Notes of Geometria Analítica e Algebra Linear

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2023/2

Lecture notes from the YEAR undergraduate course GAAL (Geometria Analítica e Algebra Linear), given by professor Nelson Gorgonio at the Departamento de Matemática of Universidade Federal de Minas Gerais in the academic year 2023.

The main goal of this course was teach the principles of the mathematical language. The topics of the course were: matrices, linear systems, analytic geometry and identification of conicals. *Disclaimer:* This document will inevitably contain some mistakes— both simple typos and legitimate errors. Keep in mind that these are the notes of an undergraduate student in the process of learning the material himself, so take what you read with a grain of salt. If you find mistakes and feel like telling me, I will be grateful and happy to hear from you, even for the most trivial of errors. You can reach me by email. ulissao@proton.me.

For more notes like this, visit link.

Ulisses Rosa,
Fall Term: 2022,
Last Update: 2023/2,
Faculty of Something

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Lecture 1: Analytic Geometry

1.1 Vectors

A vector is a mathematical object that has lenght and direction.

Theorem 1.1.

1. $v + w = w + v$
2. $v + (w + u) = (v + w) + u$
3. $v + (-v) = 0$
4. $v + \vec{0} = 0$
5. $\alpha(w + u) = \alpha w + \alpha u$
6. $(\alpha + \beta) \cdot u = \alpha u + \beta u$
7. $(\alpha \cdot \beta) \cdot u = \alpha(u \cdot \beta u)$
8. $1 \cdot u = u$

The lenght of a vector is called norm. Let $V = (v_1, v_2)$. So $\|V\| = \sqrt{v_1^2 + v_2^2}$.

Note:-

- $distance(p, q) = \|\vec{pq}\|$
- $\|\alpha v\| = |\alpha| \cdot \|v\|$

Definition 1.1

$\|v\| = 1 \to v$ is a unitary vector

$$u = \frac{1}{\|v\|} \cdot v$$
$$\|u\| = \|\frac{1}{\|v\|} \cdot v\|$$
$$\|u\| = |\frac{1}{\|v\|}| \cdot \|v\| = 1$$

Definition 1.2

The dot product of two vectors v and w is defined by:

$v \cdot w = v_1 \cdot w_1 + ... + v_n \cdot w_n$

Theorem 1.2.

1. $u \cdot v = v \cdot u$
2. $u \cdot (v + w) = u \cdot v + u \cdot w$
3. $\alpha(u \cdot v) = (\alpha \cdot u) \cdot u = u \cdot (\alpha \cdot v)$
4. $v \cdot v = \|v\|^2$
5. $\forall w : w \cdot w = 0 \leftrightarrow w = \vec{0}$

Theorem 1.3. $\cos\theta = \frac{v \cdot w}{\|v\| \cdot \|w\|}$

1. $0^\circ \leq \theta < 90^\circ = v \cdot w > 0$
2. $\theta = 90^\circ \leftrightarrow v \cdot w = 0$ (v and w are orthogonal)
3. $0^\circ < \theta \leq 180^\circ \leftrightarrow v \cdot w < 0$

Definition 1.3

Given two vectors v and w, the orthogonal projection, $proj_w v$, of v over w is the vector paralel to w that $v - proj_w v$ orthogonal to w.

Theorem 1.4. $proj_w v = (\frac{v \cdot w}{\|w\|^2}) \cdot w$

1.2 Vectors in the space

Definition 1.4

Let $v = (v_1, v_2, v_3)$ and $w = (w_1, w_2, w_3)$ two vectors in the space. So the cross product between v and w is:

$$v \times w = \left(\begin{vmatrix} v_2 & v_3 \\ w_2 & w_3 \end{vmatrix}, - \begin{vmatrix} v_1 & v_3 \\ w_1 & w_3 \end{vmatrix}, \begin{vmatrix} v_1 & v_2 \\ w_1 & w_2 \end{vmatrix} \right)$$

Theorem 1.5.

1. $v \times w = -(w \times v)$

2. $v \times w = \vec{0} \leftrightarrow v \parallel w$
3. $(v \times w) \perp v \wedge (v \times w) \perp w$
4. $v \cdot v = \|v\|^2$
5. $v \times (w + u) = (v \times w) + (v \times u)$

Note:-

The canonical vectors $\vec{\imath} = (1, 0, 0)$, $\vec{\jmath} = (0, 1, 0)$ and $\vec{k} = (0, 0, 1)$ are unit vectors paralel to the coordinate axes. They are very important because every vector in the space is a linear cobination of the unit vectors

$\forall v \in \mathbb{R}_3 : v = \alpha\vec{\imath} + \beta\vec{\jmath} + \lambda\vec{k};\ \alpha, \beta, \lambda \in \mathbb{R}_3$

Theorem 1.6.

1. $\|v \times w\| = \|v\| \cdot \|w\| \cdot \sin\theta$
2. The direction is of $v \times w$ is orthogonal to v and w

The area of a paralelogram determined by two vectors is $\|v \times w\|$

Definition 1.5

$$(v \times w) \cdot u = \begin{vmatrix} v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \\ u_1 & u_2 & u_3 \end{vmatrix}$$

Note:-

The volume of paralelephiped determined by the vectors v, w, u in the space is equal to $|(v \times w) \cdot u|$

Corollary 1.1. The vectors v, w, u are in the same plane if and only if $(v \times w) \cdot u = 0$

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Lecture 2: Analytic Geometry

2.1 Planes

Definition 2.6

In the space, the equation of a plane is determined by a normal vector and a point.

Theorem 2.1. A equation of plane π that has the point $p_0 = (x_0, y_0, z_0)$ and has a orthogonal vector $N = (a, b, c)$ is defined by

$$\pi : ax + by + cz + d = 0$$

$$d = -ax_0 - by_0 - cz_0$$

Note:-

- $p \in \pi \wedge q \in \pi \to \vec{pq} \parallel \pi$
- $p \in \pi \wedge q \in \pi \to (p \times q) \perp \pi$

Definition 2.7

Let $p_0 = (x_0, y_0, z_0)$ in the plane π, and two not colinear vectors $v = (v_1, v_2, v_3)$ and $w = (w_1, w_2, w_3)$ so:

$p = (x, y, z) \in \pi \leftrightarrow \vec{p_0p} = \alpha v + \beta w$ so:
$\vec{p_0p} = (x - x_0, y - y_0, z - z_0) = \alpha \cdot (v_1, v_2, v_3) + \beta \cdot (w_1, w_2, w_3)$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \cdot \alpha + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \cdot \beta$$

Theorem 2.2. If π_1 and π_2 are two planes with normal vectors N_1, N_2, so the angle between π_1 and π_2 is defined by:

$$cos\theta = \frac{N_1 \cdot N_2}{\|N_1\| \cdot \|N_2\|}$$

Theorem 2.3. This is a theorem.

Proof. This is a proof. ☺

Example. This is an example.

Proof. This is an explanation. 😄

Claim 2.1. This is a claim.

Corollary 2.1. This is a corollary.

Proposition 2.1. This is a proposition.

Lemma 2.1. This is a lemma.

Question 1

This is a question.

Solution:-

This is a solution.

Exercise 2.1. This is an exercise.

Definition 2.8: Definition

This is a definition.

Note:-

This is a note.

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Lecture 3: Todo Notes

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Here's another line.

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Is this correct?

I'm unsure about also!

Change this!

This can help me in chapter seven!

This really needs to be improved! What was I thinking?!

The following section needs to be rewritten!

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Lecture 4: Graphs

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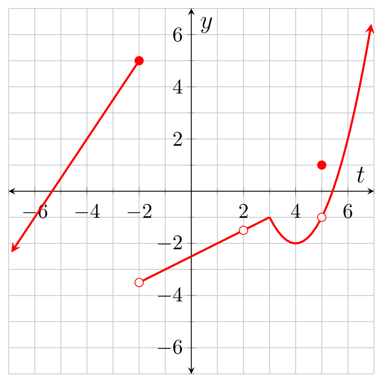

Figure 1: $y = g(t)$

Notes