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Софтуерно инженерство, 1 курс, 1 група

Домашна работа №6

a) $f(x) = x^7 e^x + \ln\left(x+\sqrt{x^2+1}\right) \cdot \sin x + \dfrac{x^3}{x^2-2x-3}$

$f^{(21)}(0) = ?$

I. Нека $g(x) = x^7 e^x$; $(x^7 e^x)^{(21)} = (-1)^{14} \cdot \binom{21}{7} \cdot 7! \cdot e^x =$

$= \dfrac{21.20.19.18.17.16.15}{7!} \cdot 7! \cdot e^x =$

$= V_{21}^{7} \cdot e^x$

$g(0)^{(21)} = V_{21}^{7} \cdot 1 = V_{21}^{7}$

II.

Нека $\ln\left(x+\sqrt{x^2+1}\right) = h(x)$

Пресмятаме $h(-x) = \ln\left(-x+\sqrt{(-x)^2+1}\right) = \ln\left(-x+\sqrt{x^2+1}\right)$

Пресмятаме $-h(x) = -\ln\left(x+\sqrt{x^2+1}\right) = \ln\left(x+\sqrt{x^2+1}\right)^{-1} \Rightarrow$

$\Rightarrow \ln\left(\dfrac{1}{x+\sqrt{x^2+1}}\right) = \ln\left(\dfrac{1}{x+\sqrt{x^2+1}} \cdot \dfrac{x-\sqrt{x^2+1}}{x-\sqrt{x^2+1}}\right) =$

$= \ln\left(\dfrac{x-\sqrt{x^2+1}}{x^2-x^2-1}\right) = \ln\left(-x+\sqrt{x^2+1}\right) \Rightarrow$

$\Rightarrow h(-x) = -h(x) \Rightarrow h(x) \rightarrow$ нечетна функ.
но $\sin x$ също е нечетна функ. $\Rightarrow$

=1=

$\Rightarrow \sin x \cdot h(x) \to$ четна функция

1 $\sin x = 0$ при $x=0$ $4k$

2 $\sin' x = \cos x = 1$ при $x=0$ $4k+1$

3 $\sin'' x = \cos' x = -\sin x = 0$ при $x=0$ $4k+2$

4 $\sin''' x = -\cos x = -1$ при $x=0$ $4k+3$

$21 = 4.5+1 \Rightarrow (\sin x)^{(21)} = 0 \Rightarrow$

$\Rightarrow (\sin 0 \cdot h(0))^{(21)} = 0$

III

Нека $\dfrac{x^3}{x^2-2x-3} = p(x)$

$p(x) = \dfrac{x^3}{(x+1)(x-3)} = \dfrac{y}{(x+1)} + \dfrac{z}{(x-3)}$

$x^3 = y\cdot(x-3) + z\cdot(x+1) \Rightarrow$ при $x=3$ $z = \dfrac{27}{4}$

при $x=-1$ $y = \dfrac{1}{4}$

$\rightarrow$

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Получаваме: $p(x) = \frac{27}{4(x-3)} + \frac{1}{4(x+1)}$

$$p^{(21)}(0) = \left(\frac{27}{4(x-3)}\right)^{(21)} + \left(\frac{1}{4(x+1)}\right)^{(21)} =$$

$$= \frac{27}{4}\left(\frac{21!\,(-1)^{21}}{(-3)^{22}}\right) + \frac{1}{4}\left(\frac{21!\,.\,(-1)^{21}}{1^{22}}\right) =$$

$$= -\frac{21!}{4}\cdot\left(\frac{3^3}{3^{22}} + 1\right) =$$

$$= -\frac{21!}{4}\cdot\left(\frac{1+3^{19}}{3^{19}}\right)$$

б) Док, че $(e^x \cdot \cos x)^{(k)} = (\sqrt{2})^k e^x \cos\left(x + k\frac{\pi}{4}\right)$

I Нека $k = 1$

$\Rightarrow (e^x \cdot \cos x)' = e^x \cos x - e^x \sin x =$

$= e^x(\cos x - \sin x) =$

$= e^x \cdot \sqrt{2} \cdot \cos\left(x + \frac{\pi}{4}\right)$ ✓

II Приемаме, че $(e^x \cdot \cos x)^{(k)} = (\sqrt{2})^k \cdot e^x \cdot \cos\left(x + k\frac{\pi}{4}\right)$ е вярно за $\forall k$.

III Доказване за $k+1$

$(\sqrt{2})^k \cdot e^x \cdot \cos\left(x + k\frac{\pi}{4}\right)$ е k-тата производна на $e^x \cdot \cos x$

За $k+1$ търсим производна на тази производна

$(e^x \cos x)^{(k+1)} = \left(\sqrt{2}^k \cdot e^x \cdot \cos\left(x + k\frac{\pi}{4}\right)\right)' =$

$= \left(\sqrt{2}^k \cdot e^x\right)' \cdot \cos\left(x + k\frac{\pi}{4}\right) - \sqrt{2}^k \cdot e^x \cdot \sin\left(x + k\frac{\pi}{4}\right) =$

$= \left(\left(\sqrt{2}^k\right)' e^x + \sqrt{2}^k (e^x)'\right) \cdot \cos\left(x + k\frac{\pi}{4}\right) - \sqrt{2}^k \cdot e^x \cdot \sin\left(x + k\frac{\pi}{4}\right) =$

$= \sqrt{2}^k \cdot e^x \cdot \cos\left(x + k\frac{\pi}{4}\right) - \sqrt{2}^k \cdot e^x \cdot \sin\left(x + k\frac{\pi}{4}\right) =$

$= \sqrt{2}^k \cdot e^x \cdot \left(\cos\left(x + k\frac{\pi}{4}\right) - \sin\left(x + k\frac{\pi}{4}\right)\right) = \longrightarrow$

$= 4F$ ФН: 0MI0600041

$= \sqrt{2}^{k} \cdot e^{x} \cdot \sqrt{2} \cdot \cos\left(x + k\frac{\pi}{4} + \frac{\pi}{4}\right) =$

$= \sqrt{2}^{k+1} e^{x} \cdot \cos\left(x + (k+1)\frac{\pi}{4}\right)$ Доказано с индукция за k

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