Филип Красимиров Файгев фон: ОМІ0600041
Софтуерно инженерство, I курс, I група

Домашна работа №2

Докажете:

a) За $x+y>1$:
$$\text{arcctg}\, x + \text{arcctg}\, y = \text{arcctg}\, \frac{xy-1}{x+y}$$

б) $\text{arcctg}\, 2 + \text{arcctg}\, 8 + \ldots + \text{arcctg}\, 2n^2 = \text{arcctg}\, \frac{n+1}{n}$

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a) $\text{arcctg}\, x + \text{arcctg}\, y = \text{arcctg}\, \frac{xy-1}{x+y}$

$\left.\begin{array}{l} \text{arcctg}\, x = \alpha \implies \text{ctg}\,\alpha = x \\ \text{arcctg}\, y = \beta \implies \text{ctg}\,\beta = y \end{array}\right\}$ 1)

$$\text{arcctg}\, \frac{xy-1}{x+y} = \text{arcctg}\, \frac{\text{ctg}\,\alpha \cdot \text{ctg}\,\beta - 1}{\text{ctg}\,\alpha + \text{ctg}\,\beta} =$$

$$= \text{arcctg}\,(\text{ctg}\,(\alpha+\beta)) = \alpha + \beta = \text{arcctg}\, x + \text{arcctg}\, y$$
$$\text{от 1)} \qquad\qquad \sqrt{} \text{ за } x+y>0 \text{ и}$$
$$x+y>1$$

$\underline{\text{Зада е в сила уравнението трябва}}\ x+y\neq 0$
Ако $x+y>0$, то $\frac{x\cdot y-1}{x+y}$ е изпълнено

$$=1=\qquad\qquad \text{фон: ОМІ0600041}$$

б) $\operatorname{arcctg}2 + \operatorname{arcctg}8 + \ldots + \operatorname{arcctg}2n^2 = \operatorname{arcctg}\dfrac{n+1}{n}$

За $n=1 \Rightarrow \operatorname{arcctg}2 = \operatorname{arcctg}2$ __Вярно е__

Метод на мат. индукция:

Допускаме, че е изпълнено за $n \Rightarrow$

$\Rightarrow \operatorname{arcctg}2 + \operatorname{arcctg}8 + \ldots + \operatorname{arcctg}2n^2 = \operatorname{arcctg}\dfrac{n+1}{n}$

Сега иде докажем, че е варно за $n+1 \Rightarrow$
имаме:

$\Rightarrow \underbrace{\operatorname{arcctg}2 + \operatorname{arcctg}8 + \ldots + \operatorname{arcctg}2n^2} + \operatorname{arcctg}2\cdot(n+1)^2 =$

$$= \operatorname{arcctg}\dfrac{n+1}{n} \Rightarrow$$

$\Rightarrow$ получаваме:

$$\operatorname{arcctg}\dfrac{n+1}{n} + \operatorname{arcctg}2\cdot(n+1)^2 =$$

$$= \text{от а)} \Rightarrow \operatorname{arcctg}\dfrac{\dfrac{n+1}{n}\cdot 2(n+1)^2 - 1}{\dfrac{n+1}{n} + 2(n+1)^2} =$$

$$= \operatorname{arcctg}\dfrac{(2n+2)(n^2+2n+1) - n}{n+1 + 2n^3 + 4n^2 + 2n} = \operatorname{arcctg}$$

$$= \operatorname{arcctg}\dfrac{}{2n^3 + 4n^3 + 3n + 1}$$

$$= 2 =$$

$$= \text{arcctg} \, \frac{\dfrac{2(n+1)^3 - n}{n}}{\dfrac{n+1+2n(n^2+2n+1)}{n}} = \text{arcctg} \, \frac{2(n^3+3n^2+3n+1) - n}{n+1+2n^3+4n^2+2n} =$$

$$= \text{arcctg} \, \frac{2n^3+6n^2+5n+2}{2n^3+4n^2+3n+1} = \text{arcctg} \, \frac{(n+2)(2n^2+2n+1)}{(n+1)(2n^2+2n+1)} =$$

$$= \text{arcctg} \, \frac{n+2}{n+1} \implies \text{изпълнено за } \underline{n+1}$$

$$\implies \text{вярно за } \forall n$$