Exercise class 2

(week 9)

Introduction to Programming and Numerical Analysis

Class 4 and 8

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Numerical optimization

My thoughts on the problem sets + tips

Exercises: Problem set 1

Constrained optimization

Exercises: Problem set 1 (continued)

I won't go through **printing** and **plotting**, as this is the "basics" of the tasks and problems. However, if you have any questions or uncertainties along the way, feel free to ask!

Plan for today's exercise class

- 15.15-15.30: recap – optimization and my thoughts on problem sets + tips
- 15.30-16.00: exercises – problem set 1
- 16.00-16.15: Break
- 16.15-16.20: recap – constrained optimization
- 16.20-16.55: exercises – problem set 1
- 16.55-17.00: Follow-up and questions

Optimization

Objective: **find** the x that **minimizes/maximizes** $f(x)$

In economics we're used to solve optimization problems analytically using first and second order conditions.

Works different on computers: we can only evaluate one point at a time – how do we find the minimum?

→ numerical optimization: use iterative methods to approximate the minimum or maximum of a function.

FYI: in computer/data science it's convention to minimize – if we want to maximize, we can simply minimize the negative of the given function.

Optimizing using grid search

One way of optimizing is just to try a lot of different x-values and pick the one that minimizes (gives the smallest values of) $f(x)$:

- Divide the range of possible values for x into a grid
- Evaluate the function at each grid point
- Choose the point with the lowest or highest function value

Pros:

- Provides a rough "sketch" of what the function looks like
- Robust against non-global minima (relatively)

Cons

- Does not check x-values from outside the grid
- Computationally expensive – especially for high-dimensional problems
- Only ever as precises as the given grid

Optimizing using solvers

A solver is some algorithm that looks for the minimum value of x by trying different values and updating guesses based on the evaluation of $f(x)$.

Which values of x to guess on is made by the chosen algorithm – except for the initial guess which must always be provided by the user.

Pros:

- Solvers are often more efficient for continuous optimization problems – faster and less computationally intensive than grid search
- More precise solution (generally)

Cons:

- The solution may depend on the chosen start value
- May not converge to a solution… initial guess provided to the solver is too far from the optimal solution, numerical precision issues (dealing with functions that involve very small or large numbers)

My thoughts on problem sets

The problem sets will be more challenging than DataCamp, but they will give you a solid foundation for solving the projects and teach you everything you should know for the exam!

If you don't make it through all tasks and problems during the exercise classes, I highly suggest you look at them and finish them at home.

Whether you're stuck or not, it's always OK to look at previous lectures or problem sets. Coding is rarely done from scratch.

OK to take a peak look at the solutions… the only one you're cheating is yourself: make sure you can solve the problem yourself as well!

The extra problems are great for preparing you for future projects, as it often requires you to do a bit more "thinking" yourself

My thoughts on problem sets

Don't rush through the exercises – focus on understanding the code to make a good foundation.

When trying to understand code or finding the root of some error, always use print-statements to check if the functions you're using acts like you expect them to. Experimentation in general is key!

Always try to understand problems conceptually before you begin programming: what are you trying to achieve, which steps is likely required? etc. ...

If you have trouble understanding any concepts, please let me know – I'm here to help you learn.

If you get an error message or forgot some syntax, try to Google it before asking. **When it comes to programming, Google (and StackOverflow) can solve a lot of minor and syntax errors!**

Tips!

Task 2:

You need to print a table looking like this:

	0	1	2	3	4
0	1.050	1.162	1.442	1.479	1.569
1	1.162	1.300	1.661	1.711	1.832
2	1.442	1.661	2.300	2.396	2.641
3	1.479	1.711	2.396	2.500	2.768
4	1.569	1.832	2.641	2.768	3.100

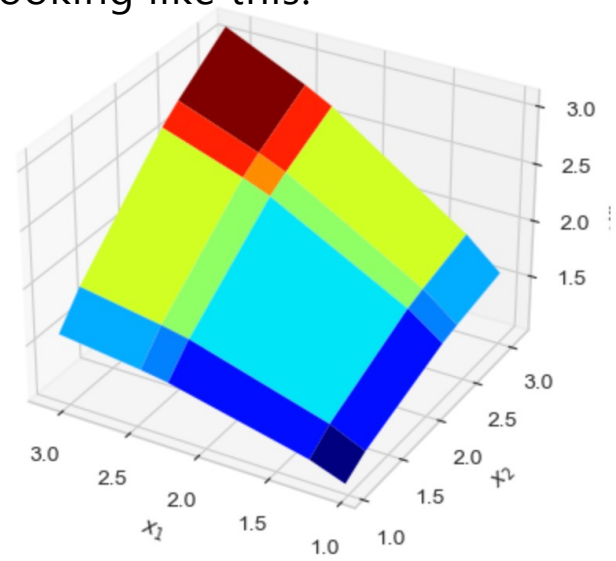
Hints:

- Create the side numbers using enumerate()
- f-strings is a great tool for printing and formatting your results. The basic syntax for printing floats is: f'{value:width.digitsf}, where width is the total width (characters) and digitsf is digits after the decimal point (if you're feeling overwhelmed, forget about this for now).

Tips!

Task 3:

You need to reproduce a figure looking like this:



Hints:

- You can look at the lecture notebook 1-plot, specifically cell 52: plot the utility function.

Tips!

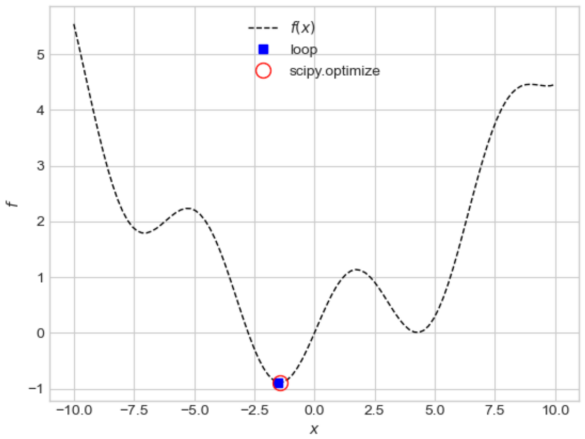
Task 4:

Hints:

- Numpy has the function np.sin()
- For Scipy optimize, I recommend you to look at previous lecture notebooks (for example "1-optimize" or refer to the documentation (i.e. google)
- I recommend you use the "Nelder-Mead" method for the solver
- You should get something like this:

```
Best with the loop method is: -0.88366802 at x = -1.51515152
Best with scipy.optimize is:  -0.88786283 at x = -1.42756250

<matplotlib.legend.Legend at 0x157b4f550>
```



Tips!

Task 5:

Hints:

- How does multiple good impact for loops?
- You should get:

```
x = [2.14 1.07 0.71 0.36 0.43 ]
utility = 0.758
E = 10.00 <= I = 10.00
expenditure shares = [0.21 0.21 0.21 0.14 0.21 ]
```

Task 6:

Hints:

- Google the documentation for the itertools product-function
- Generate all possible combinations of x1, x2, x3, x4, x5
- Use only one for loop – what should it iterate through?

Time for exercises

Problem set 1:

- Task 1-3: functions
- Problem 4: optimization
- Problem 5-8: utility maximization problem

If this is your very first-time programming or working Python and your feeling overwhelmed, I can recommend you take it slow and try and understand each line of code before moving on.

At **16:15**, I'll do a short recap on **constrained optimization**.

Constrained optimization

Constrained optimization work much in the way: both grid search and solvers can be used.

OR: The objective function can be adjusted by adding a “penalty” is the constraint is violated and use unconstrained optimization:

- Pro: Helps “guiding” the solver if it end up outside the given constraint
- Con: May introduce new local minima

Solvers - optimization

Which solver to use for optimizing? Depends on the problem, you'll grow stronger in this along the way. Some examples of methods from Scipy are:

Unconstrained optimization:

- `bfgs` (fast, especially if provided with gradient/hessian)
- `nelder-mead` (robust, but computationally slow)

Constrained optimization:

- `flsqp` (fast, especially if provided with gradient/hessian)
- … Or you can use unconstrained optimization + penalty of constraint if violated

Next time...

Video lectures:

- Random numbers basics
- Random numbers example
- Random numbers advanced (+) (optional)
- Remember **physical lecture** next week!

Exercises – Problem set 2. Finding the Walras equilibrium in a multi-agent economy:

- Drawing random numbers
- Monte Carlo integration
- User-defined modules
- Saving and loading
- … and everything you've learnt up until now!

The inaugural problem is due 24th march!

- Make sure you understand the concepts covered these previous weeks!

Questions and/or comments?