Compressive Sensing with Karhunen-Loève Transform

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1 Reference

Directly cited from:

Gwon, Y., Kung, H. T., & Vlah, D. (2012). Analyzing Interference Effects on the Performance of Heterogeneous Wireless Networks. *Proceedings of the 2012 IEEE Global Communications Conference (GLOBECOM)*, 5781-5787. Retrieved from
https://www.eecs.harvard.edu/~htk/publication/2012-globecom-gwon-kung-vlah.pdf

2 Karhunen-Loève Transform (KLT)

Consider $x \in \mathbb{C}^N$, a complex-valued, wide-sense stationary signal with mean zero (for simplicity). The covariance matrix of x can be computed numerically:

$$R_x = \mathbb{E}[xx^H],$$

where the superscript H denotes the Hermitian transpose (i.e., $x^H = (x^*)^T$). R_x is real and symmetric, and the eigen-decomposition

$$R_x = Q\Lambda Q^H,$$

gives columns of Q as the eigenvectors of R_x and Λ as a diagonal matrix of the eigenvalues. Q is a unitary matrix, thus $Q^{-1} = Q^H$.

The representation

$$s = Q^H x$$

is known as the Karhunen-Loève Transform (KLT) of x, and we call Q the KLT basis or matrix. The covariance matrix of s is diagonal:

$$R_s = \mathbb{E}[ss^H] = \mathbb{E}[Q^H xx^H Q] = Q^H R_x Q = \Lambda.$$

Thus, the KLT of x results in an uncorrelated representation s, whose covariance matrix has zero cross-correlation terms. In other words, s fully describes x without any statistical redundancy.

3 Recovering KLT Basis with Compressive Measurements

We have mentioned earlier that compressive measurements can be used to recover the KLT matrix. To explain this, we start with the connection between the compressive encoding at sensor nodes and KLT basis estimation.

Recall that the KLT matrix Q is computed from the covariance matrix of the input signal x, denoted as:

$$R_x = \mathbb{E}[xx^H]$$

Similarly, the covariance matrix of the compressive measurements y is given by:

$$R_y = \mathbb{E}[yy^H]$$

By compressive encoding, where $y = \Phi x$, we know that:

$$R_y = \mathbb{E}[\Phi x x^H \Phi^T] = \Phi \mathbb{E}[xx^H]\Phi^T = \Phi R_x \Phi^T$$

Note that Φ is not a square matrix. Using the pseudo-inverse of Φ^T, denoted as $(\Phi^T)^\dagger$, we can express the relationship as:

$$R_y (\Phi^T)^\dagger = \Phi R_x$$

Here, we find that we have been compressively measuring R_x in $R_y(\Phi^T)^\dagger$, which can be approximated from $y = \Phi x$, the data used to encode x. Thus, compressive measurements y contain sufficient information to recover R_x from the above relationship.

Below is a procedure to estimate the KLT basis Q with compressive measurements in four steps:

1. Decode X from $y = (\Phi F^{-1})X$ using fixed support F;
2. Recover x by computing $x = F^{-1}X$;
3. Repeat the previous steps l times to numerically compute: $R_x = E[xx^H] = \frac{1}{l}\sum_{i=1}^{l} x_i x_i^H$;
4. Obtain Q by eigen-decomposition $R_x = Q\Lambda Q^H$.