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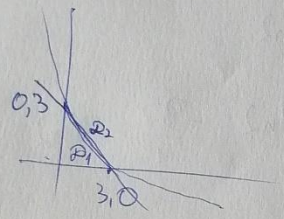
Задача 11. $0 \le z \le xy \quad x+y+z \le 3 \quad 0 \le x \quad 0 \le y$

$0 \le z \le 3-x-y \quad x+y \le 3$

$xy \le 3-x-y \; ; \; x \le \frac{3-y}{1+y} = \frac{4}{1+y} - 1$

$x' = \frac{-4}{(1+y)^2} \Rightarrow$ намаляваща; $x \ge 0, y \ge 0$ I квадрант



$D_1 \left| \begin{array}{l} 0 \le x \le 3 \\ 0 \le y \le \frac{4}{1+x} - 1 \end{array} \right. \quad D_2 \left| \begin{array}{l} 0 \le x \le 3 \\ \frac{4}{1+x} - 1 \le y \le 3-x \end{array} \right.$

$$\Rightarrow \underbrace{\iint_{D_1} xy\,dx\,dy}_{S_1} + \underbrace{\iint_{D_2} (3-x-y)\,dx\,dy}_{S_2}$$

$$S_1 = \int_0^3 \int_0^{\frac{4}{1+x}-1} xy\,dx\,dy = \frac{1}{2}\int_0^3 x\left(\frac{3-x}{1+x}\right)^2 dx = \frac{1}{2}\int_0^3 \frac{9x-6x^2+x^3}{1+2x+x^3}\,dx$$

$$\Rightarrow \frac{1}{2}\int_0^3 x-8\,dx + 4\int_0^3 \frac{x+1}{(x+1)^2} = \frac{1}{4}(x-8)^2\Big|_0^3 + 4\ln|x+1|\Big|_0^3 - 8\frac{1}{x+1}\Big|_0^3 =$$

$x^3-6x^2+9x : x^3+2x+1 = x-8$

ост. $8(3x+1)$

$$= \boxed{\frac{55}{4} + 8\ln 2 + 6}\, S_1$$

$$S_2 = \int_0^3 \int_{\frac{4}{1+x}-1}^{3-x} 3-x-y\,dy\,dx = \int_0^3 (3-x)y - \frac{y^2}{2}\Big|_{\frac{3-x}{1+x}}^{3-x} = \frac{1}{2}\int_0^3 (3-x)^2\,d(3-x) - \int_0^3 \frac{(3-x)^2}{1+x}\left(1-\frac{1}{2(1+x)}\right)dx =$$

$$= -\frac{1}{2}\frac{(3-x)^3}{3}\Big|_0^3 - \int_0^3 \frac{(3-x)^2}{1+x} + \frac{1}{2}\int_0^3 \frac{9-6x+x^2}{1+2x+x^2} = \frac{9}{2} + A_1 + A_2$$

A_1 $\quad$ A_2

$$A_1 = \int_0^3 x-7\, d(x-7) + \int_0^3 \frac{16}{(x+1)^2} d(x+1) = \frac{(x-7)^2}{2}\Big|_0^3 + \frac{16}{(x+1)}\Big|_0^3 =$$

$$= 16 - 49 + 12 = -21$$

// $x^2-6x+9 : x+1 = x+2$

ост. 16

$$A_2 = \int_0^3 \frac{x^2-6x+9}{x^2+2x+1} dx = \int_0^3 1\,dx - 8\int_0^3 \frac{x+1}{(x+1)^2} dx = 3 - 8\int_0^3 \frac{1}{x+1} d(x+1)$$

$$+16\int_0^3 \frac{1}{(x+1)^2} d(x+1) =$$

$$= 3 - 8\ln|x+1|\Big|_0^3 - 16\left(\frac{1}{x+1}\right)\Big|_0^3 = 15 - 16\ln 2$$

$$\Rightarrow S_1 + S_2 = -\frac{55}{4} + 8\ln 2 + 6 + \frac{9}{2} - 21 + 15 - 16\ln 2 = -\frac{37}{4} - 8\ln 2$$