University of Pennsylvania, ESE6190

Model Predictive Control

Chapter 1: Introduction and Overview

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F. Borrelli, A. Bemporad, and M. Morari, Predictive Control for Linear and Hybrid Systems, Cambridge University Press, 2017.

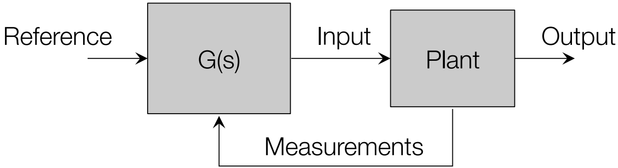
Outline

Outline

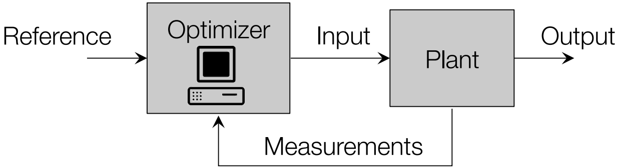
1. Optimization based Control
2. Concept of MPC
3. Applications
4. History of MPC
5. Summary
6. Literature & Acknowledgements

Optimization in the loop

Classical control loop:



The classical controller is replaced by an optimization algorithm:



The optimization uses predictions based on a model of the plant.

Optimization-based control: Motivation

Objective:

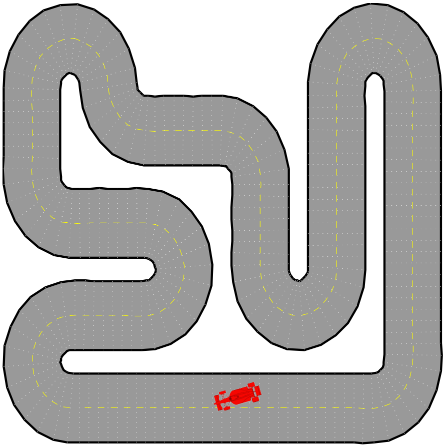
- Minimize lap time

Constraints:

- Avoid other cars
- Stay on road
- Don't skid
- Limited acceleration

Intuitive approach:

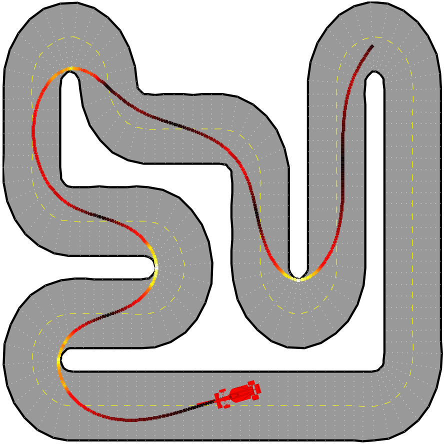
- Look forward and plan path based on
  - Road conditions
  - Upcoming corners
  - Abilities of car
  - etc...



Optimization-Based Control: Motivation

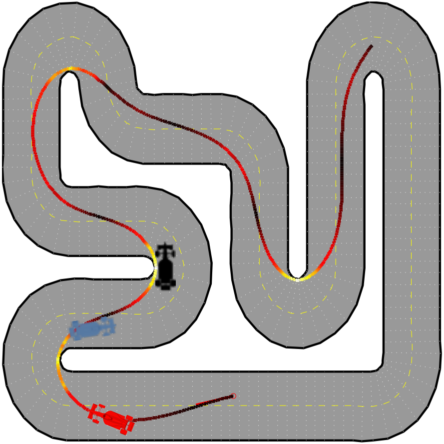
Minimize	(lap time)
while	avoid other cars
	stay on road
	...

- Solve **optimization problem** to compute minimum-time path



Optimization-Based Control: Motivation

Minimize	(lap time)
while	avoid other cars
	stay on road
	...

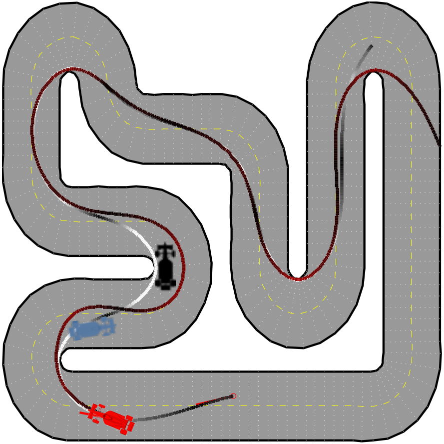


- Solve **optimization problem** to compute minimum-time path
- What to do if something unexpected happens?
  - We didn't see a car around the corner!
  - Must introduce **feedback**

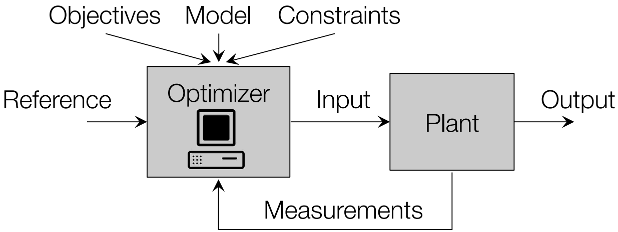
Optimization-Based Control: Motivation

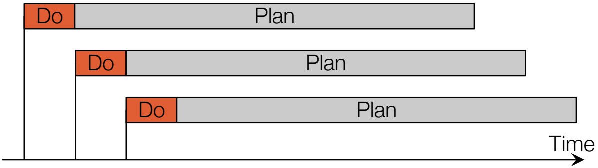
Minimize	(lap time)
while	avoid other cars
	stay on road
	...

- Solve **optimization problem** to compute minimum-time path
- Obtain series of planned control actions
- Apply **first** control action
- Repeat the planning procedure



Model Predictive Control



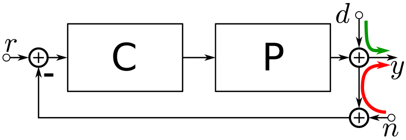


Receding horizon strategy introduces **feedback**.

Outline

Two Different Perspectives

Classical design: design C

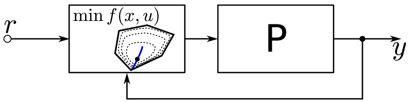


Dominant issues addressed

- Disturbance rejection ($d \to y$)
- Noise insensitivity ($n \to y$)
- Model uncertainty

(usually in **frequency domain**)

MPC: real-time, repeated optimization to choose $u(t)$ – often in supervisory mode



Dominant issues addressed

- Control constraints (limits)
- Process constraints (safety)

(usually in **time domain**)

Constraints in Control

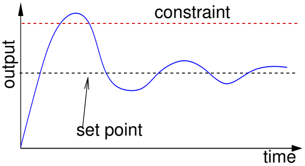
All physical systems have **constraints**:

- Physical constraints, e.g. actuator limits
- Performance constraints, e.g. overshoot
- Safety constraints, e.g. temperature/pressure limits

Optimal operating points are often near constraints.

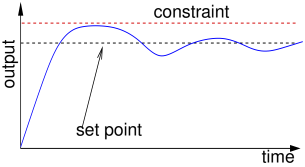
Classical control methods:

- Ad hoc constraint management
- Set point sufficiently far from constraints
- Suboptimal plant operation



Predictive control:

- Constraints included in the design
- Set point optimal
- Optimal plant operation



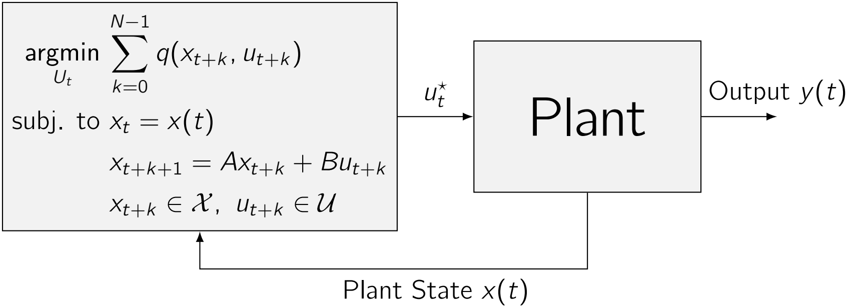
MPC: Mathematical Formulation

$$
\begin{aligned}
U_t^\star(x(t)) := \underset{U_t}{\operatorname{argmin}} \; & \sum_{k=0}^{N-1} q(x_{t+k}, u_{t+k}) && \\
\text{subj. to } \; & x_t = x(t) && \text{measurement} \\
& x_{t+k+1} = Ax_{t+k} + Bu_{t+k} && \text{system model} \\
& x_{t+k} \in \mathcal{X} && \text{state constraints} \\
& u_{t+k} \in \mathcal{U} && \text{input constraints} \\
& U_t = \{u_t, u_{t+1}, \ldots, u_{t+N-1}\} && \text{optimization variables}
\end{aligned}
$$

Problem is defined by

- **Objective** that is minimized
- Internal **system model** to predict system behavior
- **Constraints** that have to be satisfied

MPC: Mathematical Formulation



At each sample time:

- Measure / estimate current state $x(t)$
- Find the optimal input sequence for the entire planning window N: $U_t^\star = \{u_t^\star, u_{t+1}^\star, \ldots, u_{t+N-1}^\star\}$
- Implement only the **first** control action $u_t^\star$

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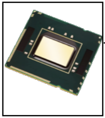
MPC: Applications



Computer control — ns

μs — Power systems

Traction control — ms

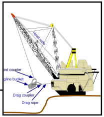
Seconds — Buildings

Refineries — Minutes

Hours — Nurse rostering

Train scheduling — Days

Weeks — Production planning



Outline

3. Applications

   Ball on Plate

   Path Following

   Autonomous Quadrocopter Flight

   Autonomous dNaNo Race Cars

   Energy Efficient Building Control

   Kite Power

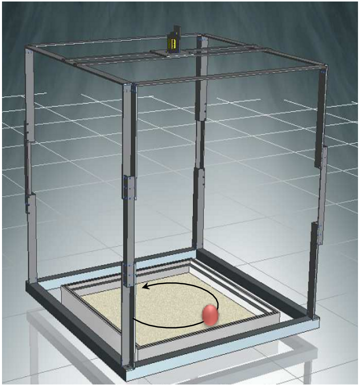
   Automotive Systems

   Catalytic Cracker

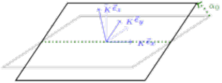
   Predictive Control in NeuroScience

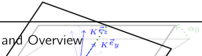
Ball on Plate

- **Movable plate** (0.66m x 0.66m)
- Can be revolved around two axis $[+17°; -17°]$ by two DC motors
- Angle is measured by potentiometers
- Position of the ball is measured by a camera
- **Model**: Linearized dynamics, 4 states, 1 input per axis
- **Input constraints**: Voltage of motors
- **State constraints**: Boundary of the plate, angle of the plate



[*R. Waldvogel. Master Thesis ETH, 2010*]





Ball on Plate

Controller comparison: LQR vs. MPC in the presence of input constraints

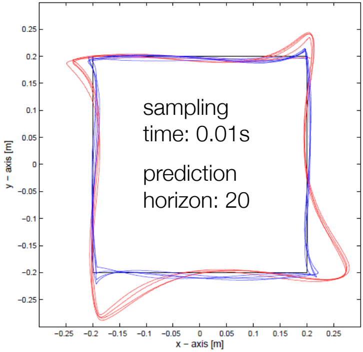


Figure: LQR (red) vs MPC (blue)

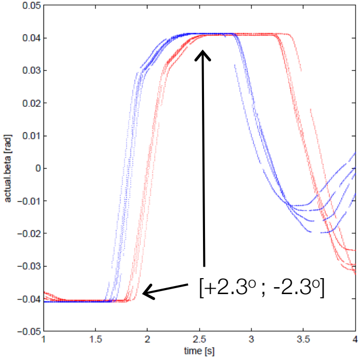


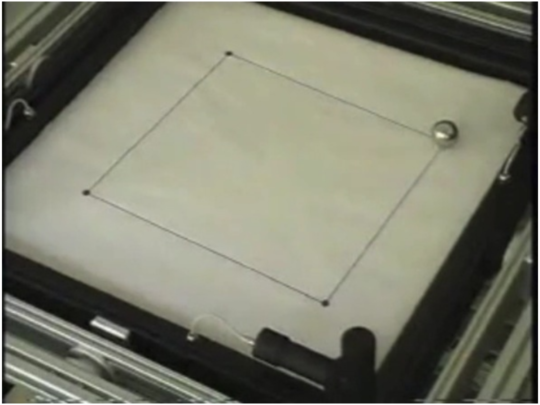
Figure: Input β for the upper left corner.

MPC introduces **preview** by predicting the state over a finite horizon

[*R. Waldvogel. Master Thesis ETH, 2010*]

Ball on Plate

MPC Control of a Ball and Plate System:



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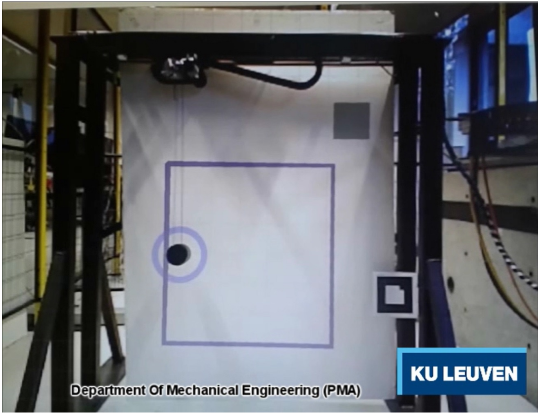
Path Following

MPC Control of a crane along a **known path**



[*Jan Sewers, KU Leuven*]

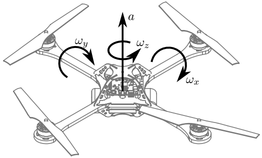
Outline

3. Applications

Autonomous Quadrocopter Flight

Quadrocopters:

- Highly agile due to fast rotational dynamics
- High thrust-to-weight ratio allows for large translational accelerations
- Motion control by altering rotation rate and/or pitch of the rotors
- High thrust motors enable high performance control

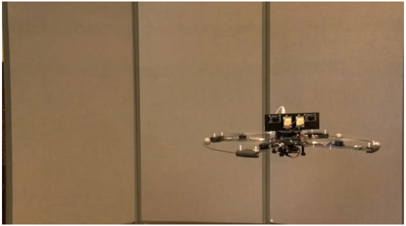


Control Problem:

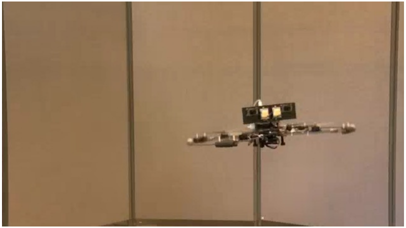
- **Nonlinear system** in 6D (position, attitude)
- **Constraints**: limited thrust, rates,...
- **Task**: Hovering, trajectory tracking
- **Challenges**: Fast unstable dynamics

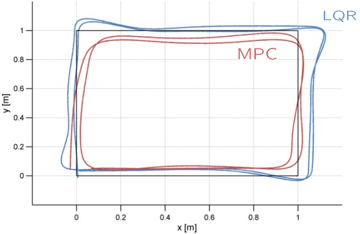
Autonomous Quadrocopter flight

LQR



MPC



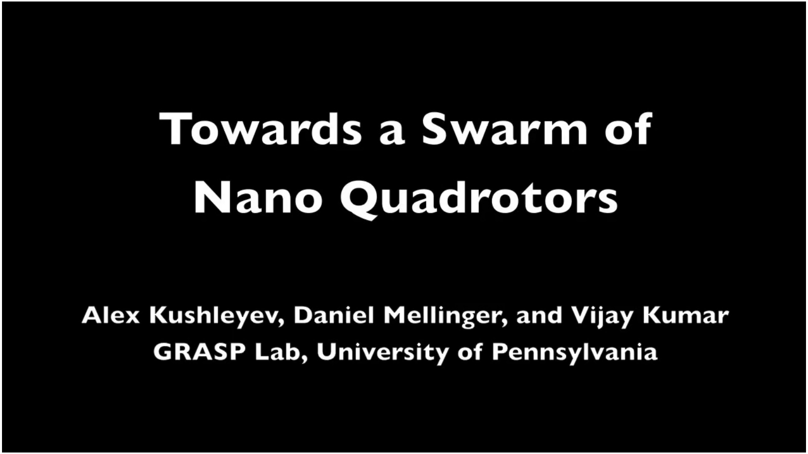


[*M. Burri. Master Thesis ETH, 2011*]

Autonomous Quadrocopter flight

[*IDSC. ETH Zurich, 2011;* `http://flyingmachinearena.org/`]

Autonomous Quadrocopter flight



[*Grasp Lab. University of Pennsylvania, 2012;* `http://www.grasp.upenn.edu/`]

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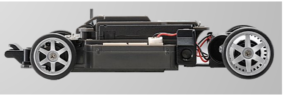
Autonomous dNaNo Race Cars

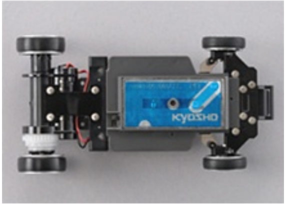
Race car:

- 1:43 scale, very light (50g) and fast
- Radio controlled
- 2.4GHz transmitter allows to run up to 40 cars

Control Problem:

- **Nonlinear model** in 4D (position, orientation)
- **Constraints:** acceleration, steering angle, race track, other cars...
- **Task:** Optimal path planning and path following
- **Challenges:** State estimation, effects that are difficult to model/measure, e.g. slip, small sampling times





Autonomous dNaNo Race Cars

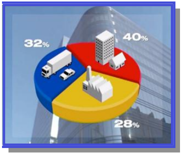
[*ORCA Racer Project. ETH, 2011;* `http://orcaracer.ethz.ch/`]

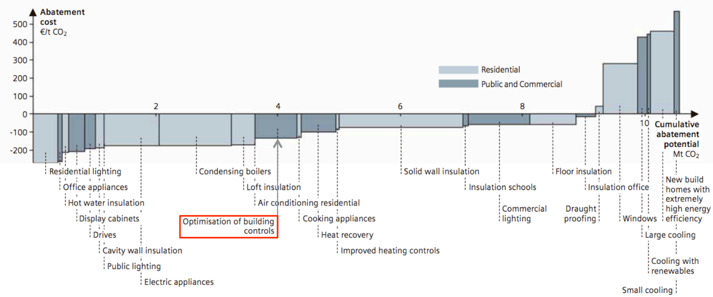
Outline

3. Applications

Energy Efficient Building Control

- Buildings account for approx. **40% of global energy use**
- Most energy is consumed during use of the buildings
- Building sector has large potential for cost-effective reduction of CO_2 emissions
- Most investments in buildings are expected to pay back through **reduced energy bills**





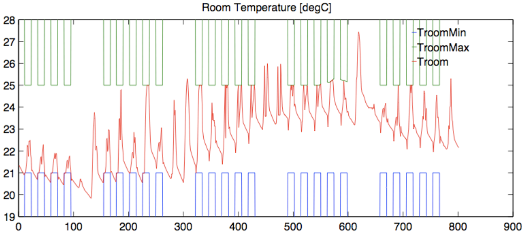
Greenhouse gas abatement cost curve for London buildings (2025, decision maker perspective)

Source: Watson, J. (ed.) (2008): Sustainable Urban Infrastructure, London Edition – a view to 2025. Siemens AG, Corporate Communications (CC) Munich, 71pp.

Energy Efficient Building Control

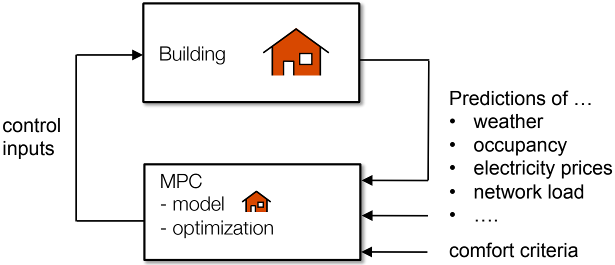
Integrated Room Automation:
Integrated control of heating, cooling, ventilation, electrical lighting, blinds,... of a single room/zone

Control Task: Use minimum amount of energy (or money) to keep room temperature, illuminance level and CO_2 concentration in **prescribed comfort ranges**



[*OptiControl Project, ETH. 2010;* `http://www.opticontrol.ethz.ch/`]

Energy Efficient Building Control



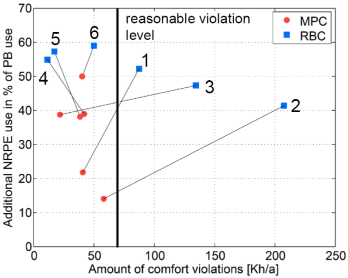
MPC opens the possibility to

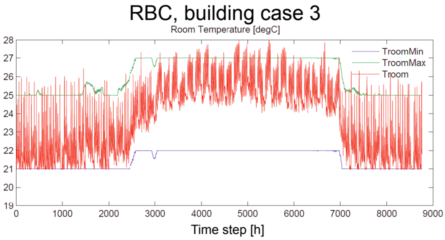
- exploit building's **thermal storage capacity**
- use **predictions** of future disturbances, e.g. weather, for better planning
- use forecasts of electricity prices to shift electricity demand for grid-friendly behavior
- offer grid-balancing services to the power network
- ...

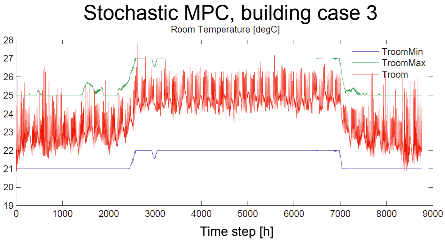
while respecting requirements for building usage (temperature, light, ...)

Energy Efficient Building Control

Optimize energy efficiency using weather predictions:





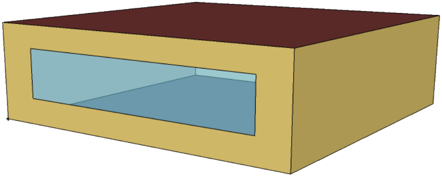


MPC: Stochastic MPC
RBC: Current best practice Rule Based Controller

[*OptiControl Project. ETH, 2010;* `http://www.opticontrol.ethz.ch/`]

Example: One-Zone Building

Model of a one-zone building for climate control



- EnergyPlus model of a single zone
- Automatic extraction and linearization: openBuild tool
- Electric heating
- Weather: Jan, 2007 in San Francisco

$$x(t+1) = Ax(t) + Bu(t) + E_{rad}v_{rad}(t) + E_{amb}T_{amb}(t)$$

States		**Input**		**Disturbance**	
x_1	Zone temp.	u	Heat flux	v_{rad}	Solar radiation
$x_2..x_4$	Wall temps.			T_{amb}	Ambient temp.

Example: One-Zone Building

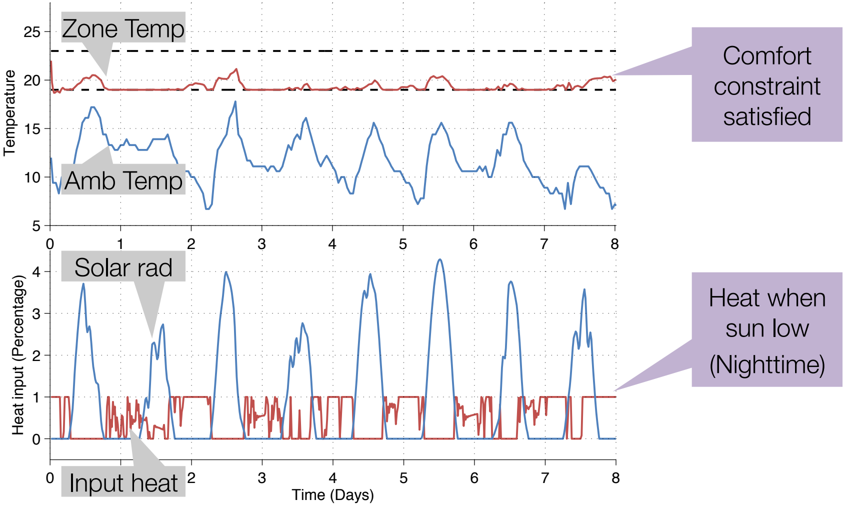
Problem formulation: Simplest Configuration

$$
\begin{aligned}
\min_{U_t} \; & \sum_{k=0}^{N-1} |u_{t+k}| \\
\text{subj. to } & x_t = x(t) \\
& x_{t+k+1} = Ax_{t+k} + Bu_{t+k} + E_{rad} v_{t+k,rad} + E_{amb} T_{t+k,amb} \\
& y_{t+k} = c^T x_{t+k} \\
& |y_{t+k} - 21| \leq 2 \\
& 0 \leq u_{t+k} \leq 1
\end{aligned}
$$

- Objective: Minimize total thermal energy use
- Forecast temperature / solar radiation N steps into the future
- Comfort constraints on room temperature
- Input: 0-100% heating

Example: One-Zone Building

Simple MPC



Annualized energy used: 81.6 kWh / m^2

Example: One-Zone Building

Problem formulation: Nighttime setbacks & pre-cooling

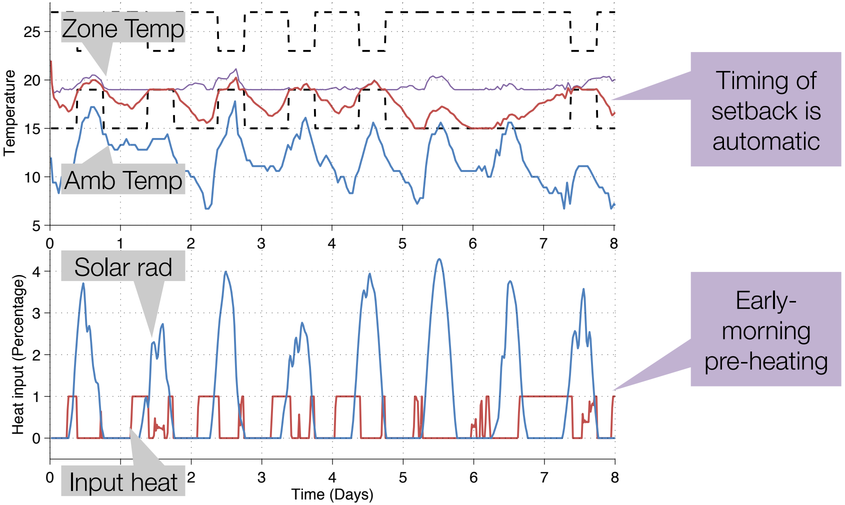
$$
\begin{aligned}
\min_{U_t} \ & \sum_{k=0}^{N-1} |u_{t+k}| \\
\text{subj. to } & x_t = x(t) \\
& x_{t+k+1} = Ax_{t+k} + Bu_{t+k} + E_{rad} v_{t+k,rad} + E_{amb} T_{t+k,amb} \\
& y_{t+k} = c^T x_{t+k} \\
& |y_{t+k} - 21| \leq 2 + \sigma_{t+k} \\
& 0 \leq u_{t+k} \leq 1
\end{aligned}
$$

Night Setbacks

$$
\sigma_{t+k} = \begin{cases} 0, & t+k \in \text{'daytime'} \\ 6, & t+k \in \text{'nighttime'} \end{cases}
$$

Example: One-Zone Building

Nighttime setback



Annualized energy used: 55.7 kWh / m^2

Example: One-Zone Building

Problem formulation: time-of-use pricing

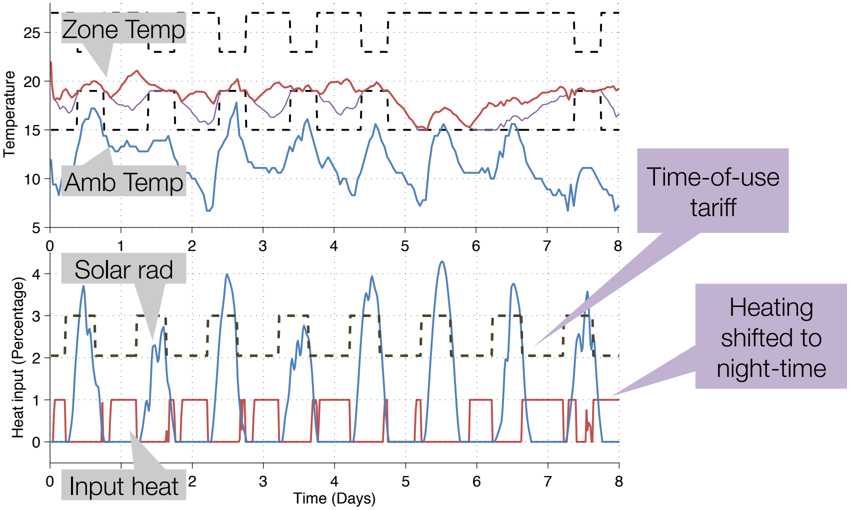
$$
\begin{aligned}
\min_{U_t} \ & \sum_{k=0}^{N-1} c_{t+k} \cdot |u_{t+k}| \\
\text{subj. to } & x_t = x(t) \\
& x_{t+k+1} = Ax_{t+k} + Bu_{t+k} + E_{rad} v_{t+k,rad} + E_{amb} T_{t+k,amb} \\
& y_{t+k} = c^T x_{t+k} \\
& |y_{t+k} - 21| \leq 2 + \sigma_{t+k} \\
& 0 \leq u_{t+k} \leq 1
\end{aligned}
$$

Time-of-Use Tariff

$$
c_{t+k} = \begin{cases} c_{day} \text{ at daytime 9h – 18h} \\ c_{day} \text{ at nightime 18h – 9h} \end{cases}
$$

Example: One-Zone Building

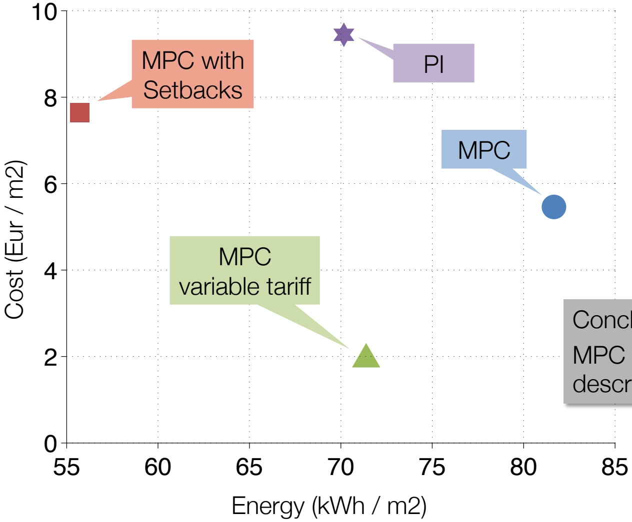
Time-of-use tariff



Annualized energy used: 71.3 kWh / m^2

Example: One-Zone Building

Annualized Comparison



Conclusion:

MPC is easy to tune / describe complex goals

Energy Efficient Building Control

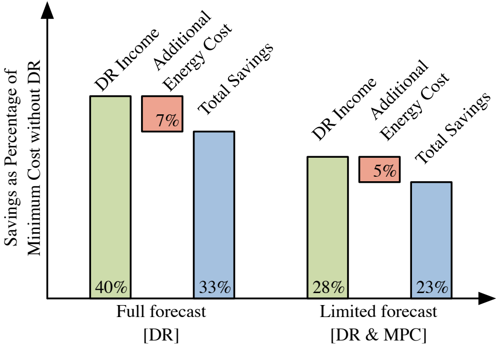
Playing the Market: New York Demand Response

Can bid ‘negawatts’ on open market - Paid to reduce consumption

Question: Reduce from what?!

Complex regulations define ‘baseline’: function of usage over x previous days

Can we ‘control’ our benchmark to gain income?



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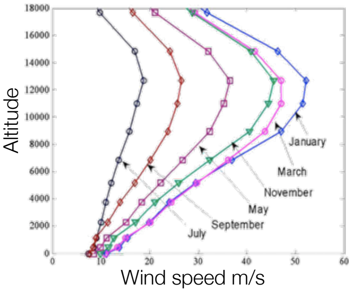
3. Applications

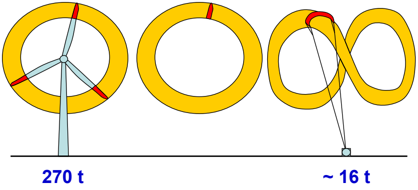
Kite Power

- Wind energy has potential to supply global energy need.
- Current wind technology is not able to exploit the potential
  - Traditional inland wind turbines are close to scaling limits
  - Economic operation only possible at a limited number of locations



Idea: Exploit the energy of high-altitude wind by means of light tethered wings (kites)

Goal: Wind power at lower cost than coal

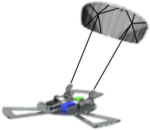


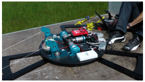
Exploit that

- Wind speed at 800m = 1.5 x speed at 80m
- Power density = $(\text{wind speed})^3$

Kite Power

- Different kites proposed: flexible vs. rigid wings (different models, nonlinear)
- On board vs. ground level generator
- Ground level seems to be more viable for large-scale
- Number of lines?

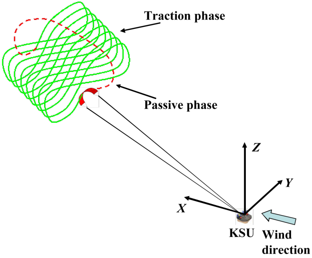




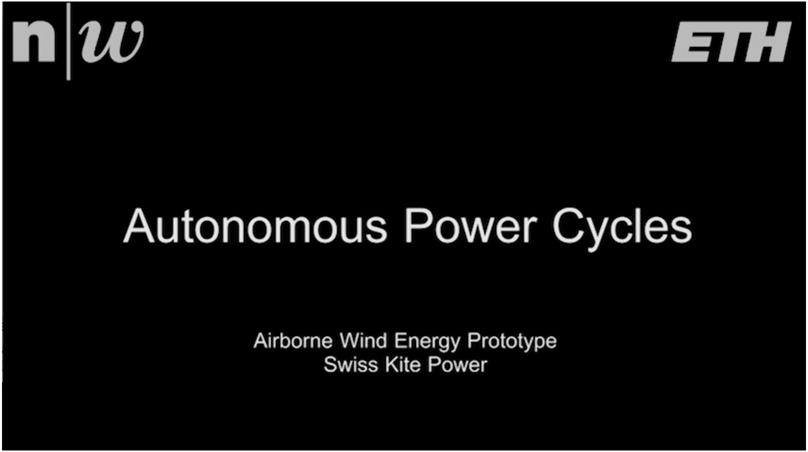
[A. Zgraggen, ETH, 2011]

Kite control problem:

- Maximize the net generated energy
- Maintain stability of the wing
- Exploit crosswind, i.e. kites fly transverse to wind at high speed
- Satisfy physical constraints: keep the kite far away from the ground, avoid line wrapping...
- Each configuration and working phase has its own performance goal



Kite Power

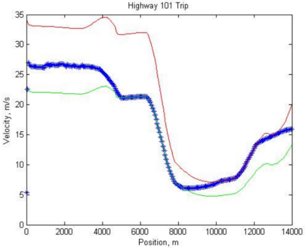


[*Airbone Wind Energy Group. ETH, 2013;* `http://control.ee.ethz.ch/~awe/`]

Outline

3. Applications

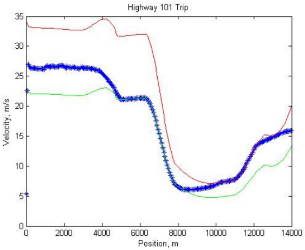
Audi Smart Engine



- **Fact:** Do not accelerate if there is a traffic jam, you will only waste fuel.
- **Idea:** Use traffic forecast to regulate the speed of a car to save fuel while getting to destination on time.

[*Khout, Borrelli and Hedrick. 15th World Congress on ITS, 2008*]

Audi Smart Engine



- MPC regulates the desired speed (through an Automatic Cruise Control) in order to reach the destination in the most fuel-efficient way, given a not-to-exceed arrival time.
- Min and Max traffic speed forecast and road grade used in the MPC constraints and model.
- Min and Max traffic speed forecast obtained from sensors embedded in the highway on each lane. (Available in the Bay Area, California).

[*Khout, Borrelli and Hedrick. 15th World Congress on ITS, 2008*]

Ford Autonomous Driving on Ice

- Autonomous double-lane change.
- Road forecast and nonlinear vehicle model (driving on ice) used in MPC.
- MPC controls differential braking and steering.
- Experimental results @ 72 km/h on ice.

[*Falcone, Borrelli et al. International Journal Vehicle Autonomous Systems, 2009*]

Volvo

- Autonomous lane keeping (minimally invasive).
- Road forecast and vehicle model used in MPC.
- MPC controls braking and steering.

[*Gray, Ali, Gao, Hedrick and Borrelli. IEEE Transactions on Intelligent Transportation Systems, 2013*]

Hyundai

- Autonomous Driving
- Road forecast and vehicle model used in MPC.
- MPC controls braking and steering.

[C:/Research/Courses/2014Fall/MPC_Course_Material/ME231A/
MPC-PartI/videos_mpg/media2.mp4]
GET MATERIAL

[*MPC Lab Project, UC Berkeley. 2010-2014;* http://www.mpc.berkeley.edu/]

Outline

3. Applications

Catalytic Cracker

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MPC-PartI/videos_mpg/catalytic_cracker.mpg]
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Outline

3. Applications

Predictive Control in NeuroScience

Outline

1. Optimization based Control
2. Concept of MPC
3. Applications
4. History of MPC
5. Summary
6. Literature & Acknowledgements

History of MPC

- **A. I. Propoi, 1963**, “Use of linear programming methods for synthesizing sampled-data automatic systems”, **Automation and Remote Control**.

- **J. Richalet et al., 1978** “Model predictive heuristic control- application to industrial processes”. **Automatica**, 14:413-428.
  - known as **IDCOM (Identification and Command)**
  - impulse response model for the plant, linear in inputs or internal variables (**only stable plants**)
  - quadratic performance objective over a finite prediction horizon
  - future plant output behavior specified by a reference trajectory
  - **ad hoc** input and output constraints
  - optimal inputs computed using a heuristic iterative algorithm, interpreted as the dual of identification
  - controller was not a transfer function, hence called **heuristic**

History of MPC

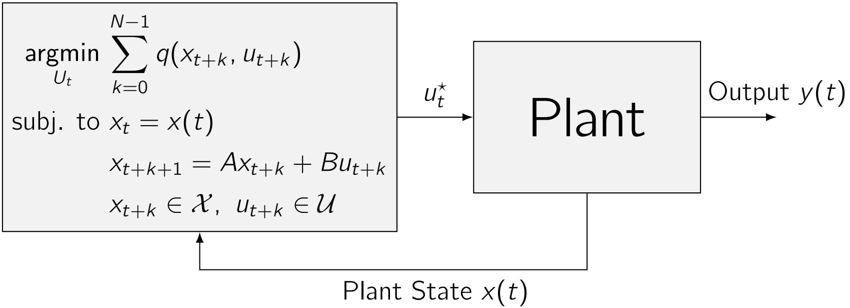
- 1970s: Cutler suggested MPC in his PhD proposal at the University of Houston in 1969 and introduced it later at Shell under the name Dynamic Matrix Control. **C. R. Cutler, B. L. Ramaker, 1979** "Dynamic matrix control – a computer control algorithm". **AICHE National Meeting**, Houston, TX.
  - successful in the petro-chemical industry
  - linear step response model for the plant
  - quadratic performance objective over a finite prediction horizon
  - future plant output behavior specified by trying to follow the set-point as closely as possible
  - input and output constraints included in the formulation
  - optimal inputs computed as the solution to a least–squares problem
  - **ad hoc** input and output constraints. Additional equation added online to account for constraints. Hence a **dynamic matrix** in the least squares problem.
- **C. Cutler, A. Morshedi, J. Haydel, 1983**. "An industrial perspective on advanced control". **AICHE Annual Meeting**, Washington, DC.
  - Standard QP problem formulated in order to systematically account for constraints.

History of MPC

- Mid 1990s: extensive theoretical effort devoted to provide conditions for guaranteeing feasibility and closed-loop stability
- 2000s: development of tractable robust MPC approaches; nonlinear and hybrid MPC; MPC for very fast systems
- 2010s: stochastic MPC; distributed large-scale MPC; economic MPC

Outline

Summary: MPC



At each sample time:

- Measure /estimate current state $x(t)$
- Find the **optimal input sequence** for the entire planning window N: $U_t^\star = \{u_t^\star, u_{t+1}^\star, \ldots, u_{t+N-1}^\star\}$
- Implement only the **first** control action $u_t^\star$

Summary

- Obtain a model of the system
- Design a state observer
- Define optimal control problem
- Set up optimization problem in optimization software
- Solve optimization problem to get optimal control sequence
- Verify that closed-loop system performs as desired,
  e.g., check performance criteria, robustness, real-time aspects,...

Important Aspects of Model Predictive Control

Main advantages:

- Systematic approach for handling **constraints**
- High **performance** controller

Main challenges:

- **Implementation**
  MPC problem has to be solved in real-time, i.e. within the sampling interval of the system, and with available hardware (storage, processor,...).
- **Stability**
  Closed-loop stability, i.e. convergence, is not automatically guaranteed
- **Robustness**
  The closed-loop system is not necessarily robust against uncertainties or disturbances
- **Feasibility**
  Optimization problem may become infeasible at some future time step, i.e. there may not exist a plan satisfying all constraints

Outline

Literature

Model Predictive Control:

- Predictive Control for linear and hybrid systems, F. Borrelli, A. Bemporad, M. Morari, 2017 Cambridge University Press
- Model Predictive Control: Theory and Design, James B. Rawlings, David Q. Mayne and Moritz M. Diehl, 2nd Edition, 2022, Nob Hill Publishing
- Predictive Control with Constraints, Jan Maciejowski, 2000 Prentice Hall

Optimization:

- Convex Optimization, Stephen Boyd and Lieven Vandenberghe, 2004 Cambridge University Press
- Numerical Optimization, Jorge Nocedal and Stephen Wright, 2006 Springer

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Parts of the slides in this lecture are based on or have been extracted from:

- Linear Dynamical Systems, Stephen Boyd, Stanford
- Convex Optimization, Stephen Boyd, Stanford
- Model Predictive Control, Manfred Morari, ETH Zurich
- Model Predictive Control, Colin Jones, EPFL
- Model Predictive Control, Francesco Borrelli, Berkeley