Chapter 2

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2.1

Def Experiment : any activity / situation w/ uncertainty about which $x \geq 2$ outcomes are possible

- coin toss, roll a die, draw a card

Def

Sample Space

- collection of all possible outcomes of a chance experiment

Notate - s or $\mathcal{U}$

- Toss a coin : heads or tails

Def

Event :

- any collection of outcomes from a sample space of a chance experiment

Notate

CAP letters : $A, B, C, \ldots$

Def Simple Event : event that consists of one outcome

Compound Event : event that consists of more than one outcome

Ex Tennis : A tennis shop carries 5 brands of rackets (Head, Prince, Sazenger, Wimbledon, Wilson). Each racket comes in midsize / oversize

a. sample space

insert diagram here

b. Let A be the event an oversized racket is piurchased

$$A = \{HO, PO, SO, WimO, WilO\}$$

c. Let B be the event the name brand starts w/ a W

$$B = \{WimM, WimO, WilM, WilO\}$$

Forming New Sets

Let A and B be any 2 events

Def Complement of A :

- all outcomes in S, not in A

Notate $A', \overline{A}, A^c$

Notate union - A or B - inclusive

$$A \cup B$$

intersection - A and B

$$A \cap B$$

Ex Tennis Cont.

d. $\overline{B}$ = brand does not start w/ W

$$\overline{B} = \{HO, HM, PO, PM, SO, SM\}$$

e. Head, Prince, and Wilson are US companies. Let C define rackets from the U.S.

$$C = \{HO, HM, PO, PM, WilO, WilM\}$$
$$B \cup C = \{HO, HM, PO, PM, WilO, WilM, WimO, WimM\}$$

f. List outcomes in $B \cap C$.

$$B \cap C = \{WilO, WilM\}$$

g. $\overline{(B \cap C)} = \{HO, HM, PO, PM, WimO, WimM\}$

Two Mutually Exclusive Events

Def mutually exclusive : no outcomes in common

Def Disjoint : no outcomes in common

include figure here

Note If A and B are disjoint, $A \cap B = \emptyset$

include figure here

Ex

$$
\begin{aligned}
A = \{4,6,8,10,12\} \quad B = \{8,10,12,14\} \quad C = \{12,14,16\} \quad D = \{16,18\} \\
A \cap B &= \{8,10,12\} \\
B \cap C &= \{12,14\} \\
A \cap (C \cap D) &= A \cap \{16\} = \emptyset \\
A \cap C &= \{12\} \\
B \cap D &= \{\} = \emptyset \\
(A \cap B) \cup C &= \{8,10,12\} \cup C = \{8,10,12,14,16\} \\
(A \cap B) \cup (B \cap C) &= \{8,10,12\} \cup \{12,14\} = \{8,10,12,14\}
\end{aligned}
$$

2.2 Classical Probability

- N equal likely outcomes
- each outcome has probability $\frac{1}{N}$.

Notate $P(E)$ is the probability of event E

$$P(E) = \frac{\text{\# of distinct outcomes in E}}{\text{\# of outcomes in sample space}}$$

Empirical Probability

- conduct a chance experiment and count the # of times E occurs
- N is the # of times the experiment was conducted

Estimate of $P(E) = \dfrac{\text{\# of times E occurs}}{N}$

Law of Large Number's :

As the number of repetitions of an experiment increases, the chance that the relative frequency of occurrences of an event will differ from the true probability of an event approaches 0.

- Problem : How large is large enough?

Note : long run stabilization will also occur

insert graph here

Subjective Probability

- a personal measure / belief
  - can be based on evaluation of facts & personal experience

Ex At a hospital, there were 645 boys born and 721 girls born

a. Find : experimental probability of having a girl

$$P(girl) = \frac{721}{645 + 721} \approx .53 = 53\%$$

b. John is @ a cookout and wants to get a drink from the cooler. The cooler contains 12 sodas, 10 waters, & 5 beers.

- Find P(water)

$$P(water) = \frac{10}{12 + 10 + 5} \approx .37 = 37\%$$

c. 2 dive rolls : what is $P(secondroll > 1^{st}roll)$

- using drawing of all probabilities $P(2^{nd} > 1^{st}) = 15/36 \approx .42 = 42\%$

Basic Properties

1. For any event, the probability it will occur is b/n 0 and 1
2. If $S(Samplespace)$, then $P(S) = 1$
3. $P(\emptyset) = 0$
4. If 2 events A & B are <u>mutually exclusive</u>, then $P(A \cup B) = P(A) + P(B)$
5. For any event, E, the $P(E) + P(\overline{E}) = 1$
   - $P(\overline{E}) = 1 - P(E)$ & $P(E) = 1 - P(\overline{E})$

Ex

1. $P($ roll 2 die & not get doubles $) = 1 - P($ roll doubles $) = 1 - \frac{6}{36} = 5/6 \approx .83$
2. Find the probability of drawing a Jack or a diamond from a standard 52 card deck

$$P(J \cup \diamondsuit) = P(\diamondsuit) + P(J) - P(J \cap \diamondsuit) = \frac{13 + 4 - 1}{52} = \frac{16}{52}$$

Note $P(J \cup \diamondsuit) \neq P(\diamondsuit) + P(J)$

3. When rolling 2 die, what is prob. of rolling @ most 1 even number?

$$P(\text{@ most one even}) = 1 - P(\text{both even}) = 1 - 9/36 = .75$$

Union

If $A_1, A_2, \ldots, A_n$ is a collection of disjoint (mutually exclusive) events, then

$$P(A_1, A_2, \ldots, A_n) = \sum_{i=1}^{\infty} P(A_i)$$

# of CD's purchased	1	2	3	4	5	6+
prob	.45	.25	.1	.1	.07	.03

Ex

a. $P(CD \leq 3) = P(1) + P(2) + P(3) = .45 + .25 + .1 = .8$

b. Prob the next purchase is at most 3 CD's $= P(\# \leq 3) P(1) + P(2) + P(3) = .45 + .25 + .1 = .8$

c. What is the prob the next customer buys 5 or more $= P(\# \geq 5) = .07 + .03 = .1$

d. What is $P(\# \geq 2)$ and what does it represent?

$$P(\# \geq 2) = 1 - P(1) = 1 - .45 = .55,$$

which is the event the next customer buys 2 or more CD's

Def General Addition Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

insert graph here

2.3

For a multistage experiment w/ n stages where the first stage has k_1 outcomes, the second k_2 outcomes, ..., the total number of possible outcomes for the sequence is the multiplication rule.

Multiplication Rule

$$k_1 \cdot k_2 \cdot \ldots \cdot k_n$$

Ex Determine # of 5 digit zip codes (digits repeat, first can't be 0)

$$\frac{\text{9 choices (1 - 9)}}{1^{st}\text{ digit}} \cdot \frac{\text{10 (0 - 9)}}{2^{nd}\text{ digit}} \cdot \frac{\text{10 (0 - 9)}}{3^{rd}\text{ digit}} \cdot \frac{\text{10 (0 - 9)}}{4^{th}\text{ digit}} \cdot \frac{\text{10 (0 - 9)}}{5^{th}\text{ digit}}$$
$$9 \cdot 10 \cdot 10 \cdot 10 \cdot 10$$