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ДИС 2

ДР №2

$$I = \int_0^{+\infty} \frac{\ln x \, dx}{x^2+2x+4}$$

1. $t = \frac{1}{x}$ $x = \frac{1}{t}$ $\frac{dx}{dt} = -\frac{1}{t^2}$

$x = 0^+ \Rightarrow t = +\infty$, $x = +\infty \Rightarrow t = 0^+$

$$I = \int_{+\infty}^{0} \frac{\ln\left(\frac{1}{t}\right) \cdot \frac{-1}{t^2} \, dt}{\frac{1}{t^2} + \frac{2}{t} + 4} = \int_0^{+\infty} \frac{\ln \frac{1}{t} \, dt}{4t^2+2t+1} = -\int_0^{+\infty} \frac{\ln t \, dt}{4t^2+2t+1}$$

2. $x = 4t$, $t = \frac{x}{4}$, $\frac{dx}{dt} = 4$

$x = 0$; $t = 0$

$x = +\infty$; $t = +\infty$

$$I = 4 \int_0^{+\infty} \frac{\ln 4t \, dt}{16t^2+8t+4} = \int_0^{+\infty} \frac{\ln 4t \, dt}{4t^2+2t+1} =$$

$$= \int_0^{+\infty} \frac{\ln 4 \, dt}{4t^2+2t+1} + \int_0^{+\infty} \frac{\ln t \, dt}{4t^2+2t+1}$$

$$I = \int_0^{+\infty} \frac{\ln x \, dx}{x^2+2x+4} = -\int_0^{+\infty} \frac{\ln t}{4t^2+2t+\cancel{1}} dt = 2\ln 2 \int_0^{+\infty} \frac{dt}{4t^2+2t+1} +$$

$$\int_0^{+\infty} \frac{\ln t \, dt}{4t^2+2t+1}$$

$$-\cancel{2}I = \cancel{2}\ln 2 \int_0^{+\infty} \frac{dt}{4t^2+2t+1}$$

$$I = -\ln 2 \int_0^{+\infty} \frac{dt}{\left(2t+\frac{1}{2}\right)^2+\frac{3}{4}} = -\frac{\ln 2}{2} \int_0^{+\infty} \frac{d\left(2t-\frac{1}{2}\right)}{\left(2t+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} =$$

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$$= -\frac{\ln 2}{2}\left(\frac{2}{\sqrt{3}} \operatorname{arctg} \frac{2}{\sqrt{3}}\left(2t+\frac{1}{2}\right)\Big|_0^{\infty}\right) = -\frac{\ln 2}{\sqrt{3}}\left(\frac{\pi}{2}-\frac{\pi}{6}\right) =$$

$$= -\frac{\ln 2}{\sqrt{3}} \frac{\pi}{3}$$