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Домашна работа №2

Зад.1:

а) $\binom{11}{3} = \frac{11.10.9}{3.2.1} = \ 165;$

б) $\binom{-3}{5} = \frac{(-3).(-4).(-5).(-6).(-7)}{5.4.3.2.1} = \ -21;$

в) $\binom{1/2}{7} = \frac{\frac{1}{2}.\left(-\frac{1}{2}\right).\left(-\frac{3}{2}\right).\left(-\frac{5}{2}\right).\left(-\frac{7}{2}\right).\left(-\frac{9}{2}\right).\left(-\frac{11}{2}\right)}{7.6.5.4.3.2.1} = \ \frac{33}{2048};$

г) $\binom{-1/3}{5} = \frac{\left(-\frac{1}{3}\right).\left(-\frac{4}{3}\right).\left(-\frac{7}{3}\right).\left(-\frac{10}{3}\right).\left(-\frac{13}{3}\right)}{5.4.3.2.1} = -\frac{91}{729};$

Зад.2:

а) $\binom{2n}{n} = (-4)^n\binom{-1/2}{n}$

Решение: $\binom{2n}{n} = \frac{(2n)!}{(n!)^2};$

$(-4)^n\binom{-\frac{1}{2}}{n} = \frac{(-4)^n\left(\left(-\frac{1}{2}\right).\left(-\frac{1}{2}-1\right)\dots\left(-\frac{1}{2}-n+1\right)\right)}{n!} = (-1)^n.(-1)^n.2^{2n}.\frac{1.3.5\dots(2n-1)}{2^n.(n!)} = \frac{(2n-1)!!(2^n.n!)}{n!.n!} = \frac{(2n-1)!!(2n)!!}{n!.n!} = \frac{(2n)!}{(n!)^2};$

б) $k\binom{\propto}{k} = \propto\binom{\propto-1}{k-1}, \propto \in R$

Решение: $k\binom{\propto}{k} = \frac{k.\propto.(\propto-1)\dots(\propto-k+1)}{k!} = \propto.\frac{(\propto-1)\dots((\propto-1)-(k-1)+1)}{(k-1)!} = \propto\binom{\propto-1}{k-1};$

Зад.3:

$$\left(1+x+\frac{1}{x^2}+\frac{1}{x^3}\right)^6 = \sum_{n=1}^{6}\binom{6}{n}(1+x)^n(x^{-2}+x^{-3})^{6-n}$$

$$= \sum_{n=0}^{6}\binom{6}{n}\left(\sum_{k=0}^{n}\binom{n}{k}1^{(n-k)}x^k\right)\left(\sum_{m=0}^{6-n}\binom{6-n}{m}x^{-3m}x^{(-2)(6-n-m)}\right)$$

$$= \left(\sum_{n=0}^{6}\ \sum_{k=0}^{n}\ \sum_{m=0}^{6-n}\binom{6}{n}\binom{n}{k}\binom{6-n}{m}x^{2n+k-m-12}\right)$$

Така получаваме системата: $2n+k-m = 12 \ \cup 0 \le n \le 6 \ \cup 0 \le k \le n \cup 0 \le m \le 6-n$

За n, m и $k \in N$. Трябва да проверим за $n = 1,2,\dots,6$.

1) $n = 0, k = 0, m = -12 \notin N$;

2) $n = 1, k = 0, m = -10 \notin N$

$k = 1, m = -9 \notin N$;

3) $n = 2, k = 0, m \notin N$

$k = 1, m \notin N$

$k = 2, m \notin N$;

4) $n = 3, k = 0,1,2,3, m \notin N$;

5) $n = 4, k = 4, m = 0$ е решение

6) $n = 5, k = 2, m = 0$ е решение

$n = 5, k = 3, m = 1$ е решение

7) $n = 6, k = m = 0$ е решение

Отговор: $\binom{6}{4}\binom{4}{4}\binom{2}{0} + \binom{6}{5}\binom{5}{2}\binom{1}{0} + \binom{6}{5}\binom{5}{3}\binom{1}{1} + \binom{6}{6}\binom{6}{0}\binom{0}{0} = 136$;

Зад.4:

$$S = \binom{n}{0} + 2\binom{n}{1} + 3\binom{n}{2} + \cdots + (k+1)\binom{n}{k} + \cdots + (n+1)\binom{n}{n} = \sum_{k=0}^{n}(k+1)\binom{n}{k}$$

$$= \sum_{k=0}^{n}\binom{n}{k} + \sum_{k=0}^{n} k\binom{n}{k} = \sum_{k=0}^{n}\binom{n}{k}1^k.1^{n-k} + \sum_{k=1}^{n} k\binom{n}{k}$$

$$= 2^n + \sum_{k=1}^{n} n\binom{n-1}{k-1} (\text{от Зад. 2 б})) = 2^n + n\sum_{k=1}^{n}\binom{n-1}{k-1} = 2^n + n.2^{n-1}$$

Зад.5:

$$S = \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \cdots + \frac{1}{k+1}\binom{n}{k} + \cdots + \frac{1}{n+1}\binom{n}{n} = \sum_{k=1}^{n}\frac{1}{k+1}\binom{n}{k}$$

$$= \left(\sum_{k=0}^{n}\frac{1}{k+1}\binom{n}{k}\right) - 1 = -1 + \sum_{k=0}^{n}\frac{n(n-1)\ldots(n-k+1)}{(k+1)!}$$

$$= -1 + \frac{1}{n+1}\left(-1 + \sum_{k=-1}^{n}\binom{n+1}{k+1}\right) = -1 - \frac{1}{n+1} + \frac{2^n}{n+1}$$