Propositions and inference

Chapter 5

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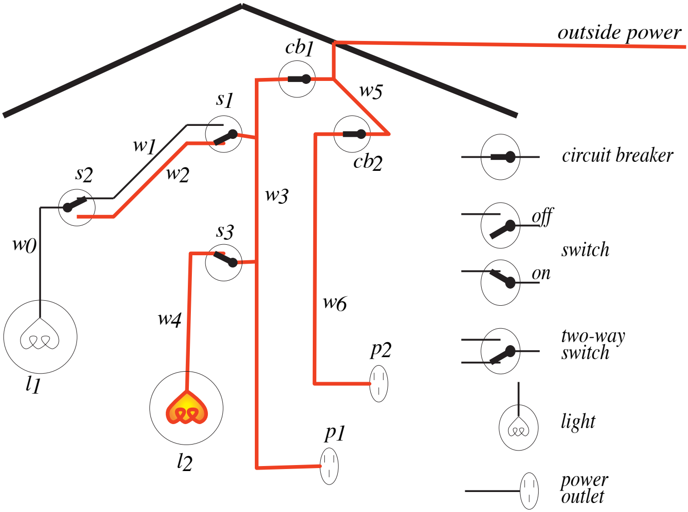
Electrical Environment



Representing the Electrical Environment

$$light\_l_1.$$
$$light\_l_2.$$
$$down\_s_1.$$
$$up\_s_2.$$
$$up\_s_3.$$
$$ok\_l_1.$$
$$ok\_l_2.$$
$$ok\_cb_1.$$
$$ok\_cb_2.$$
$$live\_outside.$$

$$lit\_l_1 \leftarrow live\_w_0 \wedge ok\_l_1$$
$$live\_w_0 \leftarrow live\_w_1 \wedge up\_s_2.$$
$$live\_w_0 \leftarrow live\_w_2 \wedge down\_s_2.$$
$$live\_w_1 \leftarrow live\_w_3 \wedge up\_s_1.$$
$$live\_w_2 \leftarrow live\_w_3 \wedge down\_s_1.$$
$$lit\_l_2 \leftarrow live\_w_4 \wedge ok\_l_2.$$
$$live\_w_4 \leftarrow live\_w_3 \wedge up\_s_3.$$
$$live\_p_1 \leftarrow live\_w_3.$$
$$live\_w_3 \leftarrow live\_w_5 \wedge ok\_cb_1.$$
$$live\_p_2 \leftarrow live\_w_6.$$
$$live\_w_6 \leftarrow live\_w_5 \wedge ok\_cb_2.$$
$$live\_w_5 \leftarrow live\_outside.$$

Role of semantics

In computer:

$$light1\_broken \leftarrow sw\_up \wedge power \wedge unlit\_light1.$$

$$sw\_up.$$

$$power \leftarrow lit\_light2.$$

$$unlit\_light1.$$

$$lit\_light2.$$

In user's mind:

- $light1\_broken$: light #1 is broken
- sw_up: switch is up
- $power$: there is power in the building
- $unlit\_light1$: light #1 isn't lit
- lit_light2: light #2 is lit

Conclusion: $light1\_broken$

- The computer doesn't know the meaning of the symbols
- The user can interpret the symbol using their meaning

Simple language: propositional definite clauses

- An atom is a symbol starting with a lower case letter
- A body is an atom or is of the form $b_1 \wedge b_2$ where b_1 and b_2 are bodies.
- A definite clause is an atom or is a rule of the form $h \leftarrow b$ where h is an atom and b is a body.
- A knowledge base is a set of definite clauses

Semantics

- An interpretation I assigns a truth value to each atom.
- A body $b_1 \wedge b_2$ is true in I if b_1 is true in I and b_2 is true in I.
- A rule $h \leftarrow b$ is false in I if b is true in I and h is false in I. The rule is true otherwise.
- A knowledge base KB is true in I if and only if every clause in KB is true in I.

Models and Logical Consequence

- A model of a set of clauses is an interpretation in which all the clauses are *true*.
- If KB is a set of clauses and g is a conjunction of atoms, g is a logical consequence of KB, written $KB \models g$, if g is *true* in every model of KB.
- That is, $KB \models g$ if there is no interpretation in which KB is *true* and g is *false*.

Simple Example

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

How many interpretations?

	p	q	r	s
I_1	*true*	*true*	*true*	*true*
I_2	*false*	*false*	*false*	*false*
I_3	*true*	*true*	*false*	*false*
I_4	*true*	*true*	*true*	*false*
I_5	*true*	*true*	*false*	*true*

model of KB?

Which of p, q, r, s logically follow from KB?

Simple Example

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

How many interpretations?

	p	q	r	s	model of KB?
I_1	*true*	*true*	*true*	*true*	yes
I_2	*false*	*false*	*false*	*false*	no
I_3	*true*	*true*	*false*	*false*	yes
I_4	*true*	*true*	*true*	*false*	yes
I_5	*true*	*true*	*false*	*true*	no

Which of p, q, r, s logically follow from KB?
$KB \models p$, $KB \models q$, $KB \not\models r$, $KB \not\models s$

Proof procedures

- A proof procedure is a - possibly non-deterministic - algorithm for deriving consequences of a knowledge base.
- Given a proof procedure, $KB \vdash g$ means g can be derived from knowledge base KB.
- Recall $KB \models g$ means g is true in all models of KB.
- A proof procedure is sound if $KB \vdash g$ implies $KB \models g$.
- A proof procedure is complete if $KB \models g$ implies $KB \vdash g$.

Bottom-up proof procedure

Rule of derivation (a generalized form of *modus ponens*):

> *If "$h \leftarrow b_1 \wedge \ldots \wedge b_m$" is a clause in the knowledge base, and each b_i has been derived, then h can be derived.*

This is forward chaining on this clause.
(This rule also covers the case when $m = 0$.)

Bottom-up proof procedure

$KB \vdash g$ if $g \in C$ at the end of this procedure:

$C := \{\}$;
repeat
    select clause "$h \leftarrow b_1 \wedge \ldots \wedge b_m$" in KB such that
        $b_i \in C$ for all i, and
        $h \notin C$;
    $C := C \cup \{h\}$
until no more clauses can be selected.

Example

$$a \leftarrow b \wedge c.$$
$$a \leftarrow e \wedge f.$$
$$b \leftarrow f \wedge k.$$
$$c \leftarrow e.$$
$$d \leftarrow k.$$
$$e.$$
$$f \leftarrow j \wedge e.$$
$$f \leftarrow c.$$
$$j \leftarrow c.$$

Soundness of bottom-up proof procedure

If $KB \vdash g$ then $KB \models g$.

- Suppose there is a g such that $KB \vdash g$ and $KB \not\models g$.
- Then there must be a first atom added to C that isn't true in every model of KB. Call it h. Suppose h isn't *true* in model I of KB.
- There must be a clause in KB of form

  $$h \leftarrow b_1 \wedge \ldots \wedge b_m$$

  Each b_i is true in I. h is false in I. So this clause is false in I. Therefore I isn't a model of KB.
- Contradiction.

Fixed Point

- The C generated at the end of the bottom-up algorithm is called a fixed point.
- Let I be the interpretation in which every element of the fixed point is true and every other atom is false.
- I is a model of KB.
  Proof: suppose $h \leftarrow b_1 \wedge \ldots \wedge b_m$ in KB is false in I. Then h is false and each b_i is true in I. Thus h can be added to C. Contradiction to C being the fixed point.
- I is called a Minimal Model.

Completeness of bottom-up proof procedure

If $KB \models g$ then $KB \vdash g$.

- Suppose $KB \models g$. Then g is true in all models of KB.
- Thus g is true in the minimal model.
- Thus g is in the fixed point.
- Thus g is generated by the bottom up algorithm.
- Thus $KB \vdash g$.

Top-down proof procedure

Idea: search backward from a query to determine if it is a logical consequence of KB.

An answer clause is of the form:

$$yes \leftarrow a_1 \wedge a_2 \wedge \ldots \wedge a_m$$

The *SLD Resolution* of this answer clause on atom a_i with the clause:

$$a_i \leftarrow b_1 \wedge \ldots \wedge b_p$$

is the answer clause

$$yes \leftarrow a_1 \wedge \ldots \wedge a_{i-1} \wedge b_1 \wedge \cdots \wedge b_p \wedge a_{i+1} \wedge \cdots \wedge a_m.$$

An answer is an answer clause with $m = 0$. That is, it is the answer clause $yes \leftarrow$.

Derivations

A **derivation** of query "$?q_1 \wedge \ldots \wedge q_k$" from KB is a sequence of answer clauses $\gamma_0, \gamma_1, \ldots, \gamma_n$ such that

- γ_0 is the answer clause $yes \leftarrow q_1 \wedge \ldots \wedge q_k$,
- γ_i is obtained by resolving γ_{i-1} with a clause in KB, and
- γ_n is an answer.

Top-down proof procedure

To solve the query $?q_1 \wedge \ldots \wedge q_k$:

> $ac :=$ "$yes \leftarrow q_1 \wedge \ldots \wedge q_k$"
> **repeat**
>> **select** atom a_i from the body of ac
>> **choose** clause C from KB with a_i as head
>> replace a_i in the body of ac by the body of C
>
> **until** ac is an answer.

Nondeterministic Choice

- Don't-care nondeterminism If one selection doesn't lead to a solution, there is no point trying other alternatives. select
- Don't-know nondeterminism If one choice doesn't lead to a solution, other choices may. choose

Example: successful derivation

$$
\begin{array}{lll}
a \leftarrow b \wedge c. & a \leftarrow e \wedge f. & b \leftarrow f \wedge k. \\
c \leftarrow e. & d \leftarrow k. & e. \\
f \leftarrow j \wedge e. & f \leftarrow c. & j \leftarrow c.
\end{array}
$$

Query: $?a$

$$
\begin{array}{llll}
\gamma_0: & yes \leftarrow a & \gamma_4: & yes \leftarrow e \\
\gamma_1: & yes \leftarrow e \wedge f & \gamma_5: & yes \leftarrow \\
\gamma_2: & yes \leftarrow f & & \\
\gamma_3: & yes \leftarrow c & &
\end{array}
$$

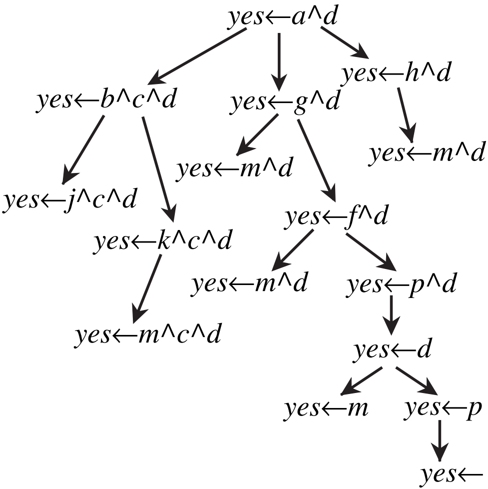
Example: failing derivation

$$
\begin{array}{lll}
a \leftarrow b \wedge c. & a \leftarrow e \wedge f. & b \leftarrow f \wedge k. \\
c \leftarrow e. & d \leftarrow k. & e. \\
f \leftarrow j \wedge e. & f \leftarrow c. & j \leftarrow c.
\end{array}
$$

Query: $?a$

$$
\begin{array}{llll}
\gamma_0 : & yes \leftarrow a & \gamma_4 : & yes \leftarrow e \wedge k \wedge c \\
\gamma_1 : & yes \leftarrow b \wedge c & \gamma_5 : & yes \leftarrow k \wedge c \\
\gamma_2 : & yes \leftarrow f \wedge k \wedge c & & \\
\gamma_3 : & yes \leftarrow c \wedge k \wedge c & &
\end{array}
$$

Search Graph for SLD Resolution



$a \leftarrow b \wedge c.$	$a \leftarrow g.$
$a \leftarrow h.$	$b \leftarrow j.$
$b \leftarrow k.$	$d \leftarrow m.$
$d \leftarrow p.$	$f \leftarrow m.$
$f \leftarrow p.$	$g \leftarrow m.$
$g \leftarrow f.$	$k \leftarrow m.$
$h \leftarrow m.$	$p.$
$?a \wedge d$	