Фалит Фолнев ОМI 0600041

(3)
$$\begin{cases} x^2+y^2 \geq 16 \\ x^{\frac{2}{3}}+y^{\frac{2}{3}} \leq 4 \end{cases}$$

Нека $x = r^2\cos^3\varphi$
$y = r^2\sin^3\varphi$

$$\Rightarrow \begin{cases} r^6(\sin^6\varphi+\cos^6\varphi)=16 \\ r^2 \leq 4 \Rightarrow r \leq 2 \end{cases} \Rightarrow r^6 = \frac{16}{\sin^6\varphi+\cos^6\varphi}$$

$$\Rightarrow \begin{cases} 0 \leq \varphi \leq 2\pi \\ \sqrt[6]{\frac{16}{\sin^6\varphi+\cos^6\varphi}} \leq r \leq 2 \end{cases}$$

$$\frac{\partial x}{\partial r} = 3r^2\cos^3\varphi \qquad \frac{\partial x}{\partial \varphi} = -3r^3\cos^2\varphi\sin\varphi$$

$$\frac{\partial y}{\partial r} = 3r^2\sin^3\varphi \qquad \frac{\partial y}{\partial \varphi} = 3r^3\sin^2\varphi\cos\varphi$$

$$\Rightarrow I = \begin{vmatrix} 3r^2\cos^3\varphi & -3r^3\cos^2\varphi\sin\varphi \\ 3r^2\sin^3\varphi & 3r^3\sin^2\varphi\cos\varphi \end{vmatrix}$$

$$= 9r^5\cos^2\varphi\sin^2\varphi$$

$$\Rightarrow \int_0^{2\pi}\int_{\sqrt[6]{\frac{16}{\sin^6\varphi+\cos^6\varphi}}}^{2} 9r^5\cos^2\sin^2\varphi\,dr\,d\varphi = 9\int_0^{2\pi}\cos^2\varphi\sin^2\varphi\,\frac{r^6}{6}\Bigg|_{\sqrt[6]{\frac{16}{\sin^6+\cos^6 x}}}^{2} d\varphi =$$

$$= \frac{9}{6}\int_0^{2\pi}\cos^2\varphi+\sin^2\varphi\left(2^6-\frac{16}{\sin^6\varphi+\cos^6\varphi}\right)d\varphi =$$

$$= \frac{3}{2}\cdot 2^5\int_0^{2\pi}\cos^2\varphi\sin^2\varphi\,d\varphi - 24\int_0^{2\pi}\frac{\cos^2\varphi+\sin^2\varphi}{(\sin\varphi+\cos\varphi)(\sin^2\varphi-\sin\varphi\cos\varphi)+\cos^2\varphi)}d\varphi =$$

$$= \frac{3}{2}\cdot 2^3\int_0^{2\pi}\sin^2 2\varphi\,d\varphi - 24\int_0^{2\pi}\frac{\sin^2\varphi+\cos^2\varphi}{(\sin\varphi+\cos\varphi)(1-\sin\varphi\cos\varphi)}d\varphi =$$

$$= 3\int_0^{2\pi}\frac{1-\cos 4\varphi}{2}d4\varphi = 6\left[\frac{\varphi-\sin\varphi}{2}\right]_0^{2\pi} = 3\pi$$