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Софтуерно инженерство, I курс, I група

ДР №10

(1) $$\int \frac{\cos x + 3\sin x - 2}{3\cos x + 5}\,dx = \int \frac{(3\cos x+5)\cdot\frac{1}{3} - \frac{5}{3} - 2 + 3\sin(x)}{3\cos x+5}\,dx =$$

$$= \frac{1}{3}\int \frac{3\cos x+5}{3\cos x+5}\,dx + \frac{11}{3}\int \frac{1}{3\cos x+5}\,dx + 3\int \frac{\sin x}{3\cos x+5}\,dx$$

A   B   C

$$A = \frac{x}{3} + C \;;$$

$$B = \int \frac{1}{3\cos x+5}\,dx = \int \frac{\frac{1}{\cos^2\frac{x}{2}}}{2\left(tg^2\frac{x}{2}+4\right)}\,dx \quad tg\,\frac{x}{2} = u \Rightarrow$$

$$\Rightarrow dx = 4\cos^2\frac{x}{2}\,du \Rightarrow \quad \frac{dx}{\cos^2 x_2} = 4du$$

$$= \int \frac{4}{2\cdot(4u^2+4)}\,du = \int \frac{2}{4u^2+4}\,du = \frac{1}{2}\int \frac{1}{u^2+1}\,du =$$

$$= \frac{arctg(u)}{2}$$

$$C = \int \frac{\sin x}{3\cos x+5}\,dx \Rightarrow \quad 3\cos x + 5 = u \Rightarrow \frac{du}{dx} = 3\sin x$$

$$dx\sin x = \frac{du}{3}$$

$$= \int \frac{\frac{1}{3}}{u}\,du = \frac{1}{3}\ln|u|$$

$$\Rightarrow \int \frac{\cos x + 3\sin x - 3}{3\cos x+5}\,dx = \frac{x}{3} + \frac{11}{6}\,arctg\left(tg\,\frac{x}{2}\right) + \ln|3x+5|$$

$$= 1 =$$

$$\sqrt{\frac{x-2}{x}} = t \Rightarrow$$

$$\Rightarrow \frac{x-2}{x} = t^2$$

$$1 - \frac{2}{x} = t^2$$

$$1 - t^2 = \frac{2}{x}$$

$$\frac{1}{x^2} = \frac{(1-t^2)^2}{4}$$

② $$\int \frac{\sqrt{x-2} - \sqrt{x}}{\sqrt{x-2} + 4\sqrt{x}}\,dx = \int \frac{\sqrt{\frac{x-2}{x}} - 1}{\sqrt{\frac{x-2}{x}} + 4}\,dx$$

$$d\sqrt{\frac{x-1}{x}} = \frac{1}{2}\sqrt{\frac{x}{x-1}} \cdot \frac{(x+2-x)}{x^2}\,dx =$$

$$= \sqrt{\frac{x}{x-1}}\,\frac{dx}{x^2} = \frac{dx\,(1-t^2)^2}{4t} = dt \Rightarrow dx = \frac{4t\,dt}{(1-t^2)^2}$$

$$\int \frac{\sqrt{\frac{x-2}{x}} - 1}{\sqrt{\frac{x-2}{x}} + 4}\,dx = \int \frac{t-1}{t+4} \cdot \frac{4t\,dt}{(1-t^2)^2} =$$

$$= \int \frac{4t(t-1)}{(t+4)(1-t)(1+t)(1-t)(1+t)}\,dx$$

$$\frac{t^2 - t}{(t+4)(t-1)(t+1)(t-1)(t+1)} = \frac{A}{t+4} + \frac{B}{t-1} + \frac{C}{(t-1)^2} + \frac{D}{t+1} + \frac{E}{(t+1)^2}$$

$t = -4 \Rightarrow 20 = A \cdot 25 \cdot 9 \Rightarrow A = \frac{4}{45}$

$t = -1 \Rightarrow 2 = E \cdot 3 \cdot 4 \Rightarrow E = \frac{1}{6}$

$t = 1 \Rightarrow 0 = C \cdot 20 = C = 0$

$t^4 = A + B + D = 0,\ \frac{4}{45} + B + D = 0 \quad D = -B - \frac{4}{45}$

$t^3 \Rightarrow 5B + 3D + C + E + C = 0 \Rightarrow 5B + 3\left(-B - \frac{4}{45}\right) + \frac{1}{6} = 0 \Rightarrow B = \frac{1}{20}$

$A + B + D = 0 \Rightarrow D = \frac{5}{36}$

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$$4\int\left(\frac{\frac{4}{45}}{t+4}+\frac{1}{20(t-1)}+\frac{5}{36(t+1)}+\frac{1}{6(t+1)^2}\right)dt$$

$$=\frac{16}{45}\int\frac{dt}{t+4}+\frac{4}{20}\int\frac{dt}{t-1}+\frac{25}{36}\int\frac{dt}{t+1}-\frac{4}{6}\int\frac{dt}{(t+1)^2}=$$

$$=\frac{16}{45}\cdot\ln\left|\frac{\sqrt{x-2}}{x}+4\right|+\frac{4}{20}\ln\left|\frac{\sqrt{x-2}}{x}-1\right|-\frac{25}{36}\ln\left|\frac{\sqrt{x-1}}{x}1\right|-\frac{4}{6}\left(\frac{\sqrt{x-2}+1}{x}\right)+C$$

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