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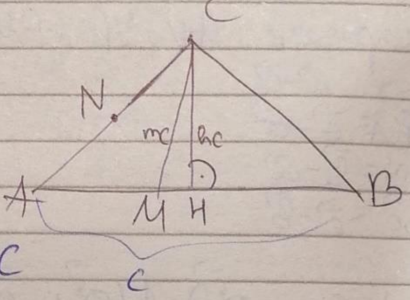
ДР №2 Геометрия

① ОКС Oxy

$\triangle ABC$ т. $A(0,-1)$

$m_c: -2x+y+7=0 \to m$

$h_c: -x+y+7=0 \to h$



a) Координати на т. B и т. C

т. $C = h_c \cap m_c$

$$\begin{cases} -2x+y+7=0 \\ -x+y+7=0 \to y=x-7 \end{cases} \Rightarrow -2x+x-7+7=0 \Rightarrow x=0 \Rightarrow y=-7$$

$\Rightarrow \boxed{\text{т. } C(0,-7)}$

Строим права $c \perp h_c$ през точка $A(0,-1)$

$h_c: -x+y+7=0$

$\Rightarrow$ права c: $x+y+c=0$

заместваме т. A $\quad 0-1+c=0 \Rightarrow c=1$

$\Rightarrow$ оконч. c: $x+y+1=0$

$m_c \cap c =$ т. M

$$\begin{cases} x+y+1=0 \Rightarrow x+2x-7+1=0 \quad 3x=6 \quad x=2 \\ -2x+y+7=0 \Rightarrow y=2x-7 \qquad y=-3 \end{cases}$$

$\boxed{\text{т. } M(2,-3)}$

т. $M \to$ среда на $AB \Rightarrow \frac{x_A+x_B}{2}=x_M$ и $\frac{y_A+y_B}{2}=y_M$

$x_B = 2x_M - x_A = 4-0=4$

$y_B = 2y_B - y_B = -6+1=-5 \Rightarrow \boxed{\text{т. } B(4,-5)}$

б) $S_{\triangle ABC} = \frac{1}{2} \cdot \begin{vmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{vmatrix} = \begin{vmatrix} 0 & -1 & 1 \\ 4 & -5 & 1 \\ 0 & -7 & 1 \end{vmatrix} =$

$=$ Ок. по първа ред: $\overset{(-1)^2}{0} \cdot 2 + \overset{(-1)^3}{(-1)} \cdot 4 + \overset{(-1)^4}{1} \cdot (-28) = 4 - 28$

$= \cancel{-32} = -24$

$S_{\triangle ABC} = \frac{24}{2} = 12$ В

в) ~~намира~~ т. Координати на център оп. окр. и дълж. R

имаме т. M $\to$ среда на AB

строим права $\perp AB$ през т. M

$AB = c: x + y + 1 = 0$

симетрала $S_c: x - y + c = 0$ в т. $M(2, -3)$

$2 + 3 + c = 0 \quad c = -5$

$S_c: x - y - 5 = 0$

намираме среда на $AC = b$. $\Rightarrow$ т. N

$x_N = \frac{x_A + x_C}{2} = \frac{0 + 0}{2} = 0 \quad y_N = \frac{y_A + y_B}{2} = \frac{-1 - 7}{2} = -4$

т. $N(0, -4)$

нека т. $A_1(x, y)$ е прав. от c

уравнение b: $\begin{vmatrix} x & y & 1 \\ 0 & -1 & 1 \\ 0 & -7 & 1 \end{vmatrix} = 0 \Rightarrow (-1)^2 \cdot x \cdot 6 + 0 \cdot y + 0 = 0$

$\Rightarrow b: x = 0$

$6 \cdot x + 0 \cdot y + 0 \cdot c = 0$

$\Rightarrow S_b$: ~~x~~ $y + c = 0 \quad c = 4 \Rightarrow S_B: y + 4 = 0$

симетрала

$y = -4$

пресичане $S_в$ и S_c $\begin{cases} y=-4 \\ x-y-5=0 \end{cases} \Rightarrow x=1$

$\Rightarrow$ т. $O(1,-4)$

4) ~~$|AB|$~~ $R=?$ $R=|\overrightarrow{AO}|$

$\overrightarrow{AO}(1,-3) = R = \overset{\text{б.о.о.}}{\sqrt{10}} = \sqrt{\overrightarrow{AO}^2}$

2) ОКС $Oxyz$

Лзг през т. $A(3,-1,1)$

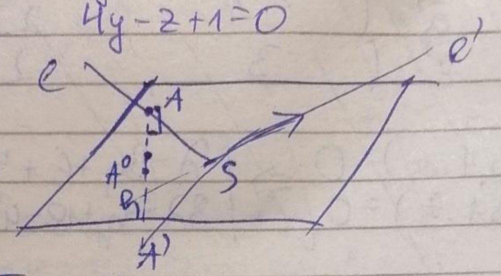
(2) ОКС $Oxyz$

Лъч през т. $A(3,-1,1)$

отрз. се през $\beta: x+2z=0$ $\quad \vec{n}_\beta(1,0,2)$

отраз. лъч е $\parallel$ на $g: \begin{cases} -x+y-z+4=0 \\ 4y-z+1=0 \end{cases}$

l и $l' = ?$



Търсим т. A', която:

$$т.A \xrightarrow{\sigma_\beta} т.A'$$

1) Търсим права $h \perp \beta$ през т. $A(3,-1,1)$

$$h: \begin{cases} x = 3+\lambda \\ y = -1+0 \\ z = 1+2\lambda \end{cases} \quad \lambda \in \mathbb{R}$$

$h \cap \beta = т.\ A_0:$

$$\begin{cases} x = 3+\lambda \\ y = -1 \\ z = 1+2\lambda \\ x+2z=0 \end{cases}$$

$3+\lambda+2+4\lambda = 0 \qquad \lambda = -1$

$\Rightarrow т.\ A_0(2,-1,-1)$

т. $A(3,-1,1)$

т. $A_0(2,-1,-1)$

т. $A'(x',y',z')$

$\dfrac{3+x'}{2} = 2 \qquad \dfrac{-1+y'}{2} = -1 \qquad \dfrac{1+z'}{2} = -1$

$x = 1 \qquad y = -1 \qquad z = -3$

т. $A'(1,-1,-3)$

$l' \parallel g$ $\qquad g\begin{cases} -x + y - z + 4 = 0 \\ 4y - z + 1 = 0 \end{cases}$

$y = s$

$$g\begin{cases} x = s - 4s + 1 + 4 = 5 - 3s \\ y = s \\ z = 4s + 1 \end{cases} \Rightarrow \vec{g}(-3, 1, 4)$$

построяваме правата $l' \begin{cases} \parallel \vec{g} \\ \ni A \end{cases}$

$$l' \begin{cases} x = 1 + q.(-3) \\ y = -1 + q.(1) \\ z = -3 + q.(4) \end{cases} \quad q \in \mathbb{R}$$

търсим т. $S = l' \cap \beta$

$$\begin{cases} x = 1 - 3q \\ y = -1 + q \\ z = -3 + 4q \\ x + 2z = 0 \end{cases}$$

$1 - 3q - 6 + 8q = 0$

$5q = 5 \quad \underline{\underline{q = 1}}$

$\Rightarrow$ т. $S(-2, 0, 1)$

Търсим правата $l \begin{cases} \ni S \\ \ni A \end{cases}$

$\cancel{\vec{AS}(3, 0, 1)}$

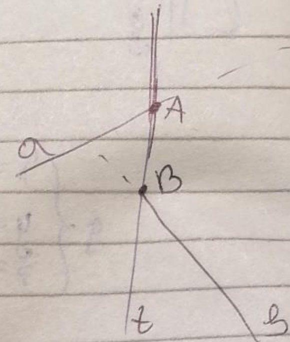
$\cancel{\vec{SA}(3, 0, 1)}$

$$\vec{SA}(5, -1, 0) \quad l\begin{cases} x = 3 + p5 \\ y = -1 + p(-1) \\ z = 1 + p0 \end{cases} \quad p \in \mathbb{R}$$

③ ОКС $K = Oxyz$

права a: $\begin{cases} 2x+y+2z-10=0 \\ 4x-y+z-11=0 \end{cases}$

права b: $\begin{cases} x=-5+3q \\ y=5-2q \\ z=3-2q \end{cases} \quad q \in \mathbb{R}$



а) разст. между $\underline{\underline{a}}$ и $\underline{\underline{b}}$

изб. т. A_1 и A_2 от правата a, т.ч.

за A_1 $z=0$

$a: \begin{cases} 2x+y-10=0 \\ 4x-y-11=0 \end{cases} \Rightarrow y=4x-11$

$6x=21 \quad x=\frac{7}{2}$

$y=3$

$\Rightarrow$ т. $A_1\left(\frac{7}{2}, 3, 0\right)$

за т. A_2 $x=0$ $\quad a: \begin{cases} y+2z-10=0 \\ -y+z-11=0 \end{cases} \quad z=y+11 \quad y=-4 \quad z=7$

$\Rightarrow$ т. $A_2(0,-4,7)$

$\overrightarrow{A_2A_1}\left(\frac{7}{2}, 7, -7\right) \Rightarrow a: \begin{cases} x=0+\frac{7}{2}p \\ y=-4+7p \\ z=7-7p \end{cases} \quad p \in \mathbb{R}$

$t \cap a = $ т. $A\left(\frac{7}{2}p, -4+7p, 7-7p\right)$

$t \cap b = $ т. $B(-5+3q, 5-2q, 3-2q)$

$\overrightarrow{BA}\left(\frac{7}{2}p-3q+5, 7p+2q-9, -7p+2q+4\right)$

$\vec{a}\left(\frac{7}{2}, 7, -7\right)$

$\vec{b}(3, -2, -2)$

Образуваме система:

$$\left|\begin{array}{l} (\vec{BA}\cdot\vec{a})=0 \\ (\vec{BA}\cdot\vec{b})=0 \end{array}\right. = \left|\begin{array}{l} 49p-42q+70+196p+56q+252+196p+56p-112=0 \\ 21p-18q+30+28p-8q+30+28p-8q-16=0 \end{array}\right.$$

$$\Rightarrow \left|\begin{array}{l} 441p-42q+294=0 \quad |:21 \\ 21p-34q+50=0 \end{array}\right.$$

$$\left|\begin{array}{l} 21p-2q-14=0 \\ 21p-34q+50=0 \end{array}\right. \xrightarrow{\text{чернова}} \begin{array}{l} q=2 \\ p=6 \end{array}$$

$$\Rightarrow \vec{BA}(3-6+5,\ 6+4-9,\ -6+4+4)$$

$$\vec{BA}(2,\ 1,\ 2)$$

$$|\vec{BA}| = \sqrt{4+1+4} = \sqrt{9} = 3$$

б) $P = a \cap \alpha: x+2y-2z-23=0$

$Q = b \cap \pi: 3x-2y-2z+14=0$

$R(2,4,-1)$ $\quad S_{\triangle PQR} = ?$

Смятаме P:
$$\left|\begin{array}{l} x=\frac{7}{2}p \\ y=-4+7p \\ z=7-7p \\ x+2y-2z-23=0 \end{array}\right.$$

$\Rightarrow \frac{7}{2}p-8+14p-14+14p-23=0$

$7p-16+28p-28+28p-46=0$

$63p=90$

$p=\frac{10}{7}$

$\cancel{\Rightarrow \frac{7}{2}p-8+14p}$

$\cancel{\Rightarrow \frac{7}{2}p-8-14p-14+14p-23=0 \quad 7p=90 \quad p=13 \quad т.P(\frac{91}{2}, 95, -84)}$

$\Rightarrow т.P(5, 6, -3)$

Смятаме т. Q

$$\begin{cases} x = -5 + 3q \\ y = 5 - 2q \\ z = 3 - 2q \\ 3x - 2y - 2z + 11 = 0 \end{cases}$$

$-15 + 9q - 10 + 4q - 6 + 4q + 11 = 0$

$17q = \frac{21}{17}$ $\quad q = \frac{21}{17}$ $\quad q = 1$

т. $Q(-2, 3, 1)$

т. $P(5, 6, -3)$

т. $R(2, 4, -1)$

$\vec{RP}(3, 2, -2)$

$\vec{RQ}(-4, -1, 2)$

$\vec{QP}(7, 3, -4)$

$$S_{\triangle RPQ} = \begin{vmatrix} 3 & 2 & -2 \\ -4 & -1 & 2 \\ 7 & 3 & -4 \end{vmatrix} \quad \frac{|\vec{RP} \times \vec{RQ}|}{2} =$$

$$= \frac{\sqrt{\begin{vmatrix} 2 & -2 \\ -1 & 2 \end{vmatrix}^2 + \begin{vmatrix} -2 & 3 \\ 2 & -4 \end{vmatrix}^2 + \begin{vmatrix} 3 & 2 \\ -4 & 1 \end{vmatrix}^2}}{2} = \frac{\sqrt{4 + 4 + 25}}{2} = \frac{\sqrt{33}}{2}$$