# Chapter 1

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# Probability

## Intro

Probability is a measure of one's belief in the occurrence of future events

**e.g.**

a policy holder making a claim in the next year

## How to assign probabilities?

1. subjectively
2. relative frequency
   - repeat the study condition (identical) for M times
   - record times event occurred (m)
   - probability = relative frequency = m / M where $\lim_{M\to\infty} m/M = P(A)$
3. Axiomatic Model Based
   - use mathematical theory to assign probabilities (what we will learn)

## Notation

Def : A random experiment is an experiment that produces outcomes that can't be predicted w/ certainty

Def : A performance of such experiment : trial

Def : A result : Outcome

Def : Sample Space : set of all possible outcomes of an experiment

- Denote $S$

Ex : Toss a coin 3 times.

$$S = \begin{Bmatrix} \text{HHH HHT HTH HTT} \\ \text{THH THT TTH TTT} \end{Bmatrix}$$

Ex : number of positive tests during a week. $S = \{0, 1, 2, \ldots\}$

Ex : highest temp. on labor day. $S = \{40 < \omega < 120\}$

Ex : An event is a subset of sample space, $S$, as $(A, B, C \ldots) \subseteq S$

Def : $\omega$ : "omega" an outcome occurs

- if $A$ contains $\omega$, we say $A$ occurred

- if $A$ does not contain $\omega$, $A$ did not occur

i.e. $\omega \in A, A$ occurred, $\omega \notin A, A$ did not occur

e.g. Toss coin 3 times. $A$ = getting @ least 2 heads

$A = \{HHH, HHT, HTH, THH\}$. If I toss the coin 3 times and get $HTH$, then $A$ occurred if I get $TTH$, $A$ did not occur

# Set Theory

- $A$ is a subset of $S$ if $\omega \in A \implies \omega \in S$
- Union : $A \cup B = \{\omega : \omega \in A \text{ or } \omega \in B\}$
- intersection : $A \cap B = \{\omega : \omega \in A \text{ and } \omega \in B\}$
- complement : $\overline{A} = \{\omega \in S : \omega \notin A\}$

Notate : $A_1 \cap A_2 \cap \ldots \cap A_n = \bigcap_{i=1}^{n} A_i$

Notate : $A_1 \cup A_2 \cup \ldots \cup A_n = \bigcup_{i=1}^{n} A_i$

Notate : $\emptyset$ = null event (empty set)

Notate : $S$ = sure event (always occur)

Def : An event is called elementary (simple) event if it contains only 1 outcome

Def : mutually exclusive (disjoint)

- $A \cap B = \emptyset \implies$ no common outcome

$A_1, A_2, \ldots, A_k$ are all mutually exclusive if they are pairwise mutually exclusive i.e. $A_i \cap A_j = \emptyset$ for all $i \neq j$

Ex : Toss a coin 3 times

A : at least 2 heads

$$A = \{HHH, HHT, HTH, THH\}$$

B : 1st toss = tails

$$B = \{THH, THT, TTH, TTT\}$$

$$
\begin{aligned}
A \cap B &= \{THH\} \implies \text{ not mutually exclusive} \\
A \cup B &= \{HHH, HHT, HTH, THH, THT, TTH, TTT\} \\
\overline{A \cup B} &= \{HTT\} \\
\overline{A} &= \{HTT\} \\
\overline{B} &= \{HTT, HHH, HHT, HTH\}
\end{aligned}
$$

**DeMorgan's Law**

$$
\begin{aligned}
\overline{A \cup B} &= \overline{A} \cap \overline{B} \\
\overline{A \cap B} &= \overline{A} \cup \overline{B}
\end{aligned}
$$