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ДИС 2

ДР №5

$$\sum_{n=0}^{\infty} \sqrt{\binom{3n+1}{2n}} \frac{x^{3n}}{\sqrt[3]{n+1}}$$

Прилагаме Даламбер

$$\sqrt{\frac{(3n+4)!}{(2n+2)!(n+2)!}}\sqrt{\frac{(2n)!(n+1)!}{(3n+1)!}}\frac{\sqrt[3]{n+1}}{\sqrt[3]{n+2}}\left|\frac{x^{3n+3}}{x^{3n}}\right| =$$

$$= \sqrt{\frac{(3n+4)(3n+3)(3n+2)\cancel{(3n+1)!}\cancel{(2n)!}\cancel{(n+1)!}}{(2n+2)(2n+1)\cancel{(2n)!}(n+2)\cancel{(n+1)!}\cancel{(3n+1)!}}}\sqrt[3]{\frac{n+1}{n+2}}\left|x^3\right| \xrightarrow[n\to\infty]{}$$

$$= \sqrt{\frac{27}{4}}\left|x^3\right|$$

$$\sqrt{\frac{27}{4}}\left|x^3\right| < 1 \Rightarrow x < \sqrt[6]{\frac{4}{27}} \Rightarrow R = \sqrt[6]{\frac{4}{27}}$$

Раабе-Дюамел

$$\lim_{n\to\infty} n\left(\frac{a_n}{a_{n+1}} - 1\right) = \lim_{n\to\infty} n\left(\sqrt{\frac{2}{3}\frac{(2n+1)(n+2)}{(3n+4)(3n+2)}\frac{9}{2}}\sqrt[3]{\frac{n+2}{n+1}} - 1\right) =$$

$$= \lim_{n\to\infty} n\left(\sqrt{\frac{18n^2+45n+18}{18n^2+36n+16}}\sqrt[3]{\frac{n+2}{n+1}} - \sqrt[3]{\frac{n+2}{n+1}} + \sqrt[3]{\frac{n+2}{n+1}} - 1\right) =$$

$$= \lim_{n\to\infty} n\sqrt[3]{\frac{n+2}{n+1}}\left(\sqrt{\frac{18n^2+45n+18}{18n^2+36n+16}} - 1\right) + \lim_{n\to\infty} n\left(\sqrt[3]{\frac{n+2}{n+1}} - 1\right) =$$

$$= \lim_{n\to\infty} \frac{9n^2+2n}{2(18n^2+36n+16)} + \lim_{n\to\infty} \frac{n}{3(n+1)} = \frac{1}{4} + \frac{1}{3} = \frac{7}{12} < 1$$

редът е условно сход. $\Rightarrow$ при $x = -R = -\sqrt[6]{\frac{4}{27}}$ е сходящ
и при $x = R = \sqrt[6]{\frac{4}{27}}$ е разходящ

редът е сходящ за $x \in \left[-\sqrt[6]{\frac{4}{27}}, \sqrt[6]{\frac{4}{27}}\right]$