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Домашна работа 5

<u>1зад</u> $z = \ln \frac{xy}{x^2-y^2} = \ln(xy) - \ln(x-y) - \ln(x+y)$

$z'_x = \frac{1}{x} - \frac{1}{x-y} - \frac{1}{x+y}$

$z''_{xx} = -\frac{1}{x^2} + \frac{1}{(x-y)^2} + \frac{1}{(x+y)^2}$

$z'''_{xxx} = +\frac{2}{x^3} - \frac{2}{(x-y)^3} - \frac{2}{(x+y)^3} = -2\left(\frac{1}{x^3} + \frac{1}{(x-y)^3} + \frac{1}{(x+y)^3}\right)$

$z'''_{xxy} = \frac{2}{(x-y)^3} - \frac{2}{(x+y)^3}$

$z''_{xy} = -\frac{1}{(x-y)^2} + \frac{1}{(x+y)^2}$

$z'''_{xyy} = -\frac{2}{(x-y)^3} - \frac{2}{(x+y)^3}$

$z'_y = \frac{1}{y} + \frac{1}{x-y} - \frac{1}{x+y}$

$z''_{yy} = -\frac{1}{y^2} + \frac{1}{(x-y)^2} + \frac{1}{(x+y)^2}$

$z'''_{yyy} = \frac{2}{y^3} + \frac{2}{(x-y)^3} - \frac{2}{(x+y)^3}$

$z'''_{xxx} + z'''_{xxy} - z'''_{xyy} - z'''_{yyy} = +\frac{2}{x^3} - \cancel{\frac{2}{(x-y)^3}} - \cancel{\frac{2}{(x+y)^3}} + \cancel{\frac{2}{(x-y)^3}} - \cancel{\frac{2}{(x+y)^3}} +$

$+ \cancel{\frac{2}{(x-y)^3}} + \cancel{\frac{2}{(x+y)^3}} - \frac{2}{y^3} - \cancel{\frac{2}{(x-y)^3}} + \cancel{\frac{2}{(x+y)^3}} = 2\left(\frac{1}{x^3} - \frac{1}{y^3}\right)$

<u>2зад</u> $J(T) = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix} = \begin{vmatrix} y^2 z & 2xyz & xy^2 \\ y^2 - y^2 z & 2xy - 2xyz & -xy^2 \\ -y^2 & 2y - 2xy & 0 \end{vmatrix} =$

$= 2xy^5(1-z)(1-x) + 2x^2y^5(1-z) + 2y^5xz(1-x) =$

$= 2x^2y^5z + 2xy^5(1-x) = 2xy^5$

$= 2x^2y^5$

<u>3зад</u> $F(x,y,z) = z^2 - x^2 - y^2 + 2xz + 2yz + ax + by + cz$

$F'_x = -2x + 2z + a$

$F'_y = -2y + 2z + b$

$F'_z = 2z + 2x + 2y + c$

$F''_{xx} = -2$

$F''_{xy} = 0$

$F''_{xz} = 2$

$F''_{yx} = 0$

$F''_{yy} = -2$

$F''_{yz} = 2$

$F''_{zx} = 2$

$F''_{zy} = 2$

$F''_{zz} = 2$

$\begin{pmatrix} -2 & 0 & 2 \\ 0 & -2 & 2 \\ 2 & 2 & 2 \end{pmatrix}$ $\Rightarrow$ няма лок. екстр.

4зад $f(x,y) = \ln\left(\sqrt{x^2+1} - \sqrt{\frac{x^2}{4} - y^2}\right)$

$f(x,y)$ – симетрична в четирите квадранта

[illegible]

Разгл. $x \geq 0,\ y \geq 0$

$x \geq 2y$

$$f'_x = \frac{1}{(\ldots)}\left(\frac{x}{\sqrt{x^2+1}} - \frac{x}{4\sqrt{\frac{x^2}{4}-y^2}}\right) = 0$$

$$4x\sqrt{\frac{x^2}{4}-y^2} - x\sqrt{x^2+1} = 0$$

① $x = 0,\ y = 0$

② $4\sqrt{\frac{x^2}{4}-y^2} = \sqrt{x^2+1}$

$4x^2 - 16y^2 = x^2+1$

$3x^2 = 16y^2 + 1$

$3x^2 = 1$

$x = \frac{1}{\sqrt{3}}$, тъй като сме в I кв.

$$f'_y = \frac{1}{(\ldots)}\left(\frac{2y}{\sqrt{\frac{x^2}{4}-y^2}}\right) = 0$$

$f(0,0) = 0$

$f\left(\frac{1}{\sqrt{3}},0\right) = \ln\left(\frac{2}{\sqrt{3}} - \frac{1}{2\sqrt{3}}\right) = \ln\frac{3}{2\sqrt{3}} = \ln\frac{\sqrt{3}}{2} < 0$

$f\left(\frac{1}{\sqrt{3}},0\right)$ – min

$f\left(-\frac{1}{\sqrt{3}},0\right)$ – min

5зад $f(x,y) = x^2\underbrace{e^{-x^2-3x-4y^2}}_{>0 \text{ за } \forall x,y}$

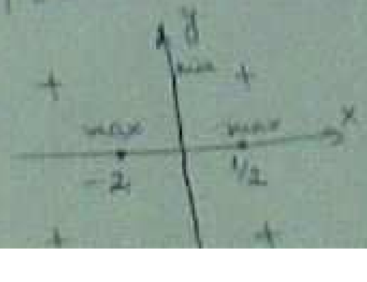
$$\begin{cases} 2x + x^2(-2x-3) = 0 \\ x\,2(-8y) = 0 \end{cases}$$

$$\begin{cases} x(2 - 2x^2 - 3x) = 0 \\ x^2 y = 0 \end{cases}$$

① $x = 0 \Rightarrow y$ – произволно

② $2x^2 + 3x - 2 = 0$

$$\begin{cases} x = \frac{1}{2} \\ y = 0 \end{cases} \qquad \begin{cases} x = -2 \\ y = 0 \end{cases}$$



∀ точки, лежащи на оса Oy са min за $f(x,y)$

а) D: $x^2+4y^2 \le 5$

$M(-2,0) \in D$

$N(\frac{1}{2},0) \in D$

При x или y, клонящо към ∞ $f \to 0$

$f(-2,0) = 4 \cdot e^{-4+6} = 4e^2$

$f(\frac{1}{2},0) = \frac{1}{4} e^{-\frac{1}{4}-\frac{3}{4}} = \frac{1}{4e}$

M – най-голяма за D

$(0,0)$ – най-малка за D

б) $f(0,y)$ – min

$f(-2,0)$ – max

$f(\frac{1}{2},0)$ – max

в) $f(-2,0) = 4e^2$ – max

$f(0,y) = 0$ – min