Задача 1

А) $\arcsin\left(\sin\frac{61518\Pi}{7}\right)=\frac{61518\Pi}{7}=\frac{2\Pi}{7}$

Б) $\arccos\left(\cos\frac{61518\Pi}{5}\right)=\frac{61518\Pi}{5}=\frac{2\Pi}{5}$

Задача 2

А) $\sin\left(arctg\frac{4}{3}\right)-\cos\left(arc\cot g\frac{12}{5}\right)$

$arctg\frac{4}{3}=\sin\alpha \quad \mathrm{arc}\cot g\frac{12}{5}=\cos\beta$

$\alpha: tg\alpha=\frac{4}{3} => \sin\alpha=\frac{4}{5} \qquad \beta:\cot g\beta=\frac{12}{5} => \cos\beta=\frac{12}{13}$

$=> \sin\alpha-\cos\beta=\frac{4}{5}-\frac{12}{13}=-\frac{8}{65}$

Б) $arc\cot g\pi+\arccos\left(-\frac{1}{2}\right)-arctg(-\pi)$

$arc\cot g\pi+arctg\pi+\arccos\left(-\frac{1}{2}\right)$

$\alpha: arc\cot g\pi=\alpha \qquad \beta: arctg\pi=\beta$

$\cot g\alpha=\pi \qquad tg\beta=\pi$

$=> \cot g\alpha=tg\beta \quad => \alpha+\beta=\frac{\pi}{2}$

$=> arc\cot g\pi+arctg\pi=\frac{\pi}{2}$

$=> arc\cot g\pi+arctg\pi+\arccos\left(-\frac{1}{2}\right)=\frac{\pi}{2}+\frac{2\pi}{3}=\frac{7\pi}{6}$

В) $\sin\ \ 2arctg\sqrt{7}\ \ -\cos\ \ 2arctg\sqrt{15}$

$$\alpha : tg\alpha = \sqrt{7} \Rightarrow \sin\left(2arctg\sqrt{7}\right) = \sin 2\alpha \qquad \sin\alpha = \frac{\sqrt{7}}{2\sqrt{2}} \Rightarrow \sin 2\alpha = \frac{\sqrt{7}}{4}$$

$$\beta : tg\beta = \sqrt{15} \Rightarrow \cos\left(2arctg\sqrt{15}\right) = \cos 2\beta \qquad \cos\beta = \frac{1}{4} \Rightarrow \cos 2\beta = -\frac{7}{8}$$

$$\Rightarrow \sin\left(2arctg\sqrt{7}\right) - \cos\left(2arctg\sqrt{15}\right) = \sin 2\alpha - \cos 2\beta = \frac{2\sqrt{7}+7}{8}$$

Задача 3

$$\arccos x = \operatorname{arctg} x, x \in \left[-1, 1\right]$$

За $x \le 0$ няма решение , защото са с различни знаци

За $x > 0$

$$\arccos x = \operatorname{arctg} x$$

$$\cos(\arccos x) = \cos(\operatorname{arctg} x)$$

$$x = \sqrt{\frac{1}{x^2+1}}$$

$$x^2 = \frac{1}{x^2+1} \qquad \Rightarrow x = \sqrt{\frac{\sqrt{5}-1}{2}}$$

Задача 5

$$S = tg\left(arctg\frac{1}{2} + arctg\frac{1}{8} + \ldots + arctg\frac{1}{2n^2}\right)$$

1) $База : n = 1 \qquad tg\left(arctg\frac{1}{2}\right) = \frac{1}{2}$

$$n = 2 \quad tg\left(arctg\frac{1}{2} + arctg\frac{1}{8}\right) = tg\left(arctg\frac{2}{3}\right) = \frac{2}{3}$$

$n=3\quad tg\left(arctg\frac{1}{2}+arctg\frac{1}{8}+arctg\frac{1}{18}\right)=tg\left(arctg\frac{3}{4}\right)=\frac{3}{4}$

2) Допускаме , че е вярно за $n=k=>S=\frac{k}{k+1}$

$$=>S_1=tg\left(S+arctg\frac{1}{2\;\;k+1\;^{2}}\right)=tg\left(arctg\frac{k}{k+1}+arctg\frac{1}{2\;\;k+1\;^{2}}\right)=tg\left(arctg\frac{k+1}{k+2}\right)=\frac{k+1}{k+2}$$

⇨ Е вярно за $n+1$ => е вярно за всяко n

=> $S=\frac{n}{n+1}$

Задача 6

А) $\lim\frac{\sqrt{n-1}-\sqrt{n+2}}{\sqrt{n+4}-\sqrt{n+3}}$

$$\lim\frac{\sqrt{n-1}-\sqrt{n+2}}{\sqrt{n+4}-\sqrt{n+3}}.\frac{\sqrt{n-1}+\sqrt{n+2}}{\sqrt{n-1}+\sqrt{n+2}}.\frac{\sqrt{n+4}+\sqrt{n+3}}{\sqrt{n+4}+\sqrt{n+3}}->-3\frac{\sqrt{n+4}+\sqrt{n+3}}{\sqrt{n-1}+\sqrt{n+2}}->$$

$$->-3\frac{\sqrt{n\left(1+\frac{4}{n}\right)}+\sqrt{n\left(1+\frac{3}{n}\right)}}{\sqrt{n\left(1-\frac{1}{n}\right)}+\sqrt{n\left(1+\frac{2}{n}\right)}}->-3\frac{\sqrt{n}\left(\sqrt{1+\frac{4}{n}}+\sqrt{1+\frac{3}{n}}\right)}{\sqrt{n}\left(\sqrt{1-\frac{1}{n}}+\sqrt{1+\frac{2}{n}}\right)}=-3.\frac{2}{2}=-3$$

Б) $\lim\frac{\sqrt{x^3-x+16}-\sqrt{8-x}}{x^2+8x+12}$

$$\lim\frac{\sqrt{x^3-x+16}-\sqrt{8-x}}{x^2+8x+12}.\frac{\sqrt{x^3-x+16}+\sqrt{8-x}}{\sqrt{x^3-x+16}+\sqrt{8-x}}->\frac{x^3-8}{x-2\quad x+6\quad\sqrt{x^3-x+16}+\sqrt{8-x}}$$

$$->\frac{x^2-x+16}{x+6\quad\sqrt{x^3-x+16}+\sqrt{8-x}}->\frac{11}{2\;\;\sqrt{26}+\sqrt{10}}$$

В) $\lim\left(\frac{n^2-3n+2}{n^2+3n+2}\right)^n$

$$->\left(\frac{n^2+3n+2}{n^2+3n+2}+\frac{-6n}{n^2+3n+2}\right)^n->\left(1+\frac{-6n}{n^2+3n+2}\right)^{n\left(\frac{-6n}{n^2+3n+2}\right)\left(\frac{n^2+3n+2}{-6n}\right)}->e^{\left(\frac{-6n}{n^2+3n+2}\right)}->e^{-6}$$

г) $\lim\left(\frac{1}{x-3}-\frac{27}{x^3-27}\right)$

$$->\frac{1}{x-3}-\frac{27}{x-3\quad x^2+3x+9}->\frac{x^2+3x+18}{x-3\quad x^2+3x+9}->\frac{x+6\quad x-3}{x-3\quad x^2+3x+9}->\frac{1}{3}$$

д)

$$\lim_{n\to\infty}\frac{1}{n^3}\lim_{x\to0}\frac{1-\cos x\cos 2x\ldots\cos nx}{x^2}\to\frac{1-(1-\frac{x^2}{2})(1-\frac{4x^2}{2})\ldots(1-\frac{n^2x^2}{2})}{n^3x^2}\to$$

$$\frac{(1+4+9+\ldots+n^2)\frac{x^2}{2}+A\frac{x^4}{2}}{n^3x^2}\to\frac{n(n+1)(2n+1)}{12n^3}\to\frac{2n^3+3n^2+n}{12n^3}\to\frac{1}{6}$$