MATH 311W Homework #6

10 point problems Sally Student

14.1 This sample document provides a template for writing a LaTeX document suitable for homework assignments in Dr. Previte's Math 311W class. The first few "problems" will explain how the document works. Then there will be some "problems" that illustrate various notations and formatting environments. Compare what is in the typeset version of this document to the file `latexsample.tex`. Note in particular that anything typed after a percent sign in the text file is treated as a comment and is ignored by the compiler. Comments in the text file refer both to LaTeX and to hints about writing good solutions and proofs.

(b) Notice that you don't have to do every part of a problem. Here, we skipped part a.

(e) Now we skip to part e.

14.4 All of your problem solutions should be contained in an "enumerate" environment. Problems with parts will also need "enumerate" environments. See the comment in the text file for the general usage format for these environments. Please use these environments so your homework will be in the standard format for the class. Also please put the assignment number, the problem type and your name at the top of the page as they are in this sample document.

14.5 The very basics of LaTeX (compare the typeset document and the text file):

(a) Extra spaces in the text file do not appear in the typeset document.

Except for a double carriage-return, that makes a new line. Single carriage returns don't do anything.

(b) Mathematical expressions are typed between dollar signs like this: $y = x^2 + 1$. To make a centered equation on its own line, use double dollar signs, like this:

$$y = x^2 + 1.$$

The percent sign above the equation in the text file just makes it so that extra space is not added between the centered equation and the main paragraph above.

(c) Many LaTeX commands and math symbols start with a backslash symbol. For example, $\sin x$ and $\{x \in \mathbb{R} : x \geq 0\}$. Notice that the set-notation parentheses need to have backslashes before them (while regular parentheses do not). This is because in LaTeX, those squiggly parentheses often have other uses.

(d) If you need to put something in italics you do it *like this*. Or maybe you need to have something in **boldface**. Or maybe ***both***.

(e) Notice that to have quotes appear "correctly" in the typeset document you may have to type them yourself using the ` and ' keys instead of the " key.

15.3 Here is some random notation you might need:

x_2, x_{25}, x^2, x^{25}, ± 4, $x \neq 17$, $x > 5$, $x < 5$, $x \geq 5$, $x \leq 5$, $\{1, 2, 3\}$, $\{x : \sqrt{x} > 2\}$, ∞.

$A \subset B$, $A \subseteq B$, $A \not\subset B$, $A \not\subseteq B$, $A \setminus B$, A^c, $A \cap B$, $A \cup B$, $x \in A$, $x \notin A$, $|A|$, $\mathcal{P}(A)$, $\emptyset$.

$\stackrel{\mathcal{P}}{\equiv}$, $\frac{5}{1+x}$, $\dfrac{5}{1+x}$, $\bigcap_{i=1}^n S_i$, $\displaystyle\bigcap_{i=1}^n S_i$, $\bigcup_{i=1}^n S_i$, $\displaystyle\bigcup_{i=1}^n S_i$, $\sum_{k=1}^{10} a_k$, $\displaystyle\sum_{k=1}^{10} a_k$, $\prod_{k=1}^{10} a_k$, $\displaystyle\prod_{k=1}^{10} a_k$.

$\mathbb{R}$, $\mathbb{Q}$, $\mathbb{Z}$, $\mathbb{N}$, $\clubsuit$, $\diamondsuit$, $\heartsuit$, $\spadesuit$, $\rightarrow$, $\leftarrow$, $\leftrightarrow$, $\longrightarrow$, $\longleftarrow$, $\longleftrightarrow$, $\Rightarrow$, $\Leftarrow$, $\Leftrightarrow$, $\Longrightarrow$, $\Longleftarrow$, $\Longleftrightarrow$, $\mapsto$, $\longmapsto$.

$\mathcal{P}$, $\mathcal{S}$, $\mathcal{F}$, $\forall$, $\exists$, $\vee$, $\wedge$, $\neg$, $\sim$, $\approx$, $\equiv$, $\times$, $*$, $\star$, $a|b$, $|x|$, $\|x\|$, $\lceil x \rceil$, $\lfloor x \rfloor$, $\{x \in \mathbb{Z} \mid x \text{ is prime}\}$.

gcd, lcm, $\binom{n}{k}$, $\binom{n+1}{k}$, $a = \binom{n+1}{k}$, $\prec$, $\preceq$, $\succ$, $\succeq$, $f\colon [0, \infty) \to \mathbb{R}$, $f \circ g$ {, }, $, %, &, _, #.

15.9 Suppose 43 students take algebra, 32 take Spanish, 7 take both.

The number taking algebra or Spanish (or both, of course) is:

$$43 + 32 - 7 = 68.$$

(We have to subtract 7 because those students are counted twice.)

15.10 You don't have to do your truth tables in LaTeX; you may write them in by hand if you like. But in case you are interested, this is how to do it:

x	y	$x \wedge y$	$\neg(x \wedge y)$	$\neg y$	$\neg(x \wedge y) \wedge \neg y$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	F	T	F	F
F	F	F	T	T	T

Since the truth-values for $\neg y$ and $\neg(x \wedge y) \wedge \neg y$ are the same for all possible truth-values of x and y, the two statements are logically equivalent.

14.2 Prove that if x and y are both odd, then $x \equiv y \mod 2$.

Proof. Let x and y be odd integers. Then by definition of odd, there exist integers m and n such that $x = 2m + 1$ and $y = 2n + 1$. Observe

$$\begin{aligned} x - y &= (2m + 1) - (2n + 1) && \text{(since } x = 2m + 1 \text{ and } y = 2n + 1\text{)} \\ &= 2m + 1 - 2n - 1 \\ &= 2m - 2n \\ &= 2(m - n). && \text{(factor out a 2)} \end{aligned}$$

So there exists $k \in \mathbb{Z}$, namely $k = m - n$, such that $x - y = 2k$. Therefore, $2|(x - y)$. Hence, $x \equiv y \mod 2$.

□

Prove that if x and y are both even, then $x \equiv y \mod 2$.

Proof. Then by definition of even, there exist integers m and n such that $x = 2m$ and $y = 2n$. Observe

$$\begin{aligned} x - y &= 2m - 2n && \text{(since } x = 2m \text{ and } y = 2n\text{)} \\ &= 2(m - n). && \text{(factor out a 2)} \end{aligned}$$

So there exists $k \in \mathbb{Z}$, namely $k = m - n$, such that $x - y = 2k$. Therefore, $2|(x - y)$. Hence, $x \equiv y \mod 2$.

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