CENG 5131 HW 7

Problem 1

Write the Fourier Series for a pulse train shifted by ~~t0~~ t_0 to the right

~~Old~~ Pulse Train Not Shifted

$$f(t) = \frac{A\tau}{T} + 2\frac{A\tau}{T}\sum_{n=1}^{\infty} \text{sinc}\left(\frac{n\omega_0\tau}{2}\right)\cos(n\omega_0 t)$$

Old Coeffs:

$$n > 0 \Rightarrow a_n = \text{sinc}\left(\frac{n\omega_0\tau}{2}\right)$$

$$a_n^{new} = a_n^{old} e^{-i\omega_0 t_0} = \text{sinc}\left(\frac{n\omega_0\tau}{2}\right) e^{-i\omega_0 t_0}$$

$$\cancel{a_n = \frac{A\tau}{T} e^{-i\omega_0 t_0}}$$

$$\boxed{f(t) = \frac{A\tau}{T} + \frac{2A\tau}{T}\sum_{n=1}^{\infty} \text{sinc}\left(\frac{n\omega_0\tau}{2}\right) e^{-i\omega_0 t_0}}$$

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Problem 2

~~Write the~~ Write the Fourier Series of the function:

$$x(t) = \alpha + \beta\cos(2\pi f_0 t) + \gamma\sin(2\pi f_0 t)$$

From the definition of the Fourier Series:

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n\cos(n w_0 t) + b_n\sin(n w_0 t)\right]$$

Where for this function, $x(t) = \alpha + \beta\cos(w_0 t) + \gamma\sin(w_0 t)$

$\alpha = a_0/2 \Rightarrow a_0 = 2\alpha$

$a_1 = \beta$

$b_1 = \gamma$ $\qquad w_0 = 2\pi f_0$

All other Fourier Coefficients are zero.

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Problem 3

a.)

$$\mathcal{F}[x(t)] = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt = X(\omega)$$

Differentiate both sides by ω:

$$\frac{d}{d\omega} \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt = \frac{d}{d\omega}[X(\omega)]$$

Differentiate under the integral sign:

$$\int_{-\infty}^{\infty} \frac{\partial}{\partial \omega}[x(t) e^{-i\omega t}] dt = \frac{d}{d\omega}[X(\omega)]$$

$$\int_{-\infty}^{\infty} -it\, x(t) e^{-i\omega t} dt = \frac{d}{d\omega}[X(\omega)]$$

$$-i \int_{-\infty}^{\infty} t\, x(t) e^{-i\omega t} dt = \frac{d}{d\omega}[X(\omega)]$$

Multiply both sides by i:

$$\int_{-\infty}^{\infty} t\, x(t) e^{-i\omega t} dt = i \frac{d}{d\omega}[X(\omega)]$$

So,

$$\mathcal{F}[t\, x(t)] = i \frac{d}{d\omega}[X(\omega)] \qquad \text{Q.E.D.}$$

b.)

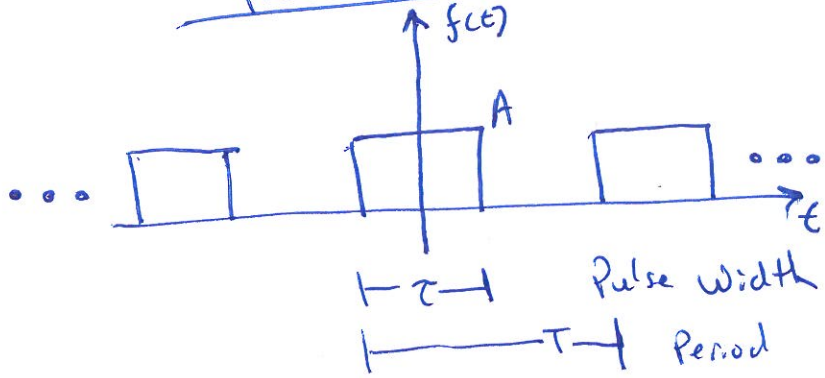
$$x(t) = 2t e^{-2t} u(t)$$

use: $\mathcal{F}[e^{-at} u(t)] = \frac{1}{a+i\omega} = \frac{1}{2+i\omega}$

$$i \frac{d}{d\omega}\left[\frac{1}{2+i\omega}\right] = \frac{-i^2}{(2+i\omega)^2} = \frac{1}{(2+i\omega)^2}$$

$$\boxed{X(\omega) = \frac{2}{(2+i\omega)^2}}$$

Problem 4



Note: Even function!

let $\omega_0 = 2\pi/T$

$$\alpha_n = \frac{1}{T}\int_{-T/2}^{T/2} A e^{-in\omega_0 t}\,dt = \frac{1}{T}\int_{-\tau/2}^{\tau/2} A e^{-in\omega_0 t}\,dt$$

$$\alpha_n = \frac{A\tau}{T}\,\mathrm{Sinc}(x) \qquad \text{w/ } x = \frac{n\omega_0\tau}{2}$$

And

$$f(t) = \frac{A\tau}{T} + \frac{2A\tau}{T}\sum_{n=1}^{\infty} \mathrm{sinc}(n\omega_0\tau/2)\cos(n\omega_0 t)$$

p 340 in book for MATLAB part!

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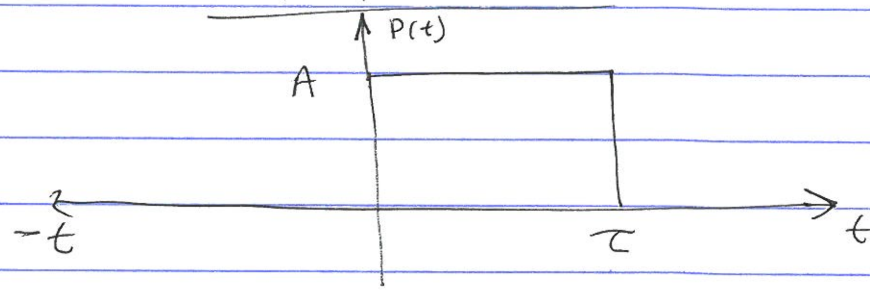
Problem 5



a.) $$p(t) = \begin{cases} A & 0 \le t \le \tau \\ 0 & \text{elsewhere} \end{cases}$$

Using results from textbook Ex 8.11

$$\boxed{P(i\omega) = \left[A\tau \operatorname{sinc}\left(\frac{\omega\tau}{2}\right)\right] e^{-i\omega\tau/2}}$$

b.) General 2nd order linear diff eqn

$$\ddot{y}(t) + \cancel{3} \, 2\zeta\omega_n \dot{y}(t) + \omega_n^2 y(t) = p(t)$$

$2\zeta\omega_n = 3 \qquad \omega_n^2 = 2 \qquad \omega_n = \sqrt{2}$

$\zeta = \frac{3}{2\sqrt{2}}$ Sub in

$$Y(i\omega) = P(i\omega) \frac{1}{\omega_n^2 \left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right] + i\left[\frac{2\zeta\omega}{\omega_n}\right]}$$

c.) $$|Y(i\omega)| = |P(i\omega)| \frac{1}{\omega_n^2} \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right] + \left[2\zeta \frac{\omega}{\omega_n}\right]^2}}$$

$$|Y(0)| = 0$$