

Banach Matchbook Problem

This is a famous probability problem posed by Hugo Steinhaus (1887 – 1972), who brought it up in a conference honoring Stefan Banach (1892 – 1945), a prodigious smoker and one of the greatest mathematicians of the twentieth century.

A smoking mathematician carries two matchboxes, one in his right pocket and one in his left pocket. Whenever he wants to smoke, he selects a pocket at random and takes a match from the box in that pocket. If each matchbox initially contains N matches, what is the probability that when the mathematician for the first time discovers that one box is empty, there are exactly m matches in the other box, $m = 0, 1, 2, \dots, N$?

The objective of this assignment is to write the user-defined **R** function `Banach.R` that simulates this problem. Given a positive integer N , which represents the number of matches in both boxes at the start, and a vector V of random numbers, simulate the drawing of a match until one matchbox is empty. The output will be the number of matches in the “other” box when one matchbox first becomes empty. Here’s how it will work. At the beginning, both the Left Matchbox and the Right Matchbox will contain N matches. For each draw, x will represent a value in V , beginning with the first value, moving to the second, etc. If $x < 0.5$, then subtract 1 match from the Left Matchbook. If $x \geq 0.5$ subtract one match from the Right Matchbook. Continue this while the number of matches in both matchbooks is greater than zero. Your function will then return the number of matches in the other matchbox.

Here are several examples.

```
> Banach(6, c(0.81,0.91,0.13,0.91,0.63,0.10,0.28,0.55,0.96,0.96,0.16,0.97))
[1] 3
> Banach(4, c(0.96,0.49,0.80,0.14,0.42,0.92,0.79,0.96))
[1] 1
> Banach(2, c(0.19,0.49,0.45,0.65))
[1] 2
```