Chapter 1

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Probability

Intro

Probability is a measure of one's belief in the occurrence of future events

e.g.

a policy holder making a claim in the next year

How to assign probabilities?

1. subjectively
2. relative frequency
    - repeat the study condition (identical) for M times
    - record times event occurred (m)
    - probability = relative frequency = m / M where $\lim_{M\to\infty} m/M = P(A)$
3. Axiomatic Model Based
    - use mathematical theory to assign probabilities (what we will learn)

Notation

Def : A random experiment is an experiment that produces outcomes that can't be predicted w/ certainty

Def : A performance of such experiment : trial

Def : A result : Outcome

Def : Sample Space : set of all possible outcomes of an experiment

- Denote S

Ex : Toss a coin 3 times.

$$S = \left\{ \begin{matrix} \text{HHH HHT HTH HTT} \\ \text{THH THT TTH TTT} \end{matrix} \right\}$$

Ex : number of positive tests during a week. $S = \{0, 1, 2, \ldots\}$

Ex : highest temp. on labor day. $S = \{40 < \omega < 120\}$

Ex : An event is a subset of sample space, S, as $(A, B, C \ldots) \subseteq S$

Def : ω : "omega" an outcome occurs

- if A contains ω, we say A occurred
- if A does not contain ω, A did not occur

i.e. $\omega \in A, A$ occurred, $\omega \notin A, A$ did not occur

e.g. Toss coin 3 times. $A =$ getting @ least 2 heads

$A = \{HHH, HHT, HTH, THH\}$. If I toss the coin 3 times and get HTH, then A occurred if I get TTH, A did not occur

Set Theory

- A is a subset of S if $\omega \in A \implies \omega \in S$

- Union : $A \cup B = \{\omega : \omega \in A \text{ or } \omega \in B\}$

- intersection : $A \cap B = \{\omega : \omega \in A \text{ and } \omega \in B\}$

- complement : $\overline{A} = \{\omega \in S : \omega \notin A\}$

Notate : $A_1 \cap A_2 \cap \ldots \cap A_n = \bigcap_{i=1}^{n} A_i$

Notate : $A_1 \cup A_2 \cup \ldots \cup A_n = \bigcup_{i=1}^{n} A_i$

Notate : $\emptyset$ = null event (empty set)

Notate : S = sure event (always occur)

Def : An event is called elementary (simple) event if it contains only 1 outcome

Def : mutually exclusive (disjoint)

- $A \cap B = \emptyset \implies$ no common outcome

$A_1, A_2, \ldots, A_k$ are all mutually exclusive if they are pairwise mutually exclusive i.e. $A_i \cap A_j = \emptyset$ for all $i \neq j$

Ex : Toss a coin 3 times

A : at least 2 heads

$$A = \{HHH, HHT, HTH, THH\}$$

B : 1st toss = tails

$$B = \{THH, THT, TTH, TTT\}$$

$$A \cap B = \{THH\} \implies \text{not mutually exclusive}$$

$$A \cup B = \{HHH, HHT, HTH, THH, THT, TTH, TTT\}$$

$$\overline{A \cup B} = \{HTT\}$$

$$\overline{A} = \{HTT\}$$

$$\overline{B} = \{HTT, HHH, HHT, HTH\}$$

DeMorgan's Law

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$