# Chapter 2

## Logan | Nick

## 2.1

**Def**   Experiment : any activity / situation w/ uncertainty about which $x \geq 2$ outcomes are possible

- coin toss, roll a die, draw a card

**Def**

**Sample Space**

- collection of all possible outcomes of a chance experiment

Notate - s or $\mathcal{U}$

- Toss a coin : heads or tails

**Def**

**Event :**

- any collection of outcomes from a sample space of a chance experiment

Notate

CAP letters : $A, B, C, \ldots$

**Def**   Simple Event : event that consists of <u>one</u> outcome

Compound Event : event that consists of <u>more than one</u> outcome

**Ex**   Tennis : A tennis shop carries 5 brands of rackets (Head, Prince, Sazenger, Wimbledon, Wilson). Each racket comes in midsize / oversize

a. sample space

*insert diagram here*

b. Let A be the event an oversized racket is piurchased

$$A = \{HO, PO, SO, WimO, WilO\}$$

c. Let B be the event the name brand starts w/ a W

$$B = \{WimM, WimO, WilM, WilO\}$$

## Forming New Sets

Let A and B be any 2 events

**Def**  Complement of A :

- all outcomes in S, not in A

**Notate**   $A', \overline{A}, A^c$

**Notate**   union - A or B - inclusive

$$A \cup B$$

intersection - A and B

$$A \cap B$$

**Ex**   Tennis Cont.

d. $\overline{B}$ = brand does not start w/ W

$$\overline{B} = \{HO, HM, PO, PM, SO, SM\}$$

e. Head, Prince, and Wilson are US companies. Let C define rackets from the U.S.

$$C = \{HO, HM, PO, PM, WilO, WilM\}$$
$$B \cup C = \{HO, HM, PO, PM, WilO, WilM, WimO, WimM\}$$

f. List outcomes in $B \cap C$.

$$B \cap C = \{WilO, WilM\}$$

g. $\overline{(B \cap C)} = \{HO, HM, PO, PM, WimO, WimM\}$

## Two Mutually Exclusive Events

**Def**   mutually exclusive : no outcomes in common

**Def**   Disjoint : no outcomes in common

*include figure here*

**Note**   If A and B are disjoint, $A \cap B = \emptyset$

*include figure here*

**Ex**

$$A = \{4, 6, 8, 10, 12\} \quad B = \{8, 10, 12, 14\} \quad C = \{12, 14, 16\} \quad D = \{16, 18\}$$
$$A \cap B = \{8, 10, 12\}$$
$$B \cap C = \{12, 14\}$$
$$A \cap (C \cap D) = A \cap \{16\} = \emptyset$$
$$A \cap C = \{12\}$$
$$B \cap D = \{\} = \emptyset$$
$$(A \cap B) \cup C = \{8, 10, 12\} \cup C = \{8, 10, 12, 14, 16\}$$
$$(A \cap B) \cup (B \cap C) = \{8, 10, 12\} \cup \{12, 14\} = \{8, 10, 12, 14\}$$

# 2.2 Classical Probability

- N equal likely outcomes
- each outcome has probability $\frac{1}{N}$.

**Notate**   $P(E)$ is the probability of event $E$

$$P(E) = \frac{\text{\# of distinct outcomes in E}}{\text{\# of outcomes in sample space}}$$

**Empirical Probability**

- conduct a chance experiment and count the # of times E occurs
- N is the # of times the experiment was conducted

$\underline{\text{Estimate}}$ of $P(E) = \dfrac{\text{\# of times E occurs}}{N}$

## Law of Large Number's :

As the number of repetitions of an experiment increases, the chance that the relative frequency of occurrences of an event will differ from the true probability of an event approaches 0.

- Problem : How large is large enough?

**Note :**   long run stabilization will also occur

*insert graph here*

## Subjective Probability

- a personal measure / belief
    - can be based on evaluation of facts & personal experience

**Ex**   At a hospital, there were 645 boys born and 721 girls born

a. Find : experimental probability of having a girl

$$P(girl) = \frac{721}{645 + 721} \approx .53 = 53\%$$

b. John is @ a cookout and wants to get a drink from the cooler. The cooler contains 12 sodas, 10 waters, & 5 beers.

- Find P(water)

$$P(water) = \frac{10}{12 + 10 + 5} \approx .37 = 37\%$$

c. 2 dive rolls : what is $P(secondroll > 1^{st} roll)$

- using drawing of all probabilities $P(2^{nd} > 1^{st}) = 15/36 \approx .42 = 42\%$

## Basic Properties

1. For any event, the probability it will occur is b/n 0 and 1

2. If $S(Samplespace)$, then $P(S) = 1$

3. $P(\emptyset) = 0$

4. If 2 events $A$ & $B$ are <u>mutually exclusive</u>, then $P(A \cup B) = P(A) + P(B)$

5. For any event, $E$, the $P(E) + P(\overline{E}) = 1$
    - $P(\overline{E}) = 1 - P(E)$ & $P(E) = 1 - P(\overline{E})$

**Ex**

1. $P($ roll 2 die & not get doubles $) = 1 - P($ roll doubles $) = 1 - \frac{6}{36} = 5/6 \approx .83$

2. Find the probability of drawing a Jack or a diamond from a standard 52 card deck

$$P(J \cup \Diamond) = P(\Diamond) + P(J) - P(J \cap \Diamond) = \frac{13 + 4 - 1}{52} = \frac{16}{52}$$

4

**Note**

$$P(J \cup \Diamond) \neq P(\Diamond) + P(J)$$

3. When rolling 2 die, what is prob. of rolling @ most 1 even number?

$$P(@ \text{ most one even}) = 1 - P(\text{both even}) = 1 - 9/36 = .75$$

## Union

If $A_1, A_2, \ldots, A_n$ is a collection of disjoint (mutually exclusive) events, then

$$P(A_1, A_2, \ldots, A_n) = \sum_{i=1}^{\infty} P(A_i)$$

| # of CD's purchased | 1 | 2 | 3 | 4 | 5 | 6+ |
|---|---|---|---|---|---|---|
| prob | .45 | .25 | .1 | .1 | .07 | .03 |

**Ex**

a. $P(CD \leq 3) = P(1) + P(2) + P(3) = .45 + .25 + .1 = .8$

b. Prob the next purchase is at most 3 CD's $= P(\# \leq 3)P(1) + P(2) + P(3) = .45 + .25 + .1 = .8$

c. What is the prob the next customer buys 5 or more $= P(\# \geq 5) = .07 + .03 = .1$

d. What is $P(\# \geq 2)$ and what does it represent?

$$P(\# \geq 2) = 1 - P(1) = 1 - .45 = .55,$$

which is the event the next customer buys 2 or more CD's

**Def** General Addition Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

*insert graph here*

## 2.3

For a multistage experiment w/ $n$ stages where the first stage has $k_1$ outcomes, the second $k_2$ outcomes, ..., the total number of possible outcomes for the sequence is the multiplication rule.

# Multiplication Rule

$$k_1 \cdot k_2 \cdot \ldots \cdot k_n$$

**Ex**  Determine # of 5 digit zip codes (digits repeat, first can't be 0)

$$\underbrace{9 \text{ choices } (1 \text{ - } 9)}_{1^{st} \text{ digit}} \cdot \underbrace{10 \ (0 \text{ - } 9)}_{2^{nd} \text{ digit}} \cdot \underbrace{10 \ (0 \text{ - } 9)}_{3^{rd} \text{ digit}} \cdot \underbrace{10 \ (0 \text{ - } 9)}_{4^{th} \text{ digit}} \cdot \underbrace{10 \ (0 \text{ - } 9)}_{5^{th} \text{ digit}}$$

$$9 \cdot 10 \cdot 10 \cdot 10 \cdot 10$$

**Ex**  odd six digit pins if digits can't be repeated

$$5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 5 \Leftarrow \text{5th digit must be odd}$$