Филип Филчев

РР №6

ДИС 2

Нека $f(u,v)$ навс. има непрек. 2ри частни произв. и за $\forall$ точка $(u,v) \in \mathbb{R}^2$ е изп. $\frac{\partial^2 f}{\partial^2 u^2}(u,v) + \frac{\partial^2 f}{\partial^2 v^2}(u,v) = 0$

Д., $\forall$ т. $(x,y) \in \mathbb{R}^3$ с $x^2+y^2 > 0$ е изп. $\frac{\partial^2 F}{\partial^2 x^2}(x,y) + \frac{\partial^2 F}{\partial^2 y^2}(x,y) = 0$

където $F(x,y) = f\left(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2}\right)$

Дво

Iви произв.

1) $u'_x = \frac{x^2+y^2-2x^2}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$

2) $u'_y = \frac{-2xy}{(x^2+y^2)^2}$

3) $v'_x = \frac{-2xy}{x^2+y^2}$

4) $v'_y = \frac{x^2-y^2}{(x^2+y^2)^2}$

$\Rightarrow u'_y = v'_x$ и $u'_x = -v'_y$

IIри произв.

5) $u''_x = \frac{-2x(x^2+y^2)^2 - (y^2-x^2) \cdot 4x(x^2+y^2)}{(x^2+y^2)^4} = \frac{-2x(3y^2-x^2)}{(x^2+y^2)^3}$

6) $u''_{yy} = \frac{\partial v_x}{\partial y} = v''_{xy} = v''_{yx} = \frac{\partial v_y}{\partial x} = \frac{\partial(-v_x)}{\partial x} = -u''_{xx}$

7) Аналог. $v''_{xx} = u''_{yx} = u''_{xy} = -v''_{yy}$

$F(x,y) = f(u,v) \Rightarrow F'_x = f''_u \cdot u'_x + f'_y v'_y$

$\Rightarrow F''_{xx} = f''_{uu} \cdot (u'_y)^2 + f'_u u''_x + f''_{vv} (v'_x)^2 + f'_v v''_{yy}$

$F''_{xx} + F''_{yy} = f''_{uu}[(u'_x)^2 + (u'_y)^2] + f''_{vv}[(v'_x)^2 + (v'_y)^2] + f'_u(u''_{xx} + u''_{yy}) +$

$f^{\Phi}_{v}{}'(v''_{xx}+v''_{yy})$

$f'_u(u''_{xx}+u''_{yy})=0$ и $f'_u(v''_{xx}+v''_{yy})=0$

Поломсе $v'_x=u'_y$ и $v'_y=-u'_x \Rightarrow$

$\Rightarrow (v'_y)^2+(v'_x)^2=(u'_y)^2+(u'_x)^2 \Rightarrow F''_{xx}+F''_{yy}=\underbrace{(f''_{uu}+f''_{vv})}\cdot$

$[(u'_x)^2+(u'_y)^2]^{0}=0.$