

# Chapter 1

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# PROBABILITY

## Intro

Probability is a measure of one's belief in the occurrence of future events

e.g.

a policy holder making a claim in the next year

## How to assign probabilities?

1. subjectively
2. relative frequency
  - repeat the study condition (identical) for  $M$  times
  - record times event occurred ( $m$ )
  - probability = relative frequency =  $m / M$  where  $\lim_{M \rightarrow \infty} m/M = P(A)$
3. Axiomatic Model Based
  - use mathematical theory to assign probabilities (what we will learn)

## Notation

Def : A random experiment is an experiment that produces outcomes that can't be predicted w/ certainty

Def : A performance of such experiment : trial

Def : A result : Outcome

Def : Sample Space : set of all possible outcomes of an experiment

- Denote  $S$

Ex : Toss a coin 3 times.

$$S = \left\{ \begin{array}{l} HHH \ HHT \ HTH \ HTT \\ THH \ THT \ TTH \ TTT \end{array} \right\}$$

Ex : number of positive tests during a week.  $S = \{0, 1, 2, \dots\}$

Ex : highest temp. on labor day.  $S = \{40 < \omega < 120\}$

Ex : An event is a subset of sample space,  $S$ , as  $(A, B, C \dots) \subseteq S$

Def :  $\omega$  : "omega" an outcome occurs

- if  $A$  contains  $\omega$ , we say  $A$  occurred
- if  $A$  does not contain  $\omega$ ,  $A$  did not occur

i.e.  $\omega \in A$ ,  $A$  occurred,  $\omega \notin A$ ,  $A$  did not occur

e.g. Toss coin 3 times.  $A$  = getting @ least 2 heads

$A = \{HHH, HHT, HTH, THH\}$ . If I toss the coin 3 times and get  $HTH$ , then  $A$  occurred if I get  $TTH$ ,  $A$  did not occur

## SET THEORY

- $A$  is a subset of  $S$  if  $\omega \in A \implies \omega \in S$
- Union :  $A \cup B = \{\omega : \omega \in A \text{ or } \omega \in B\}$
- intersection :  $A \cap B = \{\omega : \omega \in A \text{ and } \omega \in B\}$
- complement :  $\overline{A} = \{\omega \in S : \omega \notin A\}$

Notate :  $A_1 \cap A_2 \cap \dots \cap A_n = \bigcap_{i=1}^n A_i$

Notate :  $A_1 \cup A_2 \cup \dots \cup A_n = \bigcup_{i=1}^n A_i$

Notate :  $\emptyset$  = null event (empty set)

Notate :  $S$  = sure event (always occur)

Def : An event is called elementary (simple) event if it contains only 1 outcome

Def : mutually exclusive (disjoint)

- $A \cap B = \emptyset \implies$  no common outcome

$A_1, A_2, \dots, A_k$  are all mutually exclusive if they are pairwise mutually exclusive i.e.

$A_i \cap A_j = \emptyset$  for all  $i \neq j$

Ex : Toss a coin 3 times

$A$  : at least 2 heads

$$A = \{HHH, HHT, HTH, THH\}$$

$B$  : 1st toss = tails

$$B = \{THH, THT, TTH, TTT\}$$

$$A \cap B = \{THH\} \implies \text{not mutually exclusive}$$

$$A \cup B = \{HHH, HHT, HTH, THH, THT, TTH, TTT\}$$

$$\overline{A \cup B} = \{HTT\}$$

$$\overline{A} = \{HTT\}$$

$$\overline{B} = \{HTT, HHH, HHT, HTH\}$$

### DeMorgan's Law

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

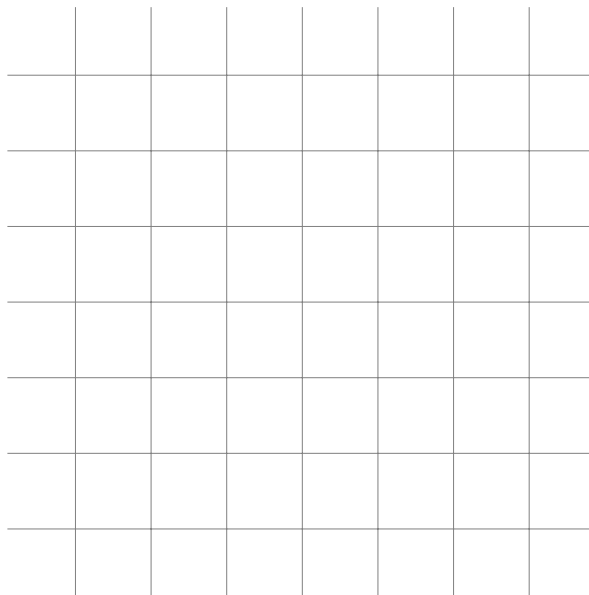
$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

### Definition of Probability

$S$  Sample Space - all outcomes of an experiment

$A$  event,  $A \subseteq S$

$B$  = set of all possible events



$P$  = probability set function if

$$P : \mathcal{B} \rightarrow [0 : 1]$$