Project 0: Inaugural Project

Vision: The inaugural project teaches you to solve a simple economic model and present the results.

- **Objectives:** In your inaugural project, you should show that you can:
  1. Apply simple numerical solution and simulation methods
  2. Structure a code project
  3. Document code
  4. Present results in text form and in figures
- **Content:** In your inaugural project, you should:
  1. Solve and simulate a pre-specified economic model (see next page)
  2. Visualize results

  Example of structure: See this repository.
- **Structure:** Your inaugural project should consist of:
  1. A README.md with a short introduction to your project
  2. A single self-contained notebook (.ipynb) presenting the analysis
  3. Fully documented Python files (.py)
- **Hand-in:** On GitHub by uploading it to the subfolder *inaugralproject*, which is located in:

  github.com/NumEconCopenhagen/projects-YEAR-YOURGROUPNAME
- **Deadline:** See Calendar.
- **Peer feedback:** After handing in, you will be asked to give peer feedback on the projects of two other groups.
- **Exam:** Your inaugural project will be a part of your exam portfolio.
  You can incorporate feedback before handing in the final version.

Exchange Economy

We consider an exchange economy with two consumers, A and B, and two goods, x_1 and x_2. The initial endowments are $\omega_1^A \geq 0$ and $\omega_2^A \geq 0$. The total endowment of each good is always one, such that

$$\begin{aligned}\omega_1^B &= 1 - \omega_1^A \\ \omega_2^B &= 1 - \omega_2^A.\end{aligned}$$

We define the vectors $\boldsymbol{p} = (p_1, p_2)$, $\boldsymbol{\omega}^A = (\omega_1^A, \omega_2^A)$, and $\boldsymbol{\omega}^B = (\omega_1^B, \omega_2^B)$.
Utility and demand functions with prices $p_1 > 0$ and $p_2 > 0$ are

$$\begin{aligned}u^A(x_1, x_2) &= x_1^{\alpha} x_2^{1-\alpha}, \;\; \alpha \in (0,1) \\ x_1^{A\star}(\boldsymbol{p}, \boldsymbol{\omega}^A) &= \alpha \frac{p_1 \omega_1^A + p_2 \omega_2^A}{p_1} \\ x_2^{A\star}(\boldsymbol{p}, \boldsymbol{\omega}^A) &= (1-\alpha) \frac{p_1 \omega_1^A + p_2 \omega_2^A}{p_2} \\ u^B(x_1, x_2) &= x_1^{\beta} x_2^{1-\beta}, \;\; \beta \in (0,1) \\ x_1^{B\star}(\boldsymbol{p}, \boldsymbol{\omega}^B) & \beta \frac{p_1 \omega_1^B + p_2 \omega_2^B}{p_1} \\ x_2^{B\star}(\boldsymbol{p}, \boldsymbol{\omega}^B) &= (1-\beta) \frac{p_1 \omega_1^B + p_2 \omega_2^B}{p_2}.\end{aligned}$$

The (Walras) market equilibrium requires market clearing for both goods,

$$\begin{aligned}x_1^{A\star}(\boldsymbol{p}, \boldsymbol{\omega}^A) + x_1^{B\star}(\boldsymbol{p}, \boldsymbol{\omega}^B) &= \omega_1^A + \omega_1^B \\ x_2^{A\star}(\boldsymbol{p}, \boldsymbol{\omega}^A) + x_2^{B\star}(\boldsymbol{p}, \boldsymbol{\omega}^B) &= \omega_2^A + \omega_2^B.\end{aligned}$$

Walras' law appliy, so if one market clears, the other one does as well.

Calibration We use the following parameter values

$$\begin{aligned}\alpha &= \frac{1}{3} \\ \beta &= \frac{2}{3}.\end{aligned}$$

Numeraire The numeraire is $p_2 = 1$.

Questions

Code to start from is provided in *IntroProg-lectures/projects/InauguralProject2024.ipynb*
The initial endowment is

$$\omega_1^A = 0.8$$
$$\omega_2^A = 0.3.$$

1. Illustrate the following set in the Edgeworth box

$$\mathcal{C} = \left\{ (x_1^A, x_2^A) \mid \begin{array}{c} u^A(x_1^A, x_2^A) \geq u^A(\omega_1^A, \omega_2^A) \\ u^B(x_1^B, x_2^B) \geq u^B(\omega_1^B, \omega_2^B) \\ x_1^B = 1 - x_1^A,\, x_2^B = 1 - x_2^A \\ x_1^A, x_2^A \in \{0, \frac{1}{N}, \frac{2}{N}, \dots, 1\}, N = 75 \end{array} \right\}$$

   That is, find the pairs of combinations of x_1^A and x_2^A that leave both players as least as well off as they were when consuming their endowments. This is thus Pareto improvements relative to the endowment.

2. For $p_1 \in \mathcal{P}_1 = \{0.5, 0.5{+}2\frac{1}{N}, 0.5{+}2\frac{2}{N}, \dots, 2.5\}$ calculate the error in the market clearing condition s

$$\epsilon_1(\boldsymbol{p}, \boldsymbol{\omega}) = x_1^{A\star}(\boldsymbol{p}, \boldsymbol{\omega}^A) - \omega_1^A + x_1^{B\star}(\boldsymbol{p}, \boldsymbol{\omega}^B) - \omega_1^B$$
$$\epsilon_2(\boldsymbol{p}, \boldsymbol{\omega}) = x_2^{A\star}(\boldsymbol{p}, \boldsymbol{\omega}^A) - \omega_2^A + x_2^{B\star}(\boldsymbol{p}, \boldsymbol{\omega}^B) - \omega_2^B$$

3. What is market clearing price?

Assume that A chooses the price to maximize her own utility.

4a. Find the allocation if only prices in $\mathcal{P}_1$ can be chosen, i.e.

$$\max_{p_1 \in \mathcal{P}_1} u^A(1 - x_1^B(\boldsymbol{p}, \boldsymbol{\omega}^B), 1 - x_2^B(\boldsymbol{p}, \boldsymbol{\omega}^B))$$

4b. Find the allocation if any positive price can be chosen, i.e.

$$\max_{p_1 > 0} u^A(1 - x_1^B(\boldsymbol{p}, \boldsymbol{\omega}^B), 1 - x_2^B(\boldsymbol{p}, \boldsymbol{\omega}^B))$$

Assume that A chooses B's consumption, but such that B is not worse of than in the initial endowment. A is thus the market maker.

5a. Find the allocation if the choice set is restricted to $\mathcal{C}$, i.e.

$$\max_{(x_1^A, x_2^A) \in \mathcal{C}} u^A(x_1^A, x_2^A)$$

5b. Find the allocation if no further restrictions are imposed, i.e.

$$\max_{(x_1^A, x_2^A) \in [0,1] \times [0,1]} u^A(x_1^A, x_2^A)$$
$$\text{s.t. } u^B(1 - x_1^A, 1 - x_2^A) \geq u^B(\omega_1^B, \omega_2^B)$$

Assume A's and B's consumption are chosen by a utilitarian social planner to maximize aggregate utility

6a. Find the resulting allocation

$$\max_{(x_1^A, x_2^A) \in [0,1] \times [0,1]} u^A(x_1^A, x_2^A) + u^B(1 - x_1^A, 1 - x_2^A)$$

6b. Illustrate and compare with your results in questions 3)-5).
Discuss the pros and cons of the various allocations.

Consider the random set

$$\mathcal{W} = \left\{ \left(\omega_1^A, \omega_2^A \right) \mid \omega_1^A \sim \mathcal{U}(0,1), \omega_2^A \sim \mathcal{U}(0,1) \right\}$$

7. Draw a set $\mathcal{W}$ with 50 elements

8. Find the market equilibrium allocation for each $\boldsymbol{\omega}^A \in \mathcal{C}$ and plot them in the Edgeworth box