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Софтуерно инженерство, I курс, I група

Домашна работа №9

① $A = \int \frac{x^4 - 4x^3 + 5x^2 + 10x - 10}{x^3 - 3x^2 + x + 5} dx \Rightarrow$

$$\begin{array}{l|l} x^4 - 4x^3 + 5x^2 + 10x - 10 & : x^3 - 3x^2 + x + 5 \\ -\;x^4 - 3x^3 + x^2 + 5x & \\ \hline -x^3 + 4x^2 + 5x - 10 & x - 1 \\ -\;-x^3 + 3x^2 - x - 5 & \\ \hline x^2 + 6x - 5 & \end{array}$$

$\Rightarrow \int \left(\frac{x^2 + 6x - 5}{x^3 - 3x^2 + x + 5} + (x-1) \right) dx =$

$= \int \frac{x^2 + 6x - 5}{x^3 - 3x^2 + x + 5} dx + \int x\, dx - \int 1\, dx$

I $\int \frac{x^2 + 6x - 5}{x^3 - 3x^2 + x + 5} dx = \int \frac{x^2 + 6x - 5}{(x+1)(x^2 - 4x + 5)} dx = \int \left(\frac{2x}{x^2 - 4x + 5} - \frac{1}{(x+1)} \right) dx =$

$= 2 \cdot \int \frac{x}{x^2 - 4x + 5} dx - \int \frac{1}{x+1} dx$

1.1 $\int \frac{1}{x+1} dx = \ln(x+1)$

1.2. $2 \cdot \int \frac{x}{x^2 - 4x + 5} dx = 2 \cdot \int \frac{(2x - 4) + 4}{2 \cdot (x^2 - 4x + 5)} dx = 2 \cdot \underbrace{\int \frac{x - 2}{x^2 - 4x + 5} dx}_{1.3} + 4 \underbrace{\int \frac{1}{x^2 - 4x + 5} dx}_{1.4}$

1.3 $2 \cdot \int \frac{x - 2}{\underbrace{x^2 - 4x + 5}_{= u}} dx \Rightarrow$

$\frac{du}{dx} = 2x - 4 \Rightarrow \quad dx = \frac{du}{2x - 4}$

$\Rightarrow 2 \cdot \int \frac{\cancel{x-2}}{\cancel{2x-4}} \cdot du = \int du = \ln(x^2 - 4x + 5)$

1.4 $4 \int \frac{1}{x^2 - 4x + 5} dx = 4 \int \frac{1}{(\underbrace{x - 2}_{u})^2 + 1} dx = 4 \int \frac{1}{u^2 + 1} du =$

$(x-2)' = 1$

$\Rightarrow u = x$

$= 4 \operatorname{arctg}(u) =$

$= 4 \operatorname{arctg}(x - 2)$

$= 1 =$

$\Rightarrow$ 1.2 $= \ln(x^2-4x+5) + 4\,\mathrm{arctg}(x+2)$

$\Rightarrow I = \ln(x^2-4x+5) + 4\,\mathrm{arctg}(x-2) + \ln(x+1)$

II $\int x\,dx = \frac{x^2}{2}$ III $\int 1\,dx = x$

$\Rightarrow$ окончателно:

$$A = \ln(x^2-4x+5) + 4\,\mathrm{arctg}(x-2) + \ln(x+1) + \frac{x^2}{2} + x + C$$

$$\frac{x^2}{2} + x = \frac{x(x-2)}{2}$$

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$$② \int \frac{dx}{(x^4+4)^2} \qquad x^4+4 = x^4+4x^2+4-4x^2 =$$
$$= (x^2+2)^2-(2x)^2 = (x^2-2x+2)(x^2+2x+2)$$

$$\int \frac{dx}{(x^4+4)^2} = \frac{1}{4}\int \frac{x^4-x^4+4}{(x^4+4)^2}dx = \frac{1}{4}\left(\int \frac{dx}{x^4+4} - \int \frac{x^4}{(x^4+4)^2}dx\right) =$$

$$= \frac{1}{4}\left(\int \frac{dx}{x^4+4} - \frac{1}{4}\int \frac{x}{(x^4+4)^2}dx^4+4\right) =$$

$$= \frac{1}{4}\left(\int \frac{dx}{x^4+4} - \underline{\frac{1}{4}\int x\, d\frac{1}{x^4+4}}\right) = \qquad = \int g(x)\, d f(x) =$$
$$= g(x)f(x) - \int f(x)\, dgx$$

$$= \frac{1}{4}\left(\int \frac{dx}{x^4+4} + \frac{x}{4(x^4+4)} - \frac{1}{4}\int \frac{dx}{x^4+4}\right) =$$

$$= \frac{x}{16(x^4+4)} - \frac{3}{16}\int \frac{dx}{x^4+4} = *$$

$$=*$$

$$\frac{1}{x^4+4} = \frac{ax+b}{(x^2-2x+2)} + \frac{cx+d}{x^2+2x+2} = (ax+b)(x^2+2x+2)+(cx+d)(x^2-2x+2)$$

$$\begin{cases} a+c=0 & a=-c \\ 2a+b-2c+d=0 & d=-4a-b \\ 2a+2b+2c-2d=0 \;:2 & a+b+c-d=0 \\ 2b+2d=1. & b=\frac{1-2d}{2} \end{cases}$$

$$\Rightarrow d=\frac{1}{4} \quad b=\frac{1}{4} \quad c=\frac{1}{8} \quad a=-\frac{1}{8}$$

$$A = \int \frac{-\frac{1}{8}x+\frac{1}{4}}{x^2-2x+2}dx + \int \frac{\frac{1}{8}x+\frac{1}{4}}{x^2+2x+2}dx = \cancel{-\frac{1}{8}\left(\int \frac{x-2}{x^2-2x+2}dx + \int \frac{x+2}{x^2+2x+2}dx\right)} =$$

$$= -\frac{1}{16}\left(\int \frac{2x-4}{x^2-2x+2} + \int \frac{2x+4}{x^2+2x+2}\right) = \qquad = 3 =$$

$$= -\frac{1}{16}\left(\int\frac{d\,x^2+2x+2}{x^2+2x+2} - 2\int\frac{dx}{\underbrace{x^2+2x+2}_{(x+1)^2+1}} + \int\frac{d\,x^2-2x+2}{x^2+2x+2} - 2\int\frac{dx}{\underbrace{x^2-2x+2}_{(x-1)^2-1}}\right) =$$

$$= -\frac{1}{16}\left(\ln|x^2+2x+2| + \ln|x^2+2x+2| + arctg(x-1) + arctg(x+1)\right)$$

цялото $\Rightarrow$ *

$$\Rightarrow \frac{1}{16}\frac{x}{x^4+4} - \frac{3}{256}\left(\ln|x^2+2x+2| + arctg(x+1) + \ln|x^2-2x+2| + arctg(x-1)\right)$$

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