Michael M. Moore

# CENG 5131 HW 6

## Problem 1

a.) Sum the following series:

$$1 + 1/2 + 1/4 + \dots$$

$$\sum_{n=0}^{n=\infty} \frac{1}{2^n} = \sum 1\left(1/2\right)^n = \frac{1}{1 - 1/2} = 2$$

The sum converges as long as $-1 < r < 1$ ... $r = 1/2$ ... so ✓

b.) Sum the following series

$$\pi + \frac{\pi}{\sqrt{2}} + \dots + \frac{\pi}{\sqrt{2^n}} + \dots$$

$$\sum_{n=0}^{n=\infty} \pi\left(\frac{1}{\sqrt{2}}\right)^n = \frac{\pi}{1 - \left(\frac{1}{\sqrt{2}}\right)} \approx 10.726$$

The sum converges because $-1 < r < 1$

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## Problem 2

a.) Find $x[n]$ by inverting the z-transform

$$X(x) = \frac{8z^3 + 2z^2 - 5z}{z^3 - 1.75z + .75}$$

$$X(z) = z\left[\frac{A}{(z+1.5)} + \frac{B}{(z-0.5)} + \frac{C}{(z-1)}\right]$$

$$8z^2 + 2z - 5 = A(z-0.5)(z-1) + B(z+1.5)(z-1) + \ldots$$
$$\ldots C(z+1.5)(z-0.5)$$

$z=1$

$$8 + 2 - 5 = C(5/4)$$
$$\boxed{C = 4}$$

$z=0.5$

$$8/4 + 4/4 - 20/4 = -B$$
$$\boxed{B = 2}$$

$z=-1.5$

$$8(9/4) + 2(-3/2) - 5 = A(-2)(-2.5)$$

$$18 - 8 = 5A$$
$$\boxed{2 = A}$$

$$X(z) = \frac{2z}{(z+1.5)} + \frac{2z}{(z-0.5)} + \frac{4z}{(z-1)}$$

$$f(n) = \left[4 + 2(1/2)^n + 2(-3/2)^n\right] u[n]$$

```
% Mike Moore
% CENG 5131
% HW 6
% Problem 2
% Description:
% Using Matlab to do an inverse z transform for problem 2
syms k z
F =  (8*z^3+2*z^2-5*z)/(z^3-1.75*z+0.75)

iztrans(F, z, k)

num = [8 2 -5 0]
den = [1 0 -1.75 0.75]
x=filter(num,den,[1 zeros(1,9)])

% output
% prob2
%
% F =
%
% (8*z^3 + 2*z^2 - 5*z)/(z^3 - (7*z)/4 + 3/4)
%
%
% ans =
%
% 2*(1/2)^k + 2*(-3/2)^k + 4
%
%
% num =
%
%      8     2    -5     0
%
%
% den =
%
%     1.0000          0   -1.7500    0.7500
%
%
% x =
%
%   Columns 1 through 5
%
%     8.0000    2.0000    9.0000   -2.5000   14.2500
%
%   Columns 6 through 10
%
%   -11.1250   26.8125  -30.1562   55.2656  -72.8828
```

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CENG 5131 HW6

## Problem 3

Find the first 5 values of e(k) from the inverse Z-transform by long division

$$E(z) = \frac{z}{z^2 - 3z + 2}$$

$$E(z) = \frac{z}{z^2 - 3z + 2} = e(0) + e(1)z^{-1} + e(2)z^{-2} + \dots$$

$$\begin{array}{r} 0 + 1 + 3 + 7 + 15 \\ z^2 - 3z + 2 \overline{)\, z} \\ -z + 3 - 2/z \\ \hline 3 - 2/z \\ -3 + 9/z - 6/z^2 \\ \hline 7/z - 6/z^2 \\ -7/z + 21/z^2 - 14/z^3 \\ \hline 15/z^2 - 14/z^3 \\ -15/z^2 + 45/z^3 - 30/z^4 \\ \hline 31/z^3 - 30/z^4 \end{array}$$

$$E(z) = \frac{1}{z} + \frac{3}{z^2} + \frac{7}{z^3} + \frac{15}{z^4}$$

$e(0) = 0$
$e(1) = 1$
$e(2) = 3$
$e(3) = 7$
$e(4) = 15$

$$e[n] = [2^n - 1]\,u[n]$$

```
% Mike Moore
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% Problem 3
% Description:
% Using Matlab to do an inverse z transform for problem 3
clear
syms k z
F =  (z)/(z^2-3*z+2)
iztrans(F, z, k)

num=[1 0 0];
den=[1 -3 2];
[K,Z,p]=residue(num,den)
x=filter(num,den,[1, zeros(1,5)])

% Output
% prob3
%
% F =
%
% z/(z^2 - 3*z + 2)
%
%
% ans =
%
% 2^k - 1
%
%
% K =
%
%      4
%     -1
%
%
% Z =
%
%      2
%      1
%
%
% p =
%
%      1
%
%
% x =
%
%      1     3     7    15    31    63
```

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CENG 5131 HW 6

## Problem 4

Solve the following difference equation using z-transforms:

$$y[n] - 0.5\,y[n-1] = 4u[n] \;; \quad y[-1] = 0$$

$b_0 = 1$, $a_0 = 1$, $a_1 = -1/2$

$$H(z) = \frac{y(z)}{X(z)} = \frac{1}{1 - \frac{1}{2z}} = \frac{z}{z - 1/2}$$

$$y(z) = X(z)\,H(z)$$

$$X(z) = \frac{4z}{z-1}$$

$$y(z) = \frac{4z}{z-1}\,\frac{z}{z-1/2}$$

$$y(z) = \frac{4 \cdot z^2}{z^2 - \frac{3}{2}z + \frac{1}{2}}$$

$$\frac{y(z)}{z} = \frac{4z}{z^2 - \frac{3}{2}z + 1/2} = \frac{A}{(z-1)} - \frac{B}{(z-1/2)}$$

P.F. Decomp $\quad A = 2 \quad B = -1$

$$y(z) = \frac{8z}{z-1} - \frac{4z}{z-1/2}$$

$$y[n] = [8 - 4(1/2)^n]\,u[n]$$

```
% Mike Moore
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% Problem 4
% Description:
% Using Matlab to verify the solution to the difference
% equation described in problem 4 (see handout)
clear; close all; clc
% declare filter input vector
x_in = 4*ones(1,11);
% initialize the filter output by applying intial conditions
y = [4, zeros(1,10)];

for n = 2:11
    % apply the filter programmatically
    y(n) = x_in(n) + 0.5*y(n-1);
end


syms k z
F =  (4*z^2)/((z-1)*(z-0.5));
iztrans(F, z, k);

num=[1];
den=[1 -1/2];
[K,Z,p]=residue(num,den);
x=filter(num,den,[4*ones(1,11)]);


% plot comparison
filter_output = [x' y']
figure(1)
stem(filter_output)
title('Stem plot of both filter outputs')
xlabel('time step')
ylabel('filter output')
legend('matlab filter', 'programmed filter')
legend
```

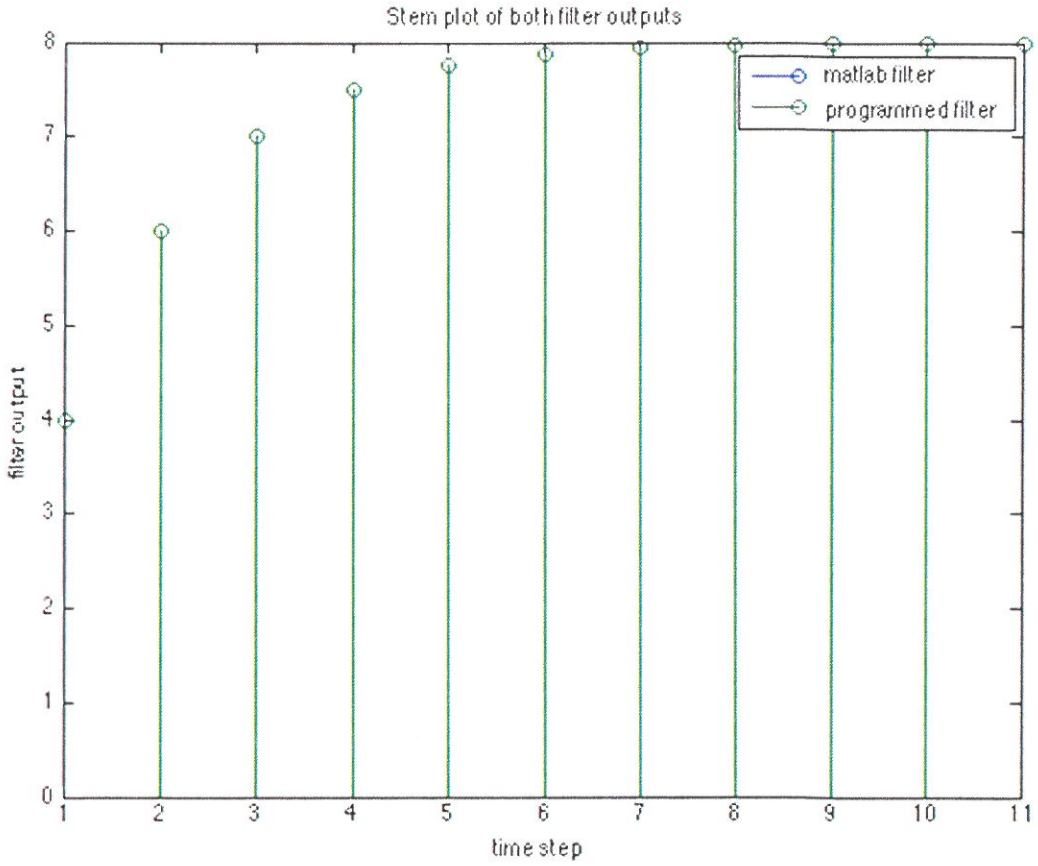
Stem plot of both filter outputs
matlab filter
programmed filter
filter output
time step
0
1
2
3
4
5
6
7
8
1
2
3
4
5
6
7
8
9
10
11

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CENG 5131 HW 6

Problem 5

$$H(z) = \frac{1}{10}\left[\frac{z}{z - 9/10}\right]$$

Find $H(z)$ for the following cases:

a.) $z = e^{i\,0.01}$

b.) $z = e^{i3}$

$$H(z) = \frac{1}{10}\left[\frac{e^{i\theta}}{e^{i\theta} - 9/10}\right]$$

$$H(z) = \frac{1}{10}\left[\frac{\cos(\theta) + i\sin(\theta)}{\cos(\theta) + i\sin(\theta) - 9/10}\right]$$

$$10\,H(z) = \left[\frac{\left(c^2\theta - 9/10\,c\theta + s^2\theta\right) + i\left(s\theta c\theta - 9/10\,s\theta - s\theta c\theta\right)}{\left(c\theta - 9/10\right)^2 + s^2\theta}\right]$$

Re Part

$$\left[\frac{c^2\theta - 9/10\,c\theta + s^2\theta}{\left(c\theta - 9/10\right)^2 + s^2\theta}\right]$$

Im Part

$$\left[\frac{-9/10\,s\theta}{\left(c\theta - 9/10\right)^2 + s^2\theta}\right]$$

```
% Mike Moore
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% HW 6
% Problem 5
% Description:
% Using Matlab to calculate the frequency response of a transfer function
% H(z) = (1/10)*[z/ (z-0.9)]
clear; close all; clc

num = [1/10 0]; den = [1 -0.9];
[h,w] = freqz(num,den);
freqz(num,den, [-pi:pi/150:pi])
% Print out values for when input frequency is 0.01 rad/sec
h(3)
% Print out values for when input frequency is 3 rad/sec
h(490)

% Output

% ans =
%
%    0.9873 - 0.1090i
%
%
% ans =
%
%    0.0526 - 0.0035i
```
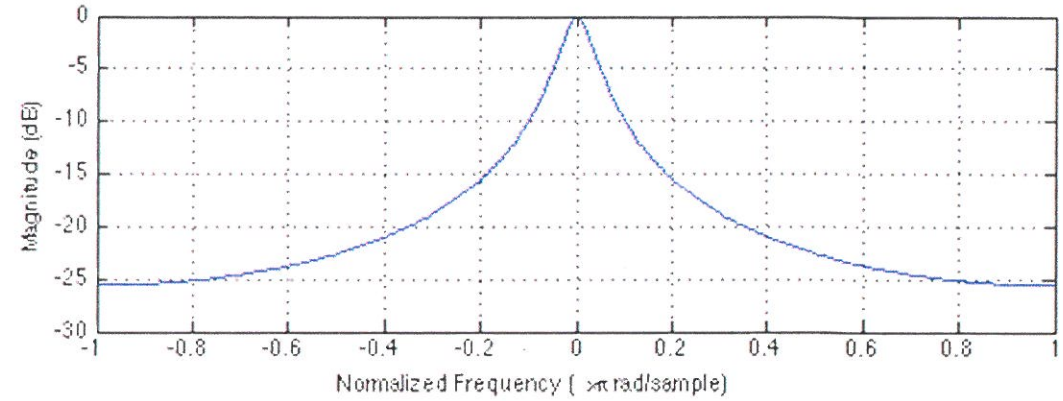
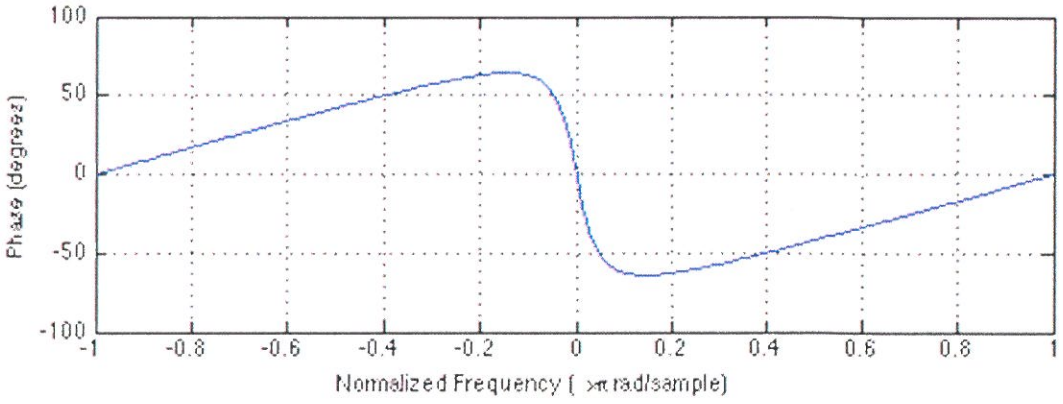


Magnitude (dB)
0
-5
-10
-15
-20
-25
-30
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
Normalized Frequency ( ×π rad/sample)


Phase (degrees)
100
50
0
-50
-100
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
Normalized Frequency ( ×π rad/sample)