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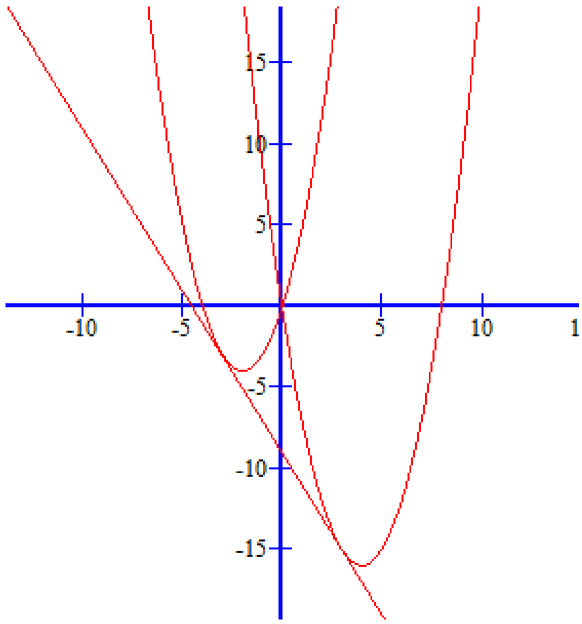
Домашна работа 2

Зад. 1



f1=x^2 +4x, доп. = kx+n

f2=x^2 -8x, доп. = kx+n

= > k=-2, n=-9

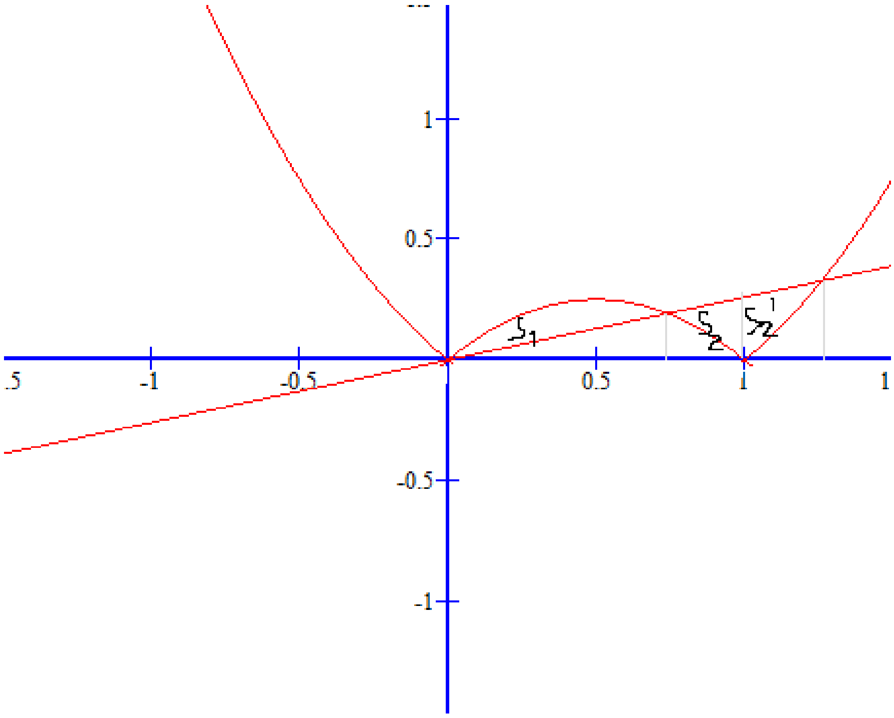
доп. = -2x-9

S=$\int_{-3}^{0} x^2 + 6x + 9\, dx + \int_{0}^{3} x^2 - 6x + 9\ dx$

=$\frac{x^3}{3} + 3x^2 + 9x|_{-3}^{0} + \frac{x^3}{3} - 3x^2 + 9x|_{0}^{3}$

=18 кв.ед.

Зад.2



kx=|x(x-1)|

I x(x-1) > 0

x=0, x=k+1

II x(x-1)<0

x=0; x=1-k

по усл – $S_1 = S_2 + S_2^I$

$\int_0^{1-k} x(1-x) - kx\, dx = \int_{1-k}^{1} kx + x(x-1)\mathrm{dx} + \int_1^{1+k} kx - x(x-1)\mathrm{dx}$

$\frac{x^2}{2} - \frac{x^3}{3} - k\frac{x^2}{2} 9x|\begin{matrix}1-k\\0\end{matrix} = -\frac{x^2}{2} + \frac{x^3}{3} + k\frac{x^2}{2}|\begin{matrix}1\\1-k\end{matrix} + \frac{x^2}{2} - \frac{x^3}{3} + k\frac{x^2}{2}x|\begin{matrix}1+k\\1\end{matrix}$

$(1+k)^3$= -2

$k_1 = \sqrt[3]{2} - 1$

$k_{2,3}$ =-1 – $\frac{1 \pm i\sqrt{3}}{2^{\frac{2}{3}}}$

Зад.3

y=sin2t, x=sint, x'=cost, y'=2cos2t

S=0,5$\int_0^{\pi} costsin2t - 2cost2tsintdt$=

0,5$\int_0^{\pi} \frac{sin3t+sint}{2} - 2\frac{sin3t-sint}{2}dt$ =

=0,5$\int_0^{\pi} \frac{-sin3t+3sint}{2}dt$ = $\frac{1}{4}$ $(\frac{\cos 3\mathrm{t}}{3} - 3cost)\Big|_0^{\pi}$=

=$\frac{4}{3}$

Зад.4

Нека преминем в полярна к.с. и y=a$\sin^4$t , x=a$\cos^4$t, x'=-4a$\cos^3$tsint

Функцията е симетрично разположена в 4те квадранта

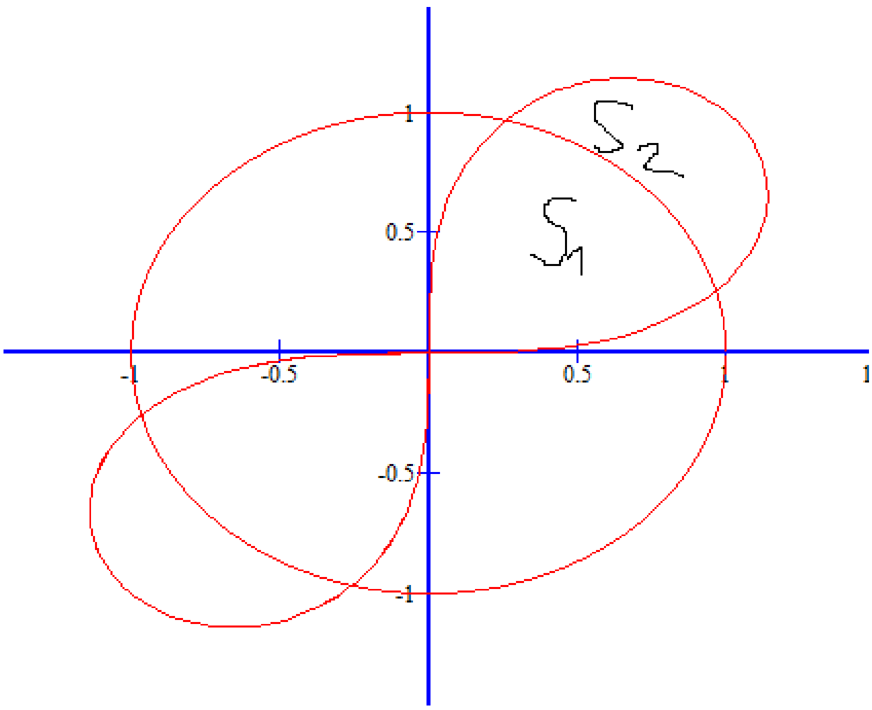
S=|4 $\int_0^{\frac{\pi}{2}} -asin^4t4cos^3tsint$ |=|-16a^2 $\int_0^{\pi/2} sin^5tcos^3t$|=

|-16a^2 $\int_0^{\pi/2} sin^5tcost + sin^7tcost$|

sint=u, dt=$\frac{du}{cost}$

$|-16a^2 \frac{sin^6 t}{6}|_{o}^{\pi/2} -16a^2 \frac{sin^8 t}{8}|_{o}^{\pi/2}| = \frac{2}{3} a^2$

Зад.5



$(x^2+y^2)^2=4xy$

$x^2+y^2=1$

Нека параметризираме – x=rcost, y=rcost

=>r^2=2sint2t, r=$\sqrt{2\text{sint}2\text{t}}$

Пресечни точки – за r=1

t= $\frac{\pi}{12}$; $\frac{5\pi}{12}$

$$S_o=\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} r^2(t)\,dt = \frac{1}{2}\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} 2\sin t\,dt = -\frac{1}{2}\cos 2t\,\Big|_{\frac{\pi}{12}}^{\frac{5\pi}{12}} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow S_2 = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$$

$$S_K = \frac{1}{2}\int_0^{\frac{\pi}{2}} r^2(t)\,dt = \frac{1}{2}\int_0^{\frac{\pi}{2}} 2\sin t\,dt = 1$$

$$\Rightarrow S_1 = 1 - S_2 = 1 - \frac{\sqrt{3}}{2} - \frac{\pi}{6}$$