

## Bisection Method

Write the **R** function `bisection.r` that calculates the root of the user-defined function  $f$  on a closed interval  $[A, B]$  using the Bisection Method.

Inputs:

1.  $A$  - the left endpoint of the interval.
2.  $B$  - the right endpoint of the interval.
3.  $t$  - a user-defined tolerance

Output:  $x$  - a close approximation of a root of the function  $f$  on the closed interval between  $a$  and  $b$ .

Basically, the algorithm calculates the midpoint  $M$  between  $A$  and  $B$ . If  $f(M)$  has the same sign as  $f(A)$ , then the root must be between  $M$  and  $B$ . Therefore, set  $A$  equal to  $M$ . On the other hand, if  $f(M)$  does not have the same sign as  $f(A)$ , then the root must be between  $A$  and  $M$ . In this case, set  $B$  equal to  $M$ . Repeat.

How long do you repeat the algorithm?

You continue while  $|B - A| > t$  and  $\frac{2 \times |B - A|}{|A| + |B|} > t$ .

You must check to see whether  $f(A)$  and  $f(B)$  have opposite signs. If they have the same sign (so there is no guarantee that there is a root between  $A$  and  $B$ ), then return NA and print an error message.

Good luck. Here are several examples. For the first two examples,  $f(x) = \cos(x) - 0.80 + 0.10x^2$ . For the third example,  $f(x) = -\sin(x) + x/50$ . And, for the last two examples,  $f(x) = (x - 3)^5$ .

```
> bisection(0,pi,0.001)
[1] 0.7267234
> bisection(0,pi,0.000001)
[1] 0.7270473
> bisection(pi/2,pi,0.0001)
[1] 3.079994
> bisection(0,5,0.0001)
[1] 2.999992
> bisection(0,2,0.0001)
Error, f(A) and f(B) must have different signs.
[1] NA
```