Notes of Geometria Analítica e Algebra Linear

Ulisses Rosa

2023/2

Lecture notes from the YEAR undergraduate course GAAL (Geometria Analítica e Algebra Linear), given by professor Nelson Gorgonio at the Departamento de Matemática of Universidade Federal de Minas Gerais in the academic year 2023.

The main goal of this course was teach the principles of the mathematical language. The topics of the course were: matrices, linear systems, analytic geometry and identification of conicals. *Disclaimer:* This document will inevitably contain some mistakes— both simple typos and legitimate errors. Keep in mind that these are the notes of an undergraduate student in the process of learning the material himself, so take what you read with a grain of salt. If you find mistakes and feel like telling me, I will be grateful and happy to hear from you, even for the most trivial of errors. You can reach me by email. ulissao@proton.me.

Contents

Lecture 1: Analytic Geometry

1.1   Vectors

A vector is a mathematical object that has lenght and direction.

> **Theorem 1.1.**
>
> 1. $v + w = w + v$
> 2. $v + (w + u) = (v + w) + u$
> 3. $v + (-v) = 0$
> 4. $v + \vec{0} = 0$
> 5. $\alpha(w + u) = \alpha w + \alpha u$
> 6. $(\alpha + \beta) \cdot u = \alpha u + \beta u$
> 7. $(\alpha \cdot \beta) \cdot u = \alpha(u \cdot \beta u)$
> 8. $1 \cdot u = u$

The lenght of a vector is called norm. Let $V = (v_1, v_2)$. So $\|V\| = \sqrt{v_1^2 + v_2^2}$.

Note:-
- $distance(p, q) = \|\vec{pq}\|$
- $\|\alpha v\| = |\alpha| \cdot \|v\|$

Definition 1.1

$\|v\| = 1 \rightarrow v$ is a unitary vector

$$u = \frac{1}{\|v\|} \cdot v$$

$$\|u\| = \|\frac{1}{\|v\|} \cdot v\|$$

$$\|u\| = |\frac{1}{\|v\|}| \cdot \|v\| = 1$$

Definition 1.2

The dot product of two vectors v and w is defined by:

$v \cdot w = v_1 \cdot w_1 + ... + v_n \cdot w_n$

Theorem 1.2.

1. $u \cdot v = v \cdot u$

2. $u \cdot (v + w) = u \cdot v + u \cdot w$

3. $\alpha(u \cdot v) = (\alpha \cdot u) \cdot u = u \cdot (\alpha \cdot v)$

4. $v \cdot v = \|v\|^2$

5. $\forall w : w \cdot w = 0 \leftrightarrow w = \vec{0}$

Theorem 1.3. $\cos \theta = \frac{v \cdot w}{\|v\| \cdot \|w\|}$

1. $0° \leq \theta < 90° = v \cdot w > 0$

2. $\theta = 90° \leftrightarrow v \cdot w = 0$ (v and w are orthogonal)

3. $0° < \theta \leq 180° \leftrightarrow v \cdot w < 0$

Definition 1.3

Given two vectors v and w, the orthogonal projection, $proj_w v$, of v over w is the vector paralel to w that $v - proj_w v$ orthogonal to w.

Theorem 1.4. $proj_w v = \left(\frac{v \cdot w}{\|w\|^2} \right) \cdot w$

1.2   Vectors in the space

Definition 1.4

Let $v = (v_1, v_2, v_3)$ and $w = (w_1, w_2, w_3)$ two vectors in the space. So the cross product between v and w is:

$$v \times w = \left(\begin{vmatrix} v_2 & v_3 \\ w_2 & w_3 \end{vmatrix}, - \begin{vmatrix} v_1 & v_3 \\ w_1 & w_3 \end{vmatrix}, \begin{vmatrix} v_1 & v_2 \\ w_1 & w_2 \end{vmatrix} \right)$$

Theorem 1.5.

1. $v \times w = -(w \times v)$

2. $v \times w = \vec{0} \leftrightarrow v \parallel w$

3. $(v \times w) \perp v \wedge (v \times w) \perp w$

4. $v \cdot v = \|v\|^2$

5. $v \times (w + u) = (v \times w) + (v \times u)$

> **Note:-**
>
> The canonical vectors $\vec{l} = (1,0,0)$, $\vec{j} = (0,1,0)$ and $\vec{k} = (0,0,1)$ are unit vectors paralel to the coordinate axes. They are very important because every vector in the space is a linear cobination of the unit vectors
>
> $$\forall v \in \mathbb{R}_3 : v = \alpha\vec{l} + \beta\vec{j} + \lambda\vec{k}; \ \alpha, \beta, \lambda \in \mathbb{R}_3$$

Theorem 1.6.

1. $\|v \times w\| = \|v\| \cdot \|w\| \cdot \sin\theta$

2. The direction is of $v \times w$ is orthogonal to v and w

The area of a paralelogram determined by two vectors is $\|v \times w\|$

Definition 1.5

$$(v \times w) \cdot u = \begin{vmatrix} v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \\ u_1 & u_2 & u_3 \end{vmatrix}$$

> **Note:-**
>
> The volume of paralelephiped determined by the vectors v, w, u in the space is equal to $|(v \times w) \cdot u|$

Corollary 1.1. The vectors v, w, u are in the same plane if and only if $(v \times w) \cdot u = 0$

Lecture 2: Lecture Title

2.1   Sub Section 2

Theorem 2.1. This is a theorem.

Proof. This is a proof.   ☺

Example. This is an example.

Proof. This is an explanation.   ☺

Claim 2.1. This is a claim.

Corollary 2.1. This is a corollary.

Proposition 2.1. This is a proposition.

Lemma 2.1. This is a lemma.

Question 1

This is a question.

Solution:-

This is a solution.

Exercise 2.1. This is an exercise.

Definition 2.6: Definition

This is a definition.

Note:-

This is a note.

Oct 17 2022 Mon (13:18:57)

Lecture 3: Todo Notes

The original todo note withouth changed colours.
Here's another line.

Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetuer eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor. Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetuer eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor. Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetuer eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor. Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetuer eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor. Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetuer eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor.

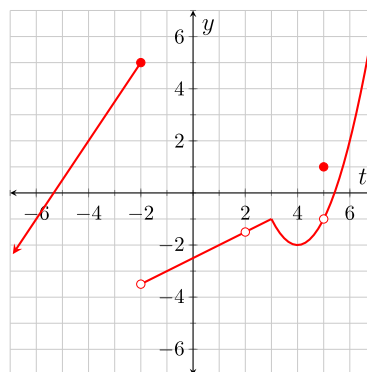Is this correct?

I'm unsure about also!

Change this!

This can help me in chapter seven!

This really needs to be improved! What was I thinking?!

The following section needs to be rewritten!

Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetuer eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor.

Lecture 4: Graphs

Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet. Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet.

Figure 1: $y = g(t)$

Notes