Zeuthen strategy

Negotiation is a form of interaction in which a group of agents with conflicting objectives and a desire to cooperate, try to come to a mutually acceptable agreement on the division of scarce resources. This typically occurs when agents have competing claims on scarce resources, not all of which can be simultaneously satisfied. The use of the word “resources” here is to be taken in the broadest possible sense. Thus, resources can be commodities, services, time, money etc. In short, anything that is needed to achieve some objective.

Given a set of agents, a set of resources, an existing resource allocation and a set of other possible allocations, the main goal of negotiation is to find an allocation that is *better* in some sense, if such allocation exists. In order to achieve this goal, agents need some mechanism. Abstractly, a mechanism specifies the *rules of encounter*: what agents are allowed to say and when, how allocations are calculated, whether calculations are done using a centralised algorithm or in a distributed fashion, and so on.

An example mechanism is *bargaining*, where two agents exchange offers (i.e. suggestions about how to exchange resources) until one agent makes an offer that is acceptable by the other. In the bargaining problem, we say that each agent i has a utility function u_i defined over the set of all possible deals Δ. That is, $u_i : \Delta \to \mathbb{R}$. We also assume that there is a special deal δ^- which is the *no-deal* deal. Without loss of generality we will assume that for all agents $u_i(\delta^-) = 0$, i.e. the agents will prefer no deal than accepting any deal with negative utility. The problem then is finding a protocol *f* which will lead the agents to the best deal. But, as with all the game theory methods, it is not obvious which deal is the best one.

The *negotiation set* consists of the set of deals that are individually rational and Pareto optimal. The intuition behind the first constraint is that there is no purpose in proposing a deal that is less preferable to some agent than the conflict deal (as this agent would prefer conflict); the intuition behind the second condition is that there is no point in making a proposal if an alternative proposal could make some agent better off at nobody's expense.

The *Nash bargaining solution* is the deal that maximizes the product of the utilities. That is:

$$\delta = \underset{\delta'}{\operatorname{argmax}} \prod u_i(\delta')$$

Figure 1 shows a visualization of the Nash bargaining deal. Each curve represents all pairs of utilities that have the same product, that is, the line $y = c / x$. As we move northeast, following the line $y = x$, we cut across indifference curves of monotonically higher products. As such, the last deal to intersect a curve is the one which maximizes the product of the utilities, so it is the Nash bargaining solution.

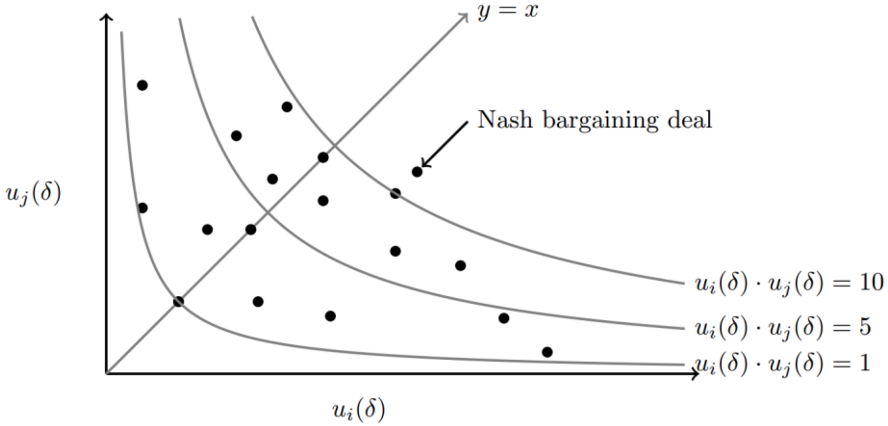

Figure 1. *Nash bargaining deal*

This idea is formalized in the *Zeuthen strategy* for the monotonic concession protocol. The idea is to measure an agent's *willingness to risk conflict*. Intuitively, an agent will be more willing to risk conflict if the difference in utility between its current proposal and the conflict deal is low.

In contrast, if the difference between the agent's current proposal and the conflict deal is high, then the agent has more to lose from conflict and is therefore less willing to risk conflict – and thus should be more willing to concede.

Agent i's willingness to risk conflict at round t, denoted $risk_i^t$, is measured in the following way:

$$risk_i^t = \frac{\text{utility } i \text{ loses by conceding and accepting } j\text{'s offer}}{\text{utility } i \text{ loses by not conceding and causing conflict}}$$

The numerator on the right-hand side of this equation is defined to be the difference between the utility to i of its current proposal, and the utility to i of j's current proposal; the denominator is defined to be the utility of agent i's current proposal. Until an agreement is reached, the value of $risk_i^t$ will be a value between 0 and 1. Higher values of $risk_i^t$ (nearer to 1) indicate that i has less to lose from conflict, and so is more willing to risk conflict. Conversely, lower values of $risk_i^t$ (nearer to 0) indicate that i has more to lose from conflict, and so is less willing to risk conflict.

Formally, we have:

$$risk_i^t = \begin{cases} 1 & \text{if } u_i(\delta_i^t) = 0 \\ \dfrac{u_i(\delta_i^t) - u_i(\delta_j^t)}{u_i(\delta_i^t)} & \text{otherwise} \end{cases}$$

The idea of assigning risk the value 1 if $u_i(\delta_i^t) = 0$ is that in this case, the utility to i of its current proposal is the same as from the conflict deal; in this case, i is completely willing to risk conflict by not conceding.

So, the Zeuthen strategy proposes that the agent to concede on round t of negotiation should be the one with the smaller value of risk.

The next question to answer is *how much should be conceded*? The simple answer to this question is just enough. If an agent does not concede enough, then on the next round, the balance of risk will indicate that it still has most to lose from conflict, and so should concede again. This is clearly inefficient. On the other hand, if an agent concedes too much, then it "wastes" some of its utility. Thus an agent should make the smallest concession necessary to change the balance of risk – so that on the next round, the other agent will concede. An agent can calculate the risks for both agents. The Zeuthen strategy tells us that the agent with the smallest risk should concede just enough so that it does not have to concede again in the next time step. That is, the agent that has the least to lose by conceding should concede. More formally, we define the new protocol as follows:

ZEUTHEN-MONOTONIC-CONCESSION

1 $\delta_i \leftarrow \arg\max_\delta u_i(\delta)$
2 Propose δ_i
3 Receive δ_j proposal
4 **if** $u_i(\delta_j) \geq u_i(\delta_i)$
5   **then** Accept δ_j
6 $\text{risk}_i \leftarrow \frac{u_i(\delta_i) - u_i(\delta_j)}{u_i(\delta_i)}$
7 $\text{risk}_j \leftarrow \frac{u_j(\delta_j) - u_j(\delta_i)}{u_j(\delta_j)}$
8 **if** $\text{risk}_i < \text{risk}_j$
9   **then** $\delta_i \leftarrow \delta'_i$ such that $\text{risk}_i(\delta'_i) > \text{risk}_j(\delta'_j)$
10   **goto** 2
11 **goto** 3

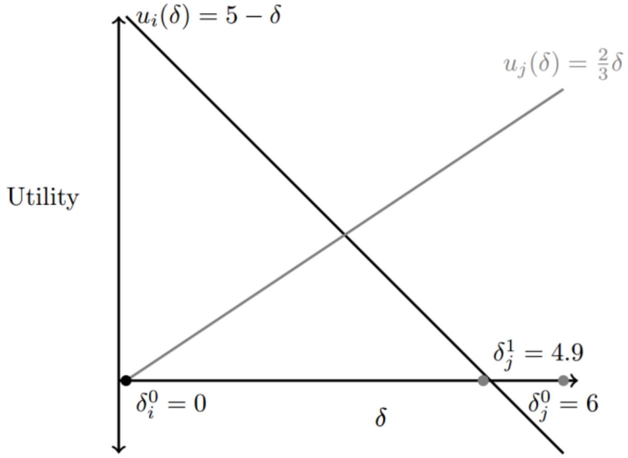

Figure 2. *Visualization of the Zeuthen strategy at work*

Figure 2 shows a graphical representation of the first step in a Zeuthen negotiation. The initial proposal are $\delta_i^0 = 0$ and $\delta_j^0 = 6$. After the initial proposal the agents calculate their risks to be

$$risk_i^0 = \frac{5-(-1)}{5} = \frac{6}{5}$$

and

$$risk_j^0 = \frac{4-0}{4} = 1$$

Since *j* has a lower risk, it must concede. The new deal must be such that *j* will not be forced to concede again. That is, it must insure that

$$risk_i = \frac{5-(5-\delta_j)}{5} < \frac{\frac{2}{3}\delta_j - 0}{\frac{2}{3}\delta_j} = risk_j$$

which simplifies to $\delta_j < 5$. As such, *j* can pick any deal *δ* less than 5. In the figure the agent chooses $\delta_j^1 = 4.9$.

The Zeuthen strategy is guaranteed to terminate and the agreement it reaches upon termination is guaranteed to be individually rational and Pareto optimal.

If both agents use the Zeuthen strategy, they will converge to a Nash bargaining solution deal, that is, a deal that maximizes the product of the utilities.

The Zeuthen strategy is also attractive because it is in a Nash equilibrium. That is, if one agent declares that it will be using the Zeuthen strategy then the other agent has nothing to gain by using any other negotiation strategy. As such, an agent that publicly states that it will be using the Zeuthen strategy can expect that every agent it negotiates with will also use the Zeuthen strategy. Thus, under the assumption that one agent is using the strategy the other can do no better than use it himself.

This is of particular interest to the designer of automated agents. It does away with any need for secrecy on the part of the programmer. An agent's strategy can be publicly known, and no other agent designer can exploit the information by choosing a different strategy. In fact, it is desirable that the strategy be known, to avoid inadvertent conflicts.

A generalization of this strategy in case of multilateral bargaining includes the proposals of all the other agents in the system:

$$risk_i = \frac{u_i(\delta_i) - \min\{u_i(\delta_k) \mid k \in A\}}{u_i(\delta_i)}.$$

References

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