p = 1+2\varepsilon$

$I_2 \sim \frac{\ln x}{x^{2p}}$

$\operatorname{arctg}\sqrt{x} \sim \frac{\pi}{2}$

$(\operatorname{arctg}\sqrt{x})^{4p} \sim \left(\frac{\pi}{2}\right)^{4p}$

$\ln(1+2x^3) \sim \ln(2x^3) = \ln 2 + 3\ln x$

$(x+x^2)^p \sim x^{2p}$

$\underbrace{\int_a^{+\infty}\frac{\ln x}{x^{1+2\varepsilon}}dx}_{\text{сх.}} < \underbrace{\int_a^{+\infty}\frac{x^{\varepsilon}}{x^{1+2\varepsilon}}dx}_{\text{сх.}}$

$\Leftarrow$

$2p=1$

$\int_a^{+\infty}\frac{\ln x}{x}dx$

$\int_a^{+\infty}\frac{1}{x}dx$ – раз х. $\Rightarrow \int_a^{+\infty}\frac{\ln x}{x}dx$ – раз х.

Задача а) $\int_0^{+\infty} \frac{\ln(1+2x^{3p})}{(x+x^2)^{4p}\,\mathrm{arctg}\sqrt{x}} = \int_0^a \dots + \int_a^{+\infty} \dots$ $\underline{\underline{p>0}}$ (по усл-е)

$\int_0^a \sim \frac{x^{3p}}{x^{4p}x^{\frac{1}{2}}} = \frac{1}{x^{p+\frac{1}{2}}}$ $p+\frac{1}{2}<1$, $p<\frac{1}{2} \Rightarrow$ сх.

$\int_a^{+\infty} \sim \frac{\ln x}{x^{8p}}$

$\mathrm{arctg}\sqrt{x} \sim \frac{\pi}{2}$

$\ln(1+2x^{3p}) \sim \ln(2x^{3p}) = \ln 2 + 3p\ln x$

$8p>1 \Rightarrow 8p = 1+2\varepsilon$

$\underbrace{\int_a^{+\infty} \frac{\ln x}{x^{1+2\varepsilon}}dx}_{\text{сх.}} < \underbrace{\int_a^{+\infty} \frac{x^{\varepsilon}}{x^{1+2\varepsilon}}dx}_{\text{сх.}}$ $\Leftarrow$

$\boxed{8p=1 \quad \int_a^{+\infty}\frac{1}{x}dx \text{ — разх.} \Rightarrow \int_a^{+\infty}\frac{\ln x}{x}dx \text{ — разх.}}$

$\frac{1}{8} < p < \frac{1}{2}$

б) $\int_0^{+\infty} \frac{\ln(1+\sqrt[3]{x}+2x^{3p})\arcsin\sqrt{\frac{x}{x+1}}}{x^{4p}+x^3}dx = \int_0^a \dots dx + \int_a^{+\infty}\dots dx$ $\boxed{p>0}$

$\int_0^a \sim \frac{x^{1/3}}{x^3} = \frac{1}{x^{8/3}}$ $\frac{8}{3}>1 \Rightarrow$ разх.

$\ln(1+\sqrt[3]{x}+2x^{3p}) \sim \sqrt[3]{x}+2x^{3p} \sim x^{1/3}$

$x^{4p}+x^3 \sim x^3$

$\arcsin\sqrt{\frac{x}{x+1}} \sim \sqrt{\frac{x}{x+1}} \to 0$

$\int_a^{+\infty} \sim \frac{\ln x}{x^{4p}}$

$\ln(1+\sqrt[3]{x}+2x^{3p}) = \ln\left(x^{3p}\left(\frac{1}{x^{3p}}+\frac{x^{1/3}}{x^{3p}}+2\right)\right) = 3p\ln x + \underbrace{\ln(\dots)}_{\ln 2}$

$\arcsin\sqrt{\frac{x}{x+1}} = \arcsin 1 = \frac{\pi}{2}$

$\sqrt{\frac{x}{x+1}} \xrightarrow[x\to\infty]{} \sqrt{1} = 1$

$x^{4p}+x^3 \sim x^{4p}$

$4p>1 \Rightarrow 4p = 1+2\varepsilon$

$\underbrace{\int_a^{+\infty}\frac{\ln x}{x^{1+2\varepsilon}}dx}_{\text{сх.}} < \underbrace{\int_a^{+\infty}\frac{x^{\varepsilon}}{x^{1+2\varepsilon}}dx}_{\text{сх.}}$ $\Leftarrow$

$4p=1$ $\int_a^{+\infty}\frac{1}{x}dx$ — разх. $\Rightarrow \int_a^{+\infty}\frac{\ln x}{x}dx$ — разх.