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(2)

Контролно 2.

$$f(x) = \frac{x^2+2x+8}{x} e^{\frac{1}{x}} \qquad ДМ \quad x \neq 0$$

$x=0$ е вертикална асимптота

Асимптоти

$$\lim_{x\to\infty} \frac{f(x)}{x} = \frac{(x^2+2x+8)e^{\frac{1}{x}}}{x^2} = 1$$

$y = ax + b$

$a = 1$

$$\lim_{x\to\infty} f(x) - ax = \frac{(x^2+2x+8)e^{\frac{1}{x}}}{x} - x =$$

$$= \frac{(x^2+2x+8)e^{\frac{1}{x}} - x^2}{x} =$$

$$= \lim_{x\to\infty} \frac{\cancel{x^2}(e^{\frac{1}{x}}-1)}{\cancel{x}} + \lim \frac{\overset{=1}{e^{\frac{1}{x}}}(2x+8)}{x} =$$

$$= \lim_{x\to\infty} \frac{(e^{\frac{1}{x}}-1)}{\frac{1}{x}} + 2 = 1+2 = 3$$

$y = x+3$

$$f'(x) = \left(\frac{x^2+2x+8}{x}\right)' e^{\frac{1}{x}} + \frac{x^2+2x+8}{x}\left(e^{\frac{1}{x}}\right)' =$$

$$= \frac{(x(2x+2) - x^2+2x+8)e^{\frac{1}{x}}}{x^2} + \frac{(x^2+2x+8)}{x} \cdot \frac{(-e^{\frac{1}{x}})}{x^2} =$$

$$= \frac{(2x^2+2x-x^2-2x-8)e^{\frac{1}{x}}}{x^2} - \frac{(x^2+2x+8)e^{\frac{1}{x}}}{x^3}$$

$$\frac{x(x^2-8)e^{\frac{1}{x}} - (x^2+2x+8)e^{\frac{1}{x}}}{x^3} = \frac{(x^3-x^2-10x-8)e^{\frac{1}{x}}}{x^3}$$

$=1=$

$$f''(x) = e^{\frac{1}{x}}\left(\frac{x^3-x^2-10x-8}{x^3}\right)' + \left(e^{\frac{1}{x}}\right)'\frac{(x^3-x^2-10x-8)}{x^3} =$$

$$= e^{\frac{1}{x}}\left(\frac{(3x^2-2x-10).x^3-3x^2(x^3-x^2-10x-8)}{x^6}\right) - \frac{e^{\frac{1}{x}}}{x^2}\cdot\frac{(x^3-x^2-10x-8)}{x^3} =$$

$$= e^{\frac{1}{x}}\left(\frac{3x^5-2x^4-10x-3x^5+3x^4+30x^3+24x^2}{x^6}\right) - \frac{e^{\frac{1}{x}}(x^3-x^2-10x-8)}{x^5}$$

$$= e^{\frac{1}{x}}\frac{(x^4+20x^3+24x^2-x^4+x^3+10x^2+8x)}{x^6} =$$

$$= \frac{(21x^2+34x+8)}{x^5}.e^{\frac{1}{x}}$$

$f'(x)=0$ $\quad \dfrac{(x^3-x^2-10x-8)e^{\frac{1}{x}}}{x^3}$ $\quad x_1=-2 \quad x_2=-1 \quad x_3=4$

$+ \quad - \quad + \quad - \quad +$

$\nearrow -2 \searrow -1 \nearrow 0 \searrow 4 \nearrow$

$x \in (-\infty; -2) \cup (-1; 0) \cup (4; +\infty) \to$ расте $\quad -2$ - локален макс.

$x \in (-2; -1) \cup (0; 4) \to$ намалява

$-1, 4$ - локален минимум

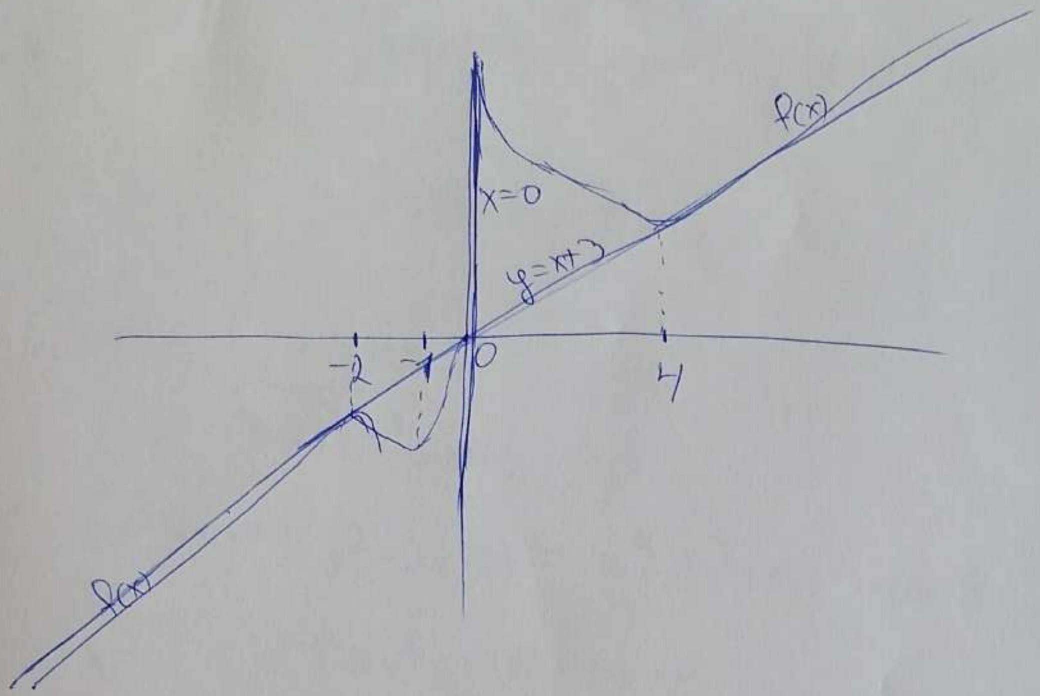
глобални няма

$f''(x)=0 \quad x_1 = -\frac{28}{21} = -\frac{4}{3}$

$x_2 = -\frac{6}{21} = -\frac{2}{7}$ $\to$ инфлексни точки

$x \in (-\infty; -\frac{4}{3}) \cup (-\frac{2}{7}; 0)$ - вдлъбната

$x \in (-\frac{4}{3}; -\frac{2}{7}) \cup (0; +\infty)$ - изпъкнала


f(x)
x=0
y=x+3
-2
-1
0
4
f(x)