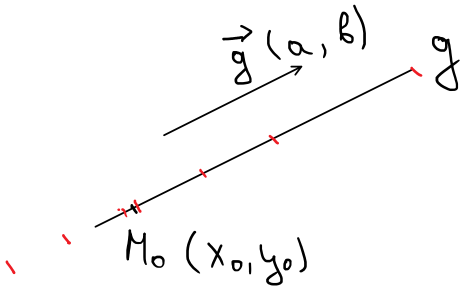
# Уравнения на права в равнината

## I Параметрични уравнения




$g \ni M_0(x_0, y_0)$

$g \parallel \vec{g}(a, b)$

$M(x, y)$ - произволна точка от $g$

$$g: \begin{cases} x = x_0 + \lambda \cdot a \\ y = y_0 + \lambda \cdot b \end{cases}, \lambda \in \mathbb{R}$$

$M \Longleftrightarrow \lambda$ - взаимно-еднозначно съответствие

## II Общо уравнение на права в равнината

$g: Ax + By + C = 0$

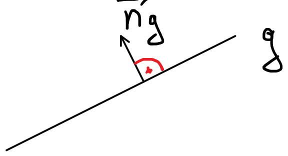
$(A, B) \neq (0, 0)$

Условие за колинеарност:

$g \parallel \vec{a}(a_1, a_2) \Longleftrightarrow A.a_1 + B.a_2 = 0$

$g \parallel \vec{g}(-B, A)$ : $A.(-B) + B.A = 0$

Спрямо ОКС: $g \perp \vec{n}_g(A, B)$ - нормален вектор на правата $g$




① ОКС $K = Oxy$

т. $A(1, -2)$; т. $B(0, -1)$

Да се намерят:

а) Уравнение на $AB$

т. $A(1,-2)$; т. $B(0,-1)$

$a: 3x+4y+2=0$

$b: 5x-12y+1=0$

$g: x+y-1=0$

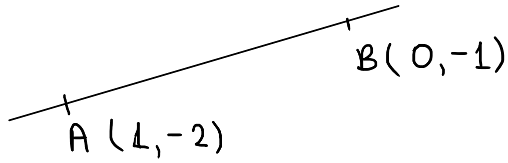
а) Уравнение на $AB$

б) у-е на $p: \begin{cases} \parallel a \\ \ni A \end{cases}$

в) у-е на $m: \begin{cases} \ni B \\ \perp b \end{cases}$

г) Координатите на т. $B' = \sigma_g(B)$

а)




т. $M(x,y)$ − произволна точка от $AB$

т. $A$, т. $B$, т. $M$ са колинеарни $\Rightarrow$ $AB: \begin{vmatrix} x & y & 1 \\ 1 & -2 & 1 \\ 0 & -1 & 1 \end{vmatrix} = 0$

$AB: x.(-2).1 + 1.(-1).1 + y.1.0 - 0.(-2).1 - (-1).1.x - 1.y.1 = 0$

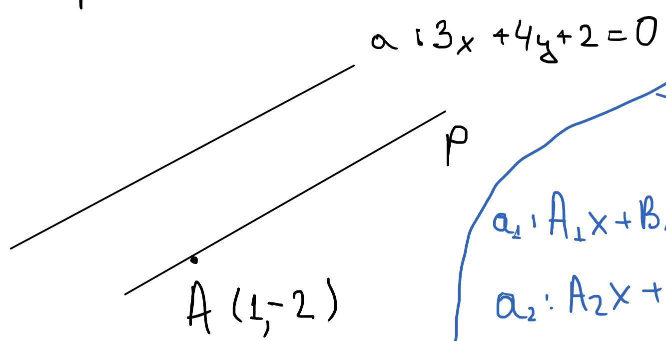
$AB: -2x - 1 + x - y = 0$

$AB: -x - y - 1 = 0$

$AB: x + y + 1 = 0$

Проверка: $1+(-2)+1=0$ ✓ $\quad 0+(-1)+1=0$ ✓

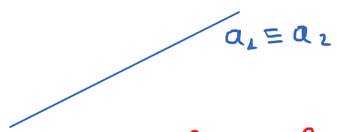
б)




1

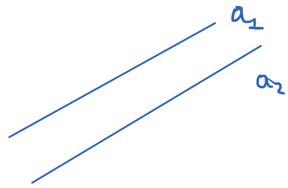
$a_1: A_1x+B_1y+C_1=0$

$a_2: A_2x+B_2y+C_2=0$



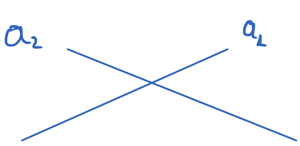

$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$

2




$\frac{A_1}{A_2} = \frac{B_1}{B_2} \neq \frac{C_1}{C_2}$

3




$a: 3x+4y+2=0$

$p: 3x+4y+C_p=0$

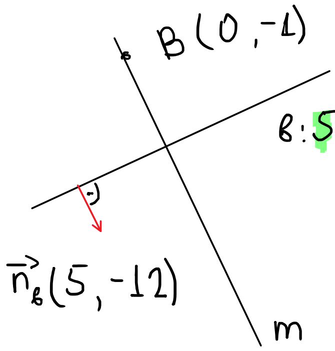
$A \ni p \Rightarrow 3.1+4.(-2)+C_p=0$

$3-8+C_p=0 \Rightarrow C_p=5$

$p: 3x+4y+5=0$

$$\frac{A_1}{A_2} \neq \frac{B_1}{B_2}$$

б) $m: \begin{cases} \ni B \\ \perp b \end{cases}$




$b: 5x - 12y + 1 = 0$

$m: A_m x + B_m y + C_m = 0$

$$\begin{aligned} B \ni m &\Rightarrow \Big| A_m \cdot 0 + B_m(-1) + C_m = 0 \\ \vec{n}_b \parallel m &\Rightarrow \Big| A_m \cdot 5 + B_m(-12) = 0 \end{aligned}$$

$$\begin{cases} -B_m + C_m = 0 \\ 5A_m - 12B_m = 0 \end{cases}$$

Нека $A_m = 12 \Rightarrow B_m = 5 \Rightarrow C_m = 5$

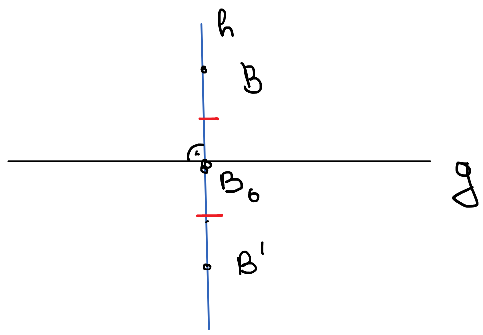
$m: 12x + 5y + 5 = 0$

$b: 5x - 12y + 1 = 0$

2) $B' = \sigma_g(B)$

$B(0,-1)$

$g: x + y - 1 = 0$




$h: \begin{cases} \perp g: 1x + 1y - 1 = 0 \\ \ni B(0,-1) \end{cases}$

$h: -1.x + 1.y + C_h = 0$

$B \ni h \Rightarrow -1.0 + 1.(-1) + C_h = 0 \Rightarrow C_h = 1$

$h: -x + y + 1 = 0$

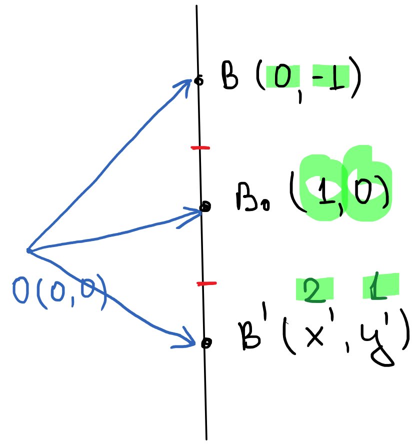
т. $B_0 = h \cap g$

$$\begin{cases} -x + y + 1 = 0 \\ x + y - 1 = 0 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 0 \end{cases}$$

$\Rightarrow B_0(1,0)$

$\overrightarrow{OB_0} = \frac{1}{2}\left(\overrightarrow{OB} + \overrightarrow{OB'}\right)$

$$\begin{cases} 1 = \frac{1}{2}\cdot(0 + x') \\ 0 = \frac{1}{2}\cdot((-1) + y') \end{cases}$$

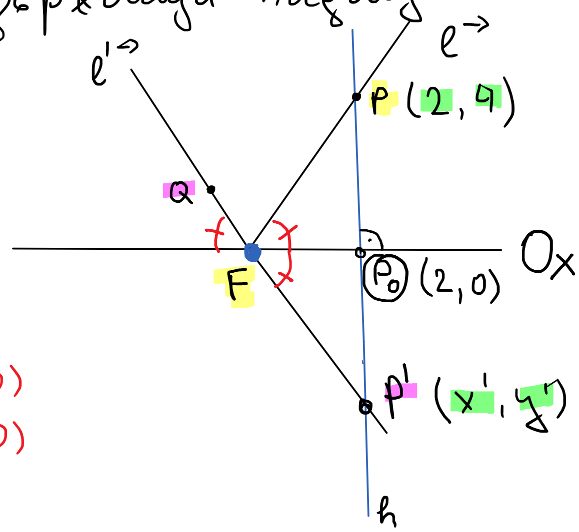



$$\begin{cases} x' = 2 \\ y' = 1 \end{cases} \Rightarrow B'(2,1)$$

(2) ОКС $K = Oxy$

т. $P(2,4)$ ; т. $Q(0,1)$

Светлинен лъч $\vec{e} \ni P$, отразява се от $Ox$ и отразения лъч $\vec{e'} \ni Q$. Да се намерят уравнения на правите, съдържащи падащия и отразения лъчи.




1) т. $P' = \sigma_{Ox}(P)$

$$h: \begin{cases} \perp Ox : y = 0 \\ \ni P(2,4) \end{cases}$$

Уравнение на $Ox$ $\begin{cases} \ni O(0,0) \\ \ni (1,0) \end{cases}$

$$Ox: \begin{vmatrix} x & y & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 0 \qquad Ox: y = 0$$

$Ox: 0.x + 1.y + 0 = 0$

$h: -1.x + 0.y + C_h = 0$

$P \ni h \Rightarrow -1.2 + 0.4 + C_h = 0 \Rightarrow C_h = 2$

$h: -x + 2 = 0$

$h: x - 2 = 0$

$T. P_0 = h \cap Ox$

$$\begin{vmatrix} x - 2 = 0 \\ y = 0 \end{vmatrix} \Rightarrow P_0(2,0)$$

$P(2,4) \qquad P_0(2,0) \qquad P'(x',y')$

$$\frac{2+x'}{2} = 2 \qquad \Rightarrow P'(2,-4)$$

$$\frac{4+y'}{2} = 0$$

2) $e' : \begin{cases} \ni Q(0,1) \\ \ni P'(2,-4) \end{cases} \qquad e' : \begin{vmatrix} x & y & 1 \\ 0 & 1 & 1 \\ 2 & -4 & 1 \end{vmatrix} = 0$

$e' : x + 2y - 2 - (-4)x = 0$

$e' : 5x + 2y - 2 = 0$

3) $T. F = e' \cap Ox$

$$\begin{vmatrix} 5x + 2y - 2 = 0 \\ y = 0 \end{vmatrix} \Rightarrow F\left(\frac{2}{5}, 0\right)$$

4) $e : \begin{cases} \ni F\left(\frac{2}{5}, 0\right) \\ \ni P(2,4) \end{cases} \qquad e : \begin{vmatrix} x & y & 1 \\ \frac{2}{5} & 0 & 1 \\ 2 & 4 & 1 \end{vmatrix} = 0$

$e : 5x - 2y - 2 = 0$

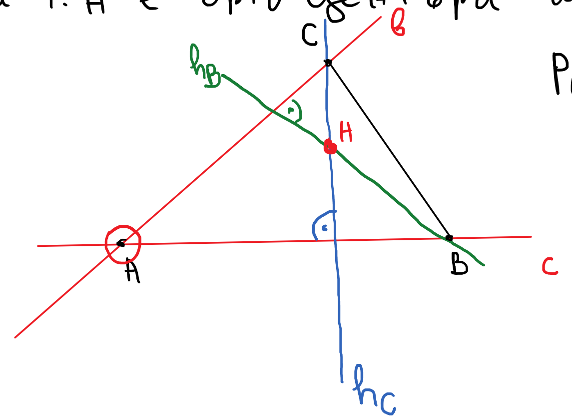
③ OKC $\quad K = Oxy$

$b : 5x + 4y - 13 = 0$

$c : x + 2y - 5 = 0$

$T. H(14, 15)$

Намерете коорд. на върховете на $\triangle ABC$, ако $b$ и $c$ съдържат страните $AC$ и $AB$ на $\triangle$-ка, а т. $H$ е ортоцентъра му. $S_{\triangle ABC} = ?$




Реш: т. $A = b \cap c$

$$\begin{vmatrix} 5x + 4y - 13 = 0 \\ x + 2y - 5 = 0 \end{vmatrix}$$

$\Rightarrow A(1, 2)$

$h_C : \begin{cases} \ni H(14, 15) \\ \perp c : 1x + 2y - 5 = 0 \end{cases}$

$-$

$h_C : -2x + y + C_{h_C} = 0$

т. $H \ni h_C \Rightarrow -2.14 + 15 + C_{h_C} = 0$

$-13 + C_{h_C} = 0$

$h_C : -2x + y + 13 = 0$

т. $C = h_C \cap b$

$$\begin{vmatrix} -2x + y + 13 = 0 \\ 5x + 4y - 13 = 0 \end{vmatrix} \Rightarrow C(5, -3)$$

$h_B : \begin{cases} \ni H(14, 15) \\ \perp b : 5x + 4y - 13 = 0 \end{cases}$

$h_B : -4x + 5y + C_{h_B} = 0$

$H \ni h_B \Rightarrow -4.14 + 5.15 + C_{h_B} = 0$

$C_{h_B} = -19$

$h_B : -4x + 5y - 19 = 0$

т. $B = h_B \cap c$

$$\Big| -4x + 5y - 19 = 0 \quad \Rightarrow B(-1, 3)$$

$$\begin{cases} -4x+5y-19=0 \\ x+2y-5=0 \end{cases} \Rightarrow B(-1,3)$$

$$S_{\triangle ABC} = \frac{1}{2}\left|\begin{vmatrix} 1 & 2 & 1 \\ -1 & 3 & 1 \\ 5 & -3 & 1 \end{vmatrix}\right| = \frac{1}{2}|3+3+10-15+3+2|$$

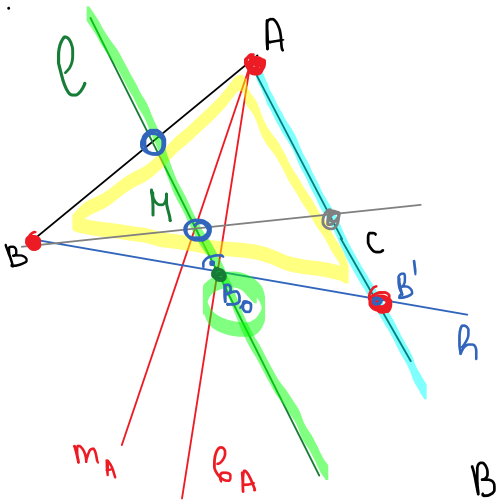
$$= \frac{1}{2}|6| = 3$$

④ ОКС $K = Oxy$

$b_A: \ x - 2y - 1 = 0$

$m_A: \ 2x - y - 8 = 0$

са съответно ъглополовяща и медианата при върха A на $\triangle ABC$. Ако т. $B(3,-4)$, да се намери $S_{\triangle ABC}$.




Реш! т. $A = m_A \cap b_A$

$$\begin{cases} 2x-y-8=0 \\ x-2y-1=0 \end{cases} \Rightarrow A(5,2)$$

$B' = \sigma_{b_A}(B)$

$h: \begin{cases} \ni B(3,-4) \\ \perp b_A: x-2y-1=0 \end{cases}$

$h: \ 2x+y+C_h=0$

т. $B \ni h \Rightarrow 2.3+(-4)+C_h=0$

$C_h = -2$

$h: \ 2x+y-2=0$

т. $B_0 = h \cap b_A$

$$\begin{cases} 2x+y-2=0 \end{cases} \Rightarrow B_0(1,0)$$

$$\begin{cases} 2x+y-2=0 \\ x-2y-1=0 \end{cases} \Rightarrow B_0(1,0)$$

т. $B(3,-4)$ т. $B_0(1,0)$ т. $B'(x',y')$

$$\begin{cases} \frac{3+x'}{2}=1 \\ -4+y' \end{cases} \Rightarrow B'(-1,4)$$

правата $AC: \begin{cases} \ni A(5,2) \\ \ni B'(-1,4) \end{cases}$

$$AC: \begin{vmatrix} x & y & 1 \\ 5 & 2 & 1 \\ -1 & 4 & 1 \end{vmatrix} = 0$$

$$AC: x+3y-11=0$$

$$\ell: \begin{cases} \parallel AC \\ \ni B_0(1,0) \end{cases}$$

$$\ell: x+3y+C_\ell=0$$

$B_0 \ni \ell \Rightarrow 1+3.0+C_\ell=0 \Rightarrow C_\ell=-1$

$$\ell: x+3y-1=0$$

т. $M = \ell \cap m_A$

$$\begin{cases} x+3y-1=0 \\ 2x-y-8=0 \end{cases} \Rightarrow M\left(\frac{25}{7}, \frac{-6}{7}\right)$$

т. $B(3,-4)$ т. $M\left(\frac{25}{7}, -\frac{6}{7}\right)$ $C(x_c, y_c)$

$$\begin{cases} \frac{3+x_c}{2}=\frac{25}{7} \end{cases} \Rightarrow C\left(\frac{29}{7}, \frac{16}{7}\right)$$

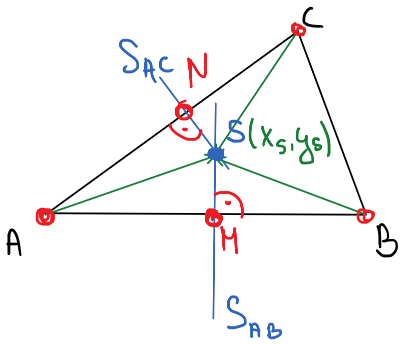
$$\left| \begin{array}{l} \frac{\ldots}{2} = \frac{\ldots}{7} \\ \frac{-4+y_c}{2} = \frac{-6}{7} \end{array} \right. \Rightarrow C\left(\frac{29}{7}, \frac{16}{7}\right)$$

$$S_{\triangle ABC} = \frac{1}{2} \left| \begin{vmatrix} 5 & 2 & 1 \\ 3 & -4 & 1 \\ \frac{29}{7} & \frac{16}{7} & 1 \end{vmatrix} \right| = \frac{20}{7}$$

ОКС  $K = Oxy$

(5) Да се намерят координатите на центъра $S$ и дължината на радиуса $R$ на описаната около $\triangle ABC$ окръжност, ако

т. $A(4,1)$; т. $B(3,-4)$; т. $C(-11,4)$




$\vec{AS}(x_s-4, y_s-1)$

$\vec{BS}(x_s-3, y_s+4)$

$\vec{CS}(x_s+11, y_s-4)$

$$\left| \begin{array}{l} |AS|^2 = |BS|^2 \\ |BS|^2 = |CS|^2 \end{array} \right.$$

$$\left| \begin{array}{l} (x_s-4)^2 + (y_s-1)^2 = (x_s-3)^2 + (y_s+4)^2 \\ (x_s-3)^2 + (y_s+4)^2 = (x_s+11)^2 + (y_s-4)^2 \end{array} \right.$$

$$\left| \begin{array}{l} x_s^2 - \underline{8x_s} + 16 + y_s^2 - \underline{2y_s} + 1 = x_s^2 - \underline{6x_s} + 9 + y_s^2 + \underline{8y_s} + 16 \\ x_s^2 - \underline{6x_s} + 9 + y_s^2 + 8y_s + 16 = x_s^2 + \underline{22x_s} + 121 + y_s^2 - 8y_s + 16 \end{array} \right.$$

$$\left| \begin{array}{l} -2x_s - 10y_s = 8 \\ -28x_s + 16y_s = 112 \end{array} \right. \Rightarrow S(-4, 0)$$

$\vec{AS}(-4-4, 0-1)$

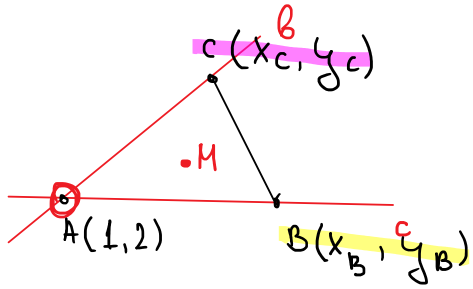
$\vec{AS}\,(-4-4,\ 0-1)$

$R = |\vec{AS}| = \sqrt{(-8)^2+(-1)^2} = \sqrt{65}$

(6) ОКС $K = Oxy$

$b:\ 2x - y = 0$

$c:\ x - 2y + 3 = 0$




т.$M(3,4)$ – медицентър на $\triangle ABC$

Коорд на $A, B, C$?

Реш: т. $A = b \cap c$ $\quad \left| \begin{array}{l} 2x - y = 0 \\ x - 2y + 3 = 0 \end{array} \right. \Rightarrow A(1,2)$

Нека т. $B(x_B, y_B)$ ; т. $C(x_C, y_C)$

т. $M(3,4)$ – медицентър $\Rightarrow$ $3 = \dfrac{1 + x_B + x_C}{3}$

$4 = \dfrac{2 + y_B + y_C}{3}$

т. $B \in c$ $\Rightarrow$ $x_B - 2y_B + 3 = 0$

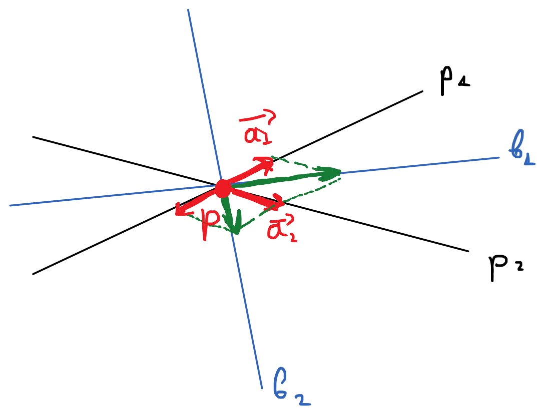
т. $C \in b$ $\Rightarrow$ $2x_C - y_C = 0$

$\Rightarrow B(5,4)\ ;\ C(3,6)$

(6) ОКС

$p_1:\ 3x - y + 5 = 0$

$p_2:\ 3x + y - 4 = 0$




Ъглополовящите $b_1, b_2$?

$p_1 \parallel \vec{p_1}(1, 3)$ $\quad \vec{a_1} \parallel \vec{p_1},\ |\vec{a_1}| = 1$

$$p_1 \parallel \vec{a_1}\left(\frac{1}{\sqrt{1^2+3^2}}, \frac{3}{\sqrt{1^2+3^2}}\right)$$

$$p_2 \parallel \vec{p_2}(-1, 3)$$

$$p_2 \parallel \vec{a_2}\left(\frac{-1}{\sqrt{(-1)^2+3^2}}, \frac{3}{\sqrt{(-1)^2+3^2}}\right)$$

$$b_1: \begin{cases} \ni P \\ \parallel (\vec{a_1}+\vec{a_2}) \end{cases} \qquad b_2: \begin{cases} \ni P \\ \parallel (-\vec{a_1}+\vec{a_2}) \end{cases}$$