Домашна работа № 3
на Петър Парушев с ФН 61620, група 1, СИ

Задача 1.

А)

$$y = \ln \sqrt[2]{\frac{1-\sin x}{1+\sin x}}$$

$$y` = \frac{1}{\sqrt[2]{\frac{1-\sin x}{1+\sin x}}} \frac{1}{2\sqrt[2]{\frac{1-\sin x}{1+\sin x}}} \frac{(-\cos x(1+\sin x)-(1-\sin x)(+\cos x))}{(1+\sin x)^2} = \frac{(1+\sin x)(-2\cos x)}{2(1-\sin x)(1+\sin x)^2} =$$

$$= \frac{-\cos x}{(1-\sin x)(1+\sin x)} = \frac{-\cos x}{(1-\sin^2 x)} = \frac{-1}{\cos x}$$

Б)

$$y == \frac{x \arcsin x}{\sqrt{1-x^2}} + \ln\sqrt{1-x^2}$$

$$y` = \frac{((\arcsin x + \frac{x}{\sqrt{1-x^2}})\sqrt{1-x^2}) - \frac{x\arcsin x(-2x)}{2\sqrt{1-x^2}}}{1-x^2} + \frac{-2x}{(\sqrt{1-x^2})(-2\sqrt{1-x^2})} =$$

$$= \frac{(\frac{\arcsin x\sqrt{1-x^2}+x}{1-x^2}) + \frac{x^2\arcsin x}{\sqrt{1-x^2}}}{1-x^2} + \frac{x}{1-x^2} =$$

$$= \frac{(\frac{\arcsin x\sqrt{1-x^2}+x}{1-x^2}) + \frac{x^2\arcsin x\sqrt{1-x^2}}{1-x^2} + x}{1-x^2} =$$

$$= \frac{\arcsin x\sqrt{1-x^2} + x + x^2\arcsin x\sqrt{1-x^2} + x(1-x^2)}{(1-x^2)^2} =$$

$$= \frac{\arcsin x\sqrt{1-x^2}(1+x^2) + 2x - x^3}{(1-x^2)^2} =$$

$$= \frac{\arcsin x\sqrt{1-x^2}(1+x^2) + 2x - x^3}{(1-x)^2(1+x)^2}$$

В)

$$y = x^3 e^{\frac{1}{x}} + \ln\cos x$$

$$y` = 3x^2 e^{\frac{1}{x}} + x^3 e^{\frac{1}{x}}(-\frac{1}{x^2}) - \frac{\sin x}{\cos x} = 2xe^{\frac{1}{x}} - \frac{\sin x}{\cos x}$$

г)

$$y=\frac{\sqrt[7]{x^2}\,(x+1)^5}{\sqrt{(3x^2+2)^3}\,(4x+3)}$$

$$\ln y=\ln\frac{\sqrt[7]{x^2}\,(x+1)^5}{\sqrt{(3x^2+2)^3}\,(4x+3)}$$

$$\ln y=\ln\sqrt[7]{x^2}+\ln(x+1)^5-\ln\sqrt{(3x^2+2)^3}-\ln(4x+3)$$

$$\ln y=\frac{2}{7}\ln x+5\ln(x+1)-\frac{3}{2}\ln(3x^2+2)-\ln(4x+3)$$

$$\frac{1}{y}y'=\frac{2}{7}.\frac{1}{x}+\frac{5}{x+1}-\frac{3}{2}.\frac{6x}{3x^2+2}-\frac{4}{4x+3}$$

$$y'=\left(\frac{2}{7x}+\frac{5}{x+1}-\frac{9x}{3x^2+2}-\frac{4}{4x+3}\right).y$$

$$y'=\left(\frac{2}{7x}+\frac{5}{x+1}-\frac{9x}{3x^2+2}-\frac{4}{4x+3}\right).\frac{\sqrt[7]{x^2}\,(x+1)^5}{\sqrt{(3x^2+2)^3}\,(4x+3)}$$

$$y'=\frac{(x+1)^4(108x^4-168x^3+69x^2+182x+12)}{7\sqrt[7]{x^5}\,(4x+3)^2\sqrt{(3x^2+2)^5}}$$

д)

$$y=\left(arctg\,\frac{1}{x^2}\right)^{\frac{1}{x}}$$

$$\ln y=\frac{1}{x}\ln\left(arctg\,\frac{1}{x^2}\right)$$

$$\frac{1}{y}y'=-\frac{\ln\left(arctg\,\frac{1}{x^2}\right)}{x^2}-\left(\frac{1}{x}.\frac{1}{arctg\,\frac{1}{x^2}}.\frac{2}{x^3}.\frac{1}{1+\frac{1}{x^4}}\right)$$

$$\frac{1}{y}y^{'}=-\frac{\ln\left(arctg\,\frac{1}{x^2}\right)}{x^2}-\frac{2}{\left(x^4+1\right)arctg\,\frac{1}{x^2}}$$

$$y^{'}=\left(-\frac{\ln\left(arctg\,\frac{1}{x^2}\right)}{x^2}-\frac{2}{\left(x^4+1\right)arctg\,\frac{1}{x^2}}\right)\left(arctg\,\frac{1}{x^2}\right)^{\frac{1}{x}}$$

E)

$$y=(x\ln x)^{x\ln x}$$

$$t=x\ln x$$

$$y=t^{t}$$

$$y`=t.t^{t-1}t`=t^{t}t`=(x\ln x)^{x\ln x}(x\ln x)`=(x\ln x)^{x\ln x}(\ln x+\frac{x}{x})=(x\ln x)^{x\ln x}(\ln x+1)$$

Задача 2.

$$f(x)=\frac{x^3+Nx^2+1}{x^2+2x+N}$$

$$f`(x)=\frac{(3x^2+2Nx)(x^2+2x+N)-(x^2+2x+N)(2x+2)}{(x^2+2x+N)^2}$$

$$f`(0)=\frac{-2}{N^2}=\frac{-2}{61620^2}$$

Задача 4.

$$f(x)=x^2-(3x+4)\ln(5x+6)$$

$$f`(x)=2x-(3\ln(5x+6)+\frac{(3x+4)5}{5x+6})$$

$$f`(-1)=-2-(3\ln(1)+5)=-7-3\ln 1=-7$$

$$f(-1)=1-\ln 1=1$$

$$t: y=-7(x-1)-1=-7x-8$$

$$g(x)=x^2+(3x-5)arctg(4-2x)$$

$$g`(x)=2x+3arctg(4-2x)+\frac{3x+5}{(4-2x)^2+1}$$

$$g`(2)=4+3arctg0+11=15$$

$$g(2)=4$$

$$l: y=15(x-2)-4=15x-34$$

$$l=t=-7x-8=15x-34$$

$$22x=-26$$

$$x=\frac{-13}{11}$$

$$M=(\frac{-13}{11};\frac{13.7-88}{11})=(\frac{-13}{11};\frac{3}{11})$$