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Софтуерно инженерство, I курс, I група

Домашна работа №1

Функциите f_n се дефинират:

$$f_1(x) = |x| - 2 \quad ; \quad f_{n+1}(x) = f_1(f_n(x))$$

I а) $f_{2021}(0) = ?$ б) $f_{2021}(1) = ?$

II $f_{2021}(x) = \frac{x}{21}$

$$\begin{aligned} \text{I а) } & \left. \begin{aligned} f_1(0) &= |0| - 2 = 0 - 2 = -2 \\ f_2(0) &= f_1(f_1(0)) = f_1(-2) = |-2| - 2 = 2 - 2 = 0 \\ f_3(0) &= f_1(f_2(0)) = f_1(0) = 0 - 2 = -2 \\ f_4(0) &= f_1(f_3(0)) = f_1(-2) = |-2| - 2 = 0 \end{aligned} \right\} \Rightarrow \end{aligned}$$

$$\Rightarrow f_{2k}(0) = 0 \quad ; \quad f_{2k+1}(0) = -2 \quad (k \in \mathbb{N})$$

$$\Rightarrow f_{\text{четно}} = 0, \quad f_{\text{нечетно}} = -2 \Rightarrow$$

$$\Rightarrow \boxed{f_{2021}(0) = -2}$$

$$\begin{aligned} \text{в)} \quad & f_1(1) = |1| - 2 = 1 - 2 = -1 \\ & f_2(1) = f_1(f_1(1)) = f_1(-1) = |-1| - 2 = 1 - 2 = -1 \\ & f_3(1) = f_1(f_2(1)) = f_1(-1) = |-1| - 2 = -1 \\ & f_4(1) = f_1(f_3(1)) = f_1(-1) = |-1| - 2 = -1 \end{aligned} \quad \left. \vphantom{\begin{aligned} f_1(1) \\ f_2(1) \\ f_3(1) \\ f_4(1) \end{aligned}} \right\} \Rightarrow$$

$$\Rightarrow f_k(1) = -1, \quad k \in \mathbb{N} \Rightarrow$$

$$\Rightarrow \boxed{f_{2021}(1) = -1}$$

$$\text{II} \quad f_{2021}(x) = \frac{x}{21}$$

$$f_1(x) = |x| - 2$$

$$f_2(x) = f_1(f_1(x)) = ||x| - 2| - 2$$

$$f_3(x) = f_1(f_2(x)) = |||x| - 2| - 2| - 2$$

График на $f_1(x) = |x| - 2$

$$f_1(0) = -2$$

$$x=0; y=-2$$

$$f_1(x) = 0$$

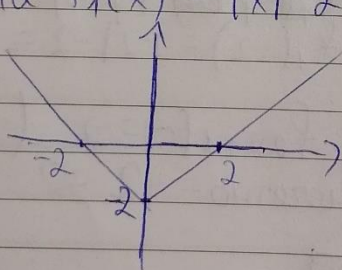
$$|x| - 2 = 0$$

$$\text{I} \quad x > 0$$

$$x - 2 = 0 \quad x = 2$$

$$\text{II} \quad x < 0$$

$$-x - 2 = 0 \quad x = -2$$



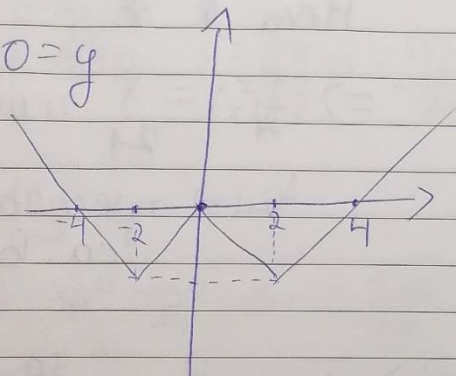
Графика на $f_2(x) = ||x|-2|-2$

при $x=0$ $f(x)=0=y$
в $x=0$

$$f_2(x) = 0$$

$$||x|-2|-2=0$$

$$||x|-2|=2$$



I $x > 0$

$$|x-2|=2$$

$$1) x-2=2$$

$$x=4$$

$$2) x-2=-2$$

$$x=0$$

II $x < 0$

$$|-x-2|=2$$

$$1) -x-2=2$$

$$x=-4$$

$$2) -x-2=-2$$

$$x=0$$

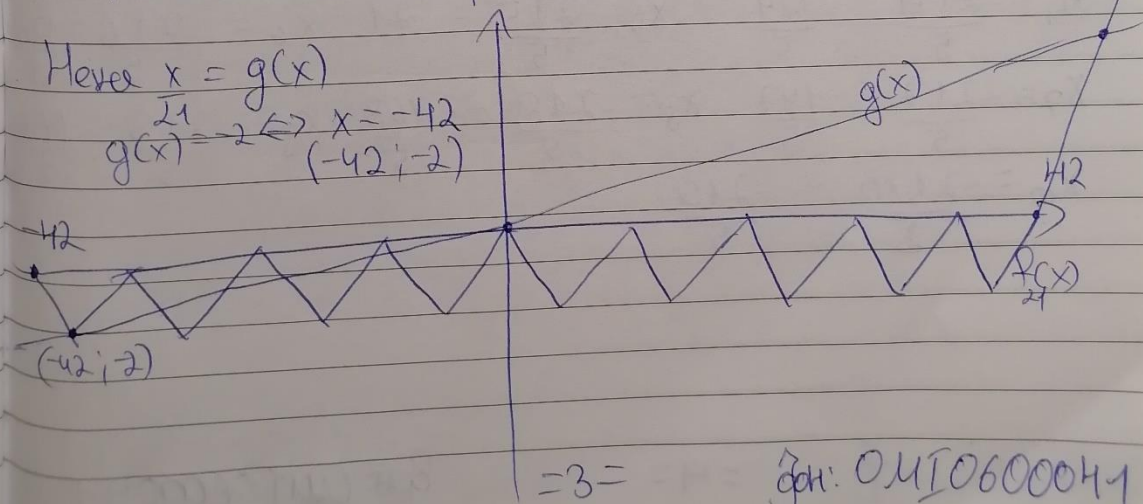
За $x=2$ $f_2(x)=y=-2$; за $x=-2$ $f_2(x)=y=-2$

Аналогично построяваме за $f_{21}(x)$

Наче $x = g(x)$

$$\frac{21}{g(x)=-2} \Leftrightarrow x=-42$$

$$(-42|-2)$$



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Класа Φ

$\Rightarrow f_1(x) = \frac{x}{21}$ има решения, там вздето

$f_2(x)$ пресича $g(x)$ и тези решения са в интервала $[-42; 0]$

\Rightarrow През Числа Φ получаваме: $\pm x \pm 4p = \frac{x}{21}$

$$\begin{aligned} \cancel{x} + \cancel{4p} &= \cancel{x} \\ x + 4p &= \frac{x}{21} \quad = 21x + 84p = x \\ x &= \frac{84p}{20} = \frac{21p}{5} \end{aligned}$$

$$x \in [-42; 0] \quad ; \quad p \in [0; 10]$$

$$x_1 = \frac{-21 \cdot 1}{5} = \frac{-21}{5} \quad x_2 = \frac{-21 \cdot 2}{5} = \frac{-42}{5} \quad x_3 = \frac{-21 \cdot 3}{5} = \frac{-63}{5}$$

$$x_4 = \frac{-21 \cdot 4}{5} = \frac{-84}{5} \quad x_5 = \frac{-21 \cdot 5}{5} = -21 \quad x_6 = \frac{-21 \cdot 6}{5} = \frac{-126}{5}$$

$$x_7 = \frac{-21 \cdot 7}{5} = \frac{-147}{5} \quad x_8 = \frac{-21 \cdot 8}{5} = \frac{-168}{5} \quad x_9 = \frac{-21 \cdot 9}{5} = \frac{-189}{5}$$

~~$x_{10} = \frac{-21 \cdot 10}{5} = -42$~~

при $p=10$ $x=-42$, което не е решение,
защото $f_2(-42) = 0$, а $g(-42) = -2$

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$$\text{II } -x - 4p = \frac{x}{21} \Rightarrow -21x - 84p = x \\ 22x = -84p \quad x = \frac{84}{22}p = \frac{42}{11}p$$

$$x \in [-42; 0); \quad p \in (0; 11]$$

$$\begin{aligned} x_1 &= -\frac{42}{11} & x_2 &= -\frac{84}{11} & x_3 &= -\frac{126}{11} & x_4 &= -\frac{168}{11} & x_5 &= -\frac{210}{11} \\ x_6 &= -\frac{252}{11} & x_7 &= -\frac{294}{11} & x_8 &= -\frac{336}{11} & x_9 &= -\frac{378}{11} & x_{10} &= -\frac{420}{11} \\ x_{11} &= -\frac{462}{11} \end{aligned}$$

При $x > 0$ $f_{2011}(x) = x - 4042$
 ~~$f_{2011}(x) = \frac{x}{21}$~~

$$x - 4042 = \frac{x}{21}$$

$$21x - 4042 \cdot 21 = x$$

$$20x = 4042 \cdot 21$$

$$x = \frac{4042 \cdot 21}{20} = \frac{2021 \cdot 21}{10} = \frac{42441}{10}$$

Итого: 20 решения за $x \in [-42; 0)$ и
 1 решение за $x > 0$ или по-точно
 $x > 4042$ ($f_{2011}(x) = x - 4042$, за $x > 0$)

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