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ДИС 2

(3) $\sum_{n=1}^{\infty} \frac{n^3}{2^n(n+3)} = \sum_{n=1}^{\infty} \frac{n^3+27-27}{2^n(n+3)} = \sum_{n=1}^{\infty} \frac{n^3+27}{2^n(n+3)} - 27\sum_{n=1}^{\infty} \frac{1}{2^n(n+3)} =$

$= \sum_{n=1}^{\infty} \frac{(n+3)(n^2-3n+9)}{2^n(n+3)} - 27\sum_{n=1}^{\infty}\frac{1}{2^n(n+3)} \quad \cdot\frac{2^3}{2^3} =$

$= \sum_{n=1}^{\infty} \frac{n^2-3n+9}{2^n} - 27.8\sum_{n=1}^{\infty}\frac{1}{2^{n+3}(n+3)} =$

$= \underbrace{\sum_{n=1}^{\infty}\frac{n^2}{2^n}}_{\Sigma_1=6} - 3\underbrace{\sum_{n=1}^{\infty}\frac{n}{2^n}}_{\Sigma_2=2} + \underbrace{9\sum_{n=1}^{\infty}\frac{1}{2^n}}_{\Sigma_3=9} - 27.8\sum_{n=4}^{\infty}\frac{1}{2^n.n} =$

$= 6-6+9-27.8\Big(\sum_{n=1}^{\infty}\frac{1}{2^n.n} - \overbrace{\frac{1}{2}-\frac{1}{8}-\frac{1}{24}}^{\frac{2}{3}}\Big) =$

$= 9 - 27.8.\sum_{n=1}^{\infty}\frac{1}{2^n.n} + 144 = \boxed{153-216.\ln 2}$

$\sum_{n=1}^{\infty}\frac{n^2}{2^n} \quad \frac{1}{2}=x \Rightarrow \sum_{n=1}^{\infty} n^2x^n = \sum_{n=1}^{\infty} n^2x^{n-1} = \frac{1}{x}\sum_{n=1}^{\infty} n^2.x^n =$

$\Big(\sum_{n=1}^{\infty} nx^n\Big)' = \Big(\frac{x}{(1-x)^2}\Big)' = \frac{(1-x)^2-(2x-2)x}{(1-x)^4} = \frac{1-2x+x^2-2x^2+2x}{1-x^4} =$

$= x\Big(\frac{1+x}{1-x}\Big|3\Big) = 6$

$9\sum_{n=1}^{\infty}\frac{1}{2^n} = 9\sum_{n=1}^{\infty}x^n = 9\frac{1}{1-x} - 9 = 9.2-9 = \underline{\underline{9}}$

$\Rightarrow$ 1) е сход. за $p \in [0,1)$

2) $g(x) = \dfrac{x^2}{\ln(x^{p^2}+x^{4p})\ln^2 x}$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \frac{\sqrt{x^4+1} - \cos^2 x}{x^2} \cdot \frac{\ln x^{p^2}+x^{4p}}{\ln 1+x^{p^2}+x^{4p}} \cdot \frac{\ln^2 x}{\ln^2(1+x^p)} = 1$$

$\Rightarrow \int_2^{+\infty} f(x)$ е сход. $\Leftrightarrow \int_2^{+\infty} g(x)$ е сходящ.

при $p \geq 0 \Rightarrow \ln x^{p^2}+x^{4p} \to x^{4p}$

$\Rightarrow \int_2^{+\infty} g(x)$ е сх. $\Leftrightarrow \int_2^{+\infty} \frac{1}{x^{p^2+4p}\ln^2 x}$ е сход.

$\Rightarrow -2+4p \geq 1$, $p \geq \frac{3}{4}$

при $<0 \Rightarrow \ln x^{p^2}+x^{4p} \to \ln x$

т.е. $\int_2^{+\infty} g(x)$ е сход. $\Leftrightarrow \int_2^{+\infty} \frac{1}{x^2 \ln^3 x}$ - разходящ

$\Rightarrow$ 2 е сходящ за $p \geq \frac{3}{4}$

$\Rightarrow$ Целият интеграл е сход. за ~~$p \in [0,1)$~~

$p \in [\frac{3}{4}; 1)$ □