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(4) Дис 2 Контролно 1

a) $\sum_{n=0}^{\infty} \sqrt[3]{\binom{2n}{n}\binom{3n+1}{n+1}} \cdot \frac{x^n}{\sqrt[4]{n^2+1}}$ $R = ?$

Търсим $\left|\frac{a_{n+1}}{a_n}\right|$ по Даламбер

$$\left.\sqrt[3]{\frac{\cancel{(2n+2)!}}{(n+1)!(n+1)!} \cdot \frac{(3n+4)!}{(n+2)!\cancel{(2n+2)!}}} \cdot \frac{x^{n+1}}{\sqrt[4]{(n+1)^2+1}}\right\} a_{n+1}$$

$$\left.\sqrt[3]{\frac{\cancel{2n!}}{n!\,n!} \cdot \frac{(3n+1)!}{(n+1)!\,\cancel{(2n!)}}} \cdot \frac{x^n}{\sqrt[4]{n^2+1}}\right\} a_n$$

$$\Rightarrow \left|\frac{a_{n+1}}{a_n}\right| = \sqrt[3]{\frac{(3n+4)!\ n!\,n!\,(n+1)!}{(n+1)!(n+1)!\,(n+2)!\,(3n+1)!}} \cdot \frac{\frac{x^{n+1}}{\sqrt[4]{n^2+2n+2}}}{\frac{x^n}{\sqrt[4]{n^2+1}}} =$$

$$= \sqrt[3]{\frac{(3n+4)(3n+3)(3n+2)}{(n+1)(n+1)(n+2)}} \cdot \frac{|x|\sqrt[4]{n^2+1}}{\sqrt[4]{n^2+2n+2}}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \sqrt[3]{27}\cdot x = 3\cdot x \Rightarrow R = 3$$

δ) $\lim n\left(\frac{a_n}{a_{n+1}}-1\right)=$

$$=\lim_{n\to\infty} n\left(\sqrt[3]{\frac{(n+1)(n+1)(n+2)}{(3n+4)(3n+3)(3n+2)}}\cdot\frac{3\cdot\sqrt[4]{n^2+2n+2}}{\sqrt[4]{n^2+1}}-1\right)=$$

$$=\lim_{n\to\infty} n\left(3\sqrt[3]{\frac{(n+1)(n+1)(n+2)}{(3n+4)(3n+3)(3n+2)}}\cdot\frac{\sqrt[4]{n^2+2n+2}}{\sqrt[4]{n^2+1}}-\sqrt[4]{\frac{n^2+2n+2}{n^2+1}}+\sqrt[4]{\frac{n^2+2n+2}{n^2+1}}-1\right)=$$

$$=\lim_{n\to\infty} n\cdot\sqrt[4]{\frac{n^2+2n+2}{n^2+1}}\left(3\sqrt[3]{\frac{\cancel{(n+1)}(n+1)(n+2)}{(3n+4)\underset{3}{\cancel{(3n+3)}}(3n+2)}}-1\right)+\lim n\left(\sqrt[4]{\frac{n^2+2n+2}{n^2+1}}-1\right)$$

$$=\lim_{n\to\infty} n\sqrt[4]{\frac{n^2+2n+2}{n^2+1}}\left(3\sqrt[3]{\frac{n^2+3n+2}{3\cdot(9n^2+18n+8)}}-1\right)+\lim_{n\to\infty} n\left(\sqrt[4]{1+\frac{2n+1}{n^2+1}}-1\right)$$

$\to 1$

$$=\lim_{n\to\infty} n\left(3\sqrt[3]{\frac{n^2+3n+2}{27n^2+54n+24}}-1\right)+\lim_{n\to\infty} n\left(\sqrt[4]{1+\frac{2n+1}{n^2+1}}-1\right)$$

$\cdot\frac{3^3}{3}$

$$=\lim_{n\to\infty} n\left(\frac{3}{3}\sqrt[3]{\frac{27n^2+81n+54}{27n^2+54n+24}}-1\right)+\lim_{n\to\infty}\frac{1}{4}n\left(\frac{2n+1}{n^2+2n+2}\right)=$$

$$=\lim_{n\to\infty} n\left(\sqrt[3]{1+\frac{27n+30}{27n^2+54n+24}}-1\right)+\frac{1}{4}\cdot(2)=$$

$$=\frac{1}{3}\cdot n\cdot\frac{27n+30}{27n^2+54n+24}+\frac{1}{2}=\frac{1}{3}+\frac{1}{2}=\frac{5}{6}\Rightarrow$$

според критерият на Раабе-Дюамел редът за
$x = R = \frac{1}{3}$ е разходящ, $\frac{5}{6} < 1$

Критерият Лайбниц за $x = -R = -\frac{1}{3}$ редът е сходящ, т.е.
е условно сходящ и $x \in \left[-\frac{1}{3}; \frac{1}{3}\right)$.