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Домашна работа 3

Задача 1.

а) $\sum_{n=0}^{\infty}(-1)^n \frac{1}{\sqrt{n}} arctg \frac{1}{2\sqrt{n}}$

$\lim_{n\to\infty}\frac{1}{\sqrt{n}} arctg \frac{1}{2\sqrt{n}} \to \frac{1}{\sqrt{n}}\frac{1}{2\sqrt{n}} \to \frac{1}{2n} \Rightarrow$ условно сходящ

б) $\sum_{n=0}^{\infty}(-1)^n \sin\frac{n^2+(-1)^n}{2n^2+5n+2011}$

$\lim_{n\to\infty}\frac{n^2+(-1)^n}{2n^2+5n+2011} \to \frac{1}{2} \Rightarrow \textit{разходящ}$

в) $\sum_{n=0}^{\infty}(-1)^n \frac{\sqrt{n+1}-\sqrt{n-1}}{\sqrt{n+69}}$

$\lim_{n\to\infty}\frac{\sqrt{n+1}-\sqrt{n-1}}{\sqrt{n+69}} \to 0$

$$\lim_{n\to\infty} n\left(\frac{a_n}{a_{n+1}}-1\right) \to n\left(\frac{\frac{\sqrt{n+1}-\sqrt{n-1}}{\sqrt{n+69}}-\frac{\sqrt{n+2}-\sqrt{n}}{\sqrt{n+70}}}{\frac{\sqrt{n+2}-\sqrt{n}}{\sqrt{n+70}}}\right) \to 0 \Rightarrow$$

$\Rightarrow$ условно сходящ

в) $\sum_{n=0}^{\infty}\binom{e-\pi}{2n}$ $\qquad e-\pi=\alpha$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| \to \left(\frac{(\alpha-2n-1)(\alpha-2n)}{(2n+1)(2n+2)}\right) \to 1$$

$$\lim_{n\to\infty} n\left(\frac{a_n}{a_{n+1}}-1\right) \to n\left(\frac{(2n+1)(2n+2)-(\alpha-2n-1)(\alpha-2n)}{(\alpha-2n-1)(\alpha-2n)}\right) \to \infty \Rightarrow$$

$\Rightarrow$ абсолютно сходящ

д) $\sum_{n=0}^{\infty}\binom{e-\pi}{3n}$ $\qquad e-\pi=\alpha$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| \to \left(\frac{(\alpha-3n-2)(\alpha-3n-1)(\alpha-3n)}{(3n+1)(3n+2)(3n+3)}\right) \to 1$$

$$\lim_{n\to\infty} n\left(\frac{a_n}{a_{n+1}}-1\right) \to n\left(\frac{(3n+1)(3n+2)(3n+3)-(\alpha-3n-2)(\alpha-3n-1)(\alpha-3n)}{(\alpha-3n-2)(\alpha-3n-1)(\alpha-3n)}\right) \to \infty \Rightarrow$$

$\Rightarrow$ абсолютно сходящ

Задача 2.

а) $\sum_{n=0}^{\infty}(\frac{(3n)!}{n!(2n+1)!}(x-3)^n$ $\qquad x-3=y$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| \to \frac{(3n+3)(3n+2)(3n+1)}{(n+1)(2n+3)(2n+2)}y \to \frac{27}{4}y \Rightarrow R=\frac{4}{27}$$

Интервалът при y = x – 3:

$$\left(-\frac{77}{27};\frac{85}{27}\right)$$

и редът е разходящ в двата си края на интервала

б) $\sum_{n=0}^{\infty}\frac{x^n}{n.5^n}\sqrt{\binom{3n+1}{n+1}}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| \to \frac{n}{(n+1)5}\sqrt{\frac{(3n+4)(3n+3)(3n+2)}{(n+2)(2n+1)(2n+2)}} \to \frac{3\sqrt{3}}{10}|x| \Rightarrow R=\frac{10}{3\sqrt{3}}$$

Интервалът е:

$$\left[-\frac{10}{3\sqrt{3}};\frac{10}{3\sqrt{3}}\right)$$

като в лявата част на интервала е условно сходящ, а в дясната – сходящ

Задача 3.

а) $arctg\dfrac{1+x}{1-x}$

$$f(0)=\frac{\pi}{4}$$
$$f'(x)=-\frac{1}{x^2+1}$$
$$f''(x)=\frac{2x}{\left(x^2+1\right)^2}$$
$$f'''(x)=-\frac{2-6x}{\left(x^2+1\right)^3}$$

$$arctg\frac{1+x}{1-x}=\frac{\pi}{4}-\frac{1}{x^2+1}+\frac{2x}{\left(x^2+1\right)^2}-\frac{2-6x}{\left(x^2+1\right)^3}+\ldots$$

б) $\ln\dfrac{2x+1}{x^2-4x+4}=\ln(2x+1)-\ln\left(x^2-4x+4\right)$

$$\ln(2x+1)=2x-\frac{(2x)^2}{2}+\frac{(2x)^3}{3}-\ldots+(-1)^n\frac{(2x)^n}{n}$$

$$\ln\left(x^2-4x+4\right)=1+\frac{2}{(x-2)}-0+\frac{2}{(x-2)^2}-\frac{4}{(x-2)^4}+\ldots+(-1)^n\frac{2(2n-2)}{(x-2)^n}$$

$$\ln\frac{2x+1}{x^2-4x+4}=\sum_{n=0}^{\infty}(-1)^n\left(\frac{(2x)^n}{n}-\frac{2(2n-2)}{(x-2)^n}\right)$$

в) $\dfrac{1}{x^2+x+1}=\dfrac{1-x}{(1-x)\left(x^2+x+1\right)}=\dfrac{1-x}{1-x^3}=1-x+x^3-x^4+x^6-x^7+\ldots=\sum\limits_{n=0}^{\infty}x^{3n}-x^{3n+1}$

Задача 4.

а) $\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}=\cosh x$

б) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{4n+1}}{(2n)!}=x\sum_{n=0}^{\infty}(-1)^n\frac{(x^2)^{2n}}{(2n)!}=x\cos^2 x$

в) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{3n+2}}{3n+1}=x\sum_{n=0}^{\infty}(-1)^n\frac{x^{3n+1}}{3n+1}=x\int\sum_{n=0}^{\infty}(-1)^n x^{3n}dx=x\int\frac{1}{1+x^3}dx$

$$\int\frac{1}{1+x^3}dx=\frac{A}{1+x}+\frac{Bx+C}{(x^2-x+1)};$$

$$\ldots A=\frac{1}{3},B=-\frac{1}{3},C=\frac{2}{3}$$

$$\Rightarrow\int\frac{1}{1+x^3}dx=\frac{1}{3}\ln|x+1|-\frac{1}{6}\ln|x^2-x+1|+\frac{\sqrt{3}}{3}arctg\left(\frac{2x-1}{\sqrt{3}}\right)$$

$$\sum_{n=0}^{\infty}(-1)^n\frac{x^{3n+2}}{3n+1}=x\left(\frac{1}{3}\ln|x+1|-\frac{1}{6}\ln|x^2-x+1|+\frac{\sqrt{3}}{3}arctg\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Задача 5.

$$\lim_{x\to 0}\frac{e^{arctgx}-e^{\sin x}}{\ln\sqrt{\frac{1+x}{1-x}}-arctg(\sin x)}$$

$$e^{arctgx}=1+x-\frac{x^3}{3}+o(x^4)$$

$$e^{\sin x}=1+x-\frac{x^3}{3!}+o(x^4)$$

$$\ln\sqrt{\frac{1+x}{1-x}}=x+\frac{x^3}{3}+o(x^4)$$

$$arctg(\sin x)=x-\frac{x^3}{2}+o(x^4)$$

$$\Rightarrow\lim_{x\to 0}\frac{e^{arctgx}-e^{\sin x}}{\ln\sqrt{\frac{1+x}{1-x}}-arctg(\sin x)}\to\frac{\frac{-x^3}{6}}{\frac{5x^3}{6}}\to-\frac{1}{5}$$