Домашна работа № 2 на Петър Парушев с ФН 61620, група 1, СИ

Задача 1.

$$\lim_{n\to\infty} \frac{\sqrt{n-1}-\sqrt{n+2}}{\sqrt{n+4}-\sqrt{n+3}} \sim \frac{\sqrt{n-1}-\sqrt{n+2}}{\sqrt{n+4}-\sqrt{n+3}} \frac{\sqrt{n-1}+\sqrt{n+2}}{\sqrt{n-1}+\sqrt{n+2}} \frac{\sqrt{n+4}+\sqrt{n+3}}{\sqrt{n+4}+\sqrt{n+3}} \sim -3 \frac{\sqrt{n+4}+\sqrt{n+3}}{\sqrt{n-1}+\sqrt{n+2}} = -3 \frac{2}{2} = -3$$

Задача 2.

$$\lim_{n \to -2} \frac{\sqrt{x^3 - x + 16} - \sqrt{8 - x}}{x^2 + 8x + 12} \sim \frac{x^3 + 8}{2\sqrt{10}(x + 2)(x + 6)} \sim \frac{(x + 2)(x^2 + 2x + 4)}{2\sqrt{10}(x + 2)(x + 6)} \sim \frac{(x^2 + 2x + 4)}{2\sqrt{10}(x + 2)} = \frac{3\sqrt{10}}{20}$$

Задача 3.

$$\lim_{n \to \infty} \left(\frac{n^2 - 3n + 2}{n^2 + 3n + 2} \right)^n$$

Полагаме $x = \frac{1}{n}$.

$$\lim_{n\to\infty} \left(\frac{n^2 - 3n + 2}{n^2 + 3n + 2}\right)^n = \lim_{x\to 0} \left(\frac{x^2 - 3x + 2}{x^2 + 3x + 2}\right)^{\frac{1}{x}}$$

$$\ln\lim_{x\to 0} (\frac{x^2-3x+2}{x^2+3x+2})^{\frac{1}{x}} = \lim_{x\to 0} \ln(\frac{x^2-3x+2}{x^2+3x+2})^{\frac{1}{x}} = \lim_{x\to 0} \frac{\ln(\frac{x^2-3x+2}{x^2+3x+2})}{x} = \lim_{x\to 0} \frac{\ln(1-\frac{6x}{x^2+3x+2})}{x} = \lim_{x\to 0} \frac{\frac{6x}{x^2+3x+2}}{x} = -6 \Rightarrow$$

$$\lim_{n \to \infty} \left(\frac{n^2 - 3n + 2}{n^2 + 3n + 2} \right)^n = e^{-6}$$

Задача 4.

$$\lim_{x \to 3} \frac{1}{x - 3} - \frac{27}{x^3 - 27} \sim \frac{1}{x - 3} - \frac{27}{(x - 3)(x^2 + 3x + 9)} \sim \frac{x^2 + 3x - 18}{(x - 3)(x^2 + 3x + 9)} \sim \frac{(x - 3)(x + 6)}{(x - 3)(x^2 + 3x + 9)} = \frac{9}{27} = \frac{1}{3}$$