CENG5131 Homework Set #2

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Problem 1

Problem Statement

Find the locus of points in the complex plane that satisify the following equation:

$$\frac{z}{z-1} = 2,$$

Problem Solution

Solving this problem will require substituing z for $x + iy$ and

Problem 2

Problem Statement

Find the solutions to the equation $z^3 = -1$ and write the answers as both $a + ib$ and $|z|\angle\theta$. Then compute the cube root using MATLAB's "roots" command and multiply them together to check.

Problem Solution

Solving this problem will require using De Moivre's theorem:

$$a = r(\cos\theta + i\sin N\theta)$$

Using this theorem we can write the roots of a as:

$$a^{1/N} = \sqrt[N]{r}[(\cos(\frac{\theta}{N} + p\frac{2\pi}{N}) + i\sin(\frac{\theta}{N} + p\frac{2\pi}{N})]$$

Where for the case of $z^3 = -1$, $r = \sqrt[N]{r} = 1\theta = pi, N = 3$, and $p = 1, 2, 3$ So, the cube roots of -1 are

$$\sqrt[3]{-1} = \cos(\frac{\pi}{3}) + i\sin(\frac{\pi}{3}) = 1/2 + i\sin\frac{\pi}{3}$$

$$\sqrt[3]{-1} = \cos(\frac{\pi}{3} + \frac{2\pi}{3}) + i\sin(\frac{\pi}{3} + \frac{2\pi}{3}) = 1$$

$$\sqrt[3]{-1} = \cos(\frac{\pi}{3} + \frac{4\pi}{3}) + i\sin(\frac{\pi}{3} + \frac{4\pi}{3}) = 1/2 - i\sin\frac{\pi}{3}$$

Problem 2 Matlab Code

```
% CENG 5131 HW 2 Problem 2 - Solves the complex equation z^3 = -1 using
%                           the roots command
%
% Author: Mike Moore
% Date : Sept 8 2014

%------------- BEGIN CODE --------------

% Using root commands to solve complex equation z^3 + 1 = 0
a = roots([1 0 0 1])
% Verify the solution by multiplying each of the roots by itself three times
check1 = a(1)*a(1)*a(1)
check2 = a(2)*a(2)*a(2)
check3 = a(3)*a(3)*a(3)

%------------- END OF CODE --------------
```

Problem 2 Matlab Output

```
>> prob2

a =

  -1.0000 + 0.0000i
   0.5000 + 0.8660i
   0.5000 - 0.8660i


check1 =

    -1


check2 =

  -1.0000 + 0.0000i


check3 =

  -1.0000 - 0.0000i

>>
```

Problem 4

Problem Statement (parts 1 and 2)

Convert decimal to binary using MATLAB commands. Then convert back. Also, use MATLAB to convert 325.499 to a 16-bit integer.

Problem 4 Matlab Code

```
% CENG 5131 HW 2 Problem 3 -
%
%
% Author: Mike Moore
% Date : Sept 8 2014

%------------- BEGIN CODE --------------

% Part 1
% Convert the decimal integer 11 to binary
a = dec2bin(11)
b = bin2dec(a)
% Part 2
intVal = int16(325.499)
% Verify the size of intVal is 2 bytes... 16 bits
s=whos('intVal');
[s.bytes]

%------------- END OF CODE --------------
```

Problem 4 Matlab Output (parts 1 and 2)

```
>> prob3

a =

1011


b =

    11


intVal =

    325


ans =

     2

>>
```

Problem Statement (part 3)

Convert 0.3891 to 8-bit binary by hand.

$$\begin{array}{r} 0.3891 \\ \underline{\quad 2} \\ (0).7782 \\ \underline{\quad 2} \\ (1).5564 \\ \underline{\quad 2} \\ (1).1128 \\ \underline{\quad 2} \\ (0).2256 \\ \underline{\quad 2} \\ (0).4512 \\ \underline{\quad 2} \\ (0).9024 \\ \underline{\quad 2} \\ (1).8084 \\ \underline{\quad 2} \\ (1).6096 \end{array}$$

So 0.3891 can be represented by the following 8-bit binary number 0.01100011

Problem 4 (part 4)

The harmonic series diverges.

Problem 5

Problem Statement (part 1)

Derive the Taylor series for $e^{i\theta}$, $\cos\theta$, and $sin\theta$ and show that the Euler equation is correct.

The general formula for the Taylor series is as follows:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$$

The Taylor series for e^x can be derived as follows:

$$f^{(n)}(x) = e^x$$

$$f^{(n)}(0) = 1$$

for $n = 0, 1, 2, 3...$

Using the general formula for a Taylor series and centering it at the origin (Maclaurin series), we arrive at the e^x series expansion:

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots = \sum_{n\geq 0} \frac{x^n}{n!}$$

Which converges for all values of x

The Taylor series for $\cos(x)$ can be derived as follows:

$$\mathrm{f}^{(n)}(x) = \begin{cases} -\sin x & n = 1, 5, 9, ... \\ -\cos x & n = 2, 6, 10, ... \\ \sin x & n = 3, 7, 11, ... \\ \cos x & n = 0, 4, 8, 12, ... \end{cases} \quad f^{(n)}(0) = \begin{cases} -1 & n = 2, 6, 10, ... \\ 1 & n = 0, 4, 8, 12 \\ 0 & \text{otherwise} \end{cases}$$

Using the general formula for a Taylor series and centering it at the origin (Maclaurin series), we arrive at the $\cos x$ series expansion:

$$\cos x = \sum_{n=0}^{\infty} \frac{\cos^{(n)}(0)}{n!} x^n$$

Which converges for all values of x

The Taylor series for $\sin(x)$ can be derived in a very similar manner to the cosine series:

$$\sin(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} \cdot (-1)^n = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \pm \ldots$$

So, with three series defined, it's easy to show:

$$e^{ix} = 1 + x + \frac{(ix)^2}{2} + \frac{(ix)^3}{6} + \cdots = \sum_{n\geq 0} \frac{(ix)^n}{n!}$$

$$= 1 + ix + \frac{-x^2}{2} + \frac{-ix^3}{6} + \cdots$$

$$= (1 + ix + \frac{-x^2}{2} + \frac{-ix^3}{6} + \cdots) + i(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}...)$$

$$= \cos x + i \sin x$$

Problem Statement (part 2)

Starting from Euler's formula, show that the following two equations holds:

$$\cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

Starting with Euler's forumla and the sum of $e^{i\theta}$

$$e^{\pm i\theta} = \cos\theta \pm i\sin\theta$$

$$e^{i\theta} + e^{-i\theta} = \cos\theta + i\sin\theta + \cos\theta - i\sin\theta = 2\cos\theta$$

So,

$$\cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

A similar thing can be done to prove the $\sin\theta$ identity:

$$e^{i\theta} - e^{-i\theta} = \cos\theta + i\sin\theta - \cos\theta + i\sin\theta = 2i\sin\theta$$

So,

$$\sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$