Karhunen-Loève Transform (KLT)

But how can we systematically find an optimal basis?

Consider $x \in \mathbb{C}^N$, a complex-valued, wide-sense stationary signal with mean zero (for simplicity). The covariance matrix of x can be computed numerically:

$$R_x = \mathbb{E}[xx^H],$$

where the superscript H denotes the Hermitian transpose (i.e., $x^H = (x^*)^T$). R_x is real and symmetric, and the eigen-decomposition

$$R_x = Q\Lambda Q^H,$$

gives columns of Q as the eigenvectors of R_x and Λ as a diagonal matrix of the eigenvalues. Q is a unitary matrix, thus $Q^{-1} = Q^H$.

The representation

$$s = Q^H x$$

is known as the Karhunen-Loève Transform (KLT) of x, and we call Q the KLT basis or matrix. The covariance matrix of s is diagonal:

$$R_s = \mathbb{E}[ss^H] = \mathbb{E}[Q^H xx^H Q] = Q^H R_x Q = \Lambda.$$

Thus, the KLT of x results in an uncorrelated representation s, whose covariance matrix has zero cross-correlation terms. In other words, s fully describes x without any statistical redundancy.