Долин Долчев ОМI0600041

(1) $\displaystyle\int_{-\infty}^{+\infty} \frac{dx}{(x^2+4)^2\sqrt{x^2+5}} = 2\int_0^{+\infty} \frac{dx}{(x^2+4)^2\sqrt{x^2+5}} = \lim_{b\to\infty} \int^{b} \frac{dx}{(x^2+4)^2\sqrt{x^2+5}}$

$(x^2+4)^2$ → четна, $\sqrt{x^2+5}$ → четна

$2\lim\limits_{b\to\infty} F(x)\Big|_a^b$

$\displaystyle\int \frac{dx}{(x^2+4)^2\sqrt{x^2+5}}$ $\quad x=\sqrt{5}\,tg\,u,\quad u = arctg\left(\frac{x}{\sqrt5}\right),\quad dx = \frac{\sqrt5}{\cos^2 u}du$ $\displaystyle = \int \frac{\sqrt5\, du}{\cos^2 u\cdot\sqrt{5tg^2u+5}\,(5tg^2u+4)^2} =$

$\displaystyle = \int \frac{\sqrt5\,du}{\cos^2 u\,(5tg^2u+4)^2\cdot\frac{\sqrt5}{\cos u}} = \int\frac{du}{\cos u(5tg^2u+4)} = \int \frac{du}{\cos u\left(5\left(\frac{\sin^2 u}{1-\sin^2 u}\right)+4\right)^2} =$

$\displaystyle = \int \frac{du}{\cos u\left(\frac{5\sin^2 u+4-4\sin^2 u}{1-\sin^2 u}\right)^2} = \int\frac{du}{\cos u\left(\frac{4+\sin^2 u}{1-\sin^2 u}\right)^2} = \int \cos u\left(-\frac{\sin^2-1}{(\sin^2 u+4)^2}\right)du =$

$= t=\sin u,\quad \frac{dt}{du}=\cos u \quad \displaystyle = -\underbrace{\int\frac{t^2-1}{(t^2+4)^2}dt}_{A} = (1)$

$\displaystyle A = \int\frac{t^2-1}{(t^2+4)^2}dt = \int\frac{t^2\cancel{+4-4}+5}{(t^2+4)^2}dt = \int\frac{(t^2+4)}{(t^2+4)^2}-\frac{5}{(t^2+4)^2}dt =$

$\displaystyle = \int\frac{1}{t^2+4}dt + 5\int\frac{dt}{(t^2+4)^2} = I$

$\downarrow$

$y=\frac{t}{2},\quad \frac{dy}{dt}=\frac12 \quad \displaystyle = \int\frac{2}{4y^2+4}dy = \frac12\int\frac{1}{y^2+1}dy = \frac{arctg\frac{t}{2}}{2}$

$$I = \int \frac{1}{(t^2+4)^2}\,dt = \frac{t}{8(t^2+4)} + \frac{1}{8}\int \frac{1}{t^2+4}\,dt =$$

$$= \frac{t}{8(t^2+4)} + \frac{\operatorname{arctg}\frac{t}{2}}{16}$$

$$\Rightarrow A = \frac{3\operatorname{arctg}\frac{t}{2}}{16} - \frac{5v}{8(v^2+4)} = \frac{5\sin u}{8(\sin^2 u+4)} - \frac{3\operatorname{arctg}\left(\frac{\sin u}{2}\right)}{16} =$$

$$= \frac{\sqrt{5}x}{8\sqrt{\frac{x^2}{5}+1}\left(\frac{x^2}{5\left(\frac{x^2}{5}+1\right)}+4\right)} - \frac{3\operatorname{arctg}\left(\frac{2\sqrt{5}\sqrt{\frac{x}{5}+1}}{x}\right)}{16} =$$

$$\Rightarrow F(x) = 3\operatorname{arctg}\left(\frac{2\sqrt{x^2+5}}{x}\right) + \frac{x\sqrt{x^2+5}}{8(x^2+4)}$$

$$\lim_{a\to\infty} F(a) = \frac{3\operatorname{artg}(2)+2}{2} \qquad F(0) = \frac{3\pi}{32}$$

$$\Rightarrow 2\lim_{a\to\infty} F(x)\Big|_0^a = \frac{3\operatorname{arctg}2}{8} - \frac{3\pi}{16} + \frac{1}{4}$$