(135) $X \sim N(\mu, \sigma^2)$. Дов. инт. с точно ниво на доверие γ

1. за μ като σ^2 е известно

$$T = \frac{\overline{X}_n^{(1)} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$$

$$\gamma = P(-q < T < q) = P\left(-q < \frac{\overline{X}_n^{(1)} - \mu}{\frac{\sigma}{\sqrt{n}}} < q\right) = P\left(\underbrace{\overline{X}_n^{(1)} - \frac{\sigma}{\sqrt{n}} q}_{I_1} < \mu < \underbrace{\overline{X}_n^{(1)} + \frac{\sigma}{\sqrt{n}} q}_{I_2}\right)$$

$\mu \in (I_1, I_2)$

2. за σ^2 като μ е известно

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{j=1}^{n}(X_j - \mu)^2 \Rightarrow \frac{n\hat{\sigma}^2}{\sigma^2} = \frac{\sum_{j=1}^{n}(X_j-\mu)^2}{\sigma^2} \sim \chi^2(n)$$

$$T = \frac{n\hat{\sigma}^2}{\sigma^2} \Rightarrow \gamma = P(q_1 < T < q_2) = P\left(q_1 < \frac{n\hat{\sigma}^2}{\sigma^2} < q_2\right) = P\left(\underbrace{\frac{\sum_{j=1}^{n}(X_j-\mu)^2}{q_2}}_{I_1} < \sigma^2 < \underbrace{\frac{\sum_{j=1}^{n}(X_j-\mu)^2}{q_1}}_{I_2}\right)$$

$\sigma^2 \in (I_1, I_2)$

3. за μ като σ^2 е неизвестно

$$\hat{\sigma} = \frac{\frac{1}{n}\sum_{j=1}^{n}(X_j - \overline{X}_n^{(1)})^2}{\sigma^2} = \frac{n-1}{n}\frac{S^2}{\sigma^2}, \text{ където } S^2 = \frac{1}{n-1}\sum_{j=1}^{n}(X_j - \overline{X}_n)^2$$

$$T = \frac{\overline{X}_n^{(1)} - \mu}{\frac{S}{\sqrt{n}}} \sim t(n-1) \Rightarrow \gamma = P\left(-q < \frac{\overline{X}_n^{(1)} - \mu}{\frac{S}{\sqrt{n}}} < q\right) = P\left(-\frac{q\sqrt{n}}{S} < \overline{X}_n^{(1)} - \mu < \frac{q\sqrt{n}}{\sqrt{S}}\right) =$$

$$= P\left(\underbrace{\overline{X}_n^{(1)} + \frac{q\sqrt{n}}{S}}_{I_2} > \mu > \underbrace{\overline{X}_n^{(1)} - \frac{q\sqrt{n}}{\sqrt{S}}}_{I_1}\right) \Rightarrow \mu \in (I_1, I_2)$$

4. за σ^2 като μ е неизв.

$$T = \frac{n\hat{\sigma}^2}{\sigma^2} = \frac{n\frac{1}{n}\sum_{j=1}^{n}(X_j - \overline{X}_n^{(1)})^2}{\sigma^2} = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

$$P\left(q_1 < \frac{(n-1)S^2}{\sigma^2} < q_2\right) = P\left(\underbrace{\frac{1}{q_1(n-1)S^2}}_{I_1} < \sigma^2 < \underbrace{\frac{1}{q_2(n-1)S^2}}_{I_2}\right)$$

$\sigma^2 \in (I_1, I_2)$