Chapter 1

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1.1 Populations, Samples, and Processes

Statistics provides methods for organizing and summarizing data and for drawing conclusions from that data

Def Data : a collection of facts

Def Population : A well defined collection of objects for which we wish to obtain info

Def Census : When desired info is obtained from every member of the population

- problems : Time, money, practical

Def Sample : A subset of the population

1. You want the home price in Edwardsville

- Fewer well trained appraisers gives better results than many poorly trained

2. Tree Age Study

Testing is destructive, so a sample is better

Def variable : any characteristic whose clue may differ from one subject to another.

- denote with low letters

Note

- Don't say $McDonald's = 10$
- Do say $x =$ the length of the tibia bone in 10 year old boys.

Def univariate data : result from making observations of 1 variable

- these variable can be qualitative / quantitative

Def Bivariate data : when observations are made on each of 2 variables for each individual

- (weight.mpg) of cars

Def Multivariate data : observations made on many variables

- patient data

Ex Labor force, sample 60,000, find population + sample

- population = labor force, sample size = 60,000 households

Branches of Stats

1. Descriptive Stats : data are collected and you wish to summarize and describe features of the data (graphs, numerical summaries)
2. Inferential stats : data is collected from a sample and used to draw a conclusion about the population
    - confidence intervals, hypothesis test, prediction, etc…

Types of sampling

- Simple random sampling : random choice / draw of the hat sampling
- Systematic sampling : selecting every k^{th} member of the population
- Cluster sampling : divide population into groups, then select some of these groups @ random
- Stratified sampling : divide population into groups. Find subgroups of groups (strata) and then draw random sample in strata
- Convenience sampling : sampling in the most convenient way
    - best to avoid , but a good starter

Notate

sample size : n

- For a dataset with n observations on some variable x, the individual observations will be denoted as $x_1, x_2, \ldots, x_n$.

1.2

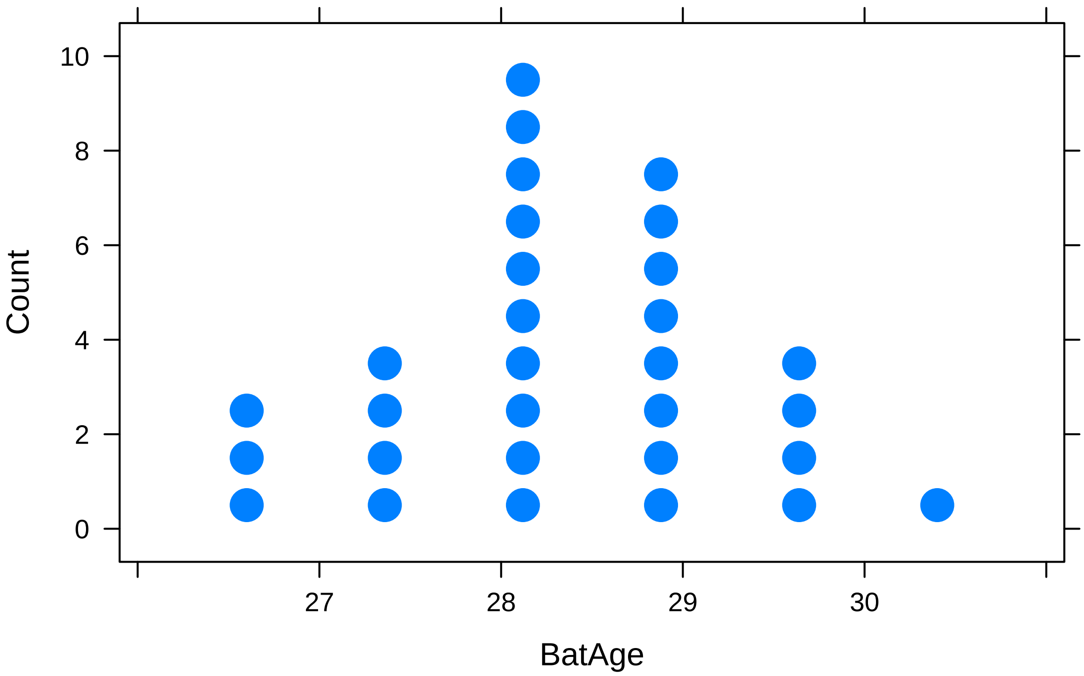
Stem and leaf plots

Ex $(54, 59, 35, 41, 46, 25, 47, 60, 54, 46, 49, 46, 41, 34, 22)$

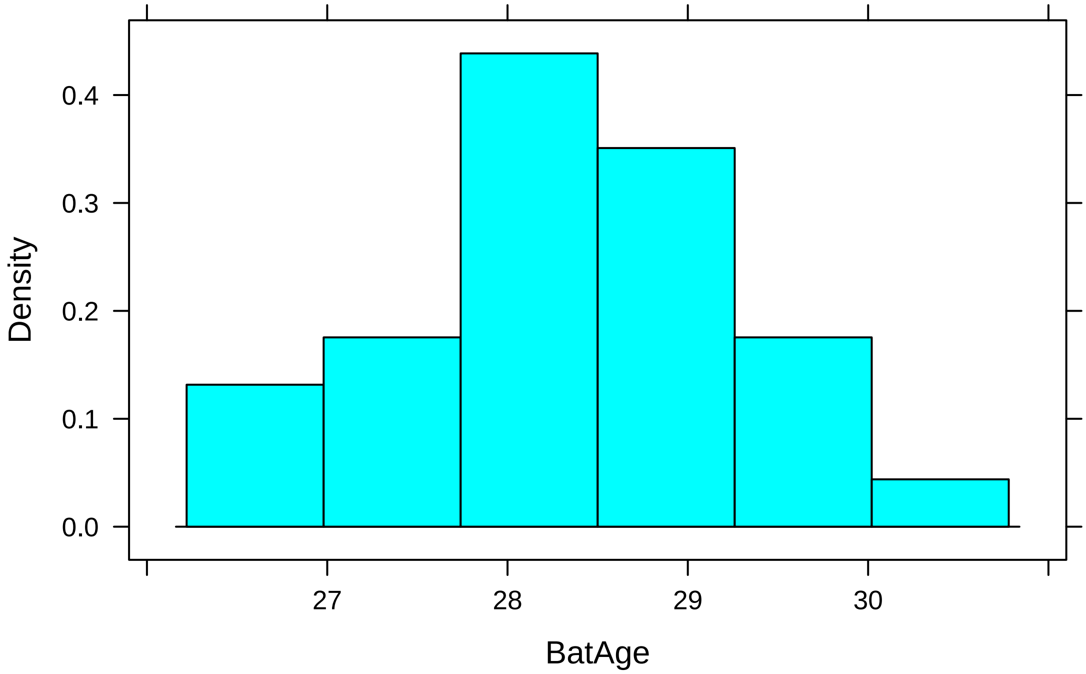
During these problems it helps to first orgranize the numebrs in the list first

$$
\begin{aligned}
2 &\mid 2, 5 \\
3 &\mid 4, 5 \\
4 &\mid 1, 1, 6, 6, 6, 7, 9 \\
5 &\mid 4, 4, 9 \\
6 &\mid 0
\end{aligned}
$$

Dot plots



Histograms



Skewed (Right and left)

add a dataset to show?

Bell

add a dataset to show?

Flat uniform

add a dataset to show?

nonsymmetric

add a dataset to show?

bimodal symmetric

add a dataset to show?

1.3

Def mean : numerical value of average

Notate

Sample mean : $\overline{x}$

$$\overline{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Notate

Population mean : μ

- avg of all values in the entire pop.

Ex $2, 2, 5, 3, 8, 9, 2, 3, 1$

$$\overline{x} = \frac{\sum_{i=1}^{10} x_i}{10} = 3.6$$

The mean is inappropriate in some cases b/c of outliers.

- this makes the mean a **nonresistant measure**

Def Median : middle value /avg of 2 middle values when sorted

Notate

Median : $\widetilde{x}$

- if $n = odd$, median is at $\dfrac{n+1}{2}$
- if $n = even$ median are b/n $\dfrac{n}{2}$ & $\dfrac{n+1}{2}$

Notate

Population Mean : $\widetilde{\mu}$

1.4 Measures of Variability

One way to describe a distribution is by using the standard deviation

Quartiles

- Q_1 - lower quartile separates bottom 25%
- Q_2 - median middle 50%
- Q_3 - upper quartile separates upper 25%

Ex 2, 2, 5, 1, 3, 8, 9, 2, 31

SORT

$$1, 1, 2, 2, 2$$
$$3, 3, 5, 8, 9$$

$\widetilde{x} = \frac{2+3}{2} = 2.5$

Five number summary

- Find min, Q_1, median, Q_3, max

Note : If median is found in list, use it in both top half and lower half.

Ex : 2 2 5 1 3 8 9 2 3 1 100 $\overline{x} = \frac{36+100}{11} \approx 12.36$

Sort to find median. $\widetilde{x} = 3$.

Mean vs. Median

- median is the equal parts point
- mean is the balance point

Notate

Trimmed mean : $\overline{x}_{tr}$

- compromise b/n the mean & median
- to find it, remove top & bottom 10%, then calculate the mean

categorical data

- the natural way to numerically summarize categorical data is by finding the proportion of successes and failures

Notate

sample proportions : $\hat{p} = \dfrac{\textbf{\# of successes}}{n}$

Notate

Population proportions : $p =$ # of successes in the population

Reporting a center of measure gives only partial info

Sets may have similar means but differ in other ways

A simple way to give more detail is to give the range

Def Range : max - min

Deviations from the mean

- a dev from the mean is the absolute diference (distance) b.n an observation and the mean

$$x_1 - \overline{x}, x_2 - \overline{x}, \ldots, x_n - \overline{x}$$

note

$$\sum_{i=1}^{n}(x_i - \overline{x}) = 0$$

proof (omitted to catch up)

Def Standard deviation = measure of how much an observation is expected to be from the mean

Notate

population std. dev = σ

Notate

sample std. dev =

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}}$$

σ is interpreted as size of typical deviation from μ w/ entire pop. of x-values

s has same units as data

Note

s is not resistant (strongly affected by outliers / skew b.c of $\overline{x}$)

$s \geq 0$

Def

variance $=$ $std.dev^2$

pop variance $= \sigma^2$

sample variance $= s^2$

Note

$$s^2 = \sqrt{\frac{\sum\limits_{i=1}^{n}(x_i - \overline{x})^2}{n-1}} = \frac{S_{xx}}{n-1}$$

$$= \frac{\sum\limits_{i=1}^{n} x_i^2 - \frac{\left(\sum\limits_{i=1}^{n} x_i\right)^2}{n}}{n-1}$$

Insert proof here if you want idc

Ex Calculate variance of 26 19.9 17.8 31.4 38.6 28.7 25

insert table here if you want

$$\sum x_i = 187.4$$

$$\sum x_i^2 = 5313.46$$

$$s^2 = \frac{5313.46 - \frac{5313.46}{7}}{6} = 49.41571$$

Constant Multiplier

let $y_i = cx_i$, then

$$s^2y = c^2s^2x$$
$$\overline{y} = c\overline{x}$$

Addition of a Constant

let $y_i = x_i + c$

$$s^2y = s^2x$$
$$\overline{y} = \overline{x} + c$$

InterQuartile Range

- also called f_s, fourth spread

$$IQR = Q_3 - Q_1$$

Def Outlier = an observation that is more than $1.5 \cdot IQR$ away from nearest quartile (end of box)

Mild Outlier :

- Upper fence $= Q_3 + 1.5 \cdot IQR$
- Lower fence $= Q_1 - 1.5 \cdot IQR$

Extreme Outlier :

- Upper fence $= Q_3 + 3 \cdot IQR$
- Lower fence $= Q_1 - 3 \cdot IQR$

Def

Modified BoxPlot :

- represents **mild outliers** w/ **solid dots** & **extreme outliers** w/ **open circles**, w/ the whiskers extending to most extreme value that is not an outlier.