Extra Credit

March 26, 2022

Chapter 3

Def 3.1.1

Let f be a fn. w/ doman $D \subseteq \mathbb{R}$. Then f has a limit as x approaches infinity iff $\exists L \in \mathbb{R}$ s.t. for every $\mathcal{E} > 0, \exists M \in \mathbb{R}^+$ s.t. $|f(x) - L| < \mathcal{E}$, if $x \geq M$ and $x \in D$. If such an L exists, then L is called the limit of the fn f as x tends to infinity and we write $\lim_{x\to\infty} f(x) = L$

Def 3.1.2

If $\lim_{x\to\infty} f(x) = L$, then the line $y = L$ is called a horizontal asymptote for the function f.

Thm 3.1.6

Suppose that $D \subseteq \mathbb{R}$ is an unbounded above domain of the function f ; that is, D contains arbitrarily large values. Then, $\lim_{x\to\infty} f(x) = L$ iff for every sequence $\{x_n\}$ in D that diverges to plus infinity, that is, $\lim_{n\to\infty} x_n = \infty$, the sequence $\{f(x_n)\}$ converges to L.

Thm 3.1.7

Suppose that the functions f, g, and h are defined on $D \subseteq \mathbb{R}$, which is unbounded above, with $\lim_{x\to\infty} f(x) = A, \lim_{x\to\infty} g(x) = B$, and $\lim_{x\to\infty} h(x) = C$. Then

(a) $\lim_{x\to\infty} f(x)$ is unique

(b) f must be eventually bounded above and below

(c) $\lim_{x\to\infty}[f(x) - A] = 0$

(d) $\lim_{x\to\infty} |f(x)| = |\lim_{x\to\infty} f(x)| = |A|$

(e) $\lim_{x\to\infty}(f \pm g)(x) = \lim_{x\to\infty} f(x) \pm \lim_{x\to\infty} g(x) = A \pm B$

(f) $\lim_{x\to\infty}(fg)(x) = [\lim_{x\to\infty} f(x)][\lim_{x\to\infty} g(x)] = AB$

(g) $\lim_{x\to\infty}[f(x)]^n = [\lim_{x\to\infty} f(x)]^n = A^n, \forall n \in \mathbb{N}$

(h) $\lim_{x\to\infty}(f/g)(x) = \dfrac{A}{B}\ if B \neq 0$

(i) $\lim_{x\to\infty}\sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to\infty} f(x)} = \sqrt[n]{A}$ if $A \geq 0$ & $f(x) \geq 0\ \forall x \in D$, with $n \in \mathbb{N}$

(j) $A \leq B\ if\ f(x) \leq g(x)$ eventually for $x \in D$

(k) $A \leq B \leq C$ if $f(x) \leq g(x) \leq h(x)$ eventually for $x \in D$. This property is called the sandwich (or squeeze) theorem

Thm 3.1.8

If the function f is defined on an unbounded above domain $D \subseteq \mathbb{R}$ and is eventually monotone and eventually bounded, then $\lim_{x\to\infty} f(x)$ is finite.

Def 3.1.9

Let f be a function with domain $D \subseteq \mathbb{R}$, which contains arbitrarily large values. We say that f tends to plus infinity as x tends to $+\infty$ iff for any real $K > 0$, there exists a real number $M > 0$ such that $f(x) > K$ provided that $x \geq M$ and $x \in D$. Whenever this is the case, we write $\lim_{x\to\infty} f(x) = +\infty$

Def 3.1.10

Let f be a function with domain $D \subseteq \mathbb{R}$, which contains arbitrarily large negative values. Then $\lim_{x\to-\infty} f(x) = L$ iff for every $\epsilon > 0$ there exists a real number $M > 0$ such that $|f(x) - L| < \epsilon$ if $x \leq M$ and $x \in D$

Def 3.2.1

Suppose that a function $f : D \to \mathbb{R}$, and suppose that a is an accumulation point of D. The function f has a limit as x approaches (or as x tends to) a iff there exists a real number L such that for every $\epsilon > 0$ there exists a real number $\delta > 0$ such that

$$|f(x) - L| < \epsilon \text{ provided that } 0 < |x - a| < \delta \text{ and } x \in D$$

write $\lim_{x\to a} f(x) = L$

Thm 3.2.5

Suppose that functions $f, g, h : D \to \mathbb{R}$, with $D \subseteq \mathbb{R}$, a is an accumulation point of D,$\lim_{x\to a} f(x) = A$,$\lim_{x\to a} g(x) = B$, and $\lim_{x\to a} h(x) = C$. Then all of the conclusions for Theorem 3.1.7 are true with ∞ replaced by a and with "eventually" replaced by "near $x = a$."

Thm 3.2.6

Let the function f be defined on some deleted neighborhood D of the real number a. The following two statements are equivalent.

(a) $\lim_{x\to a} f(x) = L$

(b) For every sequence $\{x_n\}$ converging to $x = a$, with $x_n \in D$ and $x_n \neq a$ eventually, the sequence $\{f(x_n)\}$ converges to L

Def 3.2.12

Suppose that the function $f : D \to \mathbb{R}$ with D a subset of $\mathbb{R}$ and a an accumulation point of D. Then the function f tends to plus infinity as x approaches, tends to, a iff for any given real number $K > 0$, there exists $\delta > 0$ such that $f(x) > K$, provided that $0 < |x - a| < \delta$ and $x \in D$. Write $\lim_{x\to a} f(x) = +\infty$

Thm 3.2.14

Let the functions f and g be defined on some deleted neighborhood of $x = a$. If $\lim_{x\to a} f(x) = L > 0$ and $\lim_{x\to a} g(x) = +\infty$, then $\lim_{x\to a}(fg)(x) = +\infty$.

Def 3.3.1

Suppose that the function $f : D \to \mathbb{R}$, with D a subset of $\mathbb{R}$ and a an accumulation point of the set $D \cap (a, \infty) = \{x \in D | x > a\}$ Then the function f has a right-hand limit (limit from the right) as x approaches, tends to, a iff there exists a real number L such that for every $\epsilon > 0$ there exists a positive real number $\delta > 0$ such that

$$|f(x) - L| < \epsilon \text{ provided that } 0 < x - a < \delta \text{ and } x \in D$$

we write $\lim_{x \to a^+} f(x) = L$

Def 3.3.2

Suppose that the function $f : D \to \mathbb{R}$, with D a subset of and a an accumulation point of $D \cap (a, \infty)$. Then the function f tends to infinity as x approaches, tends to, a from the right iff for any given real number $K > 0$, there exists a positive $\delta > 0$ such that $f(x) > K$, provided that $0 < x - a < \delta$ and $x \in D$. We write $\lim_{x \to a^+} f(x) = +\infty$

Def 3.3.4

If the limit from the right or from the left at $x = a$ of a function f is infinite, meaning $+\infty$or $-\infty$, then the line $x = a$ is called a vertical asymptote.

Thm 3.3.7

Let a function f be defined for $x \in (0, a)$, with $a > 0$ a real number. If

$$\lim_{x \to 0^+} f(x) \text{ or } \lim_{t \to \infty} f\left(\frac{1}{t}\right)$$

Chapter 4

Def 4.1.1

Def 4.1.2

Sequential Criterion for Continuity Thm

Def 4.1.6

Thm 4.1.7

Thm 4.1.8

Thm 4.1.9

Def 4.2.1

Def 4.2.3

Def 4.2.5

Def 4.2.7

Def 4.3.1

Def 4.3.2

Thm 4.3.3

Thm 4.3.4

Thm 4.3.5

Thm 4.3.6

Def 4.3.7

Cor 4.3.8

Cor 4.3.9