Compressive Sensing with Karhunen-Loève Transform

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1 Reference

Directly cited from:

Gwon, Y., Kung, H. T., & Vlah, D. (2012). Analyzing Interference Effects on the Performance of Heterogeneous Wireless Networks. *Proceedings of the 2012 IEEE Global Communications Conference (GLOBECOM)*, 5781-5787. Retrieved from https://www.eecs.harvard.edu/ htk/publication/2012-globecom-gwon-kung-vlah.pdf

2 Karhunen-Loève Transform (KLT)

Consider $x \in \mathbb{C}^N$, a complex-valued, wide-sense stationary signal with mean zero (for simplicity). The covariance matrix of x can be computed numerically:

$$R_x = \mathbb{E}[xx^H],$$

where the superscript H denotes the Hermitian transpose (i.e., $x^H = (x^*)^T$). R_x is real and symmetric, and the eigen-decomposition

$$R_x = Q\Lambda Q^H,$$

gives columns of Q as the eigenvectors of R_x and Λ as a diagonal matrix of the eigenvalues. Q is a unitary matrix, thus $Q^{-1} = Q^H$.

The representation

$$s = Q^H x$$

is known as the Karhunen-Loève Transform (KLT) of x, and we call Q the KLT basis or matrix. The covariance matrix of s is diagonal:

$$R_s = \mathbb{E}[ss^H] = \mathbb{E}[Q^H xx^H Q] = Q^H R_x Q = \Lambda.$$

Thus, the KLT of x results in an uncorrelated representation s, whose covariance matrix has zero cross-correlation terms. In other words, s fully describes x without any statistical redundancy.

3 Recovering KLT Basis with Compressive Measurements

We have mentioned earlier that compressive measurements can be used to recover the KLT matrix. To explain this, we start with the tie-in between the compressive encoding at sensor nodes and KLT basis estimation. Recall that the KLT matrix Q is computed from the covariance matrix of the input signal x, $R_x = E[xx^H]$. Similarly, the covariance matrix of the compressive measurements y is $R_y = E[yy^H]$. By compressive encoding $y = \Phi x$, we know: $R_y = E[\Phi x x^H \Phi^T] = \Phi E[xx^H]\Phi^T$. So, $R_y = \Phi R_x \Phi^T$. Note that Φ is not a square matrix. Using the pseudo-inverse $(\Phi^T)^\dagger$, we can have the following expression:

$$R_y(\Phi^T)^\dagger = \Phi R_x$$

Here, we find that we have been compressively measuring R_x in $R_y(\Phi^T)^\dagger$, which can be approximated from $y = \Phi x$ that we used to encode our data x. Thus, compressive measurements y have sufficient information to recover R_x from (4). Below is a procedure to estimate KLT basis Q with compressive measurements in four steps:

1. Decode X from $y = (\Phi F^{-1})X$ using fixed support F;
2. Recover x by computing $x = F^{-1}X$;
3. Repeat the previous steps l times to numerically compute: $R_x = E[xx^H] = \frac{1}{l}\sum_{i=1}^{l} x_i x_i^H$;
4. Obtain Q by eigen-decomposition $R_x = Q\Lambda Q^H$.