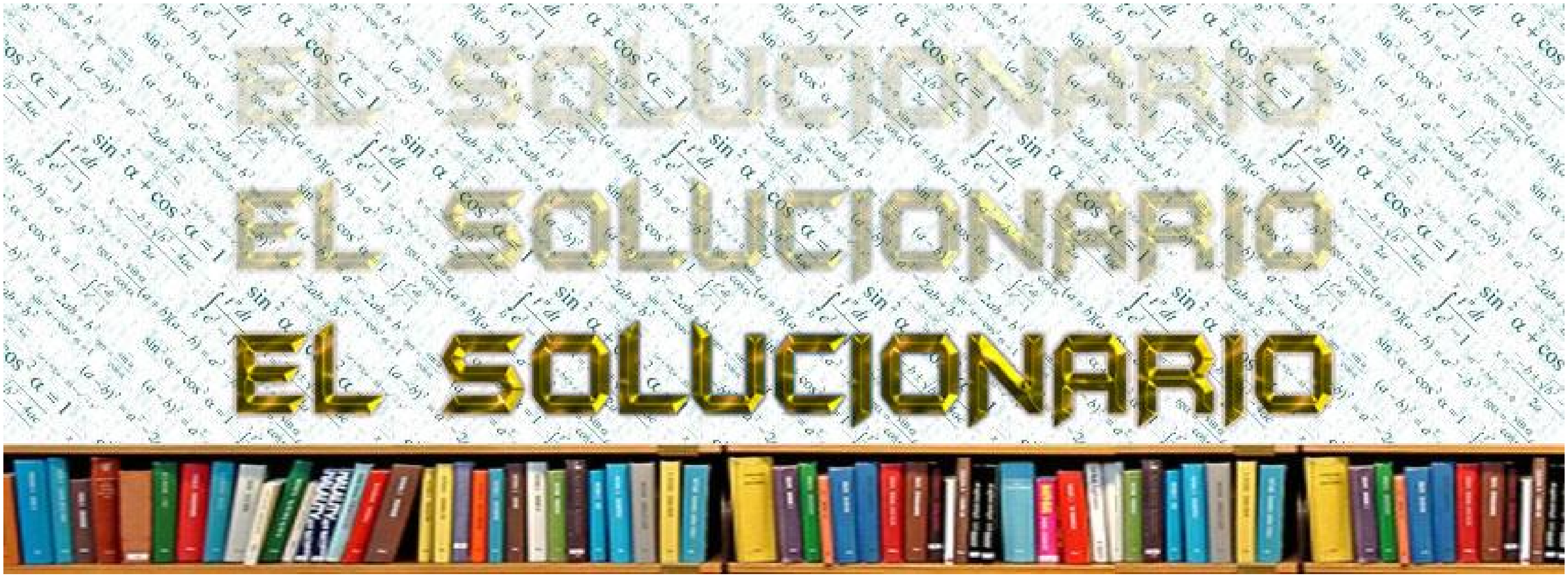
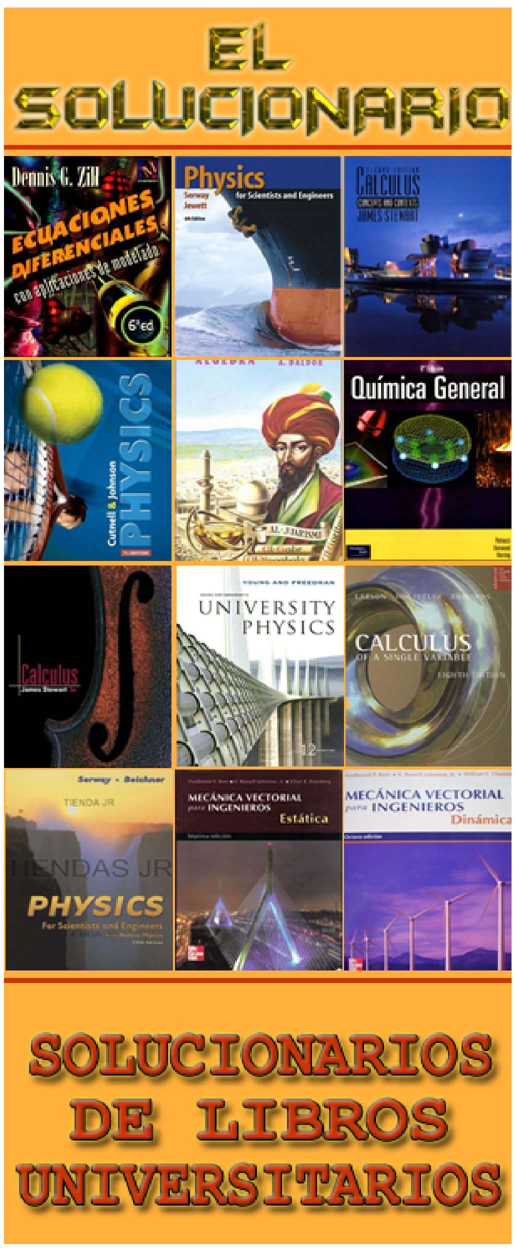


http://www.elsolucionario.blogspot.com



1

Physics and Measurement

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

** An asterisk indicates an item new to this edition.*

Q1.1 Density varies with temperature and pressure. It would be necessary to measure both mass and volume very accurately in order to use the density of water as a standard.

Q1.2 (a) 0.3 millimeters (b) 50 microseconds (c) 7.2 kilograms

***Q1.3** In the base unit we have (a) 0.032 kg (b) 0.015 kg (c) 0.270 kg (d) 0.041 kg (e) 0.27 kg. Then the ranking is c = e > d > a > b

Q1.4 No: A dimensionally correct equation need not be true. Example: 1 chimpanzee = 2 chimpanzee is dimensionally correct.
Yes: If an equation is not dimensionally correct, it cannot be correct.

***Q1.5** The answer is yes for (a), (c), and (f). You cannot add or subtract a number of apples and a number of jokes. The answer is no for (b), (d), and (e). Consider the gauge of a sausage, 4 kg/2 m, or the volume of a cube, $(2\text{ m})^3$. Thus we have (a) yes (b) no (c) yes (d) no (e) no (f) yes

***Q1.6** 41 € ≈ 41 € (1 L/1.3 €)(1 qt/1 L)(1 gal/4 qt) ≈ (10/1.3) gal ≈ 8 gallons, answer (c)

***Q1.7** The meterstick measurement, (a), and (b) can all be 4.31 cm. The meterstick measurement and (c) can both be 4.24 cm. Only (d) does not overlap. Thus (a) (b) and (c) all agree with the meterstick measurement.

***Q1.8** 0.02(1.365) = 0.03. The result is $(1.37 \pm 0.03) \times 10^7$ kg. So (d) 3 digits are significant.

SOLUTIONS TO PROBLEMS

Section 1.1 Standards of Length, Mass, and Time

P1.1 Modeling the Earth as a sphere, we find its volume as $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(6.37\times10^6\text{ m}\right)^3 = 1.08\times10^{21}\text{ m}^3$.
Its density is then $\rho = \frac{m}{V} = \frac{5.98\times10^{24}\text{ kg}}{1.08\times10^{21}\text{ m}^3} = \boxed{5.52\times10^3\text{ kg/m}^3}$. This value is intermediate between the tabulated densities of aluminum and iron. Typical rocks have densities around 2 000 to 3 000 kg/m³. The average density of the Earth is significantly higher, so higher-density material must be down below the surface.

P1.2 With $V = (\text{base area})(\text{height})$ $V = (\pi r^2)h$ and $\rho = \dfrac{m}{V}$, we have

$$\rho = \frac{m}{\pi r^2 h} = \frac{1\text{ kg}}{\pi (19.5\text{ mm})^2 (39.0\text{ mm})}\left(\frac{10^9\text{ mm}^3}{1\text{ m}^3}\right)$$

$$\rho = \boxed{2.15\times 10^4\ \text{kg/m}^3}.$$

P1.3 Let V represent the volume of the model, the same in $\rho = \dfrac{m}{V}$ for both. Then $\rho_{\text{iron}} = 9.35\ \text{kg}/V$ and $\rho_{\text{gold}} = \dfrac{m_{\text{gold}}}{V}$. Next, $\dfrac{\rho_{\text{gold}}}{\rho_{\text{iron}}} = \dfrac{m_{\text{gold}}}{9.35\text{ kg}}$ and $m_{\text{gold}} = 9.35\text{ kg}\left(\dfrac{19.3\times 10^3\ \text{kg/m}^3}{7.86\times 10^3\ \text{kg/m}^3}\right) = \boxed{23.0\text{ kg}}$.

***P1.4** $\rho = m/V$ and $V = (4/3)\pi r^3 = (4/3)\pi(d/2)^3 = \pi d^3/6$ where d is the diameter.

Then $\rho = 6m/\pi d^3 = \dfrac{6(1.67\times 10^{-27}\text{ kg})}{\pi(2.4\times 10^{-15}\text{ m})^3} = \boxed{2.3\times 10^{17}\,\text{kg/m}^3}$

$2.3\times 10^{17}\ \text{kg/m}^3/(11.3\times 10^3\ \text{kg/m}^3) = \boxed{\text{it is } 20\times 10^{12} \text{ times the density of lead}}$.

P1.5 For either sphere the volume is $V = \dfrac{4}{3}\pi r^3$ and the mass is $m = \rho V = \rho\dfrac{4}{3}\pi r^3$. We divide this equation for the larger sphere by the same equation for the smaller:

$$\frac{m_\ell}{m_s} = \frac{\rho 4\pi r_\ell^3 3}{\rho 4\pi r_s^3 3} = \frac{r_\ell^3}{r_s^3} = 5.$$

Then $r_\ell = r_s\sqrt[3]{5} = 4.50\text{ cm}(1.71) = \boxed{7.69\text{ cm}}$.

Section 1.2 **Matter and Model-Building**

P1.6 From the figure, we may see that the spacing between diagonal planes is half the distance between diagonally adjacent atoms on a flat plane. This diagonal distance may be obtained from the Pythagorean theorem, $L_{\text{diag}} = \sqrt{L^2 + L^2}$. Thus, since the atoms are separated by a distance $L = 0.200$ nm, the diagonal planes are separated by $\dfrac{1}{2}\sqrt{L^2+L^2} = \boxed{0.141\text{ nm}}$.

Section 1.3 **Dimensional Analysis**

P1.7 (a) $\boxed{\text{This is incorrect}}$ since the units of $[ax]$ are m^2/s^2, while the units of $[v]$ are m/s.

(b) $\boxed{\text{This is correct}}$ since the units of $[y]$ are m, and $\cos(kx)$ is dimensionless if $[k]$ is in m^{-1}.

P1.8 (a) Circumference has dimensions of L.

(b) Volume has dimensions of L^3.

(c) Area has dimensions of L^2.

Expression (i) has dimension $\text{L}(\text{L}^2)^{1/2} = \text{L}^2$, so this must be area (c).

Expression (ii) has dimension L, so it is (a).

Expression (iii) has dimension $\text{L}(\text{L}^2) = \text{L}^3$, so it is (b). Thus, $\boxed{(\text{a}) = \text{ii};\ (\text{b}) = \text{iii}; (\text{c}) = \text{i}}$.

P1.9 Inserting the proper units for everything except G,

$$\left[\frac{\text{kg m}}{\text{s}^2}\right]=\frac{G[\text{kg}]^2}{[\text{m}]^2}.$$

Multiply both sides by $[\text{m}]^2$ and divide by $[\text{kg}]^2$; the units of G are $\boxed{\dfrac{\text{m}^3}{\text{kg}\cdot\text{s}^2}}$.

Section 1.4 **Conversion of Units**

P1.10 Apply the following conversion factors:

$$1\text{ in}=2.54\text{ cm},\ 1\text{ d}=86\,400\text{ s},\ 100\text{ cm}=1\text{ m, and }10^9\text{ nm}=1\text{ m}$$

$$\left(\frac{1}{32}\ \text{in/day}\right)\frac{(2.54\ \text{cm/in})(10^{-2}\ \text{m/cm})(10^9\ \text{nm/m})}{86\,400\ \text{s/day}}=\boxed{9.19\ \text{nm/s}}.$$

This means the proteins are assembled at a rate of many layers of atoms each second!

P1.11 *Conceptualize*: We must calculate the area and convert units. Since a meter is about 3 feet, we should expect the area to be about $A\approx(30\text{ m})(50\text{ m})=1500\text{ m}^2$.

Categorize: We model the lot as a perfect rectangle to use Area = Length × Width. Use the conversion: $1\text{ m}=3.281\text{ ft}$.

Analyze: $A=LW=(100\text{ ft})\left(\dfrac{1\text{ m}}{3.281\text{ ft}}\right)(150\text{ ft})\left(\dfrac{1\text{ m}}{3.281\text{ ft}}\right)=1\,390\text{ m}^2=\boxed{1.39\times10^3\ \text{m}^2}$.

Finalize: Our calculated result agrees reasonably well with our initial estimate and has the proper units of m^2. Unit conversion is a common technique that is applied to many problems.

P1.12 (a) $V=(40.0\text{ m})(20.0\text{ m})(12.0\text{ m})=9.60\times10^3\ \text{m}^3$

$V=9.60\times10^3\ \text{m}^3(3.28\ \text{ft/1 m})^3=\boxed{3.39\times10^5\ \text{ft}^3}$

(b) The mass of the air is

$$m=\rho_{\text{air}}V=(1.20\ \text{kg/m}^3)(9.60\times10^3\ \text{m}^3)=1.15\times10^4\ \text{kg}.$$

The student must look up weight in the index to find

$$F_g=mg=(1.15\times10^4\ \text{kg})(9.80\ \text{m/s}^2)=1.13\times10^5\ \text{N}.$$

Converting to pounds,

$$F_g=(1.13\times10^5\ \text{N})(1\text{ lb}/4.45\text{ N})=\boxed{2.54\times10^4\ \text{lb}}.$$

***P1.13** The area of the four walls is (3.6 + 3.8 + 3.6 + 3.8)m (2.5 m) = 37 m^2. Each sheet in the book has area (0.21 m) (0.28 m) = 0.059 m^2. The number of sheets required for wallpaper is 37 m^2/0.059 m^2 = 629 sheets = 629 sheets(2 pages/1 sheet) = 1260 pages.

$\boxed{\text{The pages from volume one are inadequate, but the full version has enough pages.}}$

P1.14 (a) Seven minutes is 420 seconds, so the rate is

$$r = \frac{30.0 \text{ gal}}{420 \text{ s}} = \boxed{7.14 \times 10^{-2} \text{ gal/s}}.$$

(b) Converting gallons first to liters, then to m^3,

$$r = \left(7.14 \times 10^{-2} \text{ gal/s}\right)\left(\frac{3.786 \text{ L}}{1 \text{ gal}}\right)\left(\frac{10^{-3} \text{ m}^3}{1 \text{ L}}\right)$$

$$r = \boxed{2.70 \times 10^{-4} \text{ m}^3/\text{s}}.$$

(c) At that rate, to fill a 1-m^3 tank would take

$$t = \left(\frac{1 \text{ m}^3}{2.70 \times 10^{-4} \text{ m}^3/\text{s}}\right)\left(\frac{1 \text{ h}}{3\,600}\right) = \boxed{1.03 \text{ h}}.$$

P1.15 From Table 14.1, the density of lead is $1.13 \times 10^4 \text{ kg/m}^3$, so we should expect our calculated value to be close to this number. This density value tells us that lead is about 11 times denser than water, which agrees with our experience that lead sinks.

Density is defined as mass per volume, in $\rho = \frac{m}{V}$. We must convert to SI units in the calculation.

$$\rho = \frac{23.94 \text{ g}}{2.10 \text{ cm}^3}\left(\frac{1 \text{ kg}}{1000 \text{ g}}\right)\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 = \frac{23.94 \text{ g}}{2.10 \text{ cm}^3}\left(\frac{1 \text{ kg}}{1000 \text{ g}}\right)\left(\frac{1\,000\,000 \text{ cm}^3}{1 \text{ m}^3}\right) = \boxed{1.14 \times 10^4 \text{ kg/m}^3}$$

At one step in the calculation, we note that *one million* cubic centimeters make one cubic meter. Our result is indeed close to the expected value. Since the last reported significant digit is not certain, the difference in the two values is probably due to measurement uncertainty and should not be a concern. One important common-sense check on density values is that objects which sink in water must have a density greater than 1 g/cm^3, and objects that float must be less dense than water.

P1.16 The weight flow rate is $1\,200 \frac{\text{ton}}{\text{h}}\left(\frac{2\,000 \text{ lb}}{\text{ton}}\right)\left(\frac{1 \text{ h}}{60 \text{ min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = \boxed{667 \text{ lb/s}}$.

P1.17 (a) $\left(\frac{8 \times 10^{12} \text{ \$}}{1\,000 \text{ \$/s}}\right)\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right)\left(\frac{1 \text{ day}}{24 \text{ h}}\right)\left(\frac{1 \text{ yr}}{365 \text{ days}}\right) = \boxed{250 \text{ years}}$

(b) The circumference of the Earth at the equator is $2\pi\left(6.378 \times 10^3 \text{ m}\right) = 4.01 \times 10^7 \text{ m}$. The length of one dollar bill is 0.155 m so that the length of 8 trillion bills is $1.24 \times 10^{12} \text{ m}$. Thus, the 8 trillion dollars would encircle the Earth

$$\frac{1.24 \times 10^{12} \text{ m}}{4.01 \times 10^7 \text{ m}} = \boxed{3.09 \times 10^4 \text{ times}}.$$

P1.18
$$V = \frac{1}{3}Bh = \frac{\left[(13.0 \text{ acres})\left(43\,560 \text{ ft}^2/\text{acre}\right)\right]}{3}(481 \text{ ft})$$
$$= 9.08 \times 10^7 \text{ ft}^3,$$

or

$$V = \left(9.08 \times 10^7 \text{ ft}^3\right)\left(\frac{2.83 \times 10^{-2} \text{ m}^3}{1 \text{ ft}^3}\right)$$
$$= \boxed{2.57 \times 10^6 \text{ m}^3}$$

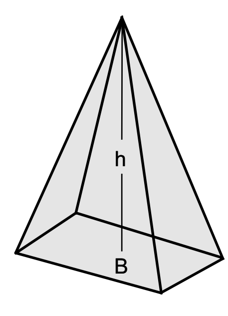


FIG. P1.18

P1.19 $F_g = (2.50 \text{ tons/block})(2.00\times10^6 \text{ blocks})(2\,000 \text{ lb/ton}) = \boxed{1.00\times10^{10} \text{ lbs}}$

P1.20 (a) $d_{\text{nucleus, scale}} = d_{\text{nucleus, real}}\left(\dfrac{d_{\text{atom, scale}}}{d_{\text{atom, real}}}\right) = (2.40\times10^{-15} \text{ m})\left(\dfrac{300 \text{ ft}}{1.06\times10^{-10} \text{ m}}\right) = 6.79\times10^{-3} \text{ ft}$, or

$$d_{\text{nucleus, scale}} = (6.79\times10^{-3} \text{ ft})(304.8 \text{ mm/1 ft}) = \boxed{2.07 \text{ mm}}$$

(b) $$\frac{V_{\text{atom}}}{V_{\text{nucleus}}} = \frac{4\pi r_{\text{atom}}^3/3}{4\pi r_{\text{nucleus}}^3/3} = \left(\frac{r_{\text{atom}}}{r_{\text{nucleus}}}\right)^3 = \left(\frac{d_{\text{atom}}}{d_{\text{nucleus}}}\right)^3 = \left(\frac{1.06\times10^{-10} \text{ m}}{2.40\times10^{-15} \text{ m}}\right)^3$$

$$= \boxed{8.62\times10^{13} \text{ times as large}}$$

P1.21 $V = At$ so $t = \dfrac{V}{A} = \dfrac{3.78\times10^{-3} \text{ m}^3}{25.0 \text{ m}^2} = \boxed{1.51\times10^{-4} \text{ m (or } 151\ \mu\text{m})}$

P1.22 (a) $$\frac{A_{\text{Earth}}}{A_{\text{Moon}}} = \frac{4\pi r_{\text{Earth}}^2}{4\pi r_{\text{Moon}}^2} = \left(\frac{r_{\text{Earth}}}{r_{\text{Moon}}}\right)^2 = \left(\frac{(6.37\times10^6 \text{ m})(100 \text{ cm/m})}{1.74\times10^8 \text{ cm}}\right)^2 = \boxed{13.4}$$

(b) $$\frac{V_{\text{Earth}}}{V_{\text{Moon}}} = \frac{4\pi r_{\text{Earth}}^3/3}{4\pi r_{\text{Moon}}^3/3} = \left(\frac{r_{\text{Earth}}}{r_{\text{Moon}}}\right)^3 = \left(\frac{(6.37\times10^6 \text{ m})(100 \text{ cm/m})}{1.74\times10^8 \text{ cm}}\right)^3 = \boxed{49.1}$$

P1.23 To balance, $m_{\text{Fe}} = m_{\text{Al}}$ or $\rho_{\text{Fe}}V_{\text{Fe}} = \rho_{\text{Al}}V_{\text{Al}}$

$$\rho_{\text{Fe}}\left(\frac{4}{3}\right)\pi r_{\text{Fe}}^{\ 3} = \rho_{\text{Al}}\left(\frac{4}{3}\right)\pi r_{\text{Al}}^{\ 3}$$

$$r_{\text{Al}} = r_{\text{Fe}}\left(\frac{\rho_{\text{Fe}}}{\rho_{\text{Al}}}\right)^{1/3} = (2.00 \text{ cm})\left(\frac{7.86}{2.70}\right)^{1/3} = \boxed{2.86 \text{ cm}}.$$

P1.24 The mass of each sphere is

$$m_{\text{Al}} = \rho_{\text{Al}}V_{\text{Al}} = \frac{4\pi\rho_{\text{Al}}r_{\text{Al}}^{\ 3}}{3}$$

and

$$m_{\text{Fe}} = \rho_{\text{Fe}}V_{\text{Fe}} = \frac{4\pi\rho_{\text{Fe}}r_{\text{Fe}}^{\ 3}}{3}.$$

Setting these masses equal,

$$\frac{4\pi\rho_{\text{Al}}r_{\text{Al}}^{\ 3}}{3} = \frac{4\pi\rho_{\text{Fe}}r_{\text{Fe}}^{\ 3}}{3} \text{ and } \boxed{r_{\text{Al}} = r_{\text{Fe}}\sqrt[3]{\frac{\rho_{\text{Fe}}}{\rho_{\text{Al}}}}}.$$

The resulting expression shows that the radius of the aluminum sphere is directly proportional to the radius of the balancing iron sphere. The sphere of lower density has larger radius. The fraction $\dfrac{\rho_{\text{Fe}}}{\rho_{\text{Al}}}$ is the factor of change between the densities, a number greater than 1. Its cube root is a number much closer to 1. The relatively small change in radius implies a change in volume sufficient to compensate for the change in density.

Section 1.5 Estimates and Order-of-Magnitude Calculations

P1.25 Model the room as a rectangular solid with dimensions 4 m by 4 m by 3 m, and each ping-pong ball as a sphere of diameter 0.038 m. The volume of the room is $4\times4\times3=48\ \text{m}^3$, while the volume of one ball is

$$\frac{4\pi}{3}\left(\frac{0.038\ \text{m}}{2}\right)^3=2.87\times10^{-5}\ \text{m}^3.$$

Therefore, one can fit about $\dfrac{48}{2.87\times10^{-5}}\sim\boxed{10^6}$ ping-pong balls in the room.

As an aside, the actual number is smaller than this because there will be a lot of space in the room that cannot be covered by balls. In fact, even in the best arrangement, the so-called "best packing fraction" is $\frac{1}{6}\pi\sqrt{2}=0.74$ so that at least 26% of the space will be empty. Therefore, the above estimate reduces to $1.67\times10^6\times0.740\sim10^6$.

P1.26 A reasonable guess for the diameter of a tire might be 2.5 ft, with a circumference of about 8 ft. Thus, the tire would make $(50\,000\ \text{mi})(5\,280\ \text{ft/mi})(1\ \text{rev}/8\ \text{ft})=3\times10^7\ \text{rev}\sim\boxed{10^7\ \text{rev}}$.

P1.27 Assume the tub measures 1.3 m by 0.5 m by 0.3 m. One-half of its volume is then

$$V=(0.5)(1.3\ \text{m})(0.5\ \text{m})(0.3\ \text{m})=0.10\ \text{m}^3.$$

The mass of this volume of water is

$$m_{\text{water}}=\rho_{\text{water}}V=(1\,000\ \text{kg/m}^3)(0.10\ \text{m}^3)=100\ \text{kg}\boxed{\sim10^2\ \text{kg}}.$$

Pennies are now mostly zinc, but consider copper pennies filling 50% of the volume of the tub. The mass of copper required is

$$m_{\text{copper}}=\rho_{\text{copper}}V=(8\,920\ \text{kg/m}^3)(0.10\ \text{m}^3)=892\ \text{kg}\boxed{\sim10^3\ \text{kg}}.$$

***P1.28** The time required for the task is

$$10^9\$\left(\frac{1\ \text{s}}{1\ \$}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)\left(\frac{1\ \text{working day}}{16\ \text{h}}\right)\left(\frac{1\ \text{bad yr}}{300\ \text{working days}}\right)=58\ \text{yr}$$

Since you are already around 20 years old, you would have a miserable life and likely die before accomplishing the task. You have better things to do. Say no.

P1.29 Assume: Total population $=10^7$; one out of every 100 people has a piano; one tuner can serve about 1 000 pianos (about 4 per day for 250 weekdays, assuming each piano is tuned once per year). Therefore,

$$\#\ \text{tuners}\sim\left(\frac{1\ \text{tuner}}{1\,000\ \text{pianos}}\right)\left(\frac{1\ \text{piano}}{100\ \text{people}}\right)(10^7\ \text{people})=\boxed{100\ \text{tuners}}.$$

Section 1.6 Significant Figures

P1.30 METHOD ONE

We treat the best value with its uncertainty as a binomial (21.3 ± 0.2) cm (9.8 ± 0.1) cm,

$$A=[21.3(9.8)\pm21.3(0.1)\pm0.2(9.8)\pm(0.2)(0.1)]\ \text{cm}^2.$$

The first term gives the best value of the area. The cross terms add together to give the uncertainty and the fourth term is negligible.

$$A=\boxed{209\ \text{cm}^2\pm4\ \text{cm}^2}.$$

METHOD TWO

We add the fractional uncertainties in the data.

$$A=(21.3\ \text{cm})(9.8\ \text{cm})\pm\left(\frac{0.2}{21.3}+\frac{0.1}{9.8}\right)=209\ \text{cm}^2\pm2\%=209\ \text{cm}^2\pm4\ \text{cm}^2$$

P1.31 (a) $\boxed{3}$ (b) $\boxed{4}$ (c) $\boxed{3}$ (d) $\boxed{2}$

P1.32 $r = (6.50 \pm 0.20)\ \text{cm} = (6.50 \pm 0.20) \times 10^{-2}\ \text{m}$

$m = (1.85 \pm 0.02)\ \text{kg}$

$$\rho = \frac{m}{\left(\frac{4}{3}\right)\pi r^3}$$

also, $\dfrac{\delta \rho}{\rho} = \dfrac{\delta m}{m} + \dfrac{3\delta r}{r}$.

In other words, the percentages of uncertainty are cumulative. Therefore,

$$\frac{\delta \rho}{\rho} = \frac{0.02}{1.85} + \frac{3(0.20)}{6.50} = 0.103,$$

$$\rho = \frac{1.85}{\left(\frac{4}{3}\right)\pi\left(6.5 \times 10^{-2}\ \text{m}\right)^3} = \boxed{1.61 \times 10^3\ \text{kg/m}^3}$$

and

$$\rho \pm \delta\rho = \boxed{(1.61 \pm 0.17) \times 10^3\ \text{kg/m}^3} = (1.6 \pm 0.2) \times 10^3\ \text{kg/m}^3.$$

P1.33 (a)

756.??
37.2?
0.83
+ 2.5?
796./5/3 = $\boxed{797}$

(b) $0.003\,2(2\text{ s.f.}) \times 356.3(4\text{ s.f.}) = 1.140\,16 = (2\text{ s.f.})\ \boxed{1.1}$

(c) $5.620(4\text{ s.f.}) \times \pi(>4\text{ s.f.}) = 17.656 = (4\text{ s.f.})\ \boxed{17.66}$

P1.34 We work to nine significant digits:

$$1\text{ yr} = 1\text{ yr}\left(\frac{365.242\,199\text{ d}}{1\text{ yr}}\right)\left(\frac{24\text{ h}}{1\text{ d}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{31\,556\,926.0\text{ s}}.$$

***P1.35** The tax amount is \$1.36 – \$1.25 = \$0.11. The tax rate is \$0.11/\$1.25 = 0.088 0 = $\boxed{8.80\%}$

***P1.36** (a) We read from the graph a vertical separation of 0.3 spaces = $\boxed{0.015\text{ g}}$.

(b) Horizontally, 0.6 spaces = $\boxed{30\text{ cm}^2}$.

(c) Because the graph line goes through the origin, the same percentage describes the vertical and the horizontal scatter: 30 cm^2/380 cm^2 = $\boxed{8\%}$.

(d) Choose a grid point on the line far from the origin: slope = 0.31 g/600 cm^2 = 0.000 52 g/cm^2 = (0.000 52 g/cm^2)(10 000 cm^2/1 m^2) = $\boxed{5.2\text{ g/m}^2}$.

(e) For any and all shapes cut from this copy paper, the mass of the cutout is proportional to its area. The proportionality constant is 5.2 g/m^2 ± 8%, where the uncertainty is estimated.

(f) This result should be expected if the paper has thickness and density that are uniform within the experimental uncertainty. The slope is the areal density of the paper, its mass per unit area.

***P1.37** 15 players = 15 players (1 shift/1.667 player) = $\boxed{\text{9 shifts}}$

***P1.38** Let o represent the number of ordinary cars and s the number of trucks. We have $o = s + 0.947s = 1.947s$, and $o = s + 18$. We eliminate o by substitution: $s + 18 = 1.947s$
$0.947s = 18$ and $s = 18/0.947 = \boxed{19}$.

***P1.39** Let s represent the number of sparrows and m the number of more interesting birds. We have $s/m = 2.25$ and $s + m = 91$. We eliminate m by substitution: $m = s/2.25$
$s + s/2.25 = 91$ $\quad 1.444s = 91$ $\quad s = 91/1.444 = \boxed{63}$.

***P1.40** For those who are not familiar with solving equations numerically, we provide a detailed solution. It goes beyond proving that the suggested answer works.

The equation $2x^4 - 3x^3 + 5x - 70 = 0$ is quartic, so we do not attempt to solve it with algebra. To find how many real solutions the equation has and to estimate them, we graph the expression:

x	-3	-2	-1	0	1	2	3	4
$y = 2x^4 - 3x^3 + 5x - 70$	158	-24	-70	-70	-66	-52	26	270

We see that the equation $y = 0$ has two roots, one around $x = -2.2$ and the other near $x = +2.7$. To home in on the first of these solutions we compute in sequence: When $x = -2.2$, $y = -2.20$. The root must be between $x = -2.2$ and $x = -3$. When $x = -2.3$, $y = 11.0$. The root is between $x = -2.2$ and $x = -2.3$. When $x = -2.23$, $y = 1.58$. The root is between $x = -2.20$ and $x = -2.23$. When $x = -2.22$, $y = 0.301$. The root is between $x = -2.20$ and -2.22. When $x = -2.215$, $y = -0.331$. The root is between $x = -2.215$ and -2.22. We could next try $x = -2.218$, but we already know to three-digit precision that the root is $x = -2.22$.

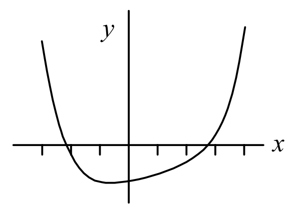


FIG. P1.40

***P1.41** We require $\sin\theta = -3\cos\theta$, or $\dfrac{\sin\theta}{\cos\theta} = -3$, or $\tan\theta = -3$.
For $\tan^{-1}(-3) = \arctan(-3)$, your calculator may return $-71.6°$, but this angle is not between 0° and 360° as the problem requires. The tangent function is negative in the second quadrant (between 90° and 180°) and in the fourth quadrant (from 270° to 360°). The solutions to the equation are then $360° - 71.6° = \boxed{288°}$ and $180° - 71.6° = \boxed{108°}$.

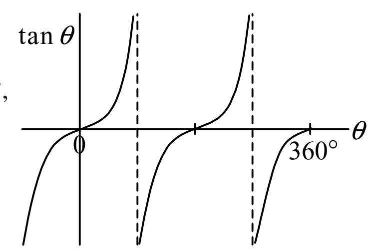


FIG. P1.41

***P1.42** We draw the radius to the initial point and the radius to the final point. The angle θ between these two radii has its sides perpendicular, right side to right side and left side to left side, to the 35° angle between the original and final tangential directions of travel. A most useful theorem from geometry then identifies these angles as equal: $\theta = 35°$. The whole circumference of a 360° circle of the same radius is $2\pi R$. By proportion, then $\frac{2\pi R}{360°} = \frac{840\text{ m}}{35°}$.

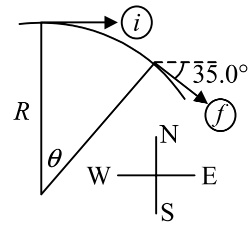


FIG. P1.42

$$R = \frac{360°}{2\pi}\frac{840\text{ m}}{35°} = \frac{840\text{ m}}{0.611} = \boxed{1.38\times10^3\text{ m}}$$

We could equally well say that the measure of the angle in radians is

$$\theta = 35° \;= 35°\left(\frac{2\pi\text{ radians}}{360°}\right) = 0.611\text{ rad} = \frac{840\text{ m}}{R}.$$

Solving yields $R = 1.38$ km.

***P1.43** Mass is proportional to cube of length: $m = k\ell^3 \quad m_f/m_i = (\ell_f/\ell_i)^3$.

Length changes by 15.8%: $\ell_f = \ell_i + 0.158\,\ell_i = 1.158\,\ell_i$.

Mass increase: $m_f = m_i + 17.3$ kg.

Eliminate by substitution: $\frac{m_f}{m_f - 17.3\text{ kg}} = 1.158^3 = 1.553$

$m_f = 1.553\,m_f - 26.9$ kg $\qquad$ $26.9\text{ kg} = 0.553\,m_f$ $\qquad$ $m_f = 26.9\text{ kg}/0.553 = \boxed{48.\ 6\text{ kg}}$.

***P1.44** We use substitution, as the most generally applicable method for solving simultaneous equations. We substitute $p = 3q$ into each of the other two equations to eliminate p:

$$\begin{cases} 3qr = qs \\ \frac{1}{2}3qr^2 + \frac{1}{2}qs^2 = \frac{1}{2}qt^2 \end{cases}.$$

These simplify to $\begin{cases} 3r = s \\ 3r^2 + s^2 = t^2 \end{cases}$. We substitute to eliminate s: $\begin{matrix} 3r^2 + (3r)^2 = t^2 \\ 12r^2 = t^2 \end{matrix}$. We solve for the combination $\frac{t}{r}$:

$$\frac{t^2}{r^2} = 12.$$

$$\frac{t}{r} = \boxed{\text{either } 3.46 \text{ or } -3.46}$$

***P1.45** Solve the given equation for Δt: $\quad \Delta t = 4QL/k\pi d^2(T_h - T_c) = [4QL/k\pi\,(T_h - T_c)]\,[1/d^2]$.

(a) Making d three times larger with d^2 in the bottom of the fraction makes Δt $\boxed{\text{nine times smaller}}$.

(b) $\boxed{\Delta t \text{ is inversely proportional to the square of } d.}$

(c) $\boxed{\text{Plot } \Delta t \text{ on the vertical axis and } 1/d^2 \text{ on the horizontal axis.}}$

(d) From the last version of the equation, the slope is $\boxed{4QL/k\pi(T_h - T_c)}$. Note that this quantity is constant as both Δt and d vary.

Additional Problems

P1.46 It is desired to find the distance x such that

$$\frac{x}{100\text{ m}}=\frac{1\,000\text{ m}}{x}$$

(i.e., such that x is the same multiple of 100 m as the multiple that 1 000 m is of x). Thus, it is seen that

$$x^2=(100\text{ m})(1\,000\text{ m})=1.00\times10^5\text{ m}^2$$

and therefore

$$x=\sqrt{1.00\times10^5\text{ m}^2}=\boxed{316\text{ m}}.$$

***P1.47** (a) The mass is equal to the mass of a sphere of radius 2.6 cm and density 4.7 g/cm^3, minus the mass of a sphere of radius a and density 4.7 g/cm^3 plus the mass of a sphere of radius a and density 1.23 g/cm^3.

$$m=\rho_1 4\pi r^3/3-\rho_1 4\pi a^3/3+\rho_2 4\pi a^3/3$$

$$=(4.7\text{ g/cm}^3)4\pi(2.6\text{ cm})^3/3-(4.7\text{ g/cm}^3)4\pi(a)^3/3+(1.23\text{ g/cm}^3)4\pi(a)^3/3$$

$$\boxed{m=346\text{ g}-(14.5\text{ g/cm}^3)a^3}$$

(b) For $\boxed{a=0}$ the mass is a maximum, (c) $\boxed{346\text{ g}}$. (d) $\boxed{\text{Yes}}$. This is the mass of the uniform sphere we considered in the first term of the calculation.

(e) For $\boxed{a=2.60\text{ cm}}$ the mass is a minimum, (f) $346-14.5(2.6)^3=\boxed{90.6\text{ g}}$. (g) $\boxed{\text{Yes}}$. This is the mass of a uniform sphere of density 1.23 g/cm^3.

(h) $(346\text{ g}+90.6\text{ g})/2=\boxed{218\text{ g}}$ (i) $\boxed{\text{No}}$. The result of part (a) gives $346\text{ g}-(14.5\text{ g/cm}^3)(1.3\text{ cm})^3=314\text{ g}$, not the same as 218 g.

(j) We should expect agreement in parts b-c-d, because those parts are about a uniform sphere of density 4.7 g/cm^3. We should expect agreement in parts e-f-g, because those parts are about a uniform liquid drop of density 1.23 g/cm^3. The function $m(a)$ is not a linear function, so a halfway between 0 and 2.6 cm does not give a value for m halfway between the minimum and maximum values. The graph of m versus a starts at $a=0$ with a horizontal tangent. Then it curves down more and more steeply as a increases. The liquid drop of radius 1.30 cm has only one eighth the volume of the whole sphere, so its presence brings down the mass by only a small amount, from 346 g to 314 g.

(k) No change, so long as the wall of the shell is unbroken.

***P1.48** (a) We have $B+C(0)=2.70\text{ g/cm}^3$ and $B+C(14\text{ cm})=19.3\text{ g/cm}^3$. We know $\boxed{B=2.70\text{ g/cm}^3}$ and we solve for C by subtracting: $C(14\text{ cm})=16.6\text{ g/cm}^3$ so $\boxed{C=1.19\text{ g/cm}^4}$.

(b)

$$m=\int_0^{14\text{ cm}}(2.70\text{ g/cm}^3+1.19\text{ g/cm}^4x)(9\text{ cm}^2)dx$$

$$=24.3\text{ g/cm}\int_0^{14\text{ cm}}dx+10.7\text{ g/cm}^2\int_0^{14\text{ cm}}xdx$$

$$=(24.3\text{ g/cm})(14\text{ cm}-0)+(10.7\text{ g/cm}^2)[(14\text{ cm})^2-0]/2$$

$$=340\text{ g}+1046\text{ g}=\boxed{1.39\text{ kg}}$$

P1.49 The scale factor used in the "dinner plate" model is

$$S = \frac{0.25\ \text{m}}{1.0\times10^{5}\ \text{lightyears}} = 2.5\times10^{-6}\ \ \text{m/lightyears}.$$

The distance to Andromeda in the scale model will be

$$D_{\text{scale}} = D_{\text{actual}}S = \left(2.0\times10^{6}\ \text{lightyears}\right)\left(2.5\times10^{-6}\ \text{m/lightyears}\right) = \boxed{5.0\ \text{m}}\ .$$

***P1.50** The rate of volume increase is

$$\frac{dV}{dt} = \frac{d}{dt}\frac{4}{3}\pi r^3 = \frac{4}{3}\pi 3r^2\frac{dr}{dt} = 4\pi r^2\frac{dr}{dt}\ .$$

(a) $dV/dt = 4\ \pi(6.5\text{ cm})^2(0.9\text{ cm/s}) = \boxed{478\text{ cm}^3\text{/s}}$

(b) $\dfrac{dr}{dt} = \dfrac{dV/dt}{4\pi r^2} = \dfrac{478\text{ cm}^3/\text{s}}{4\pi(13\text{ cm})^2} = \boxed{0.225\text{ cm}^3/\text{s}}$

(c) When the balloon radius is twice as large, its surface area is four times larger. The new volume added in one second in the inflation process is equal to this larger area times an extra radial thickness that is one-fourth as large as it was when the balloon was smaller.

P1.51 One month is

$$1\ \text{mo} = \left(30\ \text{day}\right)\left(24\ \text{h/day}\right)\left(3\,600\ \text{s/h}\right) = 2.592\times10^{6}\ \text{s}.$$

Applying units to the equation,

$$V = \left(1.50\ \ \text{Mft}^3/\text{mo}\right)t + \left(0.008\,00\ \ \text{Mft}^3/\text{mo}^2\right)t^2.$$

Since $1\ \text{Mft}^3 = 10^6\ \text{ft}^3$,

$$V = \left(1.50\times10^{6}\ \ \text{ft}^3/\text{mo}\right)t + \left(0.008\,00\times10^{6}\ \ \text{ft}^3/\text{mo}^2\right)t^2.$$

Converting months to seconds,

$$V = \frac{1.50\times10^{6}\ \ \text{ft}^3/\text{mo}}{2.592\times10^{6}\ \ \text{s/mo}}t + \frac{0.008\,00\times10^{6}\ \ \text{ft}^3/\text{mo}^2}{\left(2.592\times10^{6}\ \ \text{s/mo}\right)^2}t^2.$$

Thus, $\boxed{V\ [\text{ft}^3] = \left(0.579\ \ \text{ft}^3/\text{s}\right)t + \left(1.19\times10^{-9}\ \ \text{ft}^3/\text{s}^2\right)t^2}$.

***P1.52**

α'(deg)	α(rad)	$\tan(\alpha)$	$\sin(\alpha)$	difference between α and $\tan\alpha$
15.0	0.262	0.268	0.259	2.30%
20.0	0.349	0.364	0.342	4.09%
30.0	0.524	0.577	0.500	9.32%
33.0	0.576	0.649	0.545	11.3%
31.0	0.541	0.601	0.515	9.95%
31.1	0.543	0.603	0.516	10.02%

We see that α in radians, tan(α) and sin(α) start out together from zero and diverge only slightly in value for small angles. Thus $\boxed{31.0°}$ is the largest angle for which $\dfrac{\tan\alpha-\alpha}{\tan\alpha} < 0.1$.

P1.53 $2\pi r = 15.0\text{ m}$

$$r = 2.39\text{ m}$$

$$\frac{h}{r} = \tan 55.0°$$

$$h = (2.39\text{ m})\tan(55.0°) = \boxed{3.41\text{ m}}$$

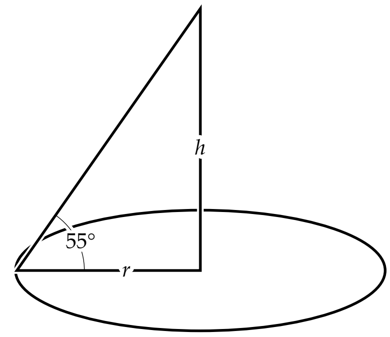


FIG. P1.53

P1.54 Let d represent the diameter of the coin and h its thickness. The mass of the gold is

$$m = \rho V = \rho A t = \rho\left(\frac{2\pi d^2}{4} + \pi d h\right)t$$

where t is the thickness of the plating.

$$m = 19.3\left[2\pi\frac{(2.41)^2}{4} + \pi(2.41)(0.178)\right](0.18\times10^{-4})$$
$$= 0.003\,64\text{ grams}$$
$$\text{cost} = 0.003\,64\text{ grams}\times\$10/\text{gram} = \$0.036\,4 = \boxed{3.64\text{ cents}}$$

This is negligible compared to \$4.98.

P1.55 The actual number of seconds in a year is

$$(86\,400\text{ s/day})(365.25\text{ day/yr}) = 31\,557\,600\text{ s/yr}.$$

The percent error in the approximation is

$$\frac{\left|(\pi\times10^7\text{ s/yr}) - (31\,557\,600\text{ s/yr})\right|}{31\,557\,600\text{ s/yr}}\times100\% = \boxed{0.449\%}.$$

P1.56 $v = \left(5.00\frac{\text{furlongs}}{\text{fortnight}}\right)\left(\frac{220\text{ yd}}{1\text{ furlong}}\right)\left(\frac{0.914\,4\text{ m}}{1\text{ yd}}\right)\left(\frac{1\text{ fortnight}}{14\text{ days}}\right)\left(\frac{1\text{ day}}{24\text{ hrs}}\right)\left(\frac{1\text{ hr}}{3\,600\text{ s}}\right) = \boxed{8.32\times10^{-4}\text{ m/s}}$

This speed is almost 1 mm/s; so we might guess the creature was a snail, or perhaps a sloth.

P1.57 (a) The speed of rise may be found from

$$v = \frac{(\text{Vol rate of flow})}{(\text{Area: }\pi D^2/4)} = \frac{16.5\text{ cm}^3/\text{s}}{\pi(6.30\text{ cm})^2/4} = \boxed{0.529\text{ cm/s}}.$$

(b) Likewise, at a 1.35 cm diameter,

$$v = \frac{16.5\text{ cm}^3/\text{s}}{\pi(1.35\text{ cm})^2/4} = \boxed{11.5\text{ cm/s}}.$$

P1.58 The density of each material is $\rho = \frac{m}{V} = \frac{m}{\pi r^2 h} = \frac{4m}{\pi D^2 h}$.

Al: $\rho = \frac{4(51.5\text{ g})}{\pi (2.52\text{ cm})^2 (3.75\text{ cm})} = \boxed{2.75\frac{\text{g}}{\text{cm}^3}}$ The tabulated value $\left(2.70\frac{\text{g}}{\text{cm}^3}\right)$ is $\boxed{2\%}$ smaller.

Cu: $\rho = \frac{4(56.3\text{ g})}{\pi (1.23\text{ cm})^2 (5.06\text{ cm})} = \boxed{9.36\frac{\text{g}}{\text{cm}^3}}$ The tabulated value $\left(8.92\frac{\text{g}}{\text{cm}^3}\right)$ is $\boxed{5\%}$ smaller.

Brass: $\rho = \frac{4(94.4\text{ g})}{\pi (1.54\text{ cm})^2 (5.69\text{ cm})} = \boxed{8.91\frac{\text{g}}{\text{cm}^3}}$

Sn: $\rho = \frac{4(69.1\text{ g})}{\pi (1.75\text{ cm})^2 (3.74\text{ cm})} = \boxed{7.68\frac{\text{g}}{\text{cm}^3}}$

Fe: $\rho = \frac{4(216.1\text{ g})}{\pi (1.89\text{ cm})^2 (9.77\text{ cm})} = \boxed{7.88\frac{\text{g}}{\text{cm}^3}}$ The tabulated value $\left(7.86\frac{\text{g}}{\text{cm}^3}\right)$ is $\boxed{0.3\%}$ smaller.

P1.59 $V_{20\text{ mpg}} = \frac{(10^8\text{ cars})(10^4\text{ mi/yr})}{20\text{ mi/gal}} = 5.0 \times 10^{10}\text{ gal/yr}$

$V_{25\text{ mpg}} = \frac{(10^8\text{ cars})(10^4\text{ mi/yr})}{25\text{ mi/gal}} = 4.0 \times 10^{10}\text{ gal/yr}$

Fuel saved $= V_{25\text{ mpg}} - V_{20\text{ mpg}} = \boxed{1.0 \times 10^{10}\text{ gal/yr}}$

P1.60 The volume of the galaxy is

$$\pi r^2 t = \pi\left(10^{21}\text{ m}\right)^2\left(10^{19}\text{ m}\right) \sim 10^{61}\text{ m}^3.$$

If the distance between stars is 4×10^{16} m, then there is one star in a volume on the order of

$$\left(4 \times 10^{16}\text{ m}\right)^3 \sim 10^{50}\text{ m}^3.$$

The number of stars is about $\frac{10^{61}\text{ m}^3}{10^{50}\text{ m}^3/\text{star}} \sim \boxed{10^{11}\text{ stars}}$.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P1.2 2.15×10^4 kg/m^3

P1.4 2.3×10^{17} kg/m^3 is twenty trillion times larger than the density of lead.

P1.6 0.141 nm

P1.8 (a) ii (b) iii (c) i

P1.10 9.19 nm/s

P1.12 (a) 3.39×10^5 ft^3 (b) 2.54×10^4 lb

P1.14 (a) 0.071 4 gal/s (b) 2.70×10^{-4} m^3/s (c) 1.03 h

P1.16 667 lb/s

P1.18 $2.57\times10^6\ \text{m}^3$

P1.20 (a) 2.07 mm (b) 8.57×10^{13} times as large

P1.22 (a) 13.4; (b) 49.1

P1.24 $r_{\text{Al}} = r_{\text{Fe}}\left(\dfrac{\rho_{\text{Fe}}}{\rho_{\text{Al}}}\right)^{1/3}$

P1.26 $\sim10^7$ rev

P1.28 No. There is a strong possibility that you would die before finishing the task, and you have much more productive things to do.

P1.30 $(209\pm4)\ \text{cm}^2$

P1.32 $(1.61 \pm 0.17) \times 10^3\ \text{kg/m}^3$

P1.34 31 556 926.0 s

P1.36 (a) 0.015 g (b) 30 cm^2 (c) 8% (d) 5.2 g/m^2 (e) For any and all shapes cut from this copy paper, the mass of the cutout is proportional to its area. The proportionality constant is 5.2 $\text{g/m}^2 \pm 8\%$, where the uncertainty is estimated. (f) This result is to be expected if the paper has thickness and density that are uniform within the experimental uncertainty. The slope is the areal density of the paper, its mass per unit area.

P1.38 19

P1.40 see the solution

P1.42 1.38 km

P1.44 either 3.46 or −3.46

P1.46 316 m

P1.48 (a) $\rho = 2.70\ \text{g/cm}^3 + 1.19\ \text{g/cm}^4\ x$ (b) 1.39 kg

P1.50 (a) 478 $\text{cm}^3\text{/s}$ (b) 0.225 cm/s (c) When the balloon radius is twice as large, its surface area is four times larger. The new volume added in one increment of time in the inflation process is equal to this larger area times an extra radial thickness that is one-fourth as large as it was when the balloon was smaller.

P1.52 0.542 rad

P1.54 3.64 cents; no

P1.56 8.32×10^{-4} m/s; a snail

P1.58 see the solution

P1.60 $\sim10^{11}$ stars

2

Motion in One Dimension

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

** An asterisk indicates an item new to this edition.*

***Q2.1** Count spaces (intervals), not dots. Count 5, not 6. The first drop falls at time zero and the last drop at $5 \times 5\text{ s} = 25\text{ s}$. The average speed is $600\text{ m}/25\text{ s} = 24\text{ m/s}$, answer (b).

Q2.2 The net displacement must be zero. The object could have moved away from its starting point and back again, but it is at its initial position again at the end of the time interval.

Q2.3 Yes. Yes. If the speed of the object varies at all over the interval, the instantaneous velocity will sometimes be greater than the average velocity and will sometimes be less.

***Q2.4** (a) It speeds up and its acceleration is positive. (b) It slows down overall, since final speed 1 m/s is slower than 3 m/s. Its acceleration is positive, meaning to the right. (c) It slows down and its acceleration is negative. (d) It speeds up to final speed 7 m/s. Its acceleration is negative, meaning toward the left or towards increasing-magnitude negative numbers on the track.

Q2.5 No: Car A might have greater acceleration than B, but they might both have zero acceleration, or otherwise equal accelerations; or the driver of B might have tramped hard on the gas pedal in the recent past to give car B greater acceleration just then.

***Q2.6** (c) A graph of velocity versus time slopes down steadily from an original positive (northward) value. The graph cuts through zero and goes through increasing-magnitude negative values, all with the same constant acceleration.

***Q2.7** (i) none. All of the disks are moving. (ii) (b) shows equal spacing, meaning constant nonzero velocity and constant zero acceleration. (iii) (b) This question has the same physical meaning as question (ii). (iv) (c) shows positive acceleration throughout. (v) (a) shows negative (leftward) acceleration in the last three images.

***Q2.8** Tramping hard on the brake at zero speed on a level road, you do not feel pushed around inside the car. The forces of rolling resistance and air resistance have dropped to zero as the car coasted to a stop, so the car's acceleration is zero at this moment and afterward.

Tramping hard on the brake at zero speed on an uphill slope, you feel thrown backward against your seat. Before, during, and after the zero-speed moment, the car is moving with a downhill acceleration if you do not tramp on the brake.

Brian Popp suggested the idea for this question.

***Q2.9** With original velocity zero, displacement is proportional to the square of time in $(1/2)at^2$. Making the time one-third as large makes the displacement one-ninth as large, answer (c).

Q2.10 No. Constant acceleration only. Yes. Zero is a constant.

Q2.11 They are the same. After the first ball reaches its apex and falls back downward past the student, it will have a downward velocity of magnitude v_i. This velocity is the same as the velocity of the second ball, so after they fall through equal heights their impact speeds will also be the same.

***Q2.12** For the release from rest we have $(4\text{ m/s})^2 = 0^2 + 2\,gh$. For case (i), we have $v_f^2 = (3\text{ m/s})^2 + 2\,gh = (3\text{ m/s})^2 + (4\text{ m/s})^2$. Thus answer (d) is true.

For case (ii) the same steps give the same answer (d).

***Q2.13** (i) Its speed is zero at b and e. Its speed is equal at a and c, and somewhat larger at d. On the bounce it is moving somewhat slower at f than at d, and slower at g than at c. The assembled answer is d > f > a = c > g > b = e.

(ii) The velocity is positive at a, f, and g, zero at b and e, and negative at c and d, with magnitudes as described in part (i). The assembled answer is f > a > g > b = e > c > d.

(iii) The acceleration has a very large positive value at e. At all the other points it is -9.8 m/s^2. The answer is e > a = b = c = d = f = g.

Q2.14 (b) Above. Your ball has zero initial speed and smaller average speed during the time of flight to the passing point. So your ball must travel a smaller distance to the passing point that the ball your friend throws.

SOLUTIONS TO PROBLEMS

Section 2.1 Position, Velocity, and Speed

P2.1 (a) $v_{avg} = \dfrac{\Delta x}{\Delta t} = \dfrac{10\text{ m}}{2\text{ s}} = \boxed{5\text{ m/s}}$

(b) $v_{avg} = \dfrac{5\text{ m}}{4\text{ s}} = \boxed{1.2\text{ m/s}}$

(c) $v_{avg} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{5\text{ m} - 10\text{ m}}{4\text{ s} - 2\text{ s}} = \boxed{-2.5\text{ m/s}}$

(d) $v_{avg} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{-5\text{ m} - 5\text{ m}}{7\text{ s} - 4\text{ s}} = \boxed{-3.3\text{ m/s}}$

(e) $v_{avg} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{0 - 0}{8 - 0} = \boxed{0\text{ m/s}}$

P2.2 (a) $v_{avg} = \boxed{2.30\text{ m/s}}$

(b) $v = \dfrac{\Delta x}{\Delta t} = \dfrac{57.5\text{ m} - 9.20\text{ m}}{3.00\text{ s}} = \boxed{16.1\text{ m/s}}$

(c) $v_{avg} = \dfrac{\Delta x}{\Delta t} = \dfrac{57.5\text{ m} - 0\text{ m}}{5.00\text{ s}} = \boxed{11.5\text{ m/s}}$

P2.3 (a) Let d represent the distance between A and B. Let t_1 be the time for which the walker has the higher speed in $5.00\ \text{m/s} = \frac{d}{t_1}$. Let t_2 represent the longer time for the return trip in $-3.00\ \text{m/s} = -\frac{d}{t_2}$. Then the times are $t_1 = \frac{d}{(5.00\ \text{m/s})}$ and $t_2 = \frac{d}{(3.00\ \text{m/s})}$. The average speed is:

$$v_{avg} = \frac{\text{Total distance}}{\text{Total time}} = \frac{d+d}{d/(5.00\ \text{m/s}) + d/(3.00\ \text{m/s})} = \frac{2d}{(8.00\ \text{m/s})d/(15.0\ \text{m}^2/\text{s}^2)}$$

$$v_{avg} = \frac{2(15.0\ \text{m}^2/\text{s}^2)}{8.00\ \text{m/s}} = \boxed{3.75\ \text{m/s}}$$

(b) She starts and finishes at the same point A. With total displacement = 0, average velocity $= \boxed{0}$.

P2.4 $x = 10t^2$: By substitution, for

$t(\text{s})$	=	2.0	2.1	3.0
$x(\text{m})$	=	40	44.1	90

(a) $v_{avg} = \frac{\Delta x}{\Delta t} = \frac{50\ \text{m}}{1.0\ \text{s}} = \boxed{50.0\ \text{m/s}}$

(b) $v_{avg} = \frac{\Delta x}{\Delta t} = \frac{4.1\ \text{m}}{0.1\ \text{s}} = \boxed{41.0\ \text{m/s}}$

Section 2.2 **Instantaneous Velocity and Speed**

P2.5 (a) at $t_i = 1.5$ s, $x_i = 8.0$ m (Point A)

at $t_f = 4.0$ s, $x_f = 2.0$ m (Point B)

$$v_{avg} = \frac{x_f - x_i}{t_f - t_i} = \frac{(2.0 - 8.0)\ \text{m}}{(4 - 1.5)\ \text{s}} = -\frac{6.0\ \text{m}}{2.5\ \text{s}} = \boxed{-2.4\ \text{m/s}}$$

(b) The slope of the tangent line can be found from points C and D. $(t_C = 1.0\ \text{s}, x_C = 9.5\ \text{m})$ and $(t_D = 3.5\ \text{s}, x_D = 0)$,

$$v \approx \boxed{-3.8\ \text{m/s}}.$$

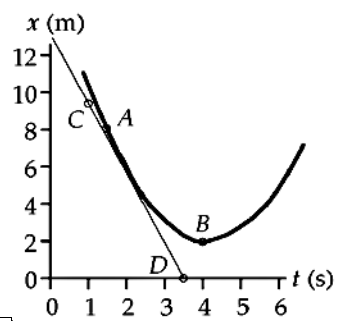


FIG. P2.5

(c) The velocity is zero when x is a minimum. This is at $t \approx \boxed{4\ \text{s}}$.

P2.6 (a) At any time, t, the position is given by $x = (3.00\ \text{m/s}^2)t^2$.

Thus, at $t_i = 3.00$ s: $x_i = (3.00\ \text{m/s}^2)(3.00\ \text{s})^2 = \boxed{27.0\ \text{m}}$.

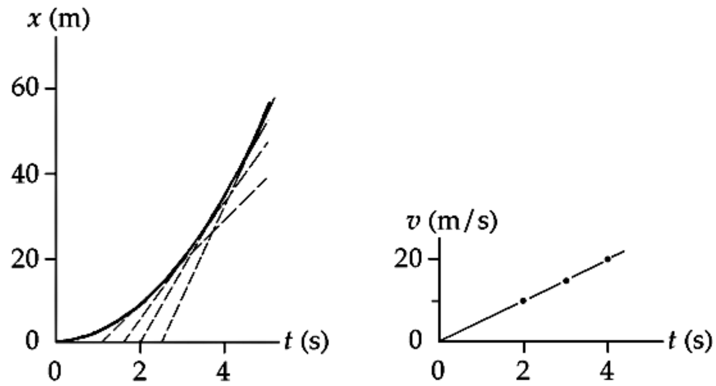
(b) At $t_f = 3.00\ \text{s} + \Delta t$: $x_f = (3.00\ \text{m/s}^2)(3.00\ \text{s} + \Delta t)^2$, or

$$x_f = \boxed{27.0\ \text{m} + (18.0\ \text{m/s})\Delta t + (3.00\ \text{m/s}^2)(\Delta t)^2}.$$

(c) The instantaneous velocity at $t = 3.00$ s is:

$$v = \lim_{\Delta t \to 0}\left(\frac{x_f - x_i}{\Delta t}\right) = \lim_{\Delta t \to 0}\left(18.0\ \text{m/s} + (3.00\ \text{m/s}^2)\Delta t\right) = \boxed{18.0\ \text{m/s}}.$$

P2.7 (a)



(b) At $t = 5.0$ s, the slope is $v \approx \dfrac{58 \text{ m}}{2.5 \text{ s}} = \boxed{23 \text{ m/s}}$.

At $t = 4.0$ s, the slope is $v \approx \dfrac{54 \text{ m}}{3 \text{ s}} = \boxed{18 \text{ m/s}}$.

At $t = 3.0$ s, the slope is $v \approx \dfrac{49 \text{ m}}{3.4 \text{ s}} = \boxed{14 \text{ m/s}}$.

At $t = 2.0$ s, the slope is $v \approx \dfrac{36 \text{ m}}{4.0 \text{ s}} = \boxed{9.0 \text{ m/s}}$.

(c) $a_{avg} = \dfrac{\Delta v}{\Delta t} \approx \dfrac{23 \text{ m/s}}{5.0 \text{ s}} = \boxed{4.6 \text{ m/s}^2}$

(d) Initial velocity of the car was $\boxed{\text{zero}}$.

P2.8 (a) $v = \dfrac{(5-0) \text{ m}}{(1-0) \text{ s}} = \boxed{5 \text{ m/s}}$

(b) $v = \dfrac{(5-10) \text{ m}}{(4-2) \text{ s}} = \boxed{-2.5 \text{ m/s}}$

(c) $v = \dfrac{(5 \text{ m} - 5 \text{ m})}{(5 \text{ s} - 4 \text{ s})} = \boxed{0}$

(d) $v = \dfrac{0 - (-5 \text{ m})}{(8 \text{ s} - 7 \text{ s})} = \boxed{+5 \text{ m/s}}$

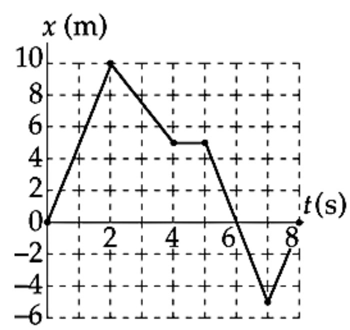


FIG. P2.8

P2.9 Once it resumes the race, the hare will run for a time of

$$t = \frac{x_f - x_i}{v_x} = \frac{1\,000 \text{ m} - 800 \text{ m}}{8 \text{ m/s}} = 25 \text{ s}.$$

In this time, the tortoise can crawl a distance

$$x_f - x_i = (0.2 \text{ m/s})(25 \text{ s}) = \boxed{5.00 \text{ m}}.$$

Section 2.3 **Acceleration**

P2.10 Choose the positive direction to be the outward direction, perpendicular to the wall.

$$v_f = v_i + at: \; a = \frac{\Delta v}{\Delta t} = \frac{22.0 \text{ m/s} - (-25.0 \text{ m/s})}{3.50 \times 10^{-3} \text{ s}} = \boxed{1.34 \times 10^4 \text{ m/s}^2}$$

P2.11 (a) Acceleration is constant over the first ten seconds, so at the end of this interval

$$v_f = v_i + at = 0 + \left(2.00 \text{ m/s}^2\right)(10.0 \text{ s}) = \boxed{20.0 \text{ m/s}}.$$

Then $a = 0$ so v is constant from $t = 10.0$ s to $t = 15.0$ s. And over the last five seconds the velocity changes to

$$v_f = v_i + at = 20.0 \text{ m/s} + \left(-3.00 \text{ m/s}^2\right)(5.00 \text{ s}) = \boxed{5.00 \text{ m/s}}.$$

(b) In the first ten seconds,

$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}\left(2.00 \text{ m/s}^2\right)(10.0 \text{ s})^2 = 100 \text{ m}.$$

Over the next five seconds the position changes to

$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 100 \text{ m} + (20.0 \text{ m/s})(5.00 \text{ s}) + 0 = 200 \text{ m}.$$

And at $t = 20.0$ s,

$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 200 \text{ m} + (20.0 \text{ m/s})(5.00 \text{ s}) + \frac{1}{2}\left(-3.00 \text{ m/s}^2\right)(5.00 \text{ s})^2 = \boxed{262 \text{ m}}.$$

P2.12 (a) Acceleration is the slope of the graph of v versus t.

For $0 < t < 5.00$ s, $a = 0$.

For $15.0 \text{ s} < t < 20.0$ s, $a = 0$.

For $5.0 \text{ s} < t < 15.0$ s, $a = \dfrac{v_f - v_i}{t_f - t_i}$.

$$a = \frac{8.00 - (-8.00)}{15.0 - 5.00} = 1.60 \text{ m/s}^2$$

We can plot $a(t)$ as shown.

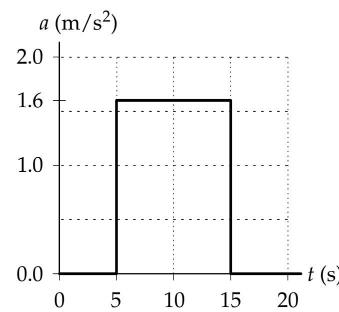


FIG. P2.12

(b) $a = \dfrac{v_f - v_i}{t_f - t_i}$

(i) For $5.00 \text{ s} < t < 15.0$ s, $t_i = 5.00$ s, $v_i = -8.00$ m/s,

$t_f = 15.0$ s

$v_f = 8.00$ m/s

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 - (-8.00)}{15.0 - 5.00} = \boxed{1.60 \text{ m/s}^2}.$$

(ii) $t_i = 0$, $v_i = -8.00$ m/s, $t_f = 20.0$ s, $v_f = 8.00$ m/s

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 - (-8.00)}{20.0 - 0} = \boxed{0.800 \text{ m/s}^2}$$

P2.13 $x = 2.00 + 3.00t - t^2$, so $v = \dfrac{dx}{dt} = 3.00 - 2.00t$, and $a = \dfrac{dv}{dt} = -2.00$

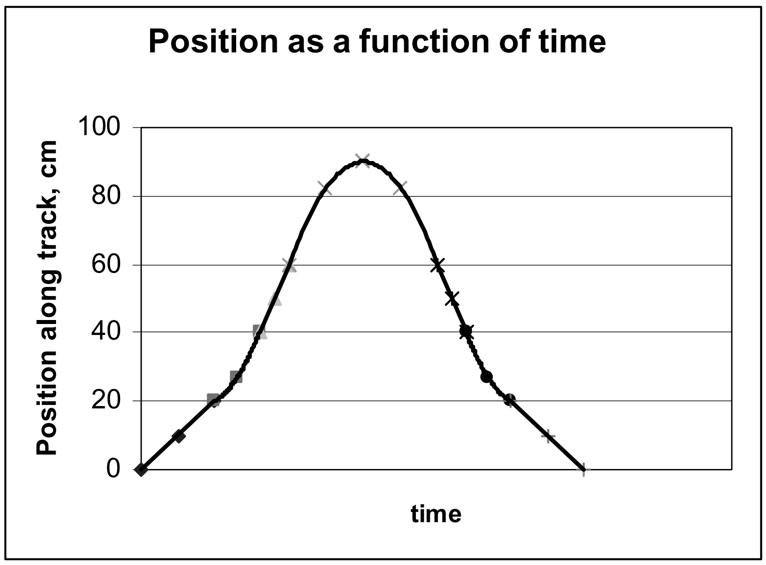
At $t = 3.00$ s:

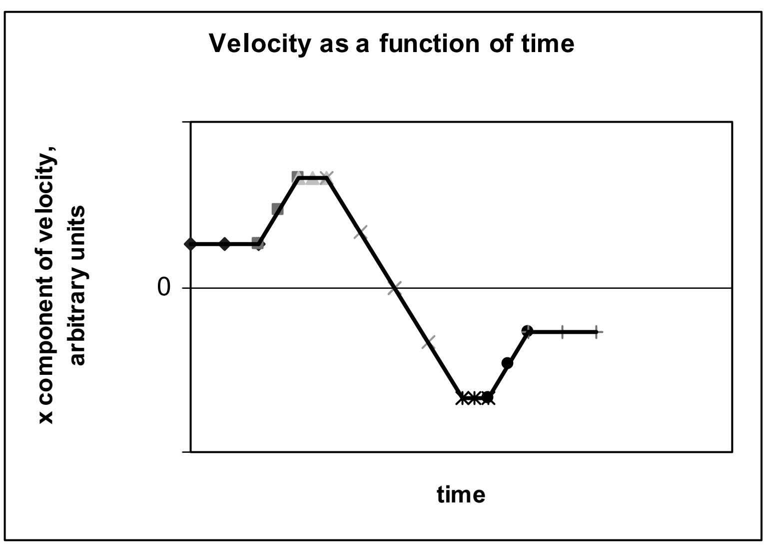
(a) $x = (2.00 + 9.00 - 9.00)\ \text{m} = \boxed{2.00\ \text{m}}$

(b) $v = (3.00 - 6.00)\ \text{m/s} = \boxed{-3.00\ \text{m/s}}$

(c) $a = \boxed{-2.00\ \text{m/s}^2}$

***P2.14** The acceleration is zero whenever the marble is on a horizontal section. The acceleration has a constant positive value when the marble is rolling on the 20-to-40-cm section and has a constant negative value when it is rolling on the second sloping section. The position graph is a straight sloping line whenever the speed is constant and a section of a parabola when the speed changes.





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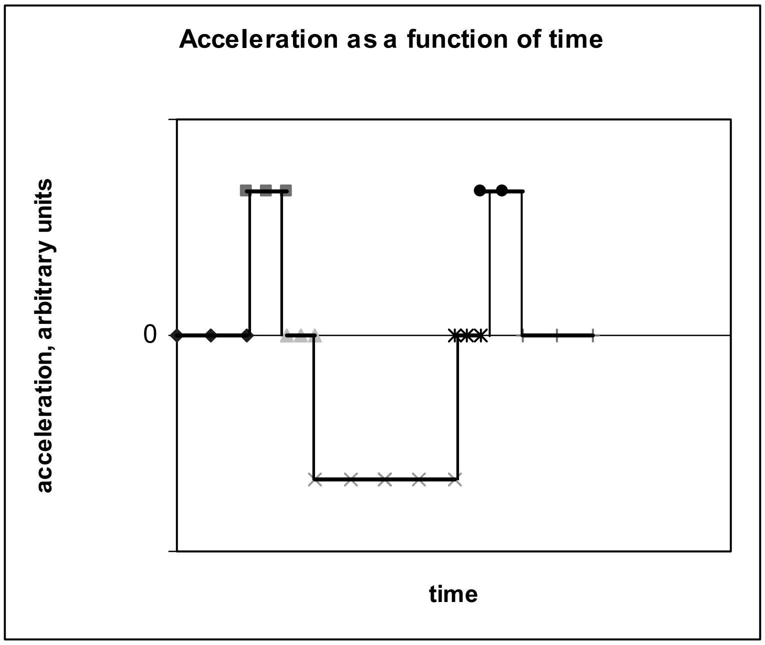


P2.15 (a) At $t = 2.00$ s, $x = \left[3.00(2.00)^2 - 2.00(2.00) + 3.00\right]$ m $= 11.0$ m.

At $t = 3.00$ s, $x = \left[3.00(9.00)^2 - 2.00(3.00) + 3.00\right]$ m $= 24.0$ m

so

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{24.0\text{ m} - 11.0\text{ m}}{3.00\text{ s} - 2.00\text{ s}} = \boxed{13.0\text{ m/s}}.$$

(b) At all times the instantaneous velocity is

$$v = \frac{d}{dt}\left(3.00t^2 - 2.00t + 3.00\right) = (6.00t - 2.00)\text{ m/s}$$

At $t = 2.00$ s, $v = \left[6.00(2.00) - 2.00\right]$ m/s $= \boxed{10.0\text{ m/s}}$.

At $t = 3.00$ s, $v = \left[6.00(3.00) - 2.00\right]$ m/s $= \boxed{16.0\text{ m/s}}$.

(c) $a_{avg} = \dfrac{\Delta v}{\Delta t} = \dfrac{16.0\text{ m/s} - 10.0\text{ m/s}}{3.00\text{ s} - 2.00\text{ s}} = \boxed{6.00\text{ m/s}^2}$

(d) At all times $a = \dfrac{d}{dt}(6.00t - 2.00) = \boxed{6.00\text{ m/s}^2}$. This includes both $t = 2.00$ s and $t = 3.00$ s.

P2.16 (a) $a = \dfrac{\Delta v}{\Delta t} = \dfrac{8.00\text{ m/s}}{6.00\text{ s}} = \boxed{1.3\text{ m/s}^2}$

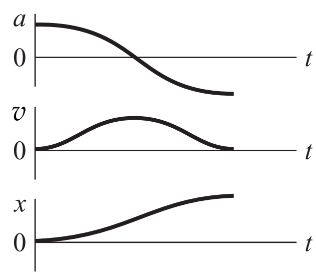
(b) Maximum positive acceleration is at $t = 3$ s, and is the slope of the graph, approximately $(6 - 2)/(4 - 2) = \boxed{2\text{ m/s}^2}$.

(c) $a = 0$ at $\boxed{t = 6\text{ s}}$, and also for $\boxed{t > 10\text{ s}}$.

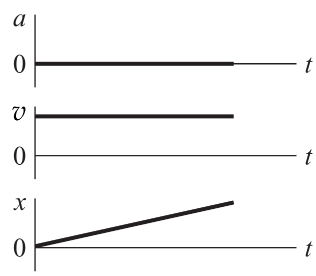
(d) Maximum negative acceleration is at $t = 8$ s, and is the slope of the graph, approximately $\boxed{-1.5\text{ m/s}^2}$.

Section 2.4 **Motion Diagrams**

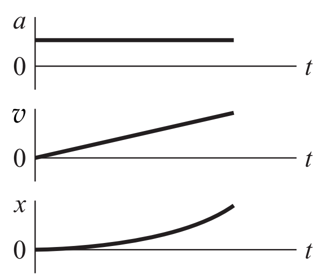
***P2.17** (a) The motion is slow at first, then fast, and then slow again.



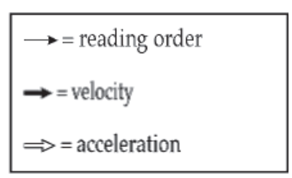
(b) The motion is constant in speed.



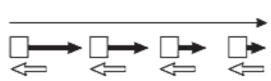
(c) The motion is speeding up, and we suppose the acceleration is constant.

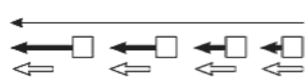


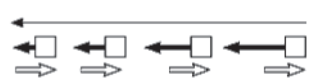
P2.18 (a) 



(b) 

(c) 

(d) 

(e) 

(f) One way of phrasing the answer: The spacing of the successive positions would change with less regularity.

Another way: The object would move with some combination of the kinds of motion shown in (a) through (e). Within one drawing, the accelerations vectors would vary in magnitude and direction.

Section 2.5 **One-Dimensional Motion with Constant Acceleration**

***P2.19** (a) $v_f = v_i + at = 13\text{ m/s} - 4\text{ m/s}^2\,(1\text{ s}) = \boxed{9.00\text{ m/s}}$

(b) $v_f = v_i + at = 13\text{ m/s} - 4\text{ m/s}^2\,(2\text{ s}) = \boxed{5.00\text{ m/s}}$

(c) $v_f = v_i + at = 13\text{ m/s} - 4\text{ m/s}^2\,(2.5\text{ s}) = \boxed{3.00\text{ m/s}}$

(d) $v_f = v_i + at = 13\text{ m/s} - 4\text{ m/s}^2\,(4\text{ s}) = \boxed{-3.00\text{ m/s}}$

(e) $v_f = v_i + at = 13\text{ m/s} - 4\text{ m/s}^2\,(-1\text{ s}) = \boxed{17.0\text{ m/s}}$

(f) The graph of velocity versus time is a slanting straight line, having the value 13 m/s at 10:05:00 a.m. on the certain date, and sloping down by 4 m/s for every second thereafter.

(g) If we also know the velocity at any one instant, then knowing the value of the constant acceleration tells us the velocity at all other instants.

P2.20 (a) $x_f - x_i = \frac{1}{2}\left(v_i + v_f\right)t$ becomes $40\text{ m} = \frac{1}{2}\left(v_i + 2.80\text{ m/s}\right)(8.50\text{ s})$ which yields

$v_i = \boxed{6.61\text{ m/s}}$.

(b) $a = \dfrac{v_f - v_i}{t} = \dfrac{2.80\text{ m/s} - 6.61\text{ m/s}}{8.50\text{ s}} = \boxed{-0.448\text{ m/s}^2}$

P2.21 Given $v_i = 12.0\text{ cm/s}$ when $x_i = 3.00\text{ cm}\,(t = 0)$, and at $t = 2.00\text{ s}$, $x_f = -5.00\text{ cm}$,

$$x_f - x_i = v_i t + \frac{1}{2}at^2 :\ -5.00 - 3.00 = 12.0(2.00) + \frac{1}{2}a(2.00)^2$$

$$-8.00 = 24.0 + 2a \qquad a = -\frac{32.0}{2} = \boxed{-16.0\text{ cm/s}^2}\,.$$

P2.22 (a) Total displacement = area under the (v, t) curve from $t = 0$ to 50 s.

$$\Delta x = \frac{1}{2}(50\text{ m/s})(15\text{ s}) + (50\text{ m/s})(40 - 15)\text{ s}$$
$$+\frac{1}{2}(50\text{ m/s})(10\text{ s})$$
$$\Delta x = 1875\text{ m} = \boxed{1.88\text{ km}}$$

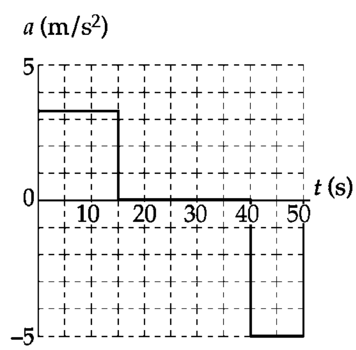


FIG. P2.22

(b) From $t = 10\text{ s}$ to $t = 40\text{ s}$, displacement is

$$\Delta x = \frac{1}{2}(50\text{ m/s} + 33\text{ m/s})(5\text{ s}) + (50\text{ m/s})(25\text{ s}) = \boxed{1.46\text{ km}}$$

(c) $0 \le t \le 15\text{ s}$: $a_1 = \dfrac{\Delta v}{\Delta t} = \dfrac{(50 - 0)\text{ m/s}}{15\text{ s} - 0} = \boxed{3.3\text{ m/s}^2}$

$15\text{ s} < t < 40\text{ s}$: $\boxed{a_2 = 0}$

$40\text{ s} \le t \le 50\text{ s}$: $a_3 = \dfrac{\Delta v}{\Delta t} = \dfrac{(0 - 50)\text{ m/s}}{50\text{ s} - 40\text{ s}} = \boxed{-5.0\text{ m/s}^2}$

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(d) (i) $x_1 = 0 + \frac{1}{2}a_1 t^2 = \frac{1}{2}(3.3\ \text{m/s}^2)t^2$ or $\boxed{x_1 = (1.67\ \text{m/s}^2)t^2}$

(ii) $x_2 = \frac{1}{2}(15\ \text{s})[50\ \text{m/s} - 0] + (50\ \text{m/s})(t - 15\ \text{s})$ or $\boxed{x_2 = (50\ \text{m/s})t - 375\ \text{m}}$

(iii) For $40\ \text{s} \le t \le 50\ \text{s}$,

$$x_3 = \begin{pmatrix} \text{area under } v \text{ vs } t \\ \text{from } t = 0 \text{ to } 40\ \text{s} \end{pmatrix} + \frac{1}{2}a_3(t - 40\ \text{s})^2 + (50\ \text{m/s})(t - 40\ \text{s})$$

or

$$x_3 = 375\ \text{m} + 1\,250\ \text{m} + \frac{1}{2}(-5.0\ \text{m/s}^2)(t - 40\ \text{s})^2 + (50\ \text{m/s})(t - 40\ \text{s})$$

which reduces to

$$\boxed{x_3 = (250\ \text{m/s})t - (2.5\ \text{m/s}^2)t^2 - 4\,375\ \text{m}}.$$

(e) $\bar{v} = \frac{\text{total displacement}}{\text{total elapsed time}} = \frac{1\,875\ \text{m}}{50\ \text{s}} = \boxed{37.5\ \text{m/s}}$

P2.23 (a) $v_i = 100\ \text{m/s}$, $a = -5.00\ \text{m/s}^2$, $v_f = v_i + at$ so $0 = 100 - 5t$, $v_f^2 = v_i^2 + 2a(x_f - x_i)$ so $0 = (100)^2 - 2(5.00)(x_f - 0)$. Thus $x_f = 1\,000\ \text{m}$ and $t = \boxed{20.0\ \text{s}}$.

(b) 1 000 m is greater than 800 m. With this acceleration $\boxed{\text{the plane would overshoot the runway: it cannot land}}$.

***P2.24** (a) For the first car the speed as a function of time is $v = v_i + at = -3.5\ \text{cm/s} + 2.4\ \text{cm/s}^2\ t$. For the second car, the speed is + 5.5 cm/s + 0. Setting the two expressions equal gives

$-3.5\ \text{cm/s} + 2.4\ \text{cm/s}^2\ t = 5.5\ \text{cm/s}$ so $t = (9\ \text{cm/s})/(2.4\ \text{cm/s}^2) = \boxed{3.75\ \text{s}}$.

(b) The first car then has speed $-3.5\ \text{cm/s} + (2.4\ \text{cm/s}^2)(3.75\ \text{s}) = \boxed{5.50\ \text{cm/s}}$, and this is the constant speed of the second car also.

(c) For the first car the position as a function of time is $x_i + v_i t + (1/2)at^2 = 15\ \text{cm} - (3.5\ \text{cm/s})t + (0.5)(2.4\ \text{cm/s}^2)\ t^2$.

For the second car, the position is $10\ \text{cm} + (5.5\ \text{cm/s})t + 0$.

At passing, the positions are equal: $15\ \text{cm} - (3.5\ \text{cm/s})t + (1.2\ \text{cm/s}^2)\ t^2 = 10\ \text{cm} + (5.5\ \text{cm/s})t\ (1.2\ \text{cm/s}^2)\ t^2 - (9\ \text{cm/s})\ t + 5\ \text{cm} = 0$.

We solve with the quadratic formula:

$$t = \frac{9 \pm \sqrt{9^2 - 4(1.2)(5)}}{2(1.2)} = \frac{9 + \sqrt{57}}{2.4} \text{ and } \frac{9 - \sqrt{57}}{2.4} = \boxed{6.90\ \text{s and } 0.604\ \text{s}}$$

(d) At 0.604 s, the second and also the first car's position is $10\ \text{cm} + (5.5\ \text{cm/s})0.604\ \text{s} = \boxed{13.3\ \text{cm}}$.
At 6.90 s, both are at position $10\ \text{cm} + (5.5\ \text{cm/s})6.90\ \text{s} = \boxed{47.9\ \text{cm}}$.

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(e) The cars are initially moving toward each other, so they soon share the same position x when their speeds are quite different, giving one answer to (c) that is not an answer to (a). The first car slows down in its motion to the left, turns around, and starts to move toward the right, slowly at first and gaining speed steadily. At a particular moment its speed will be equal to the constant rightward speed of the second car. The distance between them will at that moment be staying constant at its maximum value. The distance between the cars will be far from zero, as the accelerating car will be far to the left of the steadily moving car. Thus the answer to (a) is not an answer to (c). Eventually the accelerating car will catch up to the steadily-coasting car, whizzing past at higher speed than it has ever had before, and giving another answer to (c) that is not an answer to (a). A graph of x versus t for the two cars shows a parabola originally sloping down and then curving upward, intersecting twice with an upward-sloping straight line. The parabola and straight line are running parallel, with equal slopes, at just one point in between their intersections.

P2.25 In the simultaneous equations:

$$\left\{\begin{matrix} v_{xf} = v_{xi} + a_x t \\ x_f - x_i = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t \end{matrix}\right\} \text{ we have } \left\{\begin{matrix} v_{xf} = v_{xi} - \left(5.60\ \text{m/s}^2\right)(4.20\ \text{s}) \\ 62.4\ \text{m} = \frac{1}{2}\left(v_{xi} + v_{xf}\right)(4.20\ \text{s}) \end{matrix}\right\}.$$

So substituting for v_{xi} gives $62.4\ \text{m} = \frac{1}{2}\left[v_{xf} + \left(5.60\ \text{m/s}^2\right)(4.20\ \text{s}) + v_{xf}\right](4.20\ \text{s})$

$$14.9\ \text{m/s} = v_{xf} + \frac{1}{2}\left(5.60\ \text{m/s}^2\right)(4.20\ \text{s}).$$

Thus

$$v_{xf} = \boxed{3.10\ \text{m/s}}.$$

P2.26 Take any two of the standard four equations, such as $\left\{\begin{matrix} v_{xf} = v_{xi} + a_x t \\ x_f - x_i = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t \end{matrix}\right\}$.

Solve one for v_{xi}, and substitute into the other: $v_{xi} = v_{xf} - a_x t$

$$x_f - x_i = \frac{1}{2}\left(v_{xf} - a_x t + v_{xf}\right)t.$$

Thus

$$\boxed{x_f - x_i = v_{xf}t - \frac{1}{2}a_x t^2}.$$

We note that the equation is dimensionally correct. The units are units of length in each term. Like the standard equation $x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$, this equation represents that displacement is a quadratic function of time.

Our newly derived equation gives us for the situation back in problem 25,

$$62.4\ \text{m} = v_{xf}(4.20\ \text{s}) - \frac{1}{2}\left(-5.60\ \text{m/s}^2\right)(4.20\ \text{s})^2$$

$$v_{xf} = \frac{62.4\ \text{m} - 49.4\ \text{m}}{4.20\ \text{s}} = \boxed{3.10\ \text{m/s}}.$$

P2.27 (a) $a = \dfrac{v_f - v_i}{t} = \dfrac{632(5280/3600)}{1.40} = \boxed{-662 \text{ ft/s}^2} = -202 \text{ m/s}^2$

(b) $x_f = v_i t + \dfrac{1}{2}at^2 = (632)\left(\dfrac{5\,280}{3\,600}\right)(1.40) - \dfrac{1}{2}(662)(1.40)^2 = \boxed{649 \text{ ft}} = \boxed{198 \text{ m}}$

P2.28 (a) Compare the position equation $x = 2.00 + 3.00t - 4.00t^2$ to the general form

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

to recognize that $x_i = 2.00$ m, $v_i = 3.00$ m/s, and $a = -8.00$ m/s^2. The velocity equation, $v_f = v_i + at$, is then

$$v_f = 3.00 \text{ m/s} - (8.00 \text{ m/s}^2)t.$$

The particle changes direction when $v_f = 0$, which occurs at $t = \dfrac{3}{8}$ s. The position at this time is

$$x = 2.00 \text{ m} + (3.00 \text{ m/s})\left(\frac{3}{8} \text{ s}\right) - (4.00 \text{ m/s}^2)\left(\frac{3}{8} \text{ s}\right)^2 = \boxed{2.56 \text{ m}}.$$

(b) From $x_f = x_i + v_i t + \dfrac{1}{2}at^2$, observe that when $x_f = x_i$, the time is given by $t = -\dfrac{2v_i}{a}$. Thus, when the particle returns to its initial position, the time is

$$t = \frac{-2(3.00 \text{ m/s})}{-8.00 \text{ m/s}^2} = \frac{3}{4} \text{ s}$$

and the velocity is $v_f = 3.00 \text{ m/s} - (8.00 \text{ m/s}^2)\left(\dfrac{3}{4} \text{ s}\right) = \boxed{-3.00 \text{ m/s}}$.

P2.29 We have $v_i = 2.00 \times 10^4$ m/s, $v_f = 6.00 \times 10^6$ m/s, $x_f - x_i = 1.50 \times 10^{-2}$ m.

(a) $x_f - x_i = \dfrac{1}{2}(v_i + v_f)t$: $t = \dfrac{2(x_f - x_i)}{v_i + v_f} = \dfrac{2(1.50 \times 10^{-2} \text{ m})}{2.00 \times 10^4 \text{ m/s} + 6.00 \times 10^6 \text{ m/s}} = \boxed{4.98 \times 10^{-9} \text{ s}}$

(b) $v_f^2 = v_i^2 + 2a_x(x_f - x_i)$:

$$a_x = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{(6.00 \times 10^6 \text{ m/s})^2 - (2.00 \times 10^4 \text{ m/s})^2}{2(1.50 \times 10^{-2} \text{ m})} = \boxed{1.20 \times 10^{15} \text{ m/s}^2}$$

P2.30 (a) Along the time axis of the graph shown, let $i = 0$ and $f = t_m$. Then $v_{xf} = v_{xi} + a_x t$ gives $v_c = 0 + a_m t_m$

$$\boxed{a_m = \frac{v_c}{t_m}}.$$

(b) The displacement between 0 and t_m is

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}\frac{v_c}{t_m}t_m^2 = \frac{1}{2}v_c t_m.$$

The displacement between t_m and t_0 is

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2 = v_c(t_0 - t_m) + 0.$$

The total displacement is

$$\Delta x = \frac{1}{2}v_c t_m + v_c t_0 - v_c t_m = \boxed{v_c\left(t_0 - \frac{1}{2}t_m\right)}.$$

continued on next page

(c) For constant v_c and t_0, Δx is minimized by maximizing t_m to $t_m = t_0$. Then

$$\Delta x_{\min} = v_c\left(t_0 - \frac{1}{2}t_0\right) = \boxed{\frac{v_c t_0}{2}}.$$

(e) This is realized by having the servo motor on all the time.

(d) We maximize Δx by letting t_m approach zero. In the limit $\Delta x = v_c(t_0 - 0) = \boxed{v_c t_0}$.

(e) This cannot be attained because the acceleration must be finite.

P2.31 Let the glider enter the photogate with velocity v_i and move with constant acceleration a. For its motion from entry to exit,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$\ell = 0 + v_i \Delta t_d + \frac{1}{2}a\Delta t_d^2 = v_d \Delta t_d$$

$$v_d = v_i + \frac{1}{2}a\Delta t_d$$

(a) The speed halfway through the photogate in space is given by

$$v_{hs}^2 = v_i^2 + 2a\left(\frac{\ell}{2}\right) = v_i^2 + av_d\Delta t_d.$$

$v_{hs} = \sqrt{v_i^2 + av_d\Delta t_d}$ and this is $\boxed{\text{not equal to } v_d \text{ unless } a = 0}$.

(b) The speed halfway through the photogate in time is given by $v_{ht} = v_i + a\left(\frac{\Delta t_d}{2}\right)$ and this is $\boxed{\text{equal to } v_d}$ as determined above.

P2.32 Take the original point to be when Sue notices the van. Choose the origin of the x-axis at Sue's car. For her we have $x_{is} = 0$, $v_{is} = 30.0\ \text{m/s}$, $a_s = -2.00\ \text{m/s}^2$ so her position is given by

$$x_s(t) = x_{is} + v_{is}t + \frac{1}{2}a_s t^2 = (30.0\ \text{m/s})t + \frac{1}{2}(-2.00\ \text{m/s}^2)t^2.$$

For the van, $x_{iv} = 155\ \text{m}$, $v_{iv} = 5.00\ \text{m/s}$, $a_v = 0$ and

$$x_v(t) = x_{iv} + v_{iv}t + \frac{1}{2}a_v t^2 = 155 + (5.00\ \text{m/s})t + 0.$$

To test for a collision, we look for an instant t_c when both are at the same place:

$$30.0t_c - t_c^2 = 155 + 5.00t_c$$

$$0 = t_c^2 - 25.0t_c + 155.$$

From the quadratic formula

$$t_c = \frac{25.0 \pm \sqrt{(25.0)^2 - 4(155)}}{2} = 13.6\ \text{s or } \boxed{11.4\ \text{s}}.$$

The roots are real, not imaginary, so $\boxed{\text{there is a collision}}$. The smaller value is the collision time. (The larger value tells when the van would pull ahead again if the vehicles could move through each other). The wreck happens at position

$$155\ \text{m} + (5.00\ \text{m/s})(11.4\ \text{s}) = \boxed{212\ \text{m}}.$$

***P2.33** (a) Starting from rest and accelerating at $a_b = 13.0\ \text{mi/h}\cdot\text{s}$, the bicycle reaches its maximum speed of $v_{b,\max} = 20.0\ \text{mi/h}$ in a time

$$t_{b,1} = \frac{v_{b,\max} - 0}{a_b} = \frac{20.0\ \text{mi/h}}{13.0\ \text{mi/h}\cdot\text{s}} = 1.54\ \text{s}.$$

Since the acceleration a_c of the car is less than that of the bicycle, the car cannot catch the bicycle until some time $t > t_{b,1}$ (that is, until the bicycle is at its maximum speed and coasting). The total displacement of the bicycle at time t is

$$\begin{aligned}\Delta x_b &= \frac{1}{2}a_b t_{b,1}^2 + v_{b,\max}\left(t - t_{b,1}\right)\\ &= \left(\frac{1.47\ \text{ft/s}}{1\ \text{mi/h}}\right)\left[\frac{1}{2}\left(13.0\ \frac{\text{mi/h}}{\text{s}}\right)(1.54\ \text{s})^2 + (20.0\ \text{mi/h})(t - 1.54\ \text{s})\right]\\ &= (29.4\ \text{ft/s})t - 22.6\ \text{ft}\end{aligned}$$

The total displacement of the car at this time is

$$\Delta x_c = \frac{1}{2}a_c t^2 = \left(\frac{1.47\ \text{ft/s}}{1\ \text{mi/h}}\right)\left[\frac{1}{2}\left(9.00\ \frac{\text{mi/h}}{\text{s}}\right)t^2\right] = (6.62\ \text{ft/s})t^2$$

At the time the car catches the bicycle $\Delta x_c = \Delta x_b$. This gives

$$\left(6.62\ \text{ft/s}^2\right)t^2 = (29.4\ \text{ft/s})t - 22.6\ \text{ft} \qquad \text{or} \qquad t^2 - (4.44\ \text{s})t + 3.42\ \text{s}^2 = 0$$

that has only one physically meaningful solution $t > t_{b,1}$. This solution gives the total time the bicycle leads the car and is $t = \boxed{3.45\ \text{s}}$.

(b) The lead the bicycle has over the car continues to increase as long as the bicycle is moving faster than the car. This means until the car attains a speed of $v_c = v_{b,\max} = 20.0\ \text{mi/h}$. Thus, the elapsed time when the bicycle's lead ceases to increase is

$$t = \frac{v_{b,\max}}{a_c} = \frac{20.0\ \text{mi/h}}{9.00\ \text{mi/h}\cdot\text{s}} = 2.22\ \text{s}$$

At this time, the lead is

$$\left(\Delta x_b - \Delta x_c\right)_{\max} = \left(\Delta x_b - \Delta x_c\right)\Big|_{t=2.22\ \text{s}} = \left[(29.4\ \text{ft/s})(2.22\ \text{s}) - 22.6\ \text{ft}\right] - \left[\left(6.62\ \text{ft/s}^2\right)(2.22\ \text{s})^2\right]$$

or $\left(\Delta x_b - \Delta x_c\right)_{\max} = \boxed{10.0\ \text{ft}}$.

P2.34 As in the algebraic solution to Example 2.9, we let t represent the time the trooper has been moving. We graph

$$x_{\text{car}} = 45 + 45t$$

and

$$x_{\text{trooper}} = 1.5t^2.$$

They intersect at

$$t = \boxed{31\ \text{s}}.$$

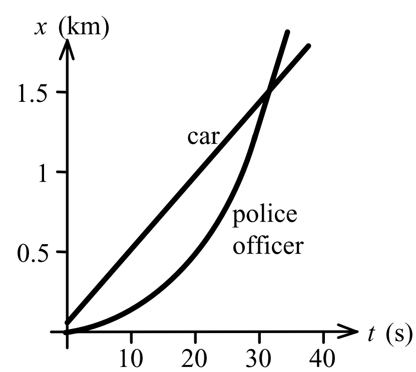

FIG. P2.34

***P2.35** (a) Let a stopwatch start from $t = 0$ as the front end of the glider passes point A. The average speed of the glider over the interval between $t = 0$ and $t = 0.628$ s is 12.4 cm/(0.628 s) = $\boxed{19.7 \text{ cm/s}}$, and this is the instantaneous speed halfway through the time interval, at $t = 0.314$ s.

(b) The average speed of the glider over the time interval between 0.628 + 1.39 = 2.02 s and 0.628 + 1.39 + 0.431 = 2.45 s is 12.4 cm/(0.431 s) = 28.8 cm/s and this is the instantaneous speed at the instant $t = (2.02 + 2.45)/2 = 2.23$ s.

Now we know the velocities at two instants, so the acceleration is found from [(28.8 − 19.7) cm/s]/[(2.23 − 0.314) s] = 9.03/1.92 cm/s^2 = $\boxed{4.70 \text{ cm/s}^2}$.

(c) The time required to pass between A and B is sufficient to find the acceleration, more directly than we could find it from the distance between the points.

Section 2.6 **Freely Falling Objects**

***P2.36** Choose the origin $(y = 0,\ t = 0)$ at the starting point of the cat and take upward as positive. Then $y_i = 0$, $v_i = 0$, and $a = -g = -9.80 \text{ m/s}^2$. The position and the velocity at time t become:

$$y_f - y_i = v_i t + \frac{1}{2}at^2: \qquad y_f = -\frac{1}{2}gt^2 = -\frac{1}{2}\left(9.80 \text{ m/s}^2\right)t^2$$

and

$$v_f = v_i + at: \qquad v_f = -gt = -\left(9.80 \text{ m/s}^2\right)t.$$

(a) at $t = 0.1$ s: $y_f = -\frac{1}{2}\left(9.80 \text{ m/s}^2\right)(0.1 \text{ s})^2 = \boxed{-0.049\,0 \text{ m}}$

at $t = 0.2$ s: $y_f = -\frac{1}{2}\left(9.80 \text{ m/s}^2\right)(0.2 \text{ s})^2 = \boxed{-0.196 \text{ m}}$

at $t = 0.3$ s: $y_f = -\frac{1}{2}\left(9.80 \text{ m/s}^2\right)(0.3 \text{ s})^2 = \boxed{-0.441 \text{ m}}$

(b) at $t = 0.1$ s: $v_f = -\left(9.80 \text{ m/s}^2\right)(0.1 \text{ s}) = \boxed{-0.980 \text{ m/s}}$

at $t = 0.2$ s: $v_f = -\left(9.80 \text{ m/s}^2\right)(0.2 \text{ s}) = \boxed{-1.96 \text{ m/s}}$

at $t = 0.3$ s: $v_f = -\left(9.80 \text{ m/s}^2\right)(0.3 \text{ s}) = \boxed{-2.94 \text{ m/s}}$

P2.37 Assume that air resistance may be neglected. Then, the acceleration at all times during the flight is that due to gravity, $a = -g = -9.80 \text{ m/s}^2$. During the flight, Goff went 1 mile (1 609 m) up and then 1 mile back down. Determine his speed just after launch by considering his upward flight:

$$v_f^2 = v_i^2 + 2a(y_f - y_i): \quad 0 = v_i^2 - 2(9.80 \text{ m/s}^2)(1\,609 \text{ m})$$
$$v_i = 178 \text{ m/s}.$$

His time in the air may be found by considering his motion from just after launch to just before impact:

$$y_f - y_i = v_i t + \frac{1}{2}at^2: \; 0 = (178 \text{ m/s})t - \frac{1}{2}(-9.80 \text{ m/s}^2)t^2.$$

The root $t = 0$ describes launch; the other root, $t = 36.2$ s, describes his flight time. His rate of pay is then

$$\text{pay rate} = \frac{\$1.00}{36.2 \text{ s}} = (0.027\,6 \text{ \$/s})(3\,600 \text{ s/h}) = \boxed{\$99.3/\text{h}}.$$

We have assumed that the workman's flight time, "a mile," and "a dollar," were measured to three-digit precision. We have interpreted "up in the sky" as referring to the free fall time, not to the launch and landing times.

Both the takeoff and landing times must be several seconds away from the job, in order for Goff to survive to resume work.

P2.38 We have $y_f = -\frac{1}{2}gt^2 + v_i t + y_i$

$$0 = -(4.90 \text{ m/s}^2)t^2 - (8.00 \text{ m/s})t + 30.0 \text{ m}.$$

Solving for t,

$$t = \frac{8.00 \pm \sqrt{64.0 + 588}}{-9.80}.$$

Using only the positive value for t, we find that $t = \boxed{1.79 \text{ s}}$.

P2.39 (a) $y_f - y_i = v_i t + \frac{1}{2}at^2$: $4.00 = (1.50)v_i - (4.90)(1.50)^2$ and $v_i = \boxed{10.0 \text{ m/s upward}}$.

(b) $v_f = v_i + at = 10.0 - (9.80)(1.50) = -4.68 \text{ m/s}$

$$v_f = \boxed{4.68 \text{ m/s downward}}$$

P2.40 The bill starts from rest $v_i = 0$ and falls with a downward acceleration of 9.80 m/s^2 (due to gravity). Thus, in 0.20 s it will fall a distance of

$$\Delta y = v_i t - \frac{1}{2}gt^2 = 0 - (4.90 \text{ m/s}^2)(0.20 \text{ s})^2 = -0.20 \text{ m}.$$

This distance is about twice the distance between the center of the bill and its top edge $(\approx 8 \text{ cm})$. $\boxed{\text{Thus, David will be unsuccessful.}}$

P2.41 (a) $v_f = v_i - gt$: $v_f = 0$ when $t = 3.00$ s, and $g = 9.80\ \text{m/s}^2$. Therefore,

$$v_i = gt = \left(9.80\ \text{m/s}^2\right)(3.00\ \text{s}) = \boxed{29.4\ \text{m/s}}.$$

(b) $y_f - y_i = \frac{1}{2}\left(v_f + v_i\right)t$

$$y_f - y_i = \frac{1}{2}(29.4\ \text{m/s})(3.00\ \text{s}) = \boxed{44.1\ \text{m}}$$

***P2.42** We can solve (a) and (b) at the same time by assuming the rock passes the top of the wall and finding its speed there. If the speed comes out imaginary, the rock will not reach this elevation.

$v_f^2 = v_i^2 + 2a(x_f - x_i) = (7.4\ \text{m/s})^2 - 2(9.8\ \text{m/s}^2)(3.65\ \text{m} - 1.55\ \text{m}) = 13.6\ \text{m}^2/\text{s}^2$

so $\boxed{\text{the rock does reach the top of the wall with } v_f = 3.69\ \text{m/s}}$.

(c) We find the final speed, just before impact, of the rock thrown down:

$v_f^2 = v_i^2 + 2a(x_f - x_i) = (-7.4\ \text{m/s})^2 - 2(9.8\ \text{m/s}^2)(1.55\ \text{m} - 3.65\ \text{m}) = 95.9\ \text{m}^2/\text{s}^2$

$v_f = -9.79$ m/s. The change in speed of the rock thrown down is $|9.79 - 7.4| = \boxed{2.39\ \text{m/s}}$

(d) The magnitude of the speed change of the rock thrown up is $|7.4 - 3.69| = 3.71$ m/s. This $\boxed{\text{does not agree}}$ with 2.39 m/s.

$\boxed{\text{The upward-moving rock spends more time in flight, so the planet has more time to change its speed.}}$

P2.43 Time to fall 3.00 m is found from the equation describing position as a function of time, with $v_i = 0$, thus: $3.00\ \text{m} = \frac{1}{2}\left(9.80\ \text{m/s}^2\right)t^2$, giving $t = 0.782$ s.

(a) With the horse galloping at $10.0\ \text{m/s}$, the horizontal distance is $vt = \boxed{7.82\ \text{m}}$.

(b) from above $t = \boxed{0.782\ \text{s}}$

P2.44 $y = 3.00t^3$: At $t = 2.00$ s, $y = 3.00(2.00)^3 = 24.0$ m and

$$v_y = \frac{dy}{dt} = 9.00t^2 = 36.0\ \text{m/s} \uparrow.$$

If the helicopter releases a small mailbag at this time, the mailbag starts its free fall with velocity 36 m/s upward. The equation of motion of the mailbag is

$$y_b = y_{bi} + v_i t - \frac{1}{2}gt^2 = 24.0 + 36.0t - \frac{1}{2}(9.80)t^2.$$

Setting $y_b = 0$,

$$0 = 24.0 + 36.0t - 4.90t^2.$$

Solving for t, (only positive values of t count), $\boxed{t = 7.96\ \text{s}}$.

P2.45 We assume the object starts from rest. Consider the last 30 m of its fall. We find its speed 30 m above the ground:

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$0 = 30\ \text{m} + v_{yi}(1.5\ \text{s}) + \frac{1}{2}\left(-9.8\ \text{m/s}^2\right)(1.5\ \text{s})^2$$

$$v_{yi} = \frac{-30\ \text{m} + 11.0\ \text{m}}{1.5\ \text{s}} = -12.6\ \text{m/s}.$$

Now consider the portion of its fall above the 30 m point:

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$$

$$\left(-12.6\ \text{m/s}\right)^2 = 0 + 2\left(-9.8\ \text{m/s}^2\right)\Delta y$$

$$\Delta y = \frac{160\ \text{m}^2/\text{s}^2}{-19.6\ \text{m/s}^2} = -8.16\ \text{m}.$$

Its original height was then $30\ \text{m} + |-8.16\ \text{m}| = \boxed{38.2\ \text{m}}$.

Section 2.7 **Kinematic Equations Derived from Calculus**

P2.46 (a) See the graphs at the right.

Choose $x = 0$ at $t = 0$.

At $t = 3\ \text{s}$, $x = \frac{1}{2}(8\ \text{m/s})(3\ \text{s}) = 12\ \text{m}$.

At $t = 5\ \text{s}$, $x = 12\ \text{m} + (8\ \text{m/s})(2\ \text{s}) = 28\ \text{m}$.

At $t = 7\ \text{s}$, $x = 28\ \text{m} + \frac{1}{2}(8\ \text{m/s})(2\ \text{s}) = 36\ \text{m}$.

(b) For $0 < t < 3\ \text{s}$, $a = \dfrac{8\ \text{m/s}}{3\ \text{s}} = 2.67\ \text{m/s}^2$.

For $3 < t < 5\ \text{s}$, $a = 0$.

(c) For $5\ \text{s} < t < 9\ \text{s}$, $a = -\dfrac{16\ \text{m/s}}{4\ \text{s}} = \boxed{-4\ \text{m/s}^2}$.

(d) At $t = 6\ \text{s}$, $x = 28\ \text{m} + (6\ \text{m/s})(1\ \text{s}) = \boxed{34\ \text{m}}$.

(e) At $t = 9\ \text{s}$, $x = 36\ \text{m} + \frac{1}{2}(-8\ \text{m/s})(2\ \text{s}) = \boxed{28\ \text{m}}$.

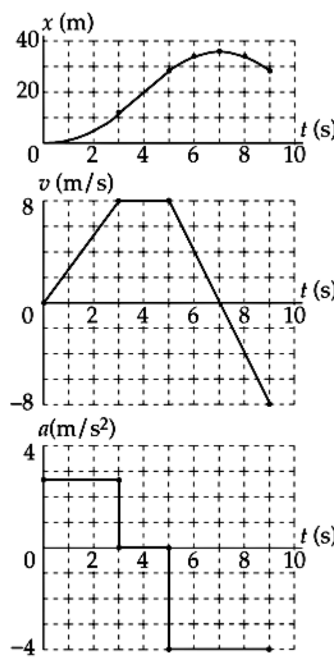

FIG. P2.46

P2.47 (a) $J = \frac{da}{dt} =$ constant

$$da = Jdt$$

$$a = J\int dt = Jt + c_1$$

but $a = a_i$ when $t = 0$ so $c_1 = a_i$. Therefore, $\boxed{a = Jt + a_i}$

$$a = \frac{dv}{dt}$$

$$dv = adt$$

$$v = \int adt = \int (Jt + a_i)dt = \frac{1}{2}Jt^2 + a_i t + c_2$$

but $v = v_i$ when $t = 0$, so $c_2 = v_i$ and $\boxed{v = \frac{1}{2}Jt^2 + a_i t + v_i}$

$$v = \frac{dx}{dt}$$

$$dx = vdt$$

$$x = \int vdt = \int \left(\frac{1}{2}Jt^2 + a_i t + v_i\right)dt$$

$$x = \frac{1}{6}Jt^3 + \frac{1}{2}a_i t^2 + v_i t + c_3$$

$$x = x_i$$

when $t = 0$, so $c_3 = x_i$. Therefore, $\boxed{x = \frac{1}{6}Jt^3 + \frac{1}{2}a_i t^2 + v_i t + x_i}$.

(b) $a^2 = (Jt + a_i)^2 = J^2t^2 + a_i^2 + 2Ja_i t$

$a^2 = a_i^2 + (J^2t^2 + 2Ja_i t)$

$a^2 = a_i^2 + 2J\left(\frac{1}{2}Jt^2 + a_i t\right)$

Recall the expression for v: $v = \frac{1}{2}Jt^2 + a_i t + v_i$. So $(v - v_i) = \frac{1}{2}Jt^2 + a_i t$. Therefore,

$$\boxed{a^2 = a_i^2 + 2J(v - v_i)}.$$

P2.48 (a) $a = \frac{dv}{dt} = \frac{d}{dt}\left[-5.00\times10^7 t^2 + 3.00\times10^5 t\right]$

$$\boxed{a = -\left(10.0\times10^7\ \text{m/s}^3\right)t + 3.00\times10^5\ \text{m/s}^2}$$

Take $x_i = 0$ at $t = 0$. Then $v = \frac{dx}{dt}$

$$x - 0 = \int_0^t v\,dt = \int_0^t \left(-5.00\times10^7 t^2 + 3.00\times10^5 t\right)dt$$

$$x = -5.00\times10^7 \frac{t^3}{3} + 3.00\times10^5 \frac{t^2}{2}$$

$$\boxed{x = -\left(1.67\times10^7\ \text{m/s}^3\right)t^3 + \left(1.50\times10^5\ \text{m/s}^2\right)t^2}.$$

(b) The bullet escapes when $a = 0$, at $-\left(10.0\times10^7\ \text{m/s}^3\right)t + 3.00\times10^5\ \text{m/s}^2 = 0$

$$t = \frac{3.00\times10^5\ \text{s}}{10.0\times10^7} = \boxed{3.00\times10^{-3}\ \text{s}}.$$

(c) New $v = \left(-5.00\times10^7\right)\left(3.00\times10^{-3}\right)^2 + \left(3.00\times10^5\right)\left(3.00\times10^{-3}\right)$

$$v = -450\ \text{m/s} + 900\ \text{m/s} = \boxed{450\ \text{m/s}}.$$

(d) $x = -\left(1.67\times10^7\right)\left(3.00\times10^{-3}\right)^3 + \left(1.50\times10^5\right)\left(3.00\times10^{-3}\right)^2$

$x = -0.450\ \text{m} + 1.35\ \text{m} = \boxed{0.900\ \text{m}}$

Additional Problems

***P2.49** (a) The velocity is constant between $t_i = 0$ and $t = 4$ s. Its acceleration is $\boxed{0}$.

(b) $a = (v_9 - v_4)/(9\ \text{s} - 4\ \text{s}) = (18 - [-12])\ (\text{m/s})/5\ \text{s} = \boxed{6.0\ \text{m/s}^2}$.

(c) $a = (v_{18} - v_{13})/(18\ \text{s} - 13\ \text{s}) = (0 - 18)\ (\text{m/s})/5\ \text{s} = \boxed{-3.6\ \text{m/s}^2}$.

(d) We read from the graph that the speed is zero $\boxed{\text{at } t = 6 \text{ s and at } 18 \text{ s}}$.

(e) and (f) The object moves away from $x = 0$ into negative coordinates from $t = 0$ to $t = 6$ s, but then comes back again, crosses the origin and moves farther into positive coordinates until $\boxed{t = 18\ \text{s}}$, then attaining its maximum distance, which is the cumulative distance under the graph line:

$(-12\ \text{m/s})(4\ \text{s}) + (-6\ \text{m/s})(2\ \text{s}) + (9\ \text{m/s})(3\ \text{s}) + (18\ \text{m/s})(4\ \text{s}) + (9\ \text{m/s})(5\ \text{s}) = -60\ \text{m} + 144\ \text{m} = \boxed{84\ \text{m}}$.

(g) To gauge the wear on the tires, we consider the total distance rather than the resultant displacement, by counting the contributions computed in part (f) as all positive:

$+ 60\ \text{m} + 144\ \text{m} = \boxed{204\ \text{m}}$.

P2.50 (a) As we see from the graph, from about -50 s to 50 s Acela is cruising at a constant positive velocity in the $+x$ direction. From 50 s to 200 s, Acela accelerates in the $+x$ direction reaching a top speed of about 170 mi/h. Around 200 s, the engineer applies the brakes, and the train, still traveling in the $+x$ direction, slows down and then stops at 350 s. Just after 350 s, Acela reverses direction (v becomes negative) and steadily gains speed in the $-x$ direction.

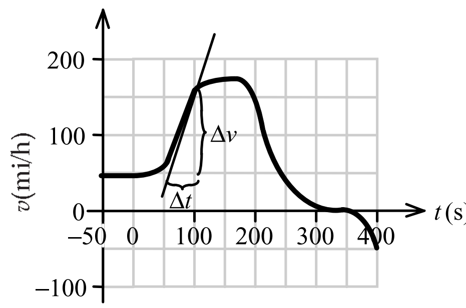

FIG. P2.50(a)

(b) The peak acceleration between 45 and 170 mi/h is given by the slope of the steepest tangent to the v versus t curve in this interval. From the tangent line shown, we find

$$a = \text{slope} = \frac{\Delta v}{\Delta t} = \frac{(155 - 45)\ \text{mi/h}}{(100 - 50)\ \text{s}} = \boxed{2.2\ (\text{mi/h})/\text{s}} = 0.98\ \text{m/s}^2.$$

(c) Let us use the fact that the area under the v versus t curve equals the displacement. The train's displacement between 0 and 200 s is equal to the area of the gray shaded region, which we have approximated with a series of triangles and rectangles.

$$\begin{aligned}
\Delta x_{0\to 200\ \text{s}} &= \text{area}_1 + \text{area}_2 + \text{area}_3 + \text{area}_4 + \text{area}_5 \\
&\approx (50\ \text{mi/h})(50\ \text{s}) + (50\ \text{mi/h})(50\ \text{s}) \\
&\quad + (160\ \text{mi/h})(100\ \text{s}) \\
&\quad + \frac{1}{2}(50\ \text{s})(100\ \text{mi/h}) \\
&\quad + \frac{1}{2}(100\ \text{s})(170\ \text{mi/h} - 160\ \text{mi/h}) \\
&= 24\,000(\text{mi/h})(\text{s})
\end{aligned}$$

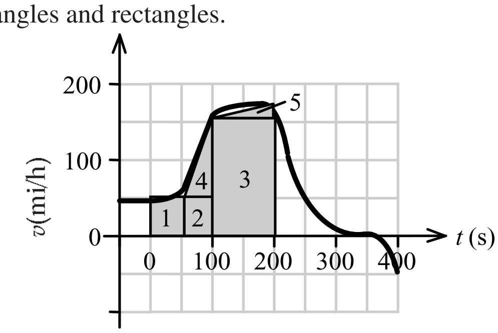

FIG. P2.50(c)

Now, at the end of our calculation, we can find the displacement in miles by converting hours to seconds. As $1\ \text{h} = 3\,600\ \text{s}$,

$$\Delta x_{0\to 200\ \text{s}} \approx \left(\frac{24\,000\ \text{mi}}{3\,600\ \text{s}}\right)(\text{s}) = \boxed{6.7\ \text{mi}}.$$

P2.51 Let point 0 be at ground level and point 1 be at the end of the engine burn. Let point 2 be the highest point the rocket reaches and point 3 be just before impact. The data in the table are found for each phase of the rocket's motion.

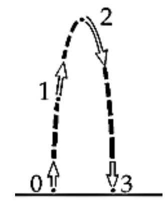

FIG. P2.51

(0 to 1) $v_f^2 - (80.0)^2 = 2(4.00)(1\,000)$ so $v_f = 120$ m/s

$120 = 80.0 + (4.00)t$ giving $t = 10.0$ s

(1 to 2) $0 - (120)^2 = 2(-9.80)(x_f - x_i)$ giving $x_f - x_i = 735$ m

$0 - 120 = -9.80t$ giving $t = 12.2$ s

This is the time of maximum height of the rocket.

(2 to 3) $v_f^2 - 0 = 2(-9.80)(-1\,735)$

$v_f = -184 = (-9.80)t$ giving $t = 18.8$ s

(a) $t_{\text{total}} = 10 + 12.2 + 18.8 = \boxed{41.0\text{ s}}$

(b) $(x_f - x_i)_{\text{total}} = \boxed{1.73\text{ km}}$

(c) $v_{\text{final}} = \boxed{-184\text{ m/s}}$

		t	x	v	a
0	Launch	0.0	0	80	+4.00
#1	End Thrust	10.0	1 000	120	+4.00
#2	Rise Upwards	22.2	1 735	0	–9.80
#3	Fall to Earth	41.0	0	–184	–9.80

P2.52 Area A_1 is a rectangle. Thus, $A_1 = hw = v_{xi}t$.

Area A_2 is triangular. Therefore $A_2 = \frac{1}{2}bh = \frac{1}{2}t(v_x - v_{xi})$.

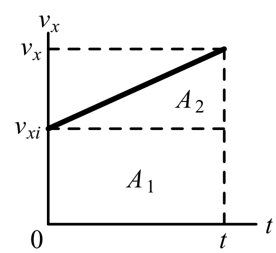

FIG. P2.52

The total area under the curve is

$$A = A_1 + A_2 = v_{xi}t + \frac{(v_x - v_{xi})t}{2}$$

and since $v_x - v_{xi} = a_x t$

$$\boxed{A = v_{xi}t + \tfrac{1}{2}a_x t^2}.$$

The displacement given by the equation is: $x = v_{xi}t + \frac{1}{2}a_x t^2$, the same result as above for the total area.

P2.53 (a) Let x be the distance traveled at acceleration a until maximum speed v is reached. If this is achieved in time t_1 we can use the following three equations:

$$x=\frac{1}{2}(v+v_i)t_1,\ 100-x=v(10.2-t_1),\text{ and } v=v_i+at_1.$$

The first two give

$$100=\left(10.2-\frac{1}{2}t_1\right)v=\left(10.2-\frac{1}{2}t_1\right)at_1$$
$$a=\frac{200}{(20.4-t_1)t_1}.$$

$$\text{For Maggie: } a=\frac{200}{(18.4)(2.00)}=\boxed{5.43\ \text{m/s}^2}$$
$$\text{For Judy: } a=\frac{200}{(17.4)(3.00)}=\boxed{3.83\ \text{m/s}^2}$$

(b) $v=at_1$

$$\text{Maggie: } v=(5.43)(2.00)=\boxed{10.9\ \text{m/s}}$$
$$\text{Judy: } v=(3.83)(3.00)=\boxed{11.5\ \text{m/s}}$$

(c) At the six-second mark

$$x=\frac{1}{2}at_1^2+v(6.00-t_1)$$

$$\text{Maggie: } x=\frac{1}{2}(5.43)(2.00)^2+(10.9)(4.00)=54.3\text{ m}$$
$$\text{Judy: } x=\frac{1}{2}(3.83)(3.00)^2+(11.5)(3.00)=51.7\text{ m}$$

Maggie is ahead by 54.3 m − 51.7 m = $\boxed{2.62\text{ m}}$. Note that your students may need a reminder that to get the answer in the back of the book they must use calculator memory or a piece of paper to save intermediate results without "rounding off" until the very end.

***P2.54** (a) We first find the distance s_{stop} over which you can stop. The car travels this distance during your reaction time: $\Delta x_1 = v_0(0.6\text{ s})$. As you brake to a stop, the average speed of the car is $v_0/2$, the interval of time is $(v_f - v_i)/a = -v_0/(-2.40\text{ m/s}^2) = v_0\text{ s}^2/2.40\text{ m}$, and the braking distance is $\Delta x_2 = v_{avg}\,\Delta t = (v_0\text{ s}^2/2.40\text{ m})(v_0/2) = v_0^2\text{ s}^2/4.80\text{ m}$. The total stopping distance is then $s_{stop} = \Delta x_1 + \Delta x_2 = v_0(0.6\text{ s}) + v_0^2\text{ s}^2/4.80\text{ m}$.

If the car is at this distance from the intersection, it can barely brake to a stop, so it should also be able to get through the intersection at constant speed while the light is yellow, moving a total distance $s_{stop} + 22\text{ m} = v_0(0.6\text{ s}) + v_0^2\text{ s}^2/4.80\text{ m} + 22\text{ m}$. This constant-speed motion requires time $\Delta t_y = (s_{stop} + 22\text{ m})/v_0 = (v_0(0.6\text{ s}) + v_0^2\text{ s}^2/4.80\text{ m} + 22\text{ m})/v_0 =$ $\boxed{0.6\text{ s} + v_0\text{ s}^2/4.80\text{ m} + 22\text{ m}/v_0}$.

continued on next page

(b) Substituting, $\Delta t_y = 0.6\text{ s} + (8\text{ m/s})\text{ s}^2/4.80\text{ m} + 22\text{ m}/(8\text{ m/s}) = 0.6\text{ s} + 1.67\text{ s} + 2.75\text{ s} = \boxed{5.02\text{ s}}$.

(c) We are asked about higher and higher speeds. For 11 m/s instead of 8 m/s, the time is $0.6\text{ s} + (11\text{ m/s})\text{ s}^2/4.80\text{ m} + 22\text{ m}/(11\text{ m/s}) = \boxed{4.89\text{ s}}$ *less* than we had at the lower speed.

(d) Now the time $0.6\text{ s} + (18\text{ m/s})\text{ s}^2/4.80\text{ m} + 22\text{ m}/(18\text{ m/s}) = \boxed{5.57\text{ s}}$ begins to increase

(e) $0.6\text{ s} + (25\text{ m/s})\text{ s}^2/4.80\text{ m} + 22\text{ m}/(25\text{ m/s}) = \boxed{6.69\text{ s}}$

(f) As v_0 goes to zero, the $22\text{ m}/v_0$ term in the expression for $\boxed{\Delta t_y \text{ becomes large, approaching infinity}}$.

(g) As v_0 grows without limit, the $v_0\text{ s}^2/4.80\text{ m}$ term in the expression for $\boxed{\Delta t_y \text{ becomes large, approaching infinity}}$.

(h) Δt_y decreases steeply from an infinite value at $v_0 = 0$, goes through a rather flat minimum, and then diverges to infinity as v_0 increases without bound. For a very slowly moving car entering the intersection and not allowed to speed up, a very long time is required to get across the intersection. A very fast-moving car requires a very long time to slow down at the constant acceleration we have assumed.

(i) To find the minimum, we set the derivative of Δt_y with respect to v_0 equal to zero:

$$\frac{d}{dv_0}\left(0.6\text{ s} + \frac{v_0\text{ s}^2}{4.8\text{ m}} + 22\text{ m }v_0^{-1}\right) = 0 + \frac{\text{s}^2}{4.8\text{ m}} - 22\text{ m }v_0^{-2} = 0$$

$22\text{ m}/v_0^2 = \text{s}^2/4.8\text{ m}$ $\qquad v_0 = (22\text{m }[4.8\text{ m/s}^2])^{1/2} = \boxed{10.3\text{ m/s}}$

(j) Evaluating again, $\Delta t_y = 0.6\text{ s} + (10.3\text{ m/s})\text{ s}^2/4.80\text{ m} + 22\text{ m}/(10.3\text{ m/s}) = \boxed{4.88\text{ s}}$, just a little less than the answer to part (c).

For some students an interesting project might be to measure the yellow-times of traffic lights on local roadways with various speed limits and compare with the minimum

$$\Delta t_{reaction} + (width/2\,|a_{braking}|)^{1/2}$$

implied by the analysis here. But do not let the students string a tape measure across the intersection.

P2.55 $a_1 = 0.100\text{ m/s}^2$ $\qquad a_2 = -0.500\text{ m/s}^2$

$$x = 1\,000\text{ m} = \frac{1}{2}a_1t_1^2 + v_1t_2 + \frac{1}{2}a_2t_2^2 \qquad t = t_1 + t_2 \text{ and } v_1 = a_1t_1 = -a_2t_2$$

$$1\,000 = \frac{1}{2}a_1t_1^2 + a_1t_1\left(-\frac{a_1t_1}{a_2}\right) + \frac{1}{2}a_2\left(\frac{a_1t_1}{a_2}\right)^2 \qquad 1\,000 = \frac{1}{2}a_1\left(1 - \frac{a_1}{a_2}\right)t_1^2$$

$$1\,000 = 0.5(0.1)[1 - (0.1/-0.5)]t_1^2 \quad 20\,000 = 1.20\,t_1^2 \qquad t_1 = \sqrt{\frac{20\,000}{1.20}} = \boxed{129\text{ s}}$$

$$t_2 = \frac{a_1t_1}{-a_2} = \frac{12.9}{0.500} \approx 26\text{ s} \qquad \text{Total time } = t = \boxed{155\text{ s}}$$

***P2.56** (a) From the information in the problem, we model the Ferrari as a particle under constant acceleration. The important "particle" for this part of the problem is the nose of the car. We use the position equation from the particle under constant acceleration model to find the velocity v_0 of the particle as it enters the intersection:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \rightarrow 28.0\text{ m} = 0 + v_0(3.10\text{ s}) + \tfrac{1}{2}(-2.10\text{ m/s}^2)(3.10\text{ s})^2$$
$$\rightarrow v_0 = 12.3\text{ m/s}$$

Now we use the velocity-position equation in the particle under constant acceleration model to find the displacement of the particle from the first edge of the intersection when the Ferrari stops:

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow x - x_0 = \Delta x = \frac{v^2 - v_0^2}{2a} = \frac{0 - (12.3\text{ m/s})^2}{2(-2.10\text{ m/s}^2)} = \boxed{35.9\text{ m}}$$

(b) The time interval during which any part of the Ferrari is in the intersection is that time interval between the instant at which the nose enters the intersection and the instant when the tail leaves the intersection. Thus, the change in position of the nose of the Ferrari is 4.52 m + 28.0 m = 32.52 m. We find the time at which the car is at position $x = 32.52$ m if it is at $x = 0$ and moving at 12.3 m/s at $t = 0$:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \rightarrow 32.52\text{ m} = 0 + (12.3\text{ m/s})t + \tfrac{1}{2}(-2.10\text{ m/s}^2)t^2$$
$$\rightarrow -1.05t^2 + 12.3t - 32.52 = 0$$

The solutions to this quadratic equation are $t = 4.04$ s and 7.66 s. Our desired solution is the lower of these, so $t = \boxed{4.04\text{ s}}$. (The later time corresponds to the Ferrari stopping and reversing, which it must do if the acceleration truly remains constant, and arriving again at the position $x = 32.52$ m.)

(c) We again define $t = 0$ as the time at which the nose of the Ferrari enters the intersection. Then at time $t = 4.04$ s, the tail of the Ferrari leaves the intersection. Therefore, to find the minimum distance from the intersection for the Corvette, its nose must enter the intersection at $t = 4.04$ s. We calculate this distance from the position equation:

$$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(5.60\text{ m/s}^2)(4.04\text{ s})^2 = \boxed{45.8\text{ m}}$$

(d) We use the velocity equation:

$$v = v_0 + at = 0 + (5.60\text{ m/s}^2)(4.04\text{ s}) = \boxed{22.6\text{ m/s}}$$

P2.57 (a) $y_f = v_{i1}t + \frac{1}{2}at^2 = 50.0 = 2.00t + \frac{1}{2}(9.80)t^2,\ 4.90t^2 + 2.00t - 50.0 = 0$

$$t = \frac{-2.00 + \sqrt{2.00^2 - 4(4.90)(-50.0)}}{2(4.90)}$$

Only the positive root is physically meaningful:

$$t = \boxed{3.00\ \text{s}} \text{ after the first stone is thrown.}$$

(b) $y_f = v_{i2}t + \frac{1}{2}at^2$ and $t = 3.00 - 1.00 = 2.00$ s

substitute $50.0 = v_{i2}(2.00) + \frac{1}{2}(9.80)(2.00)^2$:

$$v_{i2} = \boxed{15.3\ \text{m/s}} \text{ downward}$$

(c) $v_{1f} = v_{i1} + at = 2.00 + (9.80)(3.00) = \boxed{31.4\ \text{m/s}}$ downward

$v_{2f} = v_{i2} + at = 15.3 + (9.80)(2.00) = \boxed{34.8\ \text{m/s}}$ downward

P2.58 Let the ball fall freely for 1.50 m after starting from rest. It strikes at speed given by

$$v_{xf}^2 = v_{xi}^2 + 2a(x_f - x_i):$$

$$v_{xf}^2 = 0 + 2(-9.80\ \text{m/s}^2)(-1.50\ \text{m})$$

$$v_{xf} = -5.42\ \text{m/s}$$

If its acceleration were constant its stopping would be described by

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$
$$0 = (-5.42\ \text{m/s})^2 + 2a_x(-10^{-2}\ \text{m})$$
$$a_x = \frac{-29.4\ \text{m}^2/\text{s}^2}{-2.00 \times 10^{-2}\ \text{m}} = +1.47 \times 10^3\ \text{m/s}^2.$$

Upward acceleration of this same order of magnitude will continue for some additional time after the dent is at its maximum depth, to give the ball the speed with which it rebounds from the pavement. The ball's maximum acceleration will be larger than the average acceleration we estimate by imagining constant acceleration, but will still be of order of magnitude $\boxed{\sim 10^3\ \text{m/s}^2}$.

P2.59 (a) We require $x_s = x_k$ when $t_s = t_k + 1.00$

$$x_s = \frac{1}{2}(3.50\ \text{m/s}^2)(t_k + 1.00)^2 = \frac{1}{2}(4.90\ \text{m/s}^2)(t_k)^2 = x_k$$
$$t_k + 1.00 = 1.183t_k$$
$$t_k = \boxed{5.46\ \text{s}}.$$

(b) $x_k = \frac{1}{2}(4.90\ \text{m/s}^2)(5.46\ \text{s})^2 = \boxed{73.0\ \text{m}}$

(c) $v_k = (4.90\ \text{m/s}^2)(5.46\ \text{s}) = \boxed{26.7\ \text{m/s}}$

$v_s = (3.50\ \text{m/s}^2)(6.46\ \text{s}) = \boxed{22.6\ \text{m/s}}$

P2.60 (a) $d = \frac{1}{2}(9.80)t_1^2$ $\quad d = 336t_2$

$t_1 + t_2 = 2.40$ $\quad 336t_2 = 4.90(2.40 - t_2)^2$

$4.90t_2^2 - 359.5t_2 + 28.22 = 0$ $\quad t_2 = \frac{359.5 \pm \sqrt{359.5^2 - 4(4.90)(28.22)}}{9.80}$

$t_2 = \frac{359.5 \pm 358.75}{9.80} = 0.076\,5\text{ s}$ so $d = 336t_2 = \boxed{26.4\text{ m}}$

(b) Ignoring the sound travel time, $d = \frac{1}{2}(9.80)(2.40)^2 = 28.2\text{ m}$, an error of $\boxed{6.82\%}$.

***P2.61** (a) and (b) We divide each given thinking distance by the corresponding speed to test for the constancy of a proportionality constant. First, 27 ft/25(5280 ft/3600 s) = 0.736 s. Similarly,

constant speed, mi/h	25	35	45	55	65
constant speed, ft/s	36.7	51.3	66	80.7	95.3
thinking distance, ft	27	38	49	60	71
thinking time, s	0.736	0.740	0.742	0.744	0.745

The times can be summarized as 0.742 s ± 0.7%. Their near constancy means that

$\boxed{\text{the car can be modeled as traveling at constant speed, and that the reaction time is 0.742 s}}$.

The proportionality could also be displayed on a graph of thinking distance versus speed, to which a straight line through the origin could be fitted, passing very close to all the points. The line slope is the reaction time.

(c) and (d) According to $v_f^2 = v_i^2 + 2a(x_f - x_i)$ $\quad 0 = v_i^2 - 2|a|$ (*braking distance*)

$|a| = v_i^2/2$ (*braking distance*), we test for proportionality of speed squared to distance by dividing each squared speed by the given braking distance. The first is $(36.7\text{ ft/s})^2/34\text{ ft} = 39.5\text{ ft/s}^2 = -2a$. Similarly,

initial speed, ft/s	36.7	51.3	66	80.7	95.3
braking distance, ft	34	67	110	165	231
proportionality constant, ft/s^2	39.5	39.3	39.6	39.4	39.3

The constancy within experimental uncertainty of the last line indicates that the square of initial speed is indeed proportional to the braking distance, so that $\boxed{\text{the braking acceleration is constant}}$. This could be displayed also by graphing initial speed squared versus braking distance. A straight line fits the points convincingly and its slope is $-2a = 39.5\text{ ft/s}^2$, indicating that the braking acceleration is $\boxed{-19.7\text{ ft/s}^2 = -6.01\text{ m/s}^2}$.

P2.62

Time t (s)	Height h (m)	Δh (m)	Δt (s)	$\overline{v}$ (m/s)	midpt time t (s)
0.00	5.00				
		0.75	0.25	3.00	0.13
0.25	5.75				
		0.65	0.25	2.60	0.38
0.50	6.40				
		0.54	0.25	2.16	0.63
0.75	6.94				
		0.44	0.25	1.76	0.88
1.00	7.38				
		0.34	0.25	1.36	1.13
1.25	7.72				
		0.24	0.25	0.96	1.38
1.50	7.96				
		0.14	0.25	0.56	1.63
1.75	8.10				
		0.03	0.25	0.12	1.88
2.00	8.13				
		–0.06	0.25	–0.24	2.13
2.25	8.07				
		–0.17	0.25	–0.68	2.38
2.50	7.90				
		–0.28	0.25	–1.12	2.63
2.75	7.62				
		–0.37	0.25	–1.48	2.88
3.00	7.25				
		–0.48	0.25	–1.92	3.13
3.25	6.77				
		–0.57	0.25	–2.28	3.38
3.50	6.20				
		–0.68	0.25	–2.72	3.63
3.75	5.52				
		–0.79	0.25	–3.16	3.88
4.00	4.73				
		–0.88	0.25	–3.52	4.13
4.25	3.85				
		–0.99	0.25	–3.96	4.38
4.50	2.86				
		–1.09	0.25	–4.36	4.63
4.75	1.77				
		–1.19	0.25	–4.76	4.88
5.00	0.58				

TABLE P2.62

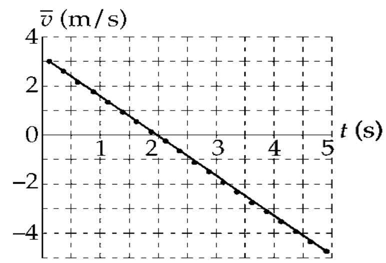

FIG. P2.62

The very convincing fit of a single straight line to the points in the graph of velocity versus time indicates that the rock does fall with constant acceleration. The acceleration is the slope of line:

$$a_{avg} = -1.63\ \text{m/s}^2 = \boxed{1.63\ \text{m/s}^2\ \text{downward}}$$

P2.63 The distance x and y are always related by $x^2 + y^2 = L^2$. Differentiating through this equation with respect to time, we have

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Now $\dfrac{dy}{dt}$ is v_B, the unknown velocity of B; and $\dfrac{dx}{dt} = -v$.

From the equation resulting from differentiation, we have

$$\frac{dy}{dt} = -\frac{x}{y}\left(\frac{dx}{dt}\right) = -\frac{x}{y}(-v).$$

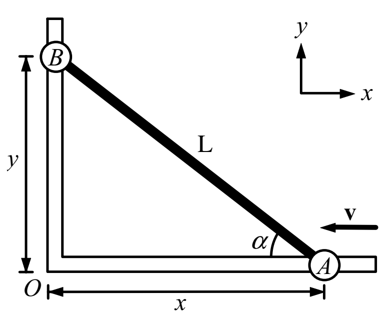

FIG. P2.63

But $\dfrac{y}{x} = \tan\alpha$ so $v_B = \left(\dfrac{1}{\tan\alpha}\right)v$. When $\alpha = 60.0°$, $v_B = \dfrac{v}{\tan 60.0°} = \dfrac{v\sqrt{3}}{3} = \boxed{0.577v}$.

ANSWERS TO EVEN PROBLEMS

P2.2 (a) 2.30 m/s (b) 16.1 m/s (c) 11.5 m/s

P2.4 (a) $50.0\ \text{m/s}$ (b) $41.0\ \text{m/s}$

P2.6 (a) $27.0\ \text{m}$ (b) $27.0\ \text{m} + (18.0\,\text{m/s})\Delta t + (3.00\,\text{m/s}^2)(\Delta t)^2$ (c) $18.0\ \text{m/s}$

P2.8 (a) 5.0 m/s (b) –2.5 m/s (c) 0 (d) 5.0 m/s

P2.10 $1.34 \times 10^4\ \text{m/s}^2$

P2.12 (a) see the solution (b) $1.60\ \text{m/s}^2;\ 0.800\ \text{m/s}^2$

P2.14 see the solution

P2.16 (a) 1.3 m/s^2 (b) 2.0 m/s^2 at 3 s (c) at $t = 6$ s and for $t > 10$ s (d) –1.5 m/s^2 at 8 s

P2.18 see the solution

P2.20 (a) $6.61\ \text{m/s}$ (b) $-0.448\ \text{m/s}^2$

P2.22 (a) 1.88 km; (b) 1.46 km (c) see the solution (d) (i) $x_1 = (1.67\ \text{m/s}^2)t^2$
(ii) $x_2 = (50\ \text{m/s})t - 375\ \text{m}$ (iii) $x_3 = (250\,\text{m/s})t - (2.5\,\text{m/s}^2)t^2 - 4\,375\,\text{m}$ (e) $37.5\ \text{m/s}$

P2.24 (a) 3.75 s after release (b) 5.50 cm/s (c) 0.604 s and 6.90 s (d) 13.3 cm and 47.9 cm (e) The cars are initially moving toward each other, so they soon share the same position x when their speeds are quite different, giving one answer to (c) that is not an answer to (a). The first car slows down in its motion to the left, turns around, and starts to move toward the right, slowly at first and gaining speed steadily. At a particular moment its speed will be equal to the constant rightward speed of the second car. The distance between them will at that moment be at its maximum value. The distance between the cars will be far from zero, as the accelerating car will be far to the left of the steadily moving car. Thus the answer to (a) is not an answer to (c). Eventually the accelerating car will catch up to the steadily-coasting car, whizzing past at higher speed than it has ever had before, and giving another answer to (c) that is not an answer to (a). A graph of x versus t for the two cars shows a parabola originally sloping down and then curving upward, intersecting twice with an upward-sloping straight line. The parabola and straight line are running parallel, with equal slopes, at just one point in between their intersections.

P2.26 $x_f - x_i = v_{xf}t - \frac{1}{2}a_x t^2;\ 3.10\ \text{m/s}$

P2.28 (a) 2.56 m (b) –3.00 m/s

P2.30 (a) v_c/t_m (c) $v_c t_o/2$ (d) $v_c t_o$ (e) The minimum displacement can be attained by having the servo motor on all the time. The maximum displacement cannot be attained because the acceleration must be finite.

P2.32 Yes; 212 m; 11.4 s

P2.34 31.0 s

P2.36 (a) $-0.049\,0$ m, -0.196 m, -0.441 m, with the negative signs all indicating downward
(b) -0.980 m/s, -1.96 m/s, -2.94 m/s

P2.38 1.79 s

P2.40 No; see the solution

P2.42 (a) Yes (b) 3.69 m/s (c) 2.39 m/s (d) +2.39 m/s does not agree with the magnitude of -3.71 m/s. The upward-moving rock spends more time in flight, so its speed change is larger.

P2.44 7.96 s

P2.46 (a) and (b) see the solution (c) $-4\ \mathrm{m/s^2}$ (d) 34 m (e) 28 m

P2.48 (a) $a = -(10.0\times10^7\ \mathrm{m/s^3})t + 3.00\times10^5\ \mathrm{m/s^2}$; $x = -(1.67\times10^7\ \mathrm{m/s^3})t^3 + (1.50\times10^5\ \mathrm{m/s^2})t^2$
(b) 3.00×10^{-3} s (c) 450 m/s (d) 0.900 m

P2.50 (a) Acela steadily cruises out of the city center at 45 mi/h. In less than a minute it smoothly speeds up to 150 mi/h; then its speed is nudged up to 170 mi/h. Next it smoothly slows to a very low speed, which it maintains as it rolls into a railroad yard. When it stops, it immediately begins backing up and smoothly speeds up to 50 mi/h in reverse, all in less than seven minutes after it started. (b) 2.2 mi/h/s = 0.98 m/s^2 (c) 6.7 mi.

P2.52 $v_{xi}t + \frac{1}{2}a_x t^2$; displacements agree

P2.54 (a) $\Delta t_y = 0.6\ \mathrm{s} + v_0\ \mathrm{s^2}/4.8\ \mathrm{m} + 22\ \mathrm{m}/v_0$ (b) 5.02 s (c) 4.89 s (d) 5.57 s (e) 6.69 s (f) $\Delta t_y \to \infty$ (g) $\Delta t_y \to \infty$ (h) Δt_y decreases steeply from an infinite value at $v_0 = 0$, goes through a rather flat minimum, and then diverges to infinity as v_0 increases without bound. For a very slowly moving car entering the intersection and not allowed to speed up, a very long time is required to get across the intersection. A very fast-moving car requires a very long time to slow down at the constant acceleration we have assumed. (i) at $v_0 = 10.3$ m/s, (j) $\Delta t_y = 4.88$ s.

P2.56 (a) 35.9 m (b) 4.04 s (c) 45.8 m (d) 22.6 m/s

P2.58 $\sim10^3\ \mathrm{m/s^2}$

P2.60 (a) 26.4 m (b) 6.82%

P2.62 see the solution; $a_x = -1.63\ \mathrm{m/s^2}$

3

Vectors

CHAPTER OUTLINE

3.1 Coordinate Systems
3.2 Vector and Scalar Quantities
3.3 Some Properties of Vectors
3.4 Components of a Vector and Unit Vectors

ANSWERS TO QUESTIONS

Q3.1 Only force and velocity are vectors. None of the other quantities requires a direction to be described. The answers are (a) yes (b) no (c) no (d) no (e) no (f) yes (g) no.

Q3.2 The book's displacement is zero, as it ends up at the point from which it started. The distance traveled is 6.0 meters.

***Q3.3** The vector $-2\vec{\mathbf{D}}_1$ will be twice as long as $\vec{\mathbf{D}}_1$ and in the opposite direction, namely northeast. Adding $\vec{\mathbf{D}}_2$, which is about equally long and southwest, we get a sum that is still longer and due east, choice (a).

***Q3.4** The magnitudes of the vectors being added are constant, and we are considering the magnitude only—not the direction—of the resultant. So we need look only at the angle between the vectors being added in each case. The smaller this angle, the larger the resultant magnitude. Thus the ranking is c = e > a > d > b.

***Q3.5** (a) leftward: negative. (b) upward: positive (c) rightward: positive (d) downward: negative (e) Depending on the signs and angles of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, the sum could be in any quadrant. (f) Now $-\vec{\mathbf{A}}$ will be in the fourth quadrant, so $-\vec{\mathbf{A}}+\vec{\mathbf{B}}$ will be in the fourth quadrant.

***Q3.6** (i) The magnitude is $\sqrt{10^2+10^2}$ m/s, answer (f). (ii) Having no y component means answer (a).

***Q3.7** The vertical component is opposite the 30° angle, so sin 30° = (vertical component)/50 m and the answer is (h).

***Q3.8** Take the difference of the coordinates of the ends of the vector. Final first means head end first. (i) $-4 - 2 = -6$ cm, answer (j) (ii) $1 - (-2) = 3$ cm, answer (c)

Q3.9 (i) If the direction-angle of $\vec{\mathbf{A}}$ is between 180 degrees and 270 degrees, its components are both negative: answer (c). If a vector is in the second quadrant or the fourth quadrant, its components have opposite signs: answer (b) or (d).

Q3.10 Vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other.

Q3.11 No, the magnitude of a vector is always positive. A minus sign in a vector only indicates direction, not magnitude.

Q3.12 Addition of a vector to a scalar is not defined. Think of numbers of apples and of clouds.

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SOLUTIONS TO PROBLEMS

Section 3.1 **Coordinate Systems**

P3.1 $x = r\cos\theta = (5.50\text{ m})\cos 240° = (5.50\text{ m})(-0.5) = \boxed{-2.75\text{ m}}$

$y = r\sin\theta = (5.50\text{ m})\sin 240° = (5.50\text{ m})(-0.866) = \boxed{-4.76\text{ m}}$

P3.2 (a) $x = r\cos\theta$ and $y = r\sin\theta$, therefore

$x_1 = (2.50\text{ m})\cos 30.0°$, $y_1 = (2.50\text{ m})\sin 30.0°$, and

$(x_1,\ y_1) = \boxed{(2.17,\ 1.25)\text{ m}}$

$x_2 = (3.80\text{ m})\cos 120°$, $y_2 = (3.80\text{ m})\sin 120°$, and

$(x_2,\ y_2) = \boxed{(-1.90,\ 3.29)\text{ m}}$

(b) $d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{4.07^2 + 2.04^2}\text{ m} = \boxed{4.55\text{ m}}$

P3.3 The x distance out to the fly is 2.00 m and the y distance up to the fly is 1.00 m.

(a) We can use the Pythagorean theorem to find the distance from the origin to the fly.

$$\text{distance} = \sqrt{x^2 + y^2} = \sqrt{(2.00\text{ m})^2 + (1.00\text{ m})^2} = \sqrt{5.00\text{ m}^2} = \boxed{2.24\text{ m}}$$

(b) $\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.6°$; $\vec{\mathbf{r}} = \boxed{2.24\text{ m},\ 26.6°}$

P3.4 We have $2.00 = r\cos 30.0°$

$$r = \frac{2.00}{\cos 30.0°} = \boxed{2.31}$$

and $y = r\sin 30.0° = 2.31\sin 30.0° = \boxed{1.15}$.

P3.5 We have $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$.

(a) The radius for this new point is

$$\sqrt{(-x)^2 + y^2} = \sqrt{x^2 + y^2} = \boxed{r}$$

and its angle is

$$\tan^{-1}\left(\frac{y}{-x}\right) = \boxed{180° - \theta}.$$

(b) $\sqrt{(-2x)^2 + (-2y)^2} = \boxed{2r}$ This point is in the third quadrant if (x, y) is in the first quadrant or in the fourth quadrant if (x, y) is in the second quadrant. It is at an angle of $\boxed{180° + \theta}$.

(c) $\sqrt{(3x)^2 + (-3y)^2} = \boxed{3r}$ This point is in the fourth quadrant if (x, y) is in the first quadrant or in the third quadrant if (x, y) is in the second quadrant. It is at an angle of $\boxed{-\theta}$.

Section 3.2 **Vector and Scalar Quantities**

Section 3.3 **Some Properties of Vectors**

P3.6 $-\vec{\mathbf{R}} = \boxed{310 \text{ km at } 57° \text{ S of W}}$

(Scale: 1 unit = 20 km)

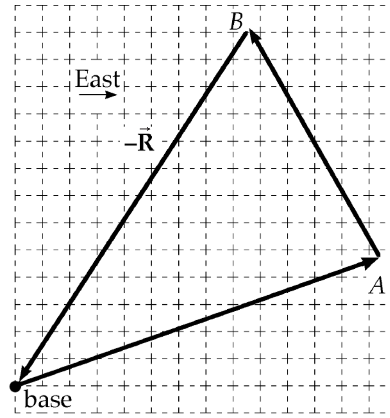


FIG. P3.6

P3.7 $\tan 35.0° = \dfrac{x}{100 \text{ m}}$

$$x = (100 \text{ m})\tan 35.0° = \boxed{70.0 \text{ m}}$$

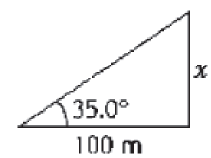


FIG. P3.7

P3.8 Find the resultant $\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$ graphically by placing the tail of $\vec{\mathbf{F}}_2$ at the head of $\vec{\mathbf{F}}_1$. The resultant force vector $\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$ is of magnitude $\boxed{9.5 \text{ N}}$ and at an angle of $\boxed{57° \text{ above the } x \text{ axis}}$.

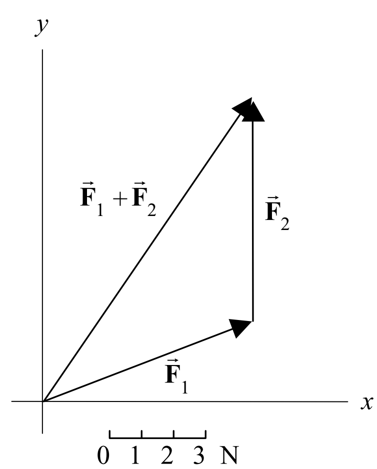


FIG. P3.8

P3.9 (a) $\left|\vec{\mathbf{d}}\right| = \left|-10.0\hat{\mathbf{i}}\right| = \boxed{10.0 \text{ m}}$ since the displacement is in a straight line from point A to point B.

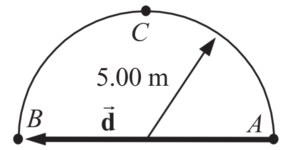


FIG. P3.9

(b) The actual distance skated is not equal to the straight-line displacement. The distance follows the curved path of the semi-circle (ACB).

$$s = \frac{1}{2}(2\pi r) = 5\pi = \boxed{15.7 \text{ m}}$$

(c) If the circle is complete, $\vec{\mathbf{d}}$ begins and ends at point **A**. Hence, $\left|\vec{\mathbf{d}}\right| = \boxed{0}$.

P3.10 (a) The large majority of people are standing or sitting at this hour. Their instantaneous foot-to-head vectors have upward vertical components on the order of 1 m and randomly oriented horizontal components. The citywide sum will be $\boxed{\sim 10^5 \text{ m upward}}$.

(b) Most people are lying in bed early Saturday morning. We suppose their beds are oriented north, south, east, and west quite at random. Then the horizontal component of their total vector height is very nearly zero. If their compressed pillows give their height vectors vertical components averaging 3 cm, and if one-tenth of one percent of the population are on-duty nurses or police officers, we estimate the total vector height as $\sim 10^5(0.03 \text{ m}) + 10^2(1 \text{ m})\ \boxed{\sim 10^3 \text{ m upward}}$.

P3.11 To find these vector expressions graphically, we draw each set of vectors. Measurements of the results are taken using a ruler and protractor. (Scale: 1 unit = 0.5 m)

(a) $\vec{\mathbf{A}} + \vec{\mathbf{B}} = 5.2$ m at 60°

(b) $\vec{\mathbf{A}} - \vec{\mathbf{B}} = 3.0$ m at 330°

(c) $\vec{\mathbf{B}} - \vec{\mathbf{A}} = 3.0$ m at 150°

(d) $\vec{\mathbf{A}} - 2\mathbf{B} = 5.2$ m at 300°

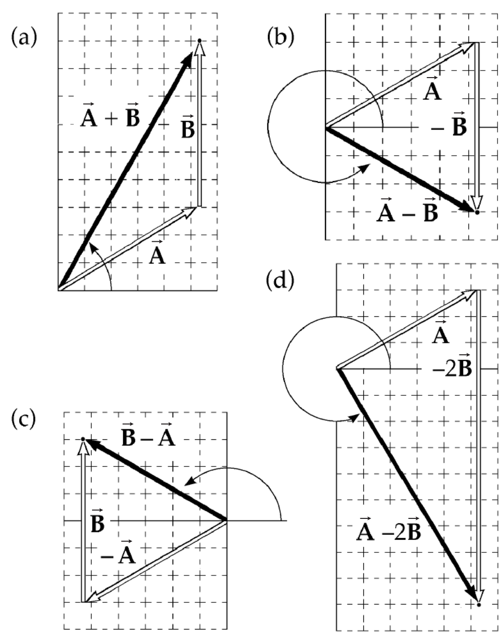


FIG. P3.11

P3.12 The three diagrams shown below represent the graphical solutions for the three vector sums: $\vec{\mathbf{R}}_1 = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$, $\vec{\mathbf{R}}_2 = \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{A}}$, and $\vec{\mathbf{R}}_3 = \vec{\mathbf{C}} + \vec{\mathbf{B}} + \vec{\mathbf{A}}$. We observe that $\vec{\mathbf{R}}_1 = \vec{\mathbf{R}}_2 = \vec{\mathbf{R}}_3$, illustrating that

$\boxed{\text{the sum of a set of vectors is not affected by the order in which the vectors are added}}$.

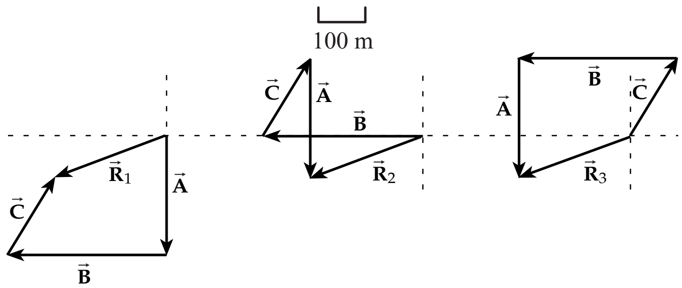
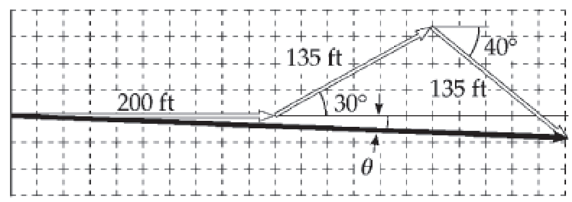


FIG. P3.12

P3.13 The scale drawing for the graphical solution should be similar to the figure to the right. The magnitude and direction of the final displacement from the starting point are obtained by measuring d and θ on the drawing and applying the scale factor used in making the drawing. The results should be

$\boxed{d = 420 \text{ ft and } \theta = -3°}$.



(Scale: 1 unit = 20 ft)

FIG. P3.13

Section 3.4 **Components of a Vector and Unit Vectors**

***P3.14** We assume the floor is level. Take the x axis in the direction of the first displacement.

$\boxed{\text{If both of the 90° turns are to the right or both to the left}}$, the displacements add like

$$40.0 \text{ m } \hat{\mathbf{i}} + 15.0 \text{ m } \hat{\mathbf{j}} - 20.0 \text{ m } \hat{\mathbf{i}} = \left(20.0\ \hat{\mathbf{i}} + 15.0\ \hat{\mathbf{j}}\right) \text{m}$$

to give (a) displacement magnitude $(20^2 + 15^2)^{1/2}$ m = $\boxed{25.0 \text{ m}}$

at (b) $\tan^{-1}(15/20) = \boxed{36.9°}$.

$\boxed{\text{If one turn is right and the other is left}}$, the displacements add like

$$40.0 \text{ m } \hat{\mathbf{i}} + 15.0 \text{ m } \hat{\mathbf{j}} + 20.0 \text{ m } \hat{\mathbf{i}} = \left(60.0\ \hat{\mathbf{i}} + 15.0\ \hat{\mathbf{j}}\right) \text{m}$$

to give (a) displacement magnitude $(60^2 + 15^2)^{1/2}$ m = $\boxed{61.8 \text{ m}}$

at (b) $\tan^{-1}(15/60) = \boxed{14.0^\circ. \text{ Just two answers are possible.}}$

P3.15 $A_x = -25.0$

$A_y = 40.0$

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0)^2 + (40.0)^2} = \boxed{47.2 \text{ units}}$

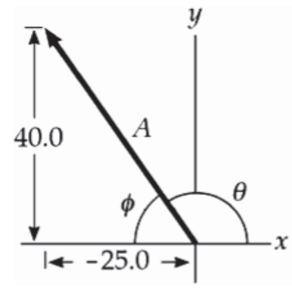


FIG. P3.15

We observe that

$$\tan\phi = \frac{|A_y|}{|A_x|}.$$

So

$$\phi = \tan^{-1}\left(\frac{A_y}{|A_x|}\right) = \tan\frac{40.0}{25.0} = \tan^{-1}(1.60) = 58.0°.$$

The diagram shows that the angle from the $+x$ axis can be found by subtracting from 180°:

$$\theta = 180° - 58° = \boxed{122°}.$$

P3.16 The person would have to walk $3.10\sin(25.0°) = \boxed{1.31 \text{ km north}}$, and

$$3.10\cos(25.0°) = \boxed{2.81 \text{ km east}}.$$

***P3.17** Let v represent the speed of the camper. The northward component of its velocity is $v \cos 8.5°$. To avoid crowding the minivan we require $v \cos 8.5° \geq 28$ m/s.

$\boxed{\text{We can satisfy this requirement simply by taking } v \geq (28 \text{ m/s})/\cos 8.5° = 28.3 \text{ m/s.}}$

P3.18 (a) Her net x (east-west) displacement is $-3.00 + 0 + 6.00 = +3.00$ blocks, while her net y (north-south) displacement is $0 + 4.00 + 0 = +4.00$ blocks. The magnitude of the resultant displacement is

$$R = \sqrt{(x_{\text{net}})^2 + (y_{\text{net}})^2} = \sqrt{(3.00)^2 + (4.00)^2} = 5.00 \text{ blocks}$$

and the angle the resultant makes with the x axis (eastward direction) is

$$\theta = \tan^{-1}\left(\frac{4.00}{3.00}\right) = \tan^{-1}(1.33) = 53.1°.$$

The resultant displacement is then $\boxed{5.00 \text{ blocks at } 53.1° \text{ N of E}}$.

(b) The total distance traveled is $3.00 + 4.00 + 6.00 = \boxed{13.0 \text{ blocks}}$.

P3.19 $x = r\cos\theta$ and $y = r\sin\theta$, therefore:

(a) $x = 12.8\cos 150°$, $y = 12.8\sin 150°$, and $(x, y) = \left(-11.1\hat{\mathbf{i}} + 6.40\hat{\mathbf{j}}\right)$ m

(b) $x = 3.30\cos 60.0°$, $y = 3.30\sin 60.0°$, and $(x, y) = \left(1.65\hat{\mathbf{i}} + 2.86\hat{\mathbf{j}}\right)$ cm

(c) $x = 22.0\cos 215°$, $y = 22.0\sin 215°$, and $(x, y) = \left(-18.0\hat{\mathbf{i}} - 12.6\hat{\mathbf{j}}\right)$ in

P3.20 $x = d\cos\theta = (50.0 \text{ m})\cos(120) = -25.0$ m

$y = d\sin\theta = (50.0 \text{ m})\sin(120) = 43.3$ m

$\vec{\mathbf{d}} = \boxed{(-25.0 \text{ m})\hat{\mathbf{i}} + (43.3 \text{ m})\hat{\mathbf{j}}}$

P3.21 Let $+x$ be East and $+y$ be North.

$$\sum x = 250 + 125\cos 30° = 358 \text{ m}$$
$$\sum y = 75 + 125\sin 30° - 150 = -12.5 \text{ m}$$
$$d = \sqrt{\left(\sum x\right)^2 + \left(\sum y\right)^2} = \sqrt{(358)^2 + (-12.5)^2} = 358 \text{ m}$$
$$\tan\theta = \frac{\left(\sum y\right)}{\left(\sum x\right)} = -\frac{12.5}{358} = -0.0349$$
$$\theta = -2.00°$$
$$\boxed{\vec{\mathbf{d}} = 358 \text{ m at } 2.00° \text{ S of E}}$$

P3.22 The east and north components of the displacement from Dallas (D) to Chicago (C) are the sums of the east and north components of the displacements from Dallas to Atlanta (A) and from Atlanta to Chicago. In equation form:

$$d_{\text{DC east}} = d_{\text{DA east}} + d_{\text{AC east}} = 730\cos 5.00° - 560\sin 21.0° = 527 \text{ miles}$$
$$d_{\text{DC north}} = d_{\text{DA north}} + d_{\text{AC north}} = 730\sin 5.00° + 560\cos 21.0° = 586 \text{ miles}$$

By the Pythagorean theorem, $d = \sqrt{(d_{\text{DC east}})^2 + (d_{\text{DC north}})^2} = 788 \text{ mi}$.

$$\text{Then } \tan\theta = \frac{d_{\text{DC north}}}{d_{\text{DC east}}} = 1.11 \text{ and } \theta = 48.0°.$$

Thus, Chicago is $\boxed{\text{788 miles at 48.0° northeast of Dallas}}$.

P3.23 We have $\vec{\mathbf{B}} = \vec{\mathbf{R}} - \vec{\mathbf{A}}$:

$$A_x = 150 \cos 120° = -75.0 \text{ cm}$$
$$A_y = 150\sin 120° = 130 \text{ cm}$$
$$R_x = 140\cos 35.0° = 115 \text{ cm}$$
$$R_y = 140\sin 35.0° = 80.3 \text{ cm}$$

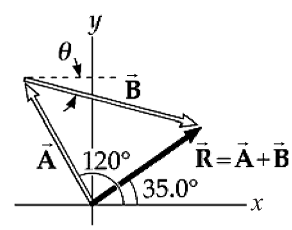


FIG. P3.23

Therefore,

$$\vec{\mathbf{B}} = [115 - (-75)]\hat{\mathbf{i}} + [80.3 - 130]\hat{\mathbf{j}} = \left(190\hat{\mathbf{i}} - 49.7\hat{\mathbf{j}}\right) \text{ cm}$$
$$\left|\vec{\mathbf{B}}\right| = \sqrt{190^2 + 49.7^2} = \boxed{196 \text{ cm}}$$
$$\theta = \tan^{-1}\left(-\frac{49.7}{190}\right) = \boxed{-14.7°}.$$

P3.24 (a) See figure to the right.

(b) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} = \boxed{5.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}}$

$\vec{\mathbf{C}} = \sqrt{25.0 + 16.0} \text{ at } \tan^{-1}\left(\frac{4}{5}\right) = \boxed{6.40 \text{ at } 38.7°}$

$\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} = \boxed{-1.00\hat{\mathbf{i}} + 8.00\hat{\mathbf{j}}}$

$\vec{\mathbf{D}} = \sqrt{(-1.00)^2 + (8.00)^2} \text{ at } \tan^{-1}\left(\frac{8.00}{-1.00}\right)$

$\vec{\mathbf{D}} = 8.06 \text{ at } (180° - 82.9°) = \boxed{8.06 \text{ at } 97.2°}$

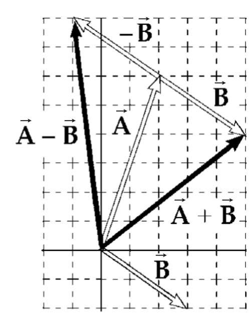


FIG. P3.24

P3.25 (a) $\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}\right)=\left(3\hat{\mathbf{i}}-2\hat{\mathbf{j}}\right)+\left(-\hat{\mathbf{i}}-4\hat{\mathbf{j}}\right)=\boxed{2\hat{\mathbf{i}}-6\hat{\mathbf{j}}}$

(b) $\left(\vec{\mathbf{A}}-\vec{\mathbf{B}}\right)=\left(3\hat{\mathbf{i}}-2\hat{\mathbf{j}}\right)-\left(-\hat{\mathbf{i}}-4\hat{\mathbf{j}}\right)=\boxed{4\hat{\mathbf{i}}+2\hat{\mathbf{j}}}$

(c) $\left|\vec{\mathbf{A}}+\vec{\mathbf{B}}\right|=\sqrt{2^2+6^2}=\boxed{6.32}$

(d) $\left|\vec{\mathbf{A}}-\vec{\mathbf{B}}\right|=\sqrt{4^2+2^2}=\boxed{4.47}$

(e) $\theta_{|\mathbf{A}+\mathbf{B}|}=\tan^{-1}\left(-\dfrac{6}{2}\right)=-71.6°=\boxed{288°}$

$\theta_{|\mathbf{A}-\mathbf{B}|}=\tan^{-1}\left(\dfrac{2}{4}\right)=\boxed{26.6°}$

***P3.26** We take the x axis along the slope uphill. Students, get used to this choice! The y axis is perpendicular to the slope, at 35° to the vertical. Then the displacement of the snow makes an angle of 90° − 35° − 20° = 35° with the x axis.

(a) Its component parallel to the surface is 5 m cos 35° = $\boxed{\text{4.10 m toward the top of the hill}}$.

(b) Its component perpendicular to the surface is 5 m sin 35° = $\boxed{\text{2.87 m}}$.

P3.27 $\vec{\mathbf{d}}_1=\left(-3.50\hat{\mathbf{j}}\right)$ m

$\vec{\mathbf{d}}_2=8.20\cos 45.0°\hat{\mathbf{i}}+8.20\sin 45.0°\hat{\mathbf{j}}=\left(5.80\hat{\mathbf{i}}+5.80\hat{\mathbf{j}}\right)$ m

$\vec{\mathbf{d}}_3=\left(-15.0\hat{\mathbf{i}}\right)$ m

$\vec{\mathbf{R}}=\vec{\mathbf{d}}_1+\vec{\mathbf{d}}_2+\vec{\mathbf{d}}_3=(-15.0+5.80)\hat{\mathbf{i}}+(5.80-3.50)\hat{\mathbf{j}}=\boxed{\left(-9.20\hat{\mathbf{i}}+2.30\hat{\mathbf{j}}\right)\text{ m}}$

(or 9.20 m west and 2.30 m north)
The magnitude of the resultant displacement is

$$\left|\vec{\mathbf{R}}\right|=\sqrt{R_x^2+R_y^2}=\sqrt{(-9.20)^2+(2.30)^2}=\boxed{9.48\text{ m}}.$$

The direction is $\theta=\arctan\left(\dfrac{2.30}{-9.20}\right)=\boxed{166°}$.

P3.28 Refer to the sketch

$$\vec{\mathbf{R}}=\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}=-10.0\hat{\mathbf{i}}-15.0\hat{\mathbf{j}}+50.0\hat{\mathbf{i}}$$
$$=40.0\hat{\mathbf{i}}-15.0\hat{\mathbf{j}}$$
$$\left|\vec{\mathbf{R}}\right|=\left[(40.0)^2+(-15.0)^2\right]^{1/2}=\boxed{42.7\text{ yards}}$$

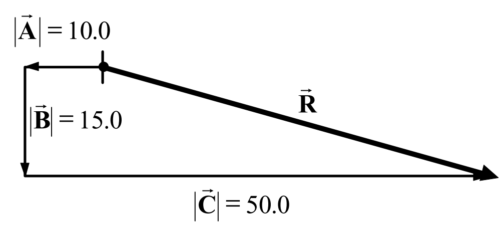


FIG. P3.28

P3.29

East	North
x	y
0 m	4.00 m
1.41	1.41
–0.500	–0.866
+0.914	4.55

$\left|\vec{\mathbf{R}}\right|=\sqrt{|x|^2+|y|^2}$ at $\tan^{-1}(y/x)=\boxed{4.64\text{ m at }78.6°\text{ N of E}}$

P3.30 $\vec{\mathbf{A}} = -8.70\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = 13.2\hat{\mathbf{i}} - 6.60\hat{\mathbf{j}}$

$\vec{\mathbf{A}} - \vec{\mathbf{B}} + 3\vec{\mathbf{C}} = 0$: $\quad 3\vec{\mathbf{C}} = \vec{\mathbf{B}} - \vec{\mathbf{A}} = 21.9\hat{\mathbf{i}} - 21.6\hat{\mathbf{j}}$

$$\vec{\mathbf{C}} = 7.30\hat{\mathbf{i}} - 7.20\hat{\mathbf{j}}$$

or $\quad C_x = \boxed{7.30 \text{ cm}}$; $C_y = \boxed{-7.20 \text{ cm}}$

P3.31 (a) $\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$

$$\vec{\mathbf{F}} = 120\cos(60.0°)\hat{\mathbf{i}} + 120\sin(60.0°)\hat{\mathbf{j}} - 80.0\cos(75.0°)\hat{\mathbf{i}} + 80.0\sin(75.0°)\hat{\mathbf{j}}$$

$$\vec{\mathbf{F}} = 60.0\hat{\mathbf{i}} + 104\hat{\mathbf{j}} - 20.7\hat{\mathbf{i}} + 77.3\hat{\mathbf{j}} = \left(39.3\hat{\mathbf{i}} + 181\hat{\mathbf{j}}\right) \text{ N}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{39.3^2 + 181^2} = \boxed{185 \text{ N}}$$

$$\theta = \tan^{-1}\left(\frac{181}{39.3}\right) = \boxed{77.8°}$$

(b) $\vec{\mathbf{F}}_3 = -\vec{\mathbf{F}} = \boxed{\left(-39.3\hat{\mathbf{i}} - 181\hat{\mathbf{j}}\right) \text{ N}}$

P3.32 $\vec{\mathbf{A}} = 3.00 \text{ m},\ \theta_A = 30.0°$ $\qquad \vec{\mathbf{B}} = 3.00 \text{ m},\ \theta_B = 90.0°$

$A_x = A\cos\theta_A = 3.00\cos 30.0° = 2.60 \text{ m}$ $\qquad A_y = A\sin\theta_A = 3.00\sin 30.0° = 1.50 \text{ m}$

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = \left(2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right) \text{ m}$$

$B_x = 0,\ B_y = 3.00 \text{ m}$ $\qquad$ so $\qquad \vec{\mathbf{B}} = 3.00\hat{\mathbf{j}} \text{ m}$

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \left(2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right) + 3.00\hat{\mathbf{j}} = \boxed{\left(2.60\hat{\mathbf{i}} + 4.50\hat{\mathbf{j}}\right) \text{ m}}$$

P3.33 $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}} = 4.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{k}}$

$$\left|\vec{\mathbf{B}}\right| = \sqrt{4.00^2 + 6.00^2 + 3.00^2} = \boxed{7.81}$$

$\alpha = \cos^{-1}\left(\frac{4.00}{7.81}\right) = \boxed{59.2°}$ is the angle with the x axis

$\beta = \cos^{-1}\left(\frac{6.00}{7.81}\right) = \boxed{39.8°}$ is the angle with the y axis

$\gamma = \cos^{-1}\left(\frac{3.00}{7.81}\right) = \boxed{67.4°}$ is the angle with the z axis

P3.34 (a) $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$

$$\left|\vec{\mathbf{D}}\right| = \sqrt{2^2 + 2^2} = \boxed{2.83 \text{ m at } \theta = 315°}$$

(b) $\vec{\mathbf{E}} = -\vec{\mathbf{A}} - \vec{\mathbf{B}} + \vec{\mathbf{C}} = -6\hat{\mathbf{i}} + 12\hat{\mathbf{j}}$

$$\left|\vec{\mathbf{E}}\right| = \sqrt{6^2 + 12^2} = \boxed{13.4 \text{ m at } \theta = 117°}$$

P3.35 (a) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = \boxed{\left(5.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}\right) \text{ m}}$

$$\left|\vec{\mathbf{C}}\right| = \sqrt{(5.00)^2 + (1.00)^2 + (3.00)^2} \text{ m} = \boxed{5.92 \text{ m}}$$

(b) $\vec{\mathbf{D}} = 2\vec{\mathbf{A}} - \vec{\mathbf{B}} = \boxed{\left(4.00\hat{\mathbf{i}} - 11.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{k}}\right) \text{ m}}$

$$\left|\vec{\mathbf{D}}\right| = \sqrt{(4.00)^2 + (11.0)^2 + (15.0)^2} \text{ m} = \boxed{19.0 \text{ m}}$$

P3.36 Let the positive x-direction be eastward, the positive y-direction be vertically upward, and the positive z-direction be southward. The total displacement is then

$$\vec{\mathbf{d}} = \left(4.80\hat{\mathbf{i}} + 4.80\hat{\mathbf{j}}\right)\text{ cm} + \left(3.70\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}\right)\text{ cm} = \left(4.80\hat{\mathbf{i}} + 8.50\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}\right)\text{ cm}.$$

(a) The magnitude is $d = \sqrt{(4.80)^2 + (8.50)^2 + (-3.70)^2}\text{ cm} = \boxed{10.4\text{ cm}}$.

(b) Its angle with the y axis follows from $\cos\theta = \dfrac{8.50}{10.4}$, giving $\boxed{\theta = 35.5°}$.

P3.37 (a) $\vec{\mathbf{A}} = \boxed{8.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}} - 4.00\hat{\mathbf{k}}}$

(b) $\vec{\mathbf{B}} = \dfrac{\vec{\mathbf{A}}}{4} = \boxed{2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}} - 1.00\hat{\mathbf{k}}}$

(c) $\vec{\mathbf{C}} = -3\vec{\mathbf{A}} = \boxed{-24.0\hat{\mathbf{i}} - 36.0\hat{\mathbf{j}} + 12.0\hat{\mathbf{k}}}$

P3.38 The y coordinate of the airplane is constant and equal to 7.60×10^3 m whereas the x coordinate is given by $x = v_i t$ where v_i is the constant speed in the horizontal direction.

At $t = 30.0$ s we have $x = 8.04 \times 10^3$, so $v_i = 8\,040\text{ m}/30\text{ s} = 268\text{ m/s}$. The position vector as a function of time is

$$\vec{\mathbf{P}} = (268\text{ m/s})t\hat{\mathbf{i}} + \left(7.60 \times 10^3\text{ m}\right)\hat{\mathbf{j}}.$$

At $t = 45.0$ s, $\vec{\mathbf{P}} = \left[1.21 \times 10^4\,\hat{\mathbf{i}} + 7.60 \times 10^3\,\hat{\mathbf{j}}\right]$ m. The magnitude is

$$\vec{\mathbf{P}} = \sqrt{\left(1.21 \times 10^4\right)^2 + \left(7.60 \times 10^3\right)^2}\text{ m} = \boxed{1.43 \times 10^4\text{ m}}$$

and the direction is

$$\theta = \arctan\left(\frac{7.60 \times 10^3}{1.21 \times 10^4}\right) = \boxed{32.2°\text{ above the horizontal}}.$$

P3.39 The position vector from radar station to ship is

$$\vec{\mathbf{S}} = \left(17.3\sin 136°\hat{\mathbf{i}} + 17.3\cos 136°\hat{\mathbf{j}}\right)\text{ km} = \left(12.0\hat{\mathbf{i}} - 12.4\hat{\mathbf{j}}\right)\text{ km}.$$

From station to plane, the position vector is

$$\vec{\mathbf{P}} = \left(19.6\sin 153°\hat{\mathbf{i}} + 19.6\cos 153°\hat{\mathbf{j}} + 2.20\hat{\mathbf{k}}\right)\text{ km},$$

or

$$\vec{\mathbf{P}} = \left(8.90\hat{\mathbf{i}} - 17.5\hat{\mathbf{j}} + 2.20\hat{\mathbf{k}}\right)\text{ km}.$$

(a) To fly to the ship, the plane must undergo displacement

$$\vec{\mathbf{D}} = \vec{\mathbf{S}} - \vec{\mathbf{P}} = \boxed{\left(3.12\hat{\mathbf{i}} + 5.02\hat{\mathbf{j}} - 2.20\hat{\mathbf{k}}\right)\text{ km}}.$$

(b) The distance the plane must travel is

$$D = \left|\vec{\mathbf{D}}\right| = \sqrt{(3.12)^2 + (5.02)^2 + (2.20)^2}\text{ km} = \boxed{6.31\text{ km}}.$$

P3.40 (a) $\vec{\mathbf{E}} = (17.0 \text{ cm})\cos 27.0°\hat{\mathbf{i}} + (17.0 \text{ cm})\sin 27.0°\hat{\mathbf{j}}$

$\vec{\mathbf{E}} = \boxed{\left(15.1\hat{\mathbf{i}} + 7.72\hat{\mathbf{j}}\right) \text{ cm}}$

(b) $\vec{\mathbf{F}} = -(17.0 \text{ cm})\sin 27.0°\hat{\mathbf{i}} + (17.0 \text{ cm})\cos 27.0°\hat{\mathbf{j}}$

$\vec{\mathbf{F}} = \boxed{\left(-7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}\right) \text{ cm}}$ Note that we do not need to explicitly identify the angle with the positive x axis.

(c) $\vec{\mathbf{G}} = +(17.0 \text{ cm})\sin 27.0°\hat{\mathbf{i}} + (17.0 \text{ cm})\cos 27.0°\hat{\mathbf{j}}$

$\vec{\mathbf{G}} = \boxed{\left(+7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}\right) \text{ cm}}$

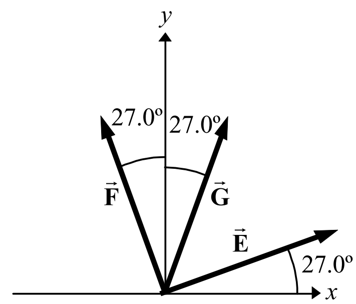


FIG. P3.40

P3.41 $A_x = -3.00,\ A_y = 2.00$

(a) $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = \boxed{-3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}}}$

(b) $\left|\vec{\mathbf{A}}\right| = \sqrt{A_x^2 + A_y^2} = \sqrt{(-3.00)^2 + (2.00)^2} = \boxed{3.61}$

$\tan\theta = \dfrac{A_y}{A_x} = \dfrac{2.00}{(-3.00)} = -0.667,\ \tan^{-1}(-0.667) = -33.7°$

θ is in the 2nd quadrant, so $\theta = 180° + (-33.7°) = \boxed{146°}$.

(c) $R_x = 0,\ R_y = -4.00,\ \vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ thus $\vec{\mathbf{B}} = \vec{\mathbf{R}} - \vec{\mathbf{A}}$ and

$B_x = R_x - A_x = 0 - (-3.00) = 3.00,\ B_y = R_y - A_y = -4.00 - 2.00 = -6.00.$

Therefore, $\vec{\mathbf{B}} = \boxed{3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}}$.

P3.42 The hurricane's first displacement is $\left(\dfrac{41.0 \text{ km}}{\text{h}}\right)(3.00 \text{ h})$ at 60.0° N of W, and its second displacement is $\left(\dfrac{25.0 \text{ km}}{\text{h}}\right)(1.50 \text{ h})$ due North. With $\hat{\mathbf{i}}$ representing east and $\hat{\mathbf{j}}$ representing north, its total displacement is:

$$\left(41.0\frac{\text{km}}{\text{h}}\cos 60.0°\right)(3.00 \text{ h})\left(-\hat{\mathbf{i}}\right) + \left(41.0\frac{\text{km}}{\text{h}}\sin 60.0°\right)(3.00 \text{ h})\hat{\mathbf{j}} + \left(25.0\frac{\text{km}}{\text{h}}\right)(1.50 \text{ h})\hat{\mathbf{j}}$$

$$= 61.5 \text{ km}\left(-\hat{\mathbf{i}}\right) + 144 \text{ km}\,\hat{\mathbf{j}}$$

with magnitude $\sqrt{(61.5 \text{ km})^2 + (144 \text{ km})^2} = \boxed{157 \text{ km}}$.

P3.43 (a) $R_x = 40.0\cos 45.0° + 30.0\cos 45.0° = 49.5$

$R_y = 40.0\sin 45.0° - 30.0\sin 45.0° + 20.0 = 27.1$

$\vec{\mathbf{R}} = \boxed{49.5\hat{\mathbf{i}} + 27.1\hat{\mathbf{j}}}$

(b) $\left|\vec{\mathbf{R}}\right| = \sqrt{(49.5)^2 + (27.1)^2} = \boxed{56.4}$

$\theta = \tan^{-1}\left(\dfrac{27.1}{49.5}\right) = \boxed{28.7°}$

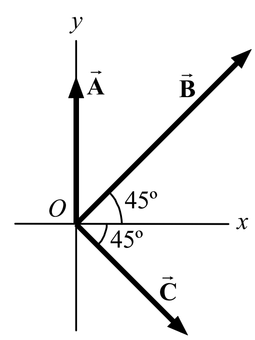


FIG. P3.43

***P3.44** (a) Taking components along $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we get two equations:

$$6.00a - 8.00b + 26.0 = 0$$

and

$$-8.00a + 3.00b + 19.0 = 0.$$

We solve simultaneously by substituting $a = 1.33\,b - 4.33$ to find $-8(1.33\,b - 4.33) + 3\,b + 19 = 0$

or $\quad 7.67b = 53.67 \quad$ so $\quad b = 7.00 \quad$ and $\quad a = 1.33(7) - 4.33.$

Thus

$$\boxed{a = 5.00,\; b = 7.00}.$$

Therefore,

$$5.00\vec{\mathbf{A}} + 7.00\vec{\mathbf{B}} + \vec{\mathbf{C}} = 0.$$

(b) In order for vectors to be equal, all of their components must be equal. A vector equation contains more information than a scalar equation.

***P3.45** The displacement from the start to the finish is $16\,\hat{\mathbf{i}} + 12\,\hat{\mathbf{j}} - (5\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}) = (11\,\hat{\mathbf{i}} + 9\,\hat{\mathbf{j}})$ meters.

The displacement from the starting point to A is $f(11\,\hat{\mathbf{i}} + 9\,\hat{\mathbf{j}})$ meters.

(a) The position vector of point A is $5\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}} + f(11\,\hat{\mathbf{i}} + 9\,\hat{\mathbf{j}}) = \boxed{(5 + 11f)\,\hat{\mathbf{i}} + (3 + 9f)\,\hat{\mathbf{j}}\text{ meters}}$.

(b) For $f = 0$ we have the position vector $(5 + 0)\,\hat{\mathbf{i}} + (3 + 0)\,\hat{\mathbf{j}}$ meters.

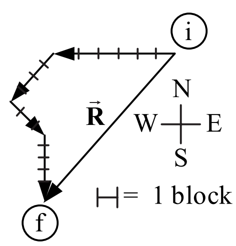
This is reasonable because it is the location of the starting point, $5\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}$ meters.

(c) For $f = 1 = 100\%$, we have position vector $(5 + 11)\,\hat{\mathbf{i}} + (3 + 9)\,\hat{\mathbf{j}}$ meters $= \boxed{16\,\hat{\mathbf{i}} + 12\,\hat{\mathbf{j}}\text{ meters}}$.

This is reasonable because we have completed the trip and this is the position vector of the endpoint.

***P3.46** We note that $-\hat{\mathbf{i}} =$ west and $-\hat{\mathbf{j}} =$ south. The given mathematical representation of the trip can be written as 6.3 b west + 4 b at 40° south of west + 3 b at 50° south of east + 5 b south.

(a)



(b) The total odometer distance is the sum of the magnitudes of the four displacements:

$$6.3\text{ b} + 4\text{ b} + 3\text{ b} + 5\text{ b} = \boxed{18.3\text{ b}}.$$

(c)
$$\begin{aligned}
\vec{\mathbf{R}} &= (-6.3 - 3.06 + 1.93)\text{ b }\hat{\mathbf{i}} + (-2.57 - 2.30 - 5)\text{ b }\hat{\mathbf{j}} \\
&= -7.44\text{ b }\hat{\mathbf{i}} - 9.87\text{ b }\hat{\mathbf{j}} \\
&= \sqrt{(7.44\text{ b})^2 + (9.87\text{ b})^2}\text{ at }\tan^{-1}\frac{9.87}{7.44}\text{ south of west} \\
&= 12.4\text{ b at }53.0°\text{ south of west} \\
&= \boxed{12.4\text{ b at }233°\text{ counterclockwise from east}}
\end{aligned}$$

Additional Problems

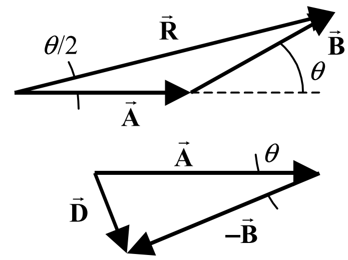


FIG. P3.47

P3.47 Let θ represent the angle between the directions of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Since $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have the same magnitudes, $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ form an isosceles triangle in which the angles are $180° - \theta$, $\dfrac{\theta}{2}$, and $\dfrac{\theta}{2}$.

The magnitude of $\vec{\mathbf{R}}$ is then $R = 2A\cos\left(\dfrac{\theta}{2}\right)$.

This can be seen from applying the law of cosines to the isosceles triangle and using the fact that $B = A$.

Again, $\vec{\mathbf{A}}$, $-\vec{\mathbf{B}}$, and $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity

$$(1 - \cos\theta) = 2\sin^2\left(\frac{\theta}{2}\right)$$

gives the magnitude of $\vec{\mathbf{D}}$ as $D = 2A\sin\left(\dfrac{\theta}{2}\right)$.

The problem requires that $R = 100D$.

Thus, $2A\cos\left(\dfrac{\theta}{2}\right) = 200A\sin\left(\dfrac{\theta}{2}\right)$. This gives $\tan\left(\dfrac{\theta}{2}\right) = 0.010$ and $\boxed{\theta = 1.15°}$.

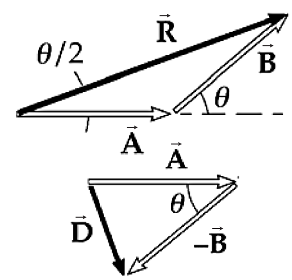


FIG. P3.48

P3.48 Let θ represent the angle between the directions of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Since $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have the same magnitudes, $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ form an isosceles triangle in which the angles are $180° - \theta$, $\dfrac{\theta}{2}$, and $\dfrac{\theta}{2}$.

The magnitude of $\vec{\mathbf{R}}$ is then $R = 2A\cos\left(\dfrac{\theta}{2}\right)$. This can be seen by applying the law of cosines to the isosceles triangle and using the fact that $B = A$.

Again, $\vec{\mathbf{A}}$, $-\vec{\mathbf{B}}$, and $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity

$$(1 - \cos\theta) = 2\sin^2\left(\frac{\theta}{2}\right)$$

gives the magnitude of $\vec{\mathbf{D}}$ as $D = 2A\sin\left(\dfrac{\theta}{2}\right)$.

The problem requires that $R = nD$ or $\cos\left(\dfrac{\theta}{2}\right) = n\sin\left(\dfrac{\theta}{2}\right)$ giving $\boxed{\theta = 2\tan^{-1}\left(\dfrac{1}{n}\right)}$.

The larger R is to be compared to D, the smaller the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ becomes.

P3.49 The position vector from the ground under the controller of the first airplane is

$$\begin{aligned}\vec{\mathbf{r}}_1 &= (19.2\text{ km})(\cos 25°)\hat{\mathbf{i}} + (19.2\text{ km})(\sin 25°)\hat{\mathbf{j}} + (0.8\text{ km})\hat{\mathbf{k}} \\ &= \left(17.4\hat{\mathbf{i}} + 8.11\hat{\mathbf{j}} + 0.8\hat{\mathbf{k}}\right)\text{ km.}\end{aligned}$$

The second is at

$$\begin{aligned}\vec{\mathbf{r}}_2 &= (17.6\text{ km})(\cos 20°)\hat{\mathbf{i}} + (17.6\text{ km})(\sin 20°)\hat{\mathbf{j}} + (1.1\text{ km})\hat{\mathbf{k}} \\ &= \left(16.5\hat{\mathbf{i}} + 6.02\hat{\mathbf{j}} + 1.1\hat{\mathbf{k}}\right)\text{ km.}\end{aligned}$$

Now the displacement from the first plane to the second is

$$\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 = \left(-0.863\hat{\mathbf{i}} - 2.09\hat{\mathbf{j}} + 0.3\hat{\mathbf{k}}\right)\text{ km}$$

with magnitude

$$\sqrt{(0.863)^2 + (2.09)^2 + (0.3)^2} = \boxed{2.29\text{ km}}.$$

P3.50 Take the x axis along the tail section of the snake. The displacement from tail to head is

$$240 \text{ m}\hat{\mathbf{i}} + (420 - 240) \text{ m}\cos(180° - 105°)\,\hat{\mathbf{i}} - 180 \text{ m}\sin 75°\,\hat{\mathbf{j}} = 287 \text{ m}\hat{\mathbf{i}} - 174 \text{ m}\hat{\mathbf{j}}.$$

Its magnitude is $\sqrt{(287)^2 + (174)^2} \text{ m} = 335 \text{ m}$. From $v = \dfrac{\text{distance}}{\Delta t}$, the time for each child's run is

$$\text{Inge: } \Delta t = \frac{\text{distance}}{v} = \frac{335 \text{ m}(\text{h})(1 \text{ km})(3\,600 \text{ s})}{(12 \text{ km})(1\,000 \text{ m})(1 \text{ h})} = 101 \text{ s}$$

$$\text{Olaf: } \Delta t = \frac{420 \text{ m}\cdot\text{s}}{3.33 \text{ m}} = 126 \text{ s}.$$

Inge wins by $126 - 101 = \boxed{25.4 \text{ s}}$.

P3.51 Let A represent the distance from island 2 to island 3. The displacement is $\vec{\mathbf{A}} = A$ at $159°$. Represent the displacement from 3 to 1 as $\vec{\mathbf{B}} = B$ at $298°$. We have $4.76 \text{ km at } 37° + \vec{\mathbf{A}} + \vec{\mathbf{B}} = 0$.

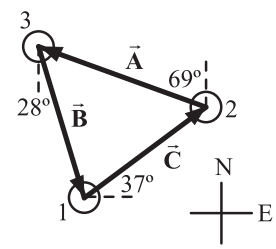


FIG. P3.51

For x components

$$(4.76 \text{ km})\cos 37° + A\cos 159° + B\cos 298° = 0$$
$$3.80 \text{ km} - 0.934A + 0.469B = 0$$
$$B = -8.10 \text{ km} + 1.99A$$

For y components

$$(4.76 \text{ km})\sin 37° + A\sin 159° + B\sin 298° = 0$$
$$2.86 \text{ km} + 0.358A - 0.883B = 0$$

(a) We solve by eliminating B by substitution:

$$2.86 \text{ km} + 0.358A - 0.883(-8.10 \text{ km} + 1.99A) = 0$$
$$2.86 \text{ km} + 0.358A + 7.15 \text{ km} - 1.76A = 0$$
$$10.0 \text{ km} = 1.40A$$
$$A = \boxed{7.17 \text{ km}}$$

(b) $B = -8.10 \text{ km} + 1.99(7.17 \text{ km}) = \boxed{6.15 \text{ km}}$

P3.52 (a) $R_x = \boxed{2.00}$, $R_y = \boxed{1.00}$, $R_z = \boxed{3.00}$

(b) $|\vec{\mathbf{R}}| = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{4.00 + 1.00 + 9.00} = \sqrt{14.0} = \boxed{3.74}$

(c) $$\cos\theta_x = \frac{R_x}{|\vec{\mathbf{R}}|} \quad\Rightarrow\quad \theta_x = \cos^{-1}\left(\frac{R_x}{|\vec{\mathbf{R}}|}\right) = \boxed{57.7° \text{ from } +x}$$

$$\cos\theta_y = \frac{R_y}{|\vec{\mathbf{R}}|} \quad\Rightarrow\quad \theta_y = \cos^{-1}\left(\frac{R_y}{|\vec{\mathbf{R}}|}\right) = \boxed{74.5° \text{ from } +y}$$

$$\cos\theta_z = \frac{R_z}{|\vec{\mathbf{R}}|} \quad\Rightarrow\quad \theta_z = \cos^{-1}\left(\frac{R_z}{|\vec{\mathbf{R}}|}\right) = \boxed{36.7° \text{ from } +z}$$

P3.53 $\vec{\mathbf{v}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = (300 + 100\cos 30.0°)\,\hat{\mathbf{i}} + (100\sin 30.0°)\,\hat{\mathbf{j}}$

$\vec{\mathbf{v}} = \left(387\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}}\right)$ mi/h

$|\vec{\mathbf{v}}| = \boxed{390 \text{ mi/h at } 7.37° \text{ N of E}}$

***P3.54** (a) $\vec{\mathbf{A}} = -60 \text{ cm } \hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = \left(80\ \cos\theta\ \hat{\mathbf{i}} + 80\ \sin\theta\ \hat{\mathbf{j}}\right)$ cm

so $\vec{\mathbf{A}} + \vec{\mathbf{B}} = 80\cos\theta\ \hat{\mathbf{i}} + (80\sin\theta - 60)\,\hat{\mathbf{j}}$ centimeters and

$$\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \left[(80\cos\theta)^2 + (80\sin\theta - 60)^2\right]^{1/2} \text{ cm} = \left[80^2\cos^2\theta + 80^2\sin^2\theta - 2(80)(60)\cos\theta + 60^2\right]^{1/2} \text{ cm}$$

Now $\sin^2\theta + \cos^2\theta = 1$ for all θ, so we have

$$\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \left[80^2 + 60^2 - 2(80)(60)\cos\theta\right]^{1/2} \text{ cm} = \boxed{[10\,000 - 9\,600\cos\theta]^{1/2} \text{ cm}}$$

(b) For $\boxed{\theta = 270°}$, $\cos\theta = -1$ and the expression takes on its maximum value, [10 000 + 9 600] 1/2 cm = $\boxed{140 \text{ cm}}$.

(c) For $\boxed{\theta = 90°}$, $\cos\theta = +1$ and the expression takes on its minimum value, [10 000 − 9 600] 1/2 cm = $\boxed{20.0 \text{ cm}}$.

(d) They do make sense. The maximum value is attained when $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are in the same direction, and it is 60 cm + 80 cm. The minimum value is attained when $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are in opposite directions, and it is 80 cm − 60 cm.

***P3.55** $$\Delta\vec{\mathbf{r}} = \int_0^{0.380\text{s}}\left(1.2\ \hat{\mathbf{i}} \text{ m/s} - 9.8t\ \hat{\mathbf{j}} \text{ m/s}^2\right)dt = 1.2t\ \hat{\mathbf{i}} \text{ m/s}\Big|_0^{0.38\text{s}} - 9.8\hat{\mathbf{j}} \text{ m/s}^2\ \frac{t^2}{2}\Big|_0^{0.38\text{s}}$$

$$= \left(1.2\ \hat{\mathbf{i}} \text{ m/s}\right)(0.38 \text{ s} - 0) - 9.8\hat{\mathbf{j}} \text{ m/s}^2\left(\frac{(0.38 \text{ s})^2 - 0}{2}\right) = \boxed{0.456\hat{\mathbf{i}} \text{ m} - 0.708\hat{\mathbf{j}} \text{ m}}$$

P3.56 Choose the + x axis in the direction of the first force, and the y axis at 90° counterclockwise from the x axis. Then each force will have only one nonzero component.

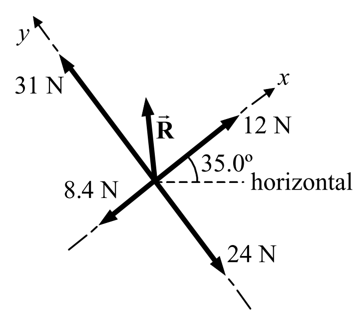


FIG. P3.56

The total force, in newtons, is then

$$12.0\hat{\mathbf{i}} + 31.0\hat{\mathbf{j}} - 8.40\hat{\mathbf{i}} - 24.0\hat{\mathbf{j}} = \boxed{\left(3.60\hat{\mathbf{i}}\right) + \left(7.00\hat{\mathbf{j}}\right) \text{ N}}.$$

The magnitude of the total force is

$$\sqrt{(3.60)^2 + (7.00)^2} \text{ N} = \boxed{7.87 \text{ N}}$$

and the angle it makes with our + x axis is given by $\tan\theta = \frac{(7.00)}{(3.60)}$, $\theta = 62.8°$.

Thus, its angle counterclockwise from the horizontal is $35.0° + 62.8° = \boxed{97.8°}$.

P3.57 $\vec{\mathbf{d}}_1 = 100\hat{\mathbf{i}}$

$\vec{\mathbf{d}}_2 = -300\hat{\mathbf{j}}$

$\vec{\mathbf{d}}_3 = -150\cos(30.0°)\hat{\mathbf{i}} - 150\sin(30.0°)\hat{\mathbf{j}} = -130\hat{\mathbf{i}} - 75.0\hat{\mathbf{j}}$

$\vec{\mathbf{d}}_4 = -200\cos(60.0°)\hat{\mathbf{i}} + 200\sin(60.0°)\hat{\mathbf{j}} = -100\hat{\mathbf{i}} + 173\hat{\mathbf{j}}$

$\vec{\mathbf{R}} = \vec{\mathbf{d}}_1 + \vec{\mathbf{d}}_2 + \vec{\mathbf{d}}_3 + \vec{\mathbf{d}}_4 = \boxed{\left(-130\hat{\mathbf{i}} - 202\hat{\mathbf{j}}\right) \text{ m}}$

$\left|\vec{\mathbf{R}}\right| = \sqrt{(-130)^2 + (-202)^2} = \boxed{240 \text{ m}}$

$\phi = \tan^{-1}\left(\dfrac{202}{130}\right) = 57.2°$

$\theta = 180 + \phi = \boxed{237°}$

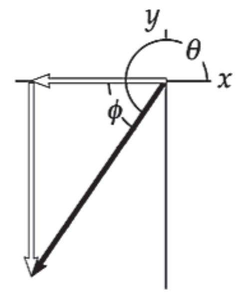


FIG. P3.57

P3.58 $\dfrac{d\vec{\mathbf{r}}}{dt} = \dfrac{d\left(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2t\hat{\mathbf{j}}\right)}{dt} = 0 + 0 - 2\hat{\mathbf{j}} = \boxed{-(2.00 \text{ m/s})\hat{\mathbf{j}}}$

The position vector at $t = 0$ is $4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$. At $t = 1$ s, the position is $4\hat{\mathbf{i}} + 1\hat{\mathbf{j}}$, and so on. The object is moving straight downward at 2 m/s, so

$$\frac{d\vec{\mathbf{r}}}{dt} \text{ represents } \boxed{\text{its velocity vector}}.$$

P3.59 (a) You start at point A: $\vec{\mathbf{r}}_1 = \vec{\mathbf{r}}_A = \left(30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}}\right)$ m.

The displacement to B is

$$\vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A = 60.0\hat{\mathbf{i}} + 80.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} + 20.0\hat{\mathbf{j}} = 30.0\hat{\mathbf{i}} + 100\hat{\mathbf{j}}.$$

You cover half of this, $\left(15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}}\right)$ to move to

$$\vec{\mathbf{r}}_2 = 30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}}.$$

Now the displacement from your current position to C is

$$\vec{\mathbf{r}}_C - \vec{\mathbf{r}}_2 = -10.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}} - 45.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} = -55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}.$$

You cover one-third, moving to

$$\vec{\mathbf{r}}_3 = \vec{\mathbf{r}}_2 + \Delta\vec{\mathbf{r}}_{23} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}} + \frac{1}{3}\left(-55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}\right) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}}.$$

The displacement from where you are to D is

$$\vec{\mathbf{r}}_D - \vec{\mathbf{r}}_3 = 40.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} - 26.7\hat{\mathbf{i}} - 16.7\hat{\mathbf{j}} = 13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}.$$

You traverse one-quarter of it, moving to

$$\vec{\mathbf{r}}_4 = \vec{\mathbf{r}}_3 + \frac{1}{4}\left(\vec{\mathbf{r}}_D - \vec{\mathbf{r}}_3\right) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}} + \frac{1}{4}\left(13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}\right) = 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}.$$

The displacement from your new location to **E** is

$$\vec{\mathbf{r}}_E - \vec{\mathbf{r}}_4 = -70.0\hat{\mathbf{i}} + 60.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}} = -100\hat{\mathbf{i}} + 55.0\hat{\mathbf{j}}$$

of which you cover one-fifth the distance, $-20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}}$, moving to

$$\vec{\mathbf{r}}_4 + \Delta\vec{\mathbf{r}}_{45} = 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}} - 20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}} = 10.0\hat{\mathbf{i}} + 16.0\hat{\mathbf{j}}.$$

The treasure is at $\boxed{(10.0 \text{ m}, 16.0 \text{ m})}$.

continued on next page

(b) Following the directions brings you to the average position of the trees. The steps we took numerically in part (a) bring you to

$$\vec{\mathbf{r}}_A + \frac{1}{2}\left(\vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A\right) = \left(\frac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B}{2}\right)$$

then to $\dfrac{\left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B\right)}{2} + \dfrac{\vec{\mathbf{r}}_C - \left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B\right)/2}{3} = \dfrac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C}{3}$

then to $\dfrac{\left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C\right)}{3} + \dfrac{\vec{\mathbf{r}}_D - \left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C\right)/3}{4} = \dfrac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D}{4}$

and last to $\dfrac{\left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D\right)}{4} + \dfrac{\vec{\mathbf{r}}_E - \left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D\right)/4}{5} = \dfrac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D + \vec{\mathbf{r}}_E}{5}$.

This center of mass of the tree distribution is the same location whatever order we take the trees in.

P3.60 (a) Let T represent the force exerted by each child. The x component of the resultant force is

$$T\cos 0 + T\cos 120° + T\cos 240° = T(1) + T(-0.5) + T(-0.5) = 0$$

The y component is

$$T\sin 0 + T\sin 120 + T\sin 240 = 0 + 0.866T - 0.866T = 0.$$

Thus,

$$\sum \vec{\mathbf{F}} = 0$$

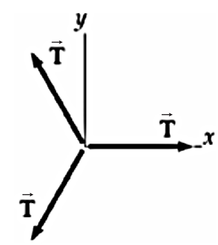


FIG. P3.60

(b) If the total force is not zero, it must point in some direction. When each child moves one space clockwise, the whole set of forces acting on the tire turns clockwise by that angle so the total force must turn clockwise by that angle, $\dfrac{360°}{N}$. Because each child exerts the same force, the new situation is identical to the old and the net force on the tire must still point in the original direction. But the force cannot have two different directions. The contradiction indicates that we were wrong in supposing that the total force is not zero. The total force *must* be zero.

P3.61 Since

$$\vec{\mathbf{A}}+\vec{\mathbf{B}}=6.00\hat{\mathbf{j}},$$

we have

$$\left(A_x+B_x\right)\hat{\mathbf{i}}+\left(A_y+B_y\right)\hat{\mathbf{j}}=0\hat{\mathbf{i}}+6.00\hat{\mathbf{j}}$$

giving

$$A_x+B_x=0 \text{ or } A_x=-B_x \qquad [1]$$

and

$$A_y+B_y=6.00. \qquad [2]$$

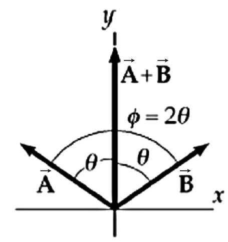


FIG. P3.61

Since both vectors have a magnitude of 5.00, we also have

$$A_x^2+A_y^2=B_x^2+B_y^2=5.00^2.$$

From $A_x=-B_x$, it is seen that

$$A_x^2=B_x^2.$$

Therefore, $A_x^2+A_y^2=B_x^2+B_y^2$ gives

$$A_y^2=B_y^2.$$

Then, $A_y=B_y$ and Equation [2] gives

$$A_y=B_y=3.00.$$

Defining θ as the angle between either $\vec{\mathbf{A}}$ or $\vec{\mathbf{B}}$ and the y axis, it is seen that

$$\cos\theta=\frac{A_y}{A}=\frac{B_y}{B}=\frac{3.00}{5.00}=0.600 \text{ and } \theta=53.1°.$$

The angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is then $\boxed{\phi=2\theta=106°}$.

P3.62 (a) From the picture, $\vec{\mathbf{R}}_1=a\hat{\mathbf{i}}+b\hat{\mathbf{j}}$ and $\left|\vec{\mathbf{R}}_1\right|=\sqrt{a^2+b^2}$.

(b) $\vec{\mathbf{R}}_2=a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$; its magnitude is

$$\sqrt{\left|\vec{\mathbf{R}}_1\right|^2+c^2}=\sqrt{a^2+b^2+c^2}.$$

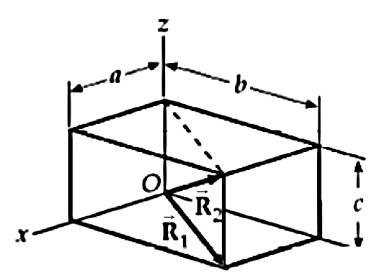


FIG. P3.62

ANSWERS TO EVEN PROBLEMS

P3.2 (a) $(2.17 \text{ m}, 1.25 \text{ m})$; $(-1.90 \text{ m}, 3.29 \text{ m})$ (b) 4.55 m

P3.4 $y=1.15$; $r=2.31$

P3.6 310 km at 57° S of W

P3.8 9.5 N at 57°

P3.10 (a) $\sim 10^5$ m vertically upward (b) $\sim 10^3$ m vertically upward

P3.12 See the solution; the sum of a set of vectors is not affected by the order in which the vectors are added.

P3.14 We assume that the shopping cart stays on the level floor. There are two possibilities. If both of the turns are right or both left, the net displacement is (a) 25.0 m (b) at 36.9°. If one turn is right and one is left, we have (a) 61.8 m (b) at 14.0°.

P3.16 1.31 km north; 2.81 km east

P3.18 (a) 5.00 blocks at 53.1° N of E (b) 13.0 blocks

P3.20 $-25.0\text{ m }\hat{\mathbf{i}} + 43.3\text{ m }\hat{\mathbf{j}}$

P3.22 788 mi at 48.0° north of east

P3.24 (a) see the solution (b) $5.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$, 6.40 at 38.7°, $-1.00\hat{\mathbf{i}} + 8.00\hat{\mathbf{j}}$, 8.06 at 97.2°

P3.26 (a) 4.10 m toward the top of the hill (b) 2.87 m

P3.28 42.7 yards

P3.30 $\vec{\mathbf{C}} = 7.30\text{ cm }\hat{\mathbf{i}} - 7.20\text{ cm }\hat{\mathbf{j}}$

P3.32 $\vec{\mathbf{A}} + \vec{\mathbf{B}} = (2.60\ \hat{\mathbf{i}} + 4.50\ \hat{\mathbf{j}})\text{m}$

P3.34 (a) 2.83 m at $\theta = 315°$ (b) 13.4 m at $\theta = 117°$

P3.36 (a) 10.4 cm; (b) 35.5°

P3.38 1.43×10^4 m at 32.2° above the horizontal

P3.40 (a) $\left(15.1\hat{\mathbf{i}} + 7.72\hat{\mathbf{j}}\right)$ cm (b) $\left(-7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}\right)$ cm (c) $\left(+7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}\right)$ cm

P3.42 157 km

P3.44 (a) $a = 5.00$ and $b = 7.00$ (b) For vectors to be equal, all of their components must be equal. A vector equation contains more information than a scalar equation.

P3.46 (a) see the solution (b) 18.3 b (c) 12.4 b at 233° counterclockwise from east

P3.48 $2\tan^{-1}\left(\dfrac{1}{n}\right)$

P3.50 25.4 s

P3.52 (a) 2.00, 1.00, 3.00 (b) 3.74 (c) $\theta_x = 57.7°$, $\theta_y = 74.5°$, $\theta_z = 36.7°$

P3.54 (a) $(10\,000 - 9\,600\cos\theta)^{1/2}$ cm (b) 270° ; 140 cm (c) 90° ; 20.0 cm (d) They do make sense. The maximum value is attained when $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are in the same direction, and it is 60 cm + 80 cm. The minimum value is attained when $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are in opposite directions, and it is 80 cm − 60 cm.

P3.56 We choose the x axis to the right at 35° above the horizontal and the y axis at 90° counterclockwise from the x axis. Then each vector has only a single nonzero component. The resultant is 7.87 N at 97.8° counterclockwise from a horizontal line to the right.

P3.58 $(-2.00\text{ m/s})\hat{\mathbf{j}}$; its velocity vector

P3.60 (a) zero (b) see the solution

P3.62 (a) $\vec{\mathbf{R}}_1 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$; $\left|\vec{\mathbf{R}}_1\right| = \sqrt{a^2 + b^2}$ (b) $\vec{\mathbf{R}}_2 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$; $\left|\vec{\mathbf{R}}_2\right| = \sqrt{a^2 + b^2 + c^2}$

4

Motion in Two Dimensions

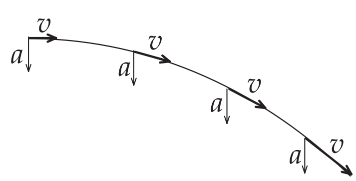
CHAPTER OUTLINE

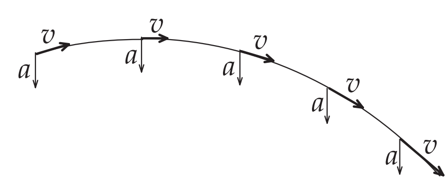
ANSWERS TO QUESTIONS

***Q4.1** The car's acceleration must have an inward component and a forward component: answer (f). Another argument: Draw a final velocity vector of two units west. Add to it a vector of one unit south. This represents subtracting the initial velocity from the final velocity, on the way to finding the acceleration. The direction of the resultant is that of vector (f).

Q4.2 No, you cannot determine the instantaneous velocity. Yes, you can determine the average velocity. The points could be widely separated. In this case, you can only determine the average velocity, which is

$$\vec{\mathbf{v}}_{avg} = \frac{\Delta \vec{\mathbf{x}}}{\Delta t}$$

Q4.3 (a) 

(b) 

***Q4.4** (i) The 45° angle means that at point *A* the horizontal and vertical velocity components are equal. The horizontal velocity component is the same at *A, B*, and *C*. The vertical velocity component is zero at *B* and negative at *C*. The assembled answer is a = b = c = e > d = 0 > f
(ii) The *x*-component of acceleration is everywhere zero and the *y*-component is everywhere -9.8 m/s^2. Then we have a = c = e = 0 > b = d = f.

Q4.5 A parabola results, because the originally forward velocity component stays constant and the rocket motor gives the spacecraft constant acceleration in a perpendicular direction.

Q4.6 (a) yes (b) no: the escaping jet exhaust exerts an extra force on the plane. (c) no (d) yes (e) no: the stone is only a few times more dense than water, so friction is a significant force on the stone. The answer is (a) and (d).

Q4.7 The projectile is in free fall. Its vertical component of acceleration is the downward acceleration of gravity. Its horizontal component of acceleration is zero.

Q4.8 (a) no (b) yes (c) yes (d) no. Answer: (b) and (c)

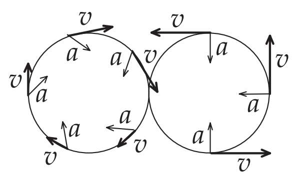
***Q4.9** The projectile on the moon is in flight for a time interval six times larger, with the same range of vertical speeds and with the same constant horizontal speed as on Earth. Then (i) its range is (d) six times larger and (ii) its maximum altitude is (d) six times larger. *Apollo* astronauts performed the experiment with golf balls.

Q4.10 (a) no. Its velocity is constant in magnitude and direction.
(b) yes. The particle is continuously changing the direction of its velocity vector.

Q4.11 (a) straight ahead (b) either in a circle or straight ahead. The acceleration magnitude can be constant either with a nonzero or with a zero value.

***Q4.12** (i) $a = v^2/r$ becomes $3^2/3 = 3$ times larger: answer (b).
(ii) $T = 2\pi r/v$ changes by a factor of $3/3 = 1$. The answer is (a).

Q4.13



The skater starts at the center of the eight, goes clockwise around the left circle and then counterclockwise around the right circle.

***Q4.14** With radius half as large, speed should be smaller by a factor of $1/\sqrt{2}$, so that $a = v^2/r$ can be the same. The answer is (d).

***Q4.15** The wrench will hit (b) at the base of the mast. If air resistance is a factor, it will hit slightly leeward of the base of the mast, displaced in the direction in which air is moving relative to the deck. If the boat is scudding before the wind, for example, the wrench's impact point can be in front of the mast.

***Q4.16** Let the positive x direction be that of the girl's motion. The x component of the velocity of the ball relative to the ground is $+5 - 12$ m/s $= -7$ m/s. The x-velocity of the ball relative to the girl is $-7 - 8$ m/s $= -15$ m/s. The relative speed of the ball is $+15$ m/s, answer (d).

SOLUTIONS TO PROBLEMS

Section 4.1 The Position, Velocity, and Acceleration Vectors

P4.1

$x(m)$	$y(m)$
0	−3 600
−3 000	0
−1 270	1 270
−4 270 m	−2 330 m

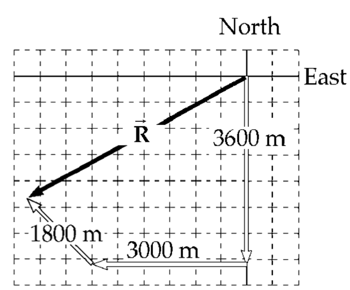


FIG. P4.1

(a) Net displacement $= \sqrt{x^2 + y^2}$ at $\tan^{-1}(y/x)$

$$\vec{\mathbf{R}} = \boxed{4.87 \text{ km at } 28.6° \text{ S of W}}$$

(b) $$\text{Average speed} = \frac{(20.0 \text{ m/s})(180 \text{ s}) + (25.0 \text{ m/s})(120 \text{ s}) + (30.0 \text{ m/s})(60.0 \text{ s})}{180 \text{ s} + 120 \text{ s} + 60.0 \text{ s}} = \boxed{23.3 \text{ m/s}}$$

(c) $$\text{Average velocity} = \frac{4.87 \times 10^3 \text{ m}}{360 \text{ s}} = \boxed{13.5 \text{ m/s along } \vec{\mathbf{R}}}$$

P4.2 (a) $\vec{\mathbf{r}} = \boxed{18.0t\hat{\mathbf{i}} + \left(4.00t - 4.90t^2\right)\hat{\mathbf{j}}}$

(b) $\vec{\mathbf{v}} = \boxed{(18.0\ \text{m/s})\hat{\mathbf{i}} + \left[4.00\ \text{m/s} - \left(9.80\ \text{m/s}^2\right)t\right]\hat{\mathbf{j}}}$

(c) $\vec{\mathbf{a}} = \boxed{\left(-9.80\ \text{m/s}^2\right)\hat{\mathbf{j}}}$

(d) by substitution, $\vec{\mathbf{r}}(3.00\ \text{s}) = \boxed{(54.0\ \text{m})\hat{\mathbf{i}} - (32.1\ \text{m})\hat{\mathbf{j}}}$

(e) $\vec{\mathbf{v}}(3.00\ \text{s}) = \boxed{(18.0\ \text{m/s})\hat{\mathbf{i}} - (25.4\ \text{m/s})\hat{\mathbf{j}}}$

(f) $\vec{\mathbf{a}}(3.00\ \text{s}) = \boxed{\left(-9.80\ \text{m/s}^2\right)\hat{\mathbf{j}}}$

P4.3 The sun projects onto the ground the x component of her velocity:

$$5.00\ \text{m/s}\ \cos(-60.0°) = \boxed{2.50\ \text{m/s}}$$

P4.4 (a) From $x = -5.00\sin\omega t$, the x component of velocity is

$$v_x = \frac{dx}{dt} = \left(\frac{d}{dt}\right)(-5.00\sin\omega t) = -5.00\omega\cos\omega t$$

and $a_x = \dfrac{dv_x}{dt} = +5.00\omega^2\sin\omega t$

similarly, $v_y = \left(\dfrac{d}{dt}\right)(4.00 - 5.00\cos\omega t) = 0 + 5.00\omega\sin\omega t$

and $a_y = \left(\dfrac{d}{dt}\right)(5.00\omega\sin\omega t) = 5.00\omega^2\cos\omega t$

At $t = 0$, $\vec{\mathbf{v}} = -5.00\omega\cos 0\hat{\mathbf{i}} + 5.00\omega\sin 0\hat{\mathbf{j}} = \boxed{\left(5.00\omega\hat{\mathbf{i}} + 0\hat{\mathbf{j}}\right)\ \text{m/s}}$

and $\vec{\mathbf{a}} = 5.00\omega^2\sin 0\hat{\mathbf{i}} + 5.00\omega^2\cos 0\hat{\mathbf{j}} = \boxed{\left(0\hat{\mathbf{i}} + 5.00\omega^2\hat{\mathbf{j}}\right)\ \text{m/s}^2}$

(b) $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = \boxed{(4.00\ \text{m})\hat{\mathbf{j}} + (5.00\ \text{m})\left(-\sin\omega t\,\hat{\mathbf{i}} - \cos\omega t\,\hat{\mathbf{j}}\right)}$

$\vec{\mathbf{v}} = \boxed{(5.00\ \text{m})\omega\left[-\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}}\right]}$

$\vec{\mathbf{a}} = \boxed{(5.00\ \text{m})\omega^2\left[\sin\omega t\,\hat{\mathbf{i}} + \cos\omega t\,\hat{\mathbf{j}}\right]}$

(c) The object moves in $\boxed{\text{a circle of radius 5.00 m centered at } (0,\ 4.00\ \text{m})}$.

Section 4.2 Two-Dimensional Motion with Constant Acceleration

P4.5 $\vec{\mathbf{v}}_i = \left(4.00\hat{\mathbf{i}} + 1.00\hat{\mathbf{j}}\right)\ \text{m/s}$ and $\vec{\mathbf{v}}(20.0) = \left(20.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}\right)\ \text{m/s}$

(a) $a_x = \dfrac{\Delta v_x}{\Delta t} = \dfrac{20.0 - 4.00}{20.0}\ \text{m/s}^2 = \boxed{0.800\ \text{m/s}^2}$

$a_y = \dfrac{\Delta v_y}{\Delta t} = \dfrac{-5.00 - 1.00}{20.0}\ \text{m/s}^2 = \boxed{-0.300\ \text{m/s}^2}$

continued on next page

(b) $\theta = \tan^{-1}\left(\frac{-0.300}{0.800}\right) = -20.6° = \boxed{339° \text{ from } +x \text{ axis}}$

(c) At $t = 25.0$ s its position is specified by its coordinates and the direction of its motion is specified by the direction angle of its velocity:

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = 10.0 + 4.00(25.0) + \frac{1}{2}(0.800)(25.0)^2 = \boxed{360 \text{ m}}$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = -4.00 + 1.00(25.0) + \frac{1}{2}(-0.300)(25.0)^2 = \boxed{-72.7 \text{ m}}$$

$$v_{xf} = v_{xi} + a_x t = 4 + 0.8(25) = 24 \text{ m/s}$$

$$v_{yf} = v_{yi} + a_y t = 1 - 0.3(25) = -6.5 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-6.50}{24.0}\right) = \boxed{-15.2°}$$

P4.6 (a) $\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \left(\frac{d}{dt}\right)\left(3.00\hat{\mathbf{i}} - 6.00t^2\hat{\mathbf{j}}\right) = \boxed{-12.0t\,\hat{\mathbf{j}} \text{ m/s}}$

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \left(\frac{d}{dt}\right)\left(-12.0t\,\hat{\mathbf{j}}\right) = \boxed{-12.0\hat{\mathbf{j}} \text{ m/s}^2}$$

(b) by substitution, $\boxed{\vec{\mathbf{r}} = \left(3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}\right) \text{ m; } \vec{\mathbf{v}} = -12.0\hat{\mathbf{j}} \text{ m/s}}$

***P4.7** (a) From $\vec{\mathbf{a}} = d\vec{\mathbf{v}}/dt$, we have $\int_i^f d\vec{\mathbf{v}} = \int_i^f \vec{\mathbf{a}}\, dt$

Then $\vec{\mathbf{v}} - 5\,\hat{\mathbf{i}} \text{ m/s} = \int_0^t 6\, t^{1/2} dt \hat{\mathbf{j}} = 6\left.\frac{t^{3/2}}{3/2}\right|_0^t \hat{\mathbf{j}} = 4\, t^{3/2}\,\hat{\mathbf{j}}$ so $\vec{\mathbf{v}} = \boxed{5\,\hat{\mathbf{i}} + 4\, t^{3/2}\,\hat{\mathbf{j}}}$

(b) From $\vec{\mathbf{v}} = d\vec{\mathbf{r}}/dt$, we have $\int_i^f d\vec{\mathbf{r}} = \int_i^f \vec{\mathbf{v}}\, dt$

Then $\vec{\mathbf{r}} - 0 = \int_0^t \left(5\,\hat{\mathbf{i}} + 4\, t^{3/2}\,\hat{\mathbf{j}}\right) dt = \left.\left(5t\hat{\mathbf{i}} + 4\frac{t^{5/2}}{5/2}\hat{\mathbf{j}}\right)\right|_0^t = \boxed{5t\hat{\mathbf{i}} + 1.6\, t^{5/2}\,\hat{\mathbf{j}}}$

P4.8 $\vec{\mathbf{a}} = 3.00\hat{\mathbf{j}} \text{ m/s}^2$; $\vec{\mathbf{v}}_i = 5.00\hat{\mathbf{i}} \text{ m/s}$; $\vec{\mathbf{r}}_i = 0\hat{\mathbf{i}} + 0\hat{\mathbf{j}}$

(a) $\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2 = \boxed{\left[5.00t\hat{\mathbf{i}} + \frac{1}{2}3.00t^2\hat{\mathbf{j}}\right] \text{ m}}$

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = \boxed{\left(5.00\hat{\mathbf{i}} + 3.00t\hat{\mathbf{j}}\right) \text{ m/s}}$$

(b) $t = 2.00$ s, $\vec{\mathbf{r}}_f = 5.00(2.00)\hat{\mathbf{i}} + \frac{1}{2}(3.00)(2.00)^2\hat{\mathbf{j}} = \left(10.0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right)$ m

so $x_f = \boxed{10.0 \text{ m}}$, $y_f = \boxed{6.00 \text{ m}}$

$$\vec{\mathbf{v}}_f = 5.00\hat{\mathbf{i}} + 3.00(2.00)\hat{\mathbf{j}} = \left(5.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right) \text{ m/s}$$

$$v_f = \left|\vec{\mathbf{v}}_f\right| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(5.00)^2 + (6.00)^2} = \boxed{7.81 \text{ m/s}}$$

Section 4.3 **Projectile Motion**

P4.9 (a) The mug leaves the counter horizontally with a velocity v_{xi} (say). If time t elapses before it hits the ground, then since there is no horizontal acceleration, $x_f = v_{xi}t$, i.e.,

$$t = \frac{x_f}{v_{xi}} = \frac{(1.40\text{ m})}{v_{xi}}$$

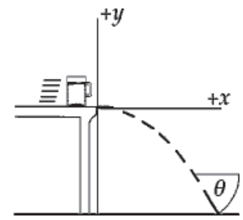


FIG. P4.9

In the same time it falls a distance of 0.860 m with acceleration downward of 9.80 m/s^2. Then

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2:\ 0 = 0.860\text{ m} + \frac{1}{2}\left(-9.80\text{ m/s}^2\right)\left(\frac{1.40\text{ m}}{v_{xi}}\right)^2$$

Thus,

$$v_{xi} = \sqrt{\frac{\left(4.90\text{ m/s}^2\right)\left(1.96\text{ m}^2\right)}{0.860\text{ m}}} = \boxed{3.34\text{ m/s}}$$

(b) The vertical velocity component with which it hits the floor is

$$v_{yf} = v_{yi} + a_y t = 0 + \left(-9.80\text{ m/s}^2\right)\left(\frac{1.40\text{ m}}{3.34\text{ m/s}}\right) = -4.11\text{ m/s}$$

Hence, the angle θ at which the mug strikes the floor is given by

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-4.11}{3.34}\right) = \boxed{-50.9^\circ}$$

P4.10 The mug is a projectile from just after leaving the counter until just before it reaches the floor. Taking the origin at the point where the mug leaves the bar, the coordinates of the mug at any time are

$$x_f = v_{xi}t + \frac{1}{2}a_x t^2 = v_{xi}t + 0 \qquad \text{and} \qquad y_f = v_{yi}t + \frac{1}{2}a_y t^2 = 0 - \frac{1}{2}gt^2$$

When the mug reaches the floor, $y_f = -h$ so

$$-h = -\frac{1}{2}gt^2$$

which gives the time of impact as

$$t = \sqrt{\frac{2h}{g}}$$

(a) Since $x_f = d$ when the mug reaches the floor, $x_f = v_{xi}t$ becomes $d = v_{xi}\sqrt{\frac{2h}{g}}$ giving the initial velocity as

$$\boxed{v_{xi} = d\sqrt{\frac{g}{2h}}}$$

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(b) Just before impact, the x component of velocity is still

$$v_{xf} = v_{xi}$$

while the y component is

$$v_{yf} = v_{yi} + a_y t = 0 - g\sqrt{\frac{2h}{g}}$$

Then the direction of motion just before impact is below the horizontal at an angle of

$$\theta = \tan^{-1}\left(\frac{|v_{yf}|}{v_{xf}}\right) = \tan^{-1}\left(\frac{g\sqrt{2h/g}}{d\sqrt{g/2h}}\right) = \boxed{\tan^{-1}\left(\frac{2h}{d}\right)}$$

The answer for v_{xi} indicates that a larger measured value for d would imply larger takeoff speed in direct proportion. A tape measure lying on the floor could be calibrated as a speedometer. A larger value for h would imply a smaller value for speed by an inverse proportionality to the square root of h. That is, if h were nine times larger, v_{xi} would be three times smaller. The answer for θ shows that the impact velocity makes an angle with the horizontal whose tangent is just twice as large as that of the elevation angle α of the edge of the table as seen from the impact point.

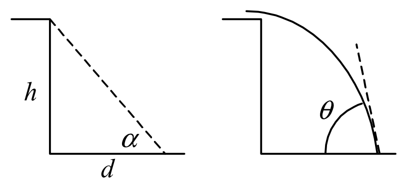

FIG. P4.10

P4.11 $x = v_{xi}t = v_i \cos\theta_i t$

$$x = (300\ \text{m/s})(\cos 55.0°)(42.0\ \text{s})$$

$$x = \boxed{7.23 \times 10^3\ \text{m}}$$

$$y = v_{yi}t - \frac{1}{2}gt^2 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

$$y = (300\ \text{m/s})(\sin 55.0°)(42.0\ \text{s}) - \frac{1}{2}(9.80\ \text{m/s}^2)(42.0\ \text{s})^2 = \boxed{1.68 \times 10^3\ \text{m}}$$

P4.12 (a) To identify the maximum height we let i be the launch point and f be the highest point:

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$$
$$0 = v_i^2 \sin^2\theta_i + 2(-g)(y_{\max} - 0)$$
$$y_{\max} = \frac{v_i^2 \sin^2\theta_i}{2g}$$

To identify the range we let i be the launch and f be the impact point; where t is not zero:

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$0 = 0 + v_i \sin\theta_i t + \frac{1}{2}(-g)t^2$$
$$t = \frac{2v_i \sin\theta_i}{g}$$
$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$
$$d = 0 + v_i \cos\theta_i \frac{2v_i \sin\theta_i}{g} + 0$$

For this rock, $d = y_{\max}$

$$\frac{v_i^2 \sin^2\theta_i}{2g} = \frac{2v_i^2 \sin\theta_i \cos\theta_i}{g}$$
$$\frac{\sin\theta_i}{\cos\theta_i} = \tan\theta_i = 4$$
$$\theta_i = \boxed{76.0°}$$

(b) Since g divides out, the answer is $\boxed{\text{the same on every planet}}$.

(c) The maximum range is attained for $\theta_i = 45°$:

$$\frac{d_{\max}}{d} = \frac{v_i \cos 45° 2v_i \sin 45° g}{g v_i \cos 76° 2v_i \sin 76°} = 2.125$$

So $d_{\max} = \boxed{\dfrac{17d}{8}}$.

P4.13 $h = \dfrac{v_i^2 \sin^2\theta_i}{2g}$; $R = \dfrac{v_i^2 (\sin 2\theta_i)}{g}$; $3h = R$

so $\dfrac{3v_i^2 \sin^2\theta_i}{2g} = \dfrac{v_i^2 (\sin 2\theta_i)}{g}$

or $\dfrac{2}{3} = \dfrac{\sin^2\theta_i}{\sin 2\theta_i} = \dfrac{\tan\theta_i}{2}$

thus $\theta_i = \tan^{-1}\left(\dfrac{4}{3}\right) = \boxed{53.1°}$

P4.14 The horizontal component of displacement is $x_f = v_{xi}t = (v_i \cos\theta_i)t$. Therefore, the time required to reach the building a distance d away is $t = \dfrac{d}{v_i \cos\theta_i}$. At this time, the altitude of the water is

$$y_f = v_{yi}t + \frac{1}{2}a_y t^2 = v_i \sin\theta_i \left(\frac{d}{v_i \cos\theta_i}\right) - \frac{g}{2}\left(\frac{d}{v_i \cos\theta_i}\right)^2$$

Therefore the water strikes the building at a height h above ground level of

$$h = y_f = \boxed{d\tan\theta_i - \frac{gd^2}{2v_i^2 \cos^2\theta_i}}$$

P4.15 (a) $x_f = v_{xi}\,t = 8.00\cos 20.0°(3.00) = \boxed{22.6 \text{ m}}$

(b) Taking y positive downwards,

$$y_f = v_{yi}t + \frac{1}{2}g\,t^2$$

$$y_f = 8.00\sin 20.0°(3.00) + \frac{1}{2}(9.80)(3.00)^2 = \boxed{52.3 \text{ m}}$$

(c) $10.0 = 8.00(\sin 20.0°)t + \frac{1}{2}(9.80)t^2$

$$4.90t^2 + 2.74t - 10.0 = 0$$

$$t = \frac{-2.74 \pm \sqrt{(2.74)^2 + 196}}{9.80} = \boxed{1.18 \text{ s}}$$

***P4.16** The time of flight of a water drop is given by $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$.

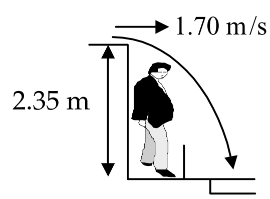

FIG. P4.16

$$0 = 2.35 \text{ m} + 0 - \frac{1}{2}(9.8 \text{ m/s}^2)t_1^2$$

For $t_1 > 0$, the root is $t_1 = \sqrt{\dfrac{2(2.35 \text{ m})}{9.8 \text{ m/s}^2}} = 0.693$ s.

(a) The horizontal range of the font is

$$x_{f1} = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$
$$= 0 + 1.70 \text{ m/s}(0.693 \text{ s}) + 0 = 1.18 \text{ m}$$

This is about the width of a town sidewalk, so $\boxed{\text{there is}}$ space for a walkway behind the waterfall. Unless the lip of the channel is well designed, water may drip on the visitors. A tall or inattentive person may get his head wet.

(b) Now the flight time t_2 is given by $0 = y_2 + 0 - \frac{1}{2}gt_2^2$.

$t_2 = \sqrt{\dfrac{2y_2}{g}} = \sqrt{\dfrac{2y_1}{g(12)}} = \dfrac{1}{\sqrt{12}}\sqrt{\dfrac{2y_1}{g}} = \dfrac{t_1}{\sqrt{12}}$ From the same equation as in part (a) for horizontal range, $x_2 = v_2 t_2$.

$$\frac{x_1}{12} = v_2\frac{t_1}{\sqrt{12}} \qquad v_2 = \frac{x_1}{t_1\sqrt{12}} = \frac{v_1}{\sqrt{12}} = \frac{1.70 \text{ m/s}}{\sqrt{12}} = \boxed{0.491 \text{ m/s}}$$

The rule that the scale factor for speed is the square root of the scale factor for distance is Froude's law, published in 1870.

P4.17 (a) We use the trajectory equation:

$$y_f = x_f \tan\theta_i - \frac{g x_f^2}{2v_i^2\cos^2\theta_i}$$

With

$$x_f = 36.0\text{ m},\ v_i = 20.0\text{ m/s, and } \theta = 53.0°$$

we find

$$y_f = (36.0\text{ m})\tan 53.0° - \frac{(9.80\text{ m/s}^2)(36.0\text{ m})^2}{2(20.0\text{ m/s})^2\cos^2(53.0°)} = 3.94\text{ m}$$

The ball clears the bar by

$$(3.94 - 3.05)\text{ m} = \boxed{0.889\text{ m}}$$

(b) The time the ball takes to reach the maximum height is

$$t_1 = \frac{v_i\sin\theta_i}{g} = \frac{(20.0\text{ m/s})(\sin 53.0°)}{9.80\text{ m/s}^2} = 1.63\text{ s}$$

The time to travel 36.0 m horizontally is $t_2 = \dfrac{x_f}{v_{ix}}$

$$t_2 = \frac{36.0\text{ m}}{(20.0\text{ m/s})(\cos 53.0°)} = 2.99\text{ s}$$

Since $t_2 > t_1$ $\boxed{\text{the ball clears the goal on its way down}}$.

P4.18 When the bomb has fallen a vertical distance 2.15 km, it has traveled a horizontal distance x_f given by

$$x_f = \sqrt{(3.25\text{ km})^2 - (2.15\text{ km})^2} = 2.437\text{ km}$$

$$y_f = x_f\tan\theta - \frac{g x_f^2}{2v_i^2\cos^2\theta_i}$$

$$-2\,150\text{ m} = (2\,437\text{ m})\tan\theta_i - \frac{(9.8\text{ m/s}^2)(2\,437\text{ m})^2}{2(280\text{ m/s})^2\cos^2\theta_i}$$

$$\therefore -2\,150\text{ m} = (2\,437\text{ m})\tan\theta_i - (371.19\text{ m})(1+\tan^2\theta_i)$$

$$\therefore \tan^2\theta - 6.565\tan\theta_i - 4.792 = 0$$

$$\therefore \tan\theta_i = \frac{1}{2}\left(6.565 \pm \sqrt{(6.565)^2 - 4(1)(-4.792)}\right) = 3.283 \pm 3.945$$

Select the negative solution, since θ_i is below the horizontal.

$$\therefore \tan\theta_i = -0.662,\ \boxed{\theta_i = -33.5°}$$

P4.19 (a) For the horizontal motion, we have

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$24\text{ m} = 0 + v_i(\cos 53°)(2.2\text{ s}) + 0$$

$$v_i = \boxed{18.1\text{ m/s}}$$

continued on next page

(b) As it passes over the wall, the ball is above the street by $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

$$y_f = 0 + (18.1\ \text{m/s})(\sin 53°)(2.2\ \text{s}) + \frac{1}{2}(-9.8\ \text{m/s}^2)(2.2\ \text{s})^2 = 8.13\ \text{m}$$

So it clears the parapet by $8.13\ \text{m} - 7\ \text{m} = \boxed{1.13\ \text{m}}$.

(c) Note that the highest point of the ball's trajectory is not directly above the wall. For the whole flight, we have from the trajectory equation

$$y_f = (\tan\theta_i)x_f - \left(\frac{g}{2v_i^2\cos^2\theta_i}\right)x_f^2$$

or

$$6\ \text{m} = (\tan 53°)x_f - \left(\frac{9.8\ \text{m/s}^2}{2(18.1\ \text{m/s})^2\cos^2 53°}\right)x_f^2$$

Solving,

$$(0.041\,2\ \text{m}^{-1})x_f^2 - 1.33x_f + 6\ \text{m} = 0$$

and

$$x_f = \frac{1.33 \pm \sqrt{1.33^2 - 4(0.0412)(6)}}{2(0.0412\ \text{m}^{-1})}$$

This yields two results:

$$x_f = 26.8\ \text{m}\ \text{ or } 5.44\ \text{m}$$

The ball passes twice through the level of the roof.
It hits the roof at distance from the wall

$$26.8\ \text{m} - 24\ \text{m} = \boxed{2.79\ \text{m}}$$

P4.20 From the instant he leaves the floor until just before he lands, the basketball star is a projectile. His vertical velocity and vertical displacement are related by the equation $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$. Applying this to the upward part of his flight gives $0 = v_{yi}^2 + 2(-9.80\ \text{m/s}^2)(1.85 - 1.02)\ \text{m}$. From this, $v_{yi} = 4.03\ \text{m/s}$. [Note that this is the answer to part (c) of this problem.]

For the downward part of the flight, the equation gives $v_{yf}^2 = 0 + 2(-9.80\ \text{m/s}^2)(0.900 - 1.85)\ \text{m}$. Thus the vertical velocity just before he lands is

$$v_{yf} = -4.32\ \text{m/s}$$

(a) His hang time may then be found from $v_{yf} = v_{yi} + a_y t$:

$$-4.32\ \text{m/s} = 4.03\ \text{m/s} + (-9.80\ \text{m/s}^2)t$$

or $t = \boxed{0.852\ \text{s}}$.

(b) Looking at the total horizontal displacement during the leap, $x = v_{xi}t$ becomes

$$2.80\ \text{m} = v_{xi}(0.852\ \text{s})$$

which yields $v_{xi} = \boxed{3.29\ \text{m/s}}$.

(c) $v_{yi} = \boxed{4.03\ \text{m/s}}$. See above for proof.

continued on next page

(d) The takeoff angle is: $\theta = \tan^{-1}\left(\dfrac{v_{yi}}{v_{xi}}\right) = \tan^{-1}\left(\dfrac{4.03\ \text{m/s}}{3.29\ \text{m/s}}\right) = \boxed{50.8°}$.

(e) Similarly for the deer, the upward part of the flight gives $v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$:

$$0 = v_{yi}^2 + 2\left(-9.80\ \text{m/s}^2\right)(2.50 - 1.20)\ \text{m}$$

so $v_{yi} = 5.04\ \text{m/s}$.

For the downward part, $v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$ yields

$v_{yf}^2 = 0 + 2\left(-9.80\ \text{m/s}^2\right)(0.700 - 2.50)\ \text{m}$ and $v_{yf} = -5.94\ \text{m/s}$

The hang time is then found as $v_{yf} = v_{yi} + a_y t$: $-5.94\ \text{m/s} = 5.04\ \text{m/s} + \left(-9.80\ \text{m/s}^2\right)t$ and

$$\boxed{t = 1.12\ \text{s}}$$

P4.21 The horizontal kick gives zero vertical velocity to the rock. Then its time of flight follows from

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$-40.0\ \text{m} = 0 + 0 + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)t^2$$
$$t = 2.86\ \text{s}$$

The extra time $3.00\ \text{s} - 2.86\ \text{s} = 0.143\ \text{s}$ is the time required for the sound she hears to travel straight back to the player. It covers distance

$$(343\ \text{m/s})0.143\ \text{s} = 49.0\ \text{m} = \sqrt{x^2 + (40.0\ \text{m})^2}$$

where x represents the horizontal distance the rock travels.

$$x = 28.3\ \text{m} = v_{xi}t + 0t^2$$
$$\therefore v_{xi} = \frac{28.3\ \text{m}}{2.86\ \text{s}} = \boxed{9.91\ \text{m/s}}$$

***P4.22** We match the given equations

$$x_f = 0 + (11.2\ \text{m/s})\cos 18.5° t$$
$$0.360\ \text{m} = 0.840\ \text{m} + (11.2\ \text{m/s})\sin 18.5° t - \frac{1}{2}\left(9.80\ \text{m/s}^2\right)t^2$$

to the equations for the coordinates of the final position of a projectile

$$x_f = x_i + v_{xi}t$$
$$y_f = y_i + v_{yi}t - \frac{1}{2}gt^2$$

For the equations to represent the same functions of time, all coefficients must agree: $x_i = 0$, $y_i = 0.840\ \text{m}$, $v_{xi} = (11.2\ \text{m/s})\cos 18.5°$, $v_{yi} = (11.2\ \text{m/s})\sin 18.5°$ and $g = 9.80\ \text{m/s}^2$.

(a) Then the original position of the athlete's center of mass is the point with coordinates $\left(x_i,\ y_i\right) = \boxed{(0,\ 0.840\ \text{m})}$. That is, his original position has position vector $\vec{\mathbf{r}} = 0\hat{\mathbf{i}} + 0.840\ \text{m}\hat{\mathbf{j}}$.

continued on next page

(b) His original velocity is $\vec{\mathbf{v}}_i = (11.2\ \text{m/s})\cos 18.5°\hat{\mathbf{i}} + (11.2\ \text{m/s})\sin 18.5°\hat{\mathbf{j}} =$

$\boxed{11.2\ \text{m/s at } 18.5°}$ above the x axis.

(c) From $4.90\ \text{m/s}^2\ t^2 - 3.55\ \text{m/s}\ \ t - 0.48\ \text{m} = 0$ we find the time of flight, which must be

positive $t = \dfrac{+3.55\ \text{m/s} + \sqrt{(3.55\ \text{m/s})^2 - 4(4.9\ \text{m/s}^2)(-0.48\ \text{m})}}{2(4.9\ \text{m/s}^2)} = 0.842$ s. Then

$$x_f = (11.2\ \text{m/s})\cos 18.5°(0.8425) = \boxed{8.94\ \text{m}}$$

(d)

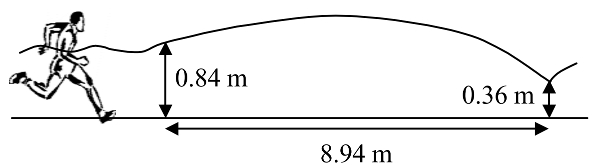

FIG. P4.22

The free-fall trajectory of the athlete is a section around the vertex of a parabola opening downward, everywhere close to horizontal and 48 cm lower on the landing side than on the takeoff side.

P4.23 For the smallest impact angle

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right)$$

we want to minimize v_{yf} and maximize $v_{xf} = v_{xi}$. The final y component of velocity is related to v_{yi} by $v_{yf}^2 = v_{yi}^2 + 2gh$, so we want to minimize v_{yi} and maximize v_{xi}. Both are accomplished by making the initial velocity horizontal. Then $v_{xi} = v$, $v_{yi} = 0$, and $v_{yf} = \sqrt{2gh}$. At last, the impact angle is

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \boxed{\tan^{-1}\left(\frac{\sqrt{2gh}}{v}\right)}$$

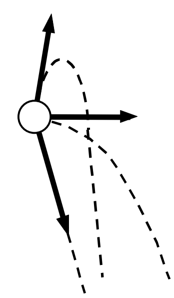

FIG. P4.23

Section 4.4 **Uniform Circular Motion**

P4.24 $a = \dfrac{v^2}{R}$, $T = 24\ \text{h}(3\,600\ \text{s/h}) = 86\,400\ \text{s}$

$$v = \frac{2\pi R}{T} = \frac{2\pi(6.37\times10^6\ \text{m})}{86\,400\ \text{s}} = 463\ \text{m/s}$$

$$a = \frac{(463\ \text{m/s})^2}{6.37\times10^6\ \text{m}} = \boxed{0.033\,7\ \text{m/s}^2\ \text{directed toward the center of Earth}}$$

P4.25 $a_c = \dfrac{v^2}{r} = \dfrac{(20.0\ \text{m/s})^2}{1.06\ \text{m}} = \boxed{377\ \text{m/s}^2}$ The mass is unnecessary information.

P4.26 $a_c = \dfrac{v^2}{r}$

$$v = \sqrt{a_c r} = \sqrt{3(9.8\ \text{m/s}^2)(9.45\ \text{m})} = 16.7\ \text{m/s}$$

Each revolution carries the astronaut over a distance of $2\pi r = 2\pi(9.45\ \text{m}) = 59.4\ \text{m}$. Then the rotation rate is

$$16.7\ \text{m/s}\left(\frac{1\ \text{rev}}{59.4\ \text{m}}\right) = \boxed{0.281\ \text{rev/s}}$$

P4.27 (a) $v = r\omega$

At $8.00\ \text{rev/s}$, $v = (0.600\ \text{m})(8.00\ \text{rev/s})(2\pi\ \text{rad/rev}) = 30.2\ \text{m/s} = 9.60\pi\ \text{m/s}$.

At $6.00\ \text{rev/s}$, $v = (0.900\ \text{m})(6.00\ \text{rev/s})(2\pi\ \text{rad/rev}) = 33.9\ \text{m/s} = 10.8\pi\ \text{m/s}$.

$\boxed{6.00\ \text{rev/s}}$ gives the larger linear speed.

(b) Acceleration $= \dfrac{v^2}{r} = \dfrac{(9.60\pi\ \text{m/s})^2}{0.600\ \text{m}} = \boxed{1.52\times10^3\ \text{m/s}^2}$.

(c) At $6.00\ \text{rev/s}$, acceleration $= \dfrac{(10.8\pi\ \text{m/s})^2}{0.900\ \text{m}} = \boxed{1.28\times10^3\ \text{m/s}^2}$. So 8 rev/s gives the higher acceleration.

Section 4.5 **Tangential and Radial Acceleration**

***P4.28** The particle's centripetal acceleration is $v^2/r = (3\ \text{m/s})^2/2\ \text{m} = 4.50\ \text{m/s}^2$. The total acceleration magnitude can be larger than or equal to this, but not smaller.

(a) Yes. The particle can be either speeding up or slowing down, with a tangential component of acceleration of magnitude $\sqrt{6^2 - 4.5^2} = 3.97\ \text{m/s}^2$.
(b) No. The magnitude of the acceleration cannot be less than $v^2/r = 4.5\ \text{m/s}^2$.

P4.29 We assume the train is still slowing down at the instant in question.

$$a_c = \frac{v^2}{r} = 1.29\ \text{m/s}^2$$

$$a_t = \frac{\Delta v}{\Delta t} = \frac{(-40.0\ \text{km/h})(10^3\ \text{m/km})(1\ \text{h}/3600\ \text{s})}{15.0\ \text{s}} = -0.741\ \text{m/s}^2$$

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{(1.29\ \text{m/s}^2)^2 + (-0.741\ \text{m/s}^2)^2}$$

at an angle of $\tan^{-1}\left(\dfrac{|a_t|}{a_c}\right) = \tan^{-1}\left(\dfrac{0.741}{1.29}\right)$

$$\vec{\mathbf{a}} = \boxed{1.48\ \text{m/s}^2\ \text{inward and } 29.9°\ \text{backward}}$$

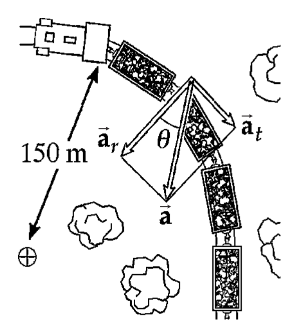

FIG. P4.29

P4.30 (a) See figure to the right.

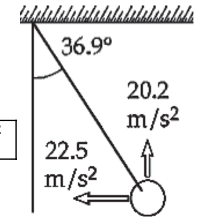

FIG. P4.30

(b) The components of the 20.2 and the 22.5 $\mathrm{m/s^2}$ along the rope together constitute the centripetal acceleration:

$$a_c = (22.5\ \mathrm{m/s^2})\cos(90.0° - 36.9°) + (20.2\ \mathrm{m/s^2})\cos 36.9° = \boxed{29.7\ \mathrm{m/s^2}}$$

(c) $a_c = \dfrac{v^2}{r}$ so $v = \sqrt{a_c r} = \sqrt{29.7\ \mathrm{m/s^2}\,(1.50\ \mathrm{m})} = 6.67\ \mathrm{m/s}$ tangent to circle

$$\vec{\mathbf{v}} = \boxed{6.67\ \mathrm{m/s}\ \text{at } 36.9° \text{ above the horizontal}}$$

P4.31 $r = 2.50\ \mathrm{m},\ a = 15.0\ \mathrm{m/s^2}$

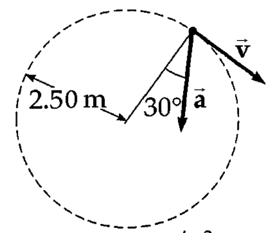

FIG. P4.31

(a) $a_c = a\cos 30.0° = (15.0\ \mathrm{m/s^2})(\cos 30°) = \boxed{13.0\ \mathrm{m/s^2}}$

(b) $a_c = \dfrac{v^2}{r}$

so $v^2 = ra_c = 2.50\ \mathrm{m}(13.0\ \mathrm{m/s^2}) = 32.5\ \mathrm{m^2/s^2}$

$v = \sqrt{32.5}\ \mathrm{m/s} = \boxed{5.70\ \mathrm{m/s}}$

(c) $a^2 = a_t^2 + a_r^2$

so $a_t = \sqrt{a^2 - a_r^2} = \sqrt{(15.0\,\mathrm{m/s^2})^2 - (13.0\,\mathrm{m/s^2})} = \boxed{7.50\,\mathrm{m/s^2}}$

P4.32 Let i be the starting point and f be one revolution later. The curvilinear motion with constant tangential acceleration is described by

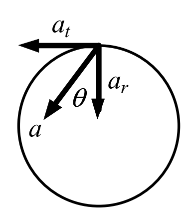

FIG. P4.32

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2$$

$$2\pi r = 0 + \frac{1}{2}a_t t^2$$

$$a_t = \frac{4\pi r}{t^2}$$

and $v_{xf} = v_{xi} + a_x t,\ v_f = 0 + a_t t = \dfrac{4\pi r}{t}$. The magnitude of the radial acceleration is $a_r = \dfrac{v_f^2}{r} = \dfrac{16\pi^2 r^2}{t^2 r}$.

Then $\tan\theta = \dfrac{a_t}{a_r} = \dfrac{4\pi r t^2}{t^2 16\pi^2 r} = \dfrac{1}{4\pi}$ $\theta = \boxed{4.55°}$

Section 4.6 Relative Velocity and Relative Acceleration

P4.33 $\vec{\mathbf{v}}_{ce}$ = the velocity of the car relative to the earth.

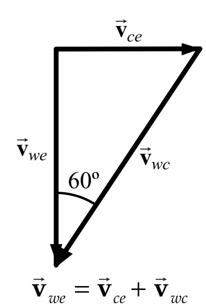

FIG. P4.33

$\vec{\mathbf{v}}_{wc}$ = the velocity of the water relative to the car.

$\vec{\mathbf{v}}_{we}$ = the velocity of the water relative to the earth.

These velocities are related as shown in the diagram at the right.

(a) Since $\vec{\mathbf{v}}_{we}$ is vertical, $v_{wc}\sin 60.0° = v_{ce} = 50.0\ \mathrm{km/h}$ or

$\vec{\mathbf{v}}_{wc} = \boxed{57.7\ \mathrm{km/h}\ \text{at } 60.0° \text{ west of vertical}}$.

(b) Since $\vec{\mathbf{v}}_{ce}$ has zero vertical component,

$$v_{we} = v_{wc}\cos 60.0° = (57.7\ \mathrm{km/h})\cos 60.0° = \boxed{28.9\ \mathrm{km/h}\ \text{downward}}$$

P4.34 (a) $\vec{\mathbf{v}}_H = 0 + \vec{\mathbf{a}}_H t = \left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right) \text{ m/s}^2 (5.00 \text{ s})$

$$\vec{\mathbf{v}}_H = \left(15.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}}\right) \text{ m/s}$$

$$\vec{\mathbf{v}}_J = 0 + \vec{\mathbf{a}}_J t = \left(1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right) \text{ m/s}^2 (5.00 \text{ s})$$

$$\vec{\mathbf{v}}_J = \left(5.00\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}\right) \text{ m/s}$$

$$\vec{\mathbf{v}}_{HJ} = v_H - v_J = \left(15.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}} - 5.00\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}}\right) \text{ m/s}$$

$$\vec{\mathbf{v}}_{HJ} = \left(10.0\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}}\right) \text{ m/s}$$

$$\left|\vec{\mathbf{v}}_{HJ}\right| = \sqrt{(10.0)^2 + (25.0)^2} \text{ m/s} = \boxed{26.9 \text{ m/s}}$$

(b) $\vec{\mathbf{r}}_H = 0 + 0 + \frac{1}{2}\vec{\mathbf{a}}_H t^2 = \frac{1}{2}\left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right) \text{ m/s}^2 (5.00 \text{ s})^2$

$$\vec{\mathbf{r}}_H = \left(37.5\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}}\right) \text{ m}$$

$$\vec{\mathbf{r}}_J = \frac{1}{2}\left(1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right) \text{ m/s}^2 (5.00\text{s})^2 = \left(12.5\hat{\mathbf{i}} + 37.5\hat{\mathbf{j}}\right) \text{ m}$$

$$\vec{\mathbf{r}}_{HJ} = \vec{\mathbf{r}}_H - \vec{\mathbf{r}}_J = \left(37.5\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}} - 12.5\hat{\mathbf{i}} - 37.5\hat{\mathbf{j}}\right) \text{ m}$$

$$\vec{\mathbf{r}}_{HJ} = \left(25.0\hat{\mathbf{i}} - 62.5\hat{\mathbf{j}}\right) \text{ m}$$

$$\left|\vec{\mathbf{r}}_{HJ}\right| = \sqrt{(25.0)^2 + (62.5)^2} \text{m} = \boxed{67.3 \text{ m}}$$

(c) $\vec{\mathbf{a}}_{HJ} = \vec{\mathbf{a}}_H - \vec{\mathbf{a}}_J = \left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} - 1.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}}\right) \text{ m/s}^2$

$$\vec{\mathbf{a}}_{HJ} = \boxed{\left(2.00\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}\right) \text{ m/s}^2}$$

P4.35 Total time in still water $t = \frac{d}{v} = \frac{2\,000}{1.20} = 1.67 \times 10^3$ s.

Total time = time upstream plus time downstream:

$$t_{\text{up}} = \frac{1000}{(1.20 - 0.500)} = 1.43 \times 10^3 \text{ s}$$

$$t_{\text{down}} = \frac{1000}{1.20 + 0.500} = 588 \text{ s}$$

Therefore, $t_{\text{total}} = 1.43 \times 10^3 + 588 = \boxed{2.02 \times 10^3 \text{ s}}$.

$\boxed{\text{This is 12.0\% larger than the time in still water.}}$

P4.36 The bumpers are initially $100 \text{ m} = 0.100 \text{ km}$ apart. After time t the bumper of the leading car travels $40.0\,t$, while the bumper of the chasing car travels $60.0t$. Since the cars are side by side at time t, we have

$$0.100 + 40.0t = 60.0t$$

yielding

$$t = 5.00 \times 10^{-3} \text{ h} = \boxed{18.0 \text{ s}}$$

P4.37 To guess the answer, think of v just a little less than the speed c of the river. Then poor Alan will spend most of his time paddling upstream making very little progress. His time-averaged speed will be low and Beth will win the race.

Now we calculate: For Alan, his speed downstream is $c + v$, while his speed upstream is $c - v$. Therefore, the total time for Alan is

$$t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \boxed{\frac{2L/c}{1-v^2/c^2}}$$

For Beth, her cross-stream speed (both ways) is

$$\sqrt{c^2 - v^2}$$

Thus, the total time for Beth is $t_2 = \dfrac{2L}{\sqrt{c^2-v^2}} = \boxed{\dfrac{2L/c}{\sqrt{1-v^2/c^2}}}$.

Since $1 - \dfrac{v^2}{c^2} < 1$, $t_1 > t_2$, or Beth, who swims cross-stream, returns first.

***P4.38** We can find the time of flight of the can by considering its horizontal motion:

$16 \text{ m} = (9.5 \text{ m/s})\, t + 0 \qquad t = 1.68 \text{ s}$

(a) For the boy to catch the can at the same location on the truck bed, he must throw it $\boxed{\text{straight up, at } 0° \text{ to the vertical}}$.

(b) For the free fall of the can, $y_f = y_i + v_{yi}t + (1/2)a_y t^2$:
$0 = 0 + v_{yi}\,(1.68 \text{ s}) - (1/2)(9.8 \text{ m/s}^2)(1.68 \text{ s})^2 \quad v_{yi} = \boxed{8.25 \text{ m/s}}$

(c) The boy sees the can always over his head, traversing $\boxed{\text{a straight line segment upward and then downward}}$.

(d) The ground observer sees the can move as a projectile on $\boxed{\text{a symmetric section of a parabola opening downward}}$. Its initial velocity is
$(9.5^2 + 8.25^2)^{1/2} \text{ m/s} = \boxed{12.6 \text{ m/s north at } \tan^{-1}(8.25/9.5) = 41.0° \text{ above the horizontal}}$

P4.39 Identify the student as the S' observer and the professor as the S observer. For the initial motion in S', we have

$$\frac{v'_y}{v'_x} = \tan 60.0° = \sqrt{3}$$

Let u represent the speed of S' relative to S. Then because there is no x-motion in S, we can write $v_x = v'_x + u = 0$ so that $v'_x = -u = -10.0 \text{ m/s}$. Hence the ball is thrown backwards in S'. Then,

$$v_y = v'_y = \sqrt{3}\left|v'_x\right| = 10.0\sqrt{3} \text{ m/s}$$

Using $v_y^2 = 2gh$ we find

$$h = \frac{\left(10.0\sqrt{3} \text{ m/s}\right)^2}{2\left(9.80 \text{ m/s}^2\right)} = \boxed{15.3 \text{ m}}$$

The motion of the ball as seen by the student in S' is shown in diagram (b). The view of the professor in S is shown in diagram (c).

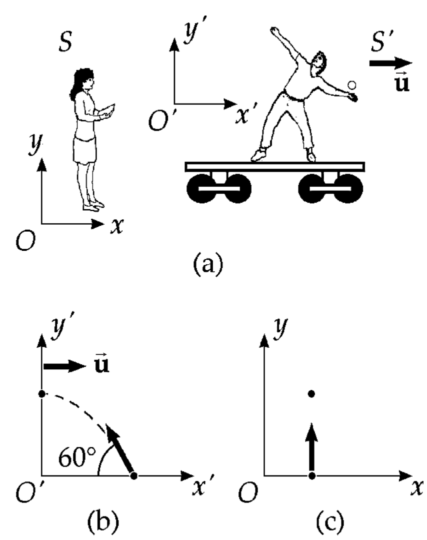

FIG. P4.39

***P4.40** (a) To an observer at rest in the train car, the bolt accelerates downward and toward the rear of the train.

$$a = \sqrt{(2.50\ \text{m/s})^2 + (9.80\ \text{m/s})^2} = \boxed{10.1\ \text{m/s}^2}$$

$$\tan\theta = \frac{2.50\ \text{m/s}^2}{9.80\ \text{m/s}^2} = 0.255$$

$$\theta = \boxed{14.3° \text{ to the south from the vertical}}$$

To this observer, the bolt moves as if it were in a gravitational field of 9.80 m/s^2 down + 2.50 m/s^2 south.

(b) $a = \boxed{9.80\ \text{m/s}^2 \text{ vertically downward}}$

(c) If it is at rest relative to the ceiling at release, the bolt moves on a straight line downward and southward at 14.3 degrees from the vertical.

(d) The bolt moves on a parabola with a vertical axis.

P4.41 Choose the x axis along the 20-km distance. The y components of the displacements of the ship and the speedboat must agree:

$$(26\ \text{km/h})t\sin(40° - 15°) = (50\ \text{km/h})t\sin\alpha$$

$$\alpha = \sin^{-1}\frac{11.0}{50} = 12.7°$$

The speedboat should head

$$15° + 12.7° = \boxed{27.7° \text{ east of north}}$$

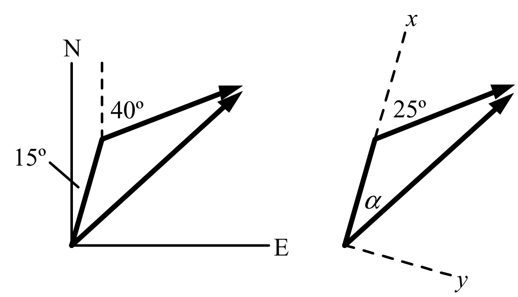

FIG. P4.41

Additional Problems

P4.42 (a) The speed at the top is $v_x = v_i\cos\theta_i = (143\ \text{m/s})\cos 45° = \boxed{101\ \text{m/s}}$.

(b) In free fall the plane reaches altitude given by

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$

$$0 = (143\ \text{m/s}\sin 45°)^2 + 2(-9.8\ \text{m/s}^2)(y_f - 31\,000\ \text{ft})$$

$$y_f = 31\,000\ \text{ft} + 522\ \text{m}\left(\frac{3.28\ \text{ft}}{1\ \text{m}}\right) = \boxed{3.27\times10^4\ \text{ft}}$$

(c) For the whole free fall motion $v_{yf} = v_{yi} + a_y t$

$$-101\ \text{m/s} = +101\ \text{m/s} - (9.8\ \text{m/s}^2)t$$

$$t = \boxed{20.6\ \text{s}}$$

(d) $a_c = \dfrac{v^2}{r}$

$$v = \sqrt{a_c r} = \sqrt{0.8(9.8\ \text{m/s}^2)4{,}130\ \text{m}} = \boxed{180\ \text{m/s}}$$

***P4.43** (a) At every point in the trajectory, including the top, the acceleration is $\boxed{9.80\text{ m/s}^2\text{ down}}$.

(b) We first find the speed of the ball just before it hits the basket rim.

$$v_{xf}^2 + v_{yf}^2 = v_{xi}^2 + v_{yi}^2 + 2a_y(y_f - y_i)$$

$$v_f^2 = v_i^2 + 2a_y(y_f - y_i) = (10.6\text{ m/s})^2 + 2(-9.8\text{ m/s}^2)(3.05\text{ m} - 0) = 52.6\text{ m}^2/\text{s}^2$$

$v_f = 7.25$ m/s. The ball's rebound speed is $v_{yi} = (7.25\text{ m/s})/2 = 3.63$ m/s

Now take the initial point just after the ball leaves the rim, and the final point at the top of its bounce.

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i): \quad 0 = (3.63\text{ m/s})^2 + 2(-9.8\text{ m/s}^2)(y_f - 3.05\text{ m})$$

$$y_f = -(3.63\text{ m/s})^2/2(-9.8\text{ m/s}^2) + 3.05\text{ m} = \boxed{3.72\text{ m}}$$

***P4.44** (a) Take the positive x axis pointing east. The ball is in free fall between the point just after it leaves the player's hands, and the point just before it bonks the bird. Its horizontal component of velocity remains constant with the value

$$(10.6\text{ m/s})\cos 55° = 6.08\text{ m/s}$$

We need to know the time of flight up to the eagle. We consider the ball's vertical motion:

$$v_{yf} = v_{yi} + a_y t \quad 0 = (10.6\text{ m/s})\sin 55° = (-9.8\text{ m/s}^2)t \quad t = -(8.68\text{ m/s})/(-9.8\text{ m/s}^2) = 0.886\text{ s}$$

The horizontal component of displacement from the player to the bird is

$$x_f = x_i + v_x t = 0 + (6.08\text{ m/s})(0.886\text{ s}) = 5.39\text{ m}$$

The downward flight takes the same time because the ball moves through the same vertical distance with the same range of vertical speeds, including zero vertical speed at one endpoint. The horizontal velocity component of the ball is $-1.5(6.08\text{ m/s}) = -9.12$ m/s. The final horizontal coordinate of the ball is

$$x_f = x_i + v_x t = 5.39\text{ m} + (-9.12\text{ m/s})(0.886\text{ s}) = 5.39\text{ m} - 8.08\text{ m/s} = -2.69\text{ m}$$

The ball lands a distance of $\boxed{2.69\text{ m behind the player}}$.

(b) The angle could be either positive or negative. Here is a conceptual argument: The horizontal bounce sends the ball 2.69 m behind the player. To shorten this distance, the bird wants to reduce the horizontal velocity component of the ball. It can do this either by sending the ball upward or downward relative to the horizontal.

Here is a mathematical argument: The height of the bird is $(1/2)(9.8\text{ m/s}^2)(0.886\text{ s})^2 = 3.85$ m. The ball's flight from the bird to the player is described by the pair of equations

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 \qquad 0 = 3.85\text{ m} + (9.12\text{ m/s})(\sin\theta)\,t + (1/2)(-9.8\text{ m/s}^2)t^2$$

and $x_f = x_i + v_x t$ $\qquad 0 = 5.39\text{ m} + (-9.12\text{ m/s})(\cos\theta)\,t$

Eliminating t by substitution gives a quadratic equation in θ. This equation has two solutions.

P4.45 Refer to the sketch. We find it convenient to solve part (b) first.

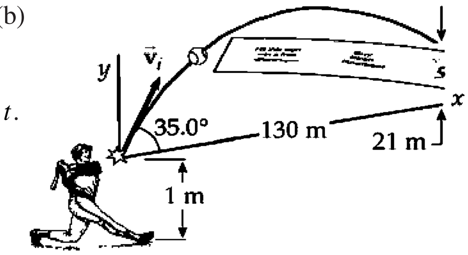

FIG. P4.45

(b) $\Delta x = v_{xi}t$; substitution yields $130 = (v_i \cos 35.0°)t$.

$\Delta y = v_{yi}t + \frac{1}{2}at^2$; substitution yields

$$20.0 = (v_i \sin 35.0°)t + \frac{1}{2}(-9.80)t^2$$

Solving the above by substituting $v_i t = 159$

gives $20 = 91 - 4.9\,t^2$ so $t = \boxed{3.81 \text{ s}}$.

(a) substituting back gives $v_i = \boxed{41.7 \text{ m/s}}$

(c) $v_{yf} = v_i \sin\theta_i - gt$, $v_x = v_i \cos\theta_i$

At $t = 3.81$ s, $v_{yf} = 41.7 \sin 35.0° - (9.80)(3.81) = \boxed{-13.4 \text{ m/s}}$

$$v_x = (41.7 \cos 35.0°) = \boxed{34.1 \text{ m/s}}$$

$$v_f = \sqrt{v_x^2 + v_{yf}^2} = \boxed{36.7 \text{ m/s}}$$

P4.46 At any time t, the two drops have identical y-coordinates. The distance between the two drops is then just twice the magnitude of the horizontal displacement either drop has undergone. Therefore,

$$d = 2|x(t)| = 2(v_{xi}t) = 2(v_i \cos\theta_i)t = \boxed{2v_i t \cos\theta_i}$$

P4.47 (a) $a_c = \frac{v^2}{r} = \frac{(5.00 \text{ m/s})^2}{1.00 \text{ m}} = \boxed{25.0 \text{ m/s}^2}$

$a_t = g = \boxed{9.80 \text{ m/s}^2}$

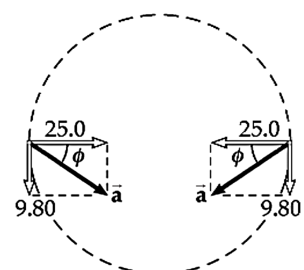

FIG. P4.47

(b) See figure to the right.

(c) $a = \sqrt{a_c^2 + a_t^2} = \sqrt{(25.0 \text{ m/s}^2)^2 + (9.80 \text{ m/s}^2)^2} = \boxed{26.8 \text{ m/s}^2}$

$\phi = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\frac{9.80 \text{ m/s}^2}{25.0 \text{ m/s}^2} = \boxed{21.4°}$

P4.48 (a) The moon's gravitational acceleration is the probe's centripetal acceleration: (For the moon's radius, see end papers of text.)

$$a = \frac{v^2}{r}$$

$$\frac{1}{6}(9.80 \text{ m/s}^2) = \frac{v^2}{1.74 \times 10^6 \text{ m}}$$

$$v = \sqrt{2.84 \times 10^6 \text{ m}^2/\text{s}^2} = \boxed{1.69 \text{ km/s}}$$

(b) $v = \frac{2\pi r}{T}$

$$T = \frac{2\pi r}{v} = \frac{2\pi(1.74 \times 10^6 \text{ m})}{1.69 \times 10^3 \text{ m/s}} = 6.47 \times 10^3 \text{ s} = \boxed{1.80 \text{ h}}$$

***P4.49** (a) We find the x coordinate from $x = 12\,t$. We find the y coordinate from $49\,t - 4.9\,t^2$. Then we find the projectile's distance from the origin as $(x^2 + y^2)^{1/2}$, with these results:

t (s)	0	1	2	3	4	5	6	7	8	9	10
r (m)	0	45.7	82.0	109	127	136	138	133	124	117	120

(b) From the table, it looks like the magnitude of r is largest at a bit less than 6 s. The vector $\vec{\mathbf{v}}$ tells how $\vec{\mathbf{r}}$ is changing. If $\vec{\mathbf{v}}$ at a particular point has a component along $\vec{\mathbf{r}}$, then $\vec{\mathbf{r}}$ will be increasing in magnitude (if $\vec{\mathbf{v}}$ is at an angle less than 90° from $\vec{\mathbf{r}}$) or decreasing (if the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{r}}$ is more than 90°). To be at a maximum, the distance from the origin must be momentarily staying constant, and the only way this can happen is for the angle between velocity and displacement to be a right angle. Then $\vec{\mathbf{r}}$ will be changing in direction at that point, but not in magnitude.

(c) The requirement for perpendicularity can be defined as equality between the tangent of the angle between $\vec{\mathbf{v}}$ and the x direction and the tangent of the angle between $\vec{\mathbf{r}}$ and the y direction. In symbols this is $(9.8t - 49)/12 = 12t/(49t - 4.9t^2)$, which has the solution $t = 5.70$ s, giving in turn $r = 138$ m. Alternatively, we can require $dr^2/dt = 0 = (d/dt)[(12t)^2 + (49t - 4.9t^2)^2]$, which results in the same equation with the same solution.

***P4.50** (a) The time of flight must be positive. It is determined by $y_f = y_i + v_{yi}t - (1/2)a_y t^2$

$0 = 1.2 + v_0 \sin 35°\, t - 4.9t^2$ from the quadratic formula as $t = \dfrac{0.574v_0 + \sqrt{0.329v_0^2 + 23.52}}{9.8}$

Then the range follows from $x = v_{xi}t + 0 = v_0 t$ as

$\boxed{x(v_0) = v_0\sqrt{0.164\ 3 + 0.002\ 299\ v_0^2} + 0.047\ 94\ v_0^2}$ where x is in meters and v_0 is in meters per second.

(b) Substituting $v_0 = 0.1$ gives $x(v_0) = \boxed{0.0410\ \text{m}}$

(c) Substituting $v_0 = 100$ gives $x(v_0) = \boxed{961\ \text{m}}$

(d) When v_0 is small, v_0^2 becomes negligible. The expression $x(v_0)$ simplifies to $v_0\sqrt{0.164\ 3 + 0} + 0 = \boxed{0.405\ v_0}$ Note that this gives nearly the answer to part (b).

(e) When v_0 is large, v_0 is negligible in comparison to v_0^2. Then $x(v_0)$ simplifies to $x(v_0) \approx v_0\sqrt{0 + 0.002\ 299\ v_0^2} + 0.047\ 94\ v_0^2 = \boxed{0.0959\ v_0^2}$ This nearly gives the answer to part (c).

(f) The graph of x versus v_0 starts from the origin as a straight line with slope 0.405 s. Then it curves upward above this tangent line, getting closer and closer to the parabola $x = (0.095\ 9\ \text{s}^2/\text{m})\ v_0^2$

P4.51 The special conditions allowing use of the horizontal range equation applies. For the ball thrown at 45°,

$$D = R_{45} = \frac{v_i^2 \sin 90}{g}$$

For the bouncing ball,

$$D = R_1 + R_2 = \frac{v_i^2 \sin 2\theta}{g} + \frac{(v_i/2)^2 \sin 2\theta}{g}$$

where θ is the angle it makes with the ground when thrown and when bouncing.

(a) We require:

$$\frac{v_i^2}{g} = \frac{v_i^2 \sin 2\theta}{g} + \frac{v_i^2 \sin 2\theta}{4g}$$

$$\sin 2\theta = \frac{4}{5}$$

$$\boxed{\theta = 26.6°}$$

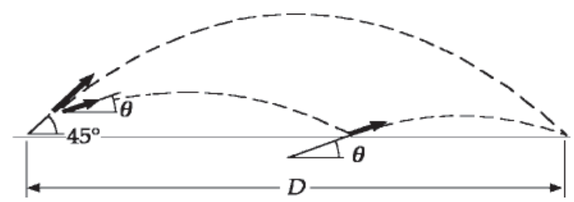

FIG. P4.51

(b) The time for any symmetric parabolic flight is given by

$$y_f = v_{yi}t - \frac{1}{2}gt^2$$

$$0 = v_i \sin\theta_i t - \frac{1}{2}gt^2.$$

If $t = 0$ is the time the ball is thrown, then $t = \frac{2v_i \sin\theta_i}{g}$ is the time at landing.

So for the ball thrown at 45.0°

$$t_{45} = \frac{2v_i \sin 45.0°}{g}$$

For the bouncing ball,

$$t = t_1 + t_2 = \frac{2v_i \sin 26.6°}{g} + \frac{2(v_i/2)\sin 26.6°}{g} = \frac{3v_i \sin 26.6°}{g}$$

The ratio of this time to that for no bounce is

$$\frac{3v_i \sin 26.6°/g}{2v_i \sin 45.0°/g} = \frac{1.34}{1.41} = \boxed{0.949}$$

P4.52 Equation of bank: $y^2 = 16x$ (1)

Equations of motion: $x = v_i t$ (2)

$$y = -\frac{1}{2} g t^2 \quad (3)$$

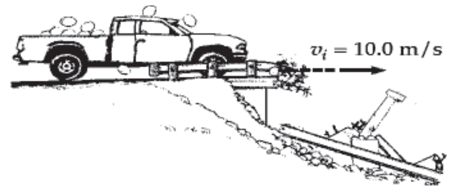

FIG. P4.52

Substitute for t from (2) into (3) $y = -\frac{1}{2} g\left(\frac{x^2}{v_i^2}\right)$. Equate y from the bank equation to y from the equations of motion:

$$16x = \left[-\frac{1}{2} g\left(\frac{x^2}{v_i^2}\right)\right]^2 \Rightarrow \frac{g^2 x^4}{4v_i^4} - 16x = x\left(\frac{g^2 x^3}{4v_i^4} - 16\right) = 0$$

From this, $x = 0$ or $x^3 = \frac{64 v_i^4}{g^2}$ and $x = 4\left(\frac{10^4}{9.80^2}\right)^{1/3} = \boxed{18.8 \text{ m}}$. Also,

$$y = -\frac{1}{2} g\left(\frac{x^2}{v_i^2}\right) = -\frac{1}{2}\frac{(9.80)(18.8)^2}{(10.0)^2} = \boxed{-17.3 \text{ m}}$$

P4.53 (a) $\Delta y = -\frac{1}{2} g t^2$; $\Delta x = v_i t$

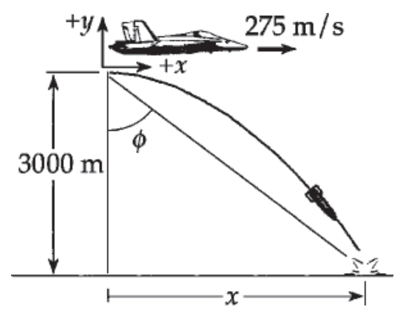

FIG. P4.53

Combine the equations eliminating t:

$$\Delta y = -\frac{1}{2} g\left(\frac{\Delta x}{v_i}\right)^2$$

From this, $(\Delta x)^2 = \left(\frac{-2\Delta y}{g}\right) v_i^2$

thus $\Delta x = v_i \sqrt{\frac{-2\Delta y}{g}} = 275\sqrt{\frac{-2(-3\,000)}{9.80}} = 6.80 \times 10^3 = \boxed{6.80 \text{ km}}$.

(b) The plane has the same velocity as the bomb in the x direction. Therefore, the plane will be $\boxed{\text{3 000 m directly above the bomb}}$ when it hits the ground.

(c) When ϕ is measured from the vertical, $\tan\phi = \frac{\Delta x}{\Delta y}$

therefore, $\phi = \tan^{-1}\left(\frac{\Delta x}{\Delta y}\right) = \tan^{-1}\left(\frac{6\,800}{3\,000}\right) = \boxed{66.2°}$.

P4.54 Measure heights above the level ground. The elevation y_b of the ball follows

$$y_b = R + 0 - \frac{1}{2}gt^2$$

with $x = v_i t$ so $y_b = R - \dfrac{gx^2}{2v_i^2}$.

(a) The elevation y_r of points on the rock is described by

$$y_r^2 + x^2 = R^2$$

We will have $y_b = y_r$ at $x = 0$, but for all other x we require the ball to be above the rock surface as in $y_b > y_r$. Then $y_b^2 + x^2 > R^2$

$$\left(R - \frac{gx^2}{2v_i^2}\right)^2 + x^2 > R^2$$

$$R^2 - \frac{gx^2 R}{v_i^2} + \frac{g^2 x^4}{4v_i^4} + x^2 > R^2$$

$$\frac{g^2 x^4}{4v_i^4} + x^2 > \frac{gx^2 R}{v_i^2}$$

If this inequality is satisfied for x approaching zero, it will be true for all x. If the ball's parabolic trajectory has large enough radius of curvature at the start, the ball will clear the whole rock: $1 > \dfrac{gR}{v_i^2}$

$$\boxed{v_i > \sqrt{gR}}$$

(b) With $v_i = \sqrt{gR}$ and $y_b = 0$, we have $0 = R - \dfrac{gx^2}{2gR}$ or $x = R\sqrt{2}$.

The distance from the rock's base is

$$x - R = \boxed{\left(\sqrt{2} - 1\right)R}$$

P4.55 (a) From Part (c), the raptor dives for $6.34 - 2.00 = 4.34$ s undergoing displacement 197 m downward and $(10.0)(4.34) = 43.4$ m forward.

$$v = \frac{\Delta d}{\Delta t}\frac{\sqrt{(197)^2 + (43.4)^2}}{4.34} = \boxed{46.5 \text{ m/s}}$$

(b) $\alpha = \tan^{-1}\left(\dfrac{-197}{43.4}\right) = \boxed{-77.6°}$

(c) $197 = \dfrac{1}{2}gt^2$, $\boxed{t = 6.34 \text{ s}}$

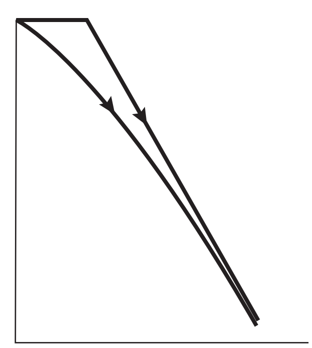

FIG. P4.55

P4.56 (a) Coyote: $\Delta x = \frac{1}{2}at^2$; $70.0 = \frac{1}{2}(15.0)t^2$

Roadrunner: $\Delta x = v_i t$; $70.0 = v_i t$

Solving the above, we get

$$v_i = \boxed{22.9 \text{ m/s}} \text{ and } t = 3.06 \text{ s}$$

(b) At the edge of the cliff,

$$v_{xi} = at = (15.0)(3.06) = 45.8 \text{ m/s}$$

Substituting into $\Delta y = \frac{1}{2}a_y t^2$, we find

$$-100 = \frac{1}{2}(-9.80)t^2$$

$$t = 4.52 \text{ s}$$

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2 = (45.8)(4.52 \text{ s}) + \frac{1}{2}(15.0)(4.52 \text{ s})^2$$

Solving,

$$\Delta x = \boxed{360 \text{ m}}$$

(c) For the Coyote's motion through the air

$$v_{xf} = v_{xi} + a_x t = 45.8 + 15(4.52) = \boxed{114 \text{ m/s}}$$

$$v_{yf} = v_{yi} + a_y t = 0 - 9.80(4.52) = \boxed{-44.3 \text{ m/s}}$$

P4.57 (a) While on the incline

$$v_f^2 - v_i^2 = 2a\Delta x$$

$$v_f - v_i = at$$

$$v_f^2 - 0 = 2(4.00)(50.0)$$

$$20.0 - 0 = 4.00t$$

$$v_f = \boxed{20.0 \text{ m/s}}$$

$$t = \boxed{5.00 \text{ s}}$$

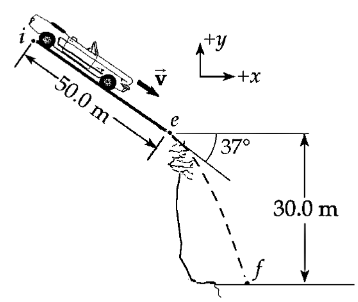

FIG. P4.57

(b) Initial free-flight conditions give us

$$v_{xi} = 20.0\cos 37.0° = 16.0 \text{ m/s}$$

and

$$v_{yi} = -20.0\sin 37.0° = -12.0 \text{ m/s}$$

$v_{xf} = v_{xi}$ since $a_x = 0$

$$v_{yf} = -\sqrt{2a_y\Delta y + v_{yi}^2} = -\sqrt{2(-9.80)(-30.0) + (-12.0)^2} = -27.1 \text{ m/s}$$

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(16.0)^2 + (-27.1)^2} = \boxed{31.5 \text{ m/s at } 59.4° \text{ below the horizontal}}$$

(c) $t_1 = 5$ s; $t_2 = \dfrac{v_{yf} - v_{yi}}{a_y} = \dfrac{-27.1 + 12.0}{-9.80} = 1.53$ s

$$t = t_1 + t_2 = \boxed{6.53 \text{ s}}$$

(d) $\Delta x = v_{xi}t_2 = 16.0(1.53) = \boxed{24.5 \text{ m}}$

P4.58 Think of shaking down the mercury in an old fever thermometer. Swing your hand through a circular arc, quickly reversing direction at the bottom end. Suppose your hand moves through one-quarter of a circle of radius 60 cm in 0.1 s. Its speed is

$$\frac{\frac{1}{4}(2\pi)(0.6\text{ m})}{0.1\text{ s}} \approx 9\text{ m/s}$$

and its centripetal acceleration is $\frac{v^2}{r} \approx \frac{(9\text{ m/s})^2}{0.6\text{ m}} \boxed{\sim 10^2\text{ m/s}^2}$.

The tangential acceleration of stopping and reversing the motion will make the total acceleration somewhat larger, but will not affect its order of magnitude.

P4.59 (a) $\Delta x = v_{xi}t$, $\Delta y = v_{yi}t + \frac{1}{2}gt^2$

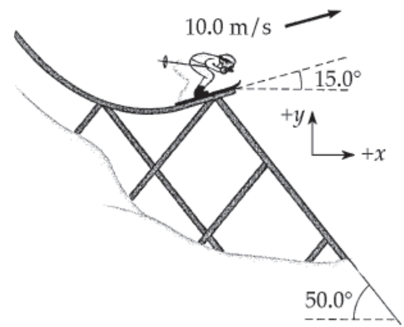

FIG. P4.59

$$d\cos 50.0° = (10.0\cos 15.0°)t$$

and

$$-d\sin 50.0° = (10.0\sin 15.0°)t + \frac{1}{2}(-9.80)t^2$$

Solving, $d = \boxed{43.2\text{ m}}$ and $t = 2.88$ s.

(b) Since $a_x = 0$,

$$v_{xf} = v_{xi} = 10.0\cos 15.0° = \boxed{9.66\text{ m/s}}$$

$$v_{yf} = v_{yi} + a_y t = 10.0\sin 15.0° - 9.80(2.88) = \boxed{-25.6\text{ m/s}}$$

Air resistance would ordinarily decrease the values of the range and landing speed. As an airfoil, he can deflect air downward so that the air deflects him upward. This means he can get some lift and increase his distance.

P4.60 (a) The ice chest floats downstream 2 km in time t, so that 2 km $= v_w t$. The upstream motion of the boat is described by $d = (v - v_w)15$ min. The downstream motion is described by $d + 2\text{ km} = (v + v_w)(t - 15\text{ min})$. We eliminate $t = \frac{2\text{ km}}{v_w}$ and d by substitution:

$$(v - v_w)15\text{ min} + 2\text{ km} = (v + v_w)\left(\frac{2\text{ km}}{v_w} - 15\text{ min}\right)$$

$$v(15\text{ min}) - v_w(15\text{ min}) + 2\text{ km} = \frac{v}{v_w}2\text{ km} + 2\text{ km} - v(15\text{ min}) - v_w(15\text{ min})$$

$$v(30\text{ min}) = \frac{v}{v_w}\ 2\text{ km}$$

$$v_w = \frac{2\text{ km}}{30\text{ min}} = \boxed{4.00\text{ km/h}}$$

(b) In the reference frame of the water, the chest is motionless. The boat travels upstream for 15 min at speed v, and then downstream at the same speed, to return to the same point. Thus it travels for 30 min. During this time, the falls approach the chest at speed v_w, traveling 2 km. Thus

$$v_w = \frac{\Delta x}{\Delta t} = \frac{2\text{ km}}{30\text{ min}} = \boxed{4.00\text{ km/h}}$$

P4.61 Find the highest firing angle θ_H for which the projectile will clear the mountain peak; this will yield the range of the closest point of bombardment. Next find the lowest firing angle; this will yield the maximum range under these conditions if both θ_H and θ_L are >45°; $x = 2500$ m, $y = 1800$ m, $v_i = 250$ m/s.

$$y_f = v_{yi}t - \frac{1}{2}gt^2 = v_i(\sin\theta)t - \frac{1}{2}gt^2$$
$$x_f = v_{xi}t = v_i(\cos\theta)t$$

Thus

$$t = \frac{x_f}{v_i\cos\theta}.$$

Substitute into the expression for y_f

$$y_f = v_i(\sin\theta)\frac{x_f}{v_i\cos\theta} - \frac{1}{2}g\left(\frac{x_f}{v_i\cos\theta}\right)^2 = x_f\tan\theta - \frac{gx_f^2}{2v_i^2\cos^2\theta}$$

but $\dfrac{1}{\cos^2\theta} = \tan^2\theta + 1$ so $y_f = x_f\tan\theta - \dfrac{gx_f^2}{2v_i^2}\left(\tan^2\theta + 1\right)$ and

$$0 = \frac{gx_f^2}{2v_i^2}\tan^2\theta - x_f\tan\theta + \frac{gx_f^2}{2v_i^2} + y_f$$

Substitute values, use the quadratic formula and find

$$\tan\theta = 3.905 \text{ or } 1.197, \text{ which gives } \theta_H = 75.6° \text{ and } \theta_L = 50.1°$$

$$\text{Range } (\text{at } \theta_H) = \frac{v_i^2\sin 2\theta_H}{g} = 3.07\times10^3 \text{ m from enemy ship}$$

$$3.07\times10^3 - 2\,500 - 300 = 270 \text{ m from shore}$$

$$\text{Range } (\text{at } \theta_L) = \frac{v_i^2\sin 2\theta_L}{g} = 6.28\times10^3 \text{ m from enemy ship}$$

$$6.28\times10^3 - 2\,500 - 300 = 3.48\times10^3 \text{ from shore}$$

Therefore, safe distance is $\boxed{< 270 \text{ m}}$ or $\boxed{> 3.48\times10^3 \text{ m}}$ from the shore.

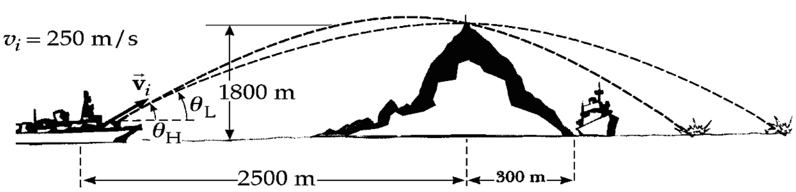

FIG. P4.61

P4.62 We follow the steps outlined in Example 4.7, eliminating $t = \dfrac{d\cos\phi}{v_i\cos\theta}$ to find

$$\frac{v_i\sin\theta\, d\cos\phi}{v_i\cos\theta} - \frac{gd^2\cos^2\phi}{2v_i^2\cos^2\theta} = -d\sin\phi$$

Clearing of fractions,

$$2v_i^2\cos\theta\sin\theta\cos\phi - gd\cos^2\phi = -2v_i^2\cos^2\theta\sin\phi$$

To maximize d as a function of θ, we differentiate through with respect to θ and set $\dfrac{dd}{d\theta} = 0$:

$$2v_i^2\cos\theta\cos\theta\cos\phi + 2v_i^2\sin\theta(-\sin\theta)\cos\phi - g\frac{dd}{d\theta}\cos^2\phi = -2v_i^2 2\cos\theta(-\sin\theta)\sin\phi$$

We use the trigonometric identities from Appendix B4 $\cos 2\theta = \cos^2\theta - \sin^2\theta$ and

$\sin 2\theta = 2\sin\theta\cos\theta$ to find $\cos\phi\cos 2\theta = \sin 2\theta\sin\phi$. Next, $\dfrac{\sin\phi}{\cos\phi} = \tan\phi$ and $\cot 2\theta = \dfrac{1}{\tan 2\theta}$

give $\cot 2\phi = \tan\phi = \tan(90° - 2\theta)$ so $\phi = 90° - 2\theta$ and $\theta = 45° - \dfrac{\phi}{2}$.

ANSWERS TO EVEN PROBLEMS

P4.2 (a) $\vec{\mathbf{r}} = 18.0t\hat{\mathbf{i}} + (4.00t - 4.90t^2)\hat{\mathbf{j}}$ (b) $\vec{\mathbf{v}} = 18.0\,\hat{\mathbf{i}} + (4.00 - 9.80t)\,\hat{\mathbf{j}}$ (c) $\vec{\mathbf{a}} = (-9.80 \text{ m/s}^2)\,\hat{\mathbf{j}}$

(d) $(54.0 \text{ m})\hat{\mathbf{i}} - (32.1 \text{ m})\hat{\mathbf{j}}$ (e) $(18.0 \text{ m/s})\hat{\mathbf{i}} - (25.4 \text{ m/s})\hat{\mathbf{j}}$ (f) $(-9.80 \text{ m/s}^2)\hat{\mathbf{j}}$

P4.4 (a) $\vec{\mathbf{v}} = (-5.00\omega\,\hat{\mathbf{i}} + 0\hat{\mathbf{j}})$ m/s; $\vec{\mathbf{a}} = (0\hat{\mathbf{i}} + 5.00\omega^2\hat{\mathbf{j}})$ m/s^2

(b) $\vec{\mathbf{r}} = 4.00 \text{ m}\,\hat{\mathbf{j}} + 5.00 \text{ m}(-\sin\omega t\,\hat{\mathbf{i}} - \cos\omega t\,\hat{\mathbf{j}})$; $\vec{\mathbf{v}} = 5.00 \text{ m}\,\omega(-\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}})$;
$\vec{\mathbf{a}} = 5.00 \text{ m}\,\omega^2(\sin\omega t\,\hat{\mathbf{i}} + \cos\omega t\,\hat{\mathbf{j}})$

(c) a circle of radius 5.00 m centered at $(0, 4.00 \text{ m})$

P4.6 (a) $\vec{\mathbf{v}} = -12.0t\,\hat{\mathbf{j}}$ m/s; $\vec{\mathbf{a}} = -12.0\hat{\mathbf{j}}$ m/s^2 (b) $\vec{\mathbf{r}} = (3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}})$ m; $\vec{\mathbf{v}} = -12.0\hat{\mathbf{j}}$ m/s

P4.8 (a) $\vec{\mathbf{r}} = (5.00t\hat{\mathbf{i}} + 1.50t^2\hat{\mathbf{j}})$ m; $\vec{\mathbf{v}} = (5.00\hat{\mathbf{i}} + 3.00t\hat{\mathbf{j}})$ m/s (b) $\vec{\mathbf{r}} = (10.0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}})$ m; 7.81 m/s

P4.10 (a) $d\sqrt{\dfrac{g}{2h}}$ horizontally (b) $\tan^{-1}\left(\dfrac{2h}{d}\right)$ below the horizontal

P4.12 (a) 76.0° (b) the same on every planet. Mathematically, this is because the acceleration of gravity divides out of the answer. (c) $\dfrac{17d}{8}$

P4.14 $d\tan\theta_i - \dfrac{gd^2}{(2v_i^2\cos^2\theta_i)}$

P4.16 (a) Yes. (b) $(1.70 \text{ m/s})/\sqrt{12} = 0.491$ m/s

P4.18 33.5° below the horizontal

P4.20 (a) 0.852 s; (b) 3.29 m/s; (c) 4.03 m/s;
(d) 50.8°; (e) 1.12 s

P4.22 (a) $\vec{\mathbf{r}}_i = 0\,\hat{\mathbf{i}} + 0.840 \text{ m}\,\hat{\mathbf{j}}$ (b) 11.2 m/s at 18.5° (c) 8.94 m (d) The free-fall trajectory of the athlete is a section around the vertex of a parabola opening downward, everywhere close to horizontal and 48 cm lower on the landing side than on the takeoff side.

P4.24 0.033 7 m/s^2 toward the center of the Earth

P4.26 0.281 rev/s

P4.28 (a) Yes. The particle can be either speeding up or slowing down, with a tangential component of acceleration of magnitude $\sqrt{6^2 - 4.5^2} = 3.97 \text{ m/s}^2$. (b) No. The magnitude of the acceleration cannot be less than $v^2/r = 4.5$ m/s^2.

P4.30 (a) see the solution (b) 29.7 m/s^2 (c) 6.67 m/s at 36.9° above the horizontal

P4.32 4.55°

P4.34 (a) 26.9 m/s (b) 67.3 m (c) $(2.00\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}})$ m/s^2

P4.36 18.0 s

P4.38 (a) 0° (b) 8.25 m/s (c) The can traverses a straight line segment upward and then downward (d) A symmetric section of a parabola opening downward; 12.6 m/s north at 41.0° above the horizontal.

P4.40 (a) 10.1 m/s^2 at 14.3° south from the vertical (b) 9.80 m/s^2 vertically downward (c) The bolt moves on a parabola with its axis downward and tilting to the south. It lands south of the point directly below its starting point. (d) The bolt moves on a parabola with a vertical axis.

P4.42 (a) 101 m/s (b) $3.27\times 10^4 \text{ ft}$ (c) 20.6 s (d) 180 m/s

P4.44 (a) 2.69 m (b) The angle could be either positive or negative. The horizontal bounce sends the ball 2.69 m behind the player. To shorten this distance, the bird wants to reduce the horizontal velocity component of the ball. It can do this either by sending the ball upward or downward relative to the horizontal.

P4.46 $2v_i t\cos\theta_i$

P4.48 (a) 1.69 km/s; (b) $6.47\times 10^3 \text{ s}$

P4.50 (a) $x = v_0(0.1643 + 0.002\,299\, v_0^2)^{1/2} + 0.047\,98\; v_0^2$ where x is in meters and v_0 is in meters per second, (b) 0.410 m (c) 961 m (d) $x \approx 0.405\, v_0$ (e) $x \approx 0.095\,9\, v_0^2$ (f) The graph of x versus v_0 starts from the origin as a straight line with slope 0.405 s. Then it curves upward above this tangent line, getting closer and closer to the parabola $x = (0.095\,9 \text{ s}^2/\text{m})\, v_0^2$.

P4.52 $(18.8 \text{ m};\ -17.3 \text{ m})$

P4.54 (a) $\sqrt{gR}$; (b) $\left(\sqrt{2}-1\right)R$

P4.56 (a) 22.9 m/s (b) 360 m from the base of the cliff (c) $\vec{\mathbf{v}} = (114\ \hat{\mathbf{i}} - 44.3\,\hat{\mathbf{j}})$ m/s

P4.58 Imagine you have a sick child and are shaking down the mercury in an old fever thermometer. Starting with your hand at the level of your shoulder, move your hand down as fast as you can and snap it around an arc at the bottom. $\sim 10^2 \text{ m/s}^2 \sim 10\, g$

P4.60 4.00 km/h

P4.62 see the solution

5

The Laws of Motion

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q5.1 (a) The force due to gravity of the earth pulling down on the ball—the reaction force is the force due to gravity of the ball pulling up on the earth. The force of the hand pushing up on the ball—reaction force is ball pushing down on the hand.

(b) The only force acting on the ball in free-fall is the gravity due to the earth—the reaction force is the gravity due to the ball pulling on the earth.

Q5.2 The resultant force is zero, as the acceleration is zero.

***Q5.3** Answer (b). An air track or air table is a wonderful thing. It exactly cancels out the force of the Earth's gravity on the gliding object, to display free motion and to imitate the effect of being far away in space.

Q5.4 When the bus starts moving, the mass of Claudette is accelerated by the force of the back of the seat on her body. Clark is standing, however, and the only force on him is the friction between his shoes and the floor of the bus. Thus, when the bus starts moving, his feet start accelerating forward, but the rest of his body experiences almost no accelerating force (only that due to his being attached to his accelerating feet!). As a consequence, his body tends to stay almost at rest, according to Newton's first law, relative to the ground. Relative to Claudette, however, he is moving toward her and falls into her lap. Both performers won Academy Awards.

***Q5.5** Shake your hands. In particular, move one hand down fast and then stop your hand abruptly. The water drops keep moving down, according to Newton's first law, and leave your hand. This method is particularly effective for fat, large-mass, high-inertia water drops.

Q5.6 First ask, "Was the bus moving forward or backing up?" If it was moving forward, the passenger is lying. A fast stop would make the suitcase fly toward the front of the bus, not toward the rear. If the bus was backing up at any reasonable speed, a sudden stop could not make a suitcase fly far. Fine her for malicious litigiousness.

***Q5.7** (a) The air inside pushes outward on each patch of rubber, exerting a force perpendicular to that section of area. The air outside pushes perpendicularly inward, but not quite so strongly. (b) As the balloon takes off, all of the sections of rubber feel essentially the same outward forces as before, but the now-open hole at the opening on the west side feels no force. The vector sum of the forces on the rubber is to the east. The small-mass balloon moves east with a large acceleration. (c) Hot combustion products in the combustion chamber push outward on all the walls of the chamber, but there is nothing for them to push on at the open rocket nozzle. The net force exerted by the gases on the chamber is up if the nozzle is pointing down. This force is larger than the gravitational force on the rocket body, and makes it accelerate upward.

***Q5.8** A portion of each leaf of grass extends above the metal bar. This portion must accelerate in order for the leaf to bend out of the way. The leaf's mass is small, but when its acceleration is very large, the force exerted by the bar on the leaf puts the leaf under tension large enough to shear it off.

Q5.9 The molecules of the floor resist the ball on impact and push the ball back, upward. The actual force acting is due to the forces between molecules that allow the floor to keep its integrity and to prevent the ball from passing through. Notice that for a ball passing through a window, the molecular forces weren't strong enough.

***Q5.10** The child exerts an upward force on the ball while it is in her hand, but the question is about the ball moving up after it leaves her hand. At those moments, her hand exerts no force on the ball, just as you exert no force on the bathroom scale when you are standing in the shower.
(a) If there were a force greater than the weight of the ball, the ball would accelerate upward, not downward. (b) If there were a force equal to the weight of the ball, the ball would move at constant velocity, but really it slows down as it moves up. (c) The "force of the throw" can be described as zero, because it shows up as zero on a force sensor stuck to her palm. (d) The ball moves up at any one moment because it was moving up the previous moment. A limited downward acceleration acting over a short time has not taken away all of its upward velocity. We could say it moves up because of 'history' or 'pigheadedness' or 'inertia.'

***Q5.11** Since they are on the order of a thousand times denser than the surrounding air, we assume the snowballs are in free fall. The net force on each is the gravitational force exerted by the Earth, which does not depend on their speed or direction of motion but only on the snowball mass. Thus we can rank the missiles just by mass: $d > a = e > b > c$.

Q5.12 It is impossible to string a horizontal cable without its sagging a bit. Since the cable has a mass, gravity pulls it downward. A vertical component of the tension must balance the weight for the cable to be in equilibrium. If the cable were completely horizontal, then there would be no vertical component of the tension to balance the weight.

Some physics teachers demonstrate this by asking a beefy student to pull on the ends of a cord supporting a can of soup at its center. Some get two burly young men to pull on opposite ends of a strong rope, while the smallest person in class gleefully mashes the center of the rope down to the table. Point out the beauty of sagging suspension-bridge cables. With a laser and an optical lever, demonstrate that the mayor makes the courtroom table sag when he sits on it, and the judge bends the bench. Give them "I make the floor sag" buttons, available to instructors who use this manual and whose classes use the textbook. Estimate the cost of an infinitely strong cable, and the truth will always win.

***Q5.13** The clever boy bends his knees to lower his body, then starts to straighten his knees to push his body up—that is when the branch breaks. When his legs are giving his body upward acceleration, the branch is exerting on him a force greater than his weight. He is just then exerting on the branch an equal-size downward force greater than his weight.

***Q5.14** Yes. The table bends down more to exert a larger upward force. The deformation is easy to see for a block of foam plastic. The sag of a table can be displayed with, for example, an optical lever.

Q5.15 As the barbell goes through the bottom of a cycle, the lifter exerts an upward force on it, and the scale reads the larger upward force that the floor exerts on them together. Around the top of the weight's motion, the scale reads less than average. If the iron is moving upward, the lifter can declare that she has thrown it, just by letting go of it for a moment, so our answer applies also to this case.

***Q5.16** (a) Yes, as exerted by a vertical wall on a ladder leaning against it. (b) Yes, as exerted by a hammer driving a tent stake into the ground. (c) Yes, as the ball accelerates upward in bouncing from the floor. (d) No; the two forces describe the same interaction.

Q5.17 As the sand leaks out, the acceleration increases. With the same driving force, a decrease in the mass causes an increase in the acceleration.

***Q5.18** (a) larger: the tension in A must accelerate two blocks and not just one. (b) equal. Whenever A moves by 1 cm, B moves by 1 cm. The two blocks have equal speeds at every instant and have equal accelerations. (c) yes, backward, equal. The force of cord B on block 1 is the tension in the cord.

Q5.19 As a man takes a step, the action is the force his foot exerts on the Earth; the reaction is the force of the Earth on his foot. In the second case, the action is the force exerted on the girl's back by the snowball; the reaction is the force exerted on the snowball by the girl's back. The third action is the force of the glove on the ball; the reaction is the force of the ball on the glove. The fourth action is the force exerted on the window by the air molecules; the reaction is the force on the air molecules exerted by the window. We could in each case interchange the terms 'action' and 'reaction.'

***Q5.20** (a) Smaller. Block 2 is not in free fall, but pulled backward by string tension. (b) The same. Whenever one block moves by 1 cm, the other block moves by 1 cm. The blocks have equal speeds at every instant and have equal accelerations. (c) The same. The light string exerts forces equal in magnitude on both blocks—the tension in the string.

Q5.21 The tension in the rope when pulling the car is twice that in the tug-of-war. One could consider the car as behaving like another team of twenty more people.

***Q5.22** (b) Newton's 3rd law describes all objects, breaking or whole. The force that the locomotive exerted on the wall is the same as that exerted by the wall on the locomotive. The framing around the wall could not exert so strong a force on the section of the wall that broke out.

Q5.23 The sack of sand moves up with the athlete, regardless of how quickly the athlete climbs. Since the athlete and the sack of sand have the same weight, the acceleration of the system must be zero.

***Q5.24** (i) b. In this case the compressional force on the bug's back only has to be large enough to accelerate the smaller block. (ii) d. and (iii) d. These two forces are equal, as described by Newton's third law.

Q5.25 An object cannot exert a force on itself. If it could, then objects would be able to accelerate themselves, without interacting with the environment. You cannot lift yourself by tugging on your bootstraps.

***Q5.26** answer (b) 200 N must be greater than the force of friction for the box's acceleration to be forward.

***Q5.27** Static friction exerted by the road is the force making the car accelerate forward. Burning gasoline can provide energy for the motion, but only external forces—forces exerted by objects outside—can accelerate the car. If the road surface were icy, the engine would make the tires spin. The rubber contacting the ice would be moving toward the rear of the car. When the road is not icy, static friction opposes this relative sliding motion by exerting a force on the rubber toward the front of the car. If the car is under control and not skidding, the relative speed is zero along the lines where the rubber meets the road, and static friction acts rather than kinetic friction.

***Q5.28** (i) answer d. The stopping distance will be the same if the mass of the truck is doubled. The normal force and the frictional force both double, so the backward acceleration remains the same as without the load. (ii) answer g. The stopping distance will decrease by a factor of four if the initial speed is cut in half.

***Q5.29** Answer (d). Formulas a, b, f, and g have the wrong units for speed. Formula c would give an imaginary answer. Formula e would imply that a more slippery table, with smaller μ, would require a larger original speed, when really it would require a smaller original speed.

***Q5.30** Answer (e). All the other possibilities would make the total force on the crate be different from zero.

Q5.31 If you slam on the brakes, your tires will skid on the road. The force of kinetic friction between the tires and the road is less than the maximum static friction force. Anti-lock brakes work by "pumping" the brakes (much more rapidly that you can) to minimize skidding of the tires on the road.

Q5.32 As you pull away from a stoplight, friction is the force that accelerates forward a box of tissues on the level floor of the car. At the same time, friction exerted by the ground on the tires of the car accelerates the car forward. When you take a step forward, friction exerted by the floor on your shoes causes your acceleration.

***Q5.33** (a) B (b) B (c) B Note that the mass of the woman is more than one-half that of the man. A free-body diagram of the pulley is the best guide for explanation. (d) A.

SOLUTIONS TO PROBLEMS

Section 5.1 **The Concept of Force**

Section 5.2 **Newton's First Law and Inertial Frames**

Section 5.3 **Mass**

Section 5.4 **Newton's Second Law**

Section 5.5 **The Gravitational Force and Weight**

Section 5.6 **Newton's Third Law**

P5.1 $m = 3.00$ kg

$$\vec{\mathbf{a}} = \left(2.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\right) \text{ m/s}^2$$

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} = \boxed{\left(6.00\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}\right) \text{ N}}$$

$$\left|\sum \vec{\mathbf{F}}\right| = \sqrt{(6.00)^2 + (15.0)^2} \text{ N} = \boxed{16.2 \text{ N}}$$

P5.2 For the same force F, acting on different masses

$$F = m_1 a_1$$

and

$$F = m_2 a_2$$

(a) $\dfrac{m_1}{m_2} = \dfrac{a_2}{a_1} = \boxed{\dfrac{1}{3}}$

(b) $F = \left(m_1 + m_2\right)a = 4m_1 a = m_1\left(3.00 \text{ m/s}^2\right)$

$a = \boxed{0.750 \text{ m/s}^2}$

P5.3 $m = 4.00\text{ kg},\ \vec{\mathbf{v}}_i = 3.00\hat{\mathbf{i}}\text{ m/s},\ \vec{\mathbf{v}}_8 = \left(8.00\hat{\mathbf{i}} + 10.0\hat{\mathbf{j}}\right)\text{ m/s},\ t = 8.00\text{ s}$

$$\vec{\mathbf{a}} = \frac{\Delta\vec{\mathbf{v}}}{t} = \frac{5.00\hat{\mathbf{i}} + 10.0\hat{\mathbf{j}}}{8.00}\text{ m/s}^2$$

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} = \boxed{\left(2.50\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\right)\text{ N}}$$

$$F = \sqrt{(2.50)^2 + (5.00)^2} = \boxed{5.59\text{ N}}$$

P5.4 (a) Let the x axis be in the original direction of the molecule's motion.

$$v_f = v_i + at:\quad -670\text{ m/s} = 670\text{ m/s} + a\left(3.00 \times 10^{-13}\text{ s}\right)$$

$$a = \boxed{-4.47 \times 10^{15}\text{ m/s}^2}$$

(b) For the molecule, $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$. Its weight is negligible.

$$\vec{\mathbf{F}}_{\text{wall on molecule}} = 4.68 \times 10^{-26}\text{ kg}\left(-4.47 \times 10^{15}\text{ m/s}^2\right) = -2.09 \times 10^{-10}\text{ N}$$

$$\vec{\mathbf{F}}_{\text{molecule on wall}} = \boxed{+2.09 \times 10^{-10}\text{ N}}$$

P5.5 (a) $\sum F = ma$ and $v_f^2 = v_i^2 + 2ax_f$ or $a = \dfrac{v_f^2 - v_i^2}{2x_f}$

Therefore,

$$\sum F = m\frac{\left(v_f^2 - v_i^2\right)}{2x_f}$$

$$\sum F = 9.11 \times 10^{-31}\text{ kg}\frac{\left[\left(7.00 \times 10^5\text{ m/s}^2\right)^2 - \left(3.00 \times 10^5\text{ m/s}^2\right)^2\right]}{2(0.050\,0\text{ m})} = \boxed{3.64 \times 10^{-18}\text{ N}}$$

(b) The gravitational force exerted by the Earth on the electron is its weight,

$$F_g = mg = \left(9.11 \times 10^{-31}\text{ kg}\right)\left(9.80\text{ m/s}^2\right) = 8.93 \times 10^{-30}\text{ N}$$

The accelerating force is $\boxed{4.08 + 10^{11}\text{ times the weight of the electron}}$.

P5.6 (a) $F_g = mg = 120\text{ lb} = (4.448\text{ N/lb})(120\text{ lb}) = \boxed{534\text{ N down}}$

(b) $m = \dfrac{F_g}{g} = \dfrac{534\text{ N}}{9.80\text{ m/s}^2} = \boxed{54.5\text{ kg}}$

P5.7 Imagine a quick trip by jet, on which you do not visit the rest room and your perspiration is just canceled out by a glass of tomato juice. By subtraction, $\left(F_g\right)_p = mg_p$ and $\left(F_g\right)_C = mg_C$ give

$$\Delta F_g = m\left(g_p - g_C\right)$$

For a person whose mass is 88.7 kg, the change in weight is

$$\Delta F_g = 88.7\text{ kg}(9.809\,5 - 9.780\,8) = \boxed{2.55\text{ N}}$$

A precise balance scale, as in a doctor's office, reads the same in different locations because it compares you with the standard masses on its beams. A typical bathroom scale is not precise enough to reveal this difference.

P5.8 We find acceleration:

$$\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i = \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$$

$$4.20\text{ m}\hat{\mathbf{i}} - 3.30\text{ m}\hat{\mathbf{j}} = 0 + \frac{1}{2}\vec{\mathbf{a}}(1.20\text{ s})^2 = 0.720\text{ s}^2\vec{\mathbf{a}}$$

$$\vec{\mathbf{a}} = \left(5.83\hat{\mathbf{i}} - 4.58\hat{\mathbf{j}}\right)\text{ m/s}^2$$

Now $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$ becomes

$$\vec{\mathbf{F}}_g + \vec{\mathbf{F}}_2 = m\vec{\mathbf{a}}$$

$$\vec{\mathbf{F}}_2 = 2.80\text{ kg}\left(5.83\hat{\mathbf{i}} - 4.58\hat{\mathbf{j}}\right)\text{ m/s}^2 + (2.80\text{ kg})\left(9.80\text{ m/s}^2\right)\hat{\mathbf{j}}$$

$$\vec{\mathbf{F}}_2 = \boxed{\left(16.3\hat{\mathbf{i}} + 14.6\hat{\mathbf{j}}\right)\text{ N}}$$

P5.9 (a) $\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \left(20.0\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}\right)\text{ N}$

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}: \quad 20.0\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}} = 5.00\vec{\mathbf{a}}$$

$$\vec{\mathbf{a}} = \left(4.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\text{ m/s}^2$$

or

$$\boxed{a = 5.00\text{ m/s}^2 \text{ at } \theta = 36.9°}$$

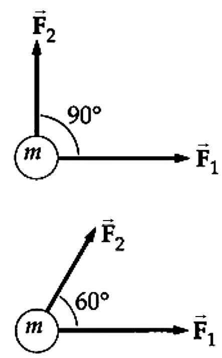

FIG. P5.9

(b) $F_{2x} = 15.0\cos 60.0° = 7.50\text{ N}$

$F_{2y} = 15.0\sin 60.0° = 13.0\text{ N}$

$$\vec{\mathbf{F}}_2 = \left(7.50\hat{\mathbf{i}} + 13.0\hat{\mathbf{j}}\right)\text{ N}$$

$$\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \left(27.5\hat{\mathbf{i}} + 13.0\hat{\mathbf{j}}\right)\text{ N} = m\vec{\mathbf{a}} = 5.00\vec{\mathbf{a}}$$

$$\vec{\mathbf{a}} = \boxed{\left(5.50\hat{\mathbf{i}} + 2.60\hat{\mathbf{j}}\right)\text{ m/s}^2 = 6.08\text{ m/s}^2 \text{ at } 25.3°}$$

***P5.10** (a) force exerted by spring on hand, to the left; force exerted by spring on wall, to the right. (b) force exerted by wagon on handle, downward to the left. Force exerted by wagon on planet, upward. Force exerted by wagon on ground, downward. (c) Force exerted by football on player, downward to the right. Force exerted by football on planet, upward. (d) Force exerted by small-mass object on large-mass object, to the left. (e) Force exerted by negative charge on positive charge, to the left. (f) Force exerted by iron on magnet, to the left.

P5.11 (a) You and the earth exert equal forces on each other: $m_y g = M_e a_e$. If your mass is 70.0 kg,

$$a_e = \frac{(70.0\text{ kg})\left(9.80\text{ m/s}^2\right)}{5.98 \times 10^{24}\text{ kg}} = \boxed{\sim 10^{-22}\text{ m/s}^2}$$

(b) You and the planet move for equal time intervals according to $x = \frac{1}{2}at^2$. If the seat is 50.0 cm high,

$$\sqrt{\frac{2x_y}{a_y}} = \sqrt{\frac{2x_e}{a_e}}$$

$$x_e = \frac{a_e}{a_y}x_y = \frac{m_y}{m_e}x_y = \frac{70.0\text{ kg}(0.500\text{ m})}{5.98 \times 10^{24}\text{ kg}}\boxed{\sim 10^{-23}\text{ m}}$$

***P5.12** The free-body diagrams (a) and (b) are included in the following diagram. The action–reaction pairs (c) are shown joined by the dashed lines.

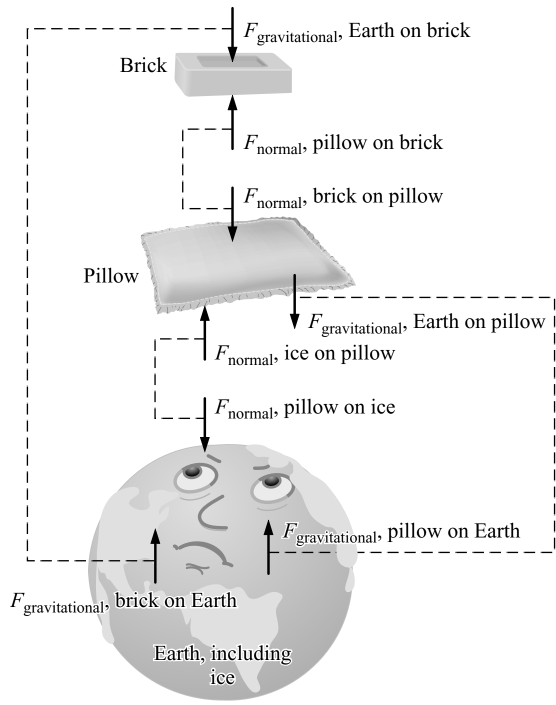

FIG. P5.12

P5.13 (a) $\boxed{15.0 \text{ lb up}}$ to counterbalance the Earth's force on the block

(b) $\boxed{5.00 \text{ lb up}}$ The forces on the block are now the Earth pulling down with 15 lb and the rope pulling up with 10 lb.

(c) $\boxed{0}$ The block now accelerates up away from the floor.

P5.14 $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$ reads

$$\left(-2.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} + 5.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}} - 45.0\hat{\mathbf{i}}\right) \text{ N} = m\left(3.75 \text{ m/s}^2\right)\hat{\mathbf{a}}$$

where $\hat{\mathbf{a}}$ represents the direction of $\vec{\mathbf{a}}$

$$\left(-42.0\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right) \text{ N} = m\left(3.75 \text{ m/s}^2\right)\hat{\mathbf{a}}$$

$$\sum \vec{\mathbf{F}} = \sqrt{(42.0)^2 + (1.00)^2} \text{ N at } \tan^{-1}\left(\frac{1.00}{42.0}\right) \text{ below the } -x \text{ axis}$$

$$\sum \vec{\mathbf{F}} = 42.0 \text{ N at } 181° = m\left(3.75 \text{ m/s}^2\right)\hat{\mathbf{a}}$$

For the vectors to be equal, their magnitudes and their directions must be equal.

(a) Therefore $\boxed{\hat{\mathbf{a}} \text{ is at } 181°}$ counterclockwise from the x axis

(b) $m = \dfrac{42.0 \text{ N}}{3.75 \text{ m/s}^2} = \boxed{11.2 \text{ kg}}$

(d) $\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = 0 + \left(3.75 \text{ m/s}^2 \text{ at } 181°\right)10.0 \text{ s}$ so $\vec{\mathbf{v}}_f = 37.5 \text{ m/s at } 181°$

$\vec{\mathbf{v}}_f = 37.5 \text{ m/s } \cos 181°\hat{\mathbf{i}} + 37.5 \text{ m/s } \sin 181°\hat{\mathbf{j}}$ so $\vec{\mathbf{v}}_f = \boxed{\left(-37.5\hat{\mathbf{i}} - 0.893\hat{\mathbf{j}}\right) \text{ m/s}}$

(c) $\left|\vec{\mathbf{v}}_f\right| = \sqrt{37.5^2 + 0.893^2} \text{ m/s} = \boxed{37.5 \text{ m/s}}$

Section 5.7 **Some Applications of Newton's Laws**

***P5.15** As the worker through the pole exerts on the lake bottom a force of 240 N downward at 35° behind the vertical, the lake bottom through the pole exerts a force of 240 N upward at 35° ahead of the vertical. With the *x* axis horizontally forward, the pole force on the boat is

$$240\text{ N}\cos 35°\hat{\mathbf{j}} + 240\text{ N}\sin 35°\hat{\mathbf{i}} = 138\text{ N}\hat{\mathbf{i}} + 197\text{ N}\hat{\mathbf{j}}$$

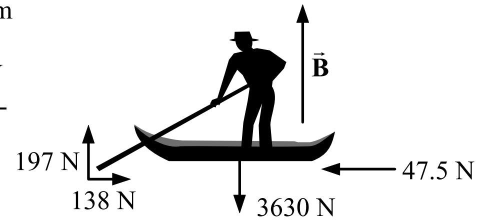

FIG. P5.15

The gravitational force of the whole Earth on boat and worker is $F_g = mg = 370\text{ kg}(9.8\text{ m/s}^2) = 3630\text{ N}$ down. The acceleration of the boat is purely horizontal, so $\sum F_y = ma_y$ gives $+B + 197\text{ N} - 3630\text{ N} = 0$.

(a) The buoyant force is $B = \boxed{3.43\times10^3\text{ N}}$.

(b) The acceleration is given by $\sum F_x = ma_x$: $+138\text{ N} - 47.5\text{ N} = (370\text{ kg})a$;
$a = \dfrac{90.2\text{ N}}{370\text{ kg}} = 0.244\text{ m/s}^2$. According to the constant-acceleration model,

$$v_{xf} = v_{xi} + a_x t = 0.857\text{ m/s} + (0.244\text{ m/s}^2)(0.450\text{ s}) = 0.967\text{ m/s}$$

$$\vec{\mathbf{v}}_f = \boxed{0.967\hat{\mathbf{i}}\text{ m/s}}$$

P5.16 $v_x = \dfrac{dx}{dt} = 10t,\ v_y = \dfrac{dy}{dt} = 9t^2$

$a_x = \dfrac{dv_x}{dt} = 10,\ a_y = \dfrac{dv_y}{dt} = 18t$

At $t = 2.00\text{ s},\ a_x = 10.0\text{ m/s}^2,\ a_y = 36.0\text{ m/s}^2$

$\sum F_x = ma_x$: $3.00\text{ kg}(10.0\text{ m/s}^2) = 30.0\text{ N}$

$\sum F_y = ma_y$: $3.00\text{ kg}(36.0\text{ m/s}^2) = 108\text{ N}$

$$\sum F = \sqrt{F_x^2 + F_y^2} = \boxed{112\text{ N}}$$

P5.17 $m = 1.00\text{ kg}$

$mg = 9.80\text{ N}$

$\tan\alpha = \dfrac{0.200\text{ m}}{25.0\text{ m}}$

$\alpha = 0.458°$

Balance forces,

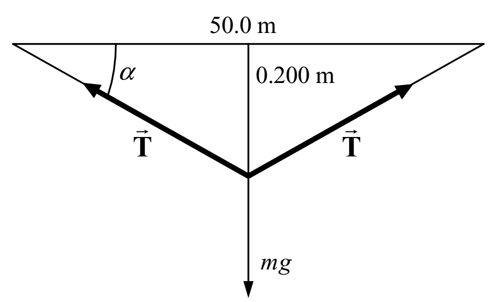

FIG. P5.17

$$2T\sin\alpha = mg$$

$$T = \frac{9.80\text{ N}}{2\sin\alpha} = \boxed{613\text{ N}}$$

***P5.18** (a)

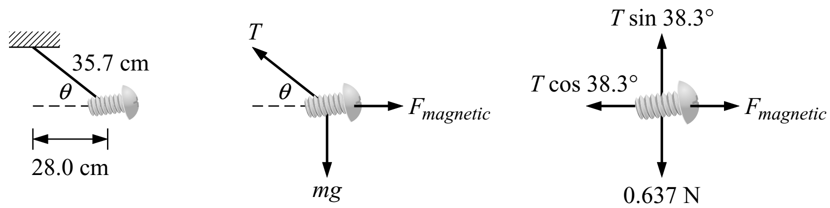

FIG. P5.18

The first diagram shows the geometry of the situation and lets us find the angle of the string with the horizontal: $\cos\theta = 28/35.7 = 0.784$ $\theta = 38.3°$. The second diagram is the free body diagram, and the third diagram is the same free body diagram with some calculated results already shown, including 0.065 kg (9.8 m/s²) = 0.637 N.

(b) $\sum F_x = ma_x$: $-T\cos 38.3° + F_{magnetic} = 0$

$\sum F_y = ma_y$: $+T\sin 38.3° - 0.637\text{ N} = 0$

from the second equation, $T = 0.637\text{ N}/\sin 38.3° = \boxed{1.03\text{ N}}$

(c) Now $F_{magnetic} = 1.03\text{ N}\cos 38.3° = \boxed{0.805\text{ N to the right}}$.

***P5.19**

(a) $P\cos 40° - n = 0$ and $P\sin 40° - 220\text{ N} = 0$
$P = 342$ N and $n = 262$ N

(b) $P - n\cos 40° - 220\text{ N}\sin 40° = 0$ and $n\sin 40 - 220\text{ N}\cos 40° = 0$
$n = 262$ N and $P = 342$ N.

(c) The results agree. The methods are basically of the same level of difficulty. Each involves one equation on one unknown and one equation in two unknowns. If we are interested in finding n without finding P, method (b) is simpler.

P5.20 From equilibrium of the sack: $T_3 = F_g$ (1)

From $\sum F_y = 0$ for the knot: $T_1\sin\theta_1 + T_2\sin\theta_2 = F_g$ (2)

From $\sum F_x = 0$ for the knot: $T_1\cos\theta_1 = T_2\cos\theta_2$ (3)

Eliminate $T_2 = T_2 = T_1\cos\theta_1/\cos\theta_2$ and solve for T_1

$$T_1 = \frac{F_g\cos\theta_2}{(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)} = \frac{F_g\cos\theta_2}{\sin(\theta_1+\theta_2)}$$

$$T_3 = F_g = \boxed{325\text{ N}}$$

$$T_1 = F_g\left(\frac{\cos 25.0°}{\sin 85.0°}\right) = \boxed{296\text{ N}}$$

$$T_2 = T_1\left(\frac{\cos\theta_1}{\cos\theta_2}\right) = 296\text{ N}\left(\frac{\cos 60.0°}{\cos 25.0°}\right) = \boxed{163\text{ N}}$$

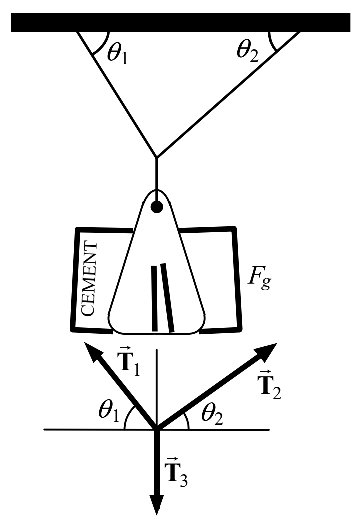

FIG. P5.20

P5.21 See the solution for T_1 in Problem 5.20. The equation indicates that the tension is directly proportional to F_g. As $\theta_1 + \theta_2$ approaches zero (as the angle between the two upper ropes approaches 180°) the tension goes to infinity. Making the right-hand rope horizontal maximizes the tension in the left-hand rope, according to the proportionality of T_1 to $\cos\theta_2$.

P5.22 (a) An explanation proceeding from fundamental physical principles will be best for the parents and for you. Consider forces on the bit of string touching the weight hanger as shown in the free-body diagram:

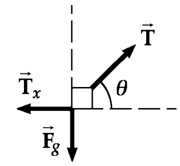

FIG. P5.22

Horizontal Forces: $\sum F_x = ma_x$: $-T_x + T\cos\theta = 0$

Vertical Forces: $\sum F_y = ma_y$: $-F_g + T\sin\theta = 0$

You need only the equation for the vertical forces to find that the tension in the string is given by $T = \boxed{\dfrac{F_g}{\sin\theta}}$. The force the child feels gets smaller, changing from T to $T\cos\theta$, while the counterweight hangs on the string. On the other hand, the kite does not notice what you are doing and the tension in the main part of the string stays constant. You do not need a level, since you learned in physics lab to sight to a horizontal line in a building. Share with the parents your estimate of the experimental uncertainty, which you make by thinking critically about the measurement, by repeating trials, practicing in advance and looking for variations and improvements in technique, including using other observers. You will then be glad to have the parents themselves repeat your measurements.

(b) $T = \dfrac{F_g}{\sin\theta} = \dfrac{0.132\text{ kg}\left(9.80\text{ m/s}^2\right)}{\sin 46.3°} = \boxed{1.79\text{ N}}$

***P5.23** (a) Isolate either mass

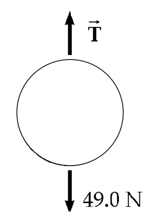

FIG. P5.23(a) and (b)

$$T + mg = ma = 0$$

$$|T| = |mg|$$

The scale reads the tension T, so

$$T = mg = 5.00\text{ kg}\left(9.80\text{ m/s}^2\right) = \boxed{49.0\text{ N}}$$

(b) The solution to part (a) is also the solution to (b).

(c) Isolate the pulley

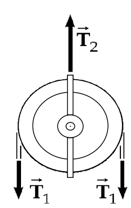

FIG. P5.23(c)

$$\vec{\mathbf{T}}_2 + 2\vec{\mathbf{T}}_1 = 0$$

$$T_2 = 2|T_1| = 2mg = \boxed{98.0\text{ N}}$$

(d) $\sum \vec{\mathbf{F}} = \vec{\mathbf{n}} + \vec{\mathbf{T}} + m\vec{\mathbf{g}} = 0$

Take the component along the incline

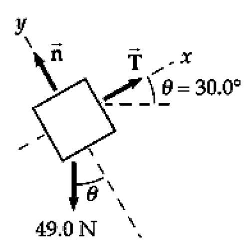

FIG. P5.23(d)

$$n_x + T_x + mg_x = 0$$

or

$$0 + T - mg\sin 30.0° = 0$$

$$T = mg\sin 30.0° = \frac{mg}{2} = \frac{5.00(9.80)}{2}$$

$$= \boxed{24.5\text{ N}}$$

P5.24 The two forces acting on the block are the normal force, n, and the weight, mg. If the block is considered to be a point mass and the x axis is chosen to be parallel to the plane, then the free body diagram will be as shown in the figure to the right. The angle θ is the angle of inclination of the plane. Applying Newton's second law for the accelerating system (and taking the direction up the plane as the positive x direction) we have

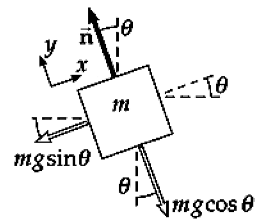

FIG. P5.24

$$\sum F_y = n - mg\cos\theta = 0: \quad n = mg\cos\theta$$

$$\sum F_x = -mg\sin\theta = ma: \quad a = -g\sin\theta$$

(a) When $\theta = 15.0°$

$$a = \boxed{-2.54 \text{ m/s}^2}$$

(b) Starting from rest

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = 2ax_f$$

$$\left|v_f\right| = \sqrt{2ax_f} = \sqrt{2\left(-2.54 \text{ m/s}^2\right)(-2.00 \text{ m})} = \boxed{3.18 \text{ m/s}}$$

P5.25 Choose a coordinate system with $\hat{\mathbf{i}}$ East and $\hat{\mathbf{j}}$ North.

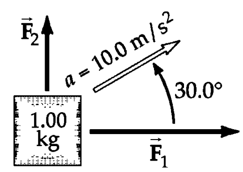

FIG. P5.25

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} = 1.00 \text{ kg}\left(10.0 \text{ m/s}^2\right) \text{ at } 30.0°$$

$$(5.00 \text{ N})\hat{\mathbf{j}} + \vec{\mathbf{F}}_1 = (10.0 \text{ N})\angle 30.0° = (5.00 \text{ N})\hat{\mathbf{j}} + (8.66 \text{ N})\hat{\mathbf{i}}$$

$$\therefore F_1 = \boxed{8.66 \text{ N (East)}}$$

P5.26 First, consider the block moving along the horizontal. The only force in the direction of movement is T. Thus, $\sum F_x = ma$

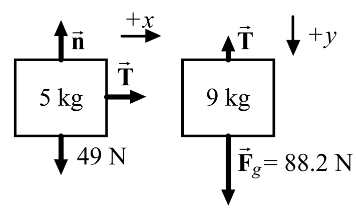

FIG. P5.26

$$T = (5 \text{ kg})a \qquad (1)$$

Next consider the block that moves vertically. The forces on it are the tension T and its weight, 88.2 N.

We have $\sum F_y = ma$

$$88.2 \text{ N} - T = (9 \text{ kg})a \qquad (2)$$

Note that both blocks must have the same magnitude of acceleration. Equations (1) and (2) can be added to give $88.2 \text{ N} = (14 \text{ kg})a$. Then

$$\boxed{a = 6.30 \text{ m/s}^2 \text{ and } T = 31.5 \text{ N}}$$

***P5.27** (a) and (b) The slope of the graph of upward velocity versus time is the acceleration of the person's body. At both time 0 and time 0.5 s, this slope is (18 cm/s)/0.6 s = 30 cm/s^2.

For the person's body, $\Sigma F_y = ma_y$: $\quad + F_{\text{bar}} - 64 \text{ kg}(9.8 \text{ m/s}^2) = 64 \text{ kg } (0.3 \text{ m/s}^2)$

Note that there is no floor touching the person to exert a normal force. Note that he does not exert any extra force 'on himself.' Solving, $F_{\text{bar}} = \boxed{646 \text{ N}}$ up.

continued on next page

(c) a_y = slope of v_y versus t graph = 0 at t = 1.1 s. The person is moving with maximum speed and is momentarily in equilibrium:

$+F_{\text{bar}} - 64\text{ kg }(9.8\text{ m/s}^2) = 0 \qquad F_{\text{bar}} = \boxed{627\text{ N}}$ up.

(d) a_y = slope of v_y versus t graph = (0 − 24 cm/s)/(1.7 s − 1.3 s) = −60 cm/s²

$+F_{\text{bar}} - 64\text{ kg}(9.8\text{ m/s}^2) = 64\text{ kg }(-0.6\text{ m/s}^2) \qquad F_{\text{bar}} = \boxed{589\text{ N}}$ up.

P5.28 $m_1 = 2.00\text{ kg},\ m_2 = 6.00\text{ kg},\ \theta = 55.0°$

(a) $\sum F_x = m_2 g\sin\theta - T = m_2 a$

and

$$T - m_1 g = m_1 a$$

$$a = \frac{m_2 g\sin\theta - m_1 g}{m_1 + m_2} = \boxed{3.57\text{ m/s}^2}$$

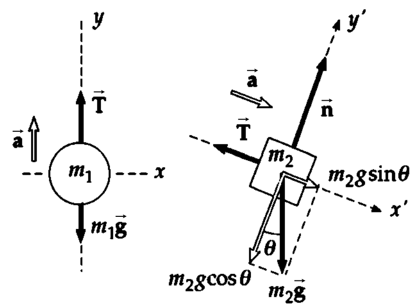

FIG. P5.28

(b) $T = m_1(a+g) = \boxed{26.7\text{ N}}$

(c) Since $v_i = 0,\ v_f = at = (3.57\text{ m/s}^2)(2.00\text{ s}) = \boxed{7.14\text{ m/s}}$.

P5.29 After it leaves your hand, the block's speed changes only because of one component of its weight:

$$\sum F_x = ma_x \qquad -mg\sin 20.0° = ma$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

Taking $v_f = 0,\ v_i = 5.00\text{ m/s}$, and $a = -g\sin(20.0°)$ gives

$$0 = (5.00)^2 - 2(9.80)\sin(20.0°)(x_f - 0)$$

or

$$x_f = \frac{25.0}{2(9.80)\sin(20.0°)} = \boxed{3.73\text{ m}}$$

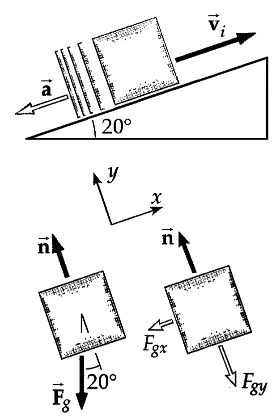

FIG. P5.29

P5.30 As the man rises steadily the pulley turns steadily and the tension in the rope is the same on both sides of the pulley. Choose man-pulley-and-platform as the system:

$$\sum F_y = ma_y$$

$$+T - 950\text{ N} = 0$$

$$T = 950\text{ N}$$

The worker must pull on the rope with force $\boxed{950\text{ N}}$.

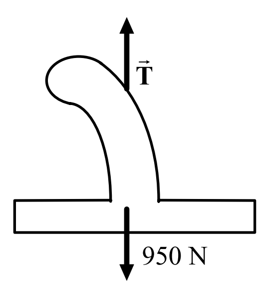

FIG. P5.30

P5.31 Forces acting on 2.00 kg block:

$$T - m_1 g = m_1 a \qquad (1)$$

Forces acting on 8.00 kg block:

$$F_x - T = m_2 a \qquad (2)$$

(a) Eliminate T and solve for a:

$$a = \frac{F_x - m_1 g}{m_1 + m_2}$$

$$\boxed{a > 0 \text{ for } F_x > m_1 g = 19.6 \text{ N}}$$

(b) Eliminate a and solve for T:

$$T = \frac{m_1}{m_1 + m_2}\left(F_x + m_2 g\right)$$

$$\boxed{T = 0 \text{ for } F_x \le -m_2 g = -78.4 \text{ N}}$$

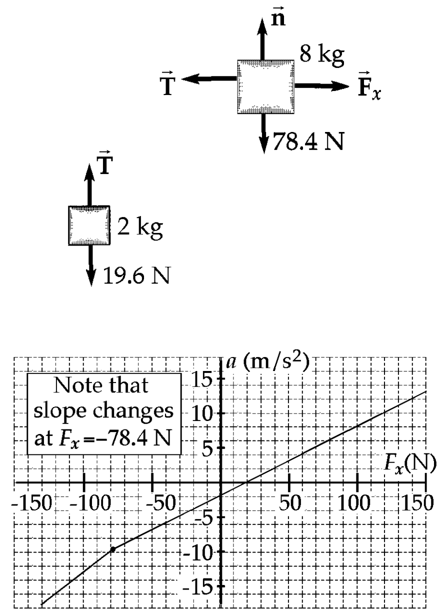

FIG. P5.31

(c)

F_x, N	−100	−78.4	−50.0	0	50.0	100
a_x, m/s²	−12.5	−9.80	−6.96	−1.96	3.04	8.04

P5.32 (a) Pulley P_1 has acceleration a_2.

Since m_1 moves *twice* the distance P_1 moves in the same time, m_1 has twice the acceleration of P_1, i.e., $\boxed{a_1 = 2a_2}$.

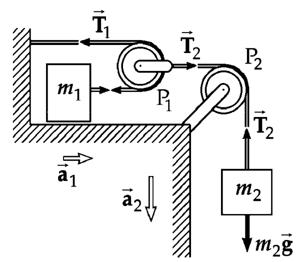

FIG. P5.32

(b) From the figure, and using

$$\sum F = ma: \quad m_2 g - T_2 = m_2 a_2 \qquad (1)$$

$$T_1 = m_1 a_1 = 2m_1 a_2 \qquad (2)$$

$$T_2 - 2T_1 = 0 \qquad (3)$$

Equation (1) becomes $m_2 g - 2T_1 = m_2 a_2$. This equation combined with Equation (2) yields

$$\frac{T_1}{m_1}\left(2m_1 + \frac{m_2}{2}\right) = m_2 g$$

$$\boxed{T_1 = \frac{m_1 m_2}{2m_1 + \frac{1}{2}m_2} g} \text{ and } \boxed{T_2 = \frac{m_1 m_2}{m_1 + \frac{1}{4}m_2} g}$$

(c) From the values of T_1 and T_2 we find that

$$a_1 = \frac{T_1}{m_1} = \boxed{\frac{m_2 g}{2m_1 + \frac{1}{2}m_2}} \text{ and } a_2 = \frac{1}{2}a_1 = \boxed{\frac{m_2 g}{4m_1 + m_2}}$$

P5.33 First, we will compute the needed accelerations:

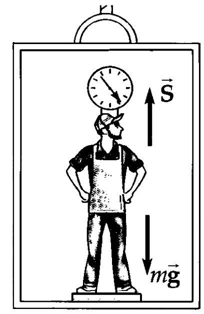

FIG. P5.33

(1) Before it starts to move: $a_y = 0$

(2) During the first 0.800 s: $a_y = \dfrac{v_{yf} - v_{yi}}{t} = \dfrac{1.20 \text{ m/s} - 0}{0.800 \text{ s}}$
$= 1.50 \text{ m/s}^2$

(3) While moving at constant velocity: $a_y = 0$

(4) During the last 1.50 s: $a_y = \dfrac{v_{yf} - v_{yi}}{t} = \dfrac{0 - 1.20 \text{ m/s}}{1.50 \text{ s}}$
$= -0.800 \text{ m/s}^2$

Newton's second law is: $\sum F_y = ma_y$

$$+S - (72.0 \text{ kg})(9.80 \text{ m/s}^2) = (72.0 \text{ kg})a_y$$
$$S = 706 \text{ N} + (72.0 \text{ kg})a_y$$

(a) When $a_y = 0$, $S = \boxed{706 \text{ N}}$.

(b) When $a_y = 1.50 \text{ m/s}^2$, $S = \boxed{814 \text{ N}}$.

(c) When $a_y = 0$, $S = \boxed{706 \text{ N}}$.

(d) When $a_y = -0.800 \text{ m/s}^2$, $S = \boxed{648 \text{ N}}$.

P5.34 Both blocks move with acceleration $a = \left(\dfrac{m_2 - m_1}{m_2 + m_1}\right)g$:

$$a = \left(\frac{7 \text{ kg} - 2 \text{ kg}}{7 \text{ kg} + 2 \text{ kg}}\right) 9.8 \text{ m/s}^2 = 5.44 \text{ m/s}^2$$

(a) Take the upward direction as positive for m_1.

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i): \quad 0 = (-2.4 \text{ m/s})^2 + 2(5.44 \text{ m/s}^2)(x_f - 0)$$
$$x_f = -\frac{5.76 \text{ m}^2/\text{s}^2}{2(5.44 \text{ m/s}^2)} = -0.529 \text{ m}$$
$$x_f = \boxed{0.529 \text{ m below its initial level}}$$

(b) $v_{xf} = v_{xi} + a_x t$: $\quad v_{xf} = -2.40 \text{ m/s} + (5.44 \text{ m/s}^2)(1.80 \text{ s})$

$$v_{xf} = \boxed{7.40 \text{ m/s upward}}$$

Section 5.8 **Forces of Friction**

P5.35 $\sum F_y = ma_y: \quad +n - mg = 0$

$$f_s \le \mu_s n = \mu_s mg$$

This maximum magnitude of static friction acts so long as the tires roll without skidding.

$$\sum F_x = ma_x: \quad -f_s = ma$$

The maximum acceleration is

$$a = -\mu_s g$$

The initial and final conditions are: $x_i = 0$, $v_i = 50.0 \text{ mi/h} = 22.4 \text{ m/s}$, $v_f = 0$

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right): \quad -v_i^2 = -2\mu_s g x_f$$

(a) $x_f = \dfrac{v_i^2}{2\mu g}$

$$x_f = \frac{(22.4 \text{ m/s})^2}{2(0.100)\left(9.80 \text{ m/s}^2\right)} = \boxed{256 \text{ m}}$$

(b) $x_f = \dfrac{v_i^2}{2\mu g}$

$$x_f = \frac{(22.4 \text{ m/s})^2}{2(0.600)\left(9.80 \text{ m/s}^2\right)} = \boxed{42.7 \text{ m}}$$

P5.36 For equilibrium: $f = F$ and $n = F_g$. Also, $f = \mu n$ i.e.,

$$\mu = \frac{f}{n} = \frac{F}{F_g}$$

$$\mu_s = \frac{75.0 \text{ N}}{25.0(9.80) \text{ N}} = \boxed{0.306}$$

and

$$\mu_k = \frac{60.0 \text{ N}}{25.0(9.80) \text{ N}} = \boxed{0.245}$$

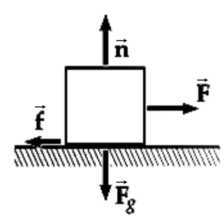

FIG. P5.36

***P5.37** (a) The car's acceleration in stopping is given by $v_{xf}^2 = v_{xi}^2 + 2\,a_x(x_f - x_i)$ $0 = (20 \text{ m/s})^2 + 2\,a_x(45 \text{ m} - 0)$ $\quad a_x = -4.44 \text{ m/s}^2$.

For the book not to slide on the horizontal seat we need

$$\sum F_x = ma_x: \quad -f_s = ma = 3.8 \text{ kg}(-4.44 \text{ m/s}^2) \quad f_s = 16.9 \text{ N}$$

$$\sum F_y = ma_y: \quad n - mg = 0 \quad n = 3.8 \text{ kg}(9.8 \text{ m/s}^2) = 37.2 \text{ N}$$

To test whether the book starts to slide, we see if this required static friction force is available in the allowed range

$$f_s \le \mu_s n = 0.65(37.2 \text{ N}) = 24.2 \text{ N}$$

Because 16.9 N is less than 24.2 N, $\boxed{\text{the book does not start to slide}}$.

(b) The actual friction force is 16.9 N backwards, and the whole force exerted by the seat on the book is $\boxed{16.9 \text{ N backward} + 37.2 \text{ N upward}}$ = 40.9 N upward and backward at 65.6° with the horizontal.

P5.38 If all the weight is on the rear wheels,

(a) $F = ma$: $\mu_s mg = ma$
But
$$\Delta x = \frac{at^2}{2} = \frac{\mu_s gt^2}{2}$$

so $\mu_s = \dfrac{2\Delta x}{gt^2}$:
$$\mu_s = \frac{2(0.250\ \text{mi})(1\,609\ \text{m/mi})}{(9.80\ \text{m/s}^2)(4.96\ \text{s})^2} = \boxed{3.34}$$

(b) Time would increase, as the wheels would skid and only kinetic friction would act; or perhaps the car would flip over.

P5.39 $m = 3.00$ kg, $\theta = 30.0°$, $x = 2.00$ m, $t = 1.50$ s

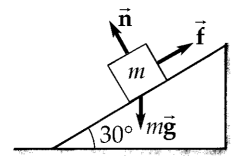

FIG. P5.39

(a) $x = \dfrac{1}{2}at^2$:
$$2.00\ \text{m} = \frac{1}{2}a(1.50\ \text{s})^2$$
$$a = \frac{4.00}{(1.50)^2} = \boxed{1.78\ \text{m/s}^2}$$

$\sum \vec{\mathbf{F}} = \vec{\mathbf{n}} + \vec{\mathbf{f}} + m\vec{\mathbf{g}} = m\vec{\mathbf{a}}$:

$$\begin{aligned} \text{Along } x:&\quad 0 - f + mg\sin 30.0° = ma \\ &\quad f = m(g\sin 30.0° - a) \\ \text{Along } y:&\quad n + 0 - mg\cos 30.0° = 0 \\ &\quad n = mg\cos 30.0° \end{aligned}$$

(b) $\mu_k = \dfrac{f}{n} = \dfrac{m(g\sin 30.0° - a)}{mg\cos 30.0°}$, $\mu_k = \tan 30.0° - \dfrac{a}{g\cos 30.0°} = \boxed{0.368}$

(c) $f = m(g\sin 30.0° - a)$, $f = 3.00(9.80\sin 30.0° - 1.78) = \boxed{9.37\ \text{N}}$

(d) $v_f^2 = v_i^2 + 2a(x_f - x_i)$

where
$$x_f - x_i = 2.00\ \text{m}$$
$$v_f^2 = 0 + 2(1.78)(2.00) = 7.11\ \text{m}^2/\text{s}^2$$
$$v_f = \sqrt{7.11\ \text{m}^2/\text{s}^2} = \boxed{2.67\ \text{m/s}}$$

P5.40 $m_{\text{suitcase}} = 20.0$ kg, $F = 35.0$ N

$$\sum F_x = ma_x:\quad -20.0\ \text{N} + F\cos\theta = 0$$
$$\sum F_y = ma_y:\quad +n + F\sin\theta - F_g = 0$$

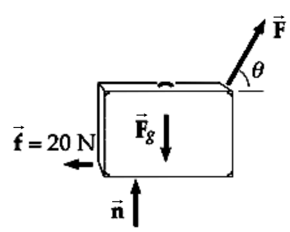

FIG. P5.40

(a) $F\cos\theta = 20.0$ N
$$\cos\theta = \frac{20.0\ \text{N}}{35.0\ \text{N}} = 0.571$$
$$\boxed{\theta = 55.2°}$$

(b) $n = F_g - F\sin\theta = [196 - 35.0(0.821)]$ N

$\boxed{n = 167\ \text{N}}$

P5.41 $T - f_k = 5.00a$ (for 5.00 kg mass)

$9.00g - T = 9.00a$ (for 9.00 kg mass)

Adding these two equations gives:

$$9.00(9.80) - 0.200(5.00)(9.80) = 14.0a$$
$$a = 5.60\ \text{m/s}^2$$
$$\therefore T = 5.00(5.60) + 0.200(5.00)(9.80)$$
$$= \boxed{37.8\ \text{N}}$$

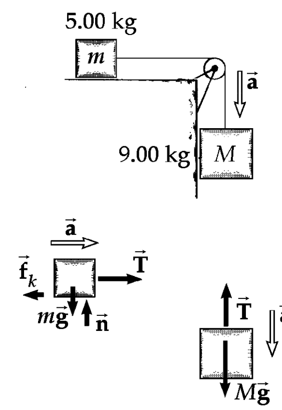

FIG. P5.41

P5.42 Let a represent the positive magnitude of the acceleration $-a\hat{\mathbf{j}}$ of m_1, of the acceleration $-a\hat{\mathbf{i}}$ of m_2, and of the acceleration $+a\hat{\mathbf{j}}$ of m_3. Call T_{12} the tension in the left rope and T_{23} the tension in the cord on the right.

For m_1, $\sum F_y = ma_y$ $+T_{12} - m_1 g = -m_1 a$

For m_2, $\sum F_x = ma_x$ $-T_{12} + \mu_k n + T_{23} = -m_2 a$

and $\sum F_y = ma_y$ $n - m_2 g = 0$

for m_3, $\sum F_y = ma_y$ $T_{23} - m_3 g = +m_3 a$

we have three simultaneous equations

$$-T_{12} + 39.2\ \text{N} = (4.00\ \text{kg})a$$
$$+T_{12} - 0.350(9.80\ \text{N}) - T_{23} = (1.00\ \text{kg})a$$
$$+T_{23} - 19.6\ \text{N} = (2.00\ \text{kg})a$$

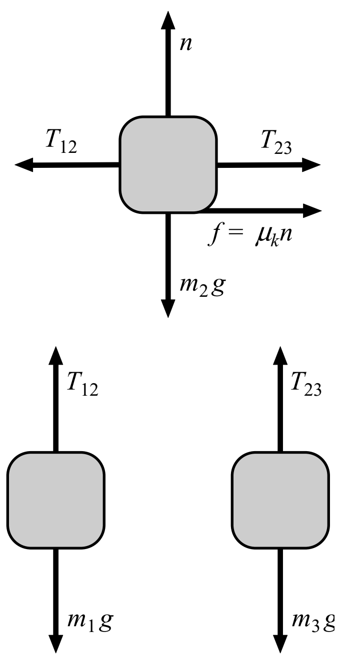

FIG. P5.42

(a) Add them up:

$$+39.2\ \text{N} - 3.43\ \text{N} - 19.6\ \text{N} = (7.00\ \text{kg})a$$

$$a = \boxed{2.31\ \text{m/s}^2\text{, down for } m_1\text{, left for } m_2\text{, and up for } m_3}$$

(b) Now $-T_{12} + 39.2\ \text{N} = (4.00\ \text{kg})(2.31\ \text{m/s}^2)$

$$\boxed{T_{12} = 30.0\ \text{N}}$$

and $T_{23} - 19.6\ \text{N} = (2.00\ \text{kg})(2.31\ \text{m/s}^2)$

$$\boxed{T_{23} = 24.2\ \text{N}}$$

P5.43 (a) See the figure adjoining

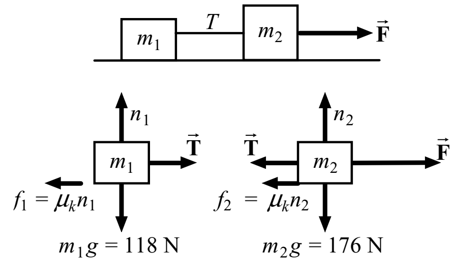

FIG. P5.43

(b) $68.0 - T - \mu m_2 g = m_2 a$ (Block #2)

$T - \mu m_1 g = m_1 a$ (Block #1)

Adding,

$$68.0 - \mu(m_1 + m_2)g = (m_1 + m_2)a$$

$$a = \frac{68.0}{(m_1 + m_2)} - \mu g = \boxed{1.29\ \text{m/s}^2}$$

$$T = m_1 a + \mu m_1 g = \boxed{27.2\ \text{N}}$$

P5.44 (a) To find the maximum possible value of P, imagine impending upward motion as case 1. Setting

$$\sum F_x = 0: \quad P\cos 50.0° - n = 0$$

$$f_{s,\,\text{max}} = \mu_s n: \quad f_{s,\,\text{max}} = \mu_s P\cos 50.0° = 0.250(0.643)P = 0.161P$$

Setting

$$\sum F_y = 0: \quad P\sin 50.0° - 0.161P - 3.00(9.80) = 0$$

$$P_{\text{max}} = \boxed{48.6\ \text{N}}$$

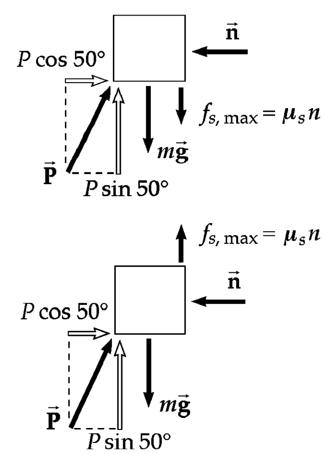

FIG. P5.44

To find the minimum possible value of P, consider impending downward motion. As in case 1,

$$f_{s,\,\text{max}} = 0.161P$$

Setting

$$\sum F_y = 0: \quad P\sin 50.0° + 0.161P - 3.00(9.80) = 0$$

$$P_{\text{min}} = \boxed{31.7\ \text{N}}$$

(b) $\boxed{\text{If } P > 48.6 \text{ N, the block slides up the wall. If } P < 31.7 \text{ N, the block slides down the wall.}}$

(c) We repeat the calculation as in part (a) with the new angle. Consider impending upward motion as case 1. Setting

$$\sum F_x = 0: \quad P\cos 13° - n = 0$$

$$f_{s,\,\text{max}} = \mu_s n: \quad f_{s,\,\text{max}} = \mu_s P\cos 13° = 0.250(0.974)P = 0.244P$$

Setting

$$\sum F_y = 0: \quad P\sin 13° - 0.244P - 3.00(9.80) = 0$$

$$P_{\text{max}} = -1580\ \text{N}$$

The push cannot really be negative. However large or small it is, it cannot produce upward motion. To find the minimum possible value of P, consider impending downward motion. As in case 1,

$$f_{s,\,\text{max}} = 0.244P$$

Setting

$$\sum F_y = 0: \quad P\sin 13° + 0.244P - 3.00(9.80) = 0$$

$$P_{\text{min}} = \boxed{62.7\ \text{N}}$$

$\boxed{P \geq 62.7 \text{ N. The block cannot slide up the wall. If } P < 62.7 \text{ N, the block slides down the wall.}}$

***P5.45** (a) If P is too small, static friction will prevent the block from moving. We will find the value of P when motion is just ready to begin:

$\Sigma F_y = ma_y$: $-P \sin 37° - 0.42 \text{ kg } 9.8 \text{ m/s}^2 + n = 0$ $\quad n = 4.12 \text{ N} + P \sin 37°$

with motion impending we read the equality sign in

$f_s = \mu_s n = 0.72(4.12 \text{ N} + P \sin 37°) = 2.96 \text{ N} + 0.433\,P$

$\Sigma F_x = ma_x$: $\quad P \cos 37° - f = 0 \quad P \cos 37° - 2.96 \text{ N} - 0.433P = 0 \quad P = 8.11 \text{ N}$

Thus $\boxed{a = 0 \text{ if } P \le 8.11 \text{ N}}$. If $P > 8.11$ N, the block starts moving. Immediately kinetic friction acts, so it controls the acceleration we measure. We have again $n = 4.12 \text{ N} + P \sin 37°$

$f_k = \mu_k n = 0.34(4.12 \text{ N} + P \sin 37°) = 1.40 \text{ N} + 0.205\,P$

$\Sigma F_x = ma_x$: $\quad P \cos 37° - f = 0.42 \text{ kg } a \quad a = (P \cos 37° - 1.40 \text{ N} - 0.205\,P)/0.42 \text{ kg}$

$$\boxed{a = 1.41\,P - 3.33 \quad \text{where } a \text{ is in m/s}^2 \text{ when } P \text{ is in N, to the right if } P > 8.11 \text{ N}}$$

(b) Since 5 N is less than 8.11 N, $a = \boxed{0}$.

(c) $f_s \le \mu_s n$ does not tell us the value of the friction force. We know that it must counterbalance 5 N cos 37° = 3.99 N, to hold the block at rest. The friction force here is $\boxed{\text{3.99 N horizontally backward}}$.

(d) $a = 1.41(10) - 3.33 = \boxed{10.8 \text{ m/s}^2 \text{ to the right}}$

(e) From part (a), $f = 1.40 \text{ N} + 0.205(10) = \boxed{3.45 \text{ N to the left}}$.

(f) The acceleration is zero for all values of P less than 8.11 N. When P passes this threshold, the acceleration jumps to its minimum nonzero value of 8.14 m/s². From there it increases linearly with P toward arbitrarily high values.

P5.46 We must consider separately the disk when it is in contact with the roof and when it has gone over the top into free fall. In the first case, we take x and y as parallel and perpendicular to the surface of the roof:

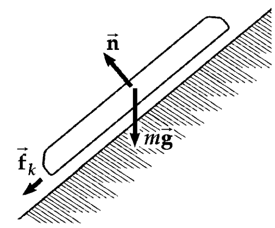

FIG. P5.46

$$\sum F_y = ma_y: \quad +n - mg\cos\theta = 0$$
$$n = mg\cos\theta$$

then friction is $f_k = \mu_k n = \mu_k mg\cos\theta$

$$\sum F_x = ma_x: \quad -f_k - mg\sin\theta = ma_x$$
$$a_x = -\mu_k g\cos\theta - g\sin\theta = (-0.4\cos 37° - \sin 37°)9.8 \text{ m/s}^2 = -9.03 \text{ m/s}^2$$

The Frisbee goes ballistic with speed given by

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right) = (15 \text{ m/s})^2 + 2\left(-9.03 \text{ m/s}^2\right)(10 \text{ m} - 0) = 44.4 \text{ m}^2/\text{s}^2$$
$$v_{xf} = 6.67 \text{ m/s}$$

For the free fall, we take x and y horizontal and vertical:

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$$
$$0 = \left(6.67 \text{ m/s } \sin 37°\right)^2 + 2\left(-9.8 \text{ m/s}^2\right)\left(y_f - 10 \text{ m } \sin 37°\right)$$
$$y_f = 6.02 \text{ m} + \frac{(4.01 \text{ m/s})^2}{19.6 \text{ m/s}^2} = \boxed{6.84 \text{ m}}$$

P5.47 Since the board is in equilibrium, $\sum F_x = 0$ and we see that the normal forces must be the same on both sides of the board. Also, if the minimum normal forces (compression forces) are being applied, the board is on the verge of slipping and the friction force on each side is

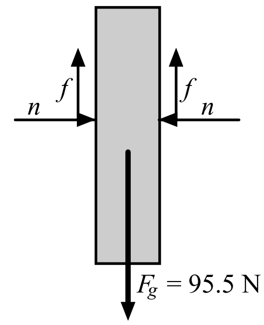

FIG. P5.47

$$f = (f_s)_{\max} = \mu_s n$$

The board is also in equilibrium in the vertical direction, so

$$\sum F_y = 2f - F_g = 0, \text{ or } f = \frac{F_g}{2}$$

The minimum compression force needed is then

$$n = \frac{f}{\mu_s} = \frac{F_g}{2\mu_s} = \frac{95.5 \text{ N}}{2(0.663)} = \boxed{72.0 \text{ N}}$$

P5.48 Take $+x$ in the direction of motion of the tablecloth. For the mug:

$$\sum F_x = ma_x \quad 0.1 \text{ N} = 0.2 \text{ kg } a_x$$
$$a_x = 0.5 \text{ m/s}^2$$

Relative to the tablecloth, the acceleration of the mug is $0.5 \text{ m/s}^2 - 3 \text{ m/s}^2 = -2.5 \text{ m/s}^2$. The mug reaches the edge of the tablecloth after time given by

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2$$
$$-0.3 \text{ m} = 0 + \frac{1}{2}(-2.5 \text{ m/s}^2)t^2$$
$$t = 0.490 \text{ s}$$

The motion of the mug relative to tabletop is over distance

$$\frac{1}{2}a_x t^2 = \frac{1}{2}(0.5 \text{ m/s}^2)(0.490 \text{ s})^2 = \boxed{0.060\,0 \text{ m}}$$

The tablecloth slides 36 cm over the table in this process.

***P5.49** (a) When the truck has the greatest acceleration it can without the box sliding, the force of friction on the box is forward and is described by $f_s = \mu_s n$.
We also have $\Sigma F_x = ma_x$: $+f_s = ma$ $\quad \Sigma F_y = ma_y$: $+n - mg = 0$
Combining by substitution gives $\mu_s mg = ma$ $\quad a = 0.3 \text{ (9.8 m/s}^2) = \boxed{2.94 \text{ m/s}^2}$ forward.

(b) The truck is accelerating forward rapidly and exerting a forward force of kinetic friction on the box, making the box accelerate forward more slowly;
$n = mg$ $\quad f_k = \mu_k mg = ma$ $\quad a = \mu_k g = 0.25 \text{ (9.8 m/s}^2) = \boxed{2.45 \text{ m/s}^2}$ forward.

(c) Now take the x axis along the direction of motion and the y axis perpendicular to the slope.
We have $\Sigma F_y = ma_y$: $+n - mg \cos 10° = 0$ $\quad +n = mg \cos 10°$ $\quad f_s = \mu_s mg \cos 10°$
$\Sigma F_x = ma_x$: $+f_s - mg \sin 10° = ma$
$a = \mu_s g \cos 10° - g \sin 10° = 0.3(9.8 \text{ m/s}^2) \cos 10° - (9.8 \text{ m/s}^2) \sin 10° =$
$\boxed{1.19 \text{ m/s}^2 \text{ up the incline}}$

(d) This time kinetic friction acts:
$\Sigma F_y = ma_y$: $+n - mg \cos 10° = 0$ $\quad +n = mg \cos 10°$ $\quad f_k = \mu_k mg \cos 10°$
$\Sigma F_x = ma_x$: $+f_k - mg \sin 10° = ma$
$a = \mu_k g \cos 10° - g \sin 10° = [0.25 \cos 10° - \sin 10°]9.8 \text{ m/s}^2 = \boxed{0.711 \text{ m/s}^2 \text{ up the incline}}$

continued on next page

(e) Model the box as in equilibrium (total force equals zero) with motion impending (static friction equals coefficient times normal force).

$\Sigma F_y = ma_y$: $\quad +n - mg\cos\theta = 0 \qquad +n = mg\cos\theta \qquad f_s = \mu_s mg\cos\theta$

$\Sigma F_x = ma_x$: $\quad +f_s - mg\sin\theta = ma = 0 \qquad \mu_s mg\cos\theta - mg\sin\theta = 0$

$\mu_s = \sin\theta/\cos\theta = \tan\theta \qquad \theta = \tan^{-1} 0.3 = \boxed{16.7°}$

(f) The mass makes no difference. Mathematically, the mass has divided out in each determination of acceleration and angle. Physically, if several packages of dishes were placed in the truck, they would all slide together, whether they were tied to one another or not.

Additional Problems

***P5.50** (a) Directly $n = 63.7\text{ N}\cos 13° = 62.1\text{ N}$

$$f_k = 0.36(62.1\text{ N}) = 22.3\text{ N}$$

Now adding $+T + 14.3\text{ N} - 22.3\text{ N} = (6.5\text{ kg})a$ and $-T + 37.2\text{ N} = (3.8\text{ kg})a$ gives

$37.2\text{ N} - 8.01\text{ N} = (10.3\text{ kg})a$

$$\boxed{a = 2.84\text{ m/s}^2}$$

Then $T = 37.2\text{ N} - 3.8\text{ kg}(2.84\text{ m/s}^2) = \boxed{26.5\text{ N}}$.

(b) We recognize the equations are describing a 6.5-kg block on an incline at 13° with the horizontal. It has coefficient of friction 0.36 with the incline. It is pulled forward, which is down the incline, by the tension in a cord running to a hanging 3.8-kg object.

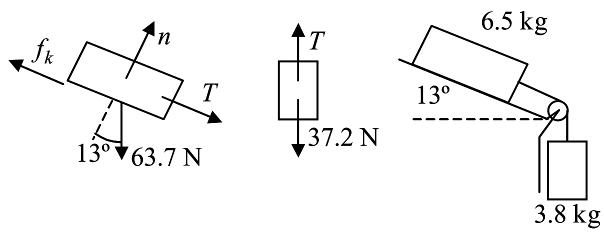

FIG. P5.50

P5.51 (a) see figure to the right

(b) First consider Pat and the chair as the system. Note that *two* ropes support the system, and T = 250 N in each rope. Applying $\sum F = ma \quad 2T - 480 = ma$, where

$$m = \frac{480}{9.80} = 49.0\text{ kg}$$

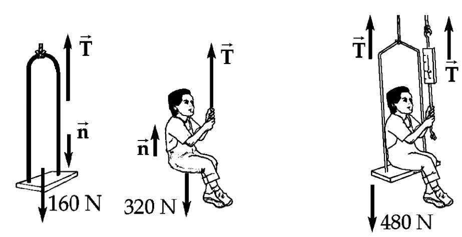

FIG. P5.51

Solving for a gives

$$a = \frac{500 - 480}{49.0} = \boxed{0.408\text{ m/s}^2}$$

(c) $\sum F = ma$ on Pat:

$$\sum F = n + T - 320 = ma, \text{ where } m = \frac{320}{9.80} = 32.7\text{ kg}$$

$n = ma + 320 - T = 32.7(0.408) + 320 - 250 = \boxed{83.3\text{ N}}$.

***P5.52** (a) As soon as Pat passes the rope to the other child,

$$\boxed{\text{Pat and the seat, with total weight 480 N, will accelerate down and the other child, only 440 N, will accelerate up.}}$$

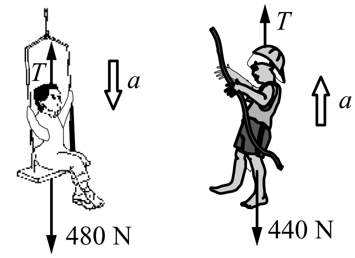

FIG. 5.52

We have $+480\text{ N} - T = \dfrac{480\text{ N}}{9.8\text{ m/s}^2}a$ and $+T - 440\text{ N} = \dfrac{440\text{ N}}{9.8\text{ m/s}^2}a$

Adding,

$$+480\text{ N} - T + T - 440\text{ N} = (49.0\text{ kg} + 44.9\text{ kg})a$$

$$a = \frac{40\text{ N}}{93.9\text{ kg}} = \boxed{0.426\text{ m/s}^2 = a}$$

The rope tension is $T = 440\text{ N} + 44.9\text{ kg}(0.426\text{ m/s}^2) = 459\text{ N}$.

(b) In problem 51, a rope tension of 250 N does not make the rope break. In part (a), the rope is strong enough to support tension 459 N. But now the tension everywhere in the rope is 480 N, so it can exceed the breaking strength of the rope.

The tension in the chain supporting the pulley is $480 + 480\text{ N} = 960\text{ N}$, so that chain may break first.

P5.53 $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$ gives the object's acceleration

$$\vec{\mathbf{a}} = \frac{\sum F}{m} = \frac{(8.00\hat{\mathbf{i}} - 4.00t\hat{\mathbf{j}})\text{ N}}{2.00\text{ kg}}$$

$$\vec{\mathbf{a}} = (4.00\text{ m/s}^2)\hat{\mathbf{i}} - (2.00\text{ m/s}^3)t\hat{\mathbf{j}} = \frac{d\vec{\mathbf{v}}}{dt}$$

Its velocity is

$$\int_{v_i}^{v} d\vec{\mathbf{v}} = \vec{\mathbf{v}} - \vec{\mathbf{v}}_i = \vec{\mathbf{v}} - 0 = \int_0^t \vec{\mathbf{a}}dt$$

$$\vec{\mathbf{v}} = \int_0^t \left[(4.00\text{ m/s}^2)\hat{\mathbf{i}} - (2.00\text{ m/s}^3)t\hat{\mathbf{j}}\right]dt$$

$$\vec{\mathbf{v}} = (4.00t\text{ m/s}^2)\hat{\mathbf{i}} - (1.00t^2\text{ m/s}^3)\hat{\mathbf{j}}$$

(a) We require $|\vec{\mathbf{v}}| = 15.0 \text{ m/s}, \ |\vec{\mathbf{v}}|^2 = 225 \text{ m}^2/\text{s}^2$

$$16.0t^2 \text{ m}^2/\text{s}^4 + 1.00t^4 \text{ m}^2/\text{s}^6 = 225 \text{ m}^2/\text{s}^2$$
$$1.00t^4 + 16.0 \text{ s}^2 t^2 - 225 \text{ s}^4 = 0$$
$$t^2 = \frac{-16.0 \pm \sqrt{(16.0)^2 - 4(-225)}}{2.00} = 9.00 \text{ s}^2$$
$$t = \boxed{3.00 \text{ s}}$$

Take $\vec{\mathbf{r}}_i = 0$ at $t = 0$. The position is

$$\vec{\mathbf{r}} = \int_0^t \vec{\mathbf{v}}dt = \int_0^t \left[\left(4.00t \text{ m/s}^2\right)\hat{\mathbf{i}} - \left(1.00t^2 \text{ m/s}^3\right)\hat{\mathbf{j}}\right]dt$$
$$\vec{\mathbf{r}} = \left(4.00 \text{ m/s}^2\right)\frac{t^2}{2}\hat{\mathbf{i}} - \left(1.00 \text{ m/s}^3\right)\frac{t^3}{3}\hat{\mathbf{j}}$$

at $t = 3$ s we evaluate.

(b) $\vec{\mathbf{r}} = \boxed{\left(18.0\hat{\mathbf{i}} - 9.00\hat{\mathbf{j}}\right) \text{ m}}$

(c) So $|\vec{\mathbf{r}}| = \sqrt{(18.0)^2 + (9.00)^2} \text{ m} = \boxed{20.1 \text{ m}}$

P5.54 (a) We write $\sum F_x = ma_x$ for each object.

$$18 \text{ N} - P = (2 \text{ kg})a$$
$$P - Q = (3 \text{ kg})a$$
$$Q = (4 \text{ kg})a$$

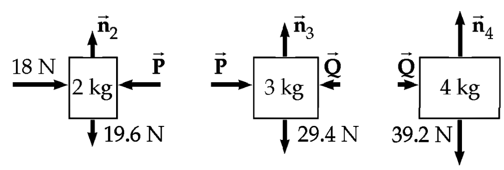

FIG. P5.54

Adding gives $18 \text{ N} = (9 \text{ kg})a$ so

.

$$a = \boxed{2.00 \text{ m/s}^2}$$

(b) $Q = 4 \text{ kg}\left(2 \text{ m/s}^2\right) = \boxed{8.00 \text{ N net force on the 4 kg}}$

$P - 8 \text{ N} = 3 \text{ kg}\left(2 \text{ m/s}^2\right) = \boxed{6.00 \text{ N net force on the 3 kg}}$ and $P = 14 \text{ N}$

$18 \text{ N} - 14 \text{ N} = 2 \text{ kg}\left(2 \text{ m/s}^2\right) = \boxed{4.00 \text{ N net force on the 2 kg}}$

continued on next page

(c) From above, $Q = \boxed{8.00\ \text{N}}$ and $P = \boxed{14.0\ \text{N}}$

(d) The 3-kg block models the heavy block of wood. The contact force on your back is represented by Q, which is much less than the force F. The difference between F and Q is the net force causing acceleration of the 5-kg pair of objects. The acceleration is real and nonzero, but lasts for so short a time that it never is associated with a large velocity. The frame of the building and your legs exert forces, small relative to the hammer blow, to bring the partition, block, and you to rest again over a time large relative to the hammer blow. This problem lends itself to interesting lecture demonstrations. One person can hold a lead brick in one hand while another hits the brick with a hammer.

***P5.55** (a) Take the rope and block together as the system:

$\sum F_x = ma_x$: $\quad +12\ \text{N} = (4\ \text{kg} + m_1)a \qquad a = \boxed{12\text{N}/(4\ \text{kg} + m_1)\ \text{forward}}$

(b) The rope tension varies along the massive rope, with the value 12 N at the front end and a value we call T_{back} at the back end. Take the block alone as the system:

$\sum F_x = ma_x$: $\quad +T_{back} = (4\ \text{kg})a = 4\ \text{kg}(12\text{N}/(4\ \text{kg} + m_1) = \boxed{12\ \text{N}/(1 + m_1/4\ \text{kg})\ \text{forward}}$

(c) We substitute $m_1 = 0.8$ kg: $a = 12\text{N}/(4\ \text{kg} + 0.8\ \text{kg}) = \boxed{2.50\ \text{m/s}^2\ \text{forward}}$

$T_{back} = 12\ \text{N}/(1 + 0.8/4) = \boxed{10.0\ \text{N forward}}$

(d) As $m_1 \to \infty$, $\qquad 12\ \text{N}/(1 + m_1/4)$ $\boxed{\text{goes to zero}}$

(e) As $m_1 \to 0$, $\qquad 12\ \text{N}/(1 + m_1/4)$ $\boxed{\text{goes to 12 N}}$

(f) $\boxed{\text{A cord of negligible mass has constant tension along its length.}}$

***P5.56** (a) Choose the black glider plus magnet as the system.

$\sum F_x = ma_x$: $\quad +0.823\ \text{N} = 0.24\ \text{kg}\ a \qquad a = \boxed{3.43\ \text{m/s}^2\ \text{toward the scrap iron}}$

(b) The analysis in part (a) applies here with no change. $a_{black} =$ $\boxed{3.43\ \text{m/s}^2\ \text{toward the scrap iron}}$.

For the green glider with the scrap iron,

$\sum F_x = ma_x$: $\quad +0.823\ \text{N} = 0.12\ \text{kg}\ a \qquad a = \boxed{6.86\ \text{m/s}^2\ \text{toward the magnet}}$

P5.57 (a) First, we note that $F = T_1$. Next, we focus on the mass M and write $T_5 = Mg$. Next, we focus on the bottom pulley and write $T_5 = T_2 + T_3$. Finally, we focus on the top pulley and write $T_4 = T_1 + T_2 + T_3$.

Since the pulleys are not starting to rotate and are frictionless, $T_1 = T_3$, and $T_2 = T_3$. From this information, we have $T_5 = 2T_2$, so $T_2 = \dfrac{Mg}{2}$.

Then $\boxed{T_1 = T_2 = T_3 = \dfrac{Mg}{2}}$, and $\boxed{T_4 = \dfrac{3Mg}{2}}$,

and $\boxed{T_5 = Mg}$.

(b) Since $F = T_1$, we have $\boxed{F = \dfrac{Mg}{2}}$.

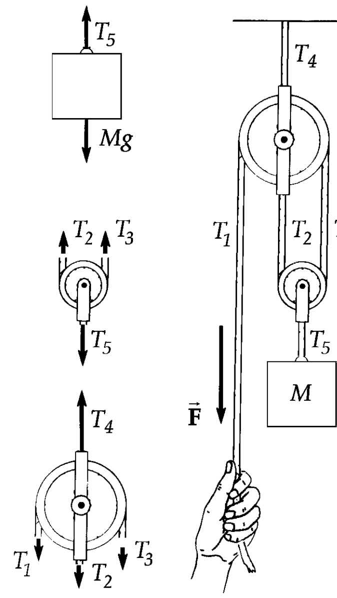

FIG. P5.57

***P5.58** (a) The cord makes angle θ with the horizontal where $\tan\theta = 0.1/0.4$ $\theta = 14.0°$.

$\Sigma F_y = ma_y$: $+10\text{ N}\sin 14.0° - 2.2\text{ kg } 9.8\text{ m/s}^2 + n = 0$ $n = 19.1\text{ N}$

$f_k = \mu_k n = 0.4(19.1\text{ N}) = 7.65\text{ N}$

$\Sigma F_x = ma_x$: $+10\text{ N}\cos 14.0° - 7.65\text{ N} = 2.2\text{ kg } a$ $a = \boxed{0.931\text{ m/s}^2}$

(b) When x is large we have $n = 21.6$ N, $f_k = 8.62$ N and $a = (10\text{ N} - 8.62\text{ N})/2.2\text{ kg} = 0.625\text{ m/s}^2$. As x decreases, the acceleration increases gradually, passes through a maximum, and then drops more rapidly, becoming negative. At $x = 0$ it reaches the value $a = [0 - 0.4(21.6\text{ N} - 10\text{ N})]/2.2\text{ kg} = -2.10\text{ m/s}^2$.

(c) We carry through the same calculations as in part (a) for a variable angle, for which $\cos\theta = x[x^2 + (.1\text{m})^2]^{-1/2}$ and $\sin\theta = 0.1\text{ m}[x^2 + (.1\text{m})^2]^{-1/2}$ We find

$$a = \frac{10\text{ N}\,x\left[x^2+0.1^2\right]^{-1/2} - 0.4\left(21.6\text{ N} - 10\text{ N}\,0.1\left[x^2+0.1^2\right]^{-1/2}\right)}{2.2\text{ kg}}$$

$$a = 4.55\,x\left[x^2+0.1^2\right]^{-1/2} - 3.92 + 0.182\left[x^2+0.1^2\right]^{-1/2}$$

Now to maximize a we take its derivative with respect to x and set it equal to zero:

$$\frac{da}{dx} = 4.55\left(x^2+0.1^2\right)^{-1/2} + 4.55x\left(-\tfrac{1}{2}\right)2x\left(x^2+0.1^2\right)^{-3/2} + 0.182\left(-\tfrac{1}{2}\right)2x\left(x^2+0.1^2\right)^{-3/2} = 0$$

$$4.55\left(x^2+0.1^2\right) - 4.55x^2 - 0.182x = 0 \qquad x = \boxed{0.250\text{ m}}$$

At this point $a = 4.55\ (0.25)\left[0.25^2+0.1^2\right]^{-1/2} - 3.92 + 0.182\left[0.25^2+0.1^2\right]^{-1/2} = \boxed{0.976\text{ m/s}^2}$

(d) We solve

$$0 = 4.55\,x\left[x^2+0.1^2\right]^{-1/2} - 3.92 + 0.182\left[x^2+0.1^2\right]^{-1/2}$$

$$3.92\left[x^2+0.1^2\right]^{1/2} = 4.55x + 0.182$$

$$15.4\left[x^2+0.1^2\right] = 20.7x^2 + 1.65x + 0.0331$$

$5.29x^2 + 1.65x - 0.121 = 0$ only the positive root is directly meaningful: $x = \boxed{0.0610\text{ m}}$

***P5.59** (a) The cable does not stretch: Whenever one car moves 1 cm, the other moves 1 cm.

$\boxed{\text{At any instant they have the same velocity and at all instants they have the same acceleration.}}$

(b) Consider the BMW as object.
$\sum F_y = ma_y$: $+T - (1461 \text{ kg})(9.8 \text{ m/s}^2) = 1461 \text{ kg } (1.25 \text{ m/s}^2)$ $T = \boxed{1.61 \times 10^4 \text{ N}}$

(c) Consider both cars as object.
$\sum F_y = ma_y$: $+T_{above} - (1461 \text{ kg} + 1207 \text{ kg})(9.8 \text{ m/s}^2) = (1461 \text{ kg} + 1207 \text{ kg})(1.25 \text{ m/s}^2)$

$T_{above} = \boxed{2.95 \times 10^4 \text{ N}}$

(d) The Ferrari pulls up on the middle section of cable with 16.1 kN. The BMW pulls down on the middle section of cable with 16.1 kN. The net force on the middle section of cable is $\boxed{0}$. The current velocity is 3.50 m/s up. After 0.01 s the acceleration of 1.25 m/s^2 gives the cable additional velocity 0.0125 m/s, for a total of $\boxed{3.51 \text{ m/s upward}}$.

> The 3.50 m/s in 3.51 m/s needs no dynamic cause; the motion of the cable continues on its own, as described by the law of 'inertia' or 'pigheadedness.' The 0.01 m/s of extra upward speed must be caused by some total upward force on the section of cable. But because the cable's mass is very small compared to a thousand kilograms, the force is very small compared to 1.61×10^4 N, the nearly uniform tension of this section of cable.

***P5.60** For the system to start to move when released, the force tending to move m_2 down the incline, $m_2 g \sin\theta$, must exceed the maximum friction force which can retard the motion:

$$f_{\max} = f_{1,\max} + f_{2,\max} = \mu_{s,1} n_1 + \mu_{s,2} n_2$$
$$f_{\max} = \mu_{s,1} m_1 g + \mu_{s,2} m_2 g \cos\theta$$

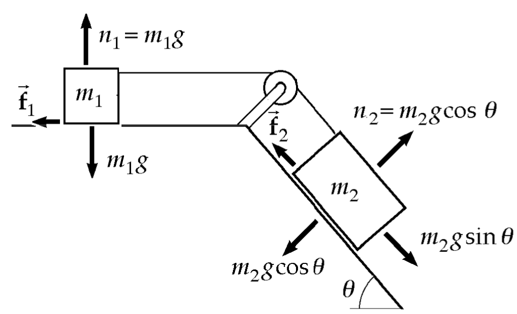

FIG. P5.60

From the table of coefficients of friction in the text, we take $\mu_{s,1} = 0.610$ (aluminum on steel) and $\mu_{s,2} = 0.530$ (copper on steel). With

$$m_1 = 2.00 \text{ kg}, \; m_2 = 6.00 \text{ kg}, \; \theta = 30.0°$$

the maximum friction force is found to be $f_{\max} = 38.9$ N. This exceeds the force tending to cause the system to move, $m_2 g \sin\theta = 6.00 \text{ kg}\left(9.80 \text{ m/s}^2\right)\sin 30° = 29.4$ N. Hence,

$\boxed{\text{the system will not start to move when released}}$.

The friction forces increase in magnitude until the total friction force retarding the motion, $f = f_1 + f_2$, equals the force tending to set the system in motion. That is, until

$$\boxed{f = m_2 g \sin\theta = 29.4 \text{ N}}$$

P5.61 (a) The crate is in equilibrium, just before it starts to move. Let the normal force acting on it be n and the friction force, f_s.

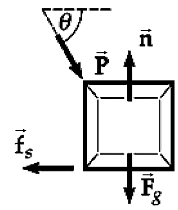

FIG. P5.61

Resolving vertically:

$$n = F_g + P\sin\theta$$

Horizontally:

$$P\cos\theta = f_s$$

But,

$$f_s \le \mu_s n$$

i.e.,

$$P\cos\theta \le \mu_s\left(F_g + P\sin\theta\right)$$

or

$$P\left(\cos\theta - \mu_s\sin\theta\right) \le \mu_s F_g$$

Divide by $\cos\theta$:

$$P\left(1-\mu_s\tan\theta\right) \le \mu_s F_g\sec\theta$$

Then

$$\boxed{P_{\text{minimum}} = \frac{\mu_s F_g\sec\theta}{1-\mu_s\tan\theta}}$$

(b) $P = \dfrac{0.400(100\text{ N})\sec\theta}{1-0.400\tan\theta}$

$\theta(\text{deg})$	0.00	15.0	30.0	45.0	60.0
$P(N)$	40.0	46.4	60.1	94.3	260

If the angle were $68.2°$ or more, the expression for P would go to infinity and motion would become impossible.

P5.62 (a) Following the in-chapter example about a block on a frictionless incline, we have

$$a = g\sin\theta = \left(9.80\text{ m/s}^2\right)\sin 30.0°$$

$$\boxed{a = 4.90\text{ m/s}^2}$$

(b) The block slides distance x on the incline, with $\sin 30.0° = \dfrac{0.500\text{ m}}{x}$

$x = 1.00\text{ m}$: $v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = 0 + 2\left(4.90\text{ m/s}^2\right)(1.00\text{ m})$

$v_f = \boxed{3.13\text{ m/s}}$ after time $t_s = \dfrac{2x_f}{v_f} = \dfrac{2(1.00\text{ m})}{3.13\text{ m/s}} = 0.639\text{ s}$

continued on next page

(c) Now in free fall $y_f - y_i = v_{yi}t + \frac{1}{2}a_y t^2$:

$$-2.00 = (-3.13\ \text{m/s})\sin 30.0° t - \frac{1}{2}(9.80\ \text{m/s}^2)t^2$$

$$(4.90\ \text{m/s}^2)t^2 + (1.56\ \text{m/s})t - 2.00\ \text{m} = 0$$

$$t = \frac{-1.56\ \text{m/s} \pm \sqrt{(1.56\ \text{m/s})^2 - 4(4.90\ \text{m/s}^2)(-2.00\ \text{m})}}{9.80\ \text{m/s}^2}$$

Only the positive root is physical

$$t = 0.499\ \text{s}$$

$$x_f = v_x t = [(3.13\ \text{m/s})\cos 30.0°](0.499\ \text{s}) = \boxed{1.35\ \text{m}}$$

(d) total time $= t_s + t = 0.639\ \text{s} + 0.499\ \text{s} = \boxed{1.14\ \text{s}}$

(e) The mass of the block makes no difference.

***P5.63** (a) The net force on the cushion is in a fixed direction, downward and forward making angle $\tan^{-1}(F/mg)$ with the vertical. Starting from rest, it will move along this line with (b) increasing speed. Its velocity changes in magnitude.

(c) Since the line of motion is in the direction of the net force, they both make the same angle with the horizontal: $x/8\ \text{m} = F/mg = 2.40\ \text{N}/(1.2\ \text{kg})(9.8\ \text{m/s}^2)$ so $x = \boxed{1.63\ \text{m}}$

(d) The cushion will move along a parabola. The axis of the parabola is parallel to the dashed line in the problem figure. If the cushion is thrown in direction above the dashed line, its path will be concave downward, to make its velocity become more and more nearly parallel to the dashed line over time. If the cushion is thrown down more steeply, its path will be concave upward, again making its velocity turn toward the fixed direction of its acceleration.

P5.64

t(s)	t^2(s^2)	x(m)
0	0	0
1.02	1.04 0	0.100
1.53	2.34 1	0.200
2.01	4.04 0	0.350
2.64	6.97 0	0.500
3.30	10.89	0.750
3.75	14.06	1.00

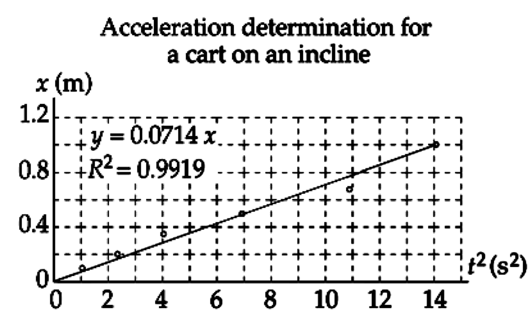

FIG. P5.64

From $x = \frac{1}{2}at^2$ the slope of a graph of x versus t^2 is $\frac{1}{2}a$, and

$$a = 2 \times \text{slope} = 2(0.071\,4\ \text{m/s}^2) = \boxed{0.143\ \text{m/s}^2}$$

From $a' = g\sin\theta$,

$$a' = 9.80\ \text{m/s}^2\left(\frac{1.77\,4}{127.1}\right) = 0.137\ \text{m/s}^2,\ \text{different by } 4\%.$$

The difference is accounted for by the uncertainty in the data, which we may estimate from the third point as

$$\frac{0.350 - (0.071\,4)(4.04)}{0.350} = 18\%$$

Thus the acceleration values agree.

P5.65 With motion impending,

$$n + T\sin\theta - mg = 0$$

$$f = \mu_s\left(mg - T\sin\theta\right)$$

and

$$T\cos\theta - \mu_s mg + \mu_s T\sin\theta = 0$$

so

$$T = \frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta}$$

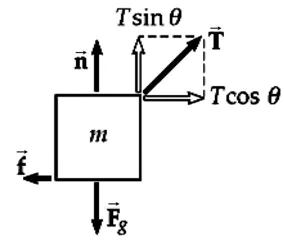

FIG. P5.65

To minimize T, we maximize $\cos\theta + \mu_s\sin\theta$

$$\frac{d}{d\theta}\left(\cos\theta + \mu_s\sin\theta\right) = 0 = -\sin\theta + \mu_s\cos\theta$$

(a) $\theta = \tan^{-1}\mu_s = \tan^{-1} 0.350 = \boxed{19.3°}$

(b) $T = \dfrac{0.350(1.30\text{ kg})(9.80\text{ m/s}^2)}{\cos 19.3° + 0.350\sin 19.3°} = \boxed{4.21\text{ N}}$

***P5.66** (a) When block 2 moves down 1 cm, block 1 moves 2 cm forward, so block 1 always has twice the speed of block 2, and $\boxed{a_1 = 2a_2}$ relates the magnitudes of the accelerations.

(b) Let T represent the uniform tension in the cord. For block 1 as object,
$\sum F_x = m_1 a_1$: $T = m_1(2a_2)$
For block 2 as object, $\sum F_y = m_1 a_1$: $T + T - (1.3\text{ kg})(9.8\text{ m/s}^2) = (1.3\text{ kg})(-a_2)$
To solve simultaneously we substitute for T: $4\,m_1 a_2 + (1.3\text{ kg})a_2 = 12.7\text{ N}$
$\boxed{a_2 = 12.7\text{ N }(1.30\text{ kg} + 4m_1)^{-1}\text{ down}}$

(c) $a_2 = 12.7\text{ N }(1.30\text{ kg} + 4[0.55\text{ kg}])^{-1}\text{ down} = \boxed{3.64\text{ m/s}^2\text{ down}}$

(d) a_2 approaches $12.7\text{ N}/1.3\text{ kg} = \boxed{9.80\text{ m/s}^2\text{ down}}$

(e) $\boxed{a_2\text{ approaches zero}}$.

(f) From $2T = 12.74\text{ N} + 0$, $\boxed{T = 6.37\text{ N}}$

(g) Yes. As m_1 approaches zero, block 2 is essentially in free fall. As m_2 becomes negligible compared to m_1, the system is nearly in equilibrium.

P5.67 $\sum F = ma$

For m_1: $T = m_1 a$

For m_2: $T - m_2 g = 0$

Eliminating T, $a = \dfrac{m_2 g}{m_1}$

For all 3 blocks:

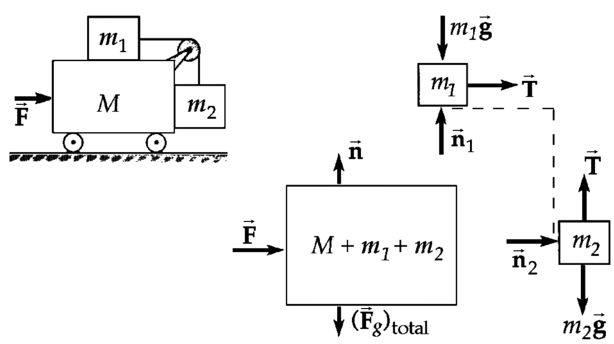

FIG. P5.67

$$F = \left(M + m_1 + m_2\right)a = \boxed{\left(M + m_1 + m_2\right)\left(\frac{m_2 g}{m_1}\right)}$$

P5.68 Throughout its up and down motion after release the block has

$$\sum F_y = ma_y: \quad +n - mg\cos\theta = 0$$
$$n = mg\cos\theta$$

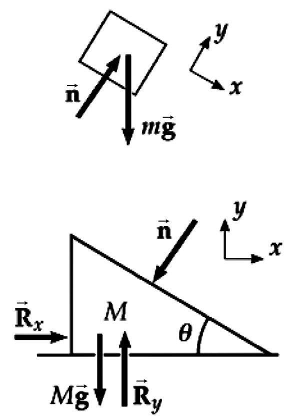

FIG. P5.68

Let $\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$ represent the force of table on incline.
We have

$$\sum F_x = ma_x: \quad +R_x - n\sin\theta = 0$$
$$R_x = mg\cos\theta\sin\theta$$
$$\sum F_y = ma_y: \quad -Mg - n\cos\theta + R_y = 0$$
$$R_y = Mg + mg\cos^2\theta$$

$$\boxed{\vec{\mathbf{R}} = mg\cos\theta\sin\theta \text{ to the right} + \left(M + m\cos^2\theta\right)g \text{ upward}}$$

P5.69 Choose the x axis pointing down the slope.

$$v_f = v_i + at: \quad 30.0\ \text{m/s} = 0 + a(6.00\ \text{s})$$
$$a = 5.00\ \text{m/s}^2$$

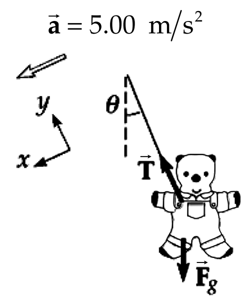

FIG. P5.69

Consider forces on the toy.

$$\sum F_x = ma_x: \quad mg\sin\theta = m\left(5.00\ \text{m/s}^2\right)$$
$$\theta = \boxed{30.7°}$$
$$\sum F_y = ma_y: \quad -mg\cos\theta + T = 0$$
$$T = mg\cos\theta = (0.100)(9.80)\cos 30.7°$$
$$T = \boxed{0.843\ \text{N}}$$

P5.70 $\sum F_y = ma_y$: $n - mg\cos\theta = 0$

or

$$n = 8.40(9.80)\cos\theta$$
$$n = (82.3\text{ N})\cos\theta$$

$\sum F_x = ma_x$: $mg\sin\theta = ma$

or

$$a = g\sin\theta$$
$$a = \left(9.80\text{ m/s}^2\right)\sin\theta$$

θ, deg	n, N	a, m/s^2
0.00	82.3	0.00
5.00	82.0	0.854
10.0	81.1	1.70
15.0	79.5	2.54
20.0	77.4	3.35
25.0	74.6	4.14
30.0	71.3	4.90
35.0	67.4	5.62
40.0	63.1	6.30
45.0	58.2	6.93
50.0	52.9	7.51
55.0	47.2	8.03
60.0	41.2	8.49
65.0	34.8	8.88
70.0	28.2	9.21
75.0	21.3	9.47
80.0	14.3	9.65
85.0	7.17	9.76
90.0	0.00	9.80

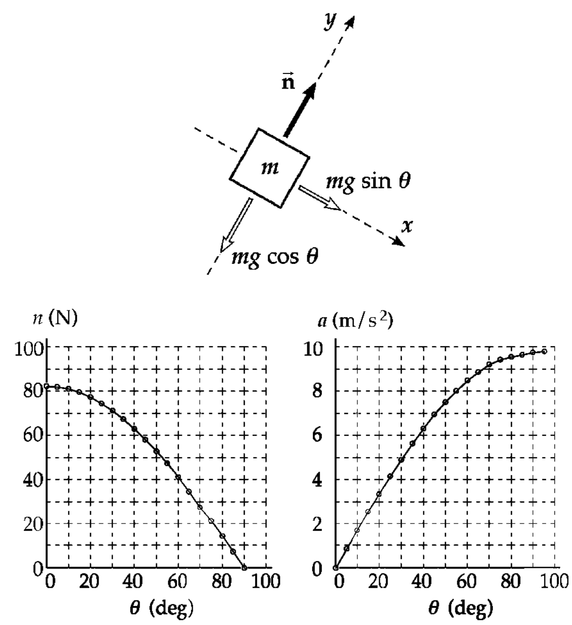

FIG. P5.70

At 0°, the normal force is the full weight and the acceleration is zero. At 90°, the mass is in free fall next to the vertical incline.

P5.71 (a) Apply Newton's second law to two points where butterflies are attached on either half of mobile (other half the same, by symmetry)

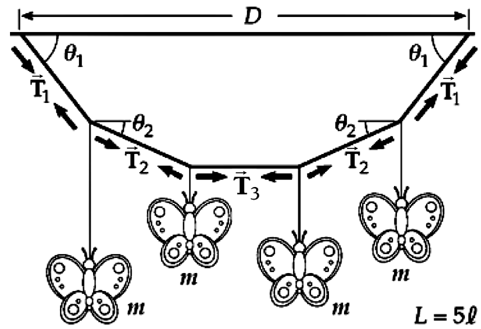

FIG. P5.71

(1) $T_2 \cos\theta_2 - T_1 \cos\theta_1 = 0$
(2) $T_1 \sin\theta_1 - T_2 \sin\theta_2 - mg = 0$
(3) $T_2 \cos\theta_2 - T_3 = 0$
(4) $T_2 \sin\theta_2 - mg = 0$

Substituting (4) into (2) for $T_2 \sin\theta_2$,

$$T_1 \sin\theta_1 - mg - mg = 0$$

Then

$$\boxed{T_1 = \frac{2mg}{\sin\theta_1}}$$

Substitute (3) into (1) for $T_2 \cos\theta_2$:

$$T_3 - T_1 \cos\theta_1 = 0,\ T_3 = T_1 \cos\theta_1$$

Substitute value of T_1:

$$T_3 = 2mg\frac{\cos\theta_1}{\sin\theta_1} = \boxed{\frac{2mg}{\tan\theta_1} = T_3}$$

From Equation (4),

$$T_2 = \frac{mg}{\sin\theta_2}$$

(b) Divide (4) by (3):

$$\frac{T_2 \sin\theta_2}{T_2 \cos\theta_2} = \frac{mg}{T_3}$$

Substitute value of T_3:

$$\tan\theta_2 = \frac{mg\tan\theta_1}{2mg},\ \boxed{\theta_2 = \tan^{-1}\left(\frac{\tan\theta_1}{2}\right)}$$

Then we can finish answering part (a):

$$\boxed{T_2 = \frac{mg}{\sin\left[\tan^{-1}\left(\frac{1}{2}\tan\theta_1\right)\right]}}$$

(c) D is the horizontal distance between the points at which the two ends of the string are attached to the ceiling.

$$D = 2\ell\cos\theta_1 + 2\ell\cos\theta_2 + \ell \text{ and } L = 5\ell$$

$$D = \frac{L}{5}\left\{2\cos\theta_1 + 2\cos\left[\tan^{-1}\left(\frac{1}{2}\tan\theta_1\right)\right] + 1\right\}$$

ANSWERS TO EVEN PROBLEMS

P5.2 (a) 1/3 (b) 0.750 m/s^2

P5.4 (a) 4.47×10^{15} m/s^2 away from the wall (b) 2.09×10^{-10} N toward the wall

P5.6 (a) 534 N down (b) 54.5 kg

P5.8 $(16.3\hat{\mathbf{i}} + 14.6\hat{\mathbf{j}})$ N

P5.10 (a) Force exerted by spring on hand, to the left; force exerted by spring on wall, to the right. (b) Force exerted by wagon on handle, downward to the left. Force exerted by wagon on planet, upward. Force exerted by wagon on ground, downward. (c) Force exerted by football on player, downward to the right. Force exerted by football on planet, upward. (d) Force exerted by small-mass object on large-mass object, to the left. (e) Force exerted by negative charge on positive charge, to the left. (f) Force exerted by iron on magnet, to the left.

P5.12 see the solution

P5.14 (a) 181° (b) 11.2 kg (c) 37.5 m/s (d) $(-37.5\hat{\mathbf{i}} - 0.893\hat{\mathbf{j}})$ m/s

P5.16 112 N

P5.18 (a) see the solution (b) 1.03 N (c) 0.805 N to the right

P5.20 $T_1 = 296$ N; $T_2 = 163$ N; $T_3 = 325$ N

P5.22 (a) see the solution (b) 1.79 N

P5.24 see the solution (a) 2.54 m/s^2 down the incline (b) 3.18 m/s

P5.26 see the solution 6.30 m/s^2; 31.5 N

P5.28 see the solution (a) 3.57 m/s^2 (b) 26.7 N (c) 7.14 m/s

P5.30 950 N

P5.32 (a) $a_1 = 2a_2$ (b) $T_1 = \dfrac{m_1 m_2 g}{2m_1 + m_2/2}$; $T_2 = \dfrac{m_1 m_2 g}{m_1 + m_2/4}$ (c) $a_1 = \dfrac{m_2 g}{2m_1 + m_2/2}$; $a_2 = \dfrac{m_2 g}{4m_1 + m_2}$

P5.34 (a) 0.529 m (b) 7.40 m/s upward

P5.36 $\mu_s = 0.306$; $\mu_k = 0.245$

P5.38 (a) 3.34 (b) time would increase

P5.40 see the solution (a) 55.2° (b) 167 N

P5.42 (a) 2.31 m/s^2 down for m_1, left for m_2 and up for m_3 (b) 30.0 N and 24.2 N

P5.44 (a) Any value between 31.7 N and 48.6 N (b) If $P > 48.6$ N, the block slides up the wall. If $P < 31.7$ N, the block slides down the wall. (c) $P \geq 62.7$ N. The block cannot slide up the wall. If $P < 62.7$ N, the block slides down the wall.

P5.46 6.84 m

P5.48 0.060 0 m

P5.50 (a) $a = 2.84\ \text{m/s}^2$; $T = 26.5$ N (b) A 3.80-kg object and a 6.50-kg object are joined by a light string passing over a light frictionless pulley. The 3.80-kg object is hanging and tows the heavier object down a ramp inclined at 13.0°, with which it has coefficient of kinetic friction 0.360. See the solution.

P5.52 (a) Pat and the seat accelerate down at $0.426\ \text{m/s}^2$. The other child accelerates up off the ground at the same rate. (b) The tension throughout the rope becomes 480 N, larger than 250 N.

P5.54 (a) $2.00\ \text{m/s}^2$ to the right (b) 8.00 N right on 4 kg; 6.00 N right on 3 kg; 4.00 N right on 2 kg (c) 8.00 N between 4 kg and 3 kg; 14.0 N between 2 kg and 3 kg (d) see the solution

P5.56 (a) $3.43\ \text{m/s}^2$ toward the endstop (b) $3.43\ \text{m/s}^2$ toward the green glider; $6.86\ \text{m/s}^2$ toward the black glider

P5.58 (a) $0.931\ \text{m/s}^2$ (b) From a value of $0.625\ \text{m/s}^2$ for large x, the acceleration gradually increases, passes through a maximum, and then drops more rapidly, becoming negative and reaching $-2.10\ \text{m/s}^2$ at $x = 0$. (c) $0.976\ \text{m/s}^2$ at $x = 25.0$ cm (d) 6.10 cm.

P5.60 They do not; 29.4 N

P5.62 (a) $4.90\ \text{m/s}^2$ (b) 3.13 m/s at 30.0° below the horizontal (c) 1.35 m (d) 1.14 s (e) No

P5.64 see the solution; $0.143\ \text{m/s}^2$ agrees with $0.137\ \text{m/s}^2$

P5.66 (a) When block 2 moves down 1 cm, block 1 moves 2 cm forward, so block 1 always has twice the speed of block 2, and $a_1 = 2\,a_2$. (b) $a_2 = 12.7\ \text{N}\,(1.30\ \text{kg} + 4m_1)^{-1}$ down (c) $3.64\ \text{m/s}^2$ down (d) a_2 approaches $9.80\ \text{m/s}^2$ down (e) a_2 approaches zero. (f) 6.37 N (g) Yes. As m_1 approaches zero, block 2 is essentially in free fall. As m_2 becomes negligible compared to m_1, the system is nearly in equilibrium.

P5.68 $mg\cos\theta\sin\theta$ to the right $+\left(M + m\cos^2\theta\right)g$ upward

P5.70 see the solution

6

Circular Motion and Other Applications of Newton's Laws

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

***Q6.1** (i) nonzero. Its direction of motion is changing. (ii) zero. Its speed is not changing. (iii) zero: when $v = 0$, $v^2/r = 0$. (iv) nonzero: its velocity is changing from, say 0.1 m/s north to 0.1 m/s south.

Q6.2 (a) The object will move in a circle at a constant speed.
(b) The object will move in a straight line at a changing speed.

Q6.3 The speed changes. The tangential force component causes tangential acceleration.

***Q6.4** (a) A > C = D > B = E. At constant speed, centripetal acceleration is largest when radius is smallest. A straight path has infinite radius of curvature. (b) Velocity is north at A, west at B, and south at C. (c) Acceleration is west at A, nonexistent at B, and east at C, to be radially inward.

***Q6.5** (a) yes, point C. Total acceleration here is centripetal acceleration, straight up. (b) yes, point A. Total acceleration here is tangential acceleration, to the right and downward perpendicular to the cord. (c) No. (d) yes, point B. Total acceleration here is to the right and upward.

Q6.6 I would not accept that statement for two reasons. First, to be "beyond the pull of gravity," one would have to be infinitely far away from all other matter. Second, astronauts in orbit are moving in a circular path. It is the gravitational pull of Earth on the astronauts that keeps them in orbit. In the space shuttle, just above the atmosphere, gravity is only slightly weaker than at the Earth's surface. Gravity does its job most clearly on an orbiting spacecraft, because the craft feels no other forces and is in free fall.

Q6.7 This is the same principle as the centrifuge. All the material inside the cylinder tends to move along a straight-line path, but the walls of the cylinder exert an inward force to keep everything moving around in a circular path.

Q6.8 The water has inertia. The water tends to move along a straight line, but the bucket pulls it in and around in a circle.

Q6.9 Blood pressure cannot supply the force necessary both to balance the gravitational force and to provide the centripetal acceleration, to keep blood flowing up to the pilot's brain.

***Q6.10** (a) The keys shift backward relative to the student's hand. The cord then pulls the keys upward and forward, to make them gain speed horizontally forward along with the airplane.
(b) The angle stays constant while the plane has constant acceleration.
This experiment is described in the book *Science from Your Airplane Window* by Elizabeth Wood.

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Q6.11 The person in the elevator is in an accelerating reference frame. The apparent acceleration due to gravity, "g," is changed inside the elevator. "g" $= g \pm a$

Q6.12 From the proportionality of the drag force to the speed squared and from Newton's second law, we derive the equation that describes the motion of the skydiver:

$$m\frac{dv_y}{dt} = mg - \frac{D\rho A}{2}v_y^2$$

where D is the coefficient of drag of the parachutist, and A is the projected area of the parachutist's body. At terminal speed,

$$a_y = \frac{dv_y}{dt} = 0 \quad \text{and} \quad v_T = \left(\frac{2mg}{D\rho A}\right)^{1/2}$$

When the parachute opens, the coefficient of drag D and the effective area A both increase, thus reducing the speed of the skydiver.

Modern parachutes also add a third term, lift, to change the equation to

$$m\frac{dv_y}{dt} = mg - \frac{D\rho A}{2}v_y^2 - \frac{L\rho A}{2}v_x^2$$

where v_y is the vertical velocity, and v_x is the horizontal velocity. The effect of lift is clearly seen in the "paraplane," an ultralight airplane made from a fan, a chair, and a parachute.

Q6.13 (a) Static friction exerted by the roadway where it meets the rubber tires accelerates the car forward and then maintains its speed by counterbalancing resistance forces. (b) The air around the propeller pushes forward on its blades. Evidence is that the propeller blade pushes the air toward the back of the plane. (c) The water pushes the blade of the oar toward the bow. Evidence is that the blade of the oar pushes the water toward the stern.

Q6.14 The larger drop has higher terminal speed. In the case of spheres, the text demonstrates that terminal speed is proportional to the square root of radius. When moving with terminal speed, an object is in equilibrium and has zero acceleration.

***Q6.15** (a) Speed increases, before she reaches terminal speed. (b) The magnitude of acceleration decreases, as the air resistance force increases to counterbalance more and more of the gravitational force.

Q6.16 The thesis is false. The moment of decay of a radioactive atomic nucleus (for example) cannot be predicted. Quantum mechanics implies that the future is indeterminate. On the other hand, our sense of free will, of being able to make choices for ourselves that can appear to be random, may be an illusion. It may have nothing to do with the subatomic randomness described by quantum mechanics.

SOLUTIONS TO PROBLEMS

Section 6.1 **Newton's Second Law Applied to Uniform Circular Motion**

P6.1 $m = 3.00$ kg, $r = 0.800$ m. The string will break if the tension exceeds the weight corresponding to 25.0 kg, so

$$T_{\max} = Mg = 25.0(9.80) = 245 \text{ N}$$

When the 3.00 kg mass rotates in a horizontal circle, the tension causes the centripetal acceleration,

so $\quad T = \dfrac{mv^2}{r} = \dfrac{(3.00)v^2}{0.800}$

Then

$$v^2 = \frac{rT}{m} = \frac{(0.800)T}{3.00} \le \frac{(0.800)T_{\max}}{3.00} = \frac{0.800(245)}{3.00} = 65.3 \text{ m}^2/\text{s}^2$$

and $\quad 0 \le v \le \sqrt{65.3}$

or $\quad \boxed{0 \le v \le 8.08 \text{ m/s}}$

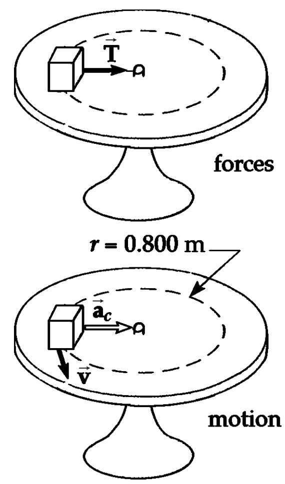

FIG. P6.1

P6.2 In $\sum F = m\dfrac{v^2}{r}$, both m and r are unknown but remain constant. Therefore, $\sum F$ is proportional to v^2 and increases by a factor of $\left(\dfrac{18.0}{14.0}\right)^2$ as v increases from 14.0 m/s to 18.0 m/s. The total force at the higher speed is then

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2 (130 \text{ N}) = 215 \text{ N}$$

Symbolically, write $\sum F_{\text{slow}} = \left(\dfrac{m}{r}\right)(14.0 \text{ m/s})^2$ and $\sum F_{\text{fast}} = \left(\dfrac{m}{r}\right)(18.0 \text{ m/s})^2$.

Dividing gives $\dfrac{\sum F_{\text{fast}}}{\sum F_{\text{slow}}} = \left(\dfrac{18.0}{14.0}\right)^2$, or

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2 \sum F_{\text{slow}} = \left(\frac{18.0}{14.0}\right)^2 (130 \text{ N}) = \boxed{215 \text{ N}}$$

This force must be $\boxed{\text{horizontally inward}}$ to produce the driver's centripetal acceleration.

P6.3 (a) $\quad F = \dfrac{mv^2}{r} = \dfrac{\left(9.11\times10^{-31} \text{ kg}\right)\left(2.20\times10^{6} \text{ m/s}\right)^2}{0.530\times10^{-10} \text{ m}} = \boxed{8.32\times10^{-8} \text{ N inward}}$

(b) $\quad a = \dfrac{v^2}{r} = \dfrac{\left(2.20\times10^{6} \text{ m/s}\right)^2}{0.530\times10^{-10} \text{ m}} = \boxed{9.13\times10^{22} \text{ m/s}^2 \text{ inward}}$

P6.4 (a) $\quad \sum F_y = ma_y,\; mg_{\text{moon}} \text{ down} = \dfrac{mv^2}{r} \text{ down}$

$$v = \sqrt{g_{\text{moon}} r} = \sqrt{\left(1.52 \text{ m/s}^2\right)\left(1.7\times10^{6} \text{ m} + 100\times10^{3} \text{ m}\right)} = \boxed{1.65\times10^{3} \text{ m/s}}$$

(b) $\quad v = \dfrac{2\pi r}{T},\; T = \dfrac{2\pi\left(1.8\times10^{6} \text{ m}\right)}{1.65\times10^{3} \text{ m/s}} = \boxed{6.84\times10^{3} \text{ s}} = 1.90 \text{ h}$

P6.5 (a) $\boxed{\text{static friction}}$

(b) $ma\hat{\mathbf{i}} = f\,\hat{\mathbf{i}} + n\hat{\mathbf{j}} + mg\left(-\hat{\mathbf{j}}\right)$

$\sum F_y = 0 = n - mg$

thus $n = mg$ and $\sum F_r = m\dfrac{v^2}{r} = f = \mu n = \mu mg$.

Then $\mu = \dfrac{v^2}{rg} = \dfrac{(50.0\ \text{cm/s})^2}{(30.0\ \text{cm})(980\ \text{cm/s}^2)} = \boxed{0.085\,0}$.

P6.6 Neglecting relativistic effects. $F = ma_c = \dfrac{mv^2}{r}$

$$F = \left(2\times1.661\times10^{-27}\ \text{kg}\right)\frac{\left(2.998\times10^{7}\ \text{m/s}\right)^2}{(0.480\ \text{m})} = \boxed{6.22\times10^{-12}\ \text{N}}$$

P6.7 Standing on the inner surface of the rim, and moving with it, each person will feel a normal force exerted by the rim. This inward force causes the $3.00\ \text{m/s}^2$ centripetal acceleration:

$a_c = v^2/r$ so $v = \sqrt{a_c r} = \sqrt{(3.00\ \text{m/s}^2)(60.0\ \text{m})} = 13.4\ \text{m/s}$

The period of rotation comes from $v = \dfrac{2\pi r}{T}$: $T = \dfrac{2\pi r}{v} = \dfrac{2\pi(60.0\ \text{m})}{13.4\ \text{m/s}} = 28.1\ \text{s}$

so the frequency of rotation is $f = \dfrac{1}{T} = \dfrac{1}{28.1\ \text{s}} = \dfrac{1}{28.1\ \text{s}}\left(\dfrac{60\ \text{s}}{1\ \text{min}}\right) = \boxed{2.14\ \text{rev/min}}$.

P6.8 $T\cos 5.00° = mg = (80.0\ \text{kg})(9.80\ \text{m/s}^2)$

(a) $T = 787$ N: $\vec{\mathbf{T}} = \boxed{(68.6\ \text{N})\hat{\mathbf{i}} + (784\ \text{N})\hat{\mathbf{j}}}$

(b) $T\sin 5.00° = ma_c$: $\boxed{a_c = 0.857\ \text{m/s}^2}$ toward the center of the circle.

The length of the wire is unnecessary information. We could, on the other hand, use it to find the radius of the circle, the speed of the bob, and the period of the motion.

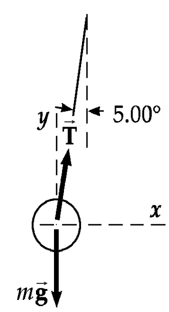

FIG. P6.8

P6.9 $n = mg$ since $a_y = 0$

The force causing the centripetal acceleration is the frictional force f.

From Newton's second law $f = ma_c = \dfrac{mv^2}{r}$.

But the friction condition is $f \le \mu_s n$

i.e., $\dfrac{mv^2}{r} \le \mu_s mg$

$v \le \sqrt{\mu_s rg} = \sqrt{0.600(35.0\ \text{m})(9.80\ \text{m/s}^2)}$ $v \le \boxed{14.3\ \text{m/s}}$

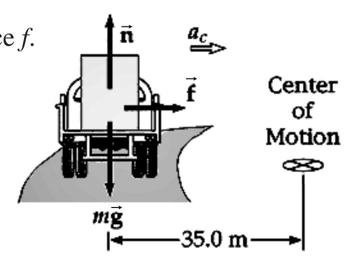

FIG. P6.9

P6.10 (a) $v = \dfrac{235\text{ m}}{36.0\text{ s}} = \boxed{6.53\text{ m/s}}$

The radius is given by $\dfrac{1}{4}2\pi r = 235\text{ m}$ so $r = 150\text{ m}$

(b) $\vec{\mathbf{a}}_r = \left(\dfrac{v^2}{r}\right)$ toward center

$$= \frac{(6.53\text{ m/s})^2}{150\text{ m}} \text{ at } 35.0° \text{ north of west}$$

$$= \left(0.285\text{ m/s}^2\right)\left(\cos 35.0°\left(-\hat{\mathbf{i}}\right) + \sin 35.0°\hat{\mathbf{j}}\right)$$

$$= \boxed{-0.233\text{ m/s}^2\,\hat{\mathbf{i}} + 0.163\text{ m/s}^2\,\hat{\mathbf{j}}}$$

(c) $\vec{\mathbf{a}}_{avg} = \dfrac{\left(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i\right)}{t}$

$$= \frac{\left(6.53\text{ m/s}\,\hat{\mathbf{j}} - 6.53\text{ m/s}\,\hat{\mathbf{i}}\right)}{36.0\text{ s}}$$

$$= \boxed{-0.181\text{ m/s}^2\,\hat{\mathbf{i}} + 0.181\text{ m/s}^2\,\hat{\mathbf{j}}}$$

P6.11 $F_g = mg = (4\text{ kg})\left(9.8\text{ m/s}^2\right) = 39.2\text{ N}$

$\sin\theta = \dfrac{1.5\text{ m}}{2\text{ m}}$

$\theta = 48.6°$

$r = (2\text{ m})\cos 48.6° = 1.32\text{ m}$

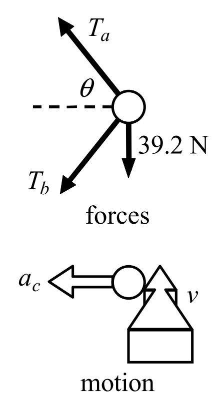

FIG. P6.11

$$\sum F_x = ma_x = \frac{mv^2}{r}$$

$$T_a\cos 48.6° + T_b\cos 48.6° = \frac{(4\text{ kg})(6\text{ m/s})^2}{1.32\text{ m}}$$

$$T_a + T_b = \frac{109\text{ N}}{\cos 48.6°} = 165\text{ N}$$

$$\sum F_y = ma_y$$

$$+T_a\sin 48.6° - T_b\sin 48.6° - 39.2\text{ N} = 0$$

$$T_a - T_b = \frac{39.2\text{ N}}{\sin 48.6°} = 52.3\text{ N}$$

(a) To solve simultaneously, we add the equations in T_a and T_b:

$$T_a + T_b + T_a - T_b = 165\text{ N} + 52.3\text{ N}$$

$$T_a = \frac{217\text{ N}}{2} = \boxed{108\text{ N}}$$

(b) $T_b = 165\text{ N} - T_a = 165\text{ N} - 108\text{ N} = \boxed{56.2\text{ N}}$

Section 6.2 Nonuniform Circular Motion

P6.12 (a) $a_c = \dfrac{v^2}{r} = \dfrac{(4.00\ \text{m/s})^2}{12.0\ \text{m}} = \boxed{1.33\ \text{m/s}^2}$

(b) $a = \sqrt{a_c^2 + a_t^2}$

$$a = \sqrt{(1.33)^2 + (1.20)^2} = \boxed{1.79\ \text{m/s}^2}$$

at an angle $\theta = \tan^{-1}\left(\dfrac{a_c}{a_t}\right) = \boxed{48.0°\ \text{inward}}$

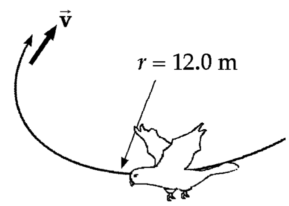

FIG. P6.12

P6.13 $M = 40.0$ kg, $R = 3.00$ m, $T = 350$ N

(a) $\sum F = 2T - Mg = \dfrac{Mv^2}{R}$

$$v^2 = (2T - Mg)\left(\frac{R}{M}\right)$$

$$v^2 = [700 - (40.0)(9.80)]\left(\frac{3.00}{40.0}\right) = 23.1\ (\text{m}^2/\text{s}^2)$$

$$\boxed{v = 4.81\ \text{m/s}}$$

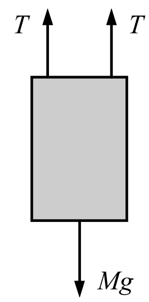

FIG. P6.13(a)

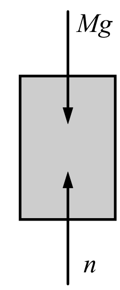

FIG. P6.13(b)

(b) $n - Mg = F = \dfrac{Mv^2}{R}$

$$n = Mg + \frac{Mv^2}{R} = 40.0\left(9.80 + \frac{23.1}{3.00}\right) = \boxed{700\ \text{N}}$$

P6.14 (a) $v = 20.0$ m/s,

n = force of track on roller coaster, and

$R = 10.0$ m.

$$\sum F = \frac{Mv^2}{R} = n - Mg$$

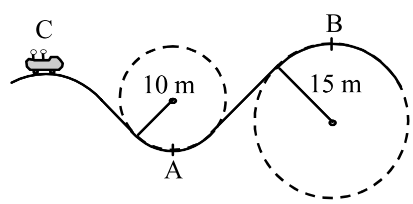

FIG. P6.14

From this we find

$$n = Mg + \frac{Mv^2}{R} = (500\ \text{kg})(9.80\ \text{m/s}^2) + \frac{(500\ \text{kg})(20.0\ \text{m/s}^2)}{10.0\ \text{m}}$$

$$n = 4\,900\ \text{N} + 20\,000\ \text{N} = \boxed{2.49 \times 10^4\ \text{N}}$$

(b) At B, $n - Mg = -\dfrac{Mv^2}{R}$

The maximum speed at B corresponds to

$$n = 0$$

$$-Mg = -\frac{Mv_{\max}^2}{R} \Rightarrow v_{\max} = \sqrt{Rg} = \sqrt{15.0(9.80)} = \boxed{12.1\ \text{m/s}}$$

P6.15 Let the tension at the lowest point be T.

$$\sum F = ma: \quad T - mg = ma_c = \frac{mv^2}{r}$$

$$T = m\left(g + \frac{v^2}{r}\right)$$

$$T = (85.0\text{ kg})\left[9.80\text{ m/s}^2 + \frac{(8.00\text{ m/s})^2}{10.0\text{ m}}\right] = 1.38\text{ kN} > 1\,000\text{ N}$$

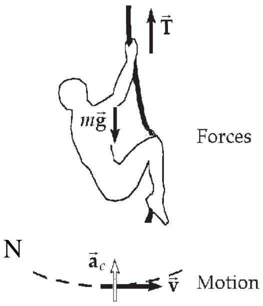

FIG. P6.15

$\boxed{\text{He doesn't make it across the river because the vine breaks.}}$

***P6.16** (a) Consider radial forces on the object, taking inward as positive.

$\Sigma F_r = ma_r$:

$T - (0.5\text{ kg})(9.8\text{ m/s}^2)\cos 20° = mv^2/r = 0.5\text{ kg}(8\text{ m/s})^2/2\text{ m}$

$T = 4.60\text{ N} + 16.0\text{ N} = \boxed{20.6\text{ N}}$

(b) We already found the radial component of acceleration,

$(8\text{ m/s})^2/2\text{ m} = \boxed{32.0\text{ m/s}^2\text{ inward}}$.

Consider tangential forces on the object.

$\Sigma F_t = ma_t$:

$(0.5\text{ kg})(9.8\text{ m/s}^2)\sin 20 = 0.5\text{ kg } a_t$

$a_t = \boxed{3.35\text{ m/s}^2\text{ downward tangent to the circle}}$

(c) $a = [32^2 + 3.35^2]^{1/2}\text{ m/s}^2$ inward and below the cord at angle $\tan^{-1}(3.35/32)$

$= \boxed{32.2\text{ m/s}^2\text{ inward and below the cord at }5.98°}$

(d) No change. If the object is swinging down it is gaining speed. If the object is swinging up it is losing speed but its acceleration is the same size and its direction can be described in the same terms.

P6.17 $$\sum F_y = \frac{mv^2}{r} = mg + n$$

But $n = 0$ at this minimum speed condition, so

$$\frac{mv^2}{r} = mg \Rightarrow v = \sqrt{gr} = \sqrt{(9.80\text{ m/s}^2)(1.00\text{ m})} = \boxed{3.13\text{ m/s}}$$

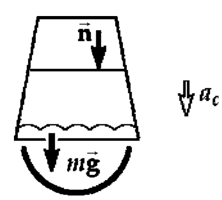

FIG. P6.17

P6.18 (a) $a_c = \dfrac{v^2}{r} \qquad r = \dfrac{v^2}{a_c} = \dfrac{(13.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = \boxed{8.62\text{ m}}$

(b) Let n be the force exerted by the rail.
Newton's second law gives

$$Mg + n = \frac{Mv^2}{r}$$

$$n = M\left(\frac{v^2}{r} - g\right) = M(2g - g) = \boxed{Mg\text{, downward}}$$

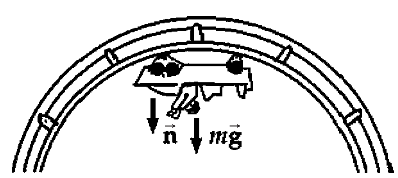

FIG. P6.18

continued on next page

(c) $a_c = \dfrac{v^2}{r}$ $\quad a_c = \dfrac{(13.0\ \text{m/s})^2}{20.0\ \text{m}} = \boxed{8.45\ \text{m/s}^2}$

If the force exerted by the rail is n_1

then $\quad n_1 + Mg = \dfrac{Mv^2}{r} = Ma_c$

$n_1 = M(a_c - g)$ which is < 0 since $a_c = 8.45\ \text{m/s}^2$

Thus, the normal force would have to point away from the center of the curve. Unless they have belts, the riders will fall from the cars.

To be safe we must require n_1 to be positive. Then $a_c > g$. We need

$$\frac{v^2}{r} > g \text{ or } v > \sqrt{rg} = \sqrt{(20.0\ \text{m})(9.80\ \text{m/s}^2)},\ v > 14.0\ \text{m/s}$$

Section 6.3 **Motion in Accelerated Frames**

P6.19 (a) $\sum F_x = Ma,$

$a = \dfrac{T}{M} = \dfrac{18.0\ \text{N}}{5.00\ \text{kg}} = \boxed{3.60\ \text{m/s}^2}$ to the right.

(b) If $v =$ const, $a = 0$, so $\boxed{T = 0}$

(This is also an equilibrium situation.)

(c) Someone in the car (noninertial observer) claims that the forces on the mass along x are T and a fictitious force ($-Ma$). Someone at rest outside the car (inertial observer) claims that T is the only force on M in the x-direction.

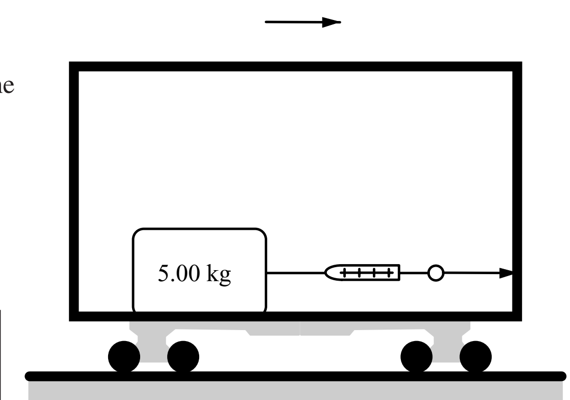

FIG. P6.19

P6.20 The water moves at speed

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.12\ \text{m})}{7.25\ \text{s}} = 0.104\ \text{m/s}.$$

The top layer of water feels a downward force of gravity mg and an outward fictitious force in the turntable frame of reference,

$$\frac{mv^2}{r} = \frac{m(0.104\ \text{m/s})^2}{0.12\ \text{m}} = m9.01\times10^{-2}\ \text{m/s}^2$$

It behaves as if it were stationary in a gravity field pointing downward and outward at

$$\tan^{-1}\frac{0.090\,1\ \text{m/s}^2}{9.8\ \text{m/s}^2} = \boxed{0.527°}$$

Its surface slopes upward toward the outside, making this angle with the horizontal.

P6.21 The only forces acting on the suspended object are the force of gravity $m\vec{g}$ and the force of tension T forward and upward at angle θ with the vertical, as shown in the free-body diagram. Applying Newton's second law in the x and y directions,

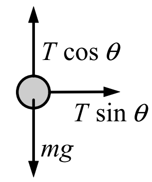

FIG. P6.21

$$\sum F_x = T\sin\theta = ma \qquad (1)$$

$$\sum F_y = T\cos\theta - mg = 0$$

or $$T\cos\theta = mg \qquad (2)$$

(a) Dividing equation (1) by (2) gives

$$\tan\theta = \frac{a}{g} = \frac{3.00\ \text{m/s}^2}{9.80\ \text{m/s}^2} = 0.306$$

Solving for θ, $\theta = \boxed{17.0°}$

(b) From Equation (1),

$$T = \frac{ma}{\sin\theta} = \frac{(0.500\ \text{kg})(3.00\ \text{m/s}^2)}{\sin(17.0°)} = \boxed{5.12\ \text{N}}$$

P6.22 Consider forces on the backpack as it slides in the Earth frame of reference.

$$\sum F_y = ma_y: \quad +n - mg = ma,\ n = m(g+a),\ f_k = \mu_k m(g+a)$$
$$\sum F_x = ma_x: \quad -\mu_k m(g+a) = ma_x$$

The motion across the floor is described by $L = vt + \frac{1}{2}a_x t^2 = vt - \frac{1}{2}\mu_k(g+a)t^2$.

We solve for μ_k: $vt - L = \frac{1}{2}\mu_k(g+a)t^2$, $\boxed{\frac{2(vt-L)}{(g+a)t^2} = \mu_k}$.

P6.23 $F_{\max} = F_g + ma = 591\ \text{N}$

$F_{\min} = F_g - ma = 391\ \text{N}$

(a) Adding, $2F_g = 982\ \text{N}$, $F_g = \boxed{491\ \text{N}}$

(b) Since $F_g = mg$, $m = \frac{491\ \text{N}}{9.80\ \text{m/s}^2} = \boxed{50.1\ \text{kg}}$

(c) Subtracting the above equations,

$2ma = 200\ \text{N} \qquad \therefore a = \boxed{2.00\ \text{m/s}^2}$

P6.24 In an inertial reference frame, the girl is accelerating horizontally inward at

$$\frac{v^2}{r}=\frac{(5.70\ \text{m/s})^2}{2.40\ \text{m}}=13.5\ \text{m/s}^2$$

In her own non-inertial frame, her head feels a horizontally outward fictitious force equal to its mass times this acceleration. Together this force and the weight of her head add to have a magnitude equal to the mass of her head times an acceleration of

$$\sqrt{g^2+\left(\frac{v^2}{r}\right)^2}=\sqrt{(9.80)^2+(13.5)^2}\ \text{m/s}^2=16.7\ \text{m/s}^2$$

This is larger than g by a factor of $\dfrac{16.7}{9.80}=1.71$.

Thus, the force required to lift her head is larger by this factor, or the required force is

$$F=1.71(55.0\ \text{N})=\boxed{93.8\ \text{N}}.$$

P6.25 $a_r=\left(\dfrac{4\pi^2R_e}{T^2}\right)\cos 35.0°=0.027\,6\ \text{m/s}^2$

We take the y axis along the local vertical.

$(a_{\text{net}})_y=9.80-(a_r)_y=9.77\ \text{m/s}^2$

$(a_{\text{net}})_x=0.015\,8\ \text{m/s}^2$

$\theta=\arctan\dfrac{a_x}{a_y}=\boxed{0.092\,8°}$

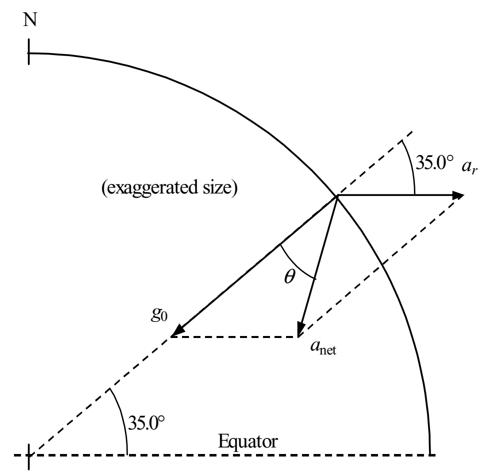

FIG. P6.25

Section 6.4 **Motion in the Presence of Resistive Forces**

P6.26 $m=80.0\ \text{kg},\ v_T=50.0\ \text{m/s},\ mg=\dfrac{D\rho A v_T^2}{2}\ \therefore\ \dfrac{D\rho A}{2}=\dfrac{mg}{v_T^2}=0.314\ \text{kg/m}$

(a) At $v=30.0\ \text{m/s}$

$$a=g-\frac{D\rho A v^2/2}{m}=9.80-\frac{(0.314)(30.0)^2}{80.0}=\boxed{6.27\ \text{m/s}^2\text{downward}}$$

(b) At $v=50.0\ \text{m/s}$, terminal velocity has been reached.

$$\sum F_y=0=mg-R$$

$$\Rightarrow R=mg=(80.0\ \text{kg})(9.80\ \text{m/s}^2)=\boxed{784\ \text{N directed up}}$$

(c) At $v=30.0\ \text{m/s}$

$$\frac{D\rho A v^2}{2}=(0.314)(30.0)^2=\boxed{283\ \text{N}}\ \text{upward}$$

P6.27 (a) $a = g - bv$

When $v = v_T$, $a = 0$ and $g = bv_T$ $\qquad b = \dfrac{g}{v_T}$

The Styrofoam falls 1.50 m at constant speed v_T in 5.00 s.

Thus,

$$v_T = \frac{y}{t} = \frac{1.50\ \text{m}}{5.00\ \text{s}} = 0.300\ \text{m/s}$$

Then

$$b = \frac{9.80\ \text{m/s}^2}{0.300\ \text{m/s}} = \boxed{32.7\ \text{s}^{-1}}$$

(b) At $t = 0$, $v = 0$ and $\quad a = g = \boxed{9.80\ \text{m/s}^2}$ down

(c) When $v = 0.150\ \text{m/s}$, $a = g - bv = 9.80\ \text{m/s}^2 - (32.7\ \text{s}^{-1})(0.150\ \text{m/s}) = \boxed{4.90\ \text{m/s}^2}$ down

P6.28 (a) $\rho = \dfrac{m}{V}$, $A = 0.020\,1\ \text{m}^2$, $R = \dfrac{1}{2}\rho_{\text{air}} A D v_T^2 = mg$

$$m = \rho_{\text{bead}} V = 0.830\ \text{g/cm}^3 \left[\frac{4}{3}\pi (8.00\ \text{cm})^3\right] = 1.78\ \text{kg}$$

Assuming a drag coefficient of $D = 0.500$ for this spherical object, and taking the density of air at 20°C from the endpapers, we have

$$v_T = \sqrt{\frac{2(1.78\ \text{kg})(9.80\ \text{m/s}^2)}{0.500(1.20\ \text{kg/m}^3)(0.020\,1\ \text{m}^2)}} = \boxed{53.8\ \text{m/s}}$$

(b) $v_f^2 = v_i^2 + 2gh = 0 + 2gh$: $h = \dfrac{v_f^2}{2g} = \dfrac{(53.8\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = \boxed{148\ \text{m}}$

P6.29 Since the upward velocity is constant, the resultant force on the ball is zero. Thus, the upward applied force equals the sum of the gravitational and drag forces (both downward): $F = mg + bv$.

The mass of the copper ball is

$$m = \frac{4\pi\rho r^3}{3} = \left(\frac{4}{3}\right)\pi (8.92 \times 10^3\ \text{kg/m}^3)(2.00 \times 10^{-2}\ \text{m})^3 = 0.299\ \text{kg}$$

The applied force is then

$$F = mg + bv = (0.299)(9.80) + (0.950)(9.00 \times 10^{-2}) = \boxed{3.01\ \text{N}}$$

P6.30 The resistive force is

$$R = \frac{1}{2} D\rho A v^2 = \frac{1}{2}(0.250)(1.20\ \text{kg/m}^3)(2.20\ \text{m}^2)(27.8\ \text{m/s})^2$$
$$R = 255\ \text{N}$$
$$a = -\frac{R}{m} = -\frac{255\ \text{N}}{1200\ \text{kg}} = \boxed{-0.212\ \text{m/s}^2}$$

P6.31 (a) At terminal velocity, $R = v_T b = mg$

$$\therefore b = \frac{mg}{v_T} = \frac{(3.00\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{2.00\times10^{-2}\ \text{m/s}} = \boxed{1.47\ \text{N}\cdot\text{s/m}}$$

(b) In the equation describing the time variation of the velocity, we have

$v = v_T\left(1-e^{-bt/m}\right)$ $\quad v = 0.632v_T$ when $e^{-bt/m} = 0.368$

or at time $\quad t = -\left(\frac{m}{b}\right)\ln(0.368) = \boxed{2.04\times10^{-3}\ \text{s}}$

(c) At terminal velocity, $R = v_T b = mg = \boxed{2.94\times10^{-2}\ \text{N}}$

***P6.32** (a) Since the window is vertical, the normal force is horizontal. $n = 4$ N
$f_k = \mu_k n = 0.9(4\ \text{N}) = 3.6\ \text{N}$ upward, to oppose downward motion
$\Sigma F_y = ma_y$: $\quad +3.6\ \text{N} - (0.16\ \text{kg})(9.8\ \text{m/s}^2) + P_y = 0 \qquad P_y = -2.03\ \text{N} = \boxed{2.03\ \text{N down}}$

(b) $\Sigma F_y = ma_y$: $\quad +3.6\ \text{N} - (0.16\ \text{kg})(9.8\ \text{m/s}^2) - 1.25(2.03\ \text{N}) = 0.16\ \text{kg}\ a_y$

$a_y = -0.508\ \text{N}/0.16\ \text{kg} = -3.18\ \text{m/s}^2 = \boxed{3.18\ \text{m/s}^2\ \text{down}}$

(c) At terminal velocity, $\Sigma F_y = ma_y$: $+(20\ \text{N}\cdot\text{s/m})v_T - (0.16\ \text{kg})(9.8\ \text{m/s}^2) - 1.25(2.03\ \text{N}) = 0$

$v_T = 4.11\ \text{N}/(20\ \text{N}\cdot\text{s/m}) = \boxed{0.205\ \text{m/s down}}$

P6.33 $v = \left(\frac{mg}{b}\right)\left[1-\exp\left(\frac{-bt}{m}\right)\right]$ where $\exp(x) = e^x$ is the exponential function.

At $t\to\infty$ $\quad v \to v_T = \frac{mg}{b}$

At $t = 5.54$ s $\quad 0.500v_T = v_T\left[1-\exp\left(\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}}\right)\right]$

$$\exp\left(\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}}\right) = 0.500$$

$$\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}} = \ln 0.500 = -0.693$$

$$b = \frac{(9.00\ \text{kg})(0.693)}{5.54\ \text{s}} = 1.13\ \text{kg/s}$$

(a) $v_T = \frac{mg}{b} \qquad v_T = \frac{(9.00\ \text{kg})(9.80\ \text{m/s}^2)}{1.13\ \text{kg/s}} = \boxed{78.3\ \text{m/s}}$

(b) $0.750v_T = v_T\left[1-\exp\left(\frac{-1.13t}{9.00\ \text{s}}\right)\right] \qquad \exp\left(\frac{-1.13t}{9.00\ \text{s}}\right) = 0.250$

$$t = \frac{9.00(\ln 0.250)}{-1.13}\ \text{s} = \boxed{11.1\ \text{s}}$$

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(c) $\frac{dx}{dt} = \left(\frac{mg}{b}\right)\left[1 - \exp\left(-\frac{bt}{m}\right)\right]; \quad \int_{x_0}^{x} dx = \int_0^t \left(\frac{mg}{b}\right)\left[1 - \exp\left(\frac{-bt}{m}\right)\right] dt$

$$x - x_0 = \frac{mgt}{b} + \left(\frac{m^2 g}{b^2}\right)\exp\left(\frac{-bt}{m}\right)\Big|_0^t = \frac{mgt}{b} + \left(\frac{m^2 g}{b^2}\right)\left[\exp\left(\frac{-bt}{m}\right) - 1\right]$$

At $t = 5.54$ s,

$$x = 9.00 \text{ kg}\left(9.80 \text{ m/s}^2\right)\frac{5.54 \text{ s}}{1.13 \text{ kg/s}} + \left(\frac{(9.00 \text{ kg})^2 \left(9.80 \text{ m/s}^2\right)}{(1.13 \text{ kg/s})^2}\right)\left[\exp(-0.693) - 1\right]$$

$$x = 434 \text{ m} + 626 \text{ m}(-0.500) = \boxed{121 \text{ m}}$$

P6.34 $\sum F = ma$

$$-kmv^2 = m\frac{dv}{dt}$$

$$-kdt = \frac{dv}{v^2}$$

$$-k\int_0^t dt = \int_{v_0}^{v} v^{-2} dv$$

$$-k(t - 0) = \frac{v^{-1}}{-1}\Big|_{v_0}^{v} = -\frac{1}{v} + \frac{1}{v_0}$$

$$\frac{1}{v} = \frac{1}{v_0} + kt = \frac{1 + v_0 kt}{v_0}$$

$$v = \frac{v_0}{1 + v_0 kt}$$

P6.35 (a) From Problem 34,

$$v = \frac{dx}{dt} = \frac{v_0}{1 + v_0 kt}$$

$$\int_0^x dx = \int_0^t v_0 \frac{dt}{1 + v_0 kt} = \frac{1}{k}\int_0^t \frac{v_0 k dt}{1 + v_0 kt}$$

$$x\Big|_0^x = \frac{1}{k}\ln\left(1 + v_0 kt\right)\Big|_0^t$$

$$x - 0 = \frac{1}{k}\left[\ln\left(1 + v_0 kt\right) - \ln 1\right]$$

$$\boxed{x = \frac{1}{k}\ln\left(1 + v_0 kt\right)}$$

(b) We have $\ln\left(1 + v_0 kt\right) = kx$

$1 + v_0 kt = e^{kx}$ so $v = \frac{v_0}{1 + v_0 kt} = \frac{v_0}{e^{kx}} = \boxed{v_0 e^{-kx} = v}$

P6.36 We write $-kmv^2 = -\frac{1}{2}D\rho Av^2$ so

$$k = \frac{D\rho A}{2m} = \frac{0.305\left(1.20\ \text{kg/m}^3\right)\left(4.2\times10^{-3}\ \text{m}^2\right)}{2\left(0.145\ \text{kg}\right)} = 5.3\times10^{-3}/\text{m}$$

$$v = v_0e^{-kx} = \left(40.2\ \text{m/s}\right)e^{-\left(5.3\times10^{-3}/\text{m}\right)(18.3\ \text{m})} = \boxed{36.5\ \text{m/s}}$$

P6.37 (a) $v(t) = v_ie^{-ct}$ $\quad v(20.0\ \text{s}) = 5.00 = v_ie^{-20.0c}$, $v_i = 10.0\ \text{m/s}$

So $5.00 = 10.0e^{-20.0c}$ and $-20.0c = \ln\left(\frac{1}{2}\right)$ $\quad c = -\frac{\ln\left(\frac{1}{2}\right)}{20.0} = \boxed{3.47\times10^{-2}\ \text{s}^{-1}}$

(b) At $t = 40.0$ s $\quad v = (10.0\ \text{m/s})e^{-40.0c} = (10.0\ \text{m/s})(0.250) = \boxed{2.50\ \text{m/s}}$

(c) $v = v_ie^{-ct}$ $\quad a = \frac{dv}{dt} = -cv_ie^{-ct} = \boxed{-cv}$

P6.38 In $R = \frac{1}{2}D\rho Av^2$, we estimate that $D = 1.00$, $\rho = 1.20\ \text{kg/m}^3$,

$A = (0.100\ \text{m})(0.160\ \text{m}) = 1.60\times10^{-2}\ \text{m}^2$ and $v = 27.0\ \text{m/s}$. The resistance force is then

$$R = \frac{1}{2}(1.00)\left(1.20\ \text{kg/m}^3\right)\left(1.60\times10^{-2}\ \text{m}^2\right)(27.0\ \text{m/s})^2 = 7.00\ \text{N}$$

or

$$R \sim \boxed{10^1\ \text{N}}$$

Additional Problems

***P6.39** Let v_0 represent the speed of the object at time 0. We have

$$\int_{v_0}^{v}\frac{dv}{v} = -\frac{b}{m}\int_0^t dt \qquad \ln v\Big|_{v_0}^{v} = -\frac{b}{m}t\Big|_0^t$$

$$\ln v - \ln v_0 = -\frac{b}{m}(t-0) \qquad \ln(v/v_0) = -\frac{bt}{m}$$

$$v/v_0 = e^{-bt/m} \qquad \boxed{v = v_0e^{-bt/m}}$$

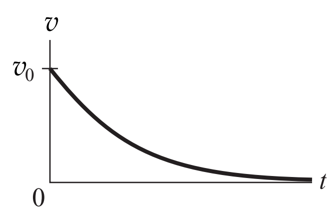

FIG. P6.39

From its original value, the speed decreases rapidly at first and then more and more slowly, asymptotically approaching zero.

In this model the object keeps losing speed forever. It travels a finite distance in stopping.

The distance it travels is given by

$$\int_0^r dr = v_0\int_0^t e^{-bt/m}dt$$

$$r = -\frac{m}{b}v_0\int_0^t e^{-bt/m}\left(-\frac{b}{m}dt\right) = -\frac{m}{b}v_0\,e^{-bt/m}\Big|_0^t = -\frac{m}{b}v_0\left(e^{-bt/m}-1\right) = \frac{mv_0}{b}\left(1-e^{-bt/m}\right)$$

As t goes to infinity, the distance approaches $\frac{mv_0}{b}(1-0) = mv_0/b$

P6.40 At the top of the vertical circle,

$$T = m\frac{v^2}{R} - mg$$

or $T = (0.400)\dfrac{(4.00)^2}{0.500} - (0.400)(9.80) = \boxed{8.88\ \text{N}}$

P6.41 (a) The speed of the bag is $\dfrac{2\pi(7.46\ \text{m})}{38\ \text{s}} = 1.23\ \text{m/s}$.

The total force on it must add to

$$ma_c = \frac{(30\ \text{kg})(1.23\ \text{m/s})^2}{7.46\ \text{m}} = 6.12\ \text{N}$$

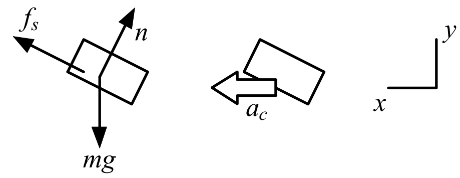

FIG. P6.41

$$\sum F_x = ma_x: \quad f_s\cos 20 - n\sin 20 = 6.12\ \text{N}$$

$$\sum F_y = ma_y: \quad f_s\sin 20 + n\cos 20 - (30\ \text{kg})(9.8\ \text{m/s}^2) = 0$$

$$n = \frac{f_s\cos 20 - 6.12\ \text{N}}{\sin 20}$$

Substitute:

$$f_s\sin 20 + f_s\frac{\cos^2 20}{\sin 20} - (6.12\ \text{N})\frac{\cos 20}{\sin 20} = 294\ \text{N}$$

$$f_s(2.92) = 294\ \text{N} + 16.8\ \text{N}$$

$$f_s = \boxed{106\ \text{N}}$$

(b) $v = \dfrac{2\pi(7.94\ \text{m})}{34\ \text{s}} = 1.47\ \text{m/s}$

$$ma_c = \frac{(30\ \text{kg})(1.47\ \text{m/s})^2}{7.94\ \text{m}} = 8.13\ \text{N}$$

$$f_s\cos 20 - n\sin 20 = 8.13\ \text{N}$$

$$f_s\sin 20 + n\cos 20 = 294\ \text{N}$$

$$n = \frac{f_s\cos 20 - 8.13\ \text{N}}{\sin 20}$$

$$f_s\sin 20 + f_s\frac{\cos^2 20}{\sin 20} - (8.13\ \text{N})\frac{\cos 20}{\sin 20} = 294\ \text{N}$$

$$f_s(2.92) = 294\ \text{N} + 22.4\ \text{N}$$

$$f_s = 108\ \text{N}$$

$$n = \frac{(108\ \text{N})\cos 20 - 8.13\ \text{N}}{\sin 20} = 273\ \text{N}$$

$$\mu_s = \frac{f_s}{n} = \frac{108\ \text{N}}{273\ \text{N}} = \boxed{0.396}$$

P6.42 When the cloth is at a lower angle θ, the radial component of $\sum F = ma$ reads

$$n + mg\sin\theta = \frac{mv^2}{r}$$

At $\theta = 68.0°$, the normal force drops to zero and $g\sin 68° = \frac{v^2}{r}$.

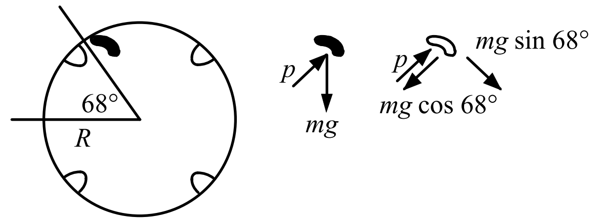

FIG. P6.42

$$v = \sqrt{rg\sin 68°} = \sqrt{(0.33\ \text{m})(9.8\ \text{m/s}^2)\sin 68°} = 1.73\ \text{m/s}$$

The rate of revolution is

$$\text{angular speed} = (1.73\ \text{m/s})\left(\frac{1\ \text{rev}}{2\pi r}\right)\left(\frac{2\pi r}{2\pi(0.33\ \text{m})}\right) = \boxed{0.835\ \text{rev/s}} = 50.1\ \text{rev/min}$$

P6.43 (a) $v = (30\ \text{km/h})\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right)\left(\frac{1\,000\ \text{m}}{1\ \text{km}}\right) = 8.33\ \text{m/s}$

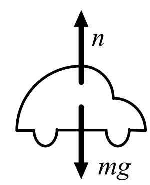

FIG. P6.43

$$\sum F_y = ma_y\text{:}\ +n - mg = -\frac{mv^2}{r}$$

$$n = m\left(g - \frac{v^2}{r}\right) = 1\,800\ \text{kg}\left[9.8\ \text{m/s}^2 - \frac{(8.33\ \text{m/s})^2}{20.4\ \text{m}}\right]$$

$$= \boxed{1.15\times10^4\ \text{N up}}$$

(b) Take $n = 0$. Then $mg = \frac{mv^2}{r}$.

$$v = \sqrt{gr} = \sqrt{(9.8\ \text{m/s}^2)(20.4\ \text{m})} = \boxed{14.1\ \text{m/s}} = 50.9\ \text{km/h}$$

P6.44 (a) $\sum F_y = ma_y = \frac{mv^2}{R}$

$$mg - n = \frac{mv^2}{R} \qquad n = \boxed{mg - \frac{mv^2}{R}}$$

(b) When $n = 0$ $\qquad mg = \frac{mv^2}{R}$

Then,

$$v = \boxed{\sqrt{gR}}$$

A more gently curved bump, with larger radius, allows the car to have a higher speed without leaving the road. This speed is proportional to the square root of the radius.

P6.45 (a) $\text{slope} = \frac{0.160\ \text{N} - 0}{9.9\ \text{m}^2/\text{s}^2} = \boxed{0.016\,2\ \text{kg/m}}$

(b) $\text{slope} = \frac{R}{v^2} = \frac{\frac{1}{2}D\rho A v^2}{v^2} = \boxed{\frac{1}{2}D\rho A}$

continued on next page

(c) $\frac{1}{2}D\rho A = 0.016\,2\ \text{kg/m}$

$$D = \frac{2(0.016\,2\ \text{kg/m})}{(1.20\ \text{kg/m}^3)\pi(0.105\ \text{m})^2} = \boxed{0.778}$$

(d) From the table, the eighth point is at force $mg = 8(1.64\times10^{-3}\ \text{kg})(9.8\ \text{m/s}^2) = 0.129\ \text{N}$ and horizontal coordinate $(2.80\ \text{m/s})^2$. The vertical coordinate of the line is here $(0.016\,2\ \text{kg/m})(2.8\ \text{m/s})^2 = 0.127\ \text{N}$. The scatter percentage is $\frac{0.129\ \text{N} - 0.127\ \text{N}}{0.127\ \text{N}} = \boxed{1.5\%}$.

(e) The interpretation of the graph can be stated thus:

For stacked coffee filters falling at terminal speed, a graph of air resistance force as a function of squared speed demonstrates that the force is proportional to the speed squared within the experimental uncertainty estimated as 2%. This proportionality agrees with that described by the theoretical equation $R = \frac{1}{2}D\rho A v^2$. The value of the constant slope of the graph implies that the drag coefficient for coffee filters is $D = 0.78 \pm 2\%$.

***P6.46** (a) The forces acting on the ice cube are the Earth's gravitational force, straight down, and the basin's normal force, upward and inward at 35° with the vertical. We choose the *x* and *y* axes to be horizontal and vertical, so that the acceleration is purely in the *x* direction. Then

$\sum F_x = ma_x$: $\quad n\sin 35° = mv^2/R$

$\sum F_y = ma_y$: $\quad n\cos 35° - mg = 0$

Dividing eliminates the normal force: $n\sin 35°/n\cos 35° = mv^2/Rmg$

$\tan 35° = v^2/Rg \qquad \boxed{v = \sqrt{Rg\tan 35.0°} = \sqrt{(6.86\ \text{m/s}^2)R}}$

(b) $\boxed{\text{The mass is unnecessary.}}$

(c) The answer to (a) indicates that the speed is proportional to the square root of the radius, so doubling the radius will make the required speed $\boxed{\text{increase by } \sqrt{2} \text{ times}}$.

(d) The period of revolution is given by $T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{Rg\tan 35.0°}} = (2.40\ \text{s}/\sqrt{\text{m}})\sqrt{R}$

When the radius doubles, the period $\boxed{\text{increases by } \sqrt{2} \text{ times}}$.

(e) On the larger circle the ice cube moves $\sqrt{2}$ times faster but also takes longer to get around, because the distance it must travel is 2 times larger. Its period is also proportional to the square root of the radius.

P6.47 Take *x*-axis up the hill

$$\sum F_x = ma_x: \quad +T\sin\theta - mg\sin\phi = ma$$

$$a = \frac{T}{m}\sin\theta - g\sin\phi$$

$$\sum F_y = ma_y: \quad +T\cos\theta - mg\cos\phi = 0$$

$$T = \frac{mg\cos\phi}{\cos\theta}$$

$$a = \frac{g\cos\phi\sin\theta}{\cos\theta} - g\sin\phi$$

$$a = \boxed{g(\cos\phi\tan\theta - \sin\phi)}$$

P6.48 (a) $v = (300 \text{ mi/h})\left(\dfrac{88.0 \text{ ft/s}}{60.0 \text{ mi/h}}\right) = 440 \text{ ft/s}$

At the lowest point, his seat exerts an upward force; therefore, his weight seems to increase. His apparent weight is

$$F_g' = mg + m\frac{v^2}{r} = 160 + \left(\frac{160}{32.0}\right)\frac{(440)^2}{1\,200} = \boxed{967 \text{ lb}}$$

(b) At the highest point, the force of the seat on the pilot is directed down and

$$F_g' = mg - m\frac{v^2}{r} = \boxed{-647 \text{ lb}}$$

Since the plane is upside down, the seat exerts this downward force as a normal force.

(c) When $F_g' = 0$, then $mg = \dfrac{mv^2}{R}$. If we vary the aircraft's R and v such that this equation is satisfied, then the pilot feels weightless.

P6.49 (a) Since the centripetal acceleration of a person is downward (toward the axis of the earth), it is equivalent to the effect of a falling elevator. Therefore,

$$F_g' = F_g - \frac{mv^2}{r} \text{ or } \boxed{F_g > F_g'}$$

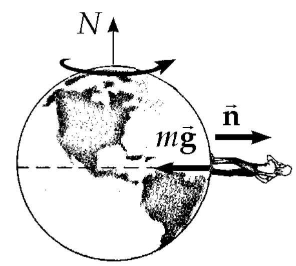

FIG. P6.49

(b) At the poles $v = 0$ and $F_g' = F_g = mg = 75.0(9.80) = \boxed{735 \text{ N}}$ down.

At the equator, $F_g' = F_g - ma_c = 735 \text{ N} - 75.0(0.033\,7) \text{ N} = \boxed{732 \text{ N}}$ down.

***P6.50** (a) Since the object of mass m_2 is in equilibrium, $\sum F_y = T - m_2 g = 0$

or

$$T = \boxed{m_2 g}$$

(b) The tension in the string provides the required centripetal acceleration of the puck.

Thus,

$$F_c = T = \boxed{m_2 g}$$

(c) From

$$F_c = \frac{m_1 v^2}{R}$$

we have

$$v = \sqrt{\frac{RF_c}{m_1}} = \boxed{\sqrt{\left(\frac{m_2}{m_1}\right) gR}}$$

(d) $\boxed{\text{The puck will spiral inward, gaining speed as it does so.}}$ It gains speed because the extra-large string tension produces forward tangential acceleration as well as inward radial acceleration of the puck, pulling at an angle of less than 90° to the direction of the inward-spiraling velocity.

(e) $\boxed{\text{The puck will spiral outward, slowing down as it does so.}}$

***P6.51**

(a) The only horizontal force on the car is the force of friction, with a maximum value determined by the surface roughness (described by the coefficient of static friction) and the normal force (here equal to the gravitational force on the car).

(b) $\sum F_x = ma_x \qquad -f = ma \qquad a = -f/m = (v^2 - v_0^2)/2(x - x_0)$

$x - x_0 = (v^2 - v_0^2)m/2f = (0^2 - [20\text{ m/s}]^2)1200\text{ kg}/2(-7000\text{ N}) = \boxed{34.3\text{ m}}$

(c) Now $f = mv^2/r \qquad r = mv^2/f = 1200\text{ kg }[20\text{ m/s}]^2/7000\text{ N} = \boxed{68.6\text{ m}}$

A top view shows that you can avoid running into the wall by turning through a quarter-circle, if you start at least this far away from the wall.

(d) Braking is better. You should not turn the wheel. If you used any of the available friction force to change the direction of the car, it would be unavailable to slow the car, and the stopping distance would be longer.

(e) The conclusion is true in general. The radius of the curve you can barely make is twice your minimum stopping distance.

P6.52 $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(9.00\text{ m})}{(15.0\text{ s})} = 3.77\text{ m/s}$

(a) $a_r = \dfrac{v^2}{r} = \boxed{1.58\text{ m/s}^2}$

(b) $F_{\text{low}} = m(g + a_r) = \boxed{455\text{ N}}$

(c) $F_{\text{high}} = m(g - a_r) = \boxed{328\text{ N}}$

(d) $F_{\text{mid}} = m\sqrt{g^2 + a_r^2} = \boxed{397\text{ N upward and}}$ at $\theta = \tan^{-1}\dfrac{a_r}{g} = \tan^{-1}\dfrac{1.58}{9.8} = \boxed{9.15°\text{ inward}}$.

P6.53 (a) The mass at the end of the chain is in vertical equilibrium. Thus $T\cos\theta = mg$.

Horizontally $T\sin\theta = ma_r = \dfrac{mv^2}{r}$

$$r = (2.50\sin\theta + 4.00)\text{ m}$$
$$r = (2.50\sin 28.0° + 4.00)\text{ m} = 5.17\text{ m}$$

Then $a_r = \dfrac{v^2}{5.17\text{ m}}$.

By division $\tan\theta = \dfrac{a_r}{g} = \dfrac{v^2}{5.17g}$

$$v^2 = 5.17g\tan\theta = (5.17)(9.80)(\tan 28.0°)\text{ m}^2/\text{s}^2$$
$$v = \boxed{5.19\text{ m/s}}$$

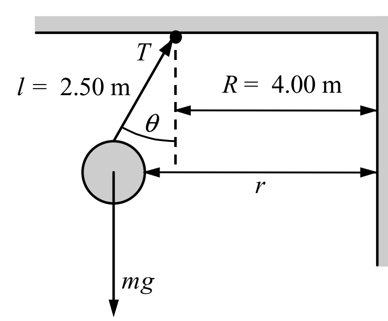

FIG. P6.53

(b) $T\cos\theta = mg$

$$T = \frac{mg}{\cos\theta} = \frac{(50.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 28.0°} = \boxed{555\text{ N}}$$

P6.54 (a) The putty, when dislodged, rises and returns to the original level in time t. To find t, we use $v_f = v_i + at$: i.e., $-v = +v - gt$ or $t = \dfrac{2v}{g}$ where v is the speed of a point on the rim of the wheel.

If R is the radius of the wheel, $v = \dfrac{2\pi R}{t}$, so $t = \dfrac{2v}{g} = \dfrac{2\pi R}{v}$.

Thus, $v^2 = \pi R g$ and $\boxed{v = \sqrt{\pi R g}}$.

(b) The putty is dislodged when F, the force holding it to the wheel, is

$$F = \frac{mv^2}{R} = \boxed{m\pi g}$$

***P6.55** (a) $n = \dfrac{mv^2}{R}$ $\quad f - mg = 0$

$f = \mu_s n \quad v = \dfrac{2\pi R}{T}$

$$T = \boxed{\sqrt{\frac{4\pi^2 R\mu_s}{g}}}$$

(b) $T = \boxed{2.54\ \text{s}}$

$$\#\ \frac{\text{rev}}{\text{min}} = \frac{1\ \text{rev}}{2.54\ \text{s}}\left(\frac{60\ \text{s}}{\text{min}}\right) = \boxed{23.6\ \frac{\text{rev}}{\text{min}}}$$

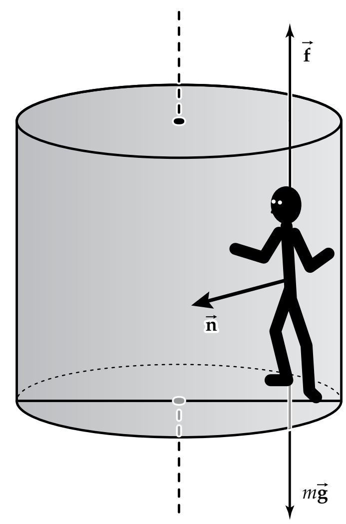

FIG. P6.55

(c) The gravitational and frictional forces remain constant. The normal force increases. The person remains in motion with the wall.

(d) The gravitational force remains constant. The normal and frictional forces decrease. The person slides relative to the wall and downward into the pit.

P6.56 Let the x-axis point eastward, the y-axis upward, and the z-axis point southward.

(a) The range is $Z = \dfrac{v_i^2 \sin 2\theta_i}{g}$

The initial speed of the ball is therefore

$$v_i = \sqrt{\frac{gZ}{\sin 2\theta_i}} = \sqrt{\frac{(9.80)(285)}{\sin 96.0°}} = 53.0\ \text{m/s}$$

The time the ball is in the air is found from $\Delta y = v_{iy}t + \dfrac{1}{2}a_y t^2$ as

$$0 = (53.0\ \text{m/s})(\sin 48.0°)t - (4.90\ \text{m/s}^2)t^2$$

giving $t = \boxed{8.04\ \text{s}}$.

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(b) $v_{ix} = \dfrac{2\pi R_e \cos\phi_i}{86\,400\text{ s}} = \dfrac{2\pi\left(6.37\times10^6\text{ m}\right)\cos 35.0°}{86\,400\text{ s}} = \boxed{379\text{ m/s}}$

(c) 360° of latitude corresponds to a distance of $2\pi R_e$, so 285 m is a change in latitude of

$$\Delta\phi = \left(\frac{S}{2\pi R_e}\right)(360°) = \left(\frac{285\text{ m}}{2\pi\left(6.37\times10^6\text{ m}\right)}\right)(360°) = 2.56\times10^{-3}\text{ degrees}$$

The final latitude is then $\phi_f = \phi_i - \Delta\phi = 35.0° - 0.002\,56° = 34.997\,4°$.

The cup is moving eastward at a speed $v_{fx} = \dfrac{2\pi R_e \cos\phi_f}{86\,400\text{ s}}$, which is larger than the eastward velocity of the tee by

$$\Delta v_x = v_{fx} - v_{fi} = \frac{2\pi R_e}{86\,400\text{ s}}\left[\cos\phi_f - \cos\phi_i\right] = \frac{2\pi R_e}{86\,400\text{ s}}\left[\cos\left(\phi_i - \Delta\phi\right) - \cos\phi_i\right]$$

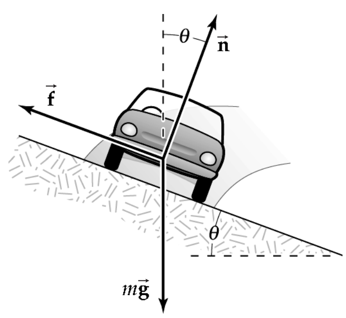
$$= \frac{2\pi R_e}{86\,400\text{ s}}\left[\cos\phi_i \cos\Delta\phi + \sin\phi_i \sin\Delta\phi - \cos\phi_i\right]$$

Since $\Delta\phi$ is such a small angle, $\cos\Delta\phi \approx 1$ and $\Delta v_x \approx \dfrac{2\pi R_e}{86\,400\text{ s}}\sin\phi_i \sin\Delta\phi$.

$$\Delta v_x \approx \frac{2\pi\left(6.37\times10^6\text{ m}\right)}{86\,400\text{ s}}\sin 35.0° \sin 0.002\,56° = \boxed{1.19\times10^{-2}\text{ m/s}}$$

(d) $\Delta x = \left(\Delta v_x\right)t = \left(1.19\times10^{-2}\text{ m/s}\right)(8.04\text{ s}) = 0.095\,5\text{ m} = \boxed{9.55\text{ cm}}$

P6.57 (a) If the car is about to slip *down* the incline, f is directed up the incline.

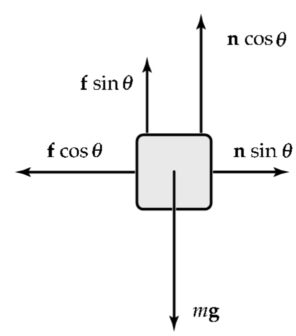


$\sum F_y = n\cos\theta + f\sin\theta - mg = 0$ where $f = \mu_s n$ gives

$$n = \frac{mg}{\cos\theta\left(1+\mu_s\tan\theta\right)} \quad\text{and}\quad f = \frac{\mu_s mg}{\cos\theta\left(1+\mu_s\tan\theta\right)}$$

Then, $\sum F_x = n\sin\theta - f\cos\theta = m\dfrac{v_{\min}^2}{R}$ yields

$$\boxed{v_{\min} = \sqrt{\frac{Rg\left(\tan\theta - \mu_s\right)}{1+\mu_s\tan\theta}}}$$

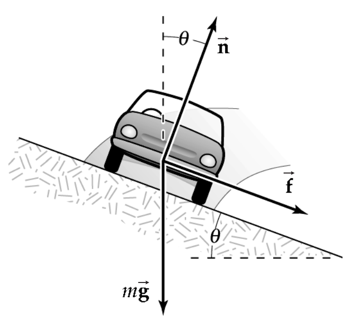


When the car is about to slip *up* the incline, f is directed down the incline. Then, $\sum F_y = n\cos\theta - f\sin\theta - mg = 0$ with $f = \mu_s n$ yields

$$n = \frac{mg}{\cos\theta\left(1-\mu_s\tan\theta\right)} \quad\text{and}\quad f = \frac{\mu_s mg}{\cos\theta\left(1-\mu_s\tan\theta\right)}$$

In this case, $\sum F_x = n\sin\theta + f\cos\theta = m\dfrac{v_{\max}^2}{R}$, which gives

$$\boxed{v_{\max} = \sqrt{\frac{Rg\left(\tan\theta + \mu_s\right)}{1-\mu_s\tan\theta}}}$$



(b) If $v_{\min} = \sqrt{\dfrac{Rg\left(\tan\theta - \mu_s\right)}{1+\mu_s\tan\theta}} = 0$, then $\boxed{\mu_s = \tan\theta}$.

continued on next page

(c) $$v_{\min} = \sqrt{\frac{(100\text{ m})(9.80\text{ m/s}^2)(\tan 10.0° - 0.100)}{1 + (0.100)\tan 10.0°}} = \boxed{8.57\text{ m/s}}$$

$$v_{\max} = \sqrt{\frac{(100\text{ m})(9.80\text{ m/s}^2)(\tan 10.0° + 0.100)}{1 - (0.100)\tan 10.0°}} = \boxed{16.6\text{ m/s}}$$

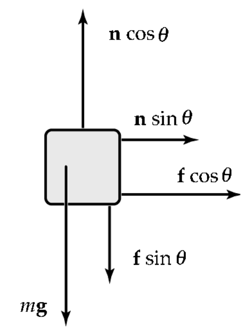


FIG. P6.57

***P6.58** (a) We let R represent the radius of the hoop and T represent the period of its rotation. The bead moves in a circle with radius $v = R\sin\theta$ at a speed of

$$v = \frac{2\pi r}{T} = \frac{2\pi R\sin\theta}{T}$$

The normal force has
an inward radial component of $n\sin\theta$
and an upward component of $n\cos\theta$

$$\sum F_y = ma_y: \quad n\cos\theta - mg = 0$$

or

$$n = \frac{mg}{\cos\theta}$$

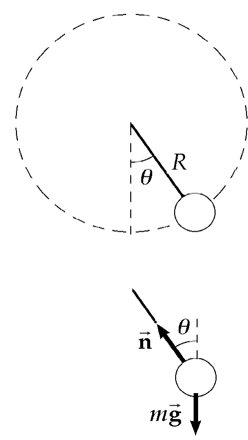


FIG. P6.58(a)

Then $\sum F_x = n\sin\theta = m\dfrac{v^2}{r}$ becomes $\quad \left(\dfrac{mg}{\cos\theta}\right)\sin\theta = \dfrac{m}{R\sin\theta}\left(\dfrac{2\pi R\sin\theta}{T}\right)^2$

which reduces to $\quad \dfrac{g\sin\theta}{\cos\theta} = \dfrac{4\pi^2 R\sin\theta}{T^2}$

This has two solutions: $\quad \sin\theta = 0 \quad \Rightarrow \quad \theta = 0° \qquad (1)$

and $\quad \cos\theta = \dfrac{gT^2}{4\pi^2 R} \qquad (2)$

If $R = 15.0$ cm and $T = 0.450$ s, the second solution yields

$$\cos\theta = \frac{(9.80\text{ m/s}^2)(0.450\text{ s})^2}{4\pi^2(0.150\text{ m})} = 0.335 \text{ and } \theta = 70.4°$$

Thus, in this case, the bead can ride at two positions $\boxed{\theta = 70.4°}$ and $\boxed{\theta = 0°}$.

(b) At this slower rotation, solution (2) above becomes

$$\cos\theta = \frac{(9.80\text{ m/s}^2)(0.850\text{ s})^2}{4\pi^2(0.150\text{ m})} = 1.20, \text{ which is impossible.}$$

In this case, the bead can ride only at the bottom of the loop, $\boxed{\theta = 0°}$.

(c) The equation that the angle must satisfy has two solutions whenever $4\pi^2 R > gT^2$ but only the solution 0° otherwise. The loop's rotation must be faster than a certain threshold value in order for the bead to move away from the lowest position. Zero is always a solution for the angle. There are never more than two solutions.

P6.59 At terminal velocity, the accelerating force of gravity is balanced by frictional drag: $mg = arv + br^2v^2$

(a) $mg = \left(3.10\times10^{-9}\right)v + \left(0.870\times10^{-10}\right)v^2$

For water, $m = \rho V = 1\,000\ \text{kg/m}^3\left[\frac{4}{3}\pi\left(10^{-5}\ \text{m}\right)^3\right]$

$$4.11\times10^{-11} = \left(3.10\times10^{-9}\right)v + \left(0.870\times10^{-10}\right)v^2$$

Assuming v is small, ignore the second term on the right hand side: $\boxed{v = 0.013\,2\ \text{m/s}}$.

(b) $mg = \left(3.10\times10^{-8}\right)v + \left(0.870\times10^{-8}\right)v^2$

Here we cannot ignore the second term because the coefficients are of nearly equal magnitude.

$$4.11\times10^{-8} = \left(3.10\times10^{-8}\right)v + \left(0.870\times10^{-8}\right)v^2$$

$$v = \frac{-3.10 \pm \sqrt{(3.10)^2 + 4(0.870)(4.11)}}{2(0.870)} = \boxed{1.03\ \text{m/s}}$$

(c) $mg = \left(3.10\times10^{-7}\right)v + \left(0.870\times10^{-6}\right)v^2$

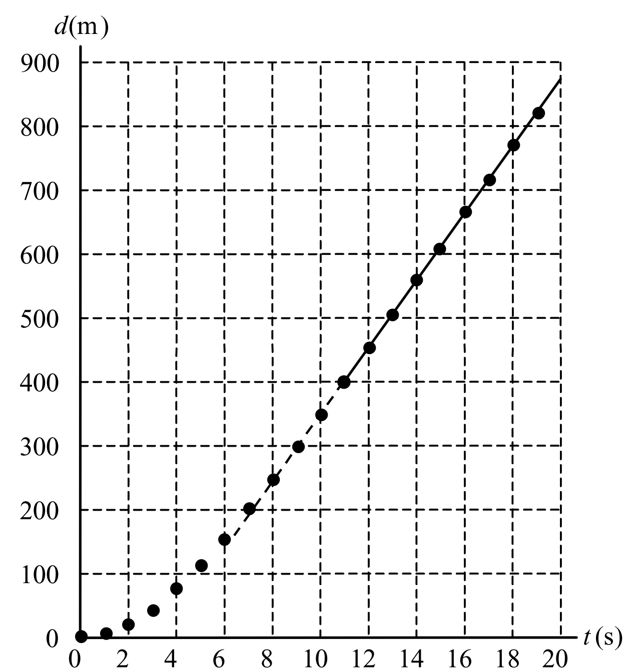
Assuming $v > 1$ m/s, and ignoring the first term:

$4.11\times10^{-5} = \left(0.870\times10^{-6}\right)v^2 \qquad v = \boxed{6.87\ \text{m/s}}$

P6.60 (a)

t(s)	d(m)
1.00	4.88
2.00	18.9
3.00	42.1
4.00	73.8
5.00	112
6.00	154
7.00	199
8.00	246
9.00	296
10.0	347
11.0	399
12.0	452
13.0	505
14.0	558
15.0	611
16.0	664
17.0	717
18.0	770
19.0	823
20.0	876

(b)



(c) A straight line fits the points from $t = 11.0$ s to 20.0 s quite precisely. Its slope is the terminal speed.

$$v_T = \text{slope} = \frac{876\ \text{m} - 399\ \text{m}}{20.0\ \text{s} - 11.0\ \text{s}} = \boxed{53.0\ \text{m/s}}$$

P6.61 $\sum F_y = L_y - T_y - mg = L\cos 20.0° - T\sin 20.0° - 7.35\text{ N} = ma_y = 0$

$$\sum F_x = L_x + T_x = L\sin 20.0° + T\cos 20.0° = m\frac{v^2}{r}$$

$$m\frac{v^2}{r} = 0.750\text{ kg}\frac{(35.0\text{ m/s})^2}{(60.0\text{ m})\cos 20.0°} = 16.3\text{ N}$$

We have the simultaneous equations

$$L\sin 20.0° + T\cos 20.0° = 16.3\text{ N}$$
$$L\cos 20.0° - T\sin 20.0° = 7.35\text{ N}$$
$$L + T\frac{\cos 20.0°}{\sin 20.0°} = \frac{16.3\text{ N}}{\sin 20.0°}$$
$$L - T\frac{\sin 20.0°}{\cos 20.0°} = \frac{7.35\text{ N}}{\cos 20.0°}$$
$$T(\cot 20.0° + \tan 20.0°) = \frac{16.3\text{ N}}{\sin 20.0°} - \frac{7.35\text{ N}}{\cos 20.0°}$$
$$T(3.11) = 39.8\text{ N}$$
$$T = \boxed{12.8\text{ N}}$$

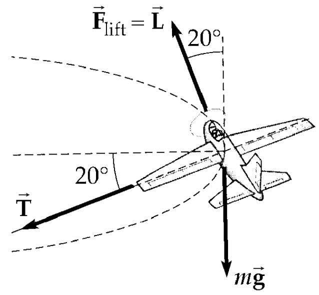


FIG. P6.61

***P6.62** (a) $v = v_i + kx$ implies the acceleration is $a = \frac{dv}{dt} = 0 + k\frac{dx}{dt} = +kv$

Then the total force is $\sum F = ma = m(+kv)$

As a vector, the force is parallel or antiparallel to the velocity: $\boxed{\sum \vec{F} = km\vec{v}}$.

(b) For k positive, some feedback mechanism could be used to impose such a force on an object for a while. The object's speed rises exponentially. Riding on such an object would be more scary than riding on a skyrocket. It would be a good opportunity for learning about exponential growth in population or in energy use.

(c) For k negative, think of a duck landing on a lake, where the water exerts a resistive force on the duck proportional to its speed.

ANSWERS TO EVEN PROBLEMS

P6.2 215 N horizontally inward

P6.4 (a) 1.65 km/s (b) $6.84\times 10^3\text{ s}$

P6.6 $6.22\times 10^{-12}\text{ N}$

P6.8 (a) 68.6 N toward the center of the circle and 784 N up (b) 0.857 m/s^2

P6.10 (a) $(-0.233\,\hat{\mathbf{i}} + 0.163\,\hat{\mathbf{j}})\text{m/s}^2$ (b) 6.53 m/s (c) $(-0.181\,\hat{\mathbf{i}} + 0.181\,\hat{\mathbf{j}})\text{m/s}^2$

P6.12 (a) 1.33 m/s^2 (b) 1.79 m/s^2 forward and 48.0° inward

P6.14 (a) 2.49×10^4 N up (b) 12.1 m/s

P6.16 (a) 20.6 N (b) 3.35 m/s^2 downward tangent to the circle; 32.0 m/s^2 radially inward (c) 32.2 m/s^2 at 5.98° to the cord, pointing toward a location below the center of the circle. (d) No change. If the object is swinging down it is gaining speed. If it is swinging up it is losing speed but its acceleration is the same size and its direction can be described in the same terms.

P6.18 (a) 8.62 m (b) Mg downward (c) $8.45\ \text{m/s}^2$ Unless they are belted in, the riders will fall from the cars.

P6.20 0.527°

P6.22 $\mu_k = \dfrac{2(vt-L)}{(g+a)t^2}$

P6.24 93.8 N

P6.26 (a) $6.27\ \text{m/s}^2$ downward (b) 784 N up (c) 283 N up

P6.28 (a) $53.8\ \text{m/s}$ (b) 148 m

P6.30 $-0.212\ \text{m/s}^2$

P6.32 (a) 2.03 N down (b) 3.18 m/s^2 down (c) 0.205 m/s down

P6.34 see the solution

P6.36 $36.5\ \text{m/s}$

P6.38 $\sim 10^1$ N

P6.40 8.88 N

P6.42 $0.835\ \text{rev/s}$

P6.44 (a) $mg - \dfrac{mv^2}{R}$ upward (b) $v = \sqrt{gR}$

P6.46 (a) $v = \sqrt{Rg\tan 35.0°} = \sqrt{(6.86\ \text{m/s}^2)R}$ (b) The mass is unnecessary. (c) Increase by $\sqrt{2}$ times (d) Increase by $\sqrt{2}$ times (e) On the larger circle the ice cube moves $\sqrt{2}$ times faster but also takes longer to get around, because the distance it must travel is 2 times larger. Its period is described by $T = \dfrac{2\pi R}{v} = \dfrac{2\pi R}{\sqrt{Rg\tan 35.0°}} = \left(2.40\ \text{s}/\sqrt{\text{m}}\right)\sqrt{R}$.

P6.48 (a) The seat exerts 967 lb up on the pilot. (b) The seat exerts 647 lb down on the pilot. (c) If the plane goes over the top of a section of a circle with $v^2 = Rg$, the pilot will feel weightless.

P6.50 (a) $m_2 g$ (b) $m_2 g$ (c) $\sqrt{\left(\dfrac{m_2}{m_1}\right)gR}$ (d) The puck will move inward along a spiral, gaining speed as it does so. (e) The puck will move outward along a spiral as it slows down.

P6.52 (a) 1.58 m/s^2 (b) 455 N (c) 329 N (d) 397 N upward and 9.15° inward

P6.54 (a) $v = \sqrt{\pi R g}$ (b) $m\pi g$

P6.56 (a) 8.04 s (b) $379\ \text{m/s}$ (c) $1.19\ \text{cm/s}$ (d) 9.55 cm

P6.58 (a) either 70.4° or 0° (b) 0° (c) The equation that the angle must satisfy has two solutions whenever $4\pi^2 R > gT^2$ but only the solution 0° otherwise. (Here R and T are the radius and period of the hoop.) Zero is always a solution for the angle. There are never more than two solutions.

P6.60 (a) and (b) see the solution (c) 53.0 m/s

P6.62 (a) $\Sigma\vec{\mathbf{F}} = mk\vec{\mathbf{v}}$ (b) For k positive, some feedback mechanism could be used to impose such a force on an object for a while. The object's speed rises exponentially. Riding on such an object would be more scary than riding on a skyrocket. It would be a good opportunity for learning about exponential growth in population or in energy use. (c) For k negative, think of a duck landing on a lake, where the water exerts a resistive force on the duck proportional to its speed.

7

Energy of a System

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q7.1 (a) Positive work is done by the chicken on the dirt.

(b) The person does no work on anything in the environment. Perhaps some extra chemical energy goes through being energy transmitted electrically and is converted into internal energy in his brain; but it would be very hard to quantify "extra."

(c) Positive work is done on the bucket.

(d) Negative work is done on the bucket.

(e) Negative work is done on the person's torso.

Q7.2 Force of tension on a ball moving in a circle on the end of a string. Normal force and gravitational force on an object at rest or moving across a level floor.

Q7.3 (a) Tension (b) Air resistance (c) The gravitational force does positive work in increasing speed on the downswing. It does negative work in decreasing speed on the upswing.

***Q7.4** Each dot product has magnitude $1 \cdot 1 \cdot \cos\theta$ where θ is the angle between the two factors. Thus for (a) and (f) we have $\cos 0 = 1$. For (b) and (g), $\cos 45° = 0.707$. For (c) and (h), $\cos 180° = -1$. For (d) and (e), $\cos 90° = 0$. The assembled answer is a = f > b = g > d = e > c = h.

Q7.5 The scalar product of two vectors is positive if the angle between them is between 0 and 90°. The scalar product is negative when $90° < \theta < 180°$.

***Q7.6** (i) The force of block on spring is equal in magnitude and opposite to the force of spring on block. The answers are (c) and (e).
(ii) The spring tension exerts equal-magnitude forces toward the center of the spring on objects at both ends. The answers are (c) and (e).

Q7.7 $k' = 2k$. To stretch the smaller piece one meter, each coil would have to stretch twice as much as one coil in the original long spring, since there would be half as many coils. Assuming that the spring is ideal, twice the stretch requires twice the force.

Q7.8 No. Kinetic energy is always positive. Mass and squared speed are both positive. A moving object can always do positive work in striking another object and causing it to move along the same direction of motion.

Q7.9 Work is only done in accelerating the ball from rest. The work is done over the effective length of the pitcher's arm—the distance his hand moves through windup and until release. He extends this distance by taking a step forward.

***Q7.10** answer (e). Kinetic energy is proportional to mass.

***Q7.11** answer (a). Kinetic energy is proportional to squared speed. Doubling the speed makes an object's kinetic energy four times larger.

***Q7.12** It is sometimes true. If the object is a particle initially at rest, the net work done on the object is equal to its final kinetic energy. If the object is not a particle, the work could go into (or come out of) some other form of energy. If the object is initially moving, its initial kinetic energy must be added to the total work to find the final kinetic energy.

***Q7.13** Yes. The floor of a rising elevator does work on a passenger. A normal force exerted by a stationary solid surface does no work.

***Q7.14** answer (c). If the total work on an object is zero in some process, its kinetic energy and so its speed must be the same at the final point as it was at the initial point.

***Q7.15** The cart's fixed kinetic energy means that it can do a fixed amount of work in stopping, namely (6 N)(6 cm) = 0.36 J. The forward force it exerts and the distance it moves in stopping must have this fixed product. answers: (i) c (ii) a (iii) d

Q7.16 As you ride an express subway train, a backpack at your feet has no kinetic energy as measured by you since, according to you, the backpack is not moving. In the frame of reference of someone on the side of the tracks as the train rolls by, the backpack is moving and has mass, and thus has kinetic energy.

***Q7.17** answer (e). $4.00\text{ J} = \frac{1}{2}k(0.100\text{ m})^2$

Therefore $k = 800\ \text{N/m}$ and to stretch the spring to 0.200 m requires extra work

$$\Delta W = \frac{1}{2}(800)(0.200)^2 - 4.00\text{ J} = \boxed{12.0\text{ J}}$$

Q7.18 (a) Not necessarily. It does if it makes the object's speed change, but not if it only makes the direction of the velocity change.

(b) Yes, according to Newton's second law.

***Q7.19** (i) The gravitational acceleration is quite precisely constant at locations separated by much less than the radius of the planet. Answer: a = b = c = d

(ii) The mass but not the elevation affects the gravitational force. Answer: c = d > a = b

(iii) Now think about the product of mass times height. Answer: c > b = d > a

Q7.20 There is no violation. Choose the book as the system. You did work and the Earth did work on the book. The average force you exerted just counterbalanced the weight of the book. The total work on the book is zero, and is equal to its overall change in kinetic energy.

Q7.21 In stirring cake batter and in weightlifting, your body returns to the same conformation after each stroke. During each stroke chemical energy is irreversibly converted into output work (and internal energy). This observation proves that muscular forces are nonconservative.

Q7.22 A graph of potential energy versus position is a straight horizontal line for a particle in neutral equilibrium. The graph represents a constant function.

***Q7.23** (c) The ice cube is in neutral equilibrium. Its zero acceleration is evidence for equilibrium.

***Q7.24** The gravitational energy of the key-Earth system is lowest when the key is on the floor letter-side-down. The average height of particles in the key is lowest in that configuration. As described by $F = -dU/dx$, a force pushes the key downhill in potential energy toward the bottom of a graph of potential energy versus orientation angle. Friction removes mechanical energy from the key-Earth system, tending to leave the key in its minimum-potential energy configuration.

Q7.25 Gaspard de Coriolis first stated the work-kinetic energy theorem. Jean Victor Poncelet, an engineer who invaded Russia with Napoleon, is most responsible for demonstrating its wide practical applicability, in his 1829 book *Industrial Mechanics*. Their work came remarkably late compared to the elucidation of momentum conservation in collisions by Descartes and to Newton's *Mathematical Principles of the Philosophy of Nature*, both in the 1600's.

SOLUTIONS TO PROBLEMS

Section 7.2 **Work Done by a Constant Force**

P7.1 (a) $W = F\Delta r\cos\theta = (16.0\text{ N})(2.20\text{ m})\cos 25.0° = \boxed{31.9\text{ J}}$

(b), (c) The normal force and the weight are both at 90° to the displacement in any time interval. Both do $\boxed{0}$ work.

(d) $\sum W = 31.9\text{ J} + 0 + 0 = \boxed{31.9\text{ J}}$

P7.2 (a) $W = mgh = \left(3.35\times10^{-5}\right)(9.80)(100)\text{ J} = \boxed{3.28\times10^{-2}\text{ J}}$

(b) Since $R = mg$, $W_{\text{air resistance}} = \boxed{-3.28\times10^{-2}\text{ J}}$

P7.3 METHOD ONE

Let ϕ represent the instantaneous angle the rope makes with the vertical as it is swinging up from $\phi_i = 0$ to $\phi_f = 60°$. In an incremental bit of motion from angle ϕ to $\phi + d\phi$, the definition of radian measure implies that $\Delta r = (12\text{ m})d\phi$. The angle θ between the incremental displacement and the force of gravity is $\theta = 90° + \phi$. Then $\cos\theta = \cos(90° + \phi) = -\sin\phi$. The work done by the gravitational force on Batman is

$$W = \int_i^f F\cos\theta dr = \int_{\phi=0}^{\phi=60°} mg(-\sin\phi)(12\text{ m})d\phi$$

$$= -mg(12\text{ m})\int_0^{60°}\sin\phi\, d\phi = (-80\text{ kg})\left(9.8\text{ m/s}^2\right)(12\text{ m})(-\cos\phi)\Big|_0^{60°}$$

$$= (-784\text{ N})(12\text{ m})(-\cos 60° + 1) = \boxed{-4.70\times10^3\text{ J}}$$

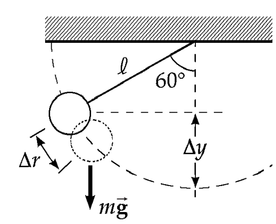


FIG. P7.3

METHOD TWO

The force of gravity on Batman is $mg = (80\text{ kg})\left(9.8\text{ m/s}^2\right) = 784\text{ N}$ down. Only his vertical displacement contributes to the work gravity does. His original y-coordinate below the tree limb is -12 m. His final y-coordinate is $(-12\text{ m})\cos 60° = -6\text{ m}$. His change in elevation is $-6\text{ m} - (-12\text{ m}) = 6\text{ m}$. The work done by gravity is

$$W = F\Delta r\cos\theta = (784\text{ N})(6\text{ m})\cos 180° = \boxed{-4.70\text{ kJ}}$$

***P7.4** Yes. Object 1 exerts some forward force on object 2 as they move through the same displacement. By Newton's third law, object 2 exerts an equal-size force in the opposite direction on object 1. In $W = F\Delta r\cos\theta$, the factors F and Δr are the same, and θ differs by 180°, so object 2 does −15.0 J of work on object 1. The energy transfer is 15 J from object 1 to object 2, which can be counted as a change in energy of −15 J for object 1 and a change in energy of +15 J for object 2.

Section 7.3 **The Scalar Product of Two Vectors**

P7.5

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right)\cdot\left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$$

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = A_xB_x\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}\right) + A_xB_y\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{j}}\right) + A_xB_z\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{k}}\right)$$

$$+A_yB_x\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{i}}\right) + A_yB_y\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}\right) + A_yB_z\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{k}}\right)$$

$$+A_zB_x\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{i}}\right) + A_zB_y\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{j}}\right) + A_zB_z\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}\right)$$

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = \boxed{A_xB_x + A_yB_y + A_zB_z}$$

P7.6 $A = 5.00;\ B = 9.00;\ \theta = 50.0°$

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = AB\cos\theta = (5.00)(9.00)\cos 50.0° = \boxed{28.9}$$

P7.7 (a) $W = \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}} = F_x x + F_y y = (6.00)(3.00)\ \text{N}\cdot\text{m} + (-2.00)(1.00)\ \text{N}\cdot\text{m} = \boxed{16.0\ \text{J}}$

(b) $\theta = \cos^{-1}\left(\dfrac{\vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}}}{F\Delta r}\right) = \cos^{-1}\dfrac{16}{\sqrt{\left((6.00)^2 + (-2.00)^2\right)\left((3.00)^2 + (1.00)^2\right)}} = \boxed{36.9°}$

P7.8 We must first find the angle between the two vectors. It is:

$$\theta = 360° - 118° - 90.0° - 132° = 20.0°$$

Then

$$\vec{\mathbf{F}}\cdot\vec{\mathbf{v}} = Fv\cos\theta = (32.8\ \text{N})(0.173\ \text{m/s})\cos 20.0°$$

or $\quad \vec{\mathbf{F}}\cdot\vec{\mathbf{v}} = 5.33\dfrac{\text{N}\cdot\text{m}}{\text{s}} = \boxed{5.33\ \dfrac{\text{J}}{\text{s}}}$

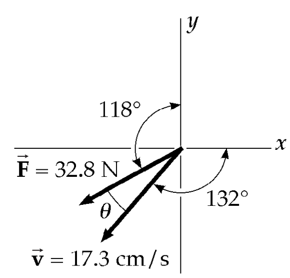


FIG. P7.8

P7.9 (a) $\vec{\mathbf{A}} = 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$

$\vec{\mathbf{B}} = 4.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}} \qquad \theta = \cos^{-1}\dfrac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB} = \cos^{-1}\dfrac{12.0 + 8.00}{\sqrt{(13.0)(32.0)}} = \boxed{11.3°}$

(b) $\vec{\mathbf{B}} = 3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$\vec{\mathbf{A}} = -2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}} \qquad \cos\theta = \dfrac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB} = \dfrac{-6.00 - 16.0}{\sqrt{(20.0)(29.0)}} \qquad \theta = \boxed{156°}$

(c) $\vec{\mathbf{A}} = \hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$\vec{\mathbf{B}} = 3.00\hat{\mathbf{j}} + 4.00\hat{\mathbf{k}} \qquad \theta = \cos^{-1}\left(\dfrac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB}\right) = \cos^{-1}\left(\dfrac{-6.00 + 8.00}{\sqrt{9.00}\cdot\sqrt{25.0}}\right) = \boxed{82.3°}$

P7.10 $\vec{\mathbf{A}}-\vec{\mathbf{B}}=\left(3.00\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)-\left(-\hat{\mathbf{i}}+2.00\hat{\mathbf{j}}+5.00\hat{\mathbf{k}}\right)$

$$\vec{\mathbf{A}}-\vec{\mathbf{B}}=4.00\hat{\mathbf{i}}-\hat{\mathbf{j}}-6.00\hat{\mathbf{k}}$$

$$\vec{\mathbf{C}}\cdot\left(\vec{\mathbf{A}}-\vec{\mathbf{B}}\right)=\left(2.00\hat{\mathbf{j}}-3.00\hat{\mathbf{k}}\right)\cdot\left(4.00\hat{\mathbf{i}}-\hat{\mathbf{j}}-6.00\hat{\mathbf{k}}\right)=0+(-2.00)+(+18.0)=\boxed{16.0}$$

***P7.11** Let θ represent the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Turning by 25° makes the dot product larger, so the angle between $\vec{\mathbf{C}}$ and $\vec{\mathbf{B}}$ must be smaller. We call it $\theta - 25°$. Then we have

$A\,5\cos\theta = 30$ and $A\,5\cos(\theta - 25°) = 35$

Then $A\cos\theta = 6$ and $A(\cos\theta\cos 25° + \sin\theta\sin 25°) = 7$

Dividing, $\cos 25° + \tan\theta\sin 25° = 7/6$ $\quad\tan\theta = (7/6 - \cos 25°)/\sin 25° = 0.616$

$\theta = 31.6°$. Then the direction angle of A is $60° - 31.6° = 28.4°$

Substituting back, A cos 31.6° = 6 so $\vec{\mathbf{A}} = \boxed{7.05\text{ m at }28.4°}$

Section 7.4 **Work Done by a Varying Force**

P7.12 $F_x = (8x - 16)$ N

(a) See figure to the right

(b) $W_{\text{net}} = \dfrac{-(2.00\text{ m})(16.0\text{ N})}{2} + \dfrac{(1.00\text{ m})(8.00\text{ N})}{2} = \boxed{-12.0\text{ J}}$

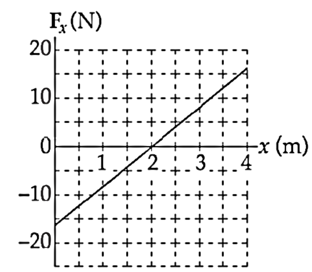


FIG. P7.12

P7.13 $W = \int_i^f F dx =$ area under curve from x_i to x_f

(a) $x_i = 0$ $\quad x_f = 8.00$ m

$W =$ area of triangle $ABC = \left(\frac{1}{2}\right)AC\times$ altitude,

$W_{0\to 8} = \left(\frac{1}{2}\right)\times 8.00\text{ m}\times 6.00\text{ N} = \boxed{24.0\text{ J}}$

(b) $x_i = 8.00$ m $\quad x_f = 10.0$ m

$W =$ area of $\Delta CDE = \left(\frac{1}{2}\right)CE\times$ altitude,

$W_{8\to 10} = \left(\frac{1}{2}\right)\times(2.00\text{ m})\times(-3.00\text{ N}) = \boxed{-3.00\text{ J}}$

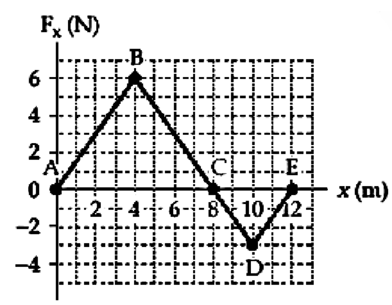


FIG. P7.13

(c) $W_{0\to 10} = W_{0\to 8} + W_{8\to 10} = 24.0 + (-3.00) = \boxed{21.0\text{ J}}$

P7.14 $W = \int_i^f \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}} = \int_0^{5\text{ m}} \left(4x\hat{\mathbf{i}} + 3y\hat{\mathbf{j}}\right)\text{ N}\cdot dx\hat{\mathbf{i}}$

$$\int_0^{5\text{ m}} (4\text{ N/m})x dx + 0 = (4\text{ N/m})\left.\frac{x^2}{2}\right|_0^{5\text{ m}} = \boxed{50.0\text{ J}}$$

P7.15 $W = \int F_x dx$

and W equals the area under the Force-Displacement curve

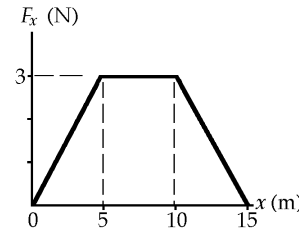


FIG. P7.15

(a) For the region $0 \le x \le 5.00$ m,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

(b) For the region $5.00 \le x \le 10.0$,

$$W = (3.00\text{ N})(5.00\text{ m}) = \boxed{15.0\text{ J}}$$

(c) For the region $10.0 \le x \le 15.0$,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

(d) For the region $0 \le x \le 15.0$

$$W = (7.50 + 7.50 + 15.0)\text{ J} = \boxed{30.0\text{ J}}$$

P7.16 (a) Spring constant is given by $F = kx$

$$k = \frac{F}{x} = \frac{(230\text{ N})}{(0.400\text{ m})} = \boxed{575\text{ N/m}}$$

(b) Work $= F_{\text{avg}} x = \frac{1}{2}(230\text{ N})(0.400\text{ m}) = \boxed{46.0\text{ J}}$

P7.17 $k = \frac{F}{y} = \frac{Mg}{y} = \frac{(4.00)(9.80)\text{ N}}{2.50 \times 10^{-2}\text{ m}} = 1.57 \times 10^{3}\text{ N/m}$

(a) For 1.50 kg mass $y = \frac{mg}{k} = \frac{(1.50)(9.80)}{1.57 \times 10^{3}} = \boxed{0.938\text{ cm}}$

(b) Work $= \frac{1}{2}ky^2$

Work $= \frac{1}{2}\left(1.57 \times 10^{3}\text{ N}\cdot\text{m}\right)\left(4.00 \times 10^{-2}\text{ m}\right)^2 = \boxed{1.25\text{ J}}$

***P7.18** In $F = -kx$, F refers to the size of the force that the spring exerts on each end. It pulls down on the doorframe in part (a) in just as real a sense at it pulls on the second person in part (b).

(a) Consider the upward force exerted by the bottom end of the spring, which undergoes a downward displacement that we count as negative:

$k = -F/x = -(7.5\text{ kg})(9.8\text{ m/s}^2)/(-0.415\text{ m} + 0.35\text{ m}) = -73.5\text{ N}/(-0.065\text{ m}) = \boxed{1.13\text{ kN/m}}$

(b) Consider the end of the spring on the right, which exerts a force to the left:

$x = -F/k = -(-190\text{ N})/(1130\text{ N/m}) = 0.168\text{ m}$

The length of the spring is then $0.35\text{ m} + 0.168\text{ m} = \boxed{0.518\text{ m}}$

***P7.19** $\Sigma F_x = ma_x$: $kx = ma$

$$k = \frac{ma}{x} = \frac{(4.70 \times 10^{-3}\text{ kg})0.800(9.80\text{ m/s}^2)}{0.500 \times 10^{-2}\text{m}}$$
$$= \boxed{7.37\text{ N/m}}$$

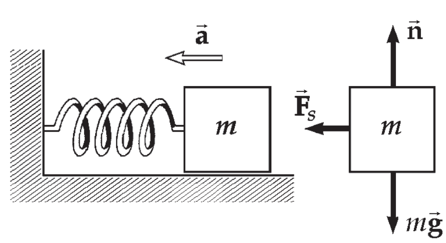


FIG. P7.19

***P7.20** The spring exerts on each block an outward force of magnitude

$$|F_s| = kx = (3.85\ \text{N/m})(0.08\ \text{m}) = 0.308\ \text{N}$$

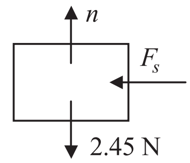


FIG. P7.20

Take the $+x$ direction to the right. For the light block on the left, the vertical forces are given by $F_g = mg = (0.25\ \text{kg})(9.8\ \text{m/s}^2) = 2.45\ \text{N}$, $\sum F_y = 0$, $n - 2.45\ \text{N} = 0$, $n = 2.45\ \text{N}$. Similarly for the heavier block $n = F_g = (0.5\ \text{kg})(9.8\ \text{m/s}^2) = 4.9\ \text{N}$

(a) For the block on the left, $\sum F_x = ma_x$, $-0.308\ \text{N} = (0.25\ \text{kg})a$, $a = \boxed{-1.23\ \text{m/s}^2}$.
For the heavier block, $+0.308\ \text{N} = (0.5\ \text{kg})a$, $a = \boxed{0.616\ \text{m/s}^2}$.

(b) For the block on the left, $f_k = \mu_k n = 0.1(2.45\ \text{N}) = 0.245\ \text{N}$

$$\sum F_x = ma_x$$

$$-0.308\ \text{m/s}^2 + 0.245\ \text{N} = (0.25\ \text{kg})a$$

$$a = \boxed{-0.252\ \text{m/s}^2 \text{ if the force of static friction is not too large.}}$$

For the block on the right, $f_k = \mu_k n = 0.490\ \text{N}$. The maximum force of static friction would be larger, so no motion would begin and the acceleration is $\boxed{\text{zero}}$.

(c) Left block: $f_k = 0.462(2.45\ \text{N}) = 1.13\ \text{N}$. The maximum static friction force would be larger, so the spring force would produce no motion of this block or of the right-hand block, which could feel even more friction force. For both $a = \boxed{0}$.

***P7.21** Compare an initial picture of the rolling car with a final picture with both springs compressed $K_i + \sum W = K_f$. Work by both springs changes the car's kinetic energy

$$K_i + \frac{1}{2}k_1\left(x_{1i}^2 - x_{1f}^2\right) + \frac{1}{2}k_2\left(x_{2i}^2 - x_{2f}^2\right) = K_f$$

$$\frac{1}{2}mv_i^2 + 0 - \frac{1}{2}(1600\ \text{N/m})(0.500\ \text{m})^2$$

$$+0 - \frac{1}{2}(3400\ \text{N/m})(0.200\ \text{m})^2 = 0$$

$$\frac{1}{2}(6\,000\ \text{kg})v_i^2 - 200\ \text{J} - 68.0\ \text{J} = 0$$

$$v_i = \sqrt{\frac{2(268\ \text{J})}{6\,000\ \text{kg}}} = \boxed{0.299\ \text{m/s}}$$

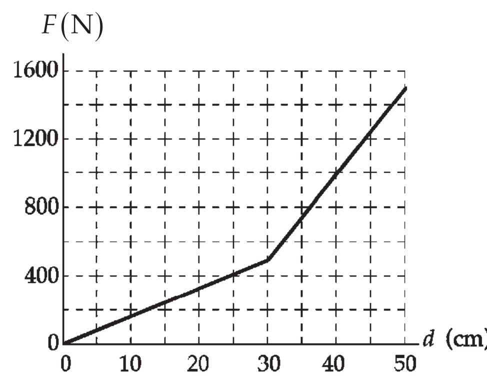


FIG. P7.21

P7.22 (a) $W = \int_i^f \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$

$$W = \int_0^{0.600\ \text{m}} \left(15\,000\ \text{N} + 10\,000x\ \text{N/m} - 25\,000x^2\ \text{N/m}^2\right) dx \cos 0°$$

$$W = 15\,000x + \frac{10\,000x^2}{2} - \frac{25\,000x^3}{3}\Bigg|_0^{0.600\ \text{m}}$$

$$W = 9.00\ \text{kJ} + 1.80\ \text{kJ} - 1.80\ \text{kJ} = \boxed{9.00\ \text{kJ}}$$

continued on next page

(b) Similarly,

$$W = (15.0\text{ kN})(1.00\text{ m}) + \frac{(10.0\text{ kN/m})(1.00\text{ m})^2}{2} - \frac{(25.0\text{ kN/m}^2)(1.00\text{ m})^3}{3}$$

$$W = \boxed{11.7\text{ kJ}}\text{, larger by }29.6\%$$

P7.23 The same force makes both light springs stretch.

(a) The hanging mass moves down by

$$x = x_1 + x_2 = \frac{mg}{k_1} + \frac{mg}{k_2} = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$= 1.5\text{ kg }9.8\text{ m/s}^2\left(\frac{1\text{ m}}{1\,200\text{ N}} + \frac{1\text{ m}}{1\,800\text{ N}}\right) = \boxed{2.04\times10^{-2}\text{ m}}$$

(b) We define the effective spring constant as

$$k = \frac{F}{x} = \frac{mg}{mg\left(1/k_1 + 1/k_2\right)} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$$

$$= \left(\frac{1\text{ m}}{1\,200\text{ N}} + \frac{1\text{ m}}{1\,800\text{ N}}\right)^{-1} = \boxed{720\text{ N/m}}$$

P7.24 See the solution to problem 7.23.

(a) $\boxed{x = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}$ Both springs stretch, so the load moves down by a larger amount than it would if either spring were missing.

(b) $\boxed{k = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}}$ The spring constant of the series combination is less than the smaller of the two individual spring constants, to describe a less stiff system, that stretches by a larger extension for any particular load.

P7.25 (a) The radius to the object makes angle θ with the horizontal, so its weight makes angle θ with the negative side of the x-axis, when we take the x-axis in the direction of motion tangent to the cylinder.

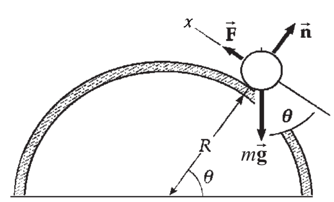


FIG. P7.25

$$\sum F_x = ma_x$$

$$F - mg\cos\theta = 0$$

$$F = \boxed{mg\cos\theta}$$

(b) $W = \int_i^f \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}$

We use radian measure to express the next bit of displacement as $dr = Rd\theta$ in terms of the next bit of angle moved through:

$$W = \int_0^{\pi/2} mg\cos\theta R d\theta = mgR\sin\theta\Big|_0^{\pi/2}$$

$$W = mgR(1-0) = \boxed{mgR}$$

P7.26 $[k] = \left[\frac{F}{x}\right] = \frac{\text{N}}{\text{m}} = \frac{\text{kg}\cdot\text{m/s}^2}{\text{m}} = \boxed{\frac{\text{kg}}{\text{s}^2}}$

***P7.27** We can write u as a function of v : (8 N −[−2 N])/(25 cm − 5 cm) = (u −[−2 N])/(v − 5 cm)

(0.5 N/cm)(v − 5 cm) = u + 2 N u = (0.5 N/cm)v − 4.5 N also v = (2 cm/N)u + 9 cm

(a) Then

$$\int_a^b u\,dv = \int_5^{25}(0.5v - 4.5)\,dv = \left[0.5v^2/2 - 4.5v\right]_5^{25} = 0.25(625-25) - 4.5(25-5)$$

$$= 150 - 90 = 60\ \text{N}\cdot\text{cm} = \boxed{0.600\ \text{J}}$$

(b) Reversing the limits of integration just gives us the negative of the quantity: $\int_b^a u\,dv = \boxed{-0.600\ \text{J}}$

(c) This is an entirely different integral. It is larger because all of the area to be counted up is positive (to the right of $v = 0$) instead of partly negative (below $u = 0$).

$$\int_a^b v\,du = \int_{-2}^{8}(2u+9)\,du = \left[2u^2/2 + 9u\right]_{-2}^{8} = 64 - (-2)^2 + 9(8+2) = 60 + 90 = 150\ \text{N}\cdot\text{cm} = \boxed{1.50\ \text{J}}$$

***P7.28** If the weight of the first tray stretches all four springs by a distance equal to the thickness of the tray, then the proportionality expressed by Hooke's law guarantees that each additional tray will have the same effect, so that the top surface of the top tray will always have the same elevation above the floor.

The weight of a tray is $0.580\ \text{kg}\left(9.8\ \text{m/s}^2\right) = 5.68\ \text{N}$. The force $\frac{1}{4}(5.68\ \text{N}) = 1.42\ \text{N}$ should stretch one spring by 0.450 cm, so its spring constant is $k = \frac{|F_s|}{x} = \frac{1.42\ \text{N}}{0.004\,5\ \text{m}} = \boxed{316\ \text{N/m}}$.

We did not need to know the length or width of the tray.

Section 7.5 Kinetic Energy and the Work-Kinetic Energy Theorem

P7.29 (a) $K_A = \frac{1}{2}(0.600\ \text{kg})(2.00\ \text{m/s})^2 = \boxed{1.20\ \text{J}}$

(b) $\frac{1}{2}mv_B^2 = K_B$: $v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{(2)(7.50)}{0.600}} = \boxed{5.00\ \text{m/s}}$

(c) $\sum W = \Delta K = K_B - K_A = \frac{1}{2}m\left(v_B^2 - v_A^2\right) = 7.50\ \text{J} - 1.20\ \text{J} = \boxed{6.30\ \text{J}}$

P7.30 (a) $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.300\ \text{kg})(15.0\ \text{m/s})^2 = \boxed{33.8\ \text{J}}$

(b) $K = \frac{1}{2}(0.300)(30.0)^2 = \frac{1}{2}(0.300)(15.0)^2(4) = 4(33.8) = \boxed{135\ \text{J}}$

P7.31 $\vec{\mathbf{v}}_i = \left(6.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right) = \text{m/s}$

(a) $v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{40.0}\ \text{m/s}$

$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(3.00\ \text{kg})\left(40.0\ \text{m}^2/\text{s}^2\right) = \boxed{60.0\ \text{J}}$

(b) $\vec{\mathbf{v}}_f = 8.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$

$v_f^2 = \vec{\mathbf{v}}_f \cdot \vec{\mathbf{v}}_f = 64.0 + 16.0 = 80.0\ \text{m}^2/\text{s}^2$

$\Delta K = K_f - K_i = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{3.00}{2}(80.0) - 60.0 = \boxed{60.0\ \text{J}}$

P7.32 (a) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 5.00$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(7.50\text{ J})}{4.00\text{ kg}}} = \boxed{1.94\text{ m/s}}$$

(b) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 10.0$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(22.5\text{ J})}{4.00\text{ kg}}} = \boxed{3.35\text{ m/s}}$$

(c) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 15.0$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(30.0\text{ J})}{4.00\text{ kg}}} = \boxed{3.87\text{ m/s}}$$

P7.33 Consider the work done on the pile driver from the time it starts from rest until it comes to rest at the end of the fall. Let $d = 5.00$ m represent the distance over which the driver falls freely, and $h = 0.12$ m the distance it moves the piling.

$$\sum W = \Delta K:\ W_{\text{gravity}} + W_{\text{beam}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

so

$$(mg)(h+d)\cos 0° + (\bar{F})(d)\cos 180° = 0 - 0$$

Thus,

$$\bar{F} = \frac{(mg)(h+d)}{d} = \frac{(2\,100\text{ kg})(9.80\text{ m/s}^2)(5.12\text{ m})}{0.120\text{ m}} = \boxed{8.78\times10^5\text{ N}}$$

The force on the pile driver is $\boxed{\text{upward}}$.

***P7.34** (a) We evaluate the kinetic energy of the cart and the work the cart would have to do to plow all the way through the pile. If the kinetic energy is larger, the cart gets through.

$K = (1/2)mv^2 = (1/2)(0.3\text{ kg})(0.6\text{ m/s})^2 = 0.054$ J

The work *done on the cart* in traveling the whole distance is the net area under the graph,

$W = (2\text{ N})(0.01\text{ m}) + [(0 - 3\text{ N})/2](0.04\text{ m}) = 0.02\text{ J} - 0.06\text{ J} = -0.04$ J

The work the cart must do is less than the original kinetic energy, so the cart does get through all the sand.

(b) The work *the cart does* is +0.04 J, so its final kinetic energy is the remaining 0.054 J – 0.04 J = 0.014 J. Another way to say it: from the work-kinetic energy theorem,

$K_i + W = K_f$ $0.054\text{ J} - 0.04\text{ J} = 0.014\text{ J} = (1/2)(0.5\text{ kg})v_f^2$

$v_f = [2(0.014\text{ kg}\cdot\text{m}^2/\text{s}^2)/(0.3\text{ kg})]^{1/2} = \boxed{0.306\text{ m/s}}$

***P7.35** (a) $K_i + \sum W = K_f = \frac{1}{2}mv_f^2$

$$0 + \sum W = \frac{1}{2}(15.0\times10^{-3}\text{ kg})(780\text{ m/s})^2 = \boxed{4.56\text{ kJ}}$$

(b) $$F = \frac{W}{\Delta r\cos\theta} = \frac{4.56\times10^3\text{ J}}{(0.720\text{ m})\cos 0°} = \boxed{6.34\text{ kN}}$$

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(c) $a = \dfrac{v_f^2 - v_i^2}{2x_f} = \dfrac{(780\ \text{m/s})^2 - 0}{2(0.720\ \text{m})} = \boxed{422\ \text{km/s}^2}$

(d) $\sum F = ma = (15\times10^{-3}\ \text{kg})(422\times10^{3}\ \text{m/s}^2) = \boxed{6.34\ \text{kN}}$

(e) $\boxed{\text{The forces are the same. The two theories agree.}}$

P7.36 (a) $v_f = 0.096(3\times10^8\ \text{m/s}) = 2.88\times10^7\ \text{m/s}$

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(9.11\times10^{-31}\ \text{kg})(2.88\times10^7\ \text{m/s})^2 = \boxed{3.78\times10^{-16}\ \text{J}}$$

(b) $K_i + W = K_f$: $\quad 0 + F\Delta r\cos\theta = K_f$

$$F(0.028\ \text{m})\cos 0° = 3.78\times10^{-16}\ \text{J}$$

$$F = \boxed{1.35\times10^{-14}\ \text{N}}$$

(c) $\sum F = ma$; $\quad a = \dfrac{\sum F}{m} = \dfrac{1.35\times10^{-14}\ \text{N}}{9.11\times10^{-31}\ \text{kg}} = \boxed{1.48\times10^{+16}\ \text{m/s}^2}$

(d) $v_{xf} = v_{xi} + a_x t$ $\quad 2.88\times10^7\ \text{m/s} = 0 + (1.48\times10^{16}\ \text{m/s}^2)t$

$$t = \boxed{1.94\times10^{-9}\ \text{s}}$$

Check: $\quad x_f = x_i + \dfrac{1}{2}(v_{xi} + v_{xf})t$

$$0.028\ \text{m} = 0 + \frac{1}{2}(0 + 2.88\times10^7\ \text{m/s})t$$

$$t = 1.94\times10^{-9}\ \text{s}$$

Section 7.6 **Potential Energy of a System**

P7.37 (a) With our choice for the zero level for potential energy when the car is at point B,

$$\boxed{U_B = 0}$$

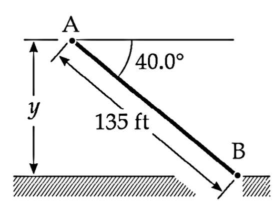

FIG. P7.37

When the car is at point A, the potential energy of the car-Earth system is given by

$$U_A = mgy$$

where y is the vertical height above zero level. With 135 ft = 41.1 m, this height is found as:

$$y = (41.1\ \text{m})\sin 40.0° = 26.4\ \text{m}$$

Thus,

$$U_A = (1\,000\ \text{kg})(9.80\ \text{m/s}^2)(26.4\ \text{m}) = \boxed{2.59\times10^5\ \text{J}}$$

The change in potential energy as the car moves from A to B is

$$U_B - U_A = 0 - 2.59\times10^5\ \text{J} = \boxed{-2.59\times10^5\ \text{J}}$$

continued on next page

(b) With our choice of the zero level when the car is at point A, we have $\boxed{U_A = 0}$. The potential energy when the car is at point B is given by $U_B = mgy$ where y is the vertical distance of point B below point A. In part (a), we found the magnitude of this distance to be 26.5 m. Because this distance is now below the zero reference level, it is a negative number.

Thus,

$$U_B = (1\,000\text{ kg})(9.80\text{ m/s}^2)(-26.5\text{ m}) = \boxed{-2.59 \times 10^5\text{ J}}$$

The change in potential energy when the car moves from A to B is

$$U_B - U_A = -2.59 \times 10^5\text{ J} - 0 = \boxed{-2.59 \times 10^5\text{ J}}$$

P7.38 (a) We take the zero configuration of system potential energy with the child at the lowest point of the arc. When the string is held horizontal initially, the initial position is 2.00 m above the zero level. Thus,

$$U_g = mgy = (400\text{ N})(2.00\text{ m}) = \boxed{800\text{ J}}$$

(b) From the sketch, we see that at an angle of 30.0° the child is at a vertical height of $(2.00\text{ m})(1 - \cos 30.0°)$ above the lowest point of the arc. Thus,

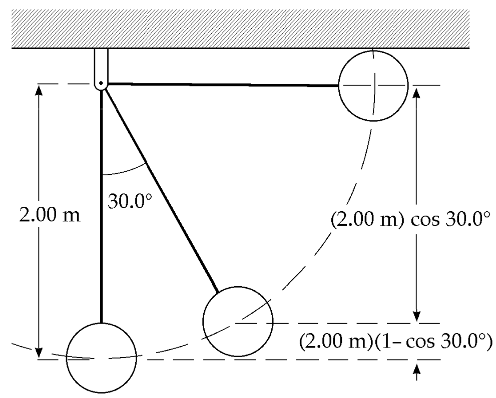

FIG. P7.38

$$U_g = mgy = (400\text{ N})(2.00\text{ m})(1 - \cos 30.0°) = \boxed{107\text{ J}}$$

(c) The zero level has been selected at the lowest point of the arc. Therefore, $\boxed{U_g = 0}$ at this location.

Section 7.7 Conservative and Nonconservative Forces

P7.39 $F_g = mg = (4.00\text{ kg})(9.80\text{ m/s}^2) = 39.2\text{ N}$

(a) Work along OAC = work along OA + work along AC

$= F_g(\text{OA})\cos 90.0° + F_g(\text{AC})\cos 180°$

$= (39.2\text{ N})(5.00\text{ m}) + (39.2\text{ N})(5.00\text{ m})(-1)$

$= \boxed{-196\text{ J}}$

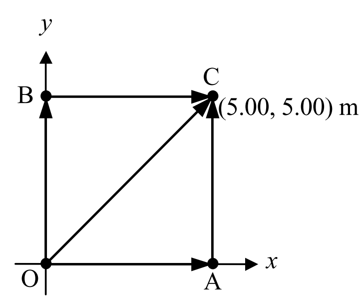

FIG. P7.39

(b) W along OBC = W along OB + W along BC

$= (39.2\text{ N})(5.00\text{ m})\cos 180° + (39.2\text{ N})(5.00\text{ m})\cos 90.0°$

$= \boxed{-196\text{ J}}$

(c) Work along OC $= F_g(\text{OC})\cos 135°$

$$= (39.2\text{ N})\left(5.00 \times \sqrt{2}\text{ m}\right)\left(-\frac{1}{\sqrt{2}}\right) = \boxed{-196\text{ J}}$$

$\boxed{\text{The results should all be the same, since gravitational forces are conservative.}}$

P7.40 (a) $W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ and if the force is constant, this can be written as

$W = \vec{\mathbf{F}} \cdot \int d\vec{\mathbf{r}} = \boxed{\vec{\mathbf{F}} \cdot \left(\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i\right)\text{, which depends only on end points, not path.}}$

(b) $W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int \left(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}\right) \cdot \left(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}\right) = (3.00\text{ N}) \int_0^{5.00\text{ m}} dx + (4.00\text{ N}) \int_0^{5.00\text{ m}} dy$

$W = (3.00\text{ N})\, x\Big|_0^{5.00\text{ m}} + (4.00\text{ N})\, y\Big|_0^{5.00\text{ m}} = 15.0\text{ J} + 20.0\text{ J} = \boxed{35.0\text{ J}}$

The same calculation applies for all paths.

P7.41 (a) The work done on the particle in its first section of motion is

$$W_{OA} = \int_0^{5.00\text{ m}} dx\hat{\mathbf{i}} \cdot \left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\text{ m}} 2y\,dx$$

and since along this path, $y = 0$ $\quad W_{OA} = 0$

In the next part of its path $\quad W_{AC} = \int_0^{5.00\text{ m}} dy\hat{\mathbf{j}} \cdot \left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\text{ m}} x^2\,dy$

For $x = 5.00$ m, $\quad W_{AC} = 125$ J

and $\quad W_{OAC} = 0 + 125 = \boxed{125\text{ J}}$

(b) Following the same steps, $\quad W_{OB} = \int_0^{5.00\text{ m}} dy\hat{\mathbf{j}} \cdot \left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\text{ m}} x^2\,dy$

since along this path, $x = 0$, $\quad W_{OB} = 0$

$$W_{BC} = \int_0^{5.00\text{ m}} dx\hat{\mathbf{i}} \cdot \left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\text{ m}} 2y\,dx$$

since $y = 5.00$ m, $\quad W_{BC} = 50.0$ J

$$W_{OBC} = 0 + 50.0 = \boxed{50.0\text{ J}}$$

(c) $$W_{OC} = \int \left(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}\right) \cdot \left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int \left(2y\,dx + x^2\,dy\right)$$

Since $x = y$ along OC, $\quad W_{OC} = \int_0^{5.00\text{ m}} \left(2x + x^2\right) dx = \boxed{66.7\text{ J}}$

(d) $\boxed{F\text{ is nonconservative since the work done is path dependent.}}$

P7.42 Along each step of motion, the frictional force is opposite in direction to the incremental displacement, so in the work $\cos 180° = -1$.

(a) $W = (3\text{ N})(5\text{ m})(-1) + (3\text{ N})(5\text{ m})(-1) = \boxed{-30.0\text{ J}}$

(b) The distance CO is $(5^2 + 5^2)^{1/2}$ m $= 7.07$ m

$W = (3\text{ N})(5\text{ m})(-1) + (3\text{ N})(5\text{ m})(-1) + (3\text{ N})(7.07\text{ m})(-1) = \boxed{-51.2\text{ J}}$

(c) $W = (3\text{ N})(7.07\text{ m})(-1) + (3\text{ N})(7.07\text{ m})(-1) = \boxed{-42.4\text{ J}}$

(d) $\boxed{\text{The force of friction is a nonconservative force.}}$

Section 7.8 Relationship Between Conservative Forces and Potential Energy

P7.43 (a) $W = \int F_x dx = \int_1^{5.00\text{ m}} (2x+4)\,dx = \left(\dfrac{2x^2}{2} + 4x\right)\Big|_1^{5.00\text{ m}} = 25.0 + 20.0 - 1.00 - 4.00 = \boxed{40.0\text{ J}}$

(b) $\Delta K + \Delta U = 0 \qquad \Delta U = -\Delta K = -W = \boxed{-40.0\text{ J}}$

(c) $\Delta K = K_f - \dfrac{mv_1^2}{2} \qquad K_f = \Delta K + \dfrac{mv_1^2}{2} = \boxed{62.5\text{ J}}$

P7.44 (a) $U = -\int_0^x \left(-Ax + Bx^2\right)dx = \boxed{\dfrac{Ax^2}{2} - \dfrac{Bx^3}{3}}$

(b) $\Delta U = -\int_{2.00\text{ m}}^{3.00\text{ m}} F\,dx = \dfrac{A\left[\left(3.00^2\right) - (2.00)^2\right]}{2} - \dfrac{B\left[(3.00)^3 - (2.00)^3\right]}{3} = \boxed{\dfrac{5.00}{2}A - \dfrac{19.0}{3}B}$

$\Delta K = \boxed{\left(-\dfrac{5.00}{2}A + \dfrac{19.0}{3}B\right)}$

P7.45 $U(r) = \dfrac{A}{r}$

$F_r = -\dfrac{\partial U}{\partial r} = -\dfrac{d}{dr}\left(\dfrac{A}{r}\right) = \boxed{\dfrac{A}{r^2}}$. If A is positive, the positive value of radial force indicates a force of repulsion.

P7.46 $F_x = -\dfrac{\partial U}{\partial x} = -\dfrac{\partial\left(3x^3y - 7x\right)}{\partial x} = -\left(9x^2y - 7\right) = 7 - 9x^2y$

$F_y = -\dfrac{\partial U}{\partial y} = -\dfrac{\partial\left(3x^3y - 7x\right)}{\partial y} = -\left(3x^3 - 0\right) = -3x^3$

Thus, the force acting at the point $(x,\ y)$ is $\vec{\mathbf{F}} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} = \boxed{\left(7 - 9x^2y\right)\hat{\mathbf{i}} - 3x^3\hat{\mathbf{j}}}$.

Section 7.9 Energy Diagrams and the Equilibrium of a System

P7.47 (a) F_x is zero at points A, C and E; F_x is positive at point B and negative at point D.

(b) A and E are unstable, and C is stable.

(c)

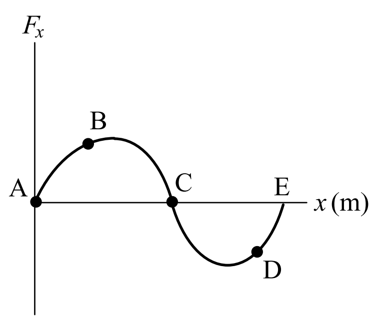

FIG. P7.47

P7.48

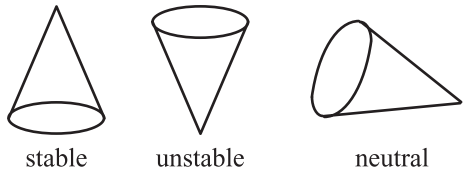

FIG. P7.48

P7.49 (a) The new length of each spring is $\sqrt{x^2+L^2}$, so its extension is $\sqrt{x^2+L^2}-L$ and the force it exerts is $k\left(\sqrt{x^2+L^2}-L\right)$ toward its fixed end. The y components of the two spring forces add to zero. Their x components add to

$$\vec{\mathbf{F}} = -2\hat{\mathbf{i}}k\left(\sqrt{x^2+L^2}-L\right)\frac{x}{\sqrt{x^2+L^2}} = \boxed{-2kx\hat{\mathbf{i}}\left(1-\frac{L}{\sqrt{x^2+L^2}}\right)}$$

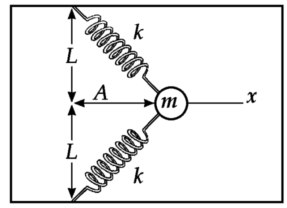

FIG. P7.49

(b) Choose $U=0$ at $x=0$. Then at any point the potential energy of the system is

$$U(x) = -\int_0^x F_x dx = -\int_0^x\left(-2kx+\frac{2kLx}{\sqrt{x^2+L^2}}\right)dx = 2k\int_0^x x\,dx - 2kL\int_0^x \frac{x}{\sqrt{x^2+L^2}}\,dx$$

$$U(x) = \boxed{kx^2+2kL\left(L-\sqrt{x^2+L^2}\right)}$$

(c) $U(x) = 40.0x^2+96.0\left(1.20-\sqrt{x^2+1.44}\right)$

For negative x, $U(x)$ has the same value as for positive x. The only equilibrium point (i.e., where $F_x=0$) is $\boxed{x=0}$.

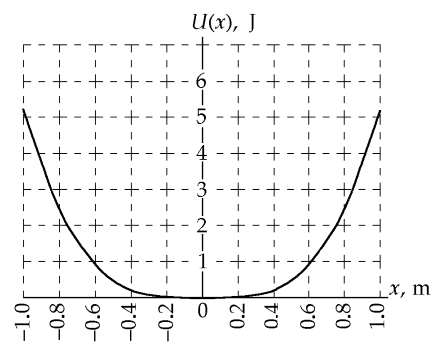

FIG. P7.49(c)

(d) $K_i+U_i+\Delta E_{\text{mech}} = K_f+U_f$

$$0+0.400\text{ J}+0 = \frac{1}{2}(1.18\text{ kg})v_f^2+0$$

$$v_f = \boxed{0.823\text{ m/s}}$$

Additional Problems

P7.50 The work done by the applied force is

$$W = \int_i^f F_{\text{applied}}dx = \int_0^{x_{\max}} -\left[-\left(k_1x+k_2x^2\right)\right]dx$$

$$= \int_0^{x_{\max}} k_1x\,dx + \int_0^{x_{\max}} k_2x^2dx = k_1\left.\frac{x^2}{2}\right|_0^{x_{\max}} + k_2\left.\frac{x^3}{3}\right|_0^{x_{\max}}$$

$$= \boxed{k_1\frac{x_{\max}^2}{2}+k_2\frac{x_{\max}^3}{3}}$$

P7.51 At start, $\vec{\mathbf{v}} = (40.0 \text{ m/s})\cos 30.0°\hat{\mathbf{i}} + (40.0 \text{ m/s})\sin 30.0°\hat{\mathbf{j}}$

At apex, $\vec{\mathbf{v}} = (40.0 \text{ m/s})\cos 30.0°\hat{\mathbf{i}} + 0\hat{\mathbf{j}} = (34.6 \text{ m/s})\hat{\mathbf{i}}$

And $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.150 \text{ kg})(34.6 \text{ m/s})^2 = \boxed{90.0 \text{ J}}$

P7.52 (a) We write

$$F = ax^b$$

$$1\,000 \text{ N} = a(0.129 \text{ m})^b$$

$$5\,000 \text{ N} = a(0.315 \text{ m})^b$$

$$5 = \left(\frac{0.315}{0.129}\right)^b = 2.44^b$$

$$\ln 5 = b \ln 2.44$$

$$b = \frac{\ln 5}{\ln 2.44} = \boxed{1.80 = b}$$

$$a = \frac{1\,000 \text{ N}}{(0.129 \text{ m})^{1.80}} = \boxed{4.01\times 10^4 \text{ N/m}^{1.8} = a}$$

(b) $$W = \int_0^{0.25 \text{ m}} F dx = \int_0^{0.25 \text{ m}} 4.01\times 10^4 \frac{\text{N}}{\text{m}^{1.8}} x^{1.8} dx$$

$$= 4.01\times 10^4 \frac{\text{N}}{\text{m}^{1.8}} \frac{x^{2.8}}{2.8}\bigg|_0^{0.25 \text{ m}} = 4.01\times 10^4 \frac{\text{N}}{\text{m}^{1.8}} \frac{(0.25 \text{ m})^{2.8}}{2.8}$$

$$= \boxed{294 \text{ J}}$$

***P7.53** (a) We assume the spring is in the horizontal plane of the motion. The radius of the puck's motion is 0.155 m + x

The spring force causes the puck's centripetal acceleration:
$(4.3 \text{ N/m})\ x = F = mv^2/r = m(2\pi r/\text{T})^2/r = 4\pi^2\ m\ r/(1.3 \text{ s})^2$

$(4.3 \text{ kg/s}^2)\ x = (23.4/\text{s}^2)\ m\ (0.155 \text{ m} + x)$

$4.3 \text{ kg}\ x = 3.62\ m\ \text{ m} + 23.4\ m\ x$

$4.3000 \text{ kg}\ x - 23.360\ m\ x = 3.6208\ m\ \text{ m}$

$\boxed{x = (3.62\ m)/(4.3 \text{ kg} - 23.4\ m) \text{ meters}}$

(b) $x = (3.62 \text{ m } 0.07 \text{ kg})/(4.30 \text{ kg} - 23.4\ [0.07 \text{ kg}]) = \boxed{0.0951 \text{ m}}$ a nice reasonable extension

(c) We double the puck mass and find
$x = (3.6208 \text{ m } 0.14 \text{ kg})/(4.30 \text{ kg} - 23.360\ [0.14 \text{ kg}]) = \boxed{0.492 \text{ m}}$ more than twice as big!

(d) $x = (3.62 \text{ m } 0.18 \text{ kg})/(4.30 \text{ kg} - 23.4\ [0.18 \text{ kg}]) = \boxed{6.85 \text{ m}}$ We have to get a bigger table!!

(e) When the denominator of the fraction goes to zero, the extension becomes infinite. This happens for $4.3 \text{ kg} - 23.4\ m = 0$; that is for $m = 0.184$ kg. For any larger mass, the spring cannot constrain the motion. $\boxed{\text{The situation is impossible.}}$

(f) The extension is directly proportional to m when m is only a few grams. Then it grows faster and faster, diverging to infinity for $m = 0.184$ kg.

***P7.54** (a) A time interval. If the interaction occupied no time, the force exerted by each ball on the other would be infinite, and that cannot happen.

(b) $k = |F|/|x| = 16\ 000\ \text{N}/0.000\ 2\ \text{m} = \boxed{80\ \text{MN/m}}$

(c) We assume that steel has the density of its main constituent, iron, shown in Table 14.1.

Then its mass is $\rho V = \rho\,(4/3)\pi r^3 = (4\pi/3)(7860\ \text{kg/m}^3)(0.0254\ \text{m}/2)^3 = 0.0674\ \text{kg}$

and $K = (1/2)\,mv^2 = (1/2)\,(0.0674\ \text{kg})(5\ \text{m/s})^2 = 0.843\ \text{J} \approx \boxed{0.8\ \text{J}}$

(d) Imagine one ball running into an infinitely hard wall and bouncing off elastically. The original kinetic energy becomes elastic potential energy

$0.843\ \text{J} = (1/2)\,(8 \times 10^7\ \text{N/m})\,x^2 \qquad x = 0.145\ \text{mm} \approx \boxed{0.15\ \text{mm}}$

(e) The ball does not really stop with constant acceleration, but imagine it moving 0.145 mm forward with average speed $(5\ \text{m/s} + 0)/2 = 2.5\ \text{m/s}$. The time interval over which it stops is then

$0.145\ \text{mm}/(2.5\ \text{m/s}) = 6 \times 10^{-5}\ \text{s}\ \boxed{\sim 10^{-4}\ \text{s}}$

***P7.55** The potential energy at point x is given by 5 plus the negative of the work the force does as a particle feeling the force is carried from $x = 0$ to location x.

$$dU = -F dx \qquad \int_5^U dU = -\int_0^x 8e^{-2x}dx \qquad U - 5 = -\left(8/\left[-2\right]\right)\int_0^x e^{-2x}\left(-2dx\right)$$

$$U = 5 - \left(8/\left[-2\right]\right)e^{-2x}\Big|_0^x = 5 + 4e^{-2x} - 4\cdot 1 = \boxed{1 + 4e^{-2x}}$$

The force must be conservative because the work the force does on the object on which it acts depends only on the original and final positions of the object, not on the path between them.

P7.56 (a) $\vec{\mathbf{F}} = -\dfrac{d}{dx}\left(-x^3 + 2x^2 + 3x\right)\hat{\mathbf{i}} = \boxed{\left(3x^2 - 4x - 3\right)\hat{\mathbf{i}}}$

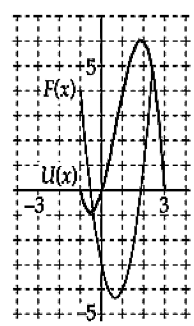

FIG. P7.56

(b) $F = 0$

when $x = \boxed{1.87 \text{ and } -0.535}$

(c) The stable point is at

$$x = -0.535 \text{ point of minimum } U(x)$$

The unstable point is at

$$x = 1.87 \text{ maximum in } U(x)$$

P7.57 $K_i + W_s + W_g = K_f$

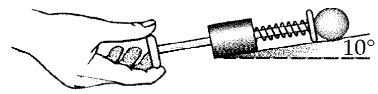

FIG. P7.57

$$\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 + mg\Delta x\cos\theta = \frac{1}{2}mv_f^2$$

$$0 + \frac{1}{2}kx_i^2 - 0 + mgx_i\cos 100° = \frac{1}{2}mv_f^2$$

$$\frac{1}{2}(1.20\ \text{N/cm})(5.00\ \text{cm})(0.050\,0\ \text{m}) - (0.100\ \text{kg})\left(9.80\ \text{m/s}^2\right)(0.050\,0\ \text{m})\sin 10.0°$$

$$= \frac{1}{2}(0.100\ \text{kg})v^2$$

$$0.150\ \text{J} - 8.51\times10^{-3}\ \text{J} = (0.050\,0\ \text{kg})v^2$$

$$v = \sqrt{\frac{0.141}{0.050\,0}} = \boxed{1.68\ \text{m/s}}$$

P7.58 (a) $\vec{\mathbf{F}}_1 = (25.0\ \text{N})\left(\cos 35.0°\hat{\mathbf{i}} + \sin 35.0°\hat{\mathbf{j}}\right) = \boxed{\left(20.5\hat{\mathbf{i}} + 14.3\hat{\mathbf{j}}\right)\ \text{N}}$

$\vec{\mathbf{F}}_2 = (42.0\ \text{N})\left(\cos 150°\hat{\mathbf{i}} + \sin 150°\hat{\mathbf{j}}\right) = \boxed{\left(-36.4\hat{\mathbf{i}} + 21.0\hat{\mathbf{j}}\right)\ \text{N}}$

(b) $\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \boxed{\left(-15.9\hat{\mathbf{i}} + 35.3\hat{\mathbf{j}}\right)\ \text{N}}$

(c) $\vec{\mathbf{a}} = \dfrac{\sum \mathbf{F}}{m} = \boxed{\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\ \text{m/s}^2}$

(d) $\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = \left(4.00\hat{\mathbf{i}} + 2.50\hat{\mathbf{j}}\right)\ \text{m/s} + \left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\left(\text{m/s}^2\right)(3.00\ \text{s})$

$\vec{\mathbf{v}}_f = \boxed{\left(-5.54\hat{\mathbf{i}} + 23.7\hat{\mathbf{j}}\right)\ \text{m/s}}$

(e) $\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \dfrac{1}{2}\vec{\mathbf{a}}t^2$

$\vec{\mathbf{r}}_f = 0 + \left(4.00\hat{\mathbf{i}} + 2.50\hat{\mathbf{j}}\right)(\text{m/s})(3.00\ \text{s}) + \dfrac{1}{2}\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\left(\text{m/s}^2\right)(3.00\ \text{s})^2$

$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f = \boxed{\left(-2.30\hat{\mathbf{i}} + 39.3\hat{\mathbf{j}}\right)\ \text{m}}$

(f) $K_f = \dfrac{1}{2}mv_f^2 = \dfrac{1}{2}(5.00\ \text{kg})\left[(5.54)^2 + (23.7)^2\right]\left(\text{m/s}^2\right) = \boxed{1.48\ \text{kJ}}$

(g) $K_f = \dfrac{1}{2}mv_i^2 + \sum \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}}$

$K_f = \dfrac{1}{2}(5.00\ \text{kg})\left[(4.00)^2 + (2.50)^2\right](\text{m/s})^2 + \left[(-15.9\ \text{N})(-2.30\ \text{m}) + (35.3\ \text{N})(39.3\ \text{m})\right]$

$K_f = 55.6\ \text{J} + 1\,426\ \text{J} = \boxed{1.48\ \text{kJ}}$

(h) The work-kinetic energy theorem is consistent with Newton's second law, used in deriving it.

P7.59 We evaluate by $\int_{12.8}^{23.7} \frac{375dx}{x^3+3.75x}$ calculating

$$\frac{375(0.100)}{(12.8)^3+3.75(12.8)}+\frac{375(0.100)}{(12.9)^3+3.75(12.9)}+\dots\frac{375(0.100)}{(23.6)^3+3.75(23.6)}=0.806$$

and

$$\frac{375(0.100)}{(12.9)^3+3.75(12.9)}+\frac{375(0.100)}{(13.0)^3+3.75(13.0)}+\dots\frac{375(0.100)}{(23.7)^3+3.75(23.7)}=0.791.$$

The answer must be between these two values. We may find it more precisely by using a value for Δx smaller than 0.100. Thus, we find the integral to be $\boxed{0.799\ \text{N}\cdot\text{m}}$.

P7.60 (a)

$F(\text{N})$	$L(\text{mm})$	$F(\text{N})$	$L(\text{mm})$
2.00	15.0	14.0	112
4.00	32.0	16.0	126
6.00	49.0	18.0	149
8.00	64.0	20.0	175
10.0	79.0	22.0	190
12.0	98.0		

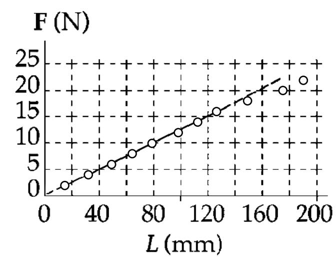

FIG. P7.60

To draw the straight line we use all the points listed and also the origin. If the coils of the spring touched each other, a bend or nonlinearity could show up at the bottom end of the graph. If the spring were stretched "too far," a nonlinearity could show up at the top end. But there is no visible evidence for a bend in the graph near either end.

(b) By least-square fitting, its slope is

$$0.125\ \text{N/mm} \pm 2\% = \boxed{125\ \text{N/m}} \pm 2\%$$

In $F = kx$, the spring constant is $k = \frac{F}{x}$, the same as the slope of the F-versus-x graph.

(c) $F = kx = (125\ \text{N/m})(0.105\ \text{m}) = \boxed{13.1\ \text{N}}$

ANSWERS TO EVEN PROBLEMS

P7.2 (a) 3.28×10^{-2} J (b) -3.28×10^{-2} J

P7.4 Yes. It exerts a force of equal magnitude in the opposite direction that acts over the same distance. −15.0 J

P7.6 28.9

P7.8 5.33 J/s

P7.10 16.0

P7.12 (a) see the solution (b) −12.0 J

P7.14 50.0 J

P7.16 (a) 575 N/m (b) 46.0 J

P7. 18 (a) 1.13 kN/m (b) 51.8 cm

P7.20 (a) −1.23 m/s² and +0.616 m/s² (b) −0.252 m/s² and 0 (c) 0 and 0

P7.22 (a) 9.00 kJ (b) 11.7 kJ, larger by 29.6%

P7.24 (a) $\frac{mg}{k_1}+\frac{mg}{k_2}$ (b) $\left(\frac{1}{k_1}+\frac{1}{k_2}\right)^{-1}$

P7.26 kg/s^2

P7. 28 If the weight of the first tray stretches all four springs by a distance equal to the thickness of the tray, then the proportionality expressed by Hooke's law guarantees that each additional tray will have the same effect, so that the top surface of the top tray will always have the same elevation. 316 N/m. We do not need to know the length and width of the tray.

P7.30 (a) 33.8 J (b) 135 J

P7.32 (a) 1.94 m/s (b) 3.35 m/s (c) 3.87 m/s

P7. 34 (a) yes. Its kinetic energy as it enters the sand is sufficient to do all of the work it must do in plowing through the pile. (b) 0.306 m/s

P7.36 (a) 3.78×10^{-16} J (b) 1.35×10^{-14} N (c) $1.48\times10^{+16}$ m/s^2 (d) 1.94 ns

P7.38 (a) 800 J (b) 107 J (c) 0

P7.40 (a) see the solution (b) 35.0 J

P7. 42 (a) –30.0 J (b) –51.2 J (c) –42.4 J (d) The force of friction is a nonconservative force.

P7.44 (a) $Ax^2/2 - Bx^3/3$ (b) $\Delta U = 2.5A - 6.33B$; $\Delta K = -2.5A + 6.33B$

P7.46 $\left(7-9x^2y\right)\hat{\mathbf{i}}-3x^3\hat{\mathbf{j}}$

P7.48 see the solution

P7.50 $k_1x_{max}{}^2/2 + k_2x_{max}{}^3/3$

P7.52 (a) $a=\dfrac{40.1\text{ kN}}{m^{1.8}}$; $b=1.80$ (b) 294 J

P7. 54 (a) A time interval. If the interaction occupied no time, the force exerted by each ball on the other would be infinite, and that cannot happen. (b) 80 MN/m (c) 0.8 J. We assume that steel has the same density as iron. (d) 0.15 mm (e) 10^{-4} s

P7.56 (a) $F_x = (3x^2 - 4x - 3)$ (b) 1.87 and −0.535 (c) see the solution. −0.535 is stable and 1.87 is unstable.

P7.58 (a) $\vec{\mathbf{F}}_1 = \left(20.5\hat{\mathbf{i}} + 14.3\hat{\mathbf{j}}\right)$ N; $\vec{\mathbf{F}}_2 = \left(-36.4\hat{\mathbf{i}} + 21.0\hat{\mathbf{j}}\right)$ N (b) $\left(-15.9\hat{\mathbf{i}} + 35.3\hat{\mathbf{j}}\right)$ N

(c) $\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)$ m/s^2 (d) $\left(-5.54\hat{\mathbf{i}} + 23.7\hat{\mathbf{j}}\right)$ m/s (e) $\left(-2.30\hat{\mathbf{i}} + 39.3\hat{\mathbf{j}}\right)$ m

(f) 1.48 kJ (g) 1.48 kJ

(h) The work-kinetic energy theorem is consistent with Newton's second law, used in deriving it.

P7. 60 (a) See the solution. We use all the points listed and also the origin. There is no visible evidence for a bend in the graph or nonlinearity near either end. (b) 125 N/m ± 2% (c) 13.1 N

8

Conservation of Energy

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

***Q8.1** Not everything has energy. A rock stationary on the floor, chosen as the $y = 0$ reference level, has no mechanical energy. In cosmic terms, think of the burnt-out core of a star far in the future after it has cooled nearly to absolute zero.

***Q8.2** answer (c). Gravitational energy is proportional to the mass of the object in the Earth's field.

***Q8.3** (i) answer b. Kinetic energy is proportional to mass.

(ii) answer c. The slide is frictionless, so $v = (2gh)^{1/2}$ in both cases.

(iii) answer a. g for the smaller child and $g \sin\theta$ for the larger.

***Q8.4** (a) yes: a block slides on the floor where we choose $y = 0$.

(b) yes: a picture on the classroom wall high above the floor.

(c) yes: an eraser hurtling across the room.

(d) yes: the block stationary on the floor.

***Q8.5** answer (d). The energy is internal energy. Energy is never "used up." The ball finally has no elevation and no compression, so it has no potential energy. There is no stove, so no heat is put in. The amount of sound energy is minuscule.

***Q8.6** answer (a). We assume the light band of the slingshot puts equal amounts of kinetic energy into the missiles. With three times more speed, the bean has nine times more squared speed, so it must have one-ninth the mass.

Q8.7 They will not agree on the original gravitational energy if they make different $y = 0$ choices. They see the same change in elevation, so they do agree on the change in gravitational energy and on the kinetic energy.

Q8.8 Lift a book from a low shelf to place it on a high shelf. The net change in its kinetic energy is zero, but the book-Earth system increases in gravitational potential energy. Stretch a rubber band to encompass the ends of a ruler. It increases in elastic energy. Rub your hands together or let a pearl drift down at constant speed in a bottle of shampoo. Each system (two hands; pearl and shampoo) increases in internal energy.

Q8.9 All the energy is supplied by foodstuffs that gained their energy from the sun.

Q8.10 The total energy of the ball-Earth system is conserved. Since the system initially has gravitational energy *mgh* and no kinetic energy, the ball will again have zero kinetic energy when it returns to its original position. Air resistance will cause the ball to come back to a point slightly below its initial position. On the other hand, if anyone gives a forward push to the ball anywhere along its path, the demonstrator will have to duck.

Q8.11 Let the gravitational energy be zero at the lowest point in the motion. If you start the vibration by pushing down on the block (2), its kinetic energy becomes extra elastic potential energy in the spring (U_s). After the block starts moving up at its lower turning point (3), this energy becomes both kinetic energy (K) and gravitational potential energy (U_g), and then just gravitational energy when the block is at its greatest height (1). The energy then turns back into kinetic and elastic potential energy, and the cycle repeats.

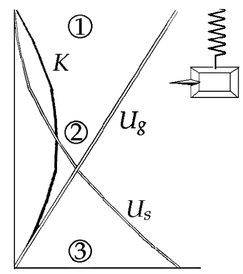

FIG. Q8.11

***Q8.12** We have $(1/2)mv^2 = \mu_k mgd$ so $d = v^2/2\mu_k g$. The quantity v^2/μ_k controls the skidding distance. In the cases quoted respectively, this quantity has the numerical value (a) 5 (b) 2.5 (c) 1.25 (d) 20 (e) 10 (f) 5. In order the ranking is then d > e > f = a > b > c.

***Q8.13** Yes, if it is exerted by an object that is moving in our frame of reference. The flat bed of a truck exerts a static friction force to start a pumpkin moving forward as it slowly starts up.

***Q8.14** (a) A campfire converts chemical energy into internal energy, within the system wood-plus-oxygen, and before energy is transferred by heat and electromagnetic radiation into the surroundings. If all the fuel burns, the process can be 100% efficient. Chemical-energy-into-internal-energy is also the conversion as iron rusts, and it is the main conversion in mammalian metabolism.

(b) An escalator motor converts electrically transmitted energy into gravitational energy. As the system we may choose motor-plus-escalator-and-riders. The efficiency could be say 90%, but in many escalators plenty of internal energy is another output. A natural process, such as atmospheric electric current in the Earth's aurora borealis raising the temperature of a particular parcel of air so that the surrounding air buoys it up, could produce the same electrically-transmitted-to-gravitational energy conversion with low efficiency.

(c) A diver jumps up from a diving board, setting it vibrating temporarily. The material in the board rises in temperature slightly as the visible vibration dies down, and then the board cools off to the constant temperature of the environment. This process for the board-plus-air system can have 100% efficiency in converting the energy of vibration into energy transferred by heat. The energy of vibration is all elastic energy at instants when the board is momentarily at rest at turning points in its motion. For a natural process, you could think of the branch of a palm tree vibrating for a while after a coconut falls from it.

(d) Some of the sound energy in a shout becomes a tiny bit of work done on a listener's ear; most of the mechanical-wave energy becomes internal energy as the sound is absorbed by all the surfaces it falls upon. We would also assign low efficiency to a train of water waves doing work to make a linear pile of shells at the high-water mark on a beach.

(e) A demonstration solar car takes in electromagnetic-wave energy in sunlight and turns some fraction of it temporarily into the car's kinetic energy. A much larger fraction becomes internal energy in the solar cells, battery, motor, and air pushed aside. Perhaps with somewhat higher net efficiency, the pressure of light from a newborn star pushes away gas and dust in the nebula surrounding it.

***Q8.15** (a) original elastic potential energy into final kinetic energy

(b) original chemical energy into final internal energy

(c) original internal energy in the batteries into final internal energy, plus a tiny bit of outgoing energy transmitted by mechanical waves

(d) original kinetic energy into final internal energy in the brakes

(e) heat input from the lower layers of the Sun, into energy transmitted by electromagnetic radiation

(f) original chemical energy into final gravitational energy

***Q8.16** Answer (k). The static friction force that each glider exerts on the other acts over no distance. The air track isolates the gliders from outside forces doing work. The gliders-Earth system keeps constant mechanical energy.

Q8.17 The larger engine is unnecessary. Consider a 30-minute commute. If you travel the same speed in each car, it will take the same amount of time, expending the same amount of energy. The extra power available from the larger engine isn't used.

SOLUTIONS TO PROBLEMS

Section 8.1 The Nonisolated System—Conservation of Energy

***P8.1** (a) The toaster coils take in energy by electrical transmission. They increase in internal energy and put out energy by heat into the air and energy by electromagnetic radiation as they start to glow. $\boxed{\Delta E_{\text{int}} = Q + T_{\text{ET}} + T_{\text{ER}}}$

(b) The car takes in energy by mass transfer. Its fund of chemical potential energy increases. As it moves, its kinetic energy increases and it puts out work on the air, energy by heat in the exhaust, and a tiny bit of energy by mechanical waves in sound. $\boxed{\Delta K + \Delta U + \Delta E_{\text{int}} = W + Q + T_{\text{MW}} + T_{\text{MT}}}$

(c) You take in energy by mass transfer. Your fund of chemical potential energy increases. You are always putting out energy by heat into the surrounding air. $\boxed{\Delta U = Q + T_{\text{MT}}}$

(d) Your house is in steady state, keeping constant energy as it takes in energy by electrical transmission to run the clocks and, we assume, an air conditioner. It absorbs sunlight, taking in energy by electromagnetic radiation. The exterior plenum of the air conditioner takes in cooler air and puts it out as warmer air, transferring out energy by mass transfer. $\boxed{0 = Q + T_{\text{MT}} + T_{\text{ET}} + T_{\text{ER}}}$

Section 8.2 The Isolated System

P8.2 (a) One child in one jump converts chemical energy into mechanical energy in the amount that her body has as gravitational energy at the top of her jump:

$$mgy = 36 \text{ kg}\left(9.81 \text{ m/s}^2\right)(0.25 \text{ m}) = 88.3 \text{ J}$$

For all of the jumps of the children the energy is $12\left(1.05\times10^6\right)88.3 \text{ J} = \boxed{1.11\times10^9 \text{ J}}$.

(b) The seismic energy is modeled as $E = \dfrac{0.01}{100}1.11\times10^9 \text{ J} = 1.11\times10^5 \text{ J}$, making the Richter magnitude $\dfrac{\log E - 4.8}{1.5} = \dfrac{\log 1.11\times10^5 - 4.8}{1.5} = \dfrac{5.05-4.8}{1.5} = \boxed{0.2}$.

P8.3 $U_i + K_i = U_f + K_f$: $\quad mgh + 0 = mg(2R) + \frac{1}{2}mv^2$

$$g(3.50R) = 2g(R) + \frac{1}{2}v^2$$

$$\boxed{v = \sqrt{3.00gR}}$$

$\sum F = m\frac{v^2}{R}$: $\quad n + mg = m\frac{v^2}{R}$

$$n = m\left[\frac{v^2}{R} - g\right] = m\left[\frac{3.00gR}{R} - g\right] = 2.00mg$$

$$n = 2.00\left(5.00\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)$$

$$= \boxed{0.098\,0\ \text{N downward}}$$

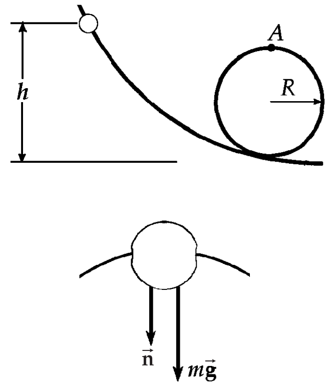

FIG. P8.3

P8.4 (a) $(\Delta K)_{A\to B} = \sum W = W_g = mg\Delta h = mg(5.00 - 3.20)$

$$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = m(9.80)(1.80)$$

$$v_B = \boxed{5.94\ \text{m/s}}$$

Similarly, $v_C = \sqrt{v_A^2 + 2g(5.00 - 2.00)} = \boxed{7.67\ \text{m/s}}$

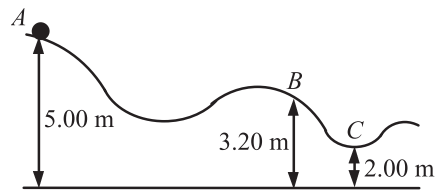

FIG. P8.4

(b) $W_g\big|_{A\to C} = mg(3.00\ \text{m}) = \boxed{147\ \text{J}}$

P8.5 From conservation of energy for the block-spring-Earth system,

$$U_{gt} = U_{si}$$

or

$$(0.250\ \text{kg})\left(9.80\ \text{m/s}^2\right)h = \left(\frac{1}{2}\right)(5\,000\ \text{N/m})(0.100\ \text{m})^2$$

This gives a maximum height $h = \boxed{10.2\ \text{m}}$.

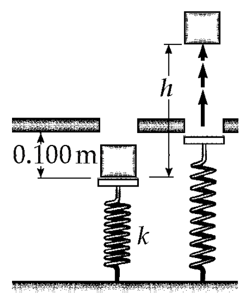

FIG. P8.5

P8.6 (a) The force needed to hang on is equal to the force F the trapeze bar exerts on the performer.

From the free-body diagram for the performer's body, as shown,

$$F - mg\cos\theta = m\frac{v^2}{\ell}$$

or

$$F = mg\cos\theta + m\frac{v^2}{\ell}$$

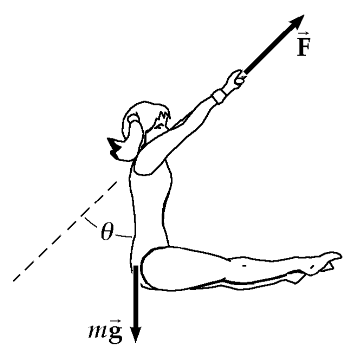

FIG. P8.6

Apply conservation of mechanical energy of the performer-Earth system as the performer moves between the starting point and any later point:

$$mg\left(\ell - \ell\cos\theta_i\right) = mg(\ell - \ell\cos\theta) + \frac{1}{2}mv^2$$

Solve for $\frac{mv^2}{\ell}$ and substitute into the force equation to obtain $F = \boxed{mg\left(3\cos\theta - 2\cos\theta_i\right)}$.

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(b) At the bottom of the swing, $\theta = 0°$

so

$$F = mg\left(3 - 2\cos\theta_i\right)$$
$$F = 2mg = mg\left(3 - 2\cos\theta_i\right)$$

which gives

$$\theta_i = \boxed{60.0°}$$

P8.7 Using conservation of energy for the system of the Earth and the two objects

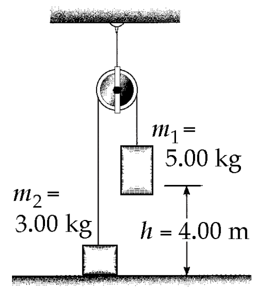

FIG. P8.7

(a) $(5.00\text{ kg})g(4.00\text{ m}) = (3.00\text{ kg})g(4.00\text{ m}) + \frac{1}{2}(5.00 + 3.00)v^2$

$v = \sqrt{19.6} = \boxed{4.43\text{ m/s}}$

(b) Now we apply conservation of energy for the system of the 3.00 kg object and the Earth during the time interval between the instant when the string goes slack and the instant at which the 3.00 kg object reaches its highest position in its free fall.

$$\frac{1}{2}(3.00)v^2 = mg\,\Delta y = 3.00g\Delta y$$
$$\Delta y = 1.00\text{ m}$$
$$y_{\max} = 4.00\text{ m} + \Delta y = \boxed{5.00\text{ m}}$$

P8.8 We assume $m_1 > m_2$

(a) $m_1gh = \frac{1}{2}\left(m_1 + m_2\right)v^2 + m_2gh$

$v = \boxed{\sqrt{\dfrac{2\left(m_1 - m_2\right)gh}{\left(m_1 + m_2\right)}}}$

(b) Since m_2 has kinetic energy $\frac{1}{2}m_2v^2$, it will rise an additional height Δh determined from

$$m_2g\,\Delta h = \frac{1}{2}m_2v^2$$

or from (a),

$$\Delta h = \frac{v^2}{2g} = \frac{\left(m_1 - m_2\right)h}{\left(m_1 + m_2\right)}$$

The total height m_2 reaches is $h + \Delta h = \boxed{\dfrac{2m_1h}{m_1 + m_2}}$.

P8.9 The force of tension and subsequent force of compression in the rod do no work on the ball, since they are perpendicular to each step of displacement. Consider energy conservation of the ball-Earth system between the instant just after you strike the ball and the instant when it reaches the top. The speed at the top is zero if you hit it just hard enough to get it there.

$$K_i + U_{gi} = K_f + U_{gf}: \qquad \frac{1}{2}mv_i^2 + 0 = 0 + mg(2L)$$

$$v_i = \sqrt{4gL} = \sqrt{4(9.80)(0.770)}$$

$$v_i = \boxed{5.49\ \text{m/s}}$$

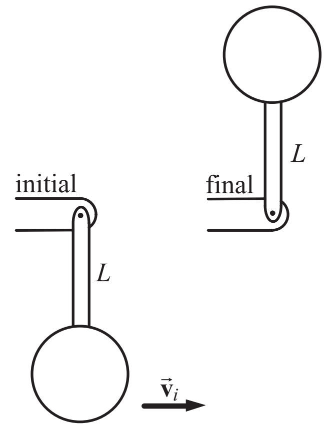

FIG. P8.9

P8.10 (a) $K_i + U_{gi} = K_f + U_{gf}$

$$\frac{1}{2}mv_i^2 + 0 = \frac{1}{2}mv_f^2 + mgy_f$$

$$\frac{1}{2}mv_{xi}^2 + \frac{1}{2}mv_{yi}^2 = \frac{1}{2}mv_{xf}^2 + mgy_f$$

Note that we have used the Pythagorean theorem to express the original kinetic energy in terms of the velocity components. Kinetic energy itself does not have components. Now $v_{xi} = v_{xf}$, so for the first ball

$$y_f = \frac{v_{yi}^2}{2g} = \frac{(1\,000\sin 37.0°)^2}{2(9.80)} = \boxed{1.85\times10^4\ \text{m}}$$

and for the second

$$y_f = \frac{(1\,000)^2}{2(9.80)} = \boxed{5.10\times10^4\ \text{m}}$$

(b) The total energy of each is constant with value

$$\frac{1}{2}(20.0\ \text{kg})(1\,000\ \text{m/s})^2 = \boxed{1.00\times10^7\ \text{J}}$$

P8.11 (a) For a 5-m cord the spring constant is described by $F = kx$, $mg = k(1.5\ \text{m})$. For a longer cord of length L the stretch distance is longer so the spring constant is smaller in inverse proportion:

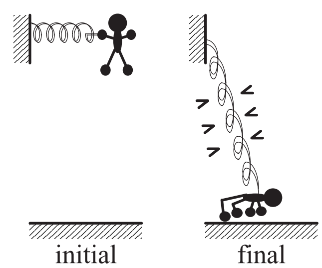

FIG. P8.11(a)

$$k = \frac{5\ \text{m}}{L}\frac{mg}{1.5\ \text{m}} = 3.33\,mg/L$$

$$(K + U_g + U_s)_i = (K + U_g + U_s)_f$$

$$0 + mgy_i + 0 = 0 + mgy_f + \frac{1}{2}kx_f^2$$

$$mg(y_i - y_f) = \frac{1}{2}kx_f^2 = \frac{1}{2}3.33\frac{mg}{L}x_f^2$$

here $y_i - y_f = 55\ \text{m} = L + x_f$

$$55.0\ \text{m}L = \frac{1}{2}3.33(55.0\ \text{m} - L)^2$$

$$55.0\ \text{m}L = 5.04\times10^3\ \text{m}^2 - 183\ \text{m}L + 1.67L^2$$

$$0 = 1.67L^2 - 238L + 5.04\times10^3 = 0$$

$$L = \frac{238 \pm \sqrt{238^2 - 4(1.67)(5.04\times10^3)}}{2(1.67)} = \frac{238\pm152}{3.33} = \boxed{25.8\ \text{m}}$$

only the value of L less than 55 m is physical.

continued on next page

(b) $k = 3.33\dfrac{mg}{25.8\text{ m}}$ $\quad x_{max} = x_f = 55.0\text{ m} - 25.8\text{ m} = 29.2\text{ m}$

$\sum F = ma \quad +kx_{max} - mg = ma$

$$3.33\frac{mg}{25.8\text{ m}}29.2\text{ m} - mg = ma$$

$$a = 2.77g = \boxed{27.1\text{ m/s}^2}$$

P8.12 When block B moves up by 1 cm, block A moves down by 2 cm and the separation becomes 3 cm. We then choose the final point to be when B has moved up by $\dfrac{h}{3}$ and has speed $\dfrac{v_A}{2}$. Then A has moved down $\dfrac{2h}{3}$ and has speed v_A:

$$(K_A + K_B + U_g)_i = (K_A + K_B + U_g)_f$$

$$0 + 0 + 0 = \frac{1}{2}mv_A^2 + \frac{1}{2}m\left(\frac{v_A}{2}\right)^2 + \frac{mgh}{3} - \frac{mg2h}{3}$$

$$\frac{mgh}{3} = \frac{5}{8}mv_A^2$$

$$\boxed{v_A = \sqrt{\frac{8gh}{15}}}$$

Section 8.3 **Situations Involving Kinetic Friction**

P8.13 $\sum F_y = ma_y$: $\quad n - 392\text{ N} = 0$

$$n = 392\text{ N}$$

$$f_k = \mu_k n = (0.300)(392\text{ N}) = 118\text{ N}$$

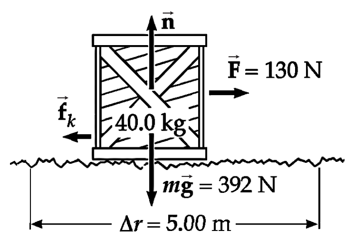

FIG. P8.13

(a) $W_F = F\Delta r\cos\theta = (130)(5.00)\cos 0° = \boxed{650\text{ J}}$

(b) $\Delta E_{int} = f_k\Delta x = (118)(5.00) = \boxed{588\text{ J}}$

(c) $W_n = n\Delta r\cos\theta = (392)(5.00)\cos 90° = \boxed{0}$

(d) $W_g = mg\Delta r\cos\theta = (392)(5.00)\cos(-90°) = \boxed{0}$

(e) $\Delta K = K_f - K_i = \sum W_{other} - \Delta E_{int}$

$$\frac{1}{2}mv_f^2 - 0 = 650\text{ J} - 588\text{ J} + 0 + 0 = \boxed{62.0\text{ J}}$$

(f) $v_f = \sqrt{\dfrac{2K_f}{m}} = \sqrt{\dfrac{2(62.0\text{ J})}{40.0\text{ kg}}} = \boxed{1.76\text{ m/s}}$

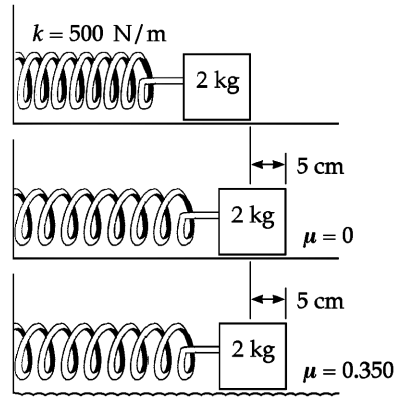

FIG. P8.14

P8.14 (a) $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$

$$= \frac{1}{2}(500)\left(5.00\times 10^{-2}\right)^2 - 0 = 0.625 \text{ J}$$

$$W_s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 - 0$$

so

$$v_f = \sqrt{\frac{2\left(\sum W\right)}{m}}$$

$$= \sqrt{\frac{2(0.625)}{2.00}} \text{ m/s} = \boxed{0.791 \text{ m/s}}$$

(b) $\frac{1}{2}mv_i^2 - f_k\Delta x + W_s = \frac{1}{2}mv_f^2$

$$0 - (0.350)(2.00)(9.80)(0.050\,0) \text{ J} + 0.625 \text{ J} = \frac{1}{2}mv_f^2$$

$$0.282 \text{ J} = \frac{1}{2}(2.00 \text{ kg})v_f^2$$

$$v_f = \sqrt{\frac{2(0.282)}{2.00}} \text{ m/s} = \boxed{0.531 \text{ m/s}}$$

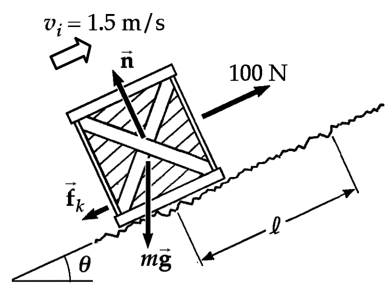

FIG. P8.15

P8.15 (a) $W_g = mg\ell\cos(90.0° + \theta)$

$$W_g = (10.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)(5.00 \text{ m})\cos 110°$$

$$= \boxed{-168 \text{ J}}$$

(b) $f_k = \mu_k n = \mu_k mg\cos\theta$

$$\Delta E_{\text{int}} = \ell f_k = \ell\mu_k mg\cos\theta$$

$$\Delta E_{\text{int}} = (5.00 \text{ m})(0.400)(10.0)(9.80)\cos 20.0°$$

$$= \boxed{184 \text{ J}}$$

(c) $W_F = F\ell = (100)(5.00) = \boxed{500 \text{ J}}$

(d) $\Delta K = \sum W_{\text{other}} - \Delta E_{\text{int}} = W_F + W_g - \Delta E_{\text{int}} = \boxed{148 \text{ J}}$

(e) $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$$v_f = \sqrt{\frac{2(\Delta K)}{m} + v_i^2} = \sqrt{\frac{2(148)}{10.0} + (1.50)^2} = \boxed{5.65 \text{ m/s}}$$

***P8.16** (i) In (a), $(kd^2)^{1/2}$ and $(mgd)^{1/2}$ both have the wrong units for speed. In (b) $(\mu_k g)^{1/2}$ has the wrong units. In (c), $(kd/m)^{1/2}$ has the wrong units. In (f) both terms have the wrong units. The answer list is $\boxed{\text{a, b, c, f}}$.

(ii) As k increases, friction becomes unimportant, so we should have $(1/2)kd^2 = (1/2)mv^2$ and $v = (kd^2/m)^{1/2}$. Possibilities $\boxed{\text{g, i, and j}}$ do not have this limit.

(iii) As μ_k goes to zero, as in (ii), we should have $v = (kd^2/m)^{1/2}$. Answer $\boxed{\text{d}}$ does not have this limit.

continued on next page

(iv) (e) cannot be true because the friction force is proportional to μ_k and not μ_k^2. And (k) cannot be true because the presence of friction will reduce the speed compared to the $\mu_k = 0$ case, and not increase the speed.

(v) If the spring force is strong enough to produce motion against static friction and if the spring energy is large enough to make the block slide the full distance d, the continuity equation for energy gives

$$(1/2)\,kd^2 + \mu_k mgd \cos 180° = (1/2)\,mv^2$$

This turns into the correct expression $\boxed{\text{h}}$.

(vi) We have $(kd^2/m - 2\,\mu_k gd)^{1/2} = [18\,(0.12)^2/0.25 - 2\,(0.6)(9.8)(0.12)]^{1/2} = [1.04 - 1.41]^{1/2}$

The expression gives an imaginary answer because the spring does not contain enough energy in this case to make the block slide the full distance d.

P8.17 $v_i = 2.00\text{ m/s}$ $\qquad \mu_k = 0.100$

$$K_i - f_k\Delta x + W_{\text{other}} = K_f: \qquad \frac{1}{2}mv_i^2 - f_k\Delta x = 0$$

$$\frac{1}{2}mv_i^2 = \mu_k mg\Delta x \qquad \Delta x = \frac{v_i^2}{2\mu_k g} = \frac{(2.00\text{ m/s})^2}{2(0.100)(9.80)} = \boxed{2.04\text{ m}}$$

Section 8.4 Changes in Mechanical Energy for Nonconservative Forces

P8.18 (a) $U_f = K_i - K_f + U_i \qquad U_f = 30.0 - 18.0 + 10.0 = \boxed{22.0\text{ J}}$

$$\boxed{E = 40.0\text{ J}}$$

(b) Yes, $\Delta E_{\text{mech}} = \Delta K + \Delta U$ is not equal to zero, some nonconservative force or forces must act. For conservative forces $\Delta K + \Delta U = 0$.

P8.19 $U_i + K_i + \Delta E_{\text{mech}} = U_f + K_f:$

$$m_2gh - fh = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2$$

$$f = \mu n = \mu m_1 g$$

$$m_2gh - \mu m_1gh = \frac{1}{2}(m_1 + m_2)v^2$$

$$v^2 = \frac{2(m_2 - \mu m_1)(hg)}{m_1 + m_2}$$

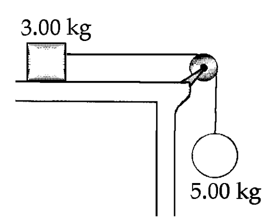

FIG. P8.19

$$v = \sqrt{\frac{2(9.80\text{ m/s}^2)(1.50\text{ m})[5.00\text{ kg} - 0.400(3.00\text{ kg})]}{8.00\text{ kg}}} = \boxed{3.74\text{ m/s}}$$

P8.20 The distance traveled by the ball from the top of the arc to the bottom is πR. The work done by the non-conservative force, the force exerted by the pitcher, is

$$\Delta E = F\Delta r\cos 0° = F(\pi R)$$

We shall assign the gravitational energy of the ball-Earth system to be zero with the ball at the bottom of the arc.

Then

$$\Delta E_{\text{mech}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i$$

becomes

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mgy_i + F(\pi R)$$

or

$$v_f = \sqrt{v_i^2 + 2gy_i + \frac{2F(\pi R)}{m}} = \sqrt{(15.0)^2 + 2(9.80)(1.20) + \frac{2(30.0)\pi(0.600)}{0.250}}$$

$$v_f = \boxed{26.5\text{ m/s}}$$

P8.21 (a) $\Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = -\frac{1}{2}mv_i^2 = \boxed{-160\text{ J}}$

(b) $\Delta U = mg(3.00\text{ m})\sin 30.0° = \boxed{73.5\text{ J}}$

(c) The mechanical energy converted due to friction is 86.5 J

$$f = \frac{86.5\text{ J}}{3.00\text{ m}} = \boxed{28.8\text{ N}}$$

(d) $f = \mu_k n = \mu_k mg\cos 30.0° = 28.8\text{ N}$

$$\mu_k = \frac{28.8\text{ N}}{(5.00\text{ kg})(9.80\text{ m/s}^2)\cos 30.0°} = \boxed{0.679}$$

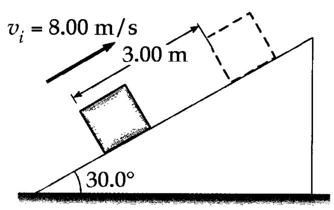

FIG. P8.21

P8.22 Consider the whole motion: $K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$

(a) $0 + mgy_i - f_1\Delta x_1 - f_2\Delta x_2 = \frac{1}{2}mv_f^2 + 0$

$$(80.0\text{ kg})(9.80\text{ m/s}^2)1\,000\text{ m} - (50.0\text{ N})(800\text{ m}) - (3\,600\text{ N})(200\text{ m}) = \frac{1}{2}(80.0\text{ kg})v_f^2$$

$$784\,000\text{ J} - 40\,000\text{ J} - 720\,000\text{ J} = \frac{1}{2}(80.0\text{ kg})v_f^2$$

$$v_f = \sqrt{\frac{2(24\,000\text{ J})}{80.0\text{ kg}}} = \boxed{24.5\text{ m/s}}$$

(b) $\boxed{\text{Yes. This is too fast for safety.}}$

(c) Now in the same energy equation as in part (a), Δx_2 is unknown, and $\Delta x_1 = 1\,000\text{ m} - \Delta x_2$:

$$784\,000\text{ J} - (50.0\text{ N})(1\,000\text{ m} - \Delta x_2) - (3\,600\text{ N})\Delta x_2 = \frac{1}{2}(80.0\text{ kg})(5.00\text{ m/s})^2$$

$$784\,000\text{ J} - 50\,000\text{ J} - (3\,550\text{ N})\Delta x_2 = 1\,000\text{ J}$$

$$\Delta x_2 = \frac{733\,000\text{ J}}{3\,550\text{ N}} = \boxed{206\text{ m}}$$

continued on next page

(d) Really the air drag will depend on the skydiver's speed. It will be larger than her 784 N weight only after the chute is opened. It will be nearly equal to 784 N before she opens the chute and again before she touches down, whenever she moves near terminal speed.

P8.23 (a) $(K+U)_i + \Delta E_{\text{mech}} = (K+U)_f$:

$$0 + \frac{1}{2}kx^2 - f\Delta x = \frac{1}{2}mv^2 + 0$$

$$\frac{1}{2}(8.00\ \text{N/m})(5.00\times10^{-2}\ \text{m})^2 - (3.20\times10^{-2}\ \text{N})(0.150\ \text{m}) = \frac{1}{2}(5.30\times10^{-3}\ \text{kg})v^2$$

$$v = \sqrt{\frac{2(5.20\times10^{-3}\ \text{J})}{5.30\times10^{-3}\ \text{kg}}} = \boxed{1.40\ \text{m/s}}$$

(b) When the spring force just equals the friction force, the ball will stop speeding up. Here $|\vec{\mathbf{F}}_s| = kx$; the spring is compressed by

$$\frac{3.20\times10^{-2}\ \text{N}}{8.00\ \text{N/m}} = 0.400\ \text{cm}$$

and the ball has moved

$$5.00\ \text{cm} - 0.400\ \text{cm} = \boxed{4.60\ \text{cm from the start}}$$

(c) Between start and maximum speed points,w

$$\frac{1}{2}kx_i^2 - f\Delta x = \frac{1}{2}mv^2 + \frac{1}{2}kx_f^2$$

$$\frac{1}{2}8.00(5.00\times10^{-2})^2 - (3.20\times10^{-2})(4.60\times10^{-2})$$

$$= \frac{1}{2}(5.30\times10^{-3})v^2 + \frac{1}{2}8.00(4.00\times10^{-3})^2$$

$$v = \boxed{1.79\ \text{m/s}}$$

P8.24 (a) There is an equilibrium point wherever the graph of potential energy is horizontal:

At $r = 1.5$ mm and 3.2 mm, the equilibrium is stable.
At $r = 2.3$ mm, the equilibrium is unstable.
A particle moving out toward $r \to \infty$ approaches neutral equilibrium.

(b) The system energy E cannot be less than -5.6 J. The particle is bound if $\boxed{-5.6\ \text{J} \le E < 1\ \text{J}}$.

(c) If the system energy is -3 J, its potential energy must be less than or equal to -3 J. Thus, the particle's position is limited to $\boxed{0.6\ \text{mm} \le r \le 3.6\ \text{mm}}$.

(d) $K + U = E$. Thus, $K_{\max} = E - U_{\min} = -3.0\ \text{J} - (-5.6\ \text{J}) = \boxed{2.6\ \text{J}}$.

(e) Kinetic energy is a maximum when the potential energy is a minimum, at $\boxed{r = 1.5\ \text{mm}}$.

(f) $-3\ \text{J} + W = 1\ \text{J}$. Hence, the binding energy is $W = \boxed{4\ \text{J}}$.

P8.25 (a) The object moved down distance $1.20\text{ m}+x$. Choose $y=0$ at its lower point.

$$K_i+U_{gi}+U_{si}+\Delta E_{\text{mech}}=K_f+U_{gf}+U_{sf}$$

$$0+mgy_i+0+0=0+0+\frac{1}{2}kx^2$$

$$(1.50\text{ kg})(9.80\text{ m/s}^2)(1.20\text{ m}+x)=\frac{1}{2}(320\text{ N/m})x^2$$

$$0=(160\text{ N/m})x^2-(14.7\text{ N})x-17.6\text{ J}$$

$$x=\frac{14.7\text{ N}\pm\sqrt{(-14.7\text{ N})^2-4(160\text{ N/m})(-17.6\text{ N}\cdot\text{m})}}{2(160\text{ N/m})}$$

$$x=\frac{14.7\text{ N}\pm 107\text{ N}}{320\text{ N/m}}$$

The negative root tells how high the object will rebound if it is instantly glued to the spring. We want

$$x=\boxed{0.381\text{ m}}$$

(b) From the same equation,

$$(1.50\text{ kg})(1.63\text{ m/s}^2)(1.20\text{ m}+x)=\frac{1}{2}(320\text{ N/m})x^2$$

$$0=160x^2-2.44x-2.93$$

The positive root is $x=\boxed{0.143\text{ m}}$.

(c) The equation expressing the energy version of the nonisolated system model has one more term:

$$mgy_i-f\Delta x=\frac{1}{2}kx^2$$

$$(1.50\text{ kg})(9.80\text{ m/s}^2)(1.20\text{ m}+x)-0.700\text{ N}(1.20\text{ m}+x)=\frac{1}{2}(320\text{ N/m})x^2$$

$$17.6\text{ J}+14.7\text{ N}x-0.840\text{ J}-0.700\text{ N}x=160\text{ N/m}\,x^2$$

$$160x^2-14.0x-16.8=0$$

$$x=\frac{14.0\pm\sqrt{(14.0)^2-4(160)(-16.8)}}{320}$$

$$x=\boxed{0.371\text{ m}}$$

P8.26 The boy converts some chemical energy in his muscles into kinetic energy. During this conversion, the energy can be measured as the work his hands do on the wheels.

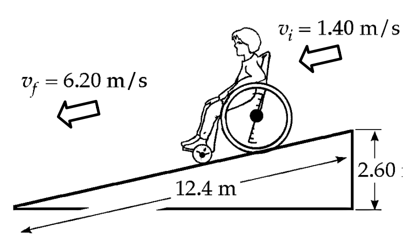

FIG. P8.26

$$K_i+U_{gi}+U_{chemical,i}-f_k\Delta x=K_f$$

$$K_i+U_{gi}+W_{hands-on-wheels}-f_k\Delta x=K_f$$

$$\tfrac{1}{2}mv_i^2+mgy_i+W_{byboy}-f_k\Delta x=\tfrac{1}{2}mv_f^2$$

or

$$W_{byboy}=\frac{1}{2}m\left(v_f^2-v_i^2\right)-mgy_i+f\Delta x$$

$$W_{byboy}=\frac{1}{2}(47.0)\left[(6.20)^2-(1.40)^2\right]-(47.0)(9.80)(2.60)+(41.0)(12.4)$$

$$W_{byboy}=\boxed{168\text{ J}}$$

P8.27 (a) Let m be the mass of the whole board. The portion on the rough surface has mass $\frac{mx}{L}$. The normal force supporting it is $\frac{mxg}{L}$ and the frictional force is $\frac{\mu_k mgx}{L} = ma$. Then

$$\boxed{a = \frac{\mu_k gx}{L} \text{ opposite to the motion}}.$$

(b) In an incremental bit of forward motion dx, the kinetic energy converted into internal energy is $f_k dx = \frac{\mu_k mgx}{L}dx$. The whole energy converted is

$$\frac{1}{2}mv^2 = \int_0^L \frac{\mu_k mgx}{L}dx = \frac{\mu_k mg}{L}\frac{x^2}{2}\bigg|_0^L = \frac{\mu_k mgL}{2}$$

$$\boxed{v = \sqrt{\mu_k gL}}$$

Section 8.5 **Power**

P8.28 $$\mathcal{P}_{\text{av}} = \frac{W}{\Delta t} = \frac{K_f}{\Delta t} = \frac{mv^2}{2\Delta t} = \frac{0.875\text{ kg}(0.620\text{ m/s})^2}{2(21\times10^{-3}\text{ s})} = \boxed{8.01\text{ W}}$$

P8.29 Power $= \frac{W}{t}$

$$\mathcal{P} = \frac{mgh}{t} = \frac{(700\text{ N})(10.0\text{ m})}{8.00\text{ s}} = \boxed{875\text{ W}}$$

***P8.30** (a) The moving sewage possesses kinetic energy in the same amount as it enters and leaves the pump. The work of the pump increases the gravitational energy of the sewage-Earth system. We take the equation $K_i + U_{gi} + W_{\text{pump}} = K_f + U_{gf}$, subtract out the K terms, and choose $U_{gi} = 0$ at the bottom of the sump, to obtain $W_{\text{pump}} = mgy_f$. Now we differentiate through with respect to time:

$$\mathcal{P}_{\text{pump}} = \frac{\Delta m}{\Delta t}gy_f = \rho\frac{\Delta V}{\Delta t}gy_f$$

$$= 1050\text{ kg/m}^3\left(1.89\times10^6\text{ L/d}\right)\left(\frac{1\text{ m}^3}{1000\text{ L}}\right)\left(\frac{1\text{ d}}{86\,400\text{ s}}\right)\left(\frac{9.80\text{ m}}{\text{s}^2}\right)(5.49\text{ m})$$

$$= \boxed{1.24\times10^3\text{ W}}$$

(b) $$\text{efficiency} = \frac{\text{useful output work}}{\text{total input work}} = \frac{\text{useful output work}/\Delta t}{\text{total input work}/\Delta t}$$

$$= \frac{\text{mechanical output power}}{\text{input electric power}} = \frac{1.24\text{ kW}}{5.90\text{ kW}} = \boxed{0.209} = 20.9\%$$

The remaining power, $5.90 - 1.24\text{ kW} = 4.66\text{ kW}$ is the rate at which internal energy is injected into the sewage and the surroundings of the pump.

Dave Barry attended the January dedication of the pumping station and was the featured speaker at a festive potluck supper to which residents of the different Grand Forks sewer districts brought casseroles, Jell-O salads, and “bars” for dessert.

P8.31 A 1 300-kg car speeds up from rest to 55.0 mi/h = 24.6 m/s in 15.0 s. The output work of the engine is equal to its final kinetic energy,

$$\frac{1}{2}(1\,300\text{ kg})(24.6\text{ m/s})^2 = 390\text{ kJ}$$

with power $\mathcal{P} = \frac{390\,000\text{ J}}{15.0\text{ s}}\boxed{\sim 10^4\text{ W}}$ around 30 horsepower.

P8.32 (a) The distance moved upward in the first 3.00 s is

$$\Delta y = \bar{v}t = \left[\frac{0+1.75\text{ m/s}}{2}\right](3.00\text{ s}) = 2.63\text{ m}$$

The motor and the earth's gravity do work on the elevator car:

$$\frac{1}{2}mv_i^2 + W_{\text{motor}} + mg\Delta y\cos 180° = \frac{1}{2}mv_f^2$$

$$W_{\text{motor}} = \frac{1}{2}(650\text{ kg})(1.75\text{ m/s})^2 - 0 + (650\text{ kg})g(2.63\text{ m}) = 1.77\times 10^4\text{ J}$$

Also, $W = \overline{\mathcal{P}}t$ so $\overline{\mathcal{P}} = \frac{W}{t} = \frac{1.77\times 10^4\text{ J}}{3.00\text{ s}} = \boxed{5.91\times 10^3\text{ W}} = 7.92\text{ hp}$.

(b) When moving upward at constant speed $(v = 1.75\text{ m/s})$ the applied force equals the weight $= (650\text{ kg})(9.80\text{ m/s}^2) = 6.37\times 10^3$ N. Therefore,

$$\mathcal{P} = Fv = (6.37\times 10^3\text{ N})(1.75\text{ m/s}) = \boxed{1.11\times 10^4\text{ W}} = 14.9\text{ hp}$$

P8.33 *energy = power × time*

For the 28.0 W bulb:

$$\text{Energy used} = (28.0\text{ W})(1.00\times 10^4\text{ h}) = 280\text{ kilowatt}\cdot\text{hrs}$$

$$\text{total cost} = \$17.00 + (280\text{ kWh})(\$0.080/\text{kWh}) = \$39.40$$

For the 100 W bulb:

$$\text{Energy used} = (100\text{ W})(1.00\times 10^4\text{ h}) = 1.00\times 10^3\text{ kilowatt}\cdot\text{hrs}$$

$$\#\text{ bulb used} = \frac{1.00\times 10^4\text{ h}}{750\text{ h/bulb}} = 13.3$$

$$\text{total cost} = 13.3(\$0.420) + (1.00\times 10^3\text{ kWh})(\$0.080/\text{kWh}) = \$85.60$$

Savings with energy-efficient bulb $= \$85.60 - \$39.40 = \boxed{\$46.2}$.

P8.34 The useful output energy is

$$120\text{ Wh}(1-0.60) = mg(y_f - y_i) = F_g\Delta y$$

$$\Delta y = \frac{120\text{ W}(3\,600\text{ s})0.40}{890\text{ N}}\left(\frac{\text{J}}{\text{W}\cdot\text{s}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right) = \boxed{194\text{ m}}$$

P8.35 The energy of the car is $E = \frac{1}{2}mv^2 + mgy$

$E = \frac{1}{2}mv^2 + mgd\sin\theta$ where d is the distance it has moved along the track.

$$\mathcal{P} = \frac{dE}{dt} = mv\frac{dv}{dt} + mgv\sin\theta$$

(a) When speed is constant,

$$\mathcal{P} = mgv\sin\theta = 950\text{ kg}\left(9.80\text{ m/s}^2\right)\left(2.20\text{ m/s}\right)\sin 30° = \boxed{1.02\times10^4\text{ W}}$$

(b) $\frac{dv}{dt} = a = \frac{2.2\text{ m/s} - 0}{12\text{ s}} = 0.183\text{ m/s}^2$

Maximum power is injected just before maximum speed is attained:

$$\mathcal{P} = mva + mgv\sin\theta = 950\text{ kg}\left(2.2\text{ m/s}\right)\left(0.183\text{ m/s}^2\right) + 1.02\times10^4\text{ W} = \boxed{1.06\times10^4\text{ W}}$$

(c) At the top end,

$$\frac{1}{2}mv^2 + mgd\sin\theta = 950\text{ kg}\left(\frac{1}{2}(2.20\text{ m/s})^2 + \left(9.80\text{ m/s}^2\right)1\,250\text{ m}\sin 30°\right) = \boxed{5.82\times10^6\text{ J}}$$

P8.36 (a) Burning 1 lb of fat releases energy

$$1\text{ lb}\left(\frac{454\text{ g}}{1\text{ lb}}\right)\left(\frac{9\text{ kcal}}{1\text{ g}}\right)\left(\frac{4\,186\text{ J}}{1\text{ kcal}}\right) = 1.71\times10^7\text{ J}$$

The mechanical energy output is

$$\left(1.71\times10^7\text{ J}\right)(0.20) = nF\Delta r\cos\theta$$

Then

$$3.42\times10^6\text{ J} = nmg\Delta y\cos 0°$$

$$3.42\times10^6\text{ J} = n(50\text{ kg})\left(9.8\text{ m/s}^2\right)(80\text{ steps})(0.150\text{ m})$$

$$3.42\times10^6\text{ J} = n\left(5.88\times10^3\text{ J}\right)$$

where the number of times she must climb the steps is $n = \frac{3.42\times10^6\text{ J}}{5.88\times10^3\text{ J}} = \boxed{582}$.

$\boxed{\text{This method is impractical compared to limiting food intake.}}$

(b) Her mechanical power output is

$$\mathcal{P} = \frac{W}{t} = \frac{5.88\times10^3\text{ J}}{65\text{ s}} = \boxed{90.5\text{ W}} = 90.5\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = \boxed{0.121\text{ hp}}$$

Additional Problems

P8.37 (a) $(K+U_g)_A = (K+U_g)_B$

$$0 + mgy_A = \frac{1}{2}mv_B^2 + 0 \qquad v_B = \sqrt{2gy_A} = \sqrt{2(9.8\ \text{m/s}^2)6.3\ \text{m}} = \boxed{11.1\ \text{m/s}}$$

(b) $$a_c = \frac{v^2}{r} = \frac{(11.1\ \text{m/s})^2}{6.3\ \text{m}} = \boxed{19.6\ \text{m/s}^2\ \text{up}}$$

(c) $\sum F_y = ma_y \qquad +n_B - mg = ma_c$

$$n_B = 76\ \text{kg}(9.8\ \text{m/s}^2 + 19.6\ \text{m/s}^2) = \boxed{2.23\times10^3\ \text{N up}}$$

(d) We compute the amount of chemical energy converted into mechanical energy as

$$W = F\Delta r\cos\theta = 2.23\times10^3\ \text{N}(0.450\ \text{m})\cos 0° = \boxed{1.01\times10^3\ \text{J}}$$

(e) $(K+U_g+U_{chemical})_B = (K+U_g)_D$

$$\frac{1}{2}mv_B^2 + 0 + 1.01\times10^3\ \text{J} = \frac{1}{2}mv_D^2 + mg(y_D - y_B)$$

$$\frac{1}{2}76\ \text{kg}(11.1\ \text{m/s})^2 + 1.01\times10^3\ \text{J} = \frac{1}{2}76\ \text{kg}\ v_D^2 + 76\ \text{kg}(9.8\ \text{m/s}^2)6.3\ \text{m}$$

$$\sqrt{\frac{(5.70\times10^3\ \text{J} - 4.69\times10^3\ \text{J})2}{76\ \text{kg}}} = v_D = \boxed{5.14\ \text{m/s}}$$

(f) $(K+U_g)_D = (K+U_g)_E$ where E is the apex of his motion

$$\frac{1}{2}mv_D^2 + 0 = 0 + mg(y_E - y_D) \qquad y_E - y_D = \frac{v_D^2}{2g} = \frac{(5.14\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = \boxed{1.35\ \text{m}}$$

(g) Consider the motion with constant acceleration between takeoff and touchdown. The time is the positive root of

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_yt^2$$

$$-2.34\ \text{m} = 0 + 5.14\ \text{m/s}\,t + \frac{1}{2}(-9.8\ \text{m/s}^2)t^2$$

$$4.9t^2 - 5.14t - 2.34 = 0$$

$$t = \frac{5.14 \pm \sqrt{5.14^2 - 4(4.9)(-2.34)}}{9.8} = \boxed{1.39\ \text{s}}$$

***P8.38** (a) $\boxed{\text{Yes, the total mechanical energy is constant.}}$ The originally hanging block loses gravitational energy, which is entirely converted into kinetic energy of both blocks.

(b) energy at release = energy just before hitting floor

$m_2gy = (1/2)\,(m_1 + m_2)v^2$

$v = [2m_2gy/(m_1 + m_2)]^{1/2} = [2(1.90\ \text{kg})(9.8\ \text{m/s}^2)0.9\ \text{m}/5.4\ \text{kg}]^{1/2} = \boxed{2.49\ \text{m/s}}$

(c) No. The kinetic energy of the impacting block turns into internal energy. But mechanical energy is conserved for the 3.50-kg block with the Earth in this block's projectile motion.

continued on next page

(d) For the 3.5-kg block from when the string goes slack until just before the block hits the floor

$$(1/2)\,(m_2)v^2 + m_2 g y = (1/2)\,(m_2)v_d^2$$

$$v_d = [2gy + v^2]^{1/2} = [2(9.8\text{ m/s}^2)1.2\text{ m} + (2.49\text{ m/s})^2]^{1/2} = \boxed{5.45\text{ m/s}}$$

(e) The 3.5-kg block takes this time in flight to the floor: from $y = (1/2)\,gt^2$ we have $t = [2(1.2)/9.8]^{1/2} = 0.495$ s. Its horizontal component of displacement at impact is then $x = v_d\,t = (2.49\text{ m/s})(0.495\text{ s}) = \boxed{1.23\text{ m}}$.

(f) $\boxed{\text{No.}}$ With the hanging block firmly stuck, the string pulls radially on the 3.5-kg block, doing no work on it.

(g) The force of static friction cannot be larger than $\mu_s n = (0.56)(3.5\text{ kg})(9.8\text{ m/s}^2) = 19.2$ N. The hanging block tends to produce string tension $(1.9\text{ kg})(9.8\text{ m/s}^2) = 18.6$ N. Then the force of static friction on the 3.5-kg block is less than its maximum value, being $\boxed{\text{18.6 N to the left}}$.

(h) $\boxed{\text{A little push is required}}$, because 18.6 N is less than 19.2 N. The motion begins with negligible speed, so $\boxed{\text{the calculated final speeds are still accurate}}$.

P8.39 (a) $x = t + 2.00t^3$

Therefore,

$$v = \frac{dx}{dt} = 1 + 6.00t^2$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(4.00)\left(1 + 6.00t^2\right)^2 = \boxed{\left(2.00 + 24.0t^2 + 72.0t^4\right)\text{ J}}$$

(b) $a = \dfrac{dv}{dt} = \boxed{(12.0t)\ \text{m/s}^2}$

$F = ma = 4.00(12.0t) = \boxed{(48.0t)\ \text{N}}$

(c) $\mathcal{P} = Fv = (48.0t)\left(1 + 6.00t^2\right) = \boxed{\left(48.0t + 288t^3\right)\text{ W}}$

(d) $W = \displaystyle\int_0^{2.00} \mathcal{P}\,dt = \int_0^{2.00}\left(48.0t + 288t^3\right)dt = \boxed{1250\text{ J}}$

***P8.40** (a) Simplified, the equation is $0 = (9\,700\ \text{N/m})x^2 - (450.8\text{ N})x - 1\,395\text{ N}\cdot\text{m}$. Then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{450.8\text{ N} \pm \sqrt{(450.8\text{ N})^2 - 4(9\,700\ \text{N/m})(-1\,395\text{ N}\cdot\text{m})}}{2(9\,700\ \text{N/m})}$$

$$= \frac{450.8\text{ N} \pm 7\,370\text{ N}}{19\,400\ \text{N/m}} = \boxed{0.403\text{ m or } -0.357\text{ m}}$$

(b) One possible problem statement: From a perch at a height of 2.80 m above the top of the pile of mattresses, a 46.0-kg child jumps nearly straight upward with speed 2.40 m/s. The mattresses behave as a linear spring with force constant 19.4 kN/m. Find the maximum amount by which they are compressed when the child lands on them. Physical meaning: The positive value of x represents the maximum spring compression. The negative value represents the maximum extension of the equivalent spring if the child sticks to the top of the mattress pile as he rebounds upward without friction.

P8.41 (a) $v=\int_0^t a\,dt=\int_0^t\left(1.16t-0.21t^2+0.24t^3\right)dt$

$$=1.16\frac{t^2}{2}-0.21\frac{t^3}{3}+0.24\frac{t^4}{4}\bigg|_0^t=0.58t^2-0.07t^3+0.06t^4$$

At $t=0$, $v_i=0$. At $t=2.5$ s,

$$v_f=\left(0.58\ \text{m/s}^3\right)(2.5\ \text{s})^2-\left(0.07\ \text{m/s}^4\right)(2.5\ \text{s})^3+\left(0.06\ \text{m/s}^5\right)(2.5\ \text{s})^4=4.88\ \text{m/s}$$

$$K_i+W=K_f$$

$$0+W=\frac{1}{2}mv_f^2=\frac{1}{2}1\,160\ \text{kg}(4.88\ \text{m/s})^2=\boxed{1.38\times10^4\ \text{J}}$$

(b) At $t=2.5$ s,

$$a=\left(1.16\ \text{m/s}^3\right)2.5\ \text{s}-\left(0.210\ \text{m/s}^4\right)(2.5\ \text{s})^2+\left(0.240\ \text{m/s}^5\right)(2.5\ \text{s})^3=5.34\ \text{m/s}^2$$

Through the axles the wheels exert on the chassis force

$$\sum F=ma=1\,160\ \text{kg}\ 5.34\ \text{m/s}^2=6.19\times10^3\ \text{N}$$

and inject power

$$\mathcal{P}=Fv=6.19\times10^3\ \text{N}\left(4.88\ \text{m/s}\right)=\boxed{3.02\times10^4\ \text{W}}$$

P8.42 (a) $\Delta E_{\text{int}}=-\Delta K=-\frac{1}{2}m\left(v_f^2-v_i^2\right)$: $\Delta E_{\text{int}}=-\frac{1}{2}(0.400\ \text{kg})\left((6.00)^2-(8.00)^2\right)(\text{m/s})^2=\boxed{5.60\ \text{J}}$

(b) $\Delta E_{\text{int}}=f\Delta r=\mu_k mg(2\pi r)$: $\quad 5.60\ \text{J}=\mu_k(0.400\ \text{kg})\left(9.80\ \text{m/s}^2\right)2\pi(1.50\ \text{m})$

Thus,

$$\mu_k=\boxed{0.152}$$

(c) After N revolutions, the object comes to rest and $K_f=0$.

Thus,

$$\Delta E_{\text{int}}=-\Delta K=-0+K_i=\frac{1}{2}mv_i^2$$

or

$$\mu_k mg\left[N(2\pi r)\right]=\frac{1}{2}mv_i^2$$

This gives

$$N=\frac{\frac{1}{2}mv_i^2}{\mu_k mg(2\pi r)}=\frac{\frac{1}{2}(8.00\ \text{m/s})^2}{(0.152)\left(9.80\ \text{m/s}^2\right)2\pi(1.50\ \text{m})}=\boxed{2.28\ \text{rev}}$$

P8.43 (a) $\sum W=\Delta K$: $\quad W_s+W_g=0$

$$\frac{1}{2}kx_i^2-0+mg\Delta x\cos(90°+60°)=0$$

$$\frac{1}{2}\left(1.40\times10^3\ \text{N/m}\right)\times(0.100)^2-(0.200)(9.80)(\sin60.0°)\Delta x=0$$

$$\Delta x=\boxed{4.12\ \text{m}}$$

continued on next page

(b) $\sum W = \Delta K + \Delta E_{\text{int}}$: $\quad W_s + W_g - \Delta E_{\text{int}} = 0$

$$\frac{1}{2}kx_i^2 + mg\Delta x\cos 150° - \mu_k mg\cos 60°\Delta x = 0$$

$$\frac{1}{2}\left(1.40\times10^3\ \text{N/m}\right)\times(0.100)^2 - (0.200)(9.80)(\sin 60.0°)\Delta x$$
$$-(0.200)(9.80)(0.400)(\cos 60.0°)\Delta x = 0$$

$$\Delta x = \boxed{3.35\ \text{m}}$$

P8.44 $\mathcal{P}\Delta t = W = \Delta K = \dfrac{(\Delta m)v^2}{2}$

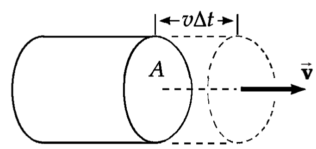

FIG. P8.44

The density is

$$\rho = \frac{\Delta m}{\text{vol}} = \frac{\Delta m}{A\Delta x}$$

Substituting this into the first equation and solving for $\mathcal{P}$, since $\dfrac{\Delta x}{\Delta t} = v$, for a constant speed, we get

$$\boxed{\mathcal{P} = \frac{\rho A v^3}{2}}$$

Also, since $\mathcal{P} = Fv$,

$$\boxed{F = \frac{\rho A v^2}{2}}$$

Our model predicts the same proportionalities as the empirical equation, and gives $D = 1$ for the drag coefficient. Air actually slips around the moving object, instead of accumulating in front of it. For this reason, the drag coefficient is not necessarily unity. It is typically less than one for a streamlined object and can be greater than one if the airflow around the object is complicated.

P8.45 $\mathcal{P} = \dfrac{1}{2}D\rho\pi r^2 v^3$

(a) $\mathcal{P}_a = \dfrac{1}{2}1\left(1.20\ \text{kg/m}^3\right)\pi\left(1.5\ \text{m}\right)^2\left(8\ \text{m/s}\right)^3 = \boxed{2.17\times10^3\ \text{W}}$

(b) $\dfrac{\mathcal{P}_b}{\mathcal{P}_a} = \dfrac{v_b^3}{v_a^3} = \left(\dfrac{24\ \text{m/s}}{8\ \text{m/s}}\right)^3 = 3^3 = 27$

$$\mathcal{P}_b = 27\left(2.17\times10^3\ \text{W}\right) = \boxed{5.86\times10^4\ \text{W}}$$

***P8.46** (a) $U_g = mgy = (64\ \text{kg})\left(9.8\ \text{m/s}^2\right)y = \boxed{(627\ \text{N})y}$

(b) At the original height and at all heights above 65 m − 25.8 m = 39.2 m, the cord is unstretched and $\boxed{U_s = 0}$. Below 39.2 m, the cord extension x is given by $x = 39.2\ \text{m} - y$, so the elastic energy is $U_s = \dfrac{1}{2}kx^2 = \boxed{\dfrac{1}{2}(81\ \text{N/m})(39.2\ \text{m} - y)^2}$.

(c) For $y > 39.2\ \text{m}$, $U_g + U_s = \boxed{(627\ \text{N})y}$

For $y \le 39.2\ \text{m}$,

$$U_g + U_s = (627\ \text{N})y + 40.5\ \text{N/m}\left(1\,537\ \text{m}^2 - (78.4\ \text{m})y + y^2\right)$$
$$= \boxed{(40.5\ \text{N/m})y^2 - (2\,550\ \text{N})y + 62\,200\ \text{J}}$$

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(d)

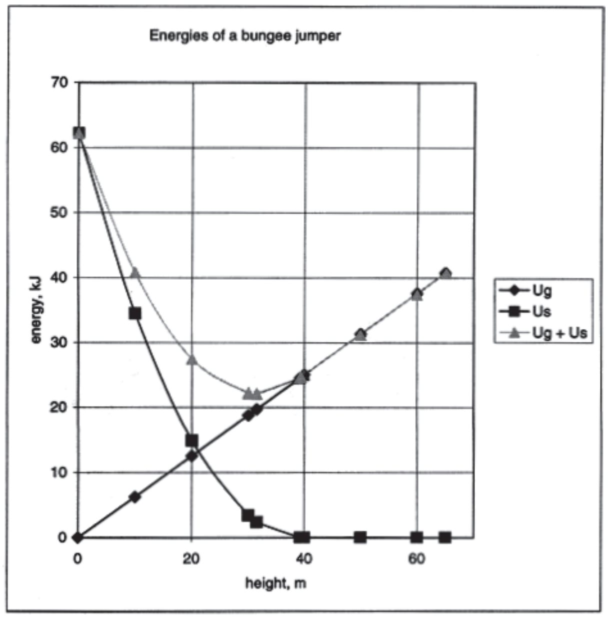

FIG. P8.46(d)

(e) At minimum height, the jumper has zero kinetic energy and the same total energy as at his starting point. $K_i + U_i = K_f + U_f$ becomes

$$627\text{ N}(65\text{ m}) = (40.5\text{ N/m})y_f^2 - (2\,550\text{ N})y_f + 62\,200\text{ J}$$
$$0 = 40.5y_f^2 - 2\,550y_f + 21\,500$$
$$y_f = \boxed{10.0\text{ m}}\quad [\text{the root 52.9 m is unphysical}]$$

(f) The total potential energy has a minimum, representing a $\boxed{\text{stable equilibrium}}$ position. To find it, we require $\dfrac{dU}{dy} = 0$:

$$\frac{d}{dy}\left(40.5y^2 - 2\,550y + 62\,200\right) = 0 = 81y - 2\,550$$
$$y = \boxed{31.5\text{ m}}$$

(g) Maximum kinetic energy occurs at minimum potential energy. Between the takeoff point and this location, we have $K_i + U_i = K_f + U_f$

$$0 + 40\,800\text{ J} = \frac{1}{2}(64\text{ kg})v_{\max}^2 + 40.5(31.5)^2 - 2\,550(31.5) + 62\,200$$
$$v_{\max} = \left(\frac{2(40\,800 - 22\,200)}{64}\right)^{1/2} = \boxed{24.1\text{ m/s}}$$

P8.47 (a) So long as the spring force is greater than the friction force, the block will be gaining speed. The block slows down when the friction force becomes the greater. It has maximum speed when $-kx_a - f_k = ma = 0$.

$$-(1.0\times10^3\text{ N/m})x_a - 4.0\text{ N} = 0 \qquad \boxed{x = -4.0\times10^{-3}\text{ m}}$$

(b) By the same logic,

$$-(1.0\times10^3\text{ N/m})x_b - 10.0\text{ N} = 0 \qquad \boxed{x = -1.0\times10^{-2}\text{ m}}$$

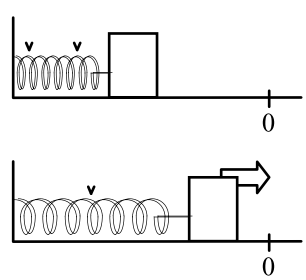

FIG. P8.47

P8.48 (a) The suggested equation $\mathcal{P}\Delta t = bwd$ implies all of the following cases:

(1) $\mathcal{P}\Delta t = b\left(\frac{w}{2}\right)(2d)$ (2) $\mathcal{P}\left(\frac{\Delta t}{2}\right) = b\left(\frac{w}{2}\right)d$

(3) $\mathcal{P}\left(\frac{\Delta t}{2}\right) = bw\left(\frac{d}{2}\right)$ and (4) $\left(\frac{\mathcal{P}}{2}\right)\Delta t = b\left(\frac{w}{2}\right)d$

These are all of the proportionalities Aristotle lists.

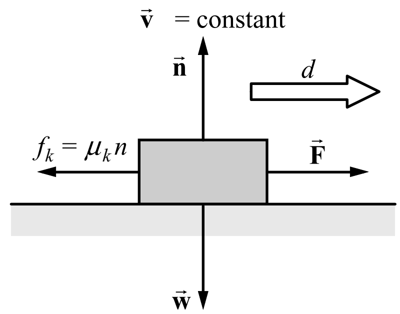

FIG. P8.48

(b) For one example, consider a horizontal force F pushing an object of weight w at constant velocity across a horizontal floor with which the object has coefficient of friction μ_k.

$\sum \vec{F} = m\vec{a}$ implies that:

$$+n - w = 0 \text{ and } F - \mu_k n = 0$$

so that $F = \mu_k w$

As the object moves a distance d, the agent exerting the force does work

$$W = Fd\cos\theta = Fd\cos 0° = \mu_k wd \text{ and puts out power } \mathcal{P} = \frac{W}{\Delta t}$$

This yields the equation $\mathcal{P}\Delta t = \mu_k wd$ which represents Aristotle's theory with $b = \mu_k$.

Our theory is more general than Aristotle's. Ours can also describe accelerated motion.

P8.49 $v = 100 \text{ km/h} = 27.8 \text{ m/s}$

The retarding force due to air resistance is

$$R = \frac{1}{2}D\rho Av^2 = \frac{1}{2}(0.330)(1.20 \text{ kg/m}^3)(2.50 \text{ m}^2)(27.8 \text{ m/s})^2 = 382 \text{ N}$$

Comparing the energy of the car at two points along the hill,

$$K_i + U_{gi} + \Delta E = K_f + U_{gf}$$

or

$$K_i + U_{gi} + \Delta W_e - R(\Delta s) = K_f + U_{gf}$$

where ΔW_e is the work input from the engine. Thus,

$$\Delta W_e = R(\Delta s) + (K_f - K_i) + (U_{gf} - U_{gi})$$

Recognizing that $K_f = K_i$ and dividing by the travel time Δt gives the required power input from the engine as

$$\mathcal{P} = \left(\frac{\Delta W_e}{\Delta t}\right) = R\left(\frac{\Delta s}{\Delta t}\right) + mg\left(\frac{\Delta y}{\Delta t}\right) = Rv + mgv\sin\theta$$

$$\mathcal{P} = (382 \text{ N})(27.8 \text{ m/s}) + (1500 \text{ kg})(9.80 \text{ m/s}^2)(27.8 \text{ m/s})\sin 3.20°$$

$$\mathcal{P} = \boxed{33.4 \text{ kW} = 44.8 \text{ hp}}$$

P8.50 (a) $U_A = mgR = (0.200 \text{ kg})(9.80 \text{ m/s}^2)(0.300 \text{ m}) = \boxed{0.588 \text{ J}}$

(b) $K_A + U_A = K_B + U_B$

$$K_B = K_A + U_A - U_B = mgR = \boxed{0.588 \text{ J}}$$

(c) $v_B = \sqrt{\dfrac{2K_B}{m}} = \sqrt{\dfrac{2(0.588 \text{ J})}{0.200 \text{ kg}}} = \boxed{2.42 \text{ m/s}}$

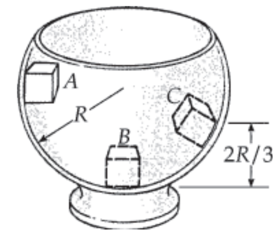

FIG. P8.50

(d) $U_C = mgh_C = (0.200 \text{ kg})(9.80 \text{ m/s}^2)(0.200 \text{ m}) = \boxed{0.392 \text{ J}}$

$$K_C = K_A + U_A - U_C = mg(h_A - h_C)$$

$$K_C = (0.200 \text{ kg})(9.80 \text{ m/s}^2)(0.300 - 0.200) \text{ m} = \boxed{0.196 \text{ J}}$$

P8.51 (a) $K_B = \dfrac{1}{2}mv_B^2 = \dfrac{1}{2}(0.200 \text{ kg})(1.50 \text{ m/s})^2 = \boxed{0.225 \text{ J}}$

(b)
$$\begin{aligned}\Delta E_{\text{mech}} &= \Delta K + \Delta U = K_B - K_A + U_B - U_A \\ &= K_B + mg(h_B - h_A) \\ &= 0.225 \text{ J} + (0.200 \text{ kg})(9.80 \text{ m/s}^2)(0 - 0.300 \text{ m}) \\ &= 0.225 \text{ J} - 0.588 \text{ J} = \boxed{-0.363 \text{ J}}\end{aligned}$$

(c) No. It is possible to find an effective coefficient of friction, but not the actual value of μ since n and f vary with position.

P8.52 m = mass of pumpkin
R = radius of silo top

$$\sum F_r = ma_r \Rightarrow n - mg\cos\theta = -m\frac{v^2}{R}$$

When the pumpkin first loses contact with the surface, $n = 0$. Thus, at the point where it leaves the surface: $v^2 = Rg\cos\theta$.

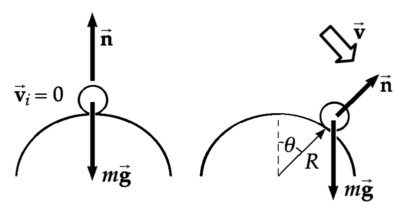

FIG. P8.52

Choose $U_g = 0$ in the $\theta = 90.0°$ plane. Then applying conservation of energy for the pumpkin-Earth system between the starting point and the point where the pumpkin leaves the surface gives

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv^2 + mgR\cos\theta = 0 + mgR$$

Using the result from the force analysis, this becomes

$\dfrac{1}{2}mRg\cos\theta + mgR\cos\theta = mgR$, which reduces to

$\cos\theta = \dfrac{2}{3}$, and gives $\theta = \cos^{-1}(2/3) = \boxed{48.2°}$

as the angle at which the pumpkin will lose contact with the surface.

P8.53 $k = 2.50 \times 10^4\ \text{N/m}$, $m = 25.0\ \text{kg}$

$x_A = -0.100\ \text{m}$, $U_g\big|_{x=0} = U_s\big|_{x=0} = 0$

(a) $E_{\text{mech}} = K_A + U_{gA} + U_{sA}$ $\quad E_{\text{mech}} = 0 + mgx_A + \frac{1}{2}kx_A^2$

$$E_{\text{mech}} = (25.0\ \text{kg})(9.80\ \text{m/s}^2)(-0.100\ \text{m}) + \frac{1}{2}(2.50 \times 10^4\ \text{N/m})(-0.100\ \text{m})^2$$

$$E_{\text{mech}} = -24.5\ \text{J} + 125\ \text{J} = \boxed{100\ \text{J}}$$

(b) Since only conservative forces are involved, the total energy of the child-pogo-stick-Earth system at point C is the same as that at point A.

$K_C + U_{gC} + U_{sC} = K_A + U_{gA} + U_{sA}$: $\quad 0 + (25.0\ \text{kg})(9.80\ \text{m/s}^2)x_C + 0 = 0 - 24.5\ \text{J} + 125\ \text{J}$

$$x_C = \boxed{0.410\ \text{m}}$$

(c) $K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$: $\quad \frac{1}{2}(25.0\ \text{kg})v_B^2 + 0 + 0 = 0 + (-24.5\ \text{J}) + 125\ \text{J}$

$$v_B = \boxed{2.84\ \text{m/s}}$$

(d) K and v are at a maximum when $a = \sum F/m = 0$ (i.e., when the magnitude of the upward spring force equals the magnitude of the downward gravitational force).

This occurs at $x < 0$ where

$$k|x| = mg$$

or

$$|x| = \frac{(25.0\ \text{kg})(9.8\ \text{m/s}^2)}{2.50 \times 10^4\ \text{N/m}} = 9.80 \times 10^{-3}\ \text{m}$$

Thus,

$$K = K_{\max} \text{ at } x = \boxed{-9.80\ \text{mm}}$$

(e) $K_{\max} = K_A + \left(U_{gA} - U_g\big|_{x=-9.80\ \text{mm}}\right) + \left(U_{sA} - U_s\big|_{x=-9.80\ \text{mm}}\right)$

or

$$\frac{1}{2}(25.0\ \text{kg})v_{\max}^2 = (25.0\ \text{kg})(9.80\ \text{m/s}^2)\left[(-0.100\ \text{m}) - (-0.009\,8\ \text{m})\right] + \frac{1}{2}(2.50 \times 10^4\ \text{N/m})\left[(-0.100\ \text{m})^2 - (-0.009\,8\ \text{m})^2\right]$$

yielding

$$v_{\max} = \boxed{2.85\ \text{m/s}}$$

P8.54 (a) Between the second and the third picture, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-\mu mgd = -\frac{1}{2}mv_i^2 + \frac{1}{2}kd^2$$

$$\frac{1}{2}(50.0 \text{ N/m})d^2 + 0.250(1.00 \text{ kg})(9.80 \text{ m/s}^2)d - \frac{1}{2}(1.00 \text{ kg})(3.00 \text{ m/s})^2 = 0$$

$$d = \frac{[-2.45 \pm 21.35] \text{ N}}{50.0 \text{ N/m}} = \boxed{0.378 \text{ m}}$$

(b) Between picture two and picture four, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-f(2d) = \frac{1}{2}mv^2 - \frac{1}{2}mv_i^2$$

$$v = \sqrt{(3.00 \text{ m/s})^2 - \frac{2}{(1.00 \text{ kg})}(2.45 \text{ N})(2)(0.378 \text{ m})}$$

$$= \boxed{2.30 \text{ m/s}}$$

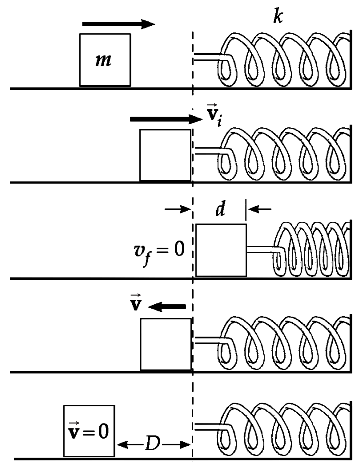

FIG. P8.54

(c) For the motion from picture two to picture five,

$$\Delta E_{\text{mech}} = \Delta K + \Delta U$$

$$-f(D + 2d) = -\frac{1}{2}(1.00 \text{ kg})(3.00 \text{ m/s})^2$$

$$D = \frac{9.00 \text{ J}}{2(0.250)(1.00 \text{ kg})(9.80 \text{ m/s}^2)} - 2(0.378 \text{ m})$$

$$= \boxed{1.08 \text{ m}}$$

P8.55

$$\Delta E_{\text{mech}} = -f\Delta x$$

$$E_f - E_i = -f \cdot d_{BC}$$

$$\frac{1}{2}kx^2 - mgh = -\mu mgd_{BC}$$

$$\mu = \frac{mgh - \frac{1}{2}kx^2}{mgd_{BC}} = \boxed{0.328}$$

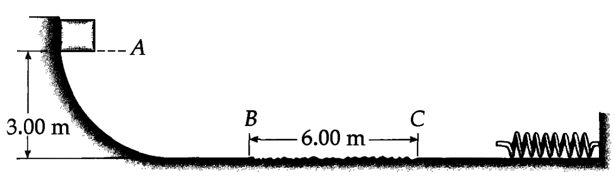

FIG. P8.55

P8.56 Let λ represent the mass of each one meter of the chain and T represent the tension in the chain at the table edge. We imagine the edge to act like a frictionless and massless pulley.

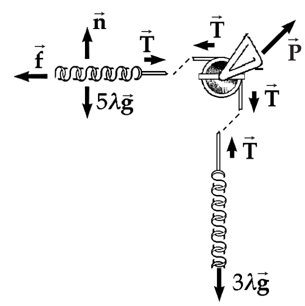

FIG. P8.56

(a) For the five meters on the table with motion impending,

$$\sum F_y = 0: \quad +n - 5\lambda g = 0 \qquad n = 5\lambda g$$

$$f_s \le \mu_s n = 0.6(5\lambda g) = 3\lambda g$$

$$\sum F_x = 0: \quad +T - f_s = 0 \qquad T = f_s$$

$$T \le 3\lambda g$$

The maximum value is barely enough to support the hanging segment according to

$$\sum F_y = 0: \quad +T - 3\lambda g = 0 \qquad T = 3\lambda g$$

so it is at this point that the chain starts to slide.

continued on next page

(b) Let x represent the variable distance the chain has slipped since the start. Then length $(5 - x)$ remains on the table, with now

$$\sum F_y = 0: \qquad +n-(5-x)\lambda g = 0 \qquad n = (5-x)\lambda g$$

$$f_k = \mu_k n = 0.4(5-x)\lambda g = 2\lambda g - 0.4x\lambda g$$

Consider energies of the chain-Earth system at the initial moment when the chain starts to slip, and a final moment when $x = 5$, when the last link goes over the brink. Measure heights above the final position of the leading end of the chain. At the moment the final link slips off, the center of the chain is at $y_f = 4$ meters.

Originally, 5 meters of chain is at height 8 m and the middle of the dangling segment is at height $8-\dfrac{3}{2}=6.5\text{ m}$.

$$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f: \qquad 0+\left(m_1gy_1+m_2gy_2\right)_i - \int_i^f f_k\,dx = \left(\frac{1}{2}mv^2+mgy\right)_f$$

$$(5\lambda g)8+(3\lambda g)6.5-\int_0^5(2\lambda g-0.4x\lambda g)\,dx = \frac{1}{2}(8\lambda)v^2+(8\lambda g)4$$

$$40.0g+19.5g-2.00g\int_0^5 dx+0.400g\int_0^5 x\,dx = 4.00v^2+32.0g$$

$$27.5g-2.00gx\Big|_0^5+0.400g\frac{x^2}{2}\Big|_0^5 = 4.00v^2$$

$$27.5g-2.00g(5.00)+0.400g(12.5) = 4.00v^2$$

$$22.5g = 4.00v^2$$

$$v=\sqrt{\frac{(22.5\text{ m})(9.80\text{ m/s}^2)}{4.00}} = \boxed{7.42\text{ m/s}}$$

P8.57 $(K+U)_i = (K+U)_f$

$$0+(30.0\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})+\frac{1}{2}(250\text{ N/m})(0.200\text{ m})^2$$

$$=\frac{1}{2}(50.0\text{ kg})v^2+(20.0\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})\sin 40.0°$$

$$58.8\text{ J}+5.00\text{ J} = (25.0\text{ kg})v^2+25.2\text{ J}$$

$$\boxed{v = 1.24\text{ m/s}}$$

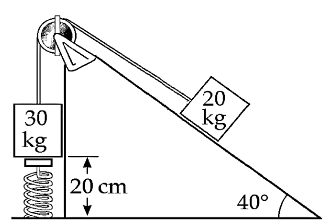

FIG. P8.57

P8.58 The geometry reveals $D = L\sin\theta + L\sin\phi$, $50.0\text{ m} = 40.0\text{ m}(\sin 50° + \sin\phi)$, $\phi = 28.9°$

(a) From takeoff to alighting for the Jane-Earth system

$$\left(K+U_g\right)_i + W_{\text{wind}} = \left(K+U_g\right)_f$$

$$\frac{1}{2}mv_i^2 + mg(-L\cos\theta) + FD(-1) = 0 + mg(-L\cos\phi)$$

$$\frac{1}{2}50\text{ kg } v_i^2 + 50\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 50°) - 110\text{ N}(50\text{ m})$$
$$= 50\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 28.9°)$$

$$\frac{1}{2}50\text{ kg } v_i^2 - 1.26\times10^4\text{ J} - 5.5\times10^3\text{ J} = -1.72\times10^4\text{ J}$$

$$v_i = \sqrt{\frac{2(947\text{ J})}{50\text{ kg}}} = \boxed{6.15\text{ m/s}}$$

(b) For the swing back

$$\frac{1}{2}mv_i^2 + mg(-L\cos\phi) + FD(+1) = 0 + mg(-L\cos\theta)$$

$$\frac{1}{2}130\text{ kg } v_i^2 + 130\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 28.9°) + 110\text{ N}(50\text{ m})$$
$$= 130\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 50°)$$

$$\frac{1}{2}130\text{ kg } v_i^2 - 4.46\times10^4\text{ J} + 5\,500\text{ J} = -3.28\times10^4\text{ J}$$

$$v_i = \sqrt{\frac{2(6\,340\text{ J})}{130\text{ kg}}} = \boxed{9.87\text{ m/s}}$$

P8.59 (a) Initial compression of spring: $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$$\frac{1}{2}(450\text{ N/m})(\Delta x)^2 = \frac{1}{2}(0.500\text{ kg})(12.0\text{ m/s})^2$$

Therefore, $\Delta x = \boxed{0.400\text{ m}}$

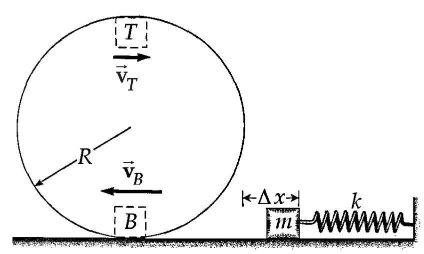

FIG. P8.59

(b) Speed of block at top of track: $\Delta E_{\text{mech}} = -f\Delta x$

$$\left(mgh_T + \frac{1}{2}mv_T^2\right) - \left(mgh_B + \frac{1}{2}mv_B^2\right) = -f(\pi R)$$

$$(0.500\text{ kg})\left(9.80\text{ m/s}^2\right)(2.00\text{ m}) + \frac{1}{2}(0.500\text{ kg})v_T^2 - \frac{1}{2}(0.500\text{ kg})(12.0\text{ m/s})^2$$
$$= -(7.00\text{ N})(\pi)(1.00\text{ m})$$

$$0.250v_T^2 = 4.21$$

$$\therefore v_T = \boxed{4.10\text{ m/s}}$$

continued on next page

(c) Does block fall off at or before top of track? Block falls if $a_c < g$

$$a_c = \frac{v_T^2}{R} = \frac{(4.10)^2}{1.00} = 16.8 \text{ m/s}^2$$

Therefore $a_c > g$ and the $\boxed{\text{block stays on the track}}$.

***P8.60** (a) Take the original point where the ball is released and the final point where its upward swing stops at height H and horizontal displacement

$$x = \sqrt{L^2 - (L-H)^2} = \sqrt{2LH - H^2}$$

Since the wind force is purely horizontal, it does work

$$W_{\text{wind}} = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = F\int dx = F\sqrt{2LH - H^2}$$

The work-energy theorem can be written:

$K_i + U_{gi} + W_{\text{wind}} = K_f + U_{gf}$, or

$0 + 0 + F\sqrt{2LH - H^2} = 0 + mgH$ giving

$F^2 2LH - F^2H^2 = m^2g^2H^2$

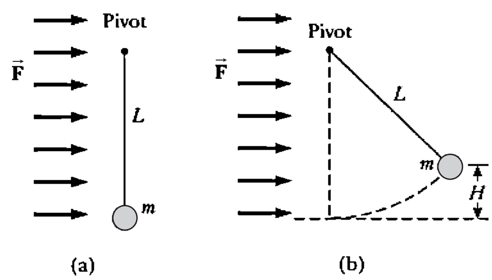

FIG. P8.60

Here the solution $H = 0$ represents the lower turning point of the ball's oscillation, and the upper limit is at $F^2(2L) = (F^2 + m^2g^2)H$. Solving for H yields

$$H = \frac{2LF^2}{F^2 + m^2g^2} = \frac{2L}{1 + (mg/F)^2} = \frac{2(0.8 \text{ m})}{1 + (0.3 \text{ kg})^2(9.8 \text{ m/s}^2)^2/F^2} = \boxed{\frac{1.6 \text{ m}}{1 + 8.64 \text{ N}^2/F^2}}$$

(b) $H = 1.6 \text{ m}[1 + 8.64/1]^{-1} = \boxed{0.166 \text{ m}}$

(c) $H = 1.6 \text{ m}[1 + 8.64/100]^{-1} = \boxed{1.47 \text{ m}}$

(d) As $F \to 0$, $H \to 0$ as is reasonable.

(e) As $F \to \infty$, $H \to 1.60$ m, which would be hard to approach experimentally.

(f) Call θ the equilibrium angle with the vertical and T the tension in the string.

$$\sum F_x = 0 \Rightarrow T\sin\theta = F, \text{ and}$$
$$\sum F_y = 0 \Rightarrow T\cos\theta = mg$$

Dividing: $\tan\theta = \dfrac{F}{mg}$

Then

$$\cos\theta = \frac{mg}{\sqrt{(mg)^2 + F^2}} = \frac{1}{\sqrt{1 + (F/mg)^2}} = \frac{1}{\sqrt{1 + F^2/8.64 \text{ N}^2}}$$

Therefore,

$$H_{\text{eq}} = L(1 - \cos\theta) = \boxed{(0.800 \text{ m})\left(1 - \frac{1}{\sqrt{1 + F^2/8.64 \text{ N}^2}}\right)}$$

continued on next page

(g) $H_{eq} = 0.8\text{ m}[1 - (1 + 100/8.64)^{-1/2}] = \boxed{0.574\text{ m}}$

(h) As $F \to \infty$, $\tan\theta \to \infty$, $\theta \to 90.0°$ $\cos\theta \to 0$ and $H_{eq} \to \boxed{0.800\text{ m}}$

A very strong wind pulls the string out horizontal, parallel to the ground.

P8.61 If the spring is just barely able to lift the lower block from the table, the spring lifts it through no noticeable distance, but exerts on the block a force equal to its weight Mg. The extension of the spring, from $|\mathbf{F}_s| = kx$, must be Mg/k. Between an initial point at release and a final point when the moving block first comes to rest, we have

$$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}: \qquad 0 + mg\left(-\frac{4mg}{k}\right) + \frac{1}{2}k\left(\frac{4mg}{k}\right)^2 = 0 + mg\left(\frac{Mg}{k}\right) + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2$$

$$-\frac{4m^2g^2}{k} + \frac{8m^2g^2}{k} = \frac{mMg^2}{k} + \frac{M^2g^2}{2k}$$

$$4m^2 = mM + \frac{M^2}{2}$$

$$\frac{M^2}{2} + mM - 4m^2 = 0$$

$$M = \frac{-m \pm \sqrt{m^2 - 4\left(\frac{1}{2}\right)\left(-4m^2\right)}}{2\left(\frac{1}{2}\right)} = -m \pm \sqrt{9m^2}$$

Only a positive mass is physical, so we take $M = m(3-1) = \boxed{2m}$.

P8.62 (a) Energy is conserved in the swing of the pendulum, and the stationary peg does no work. So the ball's speed does not change when the string hits or leaves the peg, and the ball swings equally high on both sides.

(b) Relative to the point of suspension,

$$U_i = 0,\ U_f = -mg\left[d - (L-d)\right]$$

From this we find that

$$-mg(2d - L) + \frac{1}{2}mv^2 = 0$$

Also for centripetal motion,

$$mg = \frac{mv^2}{R} \text{ where } R = L - d$$

Upon solving, we get $\boxed{d = \frac{3L}{5}}$.

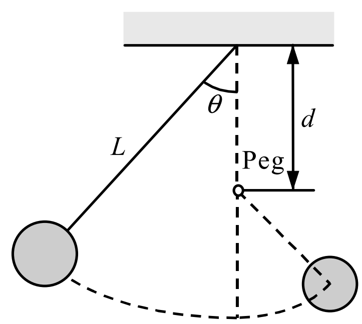

FIG. P8.62

P8.63 Applying Newton's second law at the bottom (b) and top (t) of the circle gives

$$T_b - mg = \frac{mv_b^2}{R} \text{ and } -T_t - mg = -\frac{mv_t^2}{R}$$

Adding these gives

$$T_b = T_t + 2mg + \frac{m\left(v_b^2 - v_t^2\right)}{R}$$

Also, energy must be conserved and $\Delta U + \Delta K = 0$

So, $\dfrac{m\left(v_b^2 - v_t^2\right)}{2} + \left(0 - 2mgR\right) = 0$ and $\dfrac{m\left(v_b^2 - v_t^2\right)}{R} = 4mg$

Substituting into the above equation gives $\boxed{T_b = T_t + 6mg}$.

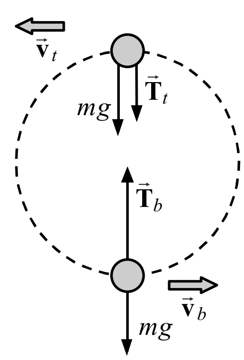

FIG. P8.63

P8.64 (a) At the top of the loop the car and riders are in free fall:

$$\sum F_y = ma_y: \qquad mg \text{ down} = \frac{mv^2}{R} \text{ down}$$

$$v = \sqrt{Rg}$$

Energy of the car-riders-Earth system is conserved between release and top of loop:

$$K_i + U_{gi} = K_f + U_{gf}: \quad 0 + mgh = \frac{1}{2}mv^2 + mg(2R)$$

$$gh = \frac{1}{2}Rg + g(2R)$$

$$\boxed{h = 2.50R}$$

(b) Let h now represent the height $\geq 2.5\,R$ of the release point. At the bottom of the loop we have

$$mgh = \frac{1}{2}mv_b^2 \qquad \text{or} \qquad v_b^2 = 2gh$$

$$\sum F_y = ma_y: \qquad n_b - mg = \frac{mv_b^2}{R}(\text{up})$$

$$n_b = mg + \frac{m(2gh)}{R}$$

At the top of the loop,

$$mgh = \frac{1}{2}mv_t^2 + mg(2R)$$

$$v_t^2 = 2gh - 4gR$$

$$\sum F_y = ma_y: \qquad -n_t - mg = -\frac{mv_t^2}{R}$$

$$n_t = -mg + \frac{m}{R}(2gh - 4gR)$$

$$n_t = \frac{m(2gh)}{R} - 5mg$$

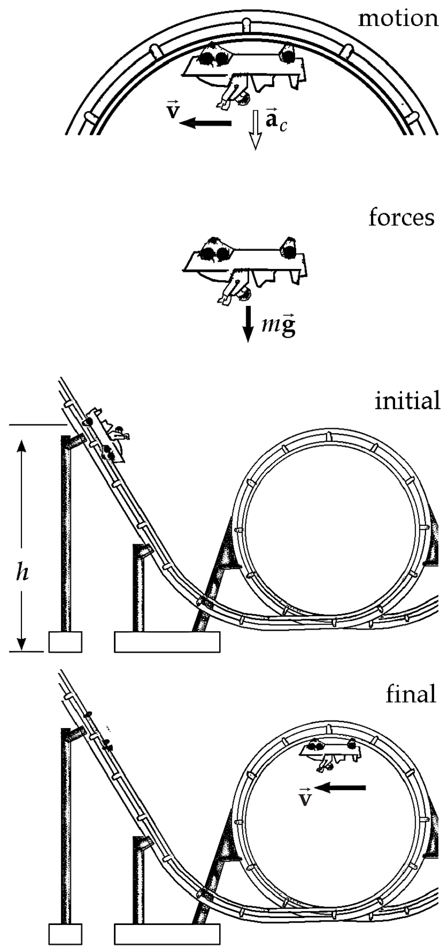

FIG. P8.64

Then the normal force at the bottom is larger by

$$n_b - n_t = mg + \frac{m(2gh)}{R} - \frac{m(2gh)}{R} + 5mg = \boxed{6mg}$$

P8.65 (a) Conservation of energy for the sled-rider-Earth system, between A and C:

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}m(2.5\ \text{m/s})^2 + m(9.80\ \text{m/s}^2)(9.76\ \text{m})$$

$$= \frac{1}{2}mv_C^2 + 0$$

$$v_C = \sqrt{(2.5\ \text{m/s})^2 + 2(9.80\ \text{m/s}^2)(9.76\ \text{m})}$$

$$= \boxed{14.1\ \text{m/s}}$$

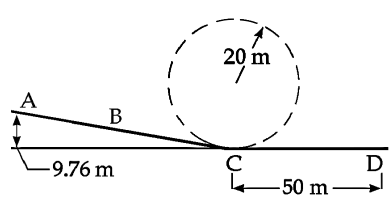

FIG. P8.65(a)

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(b) Incorporating the loss of mechanical energy during the portion of the motion in the water, we have, for the entire motion between A and D (the rider's stopping point),

$$K_i + U_{gi} - f_k \Delta x = K_f + U_{gf}:$$

$$\frac{1}{2}(80\text{ kg})(2.5\text{ m/s})^2 + (80\text{ kg})(9.80\text{ m/s}^2)(9.76\text{ m}) - f_k \Delta x = 0 + 0$$

$$-f_k \Delta x = \boxed{-7.90\times 10^3\text{ J}}$$

(c) The water exerts a frictional force

$$f_k = \frac{7.90\times 10^3\text{ J}}{\Delta x} = \frac{7.90\times 10^3\text{ N}\cdot\text{m}}{50\text{ m}} = 158\text{ N}$$

and also a normal force of

$$n = mg = (80\text{ kg})(9.80\text{ m/s}^2) = 784\text{ N}$$

The magnitude of the water force is

$$\sqrt{(158\text{ N})^2 + (784\text{ N})^2} = \boxed{800\text{ N}}$$

(d) The angle of the slide is

$$\theta = \sin^{-1}\frac{9.76\text{ m}}{54.3\text{ m}} = 10.4°$$

For forces perpendicular to the track at B,

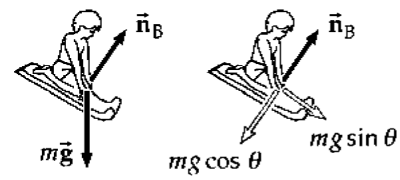

FIG. P8.65(d)

$$\sum F_y = ma_y: \qquad n_B - mg\cos\theta = 0$$

$$n_B = (80.0\text{ kg})(9.80\text{ m/s}^2)\cos 10.4° = \boxed{771\text{ N}}$$

(e) $$\sum F_y = ma_y: \qquad +n_C - mg = \frac{mv_C^2}{r}$$

$$n_C = (80.0\text{ kg})(9.80\text{ m/s}^2) + \frac{(80.0\text{ kg})(14.1\text{ m/s})^2}{20\text{ m}}$$

$$n_C = \boxed{1.57\times 10^3\text{ N up}}$$

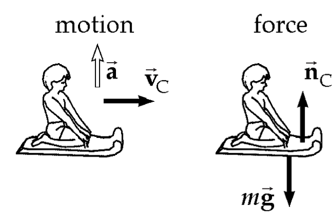

FIG. P8.65(e)

The rider pays for the thrills of a giddy height at A, and a high speed and tremendous splash at C. As a bonus, he gets the quick change in direction and magnitude among the forces we found in parts (d), (e), and (c).

***P8.66** (a) As at the end of the process analyzed in Example 8.8, we begin with a 0.800-kg block at rest on the end of a spring with stiffness constant 50.0 N/m, compressed 0.0924 m. The energy in the spring is (1/2)(50 N/m)(0.0924 m)^2 = 0.214 J. To push the block back to the unstressed spring position would require work against friction of magnitude 3.92 N(0.0924 m) = 0.362 J. Because 0.214 J is less than 0.362 J, the spring cannot push the object back to $x = 0$.

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(b) The block approaches the spring with energy $(1/2)mv^2 = (1/2)(0.8\text{ kg})(1.2\text{ m/s})^2 = 0.576$ J. It travels against friction by equal distances in compressing the spring and in being pushed back out, so it must lose one-half of this energy in its motion to the right and the rest in its motion to the left. The spring must possess one-half of this energy at its maximum compression:

$$(0.576\text{ J})/2 = (1/2)\,(50\text{ N/m})x^2$$

so

$$x = 0.107\text{ m}$$

For the compression process we have the continuity equation for energy

$$0.576\text{ J} + \mu_k\,7.84\text{ N}\,(0.107\text{ m})\cos 180° = 0.288\text{ J}$$

so

$$\mu_k = 0.288\text{ J}/0.841\text{ J}\,\boxed{= 0.342}$$

As a check, the decompression process is described by

$$0.288\text{ J} + \mu_k 7.84\text{ N}\,(0.107\text{ m})\cos 180° = 0$$

which gives the same answer for the coefficient of friction.

ANSWERS TO EVEN PROBLEMS

P8.2 (a) 1.11×10^9 J (b) 0.2

P8.4 (a) $v_B = 5.94$ m/s; $v_C = 7.67$ m/s (b) 147 J

P8.6 (a) see the solution (b) 60.0°

P8.8 (a) $\sqrt{\dfrac{2(m_1 - m_2)gh}{(m_1 + m_2)}}$ (b) $\dfrac{2m_1 h}{m_1 + m_2}$

P8.10 (a) 18.5 km, 51.0 km (b) 10.0 MJ

P8.12 $\left(\dfrac{8gh}{15}\right)^{1/2}$

P8.14 (a) 0.791 m/s (b) 0.531 m/s

P8.16 **(i)** (a), (b), (c), (f) **(ii)** (g), (i), (j) **(iii)** (d) **(iv)** (e) cannot be true because the friction force is proportional to μ_k and not μ_k^2. And (k) cannot be true because the presence of friction will reduce the speed compared to the $\mu_k = 0$ case. **(v)** Expression (h) is correct if the spring force is strong enough to produce motion against static friction and if the spring energy is large enough to make the block slide the full distance *d*. **(vi)** The expression gives an imaginary answer because the spring does not contain enough energy in this case to make the block slide the full distance *d*.

P8.18 (a) $U_f = 22.0$ J; $E = 40.0$ J (b) Yes. The total mechanical energy changes.

P8.20 26.5 m/s

P8.22 (a) $24.5\ \text{m/s}$ (b) Yes; his landing speed is too high to be safe. (c) 206 m (d) Not realistic. Air drag depends strongly on speed.

P8.24 (a) $r = 1.5$ mm and 3.2 mm, stable; 2.3 mm unstable; $r \to \infty$ neutral (b) $-5.6\ \text{J} \le E < 1\ \text{J}$ (c) $0.6\ \text{mm} \le r \le 3.6\ \text{mm}$ (d) 2.6 J (e) 1.5 mm (f) 4 J

P8.26 168 J

P8.28 8.01 W

P8.30 (a) 1.24 kW (b) 20.9%

P8.32 (a) 5.91 kW (b) 11.1 kW

P8.34 194 m

P8.36 No. (a) 582 (b) $90.5\ \text{W} = 0.121\ \text{hp}$

P8.38 (a) yes (b) 2.49 m/s (c) No, but mechanical energy is conserved for the 3.50-kg block in its projectile motion with the Earth. (d) 5.45 m/s (e) 1.23 m (f) no (g) 18.6 N to the left (h) A little push is required. The speeds are still accurate.

P8.40 (a) $x = 0.403$ m or -0.357 m (b) From a perch at a height of 2.80 m above the top of a pile of mattresses, a 46.0-kg child jumps straight upward at 2.40 m/s. The mattresses behave as a linear spring with force constant 19.4 kN/m. Find the maximum amount by which the mattresses are compressed when the child lands on them. Physical meaning of the answer: The positive value of x represents the maximum spring compression. The negative value represents the extension of the equivalent spring if the child sticks to the top of the mattress pile as the child rebounds upward without friction.

P8.42 (a) 5.60 J (b) 0.152 (c) 2.28 rev

P8.44 See the solution. Our model predicts the same proportionalities as the empirical equation, and gives $D = 1$ for the drag coefficient. Air actually slips around the moving object, instead of accumulating in front of it. For this reason, the drag coefficient is not necessarily unity. It is typically less than one for a streamlined object and can be greater than one if the airflow around the object is complicated.

P8.46 (a) $(627\ \text{N})y$ (b) $U_s = 0$ for $y > 39.2$ m and $U_s = \frac{1}{2}(81\ \text{N/m})(39.2\ \text{m} - y)^2$ for $y \le 39.2$ m

(c) $U_g + U_s = (627\ \text{N})y$, for $y > 39.2$ m and $U_g + U_s = (40.5\ \text{N/m})y^2 - (2\,550\ \text{N})y + 62\,200$ J for $y \le 39.2$ m (d) see the solution (e) 10.0 m (f) yes: stable equilibrium at 31.5 m (g) 24.1 m/s

P8.48 (a) see the solution (b) For a block of weight w pushed over a rough horizontal surface at constant velocity, $b = \mu_k$. For a load pulled vertically upward at constant velocity, $b = 1$.

P8.50 (a) 0.588 J (b) 0.588 J (c) $2.42\ \text{m/s}$ (d) $U_C = 0.392$ J, $K_C = 0.196$ J

P8.52 48.2°

P8.54 (a) 0.378 m (b) $2.30\ \text{m/s}$ (c) 1.08 m

P8.56 (a) see the solution (b) $7.42\ \text{m/s}$

P8.58 (a) 6.15 m/s (b) 9.87 m/s

P8.60 (a) $H = 1.6\text{ m}(1 + 8.64\text{ N}^2/\text{F}^2)^{-1}$ (b) 0.166 m (c) 1.47 m (d) $H \to 0$ proportionally to F^2
(e) H approaches 1.60 m (f) $H_{eq} = 0.8\text{ m}[1 - (F^2/8.64\text{ N}^2 + 1)^{-1/2}]$ (g) 0.574 m (h) 0.800 m

P8.62 see the solution

P8.64 (a) 2.5 R (b) see the solution

P8.66 (a) see the solution (b) 0.342

9

Linear Momentum and Collisions

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

***Q9.1** (a) No. Impulse, $\vec{\mathbf{F}}\Delta t$, depends on the force and the time for which it is applied.

(b) No. Work depends on the force and on the distance over which it acts.

***Q9.2** The momentum magnitude is proportional to the speed and the kinetic energy is proportional to the speed squared.

(i) The speed of the constant-mass object becomes 4 times larger and the kinetic energy 16 times larger. Answer (a).

(ii) The speed and the momentum become two times larger. Answer (d).

***Q9.3** (i) answer (c). For example, if one particle has 5 times larger mass, it will have 5 times smaller speed and 5 times smaller kinetic energy.

(ii) answer (d). Momentum is a vector.

***Q9.4** (i) Equal net work inputs imply equal kinetic energies. Answer (c).

(ii) Imagine one particle has four times more mass. For equal kinetic energy it must have half the speed. Then this more massive particle has 4(1/2) = 2 times more momentum. Answer (a).

Q9.5 (a) It does not carry force, for if it did, it could accelerate itself.

(b) It cannot deliver more kinetic energy than it possesses. This would violate the law of energy conservation.

(c) It can deliver more momentum in a collision than it possesses in its flight, by bouncing from the object it strikes.

***Q9.6** Mutual gravitation brings the ball and the Earth together. As the ball moves downward, the Earth moves upward, although with an acceleration on the order of 10^{25} times smaller than that of the ball. The two objects meet, rebound, and separate. Momentum of the ball-Earth system is conserved. Answer (d).

Q9.7 (a) Linear momentum is conserved since there are no external forces acting on the system. The fragments go off in different directions and their vector momenta add to zero.

(b) Kinetic energy is not conserved because the chemical potential energy initially in the explosive is converted into kinetic energy of the pieces of the bomb.

Q9.8 Momentum conservation is not violated if we choose as our system the planet along with you. When you receive an impulse forward, the Earth receives the same size impulse backwards. The resulting acceleration of the Earth due to this impulse is much smaller than your acceleration forward, but the planet's backward momentum is equal in magnitude to your forward momentum.

***Q9.9** (i) During the short time the collision lasts, the total system momentum is constant. Whatever momentum one loses the other gains. Answer (c).

(ii) When the car overtakes the manure spreader, the faster-moving one loses more energy than the slower one gains. Answer (a).

Q9.10 The rifle has a much lower speed than the bullet and much less kinetic energy. Also, the butt distributes the recoil force over an area much larger than that of the bullet.

***Q9.11** (i) answer (a). The ball gives more rightward momentum to the block when the ball reverses its momentum.

(ii) answer (b). In case (a) there is no temperature increase because the collision is elastic.

Q9.12 His impact speed is determined by the acceleration of gravity and the distance of fall, in $v_f^2 = v_i^2 - 2g(0 - y_i)$. The force exerted by the pad depends also on the unknown stiffness of the pad.

Q9.13 The sheet stretches and pulls the two students toward each other. These effects are larger for a faster-moving egg. The time over which the egg stops is extended, more for a faster missile, so that the force stopping it is never too large.

***Q9.14** Think about how much the vector momentum of the Frisbee changes in a horizontal plane. This will be the same in magnitude as your momentum change. Since you start from rest, this quantity directly controls your final speed. Thus f is largest and d is smallest. In between them, b is larger than c and c is larger than g and g is larger than a. Also a is equal to e, because the ice can exert a normal force to prevent you from recoiling straight down when you throw the Frisbee up. The assembled answer is f > b > c > g > a = e > d.

Q9.15 As one finger slides towards the center, the normal force exerted by the sliding finger on the ruler increases. At some point, this normal force will increase enough so that static friction between the sliding finger and the ruler will stop their relative motion. At this moment the other finger starts sliding along the ruler towards the center. This process repeats until the fingers meet at the center of the ruler.

Next step: Try a rod with a nonuniform mass distribution.

Next step: Wear a piece of sandpaper as a ring on one finger to change its coefficient of friction.

***Q9.16** (a) No: mechanical energy turns into internal energy in the coupling process.

(b) No: the Earth feeds momentum into the boxcar during the downhill rolling process.

(c) Yes: total energy is constant as it turns from gravitational into kinetic.

(d) Yes: If the boxcar starts moving north the Earth, very slowly, starts moving south.

(e) No: internal energy appears.

(f) Yes: Only forces internal to the two-car system act.

Q9.17 The center of mass of the balls is in free fall, moving up and then down with the acceleration due to gravity, during the 40% of the time when the juggler's hands are empty. During the 60% of the time when the juggler is engaged in catching and tossing, the center of mass must accelerate up with a somewhat smaller average acceleration. The center of mass moves around in a little closed loop with a parabolic top and likely a circular bottom, making three revolutions for every one revolution that one ball makes. Letting T represent the time for one cycle and F_g the weight of one ball, we have $F_J\, 0.60T = 3F_g T$ and $F_J = 5F_g$. The average force exerted by the juggler is five times the weight of one ball.

Q9.18 In empty space, the center of mass of a rocket-plus-fuel system does not accelerate during a burn, because no outside force acts on this system. The rocket body does accelerate as it blows exhaust containing momentum out the back.

According to the text's 'basic expression for rocket propulsion,' the change in speed of the rocket body will be larger than the speed of the exhaust relative to the rocket, if the final mass is less than 37% of the original mass.

Q9.19 To generalize broadly, around 1740 the English favored position (a), the Germans position (b), and the French position (c). But in France Emilie de Chatelet translated Newton's *Principia* and argued for a more inclusive view. A Frenchman, Jean D'Alembert, is most responsible for showing that each theory is consistent with the others. All the theories are equally correct. Each is useful for giving a mathematically simple and conceptually clear solution for some problems. There is another comprehensive mechanical theory, the angular impulse—angular momentum theorem, which we will glimpse in Chapter 11. It identifies the product of the torque of a force and the time it acts as the cause of a change in motion, and change in angular momentum as the effect. We have here an example of how scientific theories are different from what people call a theory in everyday life. People who think that different theories are mutually exclusive should bring their thinking up to date to around 1750.

SOLUTIONS TO PROBLEMS

Section 9.1 **Linear Momentum and Its Conservation**

P9.1 $m = 3.00 \text{ kg}, \qquad \vec{\mathbf{v}} = \left(3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}\right) \text{ m/s}$

(a) $\vec{\mathbf{p}} = m\vec{\mathbf{v}} = \left(9.00\hat{\mathbf{i}} - 12.0\hat{\mathbf{j}}\right) \text{ kg}\cdot\text{m/s}$

Thus,

$$\boxed{p_x = 9.00 \text{ kg}\cdot\text{m/s}}$$

and

$$\boxed{p_y = -12.0 \text{ kg}\cdot\text{m/s}}$$

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00)^2 + (12.0)^2} = \boxed{15.0 \text{ kg}\cdot\text{m/s}}$

$\theta = \tan^{-1}\left(\dfrac{p_y}{p_x}\right) = \tan^{-1}(-1.33) = \boxed{307°}$

***P9.2** (a) Whomever we consider the aggressor, brother and sister exert equal-magnitude oppositely-directed forces on each other, to give each other equal magnitudes of momentum. We take the eastward component of the equation

total original momentum = total final momentum for the two-sibling system

$0 = 65 \text{ kg } (-2.9 \text{ m/s}) + 40 \text{ kg } v \qquad v = 4.71 \text{ m/s}$, meaning $\boxed{\text{she moves at 4.71 m/s east}}$

continued on next page

(b) original chemical energy in girl's body = total final kinetic energy

$U_{chemical} = (1/2)(65\text{ kg})(2.9\text{ m/s})^2 + (1/2)(40\text{ kg})(4.71\text{ m/s})^2 = \boxed{717\text{ J}}$

(c) System momentum is conserved with the value zero. The net forces on the two siblings are of equal magnitude in opposite directions. Their impulses add to zero. Their final momenta are of equal magnitude in opposite directions, to add as vectors to zero.

P9.3 I have mass 85.0 kg and can jump to raise my center of gravity 25.0 cm. I leave the ground with speed given by

$$v_f^2 - v_i^2 = 2a\left(x_f - x_i\right): \qquad 0 - v_i^2 = 2\left(-9.80\text{ m/s}^2\right)(0.250\text{ m})$$

$$v_i = 2.20\text{ m/s}$$

Total momentum of the system of the Earth and me is conserved as I push the planet down and myself up:

$$0 = \left(5.98\times10^{24}\text{ kg}\right)(-v_e) + (85.0\text{ kg})\left(2.20\text{ m/s}\right)$$

$$v_e \sim \boxed{10^{-23}\text{ m/s}}$$

***P9.4** (a) For the system of two blocks $\Delta p = 0$,

or

$$p_i = p_f$$

Therefore,

$$0 = Mv_m + (3M)\left(2.00\text{ m/s}\right)$$

Solving gives $v_m = \boxed{-6.00\text{ m/s}}$ (motion toward the left).

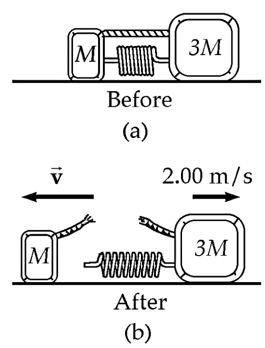

FIG. P9.4

(b) $\frac{1}{2}kx^2 = \frac{1}{2}Mv_M^2 + \frac{1}{2}(3M)v_{3M}^2 = \boxed{8.40\text{ J}}$

(c) The original energy is in the spring. A force had to be exerted over a distance to compress the spring, transferring energy into it by work. The cord exerts force, but over no distance.

(d) System momentum is conserved with the value zero. The forces on the two blocks are of equal magnitude in opposite directions. Their impulses add to zero. The final momenta of the two blocks are of equal magnitude in opposite directions.

P9.5 (a) The momentum is $p = mv$, so $v = \frac{p}{m}$ and the kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \boxed{\frac{p^2}{2m}}$$

(b) $K = \frac{1}{2}mv^2$ implies $v = \sqrt{\frac{2K}{m}}$, so $p = mv = m\sqrt{\frac{2K}{m}} = \boxed{\sqrt{2mK}}$

Section 9.2 **Impulse and Momentum**

***P9.6** From the impulse-momentum theorem, $F(\Delta t) = \Delta p = mv_f - mv_i$, the average force required to hold onto the child is

$$F = \frac{m(v_f - v_i)}{(\Delta t)} = \frac{(12\ \text{kg})(0 - 60\ \text{mi/h})}{0.050\ \text{s} - 0}\left(\frac{1\ \text{m/s}}{2.237\ \text{mi/h}}\right) = -6.44\times10^3\ \text{N}$$

> In trying to hang onto the child, he would have to exert a force of 6.44 kN (over 1 400 lb) toward the back of the car, to slow down the child's forward motion. He is not strong enough to exert so large a force. If he were belted in and his arms were firmly tied around the child, the child would exert this size force on him toward the front of the car. A person cannot safely exert or feel a force of this magnitude and a safety device should be used.

P9.7 (a) $I = \int F dt$ = area under curve

$$I = \frac{1}{2}(1.50\times10^{-3}\ \text{s})(18\ 000\ \text{N}) = \boxed{13.5\ \text{N}\cdot\text{s}}$$

(b) $$F = \frac{13.5\ \text{N}\cdot\text{s}}{1.50\times10^{-3}\ \text{s}} = \boxed{9.00\ \text{kN}}$$

(c) From the graph, we see that $F_{max} = \boxed{18.0\ \text{kN}}$

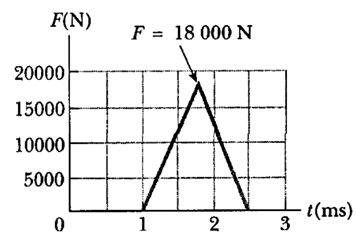

FIG. P9.7

P9.8 The impact speed is given by $\frac{1}{2}mv_1^2 = mgy_1$. The rebound speed is given by $mgy_2 = \frac{1}{2}mv_2^2$. The impulse the floor imparts to the ball is the change in the ball's momentum,

$$\begin{aligned} mv_2\ \text{up} - mv_1\ \text{down} &= m(v_2 + v_1)\ \text{up} \\ &= m\left(\sqrt{2gh_2} + \sqrt{2gh_1}\right)\ \text{up} \\ &= 0.15\ \text{kg}\sqrt{2(9.8\ \text{m/s}^2)}\left(\sqrt{0.960\ \text{m}} + \sqrt{1.25\ \text{m}}\right)\ \text{up} \\ &= \boxed{1.39\ \text{kg}\cdot\text{m/s upward}} \end{aligned}$$

P9.9

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}\Delta t$$

$$\Delta p_y = m(v_{fy} - v_{iy}) = m(v\cos 60.0°) - mv\cos 60.0° = 0$$

$$\begin{aligned} \Delta p_x &= m(-v\sin 60.0° - v\sin 60.0°) = -2mv\sin 60.0° \\ &= -2(3.00\ \text{kg})(10.0\ \text{m/s})(0.866) \\ &= -52.0\ \text{kg}\cdot\text{m/s} \end{aligned}$$

$$F_{avg} = \frac{\Delta p_x}{\Delta t} = \frac{-52.0\ \text{kg}\cdot\text{m/s}}{0.200\ \text{s}} = \boxed{-260\ \text{N}}$$

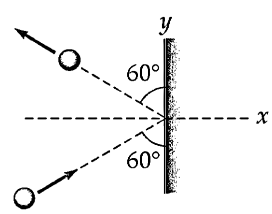

FIG. P9.9

P9.10 Assume the initial direction of the ball in the $-x$ direction.

(a) Impulse, $\vec{\mathbf{I}} = \Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i = (0.060\ 0)(40.0)\hat{\mathbf{i}} - (0.060\ 0)(50.0)(-\hat{\mathbf{i}}) = \boxed{5.40\hat{\mathbf{i}}\ \text{N}\cdot\text{s}}$

(b) Work $= K_f - K_i = \frac{1}{2}(0.060\ 0)\left[(40.0)^2 - (50.0)^2\right] = \boxed{-27.0\ \text{J}}$

***P9.11** (a) The impulse is to the right and equal to the area under the F-t graph:

$$I = [(0 + 4\text{ N})/2](2\text{ s} - 0) + (4\text{ N})(3\text{ s} - 2\text{ s}) + (2\text{ N})(2\text{ s}) = \boxed{12.0\text{ N}\cdot\text{s}\ \hat{\mathbf{i}}}$$

(b) $m\vec{\mathbf{v}}_i + \vec{\mathbf{F}}t = m\vec{\mathbf{v}}_f$ $\quad (2.5\text{ kg})(0) + 12\ \hat{\mathbf{i}}\ \text{N}\cdot\text{s} = (2.5\text{ kg})\ \vec{\mathbf{v}}_f$ $\quad \vec{\mathbf{v}}_f = \boxed{4.80\ \hat{\mathbf{i}}\ \text{m/s}}$

(c) From the same equation, $(2.5\text{ kg})(-2\ \hat{\mathbf{i}}\text{ m/s}) + 12\ \hat{\mathbf{i}}\text{ N}\cdot\text{s} = (2.5\text{ kg})\ \vec{\mathbf{v}}_f$ $\quad \vec{\mathbf{v}}_f = \boxed{2.80\ \hat{\mathbf{i}}\ \text{m/s}}$

(d) $\vec{\mathbf{F}}_{avg}\Delta t = 12.0\hat{\mathbf{i}}\ \text{N}\cdot\text{s} = \vec{\mathbf{F}}_{avg}(5\text{ s})$ $\quad \vec{\mathbf{F}}_{avg} = \boxed{2.40\hat{\mathbf{i}}\ \text{N}}$

***P9.12** (a) A graph of the expression for force shows a parabola opening down, with the value zero at the beginning and end of the 0.8 s interval.

$$I = \int_0^{0.8\text{s}} F dt = \int_0^{0.8\text{s}} \left(9200\ t\ \text{N/s}\ - 11500\ t^2\ \text{N/s}^2\right) dt$$
$$= \left[(9200\text{ N/s})t^2/2 - \left(11500\text{ N/s}^2\right)t^3/3\right]_0^{0.8\text{s}}$$
$$= (9200\text{ N/s})(0.8\text{ s})^2/2 - \left(11500\text{ N/s}^2\right)(0.8\text{ s})^3/3$$
$$= 2944\text{ N}\cdot\text{s} - 1963\text{ N}\cdot\text{s} = 981\text{ N}\cdot\text{s}$$

The athlete imparts downward impulse to the platform, so the platform imparts $\boxed{\text{981 N·s of upward impulse}}$ to her.

(b) We could find her impact speed as a free-fall calculation, but we choose to write it as a conservation-of energy calculation: $mgy_{top} = (1/2)mv_{impact}^{\ 2}$

$v_{impact} = (2gy_{top})^{1/2} = [2(9.8\text{ m/s}^2)0.6\text{ m}]^{1/2} = \boxed{3.43\text{ m/s down}}$

(c) Gravity, as well as the platform, imparts impulse to her during the interaction with the platform.

$mv_i + I_{platform} + mgt = mv_f$

$(65\text{ kg})(-3.43\text{ m/s}) + 981\text{ N}\cdot\text{s} - (65\text{ kg})(9.8\text{ m/s}^2)(0.8\text{ s}) = 65\text{ kg}\ v_f$

$-223\text{ N}\cdot\text{s} + 981\text{ N}\cdot\text{s} - 510\text{ N}\cdot\text{s} = 65\text{ kg}\ v_f$ $\quad v_f = 249\text{ N}\cdot\text{s}/65\text{ kg} = \boxed{3.83\text{ m/s up}}$

Note that the athlete is putting a lot of effort into jumping and does not exert any force "on herself." The usefulness of the force platform is to measure her effort by showing the force she exerts on the floor.

(d) Again energy is conserved in upward flight. $\quad (1/2)mv_{takeoff}^{\ 2} = mgy_{top}$

$y_{top} = v_{takeoff}^{\ 2}/2g = (3.83\text{ m/s})^2/2(9.8\text{ m/s}^2) = \boxed{0.748\text{ m}}$

P9.13 (a) Energy is conserved for the spring-mass system:

$$K_i + U_{si} = K_f + U_{sf}: \qquad 0 + \frac{1}{2}kx^2 = \frac{1}{2}mv^2 + 0$$
$$v = x\sqrt{\frac{k}{m}}$$

(b) From the equation, a $\boxed{\text{smaller}}$ value of m makes $v = x\sqrt{\frac{k}{m}}$ larger.

(c) $I = \left|\vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i\right| = mv_f - 0 = mx\sqrt{\frac{k}{m}} = x\sqrt{km}$

(d) From the equation, a $\boxed{\text{larger}}$ value of m makes $I = x\sqrt{km}$ larger.

(e) For the glider, $W = K_f - K_i = \frac{1}{2}mv^2 - 0 = \frac{1}{2}kx^2$

The mass makes $\boxed{\text{no difference}}$ to the work.

***P9.14** After 3 s of pouring, the bucket contains (3s)(0.25 L/s) = 0.75 liter of water, with mass 0.75 L(1 kg/1 L) = 0.75 kg, and feeling gravitational force 0.75 kg(9.8 m/s^2) = 7.35 N. The scale through the bucket must exert 7.35 N upward on this stationary water to support its weight. The scale must exert another 7.35 N to support the 0.75-kg bucket itself.

Water is entering the bucket with speed given by $mgy_{top} = (1/2)mv_{impact}{}^2$

$$v_{impact} = (2gy_{top})^{1/2} = [2(9.8\text{ m/s}^2)2.6\text{ m}]^{1/2} = 7.14\text{ m/s downward}$$

The scale exerts an extra upward force to stop the downward motion of this additional water, as described by

$$mv_{impact} + F_{extra}\, t = mv_f$$

The rate of change of momentum is the force itself: $(dm/dt)\, v_{impact} + F_{extra} = 0$

$$F_{extra} = -(dm/dt)\, v_{impact} = -(0.25\text{ kg/s})(-7.14\text{ m/s}) = +1.78\text{ N}$$

Altogether the scale must exert 7.35 N + 7.35 N + 1.78 N = $\boxed{16.5\text{ N}}$

Section 9.3 **Collisions in One Dimension**

P9.15 Momentum is conserved for the bullet-block system

$$\left(10.0\times10^{-3}\text{ kg}\right)v = (5.01\text{ kg})(0.600\text{ m/s})$$

$$v = \boxed{301\text{ m/s}}$$

P9.16 (a) $mv_{1i} + 3mv_{2i} = 4mv_f$ where $m = 2.50\times10^4$ kg

$$v_f = \frac{4.00+3(2.00)}{4} = \boxed{2.50\text{ m/s}}$$

(b) $$K_f - K_i = \frac{1}{2}(4m)v_f^2 - \left[\frac{1}{2}mv_{1i}^2 + \frac{1}{2}(3m)v_{2i}^2\right] = \left(2.50\times10^4\right)(12.5-8.00-6.00) = -3.75\times10^4\text{ J}$$

$$K_i = K_f + \Delta E_{int} \quad \Delta E_{int} = \boxed{+37.5\text{ kJ}}$$

P9.17 (a) The internal forces exerted by the actor do not change the total momentum of the system of the four cars and the movie actor

$$(4m)v_i = (3m)(2.00\text{ m/s}) + m(4.00\text{ m/s})$$

$$v_i = \frac{6.00\text{ m/s}+4.00\text{ m/s}}{4} = \boxed{2.50\text{ m/s}}$$

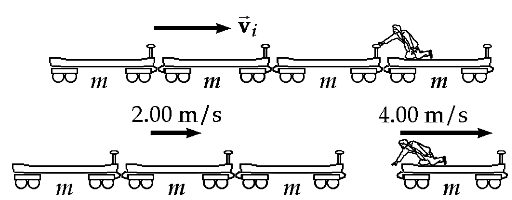

FIG. P9.17

(b) $$W_{actor} = K_f - K_i = \frac{1}{2}\left[(3m)(2.00\text{ m/s})^2 + m(4.00\text{ m/s})^2\right] - \frac{1}{2}(4\text{ m})(2.50\text{ m/s})^2$$

$$W_{actor} = \frac{\left(2.50\times10^4\text{ kg}\right)}{2}(12.0+16.0-25.0)(\text{m/s})^2 = \boxed{37.5\text{ kJ}}$$

(c) The event considered here is the time reversal of the perfectly inelastic collision in the previous problem. The same momentum conservation equation describes both processes.

P9.18 Energy is conserved for the bob-Earth system between bottom and top of swing. At the top the stiff rod is in compression and the bob nearly at rest.

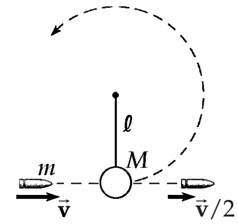

FIG. P9.18

$$K_i + U_i = K_f + U_f: \qquad \frac{1}{2}Mv_b^2 + 0 = 0 + Mg2\ell$$

$$v_b^2 = g4\ell \text{ so } v_b = 2\sqrt{g\ell}$$

Momentum of the bob-bullet system is conserved in the collision:

$$mv = m\frac{v}{2} + M\left(2\sqrt{g\ell}\right) \qquad \boxed{v = \frac{4M}{m}\sqrt{g\ell}}$$

P9.19 First we find v_1, the speed of m_1 at B before collision.

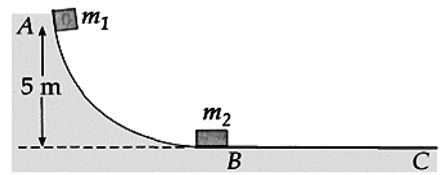

FIG. P9.19

$$\frac{1}{2}m_1v_1^2 = m_1gh$$

$$v_1 = \sqrt{2(9.80)(5.00)} = 9.90 \text{ m/s}$$

Now we use the text's analysis of one-dimensional elastic collisions to find v_{1f}, the speed of m_1 at B just after collision.

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_1 = -\frac{1}{3}(9.90) \text{ m/s} = -3.30 \text{ m/s}$$

Now the 5-kg block bounces back up to its highest point after collision according to

$$m_1gh_{\max} = \frac{1}{2}m_1(-3.30)^2 \qquad h_{\max} = \frac{(-3.30 \text{ m/s})^2}{2\left(9.80 \text{ m/s}^2\right)} = \boxed{0.556 \text{ m}}$$

***P9.20** (a) We assume that energy is conserved in the fall of the basketball and the tennis ball. Each reaches its lowest point with a speed given by

$$\left(K + U_g\right)_{\text{release}} = \left(K + U_g\right)_{\text{bottom}}$$

$$0 + mgy_i = \frac{1}{2}mv_b^2 + 0$$

$$v_b = \sqrt{2gy_i} = \sqrt{2\left(9.8 \text{ m/s}^2\right)(1.20 \text{ m})} = \boxed{4.85 \text{ m/s}}$$

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(b) The two balls exert no forces on each other as they move down. They collide with each other after the basketball has its velocity reversed by the floor. We choose upward as positive. Momentum conservation:

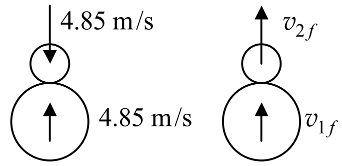

FIG. P9.20(b)

$$(57\ \text{g})(-4.85\ \text{m/s})+(590\ \text{g})(4.85\ \text{m/s})=(57\ \text{g})v_{2f}+(590\ \text{g})v_{1f}$$

To describe the elastic character of the collision, we use the relative velocity equation

$$4.85\ \text{m/s}-(-4.85\ \text{m/s})=v_{2f}-v_{1f}$$

we solve by substitution

$$v_{1f}=v_{2f}-9.70\ \text{m/s}$$

$$2\,580\ \text{gm/s}=(57\ \text{g})v_{2f}+(590\ \text{g})\left(v_{2f}-9.70\ \text{m/s}\right)$$
$$=(57\ \text{g})v_{2f}+(590\ \text{g})v_{2f}-5\,720\ \text{gm/s}$$
$$v_{2f}=\frac{8\,310\ \text{m/s}}{647}=12.8\ \text{m/s}$$

Now the tennis ball-Earth system keeps constant energy as the ball rises:

$$\frac{1}{2}(57\ \text{g})(12.8\ \text{m/s})^2=(57\ \text{g})\left(9.8\ \text{m/s}^2\right)y_f$$
$$y_f=\frac{165\ \text{m}^2/\text{s}^2}{2\left(9.8\ \text{m/s}^2\right)}=\boxed{8.41\ \text{m}}$$

P9.21 (a), (b) Let v_g and v_p be the x-components of velocity of the girl and the plank relative to the ice surface. Then we may say that v_g-v_p is the velocity of the girl relative to the plank, so that

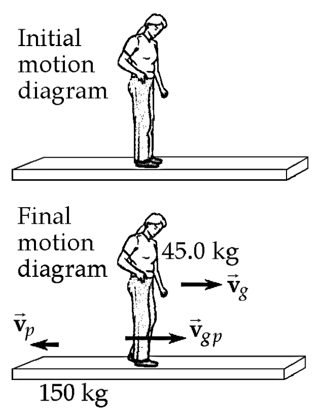

FIG. P9.21

$$v_g-v_p=1.50 \qquad (1)$$

But also we must have $m_g v_g+m_p v_p=0$, since total momentum of the girl-plank system is zero relative to the ice surface. Therefore

$$45.0v_g+150v_p=0,\ \text{or}\ v_g=-3.33v_p$$

Putting this into the equation (1) above gives

$$-3.33v_p-v_p=1.50\ \text{ or }\ v_p=\boxed{-0.346\ \hat{\mathbf{i}}\ \text{m/s}}\ \text{(answer b)}$$

$$\text{Then }v_g=-3.33(-0.346)=\boxed{1.15\ \hat{\mathbf{i}}\ \text{m/s}}\ \text{(answer a)}$$

P9.22 We assume equal firing speeds v and equal forces F required for the two bullets to push wood fibers apart. These equal forces act backward on the two bullets.

For the first,

$$K_i + \Delta E_{\text{mech}} = K_f \qquad \frac{1}{2}\left(7.00\times10^{-3}\text{ kg}\right)v^2 - F\left(8.00\times10^{-2}\text{ m}\right) = 0$$

For the second,

$$p_i = p_f \qquad \left(7.00\times10^{-3}\text{ kg}\right)v = \left(1.014\text{ kg}\right)v_f$$

$$v_f = \frac{\left(7.00\times10^{-3}\right)v}{1.014}$$

Again,

$$K_i + \Delta E_{\text{mech}} = K_f: \qquad \frac{1}{2}\left(7.00\times10^{-3}\text{ kg}\right)v^2 - Fd = \frac{1}{2}\left(1.014\text{ kg}\right)v_f^2$$

Substituting for v_f,

$$\frac{1}{2}\left(7.00\times10^{-3}\text{ kg}\right)v^2 - Fd = \frac{1}{2}\left(1.014\text{ kg}\right)\left(\frac{7.00\times10^{-3}v}{1.014}\right)^2$$

$$Fd = \frac{1}{2}\left(7.00\times10^{-3}\right)v^2 - \frac{1}{2}\frac{\left(7.00\times10^{-3}\right)^2}{1.014}v^2$$

Substituting for v,

$$Fd = F\left(8.00\times10^{-2}\text{ m}\right)\left(1 - \frac{7.00\times10^{-3}}{1.014}\right) \qquad d = \boxed{7.94\text{ cm}}$$

P9.23 (a) From the text's analysis of a one-dimensional elastic collision with an originally stationary target, the x-component of the neutron's velocity changes from v_i to $v_{1f} = (1 - 12)v_i/13 = -11v_i/13$. The x-component of the target nucleus velocity is $v_{2f} = 2v_i/13$.

The neutron started with kinetic energy $(1/2)\, m_1 v_i^2$

The target nucleus ends up with kinetic energy $(1/2)\,(12\, m_1)(\,2v_i/13)^2$

Then the fraction transferred is

$$\frac{\frac{1}{2}12m_1(2v_i/13)^2}{\frac{1}{2}m_1 v_i^2} = \frac{48}{169} = \boxed{0.284}$$ Because the collision is elastic, the other 71.6% of the original energy stays with the neutron. The carbon is functioning as a *moderator* in the reactor, slowing down neutrons to make them more likely to produce reactions in the fuel.

(b) $$K_n = (0.716)\left(1.6\times10^{-13}\text{ J}\right) = \boxed{1.15\times10^{-13}\text{ J}}$$

$$K_C = (0.284)\left(1.6\times10^{-13}\text{ J}\right) = \boxed{4.54\times10^{-14}\text{ J}}$$

P9.24 (a) Using conservation of momentum, $\left(\sum \vec{\mathbf{p}}\right)_{\text{before}} = \left(\sum \vec{\mathbf{p}}\right)_{\text{after}}$, gives

$$(4.0 \text{ kg})(5.0 \text{ m/s}) + (10 \text{ kg})(3.0 \text{ m/s}) + (3.0 \text{ kg})(-4.0 \text{ m/s}) = [(4.0 + 10 + 3.0) \text{ kg}]v$$

Therefore,

$$v = +2.24 \text{ m/s, or } \boxed{2.24 \text{ m/s toward the right}}$$

(b) $\boxed{\text{No.}}$ For example, if the 10-kg and 3.0-kg mass were to stick together first, they would move with a speed given by solving

$$(13 \text{ kg})v_1 = (10 \text{ kg})(3.0 \text{ m/s}) + (3.0 \text{ kg})(-4.0 \text{ m/s}), \text{ or } v_1 = +1.38 \text{ m/s}$$

Then when this 13 kg combined mass collides with the 4.0 kg mass, we have

$$(17 \text{ kg})v = (13 \text{ kg})(1.38 \text{ m/s}) + (4.0 \text{ kg})(5.0 \text{ m/s}), \text{ and } v = +2.24 \text{ m/s}$$

just as in part (a). $\boxed{\text{Coupling order makes no difference to the final velocity.}}$

P9.25 During impact, momentum of the clay-block system is conserved:

$$mv_1 = (m_1 + m_2)v_2$$

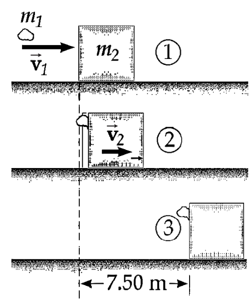

FIG. P9.25

During sliding, the change in kinetic energy of the clay-block-surface system is equal to the increase in internal energy:

$$\frac{1}{2}(m_1 + m_2)v_2^2 = f_f d = \mu(m_1 + m_2)gd$$

$$\frac{1}{2}(0.112 \text{ kg})v_2^2 = 0.650(0.112 \text{ kg})(9.80 \text{ m/s}^2)(7.50 \text{ m})$$

$$v_2^2 = 95.6 \text{ m}^2/\text{s}^2 \qquad v_2 = 9.77 \text{ m/s}$$

$$(12.0 \times 10^{-3} \text{ kg})v_1 = (0.112 \text{ kg})(9.77 \text{ m/s}) \qquad v_1 = \boxed{91.2 \text{ m/s}}$$

Section 9.4 **Two-Dimensional Collisions**

***P9.26** (a) Over a very short time interval, outside forces have no time to impart significant impulse—thus the interaction is a collision. The opponent grabs the fullback and does not let go, so the two players move together at the end of their interaction—thus the collision is completely inelastic.

continued on next page

(b) First, we conserve momentum for the system of two football players in the x direction (the direction of travel of the fullback).

$$(90.0 \text{ kg})(5.00 \text{ m/s}) + 0 = (185 \text{ kg})V\cos\theta$$

where θ is the angle between the direction of the final velocity V and the x axis. We find

$$V\cos\theta = 2.43 \text{ m/s} \qquad (1)$$

Now consider conservation of momentum of the system in the y direction (the direction of travel of the opponent).

$$(95.0 \text{ kg})(3.00 \text{ m/s}) + 0 = (185 \text{ kg})(V\sin\theta)$$

which gives

$$V\sin\theta = 1.54 \text{ m/s} \qquad (2)$$

Divide equation (2) by (1)

$$\tan\theta = \frac{1.54}{2.43} = 0.633$$

From which

$$\boxed{\theta = 32.3°}$$

Then, either (1) or (2) gives

$$V = \boxed{2.88 \text{ m/s}}$$

(c) $K_i = \frac{1}{2}(90.0 \text{ kg})(5.00 \text{ m/s})^2 + \frac{1}{2}(95.0 \text{ kg})(3.00 \text{ m/s})^2 = 1.55 \times 10^3 \text{ J}$

$K_f = \frac{1}{2}(185 \text{ kg})(2.88 \text{ m/s})^2 = 7.67 \times 10^2 \text{ J}$

Thus, the kinetic energy lost is $\boxed{\text{786 J into internal energy}}$.

P9.27 By conservation of momentum for the system of the two billiard balls (with all masses equal), in the x and y directions separately,

$$5.00 \text{ m/s} + 0 = (4.33 \text{ m/s})\cos 30.0° + v_{2fx}$$
$$v_{2fx} = 1.25 \text{ m/s}$$
$$0 = (4.33 \text{ m/s})\sin 30.0° + v_{2fy}$$
$$v_{2fy} = -2.16 \text{ m/s}$$
$$\vec{\mathbf{v}}_{2f} = \boxed{2.50 \text{ m/s at } -60.0°}$$

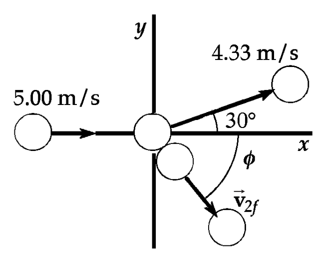

FIG. P9.27

Note that we did not need to explicitly use the fact that the collision is perfectly elastic.

P9.28 We use conservation of momentum for the system of two vehicles for both northward and eastward components, to find the original speed of car number 2.

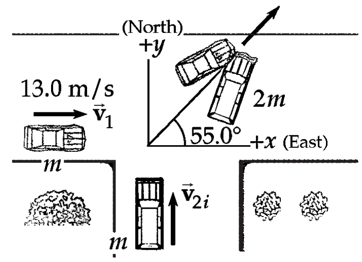

FIG. P9.28

For the eastward direction:

$$M(13.0 \text{ m/s}) = 2MV_f \cos 55.0°$$

For the northward direction:

$$Mv_{2i} = 2MV_f \sin 55.0°$$

Divide the northward equation by the eastward equation to find:

$$v_{2i} = (13.0 \text{ m/s})\tan 55.0° = 18.6 \text{ m/s} = 41.5 \text{ mi/h}$$

Thus, the driver of the north bound car was untruthful. His original speed was more than 35 mi/h.

P9.29

$$p_{xf} = p_{xi}$$

$$mv_O \cos 37.0° + mv_Y \cos 53.0° = m(5.00 \text{ m/s})$$

$$0.799v_O + 0.602v_Y = 5.00 \text{ m/s} \qquad (1)$$

$$p_{yf} = p_{yi}$$

$$mv_O \sin 37.0° - mv_Y \sin 53.0° = 0$$

$$0.602v_O = 0.799v_Y \qquad (2)$$

Solving (1) and (2) simultaneously,

$\boxed{v_O = 3.99 \text{ m/s}}$ and $\boxed{v_Y = 3.01 \text{ m/s}}$

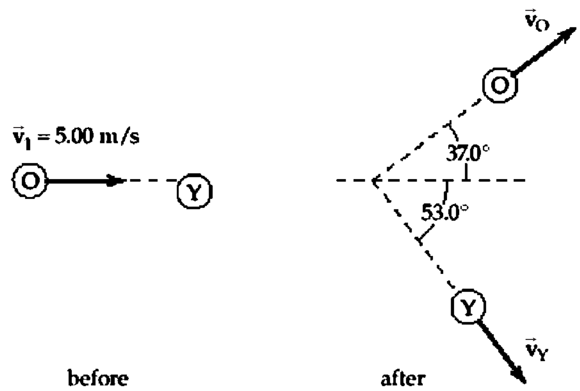

FIG. P9.29

P9.30

$$p_{xf} = p_{xi}: \quad mv_O \cos\theta + mv_Y \cos(90.0° - \theta) = mv_i$$

$$v_O \cos\theta + v_Y \sin\theta = v_i \qquad (1)$$

$$p_{yf} = p_{yi}: \quad mv_O \sin\theta - mv_Y \sin(90.0° - \theta) = 0$$

$$v_O \sin\theta = v_Y \cos\theta \qquad (2)$$

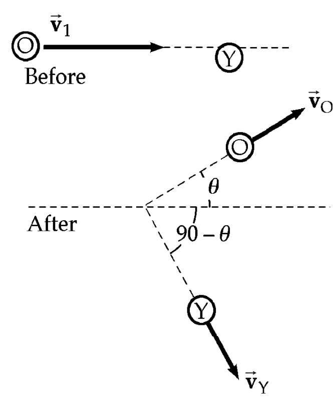

FIG. P9.30

From equation (2),

$$v_O = v_Y\left(\frac{\cos\theta}{\sin\theta}\right) \qquad (3)$$

Substituting into equation (1),

$$v_Y\left(\frac{\cos^2\theta}{\sin\theta}\right) + v_Y \sin\theta = v_i$$

so

$$v_Y(\cos^2\theta + \sin^2\theta) = v_i \sin\theta, \text{ and } \boxed{v_Y = v_i \sin\theta}$$

Then, from equation (3), $\boxed{v_O = v_i \cos\theta}$.

We did not need to write down an equation expressing conservation of mechanical energy. In the problem situation, the requirement of perpendicular final velocities is equivalent to the condition of elasticity.

P9.31 $m_1\vec{\mathbf{v}}_{1i}+m_2\vec{\mathbf{v}}_{2i}=(m_1+m_2)\vec{\mathbf{v}}_f$: $\quad 3.00(5.00)\hat{\mathbf{i}}-6.00\hat{\mathbf{j}}=5.00\vec{\mathbf{v}}$

$$\vec{\mathbf{v}}=\boxed{\left(3.00\hat{\mathbf{i}}-1.20\hat{\mathbf{j}}\right)\text{ m/s}}$$

P9.32 x-component of momentum for the system of the two objects:

$p_{1ix}+p_{2ix}=p_{1fx}+p_{2fx}$: $\quad -mv_i+3mv_i=0+3mv_{2x}$

y-component of momentum of the system: $\quad 0+0=-mv_{1y}+3mv_{2y}$

by conservation of energy of the system: $\quad +\frac{1}{2}mv_i^2+\frac{1}{2}3mv_i^2=\frac{1}{2}mv_{1y}^2+\frac{1}{2}3m\left(v_{2x}^2+v_{2y}^2\right)$

we have

$$v_{2x}=\frac{2v_i}{3}$$

also

$$v_{1y}=3v_{2y}$$

So the energy equation becomes

$$4v_i^2=9v_{2y}^2+\frac{4v_i^2}{3}+3v_{2y}^2$$

$$\frac{8v_i^2}{3}=12v_{2y}^2$$

or

$$v_{2y}=\frac{\sqrt{2}v_i}{3}$$

(a) The object of mass m has final speed

$$v_{1y}=3v_{2y}=\boxed{\sqrt{2}v_i}$$

and the object of mass 3 m moves at

$$\sqrt{v_{2x}^2+v_{2y}^2}=\sqrt{\frac{4v_i^2}{9}+\frac{2v_i^2}{9}}$$

$$\sqrt{v_{2x}^2+v_{2y}^2}=\boxed{\sqrt{\frac{2}{3}}v_i}$$

(b) $\theta=\tan^{-1}\left(\frac{v_{2y}}{v_{2x}}\right)\qquad \theta=\tan^{-1}\left(\frac{\sqrt{2}v_i}{3}\frac{3}{2v_i}\right)=\boxed{35.3°}$

P9.33 $m_0=17.0\times10^{-27}\text{ kg}\qquad \vec{\mathbf{v}}_i=0$ (the parent nucleus)

$m_1=5.00\times10^{-27}\text{ kg}\qquad \vec{\mathbf{v}}_1=6.00\times10^6\hat{\mathbf{j}}\text{ m/s}$

$m_2=8.40\times10^{-27}\text{ kg}\qquad \vec{\mathbf{v}}_2=4.00\times10^6\hat{\mathbf{i}}\text{ m/s}$

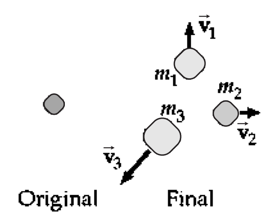

FIG. P9.33

(a) $m_1\vec{\mathbf{v}}_1+m_2\vec{\mathbf{v}}_2+m_3\vec{\mathbf{v}}_3=0$

where $m_3=m_0-m_1-m_2=3.60\times10^{-27}\text{ kg}$

$$\left(5.00\times10^{-27}\right)\left(6.00\times10^6\hat{\mathbf{j}}\right)+\left(8.40\times10^{-27}\right)\left(4.00\times10^6\hat{\mathbf{i}}\right)+\left(3.60\times10^{-27}\right)\vec{\mathbf{v}}_3=0$$

$$\vec{\mathbf{v}}_3=\boxed{\left(-9.33\times10^6\hat{\mathbf{i}}-8.33\times10^6\hat{\mathbf{j}}\right)\text{ m/s}}$$

continued on next page

(b) $E = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2$

$$E = \frac{1}{2}\Big[\left(5.00\times10^{-27}\right)\left(6.00\times10^{6}\right)^2 + \left(8.40\times10^{-27}\right)\left(4.00\times10^{6}\right)^2 + \left(3.60\times10^{-27}\right)\left(12.5\times10^{6}\right)^2\Big]$$

$$\boxed{E = 4.39\times10^{-13}\ \text{J}}$$

P9.34 The initial momentum of the system is 0. Thus,

$$(1.20m)v_{Bi} = m(10.0\ \text{m/s})$$

and

$$v_{Bi} = 8.33\ \text{m/s}$$

$$K_i = \frac{1}{2}m(10.0\ \text{m/s})^2 + \frac{1}{2}(1.20m)(8.33\ \text{m/s})^2 = \frac{1}{2}m\left(183\ \text{m}^2/\text{s}^2\right)$$

$$K_f = \frac{1}{2}m\left(v_G\right)^2 + \frac{1}{2}(1.20m)\left(v_B\right)^2 = \frac{1}{2}\left(\frac{1}{2}m\left(183\ \text{m}^2/\text{s}^2\right)\right)$$

or

$$v_G^2 + 1.20v_B^2 = 91.7\ \text{m}^2/\text{s}^2 \qquad (1)$$

From conservation of momentum,

$$mv_G = (1.20m)v_B$$

or

$$v_G = 1.20v_B \qquad (2)$$

Solving (1) and (2) simultaneously, we find

$$(1.20v_B)^2 + 1.20v_B^2 = 91.7\ \text{m}^2/\text{s}^2 \qquad v_B = (91.7\ \text{m}^2/\text{s}^2 / 2.64)^{1/2}$$

$\boxed{v_B = 5.89\ \text{m/s}}$ (speed of blue puck after collision)

and

$\boxed{v_G = 7.07\ \text{m/s}}$ (speed of green puck after collision)

Section 9.5 **The Center of Mass**

P9.35 The x-coordinate of the center of mass is

$$x_{\text{CM}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{0+0+0+0}{(2.00\ \text{kg} + 3.00\ \text{kg} + 2.50\ \text{kg} + 4.00\ \text{kg})}$$

$$\boxed{x_{\text{CM}} = 0}$$

and the y-coordinate of the center of mass is

$$y_{\text{CM}} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(2.00\ \text{kg})(3.00\ \text{m}) + (3.00\ \text{kg})(2.50\ \text{m}) + (2.50\ \text{kg})(0) + (4.00\ \text{kg})(-0.500\ \text{m})}{2.00\ \text{kg} + 3.00\ \text{kg} + 2.50\ \text{kg} + 4.00\ \text{kg}}$$

$$\boxed{y_{\text{CM}} = 1.00\ \text{m}}$$

P9.36 Let the x axis start at the Earth's center and point toward the Moon.

$$x_{\mathrm{CM}}=\frac{m_1x_1+m_2x_2}{m_1+m_2}=\frac{5.98\times10^{24}\ \mathrm{kg}\ 0+7.36\times10^{22}\ \mathrm{kg}\left(3.84\times10^{8}\ \mathrm{m}\right)}{6.05\times10^{24}\ \mathrm{kg}}$$

$$=\boxed{4.67\times10^{6}\ \mathrm{m\ from\ the\ Earth's\ center}}$$

The center of mass is within the Earth, which has radius 6.37×10^{6} m. It is 1.7 Mm below the point on the Earth's surface where the Moon is straight overhead.

P9.37 Let A_1 represent the area of the bottom row of squares, A_2 the middle square, and A_3 the top pair.

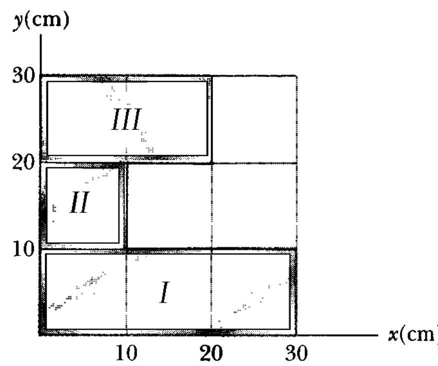

FIG. P9.37

$A=A_1+A_2+A_3$

$M=M_1+M_2+M_3$

$\dfrac{M_1}{A_1}=\dfrac{M}{A}$

$A_1=300\ \mathrm{cm}^2,\ A_2=100\ \mathrm{cm}^2,\ A_3=200\ \mathrm{cm}^2,$
$A=600\ \mathrm{cm}^2$

$$M_1=M\left(\frac{A_1}{A}\right)=\frac{300\ \mathrm{cm}^2}{600\ \mathrm{cm}^2}M=\frac{M}{2}$$

$$M_2=M\left(\frac{A_2}{A}\right)=\frac{100\ \mathrm{cm}^2}{600\ \mathrm{cm}^2}M=\frac{M}{6}$$

$$M_3=M\left(\frac{A_3}{A}\right)=\frac{200\ \mathrm{cm}^2}{600\ \mathrm{cm}^2}M=\frac{M}{3}$$

$$x_{\mathrm{CM}}=\frac{x_1M_1+x_2M_2+x_3M_3}{M}=\frac{15.0\ \mathrm{cm}\left(\frac{1}{2}M\right)+5.00\ \mathrm{cm}\left(\frac{1}{6}M\right)+10.0\ \mathrm{cm}\left(\frac{1}{3}M\right)}{M}$$

$$x_{\mathrm{CM}}=\boxed{11.7\ \mathrm{cm}}$$

$$y_{\mathrm{CM}}=\frac{\frac{1}{2}M(5.00\ \mathrm{cm})+\frac{1}{6}M(15.0\ \mathrm{cm})+\left(\frac{1}{3}M\right)(25.0\ \mathrm{cm})}{M}=13.3\ \mathrm{cm}$$

$$y_{\mathrm{CM}}=\boxed{13.3\ \mathrm{cm}}$$

P9.38 (a) Represent the height of a particle of mass dm within the object as y. Its contribution to the gravitational energy of the object-Earth system is $(dm)gy$. The total gravitational energy is $U_g=\int_{\text{all mass}} gy\,dm=g\int y\,dm$. For the center of mass we have $y_{\mathrm{CM}}=\dfrac{1}{M}\int y\,dm$, so

$U_g=gMy_{\mathrm{CM}}$

(b) The volume of the ramp is $\dfrac{1}{2}(3.6\ \mathrm{m})(15.7\ \mathrm{m})(64.8\ \mathrm{m})=1.83\times10^{3}\ \mathrm{m}^3$. Its mass is

$\rho V=\left(3\,800\ \mathrm{kg/m^3}\right)\left(1.83\times10^{3}\ \mathrm{m}^3\right)=6.96\times10^{6}\ \mathrm{kg}$. As shown in the chapter, its center

of mass is above its base by one-third of its height, $y_{\mathrm{CM}}=\dfrac{1}{3}15.7\ \mathrm{m}=5.23\ \mathrm{m}$. Then

$U_g=Mgy_{\mathrm{CM}}=6.96\times10^{6}\ \mathrm{kg}\left(9.8\ \mathrm{m/s^2}\right)5.23\ \mathrm{m}=\boxed{3.57\times10^{8}\ \mathrm{J}}$

P9.39 This object can be made by wrapping tape around a light stiff uniform rod.

(a) $M = \int_0^{0.300\text{ m}} \lambda dx = \int_0^{0.300\text{ m}} \left[50.0 \text{ g/m} + 20.0x \text{ g/m}^2\right] dx$

$M = \left[50.0x \text{ g/m} + 10.0x^2 \text{ g/m}^2\right]_0^{0.300\text{ m}} = \boxed{15.9 \text{ g}}$

(b) $x_{\text{CM}} = \dfrac{\int_{\text{all mass}} x dm}{M} = \dfrac{1}{M}\int_0^{0.300\text{ m}} \lambda x dx = \dfrac{1}{M}\int_0^{0.300\text{ m}} \left[50.0x \text{ g/m} + 20.0x^2 \text{ g/m}^2\right] dx$

$x_{\text{CM}} = \dfrac{1}{15.9 \text{ g}}\left[25.0x^2 \text{ g/m} + \dfrac{20x^3 \text{ g/m}^2}{3}\right]_0^{0.300\text{ m}} = \boxed{0.153 \text{ m}}$

P9.40 Take the origin at the center of curvature. We have $L = \frac{1}{4}2\pi r$, $r = \frac{2L}{\pi}$. An incremental bit of the rod at angle θ from the x axis has mass given by $\frac{dm}{rd\theta} = \frac{M}{L}$, $dm = \frac{Mr}{L}d\theta$ where we have used the definition of radian measure. Now

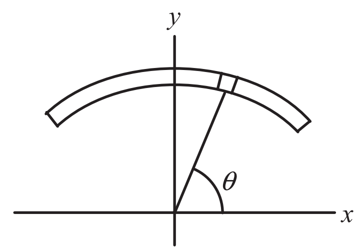

FIG. P9.40

$$y_{\text{CM}} = \frac{1}{M}\int_{\text{all mass}} y\,dm = \frac{1}{M}\int_{\theta=45°}^{135°} r\sin\theta \frac{Mr}{L}d\theta = \frac{r^2}{L}\int_{45°}^{135°} \sin\theta\, d\theta$$

$$= \left(\frac{2L}{\pi}\right)^2 \frac{1}{L}(-\cos\theta)\Big|_{45°}^{135°} = \frac{4L}{\pi^2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{4\sqrt{2}L}{\pi^2}$$

The top of the bar is above the origin by $r = \frac{2L}{\pi}$, so the center of mass is below the middle of the bar by $\frac{2L}{\pi} - \frac{4\sqrt{2}L}{\pi^2} = \frac{2}{\pi}\left(1 - \frac{2\sqrt{2}}{\pi}\right)L = \boxed{0.063\,5L}$.

Section 9.6 **Motion of a System of Particles**

P9.41 (a) $\vec{\mathbf{v}}_{\text{CM}} = \dfrac{\sum m_i \vec{\mathbf{v}}_i}{M} = \dfrac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{M}$

$$= \frac{(2.00 \text{ kg})\left(2.00\hat{\mathbf{i}} \text{ m/s} - 3.00\hat{\mathbf{j}} \text{ m/s}\right) + (3.00 \text{ kg})\left(1.00\hat{\mathbf{i}} \text{ m/s} + 6.00\hat{\mathbf{j}} \text{ m/s}\right)}{5.00 \text{ kg}}$$

$\vec{\mathbf{v}}_{\text{CM}} = \boxed{\left(1.40\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}}\right) \text{ m/s}}$

(b) $\vec{\mathbf{p}} = M\vec{\mathbf{v}}_{\text{CM}} = (5.00 \text{ kg})\left(1.40\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}}\right) \text{ m/s} = \boxed{\left(7.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}}\right) \text{ kg}\cdot\text{m/s}}$

***P9.42** $\vec{\mathbf{r}}_{CM}=\dfrac{m_1\vec{\mathbf{r}}_1+m_2\vec{\mathbf{r}}_2}{m_1+m_2}=\dfrac{3.5\left[(3\hat{\mathbf{i}}+3\hat{\mathbf{j}})t+2\hat{\mathbf{j}}t^2\right]+5.5\left[3\hat{\mathbf{i}}-2\hat{\mathbf{i}}t^2-6\hat{\mathbf{j}}t\right]}{3.5+5.5}$

$$=\left(1.83+1.17t-1.22t^2\right)\hat{\mathbf{i}}+\left(-2.5t+0.778t^2\right)\hat{\mathbf{j}}$$

(a) At $t=2.5$ s, $\vec{\mathbf{r}}_{CM}=(1.83+1.17\cdot 2.5-1.22\cdot 6.25)\hat{\mathbf{i}}+(-2.5\cdot 2.5+0.778\cdot 6.25)\hat{\mathbf{j}}=$

$\boxed{(-2.89\,\hat{\mathbf{i}}-1.39\,\hat{\mathbf{j}})\text{cm}}$ We can conveniently do part (c) on the way to part (b):

$$\vec{\mathbf{v}}_{CM}=\frac{d\vec{\mathbf{r}}_{CM}}{dt}=(1.17-2.44t)\hat{\mathbf{i}}+(-2.5+1.56t)\hat{\mathbf{j}}$$

at $t=2.5$ s, $\vec{\mathbf{v}}_{CM}=(1.17-2.44\cdot 2.5)\hat{\mathbf{i}}+(-2.5+1.56\cdot 2.5)\hat{\mathbf{j}}$

$$=\boxed{(-4.94\,\hat{\mathbf{i}}+1.39\,\hat{\mathbf{j}})\text{ cm/s}}$$

(b) Now the total linear momentum is the total mass times the velocity of the center of mass:

$$(9\text{ g})(-4.94\,\hat{\mathbf{i}}+1.39\,\hat{\mathbf{j}})\text{ cm/s}=\boxed{(-44.5\,\hat{\mathbf{i}}+12.5\,\hat{\mathbf{j}})\text{ g}\cdot\text{cm/s}}$$

(d) Differentiating again, $\vec{\mathbf{a}}_{CM}=\dfrac{d\vec{\mathbf{v}}_{CM}}{dt}=(-2.44)\hat{\mathbf{i}}+1.56\hat{\mathbf{j}}$

The center of mass acceleration is $\boxed{(-2.44\,\hat{\mathbf{i}}+1.56\,\hat{\mathbf{j}})\text{ cm/s}^2}$ at $t=2.5$ s and at all times.

(e) The net force on the system is equal to the total mass times the acceleration of the center of mass:

$$(9\text{ g})(-2.44\,\hat{\mathbf{i}}+1.56\,\hat{\mathbf{j}})\text{ cm/s}^2=\boxed{(-220\,\hat{\mathbf{i}}+140\,\hat{\mathbf{j}})\,\mu\text{N}}$$

P9.43 Let x = distance from shore to center of boat

ℓ = length of boat

x' = distance boat moves as Juliet moves toward Romeo

The center of mass stays fixed.

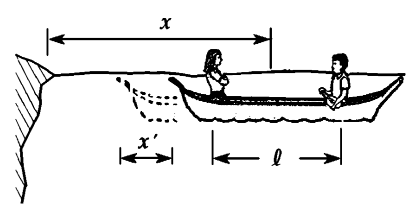

FIG. P9.43

Before: $x_{\text{CM}}=\dfrac{\left[M_b x+M_J\left(x-\frac{\ell}{2}\right)+M_R\left(x+\frac{\ell}{2}\right)\right]}{\left(M_B+M_J+M_R\right)}$

After: $x_{\text{CM}}=\dfrac{\left[M_B(x-x')+M_J\left(x+\frac{\ell}{2}-x'\right)+M_R\left(x+\frac{\ell}{2}-x'\right)\right]}{\left(M_B+M_J+M_R\right)}$

$$\ell\left(-\frac{55.0}{2}+\frac{77.0}{2}\right)=x'(-80.0-55.0-77.0)+\frac{\ell}{2}(55.0+77.0)$$

$$x'=\frac{55.0\ell}{212}=\frac{55.0(2.70)}{212}=\boxed{0.700\text{ m}}$$

P9.44 (a) Conservation of momentum for the two-ball system gives us:

$$0.200\text{ kg}(1.50\text{ m/s})+0.300\text{ kg}(-0.400\text{ m/s})=0.200\text{ kg }v_{1f}+0.300\text{ kg }v_{2f}$$

Relative velocity equation:

$$v_{2f}-v_{1f}=1.90\text{ m/s}$$

Then

$$0.300-0.120=0.200v_{1f}+0.300\left(1.90+v_{1f}\right)$$

$$v_{1f}=-0.780\text{ m/s}\qquad v_{2f}=1.12\text{ m/s}$$

$$\boxed{\vec{\mathbf{v}}_{1f}=-0.780\hat{\mathbf{i}}\text{ m/s}}\qquad\boxed{\vec{\mathbf{v}}_{2f}=1.12\hat{\mathbf{i}}\text{ m/s}}$$

continued on next page

(b) Before,

$$\vec{\mathbf{v}}_{CM} = \frac{(0.200\text{ kg})(1.50\text{ m/s})\hat{\mathbf{i}} + (0.300\text{ kg})(-0.400\text{ m/s})\hat{\mathbf{i}}}{0.500\text{ kg}}$$

$$\boxed{\vec{\mathbf{v}}_{CM} = (0.360\text{ m/s})\hat{\mathbf{i}}}$$

Afterwards, the center of mass must move at the same velocity, because the momentum of the system is conserved.

Section 9.7 **Deformable Systems**

***P9.45** (a) $\boxed{\text{Yes.}}$ The only horizontal force on the vehicle is the frictional force exerted by the floor, so it gives the vehicle all of its final momentum, (6 kg)(3 $\hat{\mathbf{i}}$ m/s) = $\boxed{18.0\ \hat{\mathbf{i}}\ \text{kg}\cdot\text{m/s}}$

(b) $\boxed{\text{No.}}$ The friction force exerted by the floor on each stationary bit of caterpillar tread acts over no distance, so it does zero work.

(c) $\boxed{\text{Yes,}}$ we could say that the final momentum of the cart came from the floor or from the planet through the floor, because the floor imparts impulse.

(d) $\boxed{\text{No.}}$ The floor does no work. The final kinetic energy came from $\boxed{\text{the original gravitational energy of the elevated load}}$, in amount (1/2)(6 kg)(3 m/s)^2 = 27.0 J.

(e) $\boxed{\text{Yes. The acceleration is caused by the static friction force exerted by the floor that prevents the caterpillar tracks from slipping backward.}}$

***P9.46** (a) $\boxed{\text{Yes.}}$ The floor exerts a force, larger than the person's weight over time as he is taking off.

(b) $\boxed{\text{No.}}$ The work by the floor on the person is zero because the force exerted by the floor acts over zero distance.

(c) He leaves the floor with a speed given by $(1/2)mv^2 = mgy_f$
$v = [2(9.8\text{ m/s}^2)0.15\text{ m}]^{1/2} = 1.71$ m/s,
so his momentum immediately after he leaves the floor is (60 kg)(1.71 m/s up) =
$\boxed{103\text{ kg}\cdot\text{m/s up}}$

(d) $\boxed{\text{Yes. You could say that it came from the planet, that gained momentum 103 kg·m/s down, but it came through the force exerted by the floor over a time interval on the person, so it came through the floor or from the floor through direct contact.}}$

(e) (1/2)(60 kg)(1.71 m/s)^2 = $\boxed{88.2\text{ J}}$

(f) $\boxed{\text{No. The energy came from chemical energy in the person's leg muscles. The floor did no work on the person.}}$

***P9.47** (a) When the cart hits the bumper it immediately stops, and the hanging particle keeps moving with its original speed v_i. The particle swings up as a pendulum on a fixed pivot, keeping constant energy. Measure elevations from the pivot:

$(1/2)mv_i^2 + mg(-L) = 0 + mg(-L\cos\theta)$ Then $\boxed{v_i = [2gL(1-\cos\theta)]^{1/2}}$

(b) $v_i = [2gL(1-\cos\theta)]^{1/2} = [2(9.8\text{ m/s}^2)(1.2\text{ m})(1-\cos 35°)]^{1/2} = \boxed{2.06\text{ m/s}}$

(c) $\boxed{\text{Yes.}}$ The bumper must provide the horizontal force to the left to slow down the swing of the particle to the right, to reverse its rightward motion, and to make it speed up to the left.

$\boxed{\text{When the particle passes its straight-down position moving to the left, the bumper stops exerting force.}}$

It is at this moment that the cart-particle system momentarily has zero horizontal acceleration for its center of mass.

***P9.48** Depending on the length of the cord and the time interval Δt for which the force is applied, the sphere may have moved very little when the force is removed, or we may have x_1 and x_2 nearly equal, or the sphere may have swung back, or it may have swung back and forth several times. Our solution applies equally to all of these cases.

(a) The applied force is constant, so the center of mass of the glider-sphere system moves with constant acceleration. It starts, we define, from $x = 0$ and moves to $(x_1 + x_2)/2$. Let v_1 and v_2 represent the horizontal components of velocity of glider and sphere at the moment the force stops. Then the velocity of the center of mass is $v_{CM} = (v_1 + v_2)/2$ and because the acceleration is constant we have $(x_1 + x_2)/2 = [(v_1 + v_2)/2]\Delta t/2$ $\Delta t = 2(x_1 + x_2)/(v_1 + v_2)$

The impulse-momentum theorem for the glider-sphere system is

$F\Delta t = mv_1 + mv_2$ $F2(x_1 + x_2)/(v_1 + v_2) = m(v_1 + v_2)$

$F2(x_1 + x_2)/m = (v_1 + v_2)^2$ $F2(x_1 + x_2)/4m = (v_1 + v_2)^2/4 = v_{CM}^2$

Then $\boxed{v_{CM} = [F(x_1 + x_2)/2m]^{1/2}}$

(b) The applied force does work that becomes, after the force is removed, kinetic energy of the constant-velocity center-of-mass motion plus kinetic energy of the vibration of the glider and sphere relative to their center of mass. The applied force acts only on the glider, so the work-energy theorem for the pushing process is

$Fx_1 = (1/2)(2\,m)\ v_{CM}^2 + E_{vib}$

Substitution gives $Fx_1 = (1/2)(2\,m)F(x_1 + x_2)/2m + E_{vib} = Fx_1/2 + Fx_2/2 + E_{vib}$

Then $E_{vib} = Fx_1/2 - Fx_2/2$

When the cord makes its largest angle with the vertical, the vibrational motion is turning around. No kinetic energy is associated with the vibration at this moment, but only gravitational energy:

$mgL(1-\cos\theta) = F(x_1 - x_2)/2$ Solving gives $\boxed{\theta = \cos^{-1}[1 - F(x_1 - x_2)/2mgL]}$

P9.49 A picture one second later differs by showing five extra kilograms of sand moving on the belt.

(a) $\dfrac{\Delta p_x}{\Delta t} = \dfrac{(5.00\text{ kg})(0.750\text{ m/s})}{1.00\text{ s}} = \boxed{3.75\text{ N}}$

(b) The only horizontal force on the sand is belt friction,

so from $p_{xi} + f\Delta t = p_{xf}$ this is $f = \dfrac{\Delta p_x}{\Delta t} = \boxed{3.75\text{ N}}$

(c) The belt is in equilibrium:

$\sum F_x = ma_x$: $+F_{\text{ext}} - f = 0$ and $F_{\text{ext}} = \boxed{3.75\text{ N}}$

continued on next page

(d) $W = F\Delta r\cos\theta = 3.75\text{ N}(0.750\text{ m})\cos 0° = \boxed{2.81\text{ J}}$

(e) $\frac{1}{2}(\Delta m)v^2 = \frac{1}{2}5.00\text{ kg}(0.750\text{ m/s})^2 = \boxed{1.41\text{ J}}$

(f) One-half of the work input becomes kinetic energy of the moving sand and the other half becomes additional internal energy. The internal energy appears when the sand does not elastically bounce under the hopper, but has friction eliminate its horizontal motion relative to the belt. By contrast, all of the impulse input becomes momentum of the moving sand.

Section 9.8 **Rocket Propulsion**

P9.50 (a) The fuel burns at a rate $\frac{dM}{dt} = \frac{12.7\text{ g}}{1.90\text{ s}} = 6.68\times10^{-3}\text{ kg/s}$

Thrust $= v_e\frac{dM}{dt}$: $5.26\text{ N} = v_e(6.68\times10^{-3}\text{ kg/s})$

$v_e = \boxed{787\text{ m/s}}$

(b) $v_f - v_i = v_e\ln\left(\frac{M_i}{M_f}\right)$: $v_f - 0 = (787\text{ m/s})\ln\left(\frac{53.5\text{ g}+25.5\text{ g}}{53.5\text{ g}+25.5\text{ g}-12.7\text{ g}}\right)$

$v_f = \boxed{138\text{ m/s}}$

P9.51 (a) Thrust $= \left|v_e\frac{dM}{dt}\right|$ Thrust $= (2.60\times10^3\text{ m/s})(1.50\times10^4\text{ kg/s}) = \boxed{3.90\times10^7\text{ N}}$

(b) $\sum F_y = \text{Thrust} - Mg = Ma$: $3.90\times10^7 - (3.00\times10^6)(9.80) = (3.00\times10^6)a$

$a = \boxed{3.20\text{ m/s}^2}$

P9.52 (a) From the equation for rocket propulsion in the text,

$$v - 0 = v_e\ln\left(\frac{M_i}{M_f}\right) = -v_e\ln\left(\frac{M_f}{M_i}\right)$$

Now, $M_f = M_i - kt$, so $v = -v_e\ln\left(\frac{M_i - kt}{M_i}\right) = -v_e\ln\left(1 - \frac{k}{M_i}t\right)$

With the definition $T_p \equiv \frac{M_i}{k}$, this becomes

$$v(t) = \boxed{-v_e\ln\left(1-\frac{t}{T_p}\right)}$$

(b) With $v_e = 1\,500\text{ m/s}$, and $T_p = 144\text{ s}$, $v = -(1\,500\text{ m/s})\ln\left(1 - \frac{t}{144\text{ s}}\right)$

$t(s)$	$v(\text{m/s})$
0	0
20	224
40	488
60	808
80	1220
100	1780
120	2690
132	3730

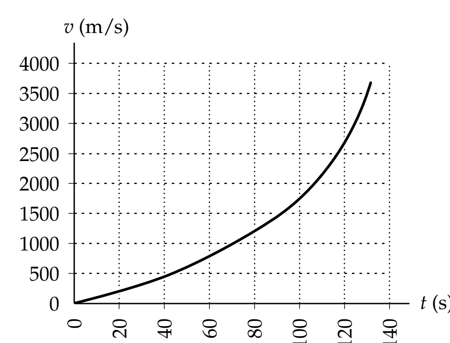

FIG. P9.52(b)

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(c) $a(t)=\dfrac{dv}{dt}=\dfrac{d\left[-v_e\ln\left(1-\frac{t}{T_p}\right)\right]}{dt}=-v_e\left(\dfrac{1}{1-\frac{t}{T_p}}\right)\left(-\dfrac{1}{T_p}\right)=\left(\dfrac{v_e}{T_p}\right)\left(\dfrac{1}{1-\frac{t}{T_p}}\right)$, or

$$a(t)=\boxed{\dfrac{v_e}{T_p-t}}$$

(d) With $v_e=1\,500$ m/s, and $T_p=144$ s, $a=\dfrac{1\,500\text{ m/s}}{144\text{ s}-t}$

$t(s)$	$a\left(\text{m/s}^2\right)$
0	10.4
20	12.1
40	14.4
60	17.9
80	23.4
100	34.1
120	62.5
132	125

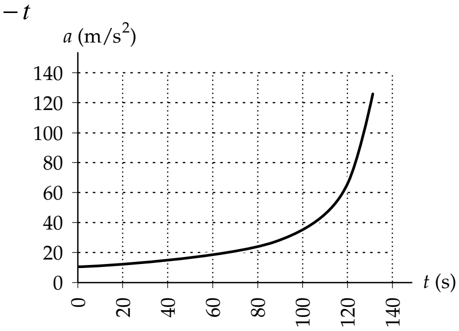

FIG. P9.52(d)

(e) $x(t)=0+\int_0^t v\,dt=\int_0^t\left[-v_e\ln\left(1-\dfrac{t}{T_p}\right)\right]dt=v_eT_p\int_0^t\ln\left[1-\dfrac{t}{T_p}\right]\left(-\dfrac{dt}{T_p}\right)$

$$x(t)=v_eT_p\left[\left(1-\frac{t}{T_p}\right)\ln\left(1-\frac{t}{T_p}\right)-\left(1-\frac{t}{T_p}\right)\right]_0^t$$

$$x(t)=\boxed{v_e\left(T_p-t\right)\ln\left(1-\frac{t}{T_p}\right)+v_et}$$

(f) With $v_e=1\,500$ m/s $=1.50$ km/s, and $T_p=144$ s,

$$x=1.50(144-t)\ln\left(1-\frac{t}{144}\right)+1.50t$$

$t(\text{s})$	$x(\text{km})$
0	0
20	2.19
40	9.23
60	22.1
80	42.2
100	71.7
120	115
132	153

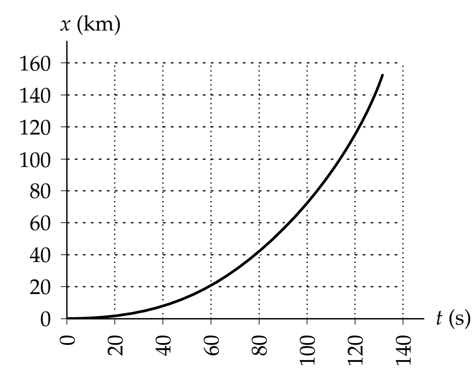

FIG. P9.52(f)

P9.53 In $v = v_e \ln \dfrac{M_i}{M_f}$ we solve for M_i.

(a) $M_i = e^{v/v_e} M_f \qquad M_i = e^5 \left(3.00 \times 10^3 \text{ kg}\right) = 4.45 \times 10^5 \text{ kg}$

The mass of fuel and oxidizer is $\Delta M = M_i - M_f = (445 - 3.00) \times 10^3 \text{ kg} = \boxed{442 \text{ metric tons}}$

(b) $\Delta M = e^2 (3.00 \text{ metric tons}) - 3.00 \text{ metric tons} = \boxed{19.2 \text{ metric tons}}$

This is much less than the suggested value of 442/2.5. Mathematically, the logarithm in the rocket propulsion equation is not a linear function. Physically, a higher exhaust speed has an extra-large cumulative effect on the rocket body's final speed, by counting again and again in the speed the body attains second after second during its burn. Because of the exponential, a relatively small increase in engine efficiency causes a large change in the amount of fuel and oxidizer required.

Additional Problems

P9.54 (a) When the spring is fully compressed, each cart moves with same velocity **v**. Apply conservation of momentum for the system of two gliders

$$\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f: \qquad m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 = (m_1 + m_2)\vec{\mathbf{v}} \qquad \boxed{\vec{\mathbf{v}} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1 + m_2}}$$

(b) Only conservative forces act; therefore $\Delta E = 0$. $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}kx_m^2$
Substitute for v from (a) and solve for x_m.

$$x_m^2 = \frac{(m_1 + m_2)m_1v_1^2 + (m_1 + m_2)m_2v_2^2 - (m_1v_1)^2 - (m_2v_2)^2 - 2m_1m_2v_1v_2}{k(m_1 + m_2)}$$

$$x_m = \sqrt{\frac{m_1m_2\left(v_1^2 + v_2^2 - 2v_1v_2\right)}{k(m_1 + m_2)}} = \boxed{(v_1 - v_2)\sqrt{\frac{m_1m_2}{k(m_1 + m_2)}}}$$

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(c) $m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$

Conservation of momentum: $m_1\left(\vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_{1f}\right) = m_2\left(\vec{\mathbf{v}}_{2f} - \vec{\mathbf{v}}_2\right)$ (1)

Conservation of energy: $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$

which simplifies to: $m_1\left(v_1^2 - v_{1f}^2\right) = m_2\left(v_{2f}^2 - v_2^2\right)$

Factoring gives

$$m_1\left(\vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_{1f}\right)\cdot\left(\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_{1f}\right) = m_2\left(\vec{\mathbf{v}}_{2f} - \vec{\mathbf{v}}_2\right)\cdot\left(\vec{\mathbf{v}}_{2f} + \vec{\mathbf{v}}_2\right)$$

and with the use of the momentum equation (equation (1)),

this reduces to $\left(\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_{1f}\right) = \left(\vec{\mathbf{v}}_{2f} + \vec{\mathbf{v}}_2\right)$

or $\vec{\mathbf{v}}_{1f} = \vec{\mathbf{v}}_{2f} + \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1$ (2)

Substituting equation (2) into equation (1) and simplifying yields:

$$\vec{\mathbf{v}}_{2f} = \boxed{\left(\frac{2m_1}{m_1 + m_2}\right)\vec{\mathbf{v}}_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{\mathbf{v}}_2}$$

Upon substitution of this expression for $\mathbf{v}_{2f}$ into equation 2, one finds

$$\vec{\mathbf{v}}_{1f} = \boxed{\left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{\mathbf{v}}_1 + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{\mathbf{v}}_2}$$

Observe that these results are the same as two equations given in the chapter text for the situation of a perfectly elastic collision in one dimension. Whatever the details of how the spring behaves, this collision ends up being just such a perfectly elastic collision in one dimension.

P9.55 We hope the momentum of the wrench provides enough recoil so that the astronaut can reach the ship before he loses life support! We might expect the elapsed time to be on the order of several minutes based on the description of the situation.

No external force acts on the system (astronaut plus wrench), so the total momentum is constant. Since the final momentum (wrench plus astronaut) must be zero, we have final momentum = initial momentum = 0.

$$m_{\text{wrench}}v_{\text{wrench}} + m_{\text{astronaut}}v_{\text{astronaut}} = 0$$

Thus

$$v_{\text{astronaut}} = -\frac{m_{\text{wrench}}v_{\text{wrench}}}{m_{\text{astronaut}}} = -\frac{(0.500\text{ kg})(20.0\text{ m/s})}{80.0\text{ kg}} = -0.125\text{ m/s}$$

At this speed, the time to travel to the ship is

$$t = \frac{30.0\text{ m}}{0.125\text{ m/s}} = \boxed{240\text{ s}} = 4.00\text{ minutes}$$

The astronaut is fortunate that the wrench gave him sufficient momentum to return to the ship in a reasonable amount of time! In this problem, we did not think of the astronaut as drifting away from the ship when he threw the wrench. However slowly, he must be drifting away since he did not encounter an external force that would reduce his velocity away from the ship. There is no air friction beyond earth's atmosphere. In a real-life situation, the astronaut would have to throw the wrench hard enough to overcome his momentum caused by his original push away from the ship.

***P9.56** Proceeding step by step, we find the real actor's speed just before collision, using energy conservation in the swing-down process: $m_a g y_i = (1/2)\, m_a v_i^2 \qquad [2(9.8 \text{ m/s}^2)(1.8 \text{ m})]^{1/2} = v_i = 5.94$ m/s
Now for the elastic collision with a stationary target we use the specialized equation from the chapter text $v_{2f} = (2\, m_1 v_{1i})/(m_1 + m_2) = 2(80 \text{ kg})(5.94 \text{ m/s})/(80 \text{ kg} + m) = (950 \text{ kg·m/s})/(80 \text{ kg} + m)$
The time for the clone's fall into the ocean is given by

$$\Delta y = v_{yi} t + (1/2) a_y t^2 - 36 \text{ m} = 0 + (1/2)(-9.8 \text{ m/s}^2) t^2 \qquad t = 2.71 \text{ s}$$

so his horizontal range is

$$R = v_{2f}\, t = (2.71 \text{ s})(950 \text{ kg} \cdot \text{m/s})/(80 \text{ kg} + m) = \boxed{2.58 \times 10^3 \text{ kg} \cdot \text{m}/(80 \text{ kg} + m)}$$

(b) By substitution, $2576 \text{ kg} \cdot \text{m}\, (80 \text{ kg} + 79 \text{ kg})^{-1} = \boxed{16.2 \text{ m}}$

(c) A little heavier and he does not go so far: $2576 \text{ kg} \cdot \text{m}\, (80 \text{ kg} + 81 \text{ kg})^{-1} = \boxed{16.0 \text{ m}}$

(d) We solve $30 \text{ m} = 2580 \text{ kg·m}\, (80 \text{ kg} + m)^{-1} \quad 80 \text{ kg} + m = 85.87 \text{ kg} \quad m = \boxed{5.87 \text{ kg}}$

(e) The maximum value for R is $2576/80 = \boxed{32.2 \text{ m}}$, obtained in the limit as

(f) we make m go to $\boxed{\text{zero}}$.

(g) The minimum value of R is approaching $\boxed{\text{zero}}$, obtained in the limit as

(h) we make m go to $\boxed{\text{infinity}}$.

(i) Yes, mechanical energy is conserved until the clone splashes down. This principle is not sufficient to solve the problem. We need also conservation of momentum in the collision.

(j) Yes, but it is not useful to include the planet in the analysis of momentum. We use instead momentum conservation for the actor-clone system while they are in contact.

(k) In symbols we have $v_i = [2\, g\, (1.8 \text{ m})]^{1/2}$
$v_{2f} = 2(80 \text{ kg})\, [2\, g\, (1.8 \text{ m})]^{1/2}/(80 \text{ kg} + m)$
$t = [2(36 \text{ m})/g]^{1/2}$
and $R = [2(36 \text{ m})/g]^{1/2}\, 2(80 \text{ kg})\, [2\, g\, (1.8 \text{ m})]^{1/2}/(80 \text{ kg} + m)$

Here g divides out. At a location with weaker gravity, the actor would be moving more slowly before the collision, but the clone would follow the same trajectory, moving more slowly over a longer time interval.

P9.57 Using conservation of momentum from just before to just after the impact of the bullet with the block:

$$m v_i = (M + m) v_f$$

or

$$v_i = \left(\frac{M + m}{m}\right) v_f \qquad (1)$$

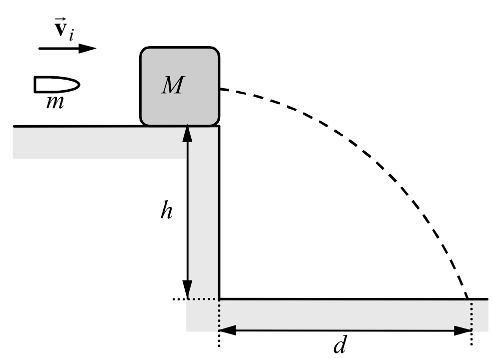

FIG. P9.57

The speed of the block and embedded bullet just after impact may be found using kinematic equations:

$$d = v_f t \text{ and } h = \frac{1}{2} g t^2$$

Thus,

$$t = \sqrt{\frac{2h}{g}} \quad \text{and} \quad v_f = \frac{d}{t} = d\sqrt{\frac{g}{2h}} = \sqrt{\frac{g d^2}{2h}}$$

Substituting into (1) from above gives $v_i = \boxed{\left(\frac{M + m}{m}\right)\sqrt{\frac{g d^2}{2h}}}$.

P9.58 (a) The initial momentum of the system is zero, which remains constant throughout the motion. Therefore, when m_1 leaves the wedge, we must have

$$m_2 v_{\text{wedge}} + m_1 v_{\text{block}} = 0$$

or

$$(3.00\text{ kg})v_{\text{wedge}} + (0.500\text{ kg})(+4.00\text{ m/s}) = 0$$

so

$$v_{\text{wedge}} = \boxed{-0.667\text{ m/s}}$$

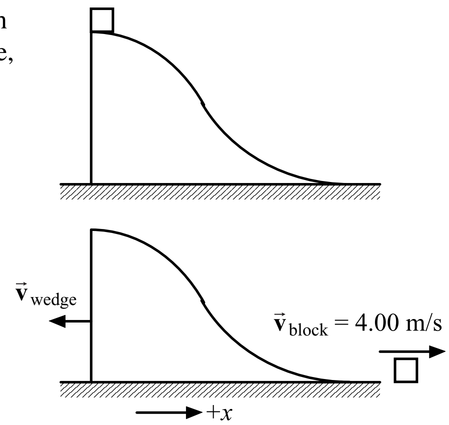

FIG. P9.58

(b) Using conservation of energy for the block-wedge-Earth system as the block slides down the smooth (frictionless) wedge, we have

$$\left[K_{\text{block}} + U_{\text{system}}\right]_i + \left[K_{\text{wedge}}\right]_i = \left[K_{\text{block}} + U_{\text{system}}\right]_f + \left[K_{\text{wedge}}\right]_f$$

or

$$\left[0 + m_1 gh\right] + 0 = \left[\frac{1}{2}m_1(4.00)^2 + 0\right] + \frac{1}{2}m_2(-0.667)^2 \text{ which gives } \boxed{h = 0.952\text{ m}}$$

P9.59 (a) Conservation of momentum:

$$0.5\text{ kg}\left(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 1\hat{\mathbf{k}}\right)\text{ m/s} + 1.5\text{ kg}\left(-1\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}\right)\text{ m/s}$$

$$= 0.5\text{ kg}\left(-1\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 8\hat{\mathbf{k}}\right)\text{ m/s} + 1.5\text{ kg }\vec{\mathbf{v}}_{2f}$$

$$\vec{\mathbf{v}}_{2f} = \frac{\left(-0.5\hat{\mathbf{i}} + 1.5\hat{\mathbf{j}} - 4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s} + \left(0.5\hat{\mathbf{i}} - 1.5\hat{\mathbf{j}} + 4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}}{1.5\text{ kg}} = \boxed{0}$$

The original kinetic energy is

$$\frac{1}{2}0.5\text{ kg}\left(2^2 + 3^2 + 1^2\right)\text{ m}^2/\text{s}^2 + \frac{1}{2}1.5\text{ kg}\left(1^2 + 2^2 + 3^2\right)\text{ m}^2/\text{s}^2 = 14.0\text{ J}$$

The final kinetic energy is $\frac{1}{2}0.5\text{ kg}\left(1^2 + 3^2 + 8^2\right)\text{ m}^2/\text{s}^2 + 0 = 18.5\text{ J}$ different from the original energy so the collision is $\boxed{\text{inelastic}}$.

(b) We follow the same steps as in part (a):

$$\left(-0.5\hat{\mathbf{i}} + 1.5\hat{\mathbf{j}} - 4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s} = 0.5\text{ kg}\left(-0.25\hat{\mathbf{i}} + 0.75\hat{\mathbf{j}} - 2\hat{\mathbf{k}}\right)\text{ m/s} + 1.5\text{ kg }\vec{\mathbf{v}}_{2f}$$

$$\vec{\mathbf{v}}_{2f} = \frac{\left(-0.5\hat{\mathbf{i}} + 1.5\hat{\mathbf{j}} - 4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s} + \left(0.125\hat{\mathbf{i}} - 0.375\hat{\mathbf{j}} + 1\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}}{1.5\text{ kg}}$$

$$= \boxed{\left(-0.250\hat{\mathbf{i}} + 0.750\hat{\mathbf{j}} - 2.00\hat{\mathbf{k}}\right)\text{ m/s}}$$

We see $\vec{\mathbf{v}}_{2f} = \vec{\mathbf{v}}_{1f}$, so the collision is $\boxed{\text{perfectly inelastic}}$.

continued on next page

(c) Conservation of momentum:

$$\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}=0.5\text{ kg}\left(-1\hat{\mathbf{i}}+3\hat{\mathbf{j}}+a\hat{\mathbf{k}}\right)\text{ m/s}+1.5\text{ kg }\vec{\mathbf{v}}_{2f}$$

$$\vec{\mathbf{v}}_{2f}=\frac{\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}+\left(0.5\hat{\mathbf{i}}-1.5\hat{\mathbf{j}}-0.5a\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}}{1.5\text{ kg}}$$

$$=\boxed{(-2.67-0.333a)\hat{\mathbf{k}}\text{ m/s}}$$

Conservation of energy:

$$14.0\text{ J}=\frac{1}{2}0.5\text{ kg}\left(1^2+3^2+a^2\right)\text{ m}^2/\text{s}^2+\frac{1}{2}1.5\text{ kg}(2.67+0.333a)^2\text{ m}^2/\text{s}^2$$

$$=2.5\text{ J}+0.25a^2+5.33\text{ J}+1.33a+0.083\,3a^2$$

$$0=0.333a^2+1.33a-6.167$$

$$a=\frac{-1.33\pm\sqrt{1.33^2-4(0.333)(-6.167)}}{0.667}$$

$a=2.74$ or -6.74. Either value is possible.

with $\boxed{a=2.74}$, $\quad\vec{\mathbf{v}}_{2f}=\left(-2.67-0.333(2.74)\right)\hat{\mathbf{k}}\text{ m/s}=\boxed{-3.58\hat{\mathbf{k}}\text{ m/s}}$

with $\boxed{a=-6.74}$, $\quad\vec{\mathbf{v}}_{2f}=\left(-2.67-0.333(-6.74)\right)\hat{\mathbf{k}}\text{ m/s}=\boxed{-0.419\hat{\mathbf{k}}\text{ m/s}}$

P9.60 Consider the motion of the firefighter during the three intervals: (1) before, (2) during, and (3) after collision with the platform.

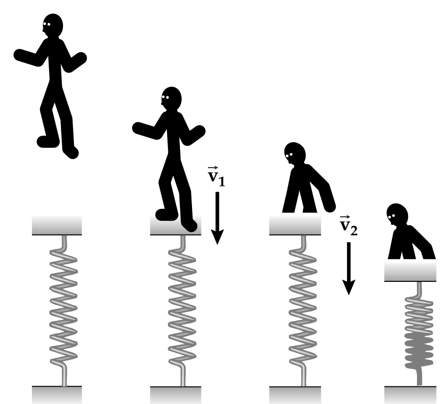

FIG. P9.60

(a) While falling a height of 4.00 m, her speed changes from $v_i=0$ to v_1 as found from

$$\Delta E=\left(K_f+U_f\right)-\left(K_i-U_i\right),\text{ or}$$

$$K_f=\Delta E-U_f+K_i+U_i$$

When the initial position of the platform is taken as the zero level of gravitational potential, we have

$$\frac{1}{2}mv_1^2=fh\cos(180°)-0+0+mgh$$

Solving for v_1 gives

$$v_1=\sqrt{\frac{2(-fh+mgh)}{m}}=\sqrt{\frac{2(-300(4.00)+75.0(9.80)4.00)}{75.0}}=\boxed{6.81\text{ m/s}}$$

continued on next page

(b) During the inelastic collision, momentum is conserved; and if v_2 is the speed of the firefighter and platform just after collision, we have $mv_1 = (m+M)v_2$ or

$$v_2 = \frac{m_1 v_1}{m+M} = \frac{75.0(6.81)}{75.0+20.0} = 5.38 \text{ m/s}$$

Following the collision and again solving for the work done by non-conservative forces, using the distances as labeled in the figure, we have (with the zero level of gravitational potential at the initial position of the platform):

$$\Delta E = K_f + U_{fg} + U_{fs} - K_i - U_{ig} - U_{is}, \text{ or}$$

$$-fs = 0 + (m+M)g(-s) + \frac{1}{2}ks^2 - \frac{1}{2}(m+M)v^2 - 0 - 0$$

This results in a quadratic equation in s:

$$2\,000s^2 - (931)s + 300s - 1\,375 = 0 \text{ or } \boxed{s = 1.00 \text{ m}}$$

P9.61 (a) Each primate swings down according to

$$mgR = \frac{1}{2}mv_1^2 \qquad MgR = \frac{1}{2}Mv_1^2 \qquad v_1 = \sqrt{2gR}$$

The collision: $-mv_1 + Mv_1 = +(m+M)v_2$

$$v_2 = \frac{M-m}{M+m}v_1$$

Swinging up: $\frac{1}{2}(M+m)v_2^2 = (M+m)gR(1-\cos 35°)$

$$v_2 = \sqrt{2gR(1-\cos 35°)}$$

$$\sqrt{2gR(1-\cos 35°)}(M+m) = (M-m)\sqrt{2gR}$$

$$0.425M + 0.425m = M - m$$

$$1.425m = 0.575M$$

$$\boxed{\frac{m}{M} = 0.403}$$

(b) No change is required if the force is different. The nature of the forces within the system of colliding objects does not affect the total momentum of the system. With strong magnetic attraction, the heavier object will be moving somewhat faster and the lighter object faster still. Their extra kinetic energy will all be immediately converted into extra internal energy when the objects latch together. Momentum conservation guarantees that none of the extra kinetic energy remains after the objects join to make them swing higher.

P9.62 (a) Utilizing conservation of momentum,

$$m_1 v_{1A} = (m_1 + m_2) v_B$$

$$v_{1A} = \frac{m_1 + m_2}{m_1}\sqrt{2gh}$$

$$v_{1A} \cong \boxed{6.29 \text{ m/s}}$$

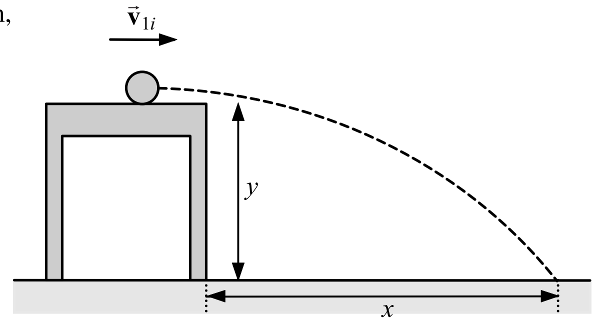

FIG. P9.62

(b) Utilizing the two equations

$$\frac{1}{2}gt^2 = y \text{ and } x = v_{1A}t$$

we combine them to find

$$v_{1A} = \frac{x}{\sqrt{2y/g}} = x\sqrt{\frac{g}{2y}}$$

From the data, $v_{1A} = \boxed{6.16 \text{ m/s}}$

Most of the 2% difference between the values for speed is accounted for by the uncertainty in the data, estimated as $\frac{0.01}{8.68} + \frac{0.1}{68.8} + \frac{1}{263} + \frac{1}{257} + \frac{0.1}{85.3} = 1.1\%$.

***P9.63** (a) In the same symbols as in the text's Example, the original kinetic energy is $K_A = (1/2)m_1 v_{1A}^2$. The example shows that the kinetic energy immediately after latching together is $K_B = (1/2)\, m_1^2 v_{1A}^2/(m_1 + m_2)$ so the fraction of kinetic energy remaining as kinetic energy is $\boxed{K_B/K_A = m_1/(m_1 + m_2)}$

(b) $K_B/K_A = 9.6 \text{ kg}/(9.6 \text{ kg} + 214 \text{ kg}) = \boxed{0.0429}$

(c) Momentum is conserved in the collision so momentum after divided by momentum before is $\boxed{1.00}$.

(d) Energy is an entirely different thing from momentum. A comparison: When a photographer's single-use flashbulb flashes, a magnesium filament oxidizes. Chemical energy disappears. (Internal energy appears and light carries some energy away.) The measured mass of the flashbulb is the same before and after. It can be the same in spite of the 100% energy conversion, because energy and mass are totally different things in classical physics. In the ballistic pendulum, conversion of energy from mechanical into internal does not upset conservation of mass or conservation of momentum.

***P9.64** (a) The mass of the sleigh plus you is 270 kg. Your velocity is 7.50 m/s in the *x* direction. You unbolt a 15.0-kg seat and throw it back at the ravening wolves, giving it a speed of 8.00 m/s relative to you. Find the velocity of the seat relative to the ground after your action, and the velocity of the sleigh.

(b) We substitute $v_{1f} = 8\ \text{m/s} - v_{2f}$

$$270\ \text{kg}(7.5\ \text{m/s}) = 15\ \text{kg}\left(-8\ \text{m/s} + v_{2f}\right) + (255\ \text{kg})v_{2f}$$
$$2\,025\ \text{kg}\cdot\text{m/s} = -120\ \text{kg}\cdot\text{m/s} + (270\ \text{kg})v_{2f}$$
$$v_{2f} = \frac{2\,145\ \text{m/s}}{270} = 7.94\ \text{m/s}$$
$$v_{1f} = 8\ \text{m/s} - 7.94\ \text{m/s} = 0.055\,6\ \text{m/s}$$

$\boxed{\text{The final velocity of the seat is } -0.055\,6\ \text{m/s}\,\hat{\mathbf{i}}\text{. That of the sleigh is } 7.94\ \text{m/s}\,\hat{\mathbf{i}}\text{.}}$

(c) You do work on both the sleigh and the seat, to change their kinetic energy according to

$$K_i + W = K_{1f} + K_{2f}$$
$$\frac{1}{2}(270\ \text{kg})(7.5\ \text{m/s})^2 + W = \frac{1}{2}(15\ \text{kg})(0.055\,6\ \text{m/s})^2 + \frac{1}{2}(255\ \text{kg})(7.94\ \text{m/s})^2$$
$$7\,594\ \text{J} + W = 0.023\,1\ \text{J} + 8\,047\ \text{J}$$
$$W = \boxed{453\ \text{J}}$$

***P9.65** The force exerted by the spring on each block is in magnitude $|F_s| = kx = (3.85\ \text{N/m})(0.08\ \text{m}) = 0.308\ \text{N}$.

(a) With no friction, the elastic energy in the spring becomes kinetic energy of the blocks, which have momenta of equal magnitude in opposite directions. The blocks move with constant speed after they leave the spring.

$$(K+U)_i = (K+U)_f$$

$$\frac{1}{2}kx^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

$$\frac{1}{2}(3.85\ \text{N/m})(0.08\ \text{m})^2 = \frac{1}{2}(0.25\ \text{kg})v_{1f}^2 + \frac{1}{2}(0.50\ \text{kg})v_{2f}^2$$

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

$$0 = (0.25\ \text{kg})v_{1f}\left(-\hat{\mathbf{i}}\right) + (0.50\ \text{kg})v_{2f}\hat{\mathbf{i}}$$

$$v_{1f} = 2v_{2f}$$

$$0.012\,3\ \text{J} = \frac{1}{2}(0.25\ \text{kg})\left(2v_{2f}\right)^2 + \frac{1}{2}(0.50\ \text{kg})v_{2f}^2 = \frac{1}{2}(1.5\ \text{kg})v_{2f}^2$$

$$v_{2f} = \left(\frac{0.123\ \text{J}}{0.75\ \text{kg}}\right)^{1/2} = 0.128\ \text{m/s} \qquad \boxed{\vec{\mathbf{v}}_{2f} = 0.128\ \text{m/s}\,\hat{\mathbf{i}}}$$

$$v_{1f} = 2(0.128\ \text{m/s}) = 0.256\ \text{m/s} \qquad \boxed{\vec{\mathbf{v}}_{1f} = 0.256\ \text{m/s}\left(-\hat{\mathbf{i}}\right)}$$

(b) For the lighter block, $\sum F_y = ma_y$, $\quad n - 0.25\ \text{kg}\left(9.8\ \text{m/s}^2\right) = 0$, $\quad n = 2.45\ \text{N}$, $f_k = \mu_k n = 0.1(2.45\ \text{N}) = 0.245\ \text{N}$. We assume that the maximum force of static friction is a similar size. Since 0.308 N is larger than 0.245 N, this block moves. For the heavier block, the normal force and the frictional force are twice as large: $f_k = 0.490\ \text{N}$. Since 0.308 N is less than this, the heavier block stands still. In this case, the frictional forces exerted by the floor change the momentum of the two-block system. The lighter block will gain speed as long as the spring force is larger than the friction force: that is until the spring compression becomes x_f given by $|F_s| = kx$, $\quad 0.245\ \text{N} = (3.85\ \text{N/m})x_f$, $\quad 0.063\,6\ \text{m} = x_f$. Now for the energy of the lighter block as it moves to this maximum-speed point we have

$$K_i + U_i - f_k d = K_f + U_f$$

$$0 + 0.012\,3\ \text{J} - 0.245\ \text{N}(0.08 - 0.063\,6\ \text{m}) = \frac{1}{2}(0.25\ \text{kg})v_f^2 + \frac{1}{2}(3.85\ \text{N/m})(0.063\,6\ \text{m})^2$$

$$0.012\,3\ \text{J} - 0.004\,01\ \text{J} = \frac{1}{2}(0.25\ \text{kg})v_f^2 + 0.007\,80\ \text{J}$$

$$\left(\frac{2(0.000\,515\ \text{J})}{0.25\ \text{kg}}\right)^{1/2} = v_f = 0.064\,2\ \text{m/s}$$

Thus for the heavier block the maximum velocity is $\boxed{0}$ and for the lighter

$\boxed{0.064\,2\ \text{m/s}\left(-\hat{\mathbf{i}}\right)}$.

(c) For the lighter block, $f_k = 0.462(2.45\ \text{N}) = 1.13\ \text{N}$. The force of static friction must be at least as large. The 0.308-N spring force is too small to produce motion of either block. Each has $\boxed{0}$ maximum speed.

P9.66 The orbital speed of the Earth is

$$v_E = \frac{2\pi r}{T} = \frac{2\pi 1.496\times10^{11}\text{ m}}{3.156\times10^{7}\text{ s}} = 2.98\times10^{4}\text{ m/s}$$

In six months the Earth reverses its direction, to undergo momentum change

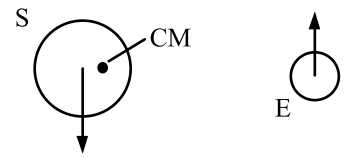

FIG. P9.66

$$m_E\left|\Delta\vec{\mathbf{v}}_E\right| = 2m_E v_E = 2\left(5.98\times10^{24}\text{ kg}\right)\left(2.98\times10^{4}\text{ m/s}\right) = 3.56\times10^{29}\text{ kg}\cdot\text{m/s}$$

Relative to the center of mass, the sun always has momentum of the same magnitude in the opposite direction. Its 6-month momentum change is the same size, $m_S\left|\Delta\vec{\mathbf{v}}_S\right| = 3.56\times10^{29}\text{ kg}\cdot\text{m/s}$.

Then $\left|\Delta\vec{\mathbf{v}}_S\right| = \dfrac{3.56\times10^{29}\text{ kg}\cdot\text{m/s}}{1.991\times10^{30}\text{ kg}} = \boxed{0.179\text{ m/s}}$

P9.67 (a) Find the speed when the bullet emerges from the block by using momentum conservation:

$$mv_i = MV_i + mv$$

The block moves a distance of 5.00 cm. Assume for an approximation that the block quickly reaches its maximum velocity, V_i, and the bullet kept going with a constant velocity, v. The block then compresses the spring and stops.

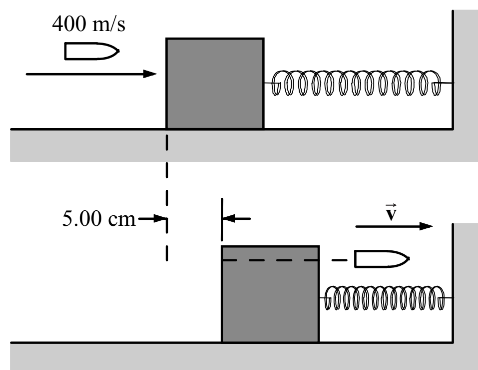

FIG. P9.67

$$\frac{1}{2}MV_i^2 = \frac{1}{2}kx^2$$

$$V_i = \sqrt{\frac{(900\text{ N/m})\left(5.00\times10^{-2}\text{ m}\right)^2}{1.00\text{ kg}}} = 1.50\text{ m/s}$$

$$v = \frac{mv_i - MV_i}{m} = \frac{\left(5.00\times10^{-3}\text{ kg}\right)(400\text{ m/s}) - (1.00\text{ kg})(1.50\text{ m/s})}{5.00\times10^{-3}\text{ kg}}$$

$$v = \boxed{100\text{ m/s}}$$

(b) $$\Delta E = \Delta K + \Delta U = \frac{1}{2}\left(5.00\times10^{-3}\text{ kg}\right)(100\text{ m/s})^2 - \frac{1}{2}\left(5.00\times10^{-3}\text{ kg}\right)(400\text{ m/s})^2 + \frac{1}{2}(900\text{ N/m})\left(5.00\times10^{-2}\text{ m}\right)^2$$

$\Delta E = -374$ J, or there is a mechanical energy loss of $\boxed{374\text{ J}}$.

P9.68 (a) $\vec{\mathbf{p}}_i + \vec{\mathbf{F}}t = \vec{\mathbf{p}}_f$: $\quad (3.00\text{ kg})(7.00\text{ m/s})\hat{\mathbf{j}} + (12.0\text{ N}\hat{\mathbf{i}})(5.00\text{ s}) = (3.00\text{ kg})\vec{\mathbf{v}}_f$

$$\vec{\mathbf{v}}_f = \boxed{(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}})\text{ m/s}}$$

(b) $\vec{\mathbf{a}} = \dfrac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t}$: $\quad \vec{\mathbf{a}} = \dfrac{(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}} - 7.00\hat{\mathbf{j}})\text{ m/s}}{5.00\text{ s}} = \boxed{4.00\hat{\mathbf{i}}\text{ m/s}^2}$

(c) $\vec{\mathbf{a}} = \dfrac{\sum \vec{\mathbf{F}}}{m}$: $\quad \vec{\mathbf{a}} = \dfrac{12.0\text{ N}\hat{\mathbf{i}}}{3.00\text{ kg}} = \boxed{4.00\hat{\mathbf{i}}\text{ m/s}^2}$

(d) $\Delta\vec{\mathbf{r}} = \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$: $\quad \Delta\vec{\mathbf{r}} = (7.00\text{ m/s}\,\hat{\mathbf{j}})(5.00\text{ s}) + \frac{1}{2}(4.00\text{ m/s}^2\,\hat{\mathbf{i}})(5.00\text{ s})^2$

$$\Delta\vec{\mathbf{r}} = \boxed{(50.0\hat{\mathbf{i}} + 35.0\hat{\mathbf{j}})\text{ m}}$$

(e) $W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$: $\quad W = (12.0\text{ N}\hat{\mathbf{i}}) \cdot (50.0\text{ m}\hat{\mathbf{i}} + 35.0\text{ m}\hat{\mathbf{j}}) = \boxed{600\text{ J}}$

(f) $\frac{1}{2}mv_f^2 = \frac{1}{2}(3.00\text{ kg})(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}) \cdot (20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}})\text{ m}^2/\text{s}^2$

$$\frac{1}{2}mv_f^2 = (1.50\text{ kg})(449\text{ m}^2/\text{s}^2) = \boxed{674\text{ J}}$$

(g) $\frac{1}{2}mv_i^2 + W = \frac{1}{2}(3.00\text{ kg})(7.00\text{ m/s})^2 + 600\text{ J} = \boxed{674\text{ J}}$

(h) The accelerations computed in different ways agree. The kinetic energies computed in different ways agree. The three theories are consistent.

P9.69 The force exerted by the table is equal to the change in momentum of each of the links in the chain.

By the calculus chain rule of derivatives,

$$F_1 = \frac{dp}{dt} = \frac{d(mv)}{dt} = v\frac{dm}{dt} + m\frac{dv}{dt}$$

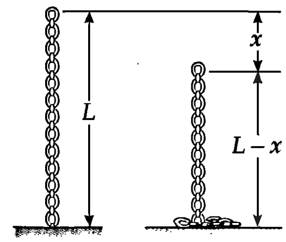

FIG. P9.69

We choose to account for the change in momentum of each link by having it pass from our area of interest just before it hits the table, so that

$$v\frac{dm}{dt} \neq 0 \text{ and } m\frac{dv}{dt} = 0$$

Since the mass per unit length is uniform, we can express each link of length dx as having a mass dm:

$$dm = \frac{M}{L}dx$$

The magnitude of the force on the falling chain is the force that will be necessary to stop each of the elements dm.

$$F_1 = v\frac{dm}{dt} = v\left(\frac{M}{L}\right)\frac{dx}{dt} = \left(\frac{M}{L}\right)v^2$$

After falling a distance x, the square of the velocity of each link $v^2 = 2gx$ (from kinematics), hence

$$F_1 = \frac{2Mgx}{L}$$

The links already on the table have a total length x, and their weight is supported by a force F_2:

$$F_2 = \frac{Mgx}{L}$$

Hence, the *total* force on the chain is

$$F_{\text{total}} = F_1 + F_2 = \boxed{\frac{3Mgx}{L}}$$

That is, *the total force is three times the weight of the chain on the table at that instant*.

ANSWERS TO EVEN PROBLEMS

P9.2 (a) She moves at 4.71 m/s east. (b) 717 J (c) System momentum is conserved with the value zero. The forces on the two siblings are of equal magnitude in opposite directions. Their impulses add to zero. Their final momenta are of equal magnitude in opposite directions.

P9.4 (a) $6.00(-\hat{\mathbf{i}})$ m/s (b) 8.40 J (c) The original energy is in the spring. A force had to be exerted over a distance to compress the spring, transferring energy into it by work. The cord exerts force, but over no distance. (d) System momentum is conserved with the value zero. The forces on the two blocks are of equal magnitude in opposite directions. Their impulses add to zero. The final momenta of the two blocks are of equal magnitude in opposite directions.

P9.6 In trying to hang onto the child, he would have to exert a force of 6.44 kN toward the back of the car, to slow down the child's forward motion. He is not strong enough to exert so large a force. If he were solidly belted in and tied to the child, the child would exert this size force on him toward the front of the car.

P9.8 $1.39\ \text{kg}\cdot\text{m/s}$ upward

P9.10 (a) $5.40\ \text{N}\cdot\text{s}$ toward the net (b) $-27.0\ \text{J}$

P9.12 (a) 981 N·s up (b) 3.43 m/s (c) 3.83 m/s (d) 0.748 m

P9.14 16.5 N

P9.16 (a) $2.50\ \text{m/s}$ (b) $3.75\times10^{4}\ \text{J}$

P9.18 $v=\dfrac{4M}{m}\sqrt{g\ell}$

P9.20 (a) 4.85 m/s (b) 8.41 m

P9.22 7.94 cm

P2.24 (a) $2.24\ \text{m/s}$ toward the right (b) No, coupling order makes no difference

P9.26 (a) Over a very short time interval, outside forces have no time to impart significant impulse—thus the interaction is a collision. The opponent grabs the fullback and does not let go, so the two players move together at the end of their interaction—thus the collision is completely inelastic. (b) 2.88 m/s at 32.3° (c) 783 J becomes internal energy.

P9.28 No; his speed was $41.5\ \text{mi/h}$

P9.30 $v_Y=v_i\sin\theta;\ v_O=v_i\cos\theta$

P9.32 (a) $\sqrt{2}v_i;\ \sqrt{\dfrac{2}{3}}v_i$ (b) 35.3°

P9.34 $v_{Blue}=5.89$ m/s and $v_{Green}=7.07$ m/s

P9.36 4.67×10^{6} m from the Earth's center

P9.38 (a) see the solution (b) $3.57\times10^{8}\ \text{J}$

P9.40 $0.063\,5L$

P9.42 (a) $(-2.89\,\hat{\mathbf{i}}-1.39\,\hat{\mathbf{j}})$cm (b) $(-44.5\,\hat{\mathbf{i}}+12.5\,\hat{\mathbf{j}})$ g·cm/s (c) $(-4.94\,\hat{\mathbf{i}}+1.39\,\hat{\mathbf{j}})$ cm/s (d) $(-2.44\,\hat{\mathbf{i}}+1.56\,\hat{\mathbf{j}})$ cm/s^2 (e) $(-220\,\hat{\mathbf{i}}+140\,\hat{\mathbf{j}})$ μN

P9.44 (a) $-0.780\hat{\mathbf{i}}$ m/s; $1.12\hat{\mathbf{i}}$ m/s (b) $0.360\hat{\mathbf{i}}$ m/s

P9.46 (a) Yes (b) No. The work by the floor on the person is zero. (c) 103 kg·m/s up (d) Yes. You could say that it came from the planet, that gained momentum 103 kg·m/s down, but it came through the force exerted by the floor over a time interval on the person, so it came through the floor or from the floor through direct contact. (e) 88.2 J (f) No. The energy came from chemical energy in the person's leg muscles. The floor did no work on the person.

P9.48 (a) $[F(x_1+x_2)/2m]^{1/2}$ (b) $\cos^{-1}[1-F(x_1-x_2)/2mgL]$

P9.50 (a) $787\ \text{m/s}$ (b) $138\ \text{m/s}$

P9.52 see the solution

P9.54 (a) $\dfrac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1 + m_2}$ (b) $(v_1 - v_2)\sqrt{\dfrac{m_1 m_2}{k(m_1 + m_2)}}$ (c) $\vec{\mathbf{v}}_{1f} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)\vec{\mathbf{v}}_1 + \left(\dfrac{2m_2}{m_1 + m_2}\right)\vec{\mathbf{v}}_2$;

$$\vec{\mathbf{v}}_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{\mathbf{v}}_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{\mathbf{v}}_2$$

P9.56 (a) $R = 2580\ \text{kg} \cdot \text{m}\ (80\ \text{kg} + m)^{-1}$ (b) 16.2 m (c) 16.0 m (d) 5.87 kg (e) 32.2 m (f) $m \to 0$ (g) 0 (h) $m \to \infty$ (i) Yes, until the clone splashes down. No; we need also conservation of momentum in the collision. (j) Yes, but it is not useful to include the planet in the analysis of momentum. We use instead momentum conservation for the actor-clone system while they are in contact. (k) At a location with weaker gravity, the actor would be moving more slowly before the collision, but the clone would follow the same trajectory, moving more slowly over a longer time interval.

P9.58 (a) -0.667 m/s (b) 0.952 m

P9.60 (a) 6.81 m/s (b) 1.00 m

P9.62 (a) 6.29 m/s (b) 6.16 m/s (c) Most of the 2% difference between the values for speed is accounted for by the uncertainty in the data, estimated as $\dfrac{0.01}{8.68} + \dfrac{0.1}{68.8} + \dfrac{1}{263} + \dfrac{1}{257} + \dfrac{0.1}{85.3} = 1.1\%$

P9.64 (a) The mass of the sleigh plus you is 270 kg and your velocity is 7.50 m/s in the x direction. You unbolt a 15.0-kg seat and throw it back at the wolves, giving it a speed of 8.00 m/s relative to you. Find the velocity of the seat relative to the ground afterward, and the velocity of the sleigh afterward. (b) 0.055 6 m/s in the $-x$ direction; 7.94 m/s in the $+x$ direction (c) 453 J

P9.66 0.179 m/s

P9.68 (a) $(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}})$ m/s (b) $4.00\hat{\mathbf{i}}$ m/s^2 (c) $4.00\hat{\mathbf{i}}$ m/s^2 (d) $(50.0\hat{\mathbf{i}} + 35.0\hat{\mathbf{j}})$ m (e) 600 J (f) 674 J (g) 674 J (h) The accelerations computed in different ways agree. The kinetic energies computed in different ways agree. The three theories are consistent.

10

Rotation of a Rigid Object About a Fixed Axis

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q10.1 1 rev/min, or $\frac{\pi}{30}$ rad/s. The direction is horizontally into the wall to represent clockwise rotation. The angular velocity is constant so $\alpha = 0$.

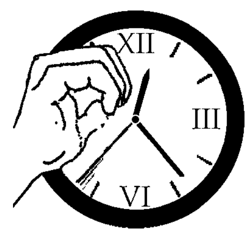

FIG. Q10.1

Q10.2 The vector angular velocity is in the direction $+\hat{\mathbf{k}}$. The vector angular acceleration has the direction $-\hat{\mathbf{k}}$.

***Q10.3** The tangential acceleration has magnitude $(3/\text{s}^2)r$ where r is the radius. It is constant in time. The radial acceleration has magnitude $\omega^2 r$, so it is $(4/\text{s}^2)r$ at the first and last moments mentioned and it is zero at the moment the wheel reverses. Thus we have b = f > a = c = e > d = 0.

***Q10.4** (i) answer (d). The speedometer measures the number of revolutions per second of the tires. A larger tire will travel more distance in one full revolution as $2\pi r$.

(ii) answer (c). If the driver uses the gearshift and the gas pedal to keep the tachometer readings and the air speeds comparable before and after the tire switch, there should be no effect.

***Q10.5** (i) answer (a). Smallest I is about x axis, along which the larger-mass balls lie.

(ii) answer (c). The balls all lie at a distance from the z axis, which is perpendicular to both the x and y axes and passes through the origin.

Q10.6 The object will start to rotate if the two forces act along different lines. Then the torques of the forces will not be equal in magnitude and opposite in direction.

***Q10.7** The accelerations are not equal, but greater in case (a). The string tension above the 5.1-kg object is less than its weight while the object is accelerating down.

Q10.8 You could measure the time that it takes the hanging object, of known mass m, to fall a measured distance after being released from rest. Using this information, the linear acceleration of the mass can be calculated, and then the torque on the rotating object and its angular acceleration.

***Q10.9** answers (a), (b), and (e). The object must rotate with nonzero angular acceleration. The center of mass can be constant in location if it is on the axis of rotation.

Q10.10 You could use $\omega = \alpha t$ and $v = at$. The equation $v = R\omega$ is valid in this situation since $a = R\alpha$.

Q10.11 The angular speed ω would decrease. The center of mass is farther from the pivot, but the moment of inertia increases also.

***Q10.12** answer (f). The sphere of twice the radius has eight times the volume and eight times the mass. Then r^2 in $I = (2/5)\,mr^2$ also gets four times larger.

Q10.13 The moment of inertia depends on the distribution of mass with respect to a given axis. If the axis is changed, then each bit of mass that makes up the object is a different distance from the axis. In an example in section 10.5 in the text, the moment of inertia of a uniform rigid rod about an axis perpendicular to the rod and passing through the center of mass is derived. If you spin a pencil back and forth about this axis, you will get a feeling for its stubbornness against changing rotation. Now change the axis about which you rotate it by spinning it back and forth about the axis that goes down the middle of the graphite. Easier, isn't it? The moment of inertia about the graphite is much smaller, as the mass of the pencil is concentrated near this axis.

Q10.14 A quick flip will set the hard–boiled egg spinning faster and more smoothly. Inside the raw egg, the yolk takes some time to start rotating. The raw egg also loses mechanical energy to internal fluid friction.

Q10.15 Sewer pipe: $I_{CM} = MR^2$. Embroidery hoop: $I_{CM} = MR^2$. Door: $I = \frac{1}{3}MR^2$. Coin: $I_{CM} = \frac{1}{2}MR^2$. The distribution of mass along lines parallel to the axis makes no difference to the moment of inertia.

Q10.16 Yes. If you drop an object, it will gain translational kinetic energy from decreasing gravitational potential energy.

Q10.17 No, just as an object need not be moving to have mass.

Q10.18 No, only if its angular velocity changes.

***Q10.19** (i) answer (c). It is no longer speeding up and not yet slowing down.

(ii) answer (b). It is reversing its angular velocity from positive to negative, and reversal counts as a change.

Q10.20 The moment of inertia would decrease. Matter would be moved toward the axis. This would result in a higher angular speed of the Earth, shorter days, and more days in the year!

***Q10.21** (i) answer (a). The basketball has rotational as well as translational kinetic energy.

(ii) answer (c). The motions of their centers of mass are identical.

(iii) answer (a). The kinetic energy controls the gravitational energy it attains.

Q10.22 There is very little resistance to motion that can reduce the kinetic energy of the rolling ball. Even though there is static friction between the ball and the floor (if there were none, then no rotation would occur and the ball would slide), there is no relative motion of the two surfaces—by the definition of "rolling without slipping"—and so no force of kinetic friction acts to reduce K. Air resistance and friction associated with deformation of the ball eventually stop the ball.

Q10.23 The sphere would reach the bottom first; the hoop would reach the bottom last. First imagine that each object has the same mass and the same radius. Then they all have the same torque due to gravity acting on them. The one with the smallest moment of inertia will thus have the largest angular acceleration and reach the bottom of the plane first. But the mass and the radius divide out in the equation about conversion of gravitational energy to total kinetic energy. This experiment is a test about the numerical factor in the tabulated formula relating the moment of inertia to the mass and radius.

***Q10.24** (a) The tricycle rolls forward. (b) The tricycle rolls forward. (c) The tricycle rolls backward. (d) The tricycle does not roll, but may skid forward. (e) The tricycle rolls backward.
To answer these questions, think about the torque of the string tension about an axis at the bottom of the wheel, where the rubber meets the road. This is the instantaneous axis of rotation in rolling. Cords a and b produce clockwise torques about this axis. Cords c and e produce counter clockwise torques. Cord d has zero lever arm.

SOLUTIONS TO PROBLEMS

Section 10.1 Angular Position, Velocity, and Acceleration

P10.1 (a) $\theta|_{t=0} = \boxed{5.00 \text{ rad}}$

$$\omega|_{t=0} = \left.\frac{d\theta}{dt}\right|_{t=0} = 10.0 + 4.00t|_{t=0} = \boxed{10.0 \text{ rad/s}}$$

$$\alpha_{t=0} = \left.\frac{d\omega}{dt}\right|_{t=0} = \boxed{4.00 \text{ rad/s}^2}$$

(b) $\theta|_{t=3.00 \text{ s}} = 5.00 + 30.0 + 18.0 = \boxed{53.0 \text{ rad}}$

$$\omega|_{t=3.00 \text{ s}} = \left.\frac{d\theta}{dt}\right|_{t=3.00 \text{ s}} = 10.0 + 4.00t|_{t=3.00 \text{ s}} = \boxed{22.0 \text{ rad/s}}$$

$$\alpha|_{t=3.00 \text{ s}} = \left.\frac{d\omega}{dt}\right|_{t=3.00 \text{ s}} = \boxed{4.00 \text{ rad/s}^2}$$

***P10.2** $\alpha = \frac{d\omega}{dt} = 10 + 6t \qquad \int_0^\omega d\omega = \int_0^t (10 + 6t)dt \qquad \omega - 0 = 10t + 6t^2/2$

$\omega = \frac{d\theta}{dt} = 10t + 3t^2 \qquad \int_0^\theta d\theta = \int_0^t (10t + 3t^2)dt \qquad \theta - 0 = 10t^2/2 + 3t^3/3$

$\theta = 5t^2 + t^3$. At $t = 4$ s, $\theta = 5(4)^2 + (4)^3 = \boxed{144 \text{ rad}}$

Section 10.2 Rotational Kinematics: Rotational Motion with Constant Angular Acceleration

P10.3 (a) $\alpha = \frac{\omega - \omega_i}{t} = \frac{12.0 \text{ rad/s}}{3.00 \text{ s}} = \boxed{4.00 \text{ rad/s}^2}$

(b) $\theta = \omega_i t + \frac{1}{2}\alpha t^2 = \frac{1}{2}(4.00 \text{ rad/s}^2)(3.00 \text{ s})^2 = \boxed{18.0 \text{ rad}}$

P10.4 $\omega_i = 3\,600 \text{ rev/min} = 3.77 \times 10^2 \text{ rad/s}$

$\theta = 50.0 \text{ rev} = 3.14 \times 10^2 \text{ rad and } \omega_f = 0$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$0 = (3.77 \times 10^2 \text{ rad/s})^2 + 2\alpha(3.14 \times 10^2 \text{ rad})$

$\alpha = \boxed{-2.26 \times 10^2 \text{ rad/s}^2}$

P10.5 $\omega_i = \dfrac{100\text{ rev}}{1.00\text{ min}}\left(\dfrac{1\text{ min}}{60.0\text{ s}}\right)\left(\dfrac{2\pi\text{ rad}}{1.00\text{ rev}}\right) = \dfrac{10\pi}{3}\text{ rad/s},\ \omega_f = 0$

(a) $t = \dfrac{\omega_f - \omega_i}{\alpha} = \dfrac{0 - (10\pi/3)}{-2.00}s = \boxed{5.24\text{ s}}$

(b) $\theta_f = \bar{\omega}t = \left(\dfrac{\omega_f + \omega_i}{2}\right)t = \left(\dfrac{10\pi}{6}\text{ rad/s}\right)\left(\dfrac{10\pi}{6}\text{ s}\right) = \boxed{27.4\text{ rad}}$

P10.6 $\theta_f - \theta_i = \omega_i t + \dfrac{1}{2}\alpha t^2$ and $\omega_f = \omega_i + \alpha t$ are two equations in two unknowns ω_i and α

$\omega_i = \omega_f - \alpha t$: $\qquad \theta_f - \theta_i = (\omega_f - \alpha t)t + \dfrac{1}{2}\alpha t^2 = \omega_f t - \dfrac{1}{2}\alpha t^2$

$$37.0\text{ rev}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 98.0\text{ rad/s}(3.00\text{ s}) - \frac{1}{2}\alpha(3.00\text{ s})^2$$

$232\text{ rad} = 294\text{ rad} - (4.50\text{ s}^2)\alpha$: $\qquad \alpha = \dfrac{61.5\text{ rad}}{4.50\text{ s}^2} = \boxed{13.7\text{ rad/s}^2}$

P10.7 (a) $\omega = \dfrac{\Delta\theta}{\Delta t} = \dfrac{1\text{ rev}}{1\text{ day}} = \dfrac{2\pi\text{ rad}}{86\,400\text{ s}} = \boxed{7.27\times10^{-5}\text{ rad/s}}$

(b) $\Delta t = \dfrac{\Delta\theta}{\omega} = \dfrac{107°}{7.27\times10^{-5}\text{ rad/s}}\left(\dfrac{2\pi\text{ rad}}{360°}\right) = \boxed{2.57\times10^4\text{ s}}$ or 428 min

P10.8 The location of the dog is described by $\theta_d = (0.750\text{ rad/s})t$. For the bone,

$$\theta_b = \frac{1}{3}2\pi\text{ rad} + \frac{1}{2}0.015\text{ rad/s}^2 t^2$$

We look for a solution to

$$0.75t = \frac{2\pi}{3} + 0.007\,5t^2$$
$$0 = 0.007\,5t^2 - 0.75t + 2.09 = 0$$
$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4(0.007\,5)2.09}}{0.015} = 2.88\text{ s or }97.1\text{ s}$$

The dog and bone will also pass if $0.75t = \dfrac{2\pi}{3} - 2\pi + 0.007\,5t^2$ or if $0.75t = \dfrac{2\pi}{3} + 2\pi + 0.007\,5t^2$ that is, if either the dog or the turntable gains a lap on the other. The first equation has

$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4(0.007\,5)(-4.19)}}{0.015} = 105\text{ s or } -5.30\text{ s}$$

only one positive root representing a physical answer. The second equation has

$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4(0.007\,5)8.38}}{0.015} = 12.8\text{ s or }87.2\text{ s}$$

In order, the dog passes the bone at $\boxed{2.88\text{ s}}$ after the merry-go-round starts to turn, and again at $\boxed{12.8\text{ s}}$ and 26.6 s, after gaining laps on the bone. The bone passes the dog at 73.4 s, 87.2 s, 97.1 s, 105 s, and so on, after the start.

P10.9 $\omega = 5.00 \text{ rev/s} = 10.0\pi \text{ rad/s}$. We will break the motion into two stages: (1) a period during which the tub speeds up and (2) a period during which it slows down.

While speeding up, $\theta_1 = \bar{\omega}t = \dfrac{0 + 10.0\pi \text{ rad/s}}{2}(8.00 \text{ s}) = 40.0\pi \text{ rad}$

While slowing down, $\theta_2 = \bar{\omega}t = \dfrac{10.0\pi \text{ rad/s} + 0}{2}(12.0 \text{ s}) = 60.0\pi \text{ rad}$

So, $\theta_{\text{total}} = \theta_1 + \theta_2 = 100\pi \text{ rad} = \boxed{50.0 \text{ rev}}$

Section 10.3 **Angular and Translational Quantities**

P10.10 (a) $v = r\omega;\ \omega = \dfrac{v}{r} = \dfrac{45.0 \text{ m/s}}{250 \text{ m}} = \boxed{0.180 \text{ rad/s}}$

(b) $a_r = \dfrac{v^2}{r} = \dfrac{(45.0 \text{ m/s})^2}{250 \text{ m}} = \boxed{8.10 \text{ m/s}^2 \text{ toward the center of track}}$

P10.11 Estimate the tire's radius at 0.250 m and miles driven as 10 000 per year.

$$\theta = \frac{s}{r} = \frac{1.00 \times 10^4 \text{ mi}}{0.250 \text{ m}}\left(\frac{1\,609 \text{ m}}{1 \text{ mi}}\right) = 6.44 \times 10^7 \text{ rad/yr}$$

$$\theta = 6.44 \times 10^7 \text{ rad/yr}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 1.02 \times 10^7 \text{ rev/yr or } \boxed{\sim 10^7 \text{ rev/yr}}$$

P10.12 (a) Consider a tooth on the front sprocket. It gives this speed, relative to the frame, to the link of the chain it engages:

$$v = r\omega = \left(\frac{0.152 \text{ m}}{2}\right)76 \text{ rev/min}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = \boxed{0.605 \text{ m/s}}$$

(b) Consider the chain link engaging a tooth on the rear sprocket:

$$\omega = \frac{v}{r} = \frac{0.605 \text{ m/s}}{(0.07 \text{ m})/2} = \boxed{17.3 \text{ rad/s}}$$

(c) Consider the wheel tread and the road. A thread could be unwinding from the tire with this speed relative to the frame:

$$v = r\omega = \left(\frac{0.673 \text{ m}}{2}\right)17.3 \text{ rad/s} = \boxed{5.82 \text{ m/s}}$$

(d) $\boxed{\text{We did not need to know the length of the pedal cranks}}$, but we could use that information to find the linear speed of the pedals:

$$v = r\omega = 0.175 \text{ m } 7.96 \text{ rad/s}\left(\frac{1}{1 \text{ rad}}\right) = 1.39 \text{ m/s}$$

P10.13 Given $r = 1.00\text{ m}, \alpha = 4.00\ \text{rad/s}^2, \omega_i = 0$ and $\theta_i = 57.3° = 1.00$ rad

(a) $\omega_f = \omega_i + \alpha t = 0 + \alpha t$

At $t = 2.00$ s, $\omega_f = 4.00\ \text{rad/s}^2(2.00\text{ s}) = \boxed{8.00\ \text{rad/s}}$

(b) $v = r\omega = 1.00\text{ m}(8.00\ \text{rad/s}) = \boxed{8.00\ \text{m/s}}$

$|a_r| = a_c = r\omega^2 = 1.00\text{ m}(8.00\ \text{rad/s})^2 = 64.0\ \text{m/s}^2$

$a_t = r\alpha = 1.00\text{ m}(4.00\ \text{rad/s}^2) = 4.00\ \text{m/s}^2$

The magnitude of the total acceleration is:

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(64.0\ \text{m/s}^2)^2 + (4.00\ \text{m/s}^2)^2} = \boxed{64.1\ \text{m/s}^2}$$

The direction of the total acceleration vector makes an angle ϕ with respect to the radius to point P:

$$\phi = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\left(\frac{4.00}{64.0}\right) = \boxed{3.58°}$$

(c) $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 = (1.00\text{ rad}) + \frac{1}{2}(4.00\ \text{rad/s}^2)(2.00\text{ s})^2 = \boxed{9.00\text{ rad}}$

P10.14 (a) $\omega = \frac{v}{r} = \frac{25.0\ \text{m/s}}{1.00\text{ m}} = \boxed{25.0\ \text{rad/s}}$

(b) $\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta)$

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2(\Delta\theta)} = \frac{(25.0\ \text{rad/s})^2 - 0}{2\left[(1.25\text{ rev})(2\pi\ \text{rad/rev})\right]} = \boxed{39.8\ \text{rad/s}^2}$$

(c) $\Delta t = \frac{\Delta\omega}{\alpha} = \frac{25.0\ \text{rad/s}}{39.8\ \text{rad/s}^2} = \boxed{0.628\text{ s}}$

***P10.15** The object starts with $\theta_i = 0$. As far as location on the circle and instantaneous motion is concerned, we can think of its final position as 9 rad – 2π = 2.72 rad = 156°.

(a) Its position vector is 3.00 m at 156° = 3 m cos 156° $\hat{\mathbf{i}}$ + 3 m sin 156° $\hat{\mathbf{j}}$ = $\boxed{(-2.73\,\hat{\mathbf{i}} + 1.24\,\hat{\mathbf{j}})\text{ m}}$

(b) $\boxed{\text{It is in the second quadrant, at 156°}}$

(c) Its original velocity is 4.5 m/s at 90°. After the displacement, its velocity is

4.5 m/s at (90° + 156°) = 4.5 m/s at 246°

$= \boxed{(-1.85\,\hat{\mathbf{i}} - 4.10\,\hat{\mathbf{j}})\text{ m/s}}$

(d) $\boxed{\text{It is moving toward the third quadrant, at 246°}}$

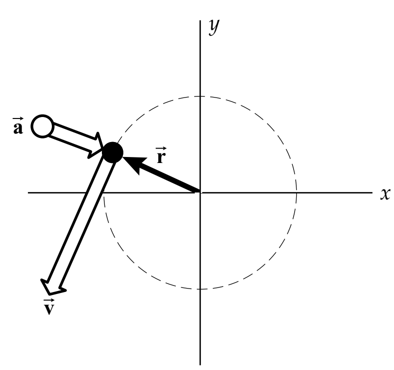

FIG. 10.15d

continued on next page

(e) Its acceleration is v^2/r opposite in direction to its position vector.

This is $(4.5 \text{ m/s})^2/3\text{m}$ at $(156° + 180°) = 6.75 \text{ m/s}^2$ at $336° = \boxed{(6.15\,\hat{\mathbf{i}} - 2.78\,\hat{\mathbf{j}}) \text{ m/s}^2}$

(f) The total force is given by $ma = 4 \text{ kg } (6.15\,\hat{\mathbf{i}} - 2.78\,\hat{\mathbf{j}}) \text{ m/s}^2 = \boxed{(24.6\,\hat{\mathbf{i}} - 11.1\,\hat{\mathbf{j}}) \text{ N}}$

P10.16 (a) $s = \bar{v}t = (11.0 \text{ m/s})(9.00 \text{ s}) = 99.0 \text{ m}$

$$\theta = \frac{s}{r} = \frac{99.0 \text{ m}}{0.290 \text{ m}} = 341 \text{ rad} = \boxed{54.3 \text{ rev}}$$

(b) $$\omega_f = \frac{v_f}{r} = \frac{22.0 \text{ m/s}}{0.290 \text{ m}} = 75.9 \text{ rad/s} = \boxed{12.1 \text{ rev/s}}$$

P10.17 (a) $$\omega = 2\pi f = \frac{2\pi \text{ rad}}{1 \text{ rev}}\left(\frac{1\,200 \text{ rev}}{60.0 \text{ s}}\right) = \boxed{126 \text{ rad/s}}$$

(b) $v = \omega r = (126 \text{ rad/s})(3.00 \times 10^{-2} \text{ m}) = \boxed{3.77 \text{ m/s}}$

(c) $a_c = \omega^2 r = (126)^2 (8.00 \times 10^{-2}) = 1\,260 \text{ m/s}^2$ so $\mathbf{a}_r = \boxed{1.26 \text{ km/s}^2 \text{ toward the center}}$

(d) $s = r\theta = \omega r t = (126 \text{ rad/s})(8.00 \times 10^{-2} \text{ m})(2.00 \text{ s}) = \boxed{20.1 \text{ m}}$

***P10.18** An object of any shape can rotate. The ladder undergoes pure rotation about its right foot. Its angular displacement in radians is $\theta = s/r = 0.690 \text{ m}/4.90 \text{ m} = t/0.410 \text{ m}$ where t is the thickness of the rock. Solving gives (a) $\boxed{5.77 \text{ cm}}$.

$\boxed{\text{(b) Yes. We used the idea of rotational motion measured by angular displacement in the solution.}}$

P10.19 The force of static friction must act forward and then more and more inward on the tires, to produce both tangential and centripetal acceleration. Its tangential component is $m(1.70 \text{ m/s}^2)$.

Its radially inward component is $\frac{mv^2}{r}$. This takes the maximum value

$$m\omega_f^2 r = mr(\omega_i^2 + 2\alpha\Delta\theta) = mr\left(0 + 2\alpha\frac{\pi}{2}\right) = m\pi r\alpha = m\pi a_t = m\pi(1.70 \text{ m/s}^2)$$

With skidding impending we have $\sum F_y = ma_y, \quad +n - mg = 0, \quad n = mg$

$$f_s = \mu_s n = \mu_s mg = \sqrt{m^2(1.70 \text{ m/s}^2)^2 + m^2\pi^2(1.70 \text{ m/s}^2)^2}$$

$$\mu_s = \frac{1.70 \text{ m/s}^2}{g}\sqrt{1+\pi^2} = \boxed{0.572}$$

***P10.20** (a) If we number the loops of the spiral track with an index n, with the innermost loop having $n = 0$, the radii of subsequent loops as we move outward on the disc is given by $r = r_i + hn$. Along a given radial line, each new loop is reached by rotating the disc through 2π rad. Therefore, the ratio $\theta/2\pi$ is the number of revolutions of the disc to get to a certain loop. This is also the number of that loop, so $n = \theta/2\pi$. Therefore, $r = r_i + h\theta/2\pi$.

(b) Starting from $\omega = v/r$, we substitute the definition of angular speed on the left and the result for r from part (a) on the right:

$$\omega = \frac{v}{r} \quad \rightarrow \quad \frac{d\theta}{dt} = \frac{v}{r_i + \dfrac{h}{2\pi}\theta}$$

continued on next page

(c) Rearrange terms in preparation for integrating both sides:

$$\left(r_i + \frac{h}{2\pi}\theta\right)d\theta = v\,dt$$

and integrate from $\theta = 0$ to $\theta = \theta$ and from $t = 0$ to $t = t$:

$$r_i\theta + \frac{h}{4\pi}\theta^2 = vt$$

We rearrange this equation to form a standard quadratic equation in θ:

$$\frac{h}{4\pi}\theta^2 + r_i\theta - vt = 0$$

The solution to this equation is

$$\theta = \frac{-r_i \pm \sqrt{r_i^2 + \frac{h}{\pi}vt}}{\frac{h}{2\pi}} = \boxed{\frac{2\pi r_i}{h}\left(\sqrt{1+\frac{vh}{\pi r_i^2}t} - 1\right)}$$

where we have chosen the positive root in order to make the angle θ positive.

(d) We differentiate the result in (c) twice with respect to time to find the angular acceleration, resulting in

$$\boxed{\alpha = -\frac{hv^2}{2\pi r_i^3\left(1+\frac{vh}{\pi r_i^2}t\right)^{3/2}}}$$

Because this expression involves the time t, the angular acceleration is not constant.

Section 10.4 **Rotational Energy**

P10.21 (a) $I = \sum_j m_j r_j^2$

In this case,

$$r_1 = r_2 = r_3 = r_4$$

$$r = \sqrt{(3.00\text{ m})^2 + (2.00\text{ m})^2} = \sqrt{13.0}\text{ m}$$

$$I = \left[\sqrt{13.0}\text{ m}\right]^2[3.00 + 2.00 + 2.00 + 4.00]\text{ kg}$$

$$= \boxed{143\text{ kg}\cdot\text{m}^2}$$

(b) $K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(143\text{ kg}\cdot\text{m}^2\right)\left(6.00\text{ rad/s}\right)^2$

$$= \boxed{2.57\times 10^3\text{ J}}$$

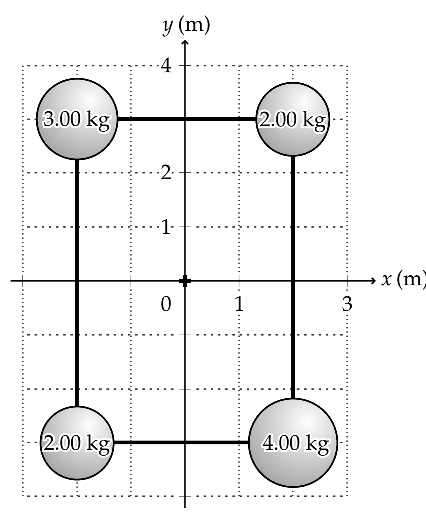

FIG. P10.21

***P10.22** $m_1 = 4.00\text{ kg}, r_1 = |y_1| = 3.00\text{ m}$

$m_2 = 2.00\text{ kg}, r_2 = |y_2| = 2.00\text{ m}$

$m_3 = 3.00\text{ kg}, \ r_3 = |y_3| = 4.00\text{ m}$

$\omega = 2.00\text{ rad/s}$ about the x-axis

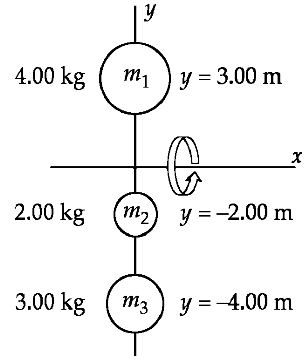

FIG. P10.22

(a) $I_x = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$

$$I_x = 4.00(3.00)^2 + 2.00(2.00)^2 + 3.00(4.00)^2 = \boxed{92.0\text{ kg}\cdot\text{m}^2}$$

$$K_R = \frac{1}{2} I_x \omega^2 = \frac{1}{2}(92.0)(2.00)^2 = \boxed{184\text{ J}}$$

(b) $v_1 = r_1\omega = 3.00(2.00) = \boxed{6.00\text{ m/s}}$ $\qquad K_1 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2}(4.00)(6.00)^2 = 72.0\text{ J}$

$v_2 = r_2\omega = 2.00(2.00) = \boxed{4.00\text{ m/s}}$ $\qquad K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2}(2.00)(4.00)^2 = 16.0\text{ J}$

$v_3 = r_3\omega = 4.00(2.00) = \boxed{8.00\text{ m/s}}$ $\qquad K_3 = \frac{1}{2} m_3 v_3^2 = \frac{1}{2}(3.00)(8.00)^2 = 96.0\text{ J}$

$$K = K_1 + K_2 + K_3 = 72.0 + 16.0 + 96.0 = \boxed{184\text{ J}} = \frac{1}{2} I_x \omega^2$$

(c) $\boxed{\text{The kinetic energies computed in parts (a) and (b) are the same.}}$ Rotational kinetic energy can be viewed as the total translational kinetic energy of the particles in the rotating object.

P10.23 $I = Mx^2 + m(L-x)^2$

$\frac{dI}{dx} = 2Mx - 2m(L-x) = 0$ (for an extremum)

$\therefore x = \frac{mL}{M+m}$

$\frac{d^2I}{dx^2} = 2m + 2M$; therefore I is at a minimum when the axis of rotation passes through $x = \frac{mL}{M+m}$ which is also the center of mass of the system. The moment of inertia about an axis passing through x is

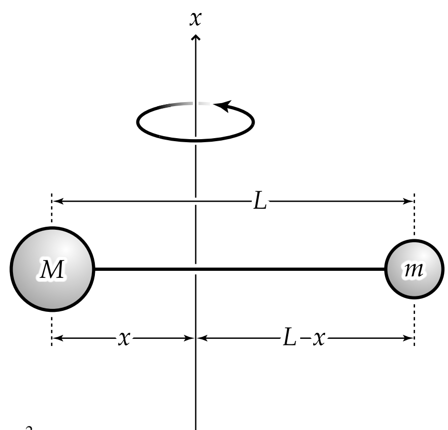

FIG. P10.23

$$I_{\text{CM}} = M\left[\frac{mL}{M+m}\right]^2 + m\left[1 - \frac{m}{M+m}\right]^2 L^2 = \frac{Mm}{M+m} L^2 = \mu L^2$$

where

$$\mu = \frac{Mm}{M+m}$$

***P10.24** For large energy storage at a particular rotation rate, we want a large moment of inertia. To combine this requirement with small mass, we place the mass as far away from the axis as possible.

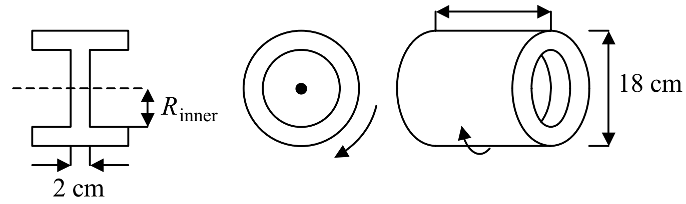

FIG. P10.24

We choose to make the flywheel as a hollow cylinder 18 cm in diameter and 8 cm long. To support this rim, we place a disk across its center. We assume that a disk 2 cm thick will be sturdy enough to support the hollow cylinder securely.

The one remaining adjustable parameter is the thickness of the wall of the hollow cylinder. From Table 10.2, the moment of inertia can be written as

$$\begin{aligned}
I_{\text{disk}} + I_{\text{hollow cylinder}} &= \frac{1}{2}M_{\text{disk}}R_{\text{disk}}^2 + \frac{1}{2}M_{\text{wall}}\left(R_{\text{outer}}^2 + R_{\text{inner}}^2\right) \\
&= \frac{1}{2}\rho V_{\text{disk}}R_{\text{outer}}^2 + \frac{1}{2}\rho V_{\text{wall}}\left(R_{\text{outer}}^2 + R_{\text{inner}}^2\right) \\
&= \frac{\rho}{2}\pi R_{\text{outer}}^2(2\text{ cm})R_{\text{outer}}^2 + \frac{\rho}{2}\left[\pi R_{\text{outer}}^2 - \pi R_{\text{inner}}^2\right](6\text{ cm})\left(R_{\text{outer}}^2 + R_{\text{inner}}^2\right) \\
&= \frac{\rho\pi}{2}\left[(9\text{ cm})^4(2\text{ cm}) + (6\text{ cm})\left((9\text{ cm})^2 - R_{\text{inner}}^2\right)\left((9\text{ cm})^2 + R_{\text{inner}}^2\right)\right] \\
&= \rho\pi\left[6\,561\text{ cm}^5 + (3\text{ cm})\left((9\text{ cm})^4 - R_{\text{inner}}^4\right)\right] \\
&= \rho\pi\left[26\,244\text{ cm}^5 - (3\text{ cm})R_{\text{inner}}^4\right]
\end{aligned}$$

For the required energy storage,

$$\frac{1}{2}I\omega_1^2 = \frac{1}{2}I\omega_2^2 + W_{\text{out}}$$

$$\frac{1}{2}I\left[(800\text{ rev/min})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]^2 - \frac{1}{2}I\left[(600)\left(\frac{2\pi\text{ rad}}{60\text{ s}}\right)\right]^2 = 60\text{ J}$$

$$I = \frac{60\text{ J}}{1\,535/\text{s}^2} = \left(7.86\times10^3\text{ kg/m}^3\right)\pi\left[26\,244\text{ cm}^5 - (3\text{ cm})R_{\text{inner}}^4\right]$$

$$1.58\times10^{-6}\text{ m}^5\left(\frac{100\text{ cm}}{1\text{ m}}\right)^5 = 26\,244\text{ cm}^5 - (3\text{ cm})R_{\text{inner}}^4$$

$$R_{\text{inner}} = \left(\frac{26\,244\text{ cm}^4 - 15\,827\text{ cm}^4}{3}\right)^{1/4} = 7.68\text{ cm}$$

$\boxed{\text{The inner radius of the flywheel is 7.68 cm. The mass of the flywheel is then 7.27 kg}}$, found as follows:

$$\begin{aligned}
M_{\text{disk}} + M_{\text{wall}} &= \rho\pi R_{\text{outer}}^2(2\text{ cm}) + \rho\left[\pi R_{\text{outer}}^2 - \pi R_{\text{inner}}^2\right](6\text{ cm}) \\
&= \left(7.86\times10^3\text{ kg/m}^3\right)\pi\left[(0.09\text{ m})^2(0.02\text{ m}) + \left[(0.09\text{ m})^2 - (0.076\,8\text{ m})^2\right](0.06\text{ m})\right] \\
&= 7.27\text{ kg}
\end{aligned}$$

If we made the thickness of the disk somewhat less than 2 cm and the inner radius of the cylindrical wall less than 7.68 cm to compensate, the mass could be a bit less than 7.27 kg.

***P10.25** Note that the torque on the trebuchet is not constant, so its angular acceleration changes in time. At our mathematical level it would be unproductive to calculate values for α on the way to find ω_f. Instead, we consider that gravitational energy of the 60-kg-Earth system becomes gravitational energy of the lighter mass plus kinetic energy of both masses.

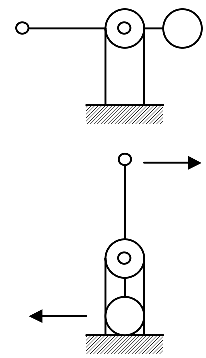

FIG. P10.25

(a) The maximum speed appears as the rod passes through the vertical. Let v_1 represent the speed of the small-mass particle m_1. Then here the rod is turning at $\omega_1 = \dfrac{v_1}{2.86\text{ m}}$. The larger-mass particle is moving at

$$v_2 = (0.14\text{ m})\omega_1 = \frac{0.14v_1}{2.86}$$

Now the energy-conservation equation becomes

$$\left(K_1 + K_2 + U_{g1} + U_{g2}\right)_i = \left(K_1 + K_2 + U_{g1} + U_{g2}\right)_f$$

$$0 + 0 + 0 + m_2 g y_{2i} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + m_1 g y_{1f} + 0$$

$$(60\text{ kg})(9.8\text{ m/s}^2)(0.14\text{ m}) = \frac{1}{2}(0.12\text{ kg})v_1^2 + \frac{1}{2}(60\text{ kg})\left(\frac{0.14v_1}{2.86}\right)^2 + (0.12\text{ kg})(9.8\text{ m/s}^2)(2.86\text{ m})$$

$$82.32\text{ J} = \frac{1}{2}(0.12\text{ kg})v_1^2 + \frac{1}{2}(0.144\text{ kg})v_1^2 + 3.36\text{ J}$$

$$v_1 = \left(\frac{2(79.0\text{ J})}{0.264\text{ kg}}\right)^{1/2} = \boxed{24.5\text{ m/s}}$$

(b) The lever arm of the gravitational force on the 60-kg particle changes during the motion, so the torque changes, so the angular acceleration changes. The projectile moves with changing net acceleration and changing tangential acceleration. The ratio of the particles' distances from the axis controls the ratio of their speeds, and this is different from the ratio of their masses, so the total momentum changes during the motion. But the mechanical energy stays constant, and that is how we solved the problem.

Section 10.5 **Calculation of Moments of Inertia**

P10.26 We assume the rods are thin, with radius much less than L. Call the junction of the rods the origin of coordinates, and the axis of rotation the z-axis.

For the rod along the y-axis, $I = \frac{1}{3}mL^2$ from the table.

For the rod parallel to the z-axis, the parallel-axis theorem gives

$$I = \frac{1}{2}mr^2 + m\left(\frac{L}{2}\right)^2 \cong \frac{1}{4}mL^2$$

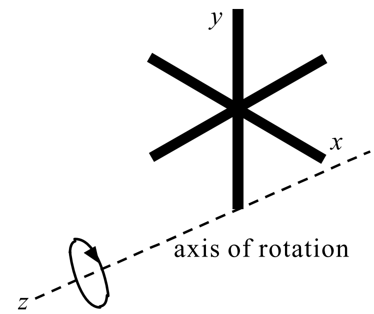

FIG. P10.26

In the rod along the x-axis, the bit of material between x and $x + dx$ has mass $\left(\frac{m}{L}\right)dx$ and is at distance $r = \sqrt{x^2 + \left(\frac{L}{2}\right)^2}$ from the axis of rotation. The total rotational inertia is:

$$I_{\text{total}} = \frac{1}{3}mL^2 + \frac{1}{4}mL^2 + \int_{-L/2}^{L/2}\left(x^2 + \frac{L^2}{4}\right)\left(\frac{m}{L}\right)dx$$

$$= \frac{7}{12}mL^2 + \left(\frac{m}{L}\right)\frac{x^3}{3}\bigg|_{-L/2}^{L/2} + \frac{mL}{4}x\bigg|_{-L/2}^{L/2}$$

$$= \frac{7}{12}mL^2 + \frac{mL^2}{12} + \frac{mL^2}{4} = \boxed{\frac{11mL^2}{12}}$$

Note: The moment of inertia of the rod along the x axis can also be calculated from the parallel-axis theorem as $\frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2$.

P10.27 Treat the tire as consisting of three parts. The two sidewalls are each treated as a hollow cylinder of inner radius 16.5 cm, outer radius 30.5 cm, and height 0.635 cm. The tread region is treated as a hollow cylinder of inner radius 30.5 cm, outer radius 33.0 cm, and height 20.0 cm.

Use $I = \frac{1}{2}m\left(R_1^2 + R_2^2\right)$ for the moment of inertia of a hollow cylinder.

Sidewall:

$$m = \pi\left[(0.305\text{ m})^2 - (0.165\text{ m})^2\right]\left(6.35\times10^{-3}\text{ m}\right)\left(1.10\times10^3\text{ kg/m}^3\right) = 1.44\text{ kg}$$

$$I_{\text{side}} = \frac{1}{2}(1.44\text{ kg})\left[(0.165\text{ m})^2 + (0.305\text{ m})^2\right] = 8.68\times10^{-2}\text{ kg}\cdot\text{m}^2$$

Tread:

$$m = \pi\left[(0.330\text{ m})^2 - (0.305\text{ m})^2\right](0.200\text{ m})\left(1.10\times10^3\text{ kg/m}^3\right) = 11.0\text{ kg}$$

$$I_{\text{tread}} = \frac{1}{2}(11.0\text{ kg})\left[(0.330\text{ m})^2 + (0.305\text{ m})^2\right] = 1.11\text{ kg}\cdot\text{m}^2$$

Entire Tire:

$$I_{\text{total}} = 2I_{\text{side}} + I_{\text{tread}} = 2\left(8.68\times10^{-2}\text{ kg}\cdot\text{m}^2\right) + 1.11\text{ kg}\cdot\text{m}^2 = \boxed{1.28\text{ kg}\cdot\text{m}^2}$$

P10.28 Every particle in the door could be slid straight down into a high-density rod across its bottom, without changing the particle's distance from the rotation axis of the door. Thus, a rod 0.870 m long with mass 23.0 kg, pivoted about one end, has the same rotational inertia as the door:

$$I = \frac{1}{3}ML^2 = \frac{1}{3}(23.0 \text{ kg})(0.870 \text{ m})^2 = \boxed{5.80 \text{ kg}\cdot\text{m}^2}$$

The $\boxed{\text{height of the door is unnecessary}}$ data.

P10.29 Model your body as a cylinder of mass 60.0 kg and circumference 75.0 cm. Then its radius is

$$\frac{0.750 \text{ m}}{2\pi} = 0.120 \text{ m}$$

and its moment of inertia is

$$\frac{1}{2}MR^2 = \frac{1}{2}(60.0 \text{ kg})(0.120 \text{ m})^2 = 0.432 \text{ kg}\cdot\text{m}^2 \sim \boxed{10^0 \text{ kg}\cdot\text{m}^2 = 1 \text{ kg}\cdot\text{m}^2}$$

P10.30 We consider the cam as the superposition of the original solid disk and a disk of negative mass cut from it. With half the radius, the cut-away part has one-quarter the face area and one-quarter the volume and one-quarter the mass M_0 of the original solid cylinder:

$$M_0 - \frac{1}{4}M_0 = M \qquad M_0 = \frac{4}{3}M$$

By the parallel-axis theorem, the original cylinder had moment of inertia

$$I_{\text{CM}} + M_0\left(\frac{R}{2}\right)^2 = \frac{1}{2}M_0R^2 + M_0\frac{R^2}{4} = \frac{3}{4}M_0R^2$$

The negative-mass portion has $I = \frac{1}{2}\left(-\frac{1}{4}M_0\right)\left(\frac{R}{2}\right)^2 = -\frac{M_0R^2}{32}$. The whole cam has

$$I = \frac{3}{4}M_0R^2 - \frac{M_0R^2}{32} = \frac{23}{32}M_0R^2 = \frac{23}{32}\frac{4}{3}MR^2 = \frac{23}{24}MR^2 \text{ and}$$

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{23}{24}MR^2\omega^2 = \boxed{\frac{23}{48}MR^2\omega^2}$$

***P10.31** We measure the distance of each particle in the rod from the y' axis:

$$I_{y'} = \int_{\text{all mass}} r^2 dm = \int_0^L x^2 \frac{M}{L}dx = \frac{M}{L}\frac{x^3}{3}\bigg|_0^L = \frac{1}{3}ML^2$$

Section 10.6 **Torque**

P10.32 Resolve the 100 N force into components perpendicular to and parallel to the rod, as

$$F_{\text{par}} = (100 \text{ N})\cos 57.0° = 54.5 \text{ N}$$

and

$$F_{\text{perp}} = (100 \text{ N})\sin 57.0° = 83.9 \text{ N}$$

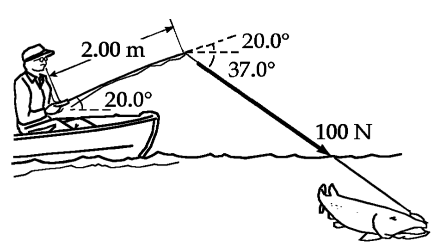

FIG. P10.32

The torque of F_{par} is zero since its line of action passes through the pivot point.

The torque of F_{perp} is $\tau = 83.9 \text{ N}(2.00 \text{ m}) = \boxed{168 \text{ N}\cdot\text{m}}$ (clockwise)

P10.33 $\sum \tau = 0.100 \text{ m}(12.0 \text{ N}) - 0.250 \text{ m}(9.00 \text{ N}) - 0.250 \text{ m}(10.0 \text{ N}) = \boxed{-3.55 \text{ N}\cdot\text{m}}$

The thirty-degree angle is unnecessary information.

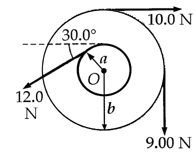

FIG. P10.33

Section 10.7 **Relationship Between Torque and Angular Acceleration**

P10.34 (a) $I = \frac{1}{2}MR^2 = \frac{1}{2}(2.00 \text{ kg})(7.00\times10^{-2} \text{ m})^2 = 4.90\times10^{-3} \text{ kg}\cdot\text{m}^2$

$$\alpha = \frac{\Sigma\tau}{I} = \frac{0.600}{4.90\times10^{-3}} = 122 \text{ rad/s}^2$$

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$\Delta t = \frac{\Delta\omega}{\alpha} = \frac{1\,200(2\pi/60)}{122} = \boxed{1.03 \text{ s}}$$

(b) $\Delta\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(122 \text{ rad/s})(1.03 \text{ s})^2 = 64.7 \text{ rad} = \boxed{10.3 \text{ rev}}$

P10.35 $m = 0.750 \text{ kg}, \ F = 0.800 \text{ N}$

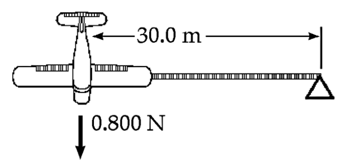

FIG. P10.35

(a) $\tau = rF = 30.0 \text{ m}(0.800 \text{ N}) = \boxed{24.0 \text{ N}\cdot\text{m}}$

(b) $\alpha = \frac{\tau}{I} = \frac{rF}{mr^2} = \frac{24.0}{0.750(30.0)^2} = \boxed{0.035\,6 \text{ rad/s}^2}$

(c) $a_t = \alpha r = 0.035\,6(30.0) = \boxed{1.07 \text{ m/s}^2}$

P10.36 $\omega_f = \omega_i + \alpha t$: $\quad 10.0 \text{ rad/s} = 0 + \alpha(6.00 \text{ s})$

$$\alpha = \frac{10.00}{6.00} \text{ rad/s}^2 = 1.67 \text{ rad/s}^2$$

(a) $\sum\tau = 36.0 \text{ N}\cdot\text{m} = I\alpha$: $I = \frac{\sum\tau}{\alpha} = \frac{36.0 \text{ N}\cdot\text{m}}{1.67 \text{ rad/s}^2} = \boxed{21.6 \text{ kg}\cdot\text{m}^2}$

(b) $\omega_f = \omega_i + \alpha t$: $\quad 0 = 10.0 + \alpha(60.0)$

$$\alpha = -0.167 \text{ rad/s}^2$$

$$|\tau| = |I\alpha| = (21.6 \text{ kg}\cdot\text{m}^2)(0.167 \text{ rad/s}^2) = \boxed{3.60 \text{ N}\cdot\text{m}}$$

(c) Number of revolutions $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

During first 6.00 s $\quad \theta_f = \frac{1}{2}(1.67)(6.00)^2 = 30.1 \text{ rad}$

During next 60.0 s $\quad \theta_f = 10.0(60.0) - \frac{1}{2}(0.167)(60.0)^2 = 299 \text{ rad}$

$$\theta_{\text{total}} = 329 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{52.4 \text{ rev}}$$

P10.37 For m_1,

$$\sum F_y = ma_y: \qquad +n - m_1 g = 0$$

$$n_1 = m_1 g = 19.6 \text{ N}$$

$$f_{k1} = \mu_k n_1 = 7.06 \text{ N}$$

$$\sum F_x = ma_x: \qquad -7.06 \text{ N} + T_1 = (2.00 \text{ kg})a \qquad (1)$$

For the pulley,

$$\sum \tau = I\alpha: \qquad -T_1 R + T_2 R = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$$

$$-T_1 + T_2 = \frac{1}{2}(10.0 \text{ kg})a$$

$$-T_1 + T_2 = (5.00 \text{ kg})a \qquad (2)$$

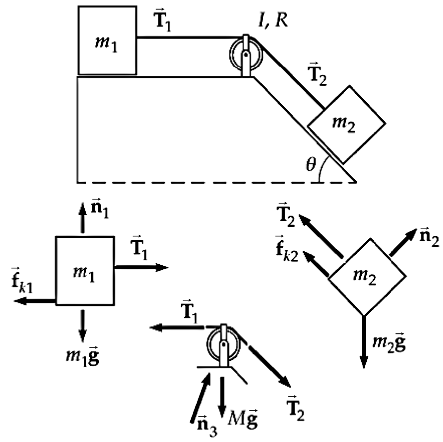

FIG. P10.37

For m_2, $\qquad +n_2 - m_2 g\cos\theta = 0$

$$n_2 = 6.00 \text{ kg}\left(9.80 \text{ m/s}^2\right)(\cos 30.0°)$$

$$= 50.9 \text{ N}$$

$$f_{k2} = \mu_k n_2$$

$$= 18.3 \text{ N}: \quad -18.3 \text{ N} - T_2 + m_2 \sin\theta = m_2 a$$

$$-18.3 \text{ N} - T_2 + 29.4 \text{ N} = (6.00 \text{ kg})a \qquad (3)$$

(a) Add equations (1), (2), and (3):

$$-7.06 \text{ N} - 18.3 \text{ N} + 29.4 \text{ N} = (13.0 \text{ kg})a$$

$$a = \frac{4.01 \text{ N}}{13.0 \text{ kg}} = \boxed{0.309 \text{ m/s}^2}$$

(b) $T_1 = 2.00 \text{ kg}\left(0.309 \text{ m/s}^2\right) + 7.06 \text{ N} = \boxed{7.67 \text{ N}}$

$T_2 = 7.67 \text{ N} + 5.00 \text{ kg}\left(0.309 \text{ m/s}^2\right) = \boxed{9.22 \text{ N}}$

P10.38 $I = \frac{1}{2}mR^2 = \frac{1}{2}(100 \text{ kg})(0.500 \text{ m})^2 = 12.5 \text{ kg}\cdot\text{m}^2$

$$\omega_i = 50.0 \text{ rev/min} = 5.24 \text{ rad/s}$$

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{0 - 5.24 \text{ rad/s}}{6.00 \text{ s}} = -0.873 \text{ rad/s}^2$$

$$\tau = I\alpha = 12.5 \text{ kg}\cdot\text{m}^2\left(-0.873 \text{ rad/s}^2\right) = -10.9 \text{ N}\cdot\text{m}$$

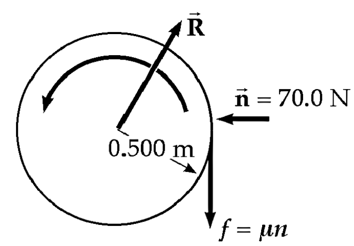

FIG. P10.38

The magnitude of the torque is given by $fR = 10.9 \text{ N}\cdot\text{m}$, where f is the force of friction.

Therefore,

$$f = \frac{10.9 \text{ N}\cdot\text{m}}{0.500 \text{ m}} \qquad \text{and} \qquad f = \mu_k n$$

yields

$$\mu_k = \frac{f}{n} = \frac{21.8 \text{ N}}{70.0 \text{ N}} = \boxed{0.312}$$

P10.39 $\sum \tau = I\alpha = \frac{1}{2}MR^2\alpha$

$$-135\text{ N}(0.230\text{ m}) + T(0.230\text{ m}) = \frac{1}{2}(80\text{ kg})\left(\frac{1.25}{2}\text{ m}\right)^2\left(-1.67\text{ rad/s}^2\right)$$

$$T = \boxed{21.5\text{ N}}$$

***P10.40** The chosen tangential force produces constant torque and so constant angular acceleration.

$\theta = 0 + 0 + (1/2)\alpha t^2 \qquad 2(2\pi\text{ rad}) = (1/2)\ \alpha(10\text{ s})^2 \qquad \alpha = 0.251\text{ rad/s}^2$

$\sum\tau = I\alpha \qquad TR = 100\text{ kg}\cdot\text{m}^2(0.251\text{ /s}^2) = 25.1\text{ N}\cdot\text{m}$

Infinitely many pairs of values that satisfy this requirement exist, such as $T = 25.1$ N and $R = 1.00$ m

Section 10.8 **Work, Power, and Energy in Rotational Motion**

P10.41 The power output of the bus is $\mathcal{P} = \frac{E}{\Delta t}$ where $E = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{1}{2}MR^2\omega^2$ is the stored energy and $\Delta t = \frac{\Delta x}{v}$ is the time it can roll. Then $\frac{1}{4}MR^2\omega^2 = \mathcal{P}\Delta t = \frac{\mathcal{P}\Delta x}{v}$ and

$$\Delta x = \frac{MR^2\omega^2 v}{4\mathcal{P}} = \frac{1\,600\text{ kg}(0.65\text{ m})^2(4\,000\cdot 2\pi/60\text{ s})^2 11.1\text{ m/s}}{4(18\cdot 746\text{ W})} = \boxed{24.5\text{ km}}$$

P10.42 The moment of inertia of a thin rod about an axis through one end is $I = \frac{1}{3}ML^2$. The total rotational kinetic energy is given as

$$K_R = \frac{1}{2}I_h\omega_h^2 + \frac{1}{2}I_m\omega_m^2$$

with

$$I_h = \frac{m_h L_h^2}{3} = \frac{60.0\text{ kg}(2.70\text{ m})^2}{3} = 146\text{ kg}\cdot\text{m}^2$$

and

$$I_m = \frac{m_m L_m^2}{3} = \frac{100\text{ kg}(4.50\text{ m})^2}{3} = 675\text{ kg}\cdot\text{m}^2$$

In addition,

$$\omega_h = \frac{2\pi\text{ rad}}{12\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.45\times10^{-4}\text{ rad/s}$$

while

$$\omega_m = \frac{2\pi\text{ rad}}{1\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.75\times10^{-3}\text{ rad/s}$$

Therefore,

$$K_R = \frac{1}{2}(146)\left(1.45\times10^{-4}\right)^2 + \frac{1}{2}(675)\left(1.75\times10^{-3}\right)^2 = \boxed{1.04\times10^{-3}\text{ J}}$$

P10.43 Work done $= F\Delta r = (5.57\ \text{N})(0.800\ \text{m}) = 4.46\ \text{J}$

and Work $= \Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$

(The last term is zero because the top starts from rest.)

Thus, $4.46\ \text{J} = \frac{1}{2}\left(4.00\times10^{-4}\ \text{kg}\cdot\text{m}^2\right)\omega_f^2$

and from this, $\omega_f = \boxed{149\ \text{rad/s}}$.

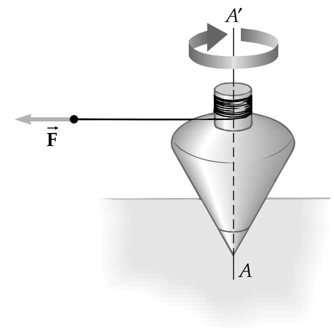

FIG. P10.43

***P10.44** Let T_1 represent the tension in the cord above m_1 and T_2 the tension in the cord above the lighter mass. The two blocks move with the same acceleration because the cord does not stretch, and the angular acceleration of the pulley is a/R For the heavier mass we have

$$\Sigma F = m_1a \qquad T_1 - m_1g = m_1(-a) \quad \text{or} \quad -T_1 + m_1g = m_1a$$

For the lighter mass,

$$\Sigma F = m_2a \quad T_2 - m_2g = m_2a$$

We assume the pulley is a uniform disk: $I = (1/2)MR^2$

$$\Sigma\tau = I\alpha \qquad +T_1R - T_2R = (1/2)MR^2(a/R) \quad \text{or} \quad T_1 - T_2 = (1/2)Ma$$

Add up the three equations in a

$$-T_1 + m_1g + T_2 - m_2g + T_1 - T_2 = m_1a + m_2a + (1/2)Ma$$

$$a = (m_1 - m_2)g/[m_1 + m_2 + (1/2)M] = (20 - 12.5)(9.8\ \text{m/s}^2)/[20 + 12.5 + 2.5] = 2.1\ \text{m/s}^2$$

Next, $x = 0 + 0 + (1/2)\,at^2 \qquad 4.00\ \text{m} = (1/2)(2.1\ \text{m/s}^2)\,t^2 \qquad t = \boxed{1.95\ \text{s}}$

If the pulley were massless, the acceleration would be larger by a factor 35/32.5 and the time shorter by the square root of the factor 32.5/35. That is, the time would be reduced by 3.64%.

P10.45 (a) $I = \frac{1}{2}M\left(R_1^2 + R_2^2\right) = \frac{1}{2}(0.35\ \text{kg})\left[(0.02\ \text{m})^2 + (0.03\ \text{m})^2\right] = 2.28\times10^{-4}\ \text{kg}\cdot\text{m}^2$

$$\left(K_1 + K_2 + K_{\text{rot}} + U_{g2}\right)_i - f_k\Delta x = \left(K_1 + K_2 + K_{\text{rot}}\right)_f$$

$$\frac{1}{2}(0.850\ \text{kg})(0.82\ \text{m/s})^2 + \frac{1}{2}(0.42\ \text{kg})(0.82\ \text{m/s})^2 + \frac{1}{2}\left(2.28\times10^{-4}\ \text{kg}\cdot\text{m}^2\right)\left(\frac{0.82\ \text{m/s}}{0.03\ \text{m}}\right)^2$$
$$+0.42\ \text{kg}\left(9.8\ \text{m/s}^2\right)(0.7\ \text{m}) - 0.25(0.85\ \text{kg})\left(9.8\ \text{m/s}^2\right)(0.7\ \text{m})$$
$$= \frac{1}{2}(0.85\ \text{kg})v_f^2 + \frac{1}{2}(0.42\ \text{kg})v_f^2 + \frac{1}{2}\left(2.28\times10^{-4}\ \text{kg}\cdot\text{m}^2\right)\left(\frac{v_f}{0.03\ \text{m}}\right)^2$$

$$0.512\ \text{J} + 2.88\ \text{J} - 1.46\ \text{J} = (0.761\ \text{kg})v_f^2$$

$$v_f = \sqrt{\frac{1.94\ \text{J}}{0.761\ \text{kg}}} = \boxed{1.59\ \text{m/s}}$$

(b) $\omega = \frac{v}{r} = \frac{1.59\ \text{m/s}}{0.03\ \text{m}} = \boxed{53.1\ \text{rad/s}}$

P10.46 We assume the rod is thin. For the compound object

$$I = \frac{1}{3}M_{\text{rod}}L^2 + \left[\frac{2}{5}m_{\text{ball}}R^2 + M_{\text{ball}}D^2\right]$$

$$I = \frac{1}{3}1.20\ \text{kg}(0.240\ \text{m})^2 + \frac{2}{5}2.00\ \text{kg}\left(4.00\times10^{-2}\ \text{m}\right)^2 + 2.00\ \text{kg}(0.280\ \text{m})^2$$

$$I = 0.181\ \text{kg}\cdot\text{m}^2$$

(a) $K_f + U_f = K_i + U_i + \Delta E$

$$\frac{1}{2}I\omega^2 + 0 = 0 + M_{\text{rod}}g\left(\frac{L}{2}\right) + M_{\text{ball}}g(L+R) + 0$$

$$\frac{1}{2}\left(0.181\ \text{kg}\cdot\text{m}^2\right)\omega^2 = 1.20\ \text{kg}\left(9.80\ \text{m/s}^2\right)(0.120\ \text{m}) + 2.00\ \text{kg}\left(9.80\ \text{m/s}^2\right)(0.280\ \text{m})$$

$$\frac{1}{2}\left(0.181\ \text{kg}\cdot\text{m}^2\right)\omega^2 = \boxed{6.90\ \text{J}}$$

(b) $\omega = \boxed{8.73\ \text{rad/s}}$

(c) $v = r\omega = (0.280\ \text{m})8.73\ \text{rad/s} = \boxed{2.44\ \text{m/s}}$

(d) $v_f^2 = v_i^2 + 2a\left(y_f - y_i\right)$

$$v_f = \sqrt{0 + 2\left(9.80\ \text{m/s}^2\right)(0.280\ \text{m})} = 2.34\ \text{m/s}$$

$$\boxed{\text{The speed it attains in swinging is greater by}}\ \frac{2.44}{2.34} = \boxed{1.043\,2\ \text{times}}$$

P10.47 (a) For the counterweight,

$\sum F_y = ma_y$ becomes: $\quad 50.0 - T = \left(\dfrac{50.0}{9.80}\right)a$

For the reel $\sum \tau = I\alpha$ reads

$$TR = I\alpha = I\frac{a}{R}$$

where

$$I = \frac{1}{2}MR^2 = 0.093\,8\ \text{kg}\cdot\text{m}^2$$

We substitute to eliminate the acceleration:

$$50.0 - T = 5.10\left(\frac{TR^2}{I}\right)$$

$$T = \boxed{11.4\ \text{N}} \quad \text{and}$$

$$a = \frac{50.0 - 11.4}{5.10} = \boxed{7.57\ \text{m/s}^2}$$

$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$: $\quad v_f = \sqrt{2(7.57)6.00} = \boxed{9.53\ \text{m/s}}$

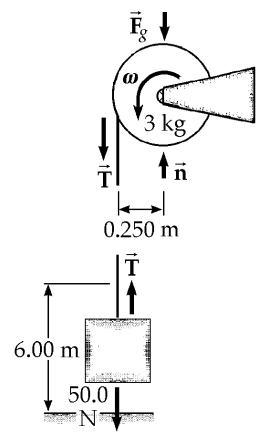

FIG. P10.47

(b) Use conservation of energy for the system of the object, the reel, and the Earth:

$(K+U)_i = (K+U)_f$: $\quad mgh = \dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2$

$$2mgh = mv^2 + I\left(\frac{v^2}{R^2}\right) = v^2\left(m + \frac{I}{R^2}\right)$$

$$v = \sqrt{\frac{2mgh}{m + (I/R^2)}} = \sqrt{\frac{2(50.0\ \text{N})(6.00\ \text{m})}{5.10\ \text{kg} + (0.0938/0.25^2)}} = \boxed{9.53\ \text{m/s}}$$

The two methods agree on the final speed.

P10.48 The moment of inertia of the cylinder is

$$I = \frac{1}{2}mr^2 = \frac{1}{2}(81.6\text{ kg})(1.50\text{ m})^2 = 91.8\text{ kg}\cdot\text{m}^2$$

and the angular acceleration of the merry-go-round is found as

$$\alpha = \frac{\tau}{I} = \frac{(Fr)}{I} = \frac{(50.0\text{ N})(1.50\text{ m})}{(91.8\text{ kg}\cdot\text{m}^2)} = 0.817\text{ rad/s}^2$$

At $t = 3.00$ s, we find the angular velocity

$$\omega = \omega_i + \alpha t$$

$$\omega = 0 + (0.817\text{ rad/s}^2)(3.00\text{ s}) = 2.45\text{ rad/s}$$

and

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(91.8\text{ kg}\cdot\text{m}^2)(2.45\text{ rad/s})^2 = \boxed{276\text{ J}}$$

P10.49 From conservation of energy for the object-turntable-cylinder-Earth system,

$$\frac{1}{2}I\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2 = mgh$$

$$I\frac{v^2}{r^2} = 2mgh - mv^2$$

$$I = \boxed{mr^2\left(\frac{2gh}{v^2} - 1\right)}$$

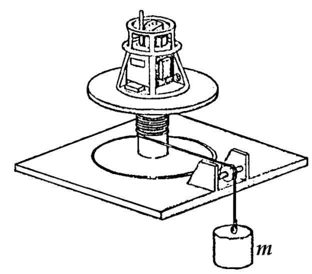

FIG. P10.49

P10.50 (a) The moment of inertia of the cord on the spool is

$$\frac{1}{2}M\left(R_1^2 + R_2^2\right) = \frac{1}{2}0.1\text{ kg}\left((0.015\text{ m})^2 + (0.09\text{ m})^2\right) = 4.16\times10^{-4}\text{ kg}\cdot\text{m}^2$$

The protruding strand has mass $(10^{-2}\text{ kg/m})0.16\text{ m} = 1.6\times10^{-3}\text{ kg}$ and

$$I = I_{\text{CM}} + Md^2 = \frac{1}{12}ML^2 + Md^2 = 1.6\times10^{-3}\text{ kg}\left(\frac{1}{12}(0.16\text{ m})^2 + (0.09\text{ m} + 0.08\text{ m})^2\right)$$

$$= 4.97\times10^{-5}\text{ kg}\cdot\text{m}^2$$

For the whole cord, $I = 4.66\times10^{-4}\text{ kg}\cdot\text{m}^2$. In speeding up, the average power is

$$\mathcal{P} = \frac{E}{\Delta t} = \frac{\frac{1}{2}I\omega^2}{\Delta t} = \frac{4.66\times10^{-4}\text{ kg}\cdot\text{m}^2}{2(0.215\text{ s})}\left(\frac{2\,500\cdot2\pi}{60\text{ s}}\right)^2 = \boxed{74.3\text{ W}}$$

(b) $$\mathcal{P} = \tau\omega = (7.65\text{ N})(0.16\text{ m} + 0.09\text{ m})\left(\frac{2\,000\cdot2\pi}{60\text{ s}}\right) = \boxed{401\text{ W}}$$

P10.51 (a) Find the velocity of the CM

$$(K+U)_i = (K+U)_f$$

$$0 + mgR = \frac{1}{2}I\omega^2$$

$$\omega = \sqrt{\frac{2mgR}{I}} = \sqrt{\frac{2mgR}{\frac{3}{2}mR^2}}$$

$$v_{\text{CM}} = R\sqrt{\frac{4g}{3R}} = \boxed{2\sqrt{\frac{Rg}{3}}}$$

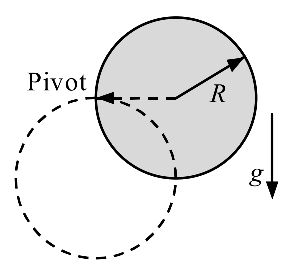

FIG. P10.51

continued on next page

(b) $v_L = 2v_{\text{CM}} = \boxed{4\sqrt{\dfrac{Rg}{3}}}$

(c) $v_{\text{CM}} = \sqrt{\dfrac{2mgR}{2m}} = \boxed{\sqrt{Rg}}$

Section 10.9 **Rolling Motion of a Rigid Object**

***P10.52** Conservation of energy for the sphere rolling without slipping:

$$U_i = K_{translation,f} + K_{rotation,f}$$

$$mgh = (1/2)\,mv^2 + (1/2)(2/5)mR^2(v/R)^2 = (7/10)mv^2 \qquad \boxed{v_f = [10gh/7]^{1/2}}$$

Conservation of energy for the sphere sliding without friction, with $\omega = 0$:

$$mgh = (1/2)\,mv^2 \quad \boxed{v_f = [2gh]^{1/2}}$$

The time intervals required for the trips follow from $x = 0 + v_{avg}t$

$$h/\sin\theta = [(0 + v_f)/2]t \qquad t = 2h/v_f \sin\theta$$

For rolling we have $t = (2h/\sin\theta)(7/10gh)^{1/2}$
and for sliding, $t = (2h/\sin\theta)(1/2gh)^{1/2}$

$\boxed{\text{The time to roll is longer by a factor of } (0.7/0.5)^{1/2} = 1.18}$

P10.53 (a) $K_{\text{trans}} = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{500\text{ J}}$

(b) $K_{\text{rot}} = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}\left(\dfrac{1}{2}mr^2\right)\left(\dfrac{v^2}{r^2}\right) = \dfrac{1}{4}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{250\text{ J}}$

(c) $K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}} = \boxed{750\text{ J}}$

***P10.54** (a) $\boxed{\text{The cylinder}}$ has extra kinetic energy, so it travels farther up the incline.

(b) Energy conservation for the smooth cube:

$$K_i = U_f \qquad (1/2)\,mv^2 = mgd\,\sin\theta \qquad d = v^2/2g\sin\theta$$

The same principle for the cylinder:

$$K_{translation,i} + K_{rotation,i} = U_f \qquad (1/2)\,mv^2 + (1/2)[(1/2)mr^2](v/r)^2 = mgd\,\sin\theta$$

$$d = 3v^2/4g\sin\theta$$

The difference in distance is $3v^2/4g\sin\theta - v^2/2g\sin\theta = \boxed{v^2/4g\sin\theta,}$ or the cylinder travels 50% farther.

(c) The cylinder does not lose mechanical energy because static friction does no work on it. Its rotation means that it has 50% more kinetic energy than the cube at the start, and so it travels 50% farther up the incline.

P10.55 (a) $\tau = I\alpha$

$$mgR\sin\theta = \left(I_{CM} + mR^2\right)\alpha$$

$$a = \frac{mgR^2\sin\theta}{I_{CM} + mR^2}$$

$$a_{\text{hoop}} = \frac{mgR^2\sin\theta}{2mR^2} = \boxed{\frac{1}{2}g\sin\theta}$$

$$a_{\text{disk}} = \frac{mgR^2\sin\theta}{\frac{3}{2}mR^2} = \boxed{\frac{2}{3}g\sin\theta}$$

$$\boxed{\text{The disk moves with } \frac{4}{3} \text{ the acceleration of the hoop.}}$$

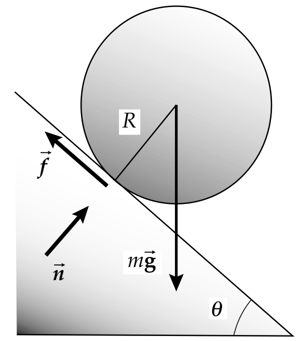

FIG. P10.55

(b) $Rf = I\alpha$

$$f = \mu n = \mu mg\cos\theta$$

$$\mu = \frac{f}{mg\cos\theta} = \frac{I\alpha/R}{mg\cos\theta} = \frac{\left(\frac{2}{3}g\sin\theta\right)\left(\frac{1}{2}mR^2\right)}{R^2 mg\cos\theta} = \boxed{\frac{1}{3}\tan\theta}$$

P10.56 $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2$ where $\omega = \frac{v}{R}$ since no slipping occurs.

Also, $U_i = mgh,\ U_f = 0,$ and $v_i = 0$

Therefore,

$$\frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2 = mgh$$

Thus,

$$v^2 = \frac{2gh}{[1 + (I/mR^2)]}$$

For a disk,

$$I = \frac{1}{2}mR^2$$

So

$$v^2 = \frac{2gh}{1 + \frac{1}{2}} \quad \text{or} \quad \boxed{v_{\text{disk}} = \sqrt{\frac{4gh}{3}}}$$

For a ring,

$$I = mR^2 \text{ so } v^2 = \frac{2gh}{2} \quad \text{or} \quad \boxed{v_{\text{ring}} = \sqrt{gh}}$$

Since $v_{\text{disk}} > v_{\text{ring}}$, $\boxed{\text{the disk}}$ reaches the bottom first.

P10.57 $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{3.00 \text{ m}}{1.50 \text{ s}} = 2.00 \text{ m/s} = \dfrac{1}{2}\left(0 + v_f\right)$

$$v_f = 4.00 \text{ m/s and } \omega_f = \frac{v_f}{r} = \frac{4.00 \text{ m/s}}{\left(6.38 \times 10^{-2} \text{ m}\right)/2} = \frac{8.00}{6.38 \times 10^{-2}} \text{ rad/s}$$

We ignore internal friction and suppose the can rolls without slipping.

$$\left(K_{\text{trans}} + K_{\text{rot}} + U_g\right)_i + \Delta E_{\text{mech}} = \left(K_{\text{trans}} + K_{\text{rot}} + U_g\right)_f$$

$$\left(0 + 0 + mgy_i\right) + 0 = \left(\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + 0\right)$$

$$0.215 \text{ kg}\left(9.80 \text{ m/s}^2\right)\left[(3.00 \text{ m})\sin 25.0°\right] = \frac{1}{2}(0.215 \text{ kg})(4.00 \text{ m/s})^2 + \frac{1}{2}I\left(\frac{8.00}{6.38 \times 10^{-2}} \text{ rad/s}\right)^2$$

$$2.67 \text{ J} = 1.72 \text{ J} + \left(7\,860 \text{ s}^{-2}\right)t$$

$$I = \frac{0.951 \text{ kg}\cdot\text{m}^2/\text{s}^2}{7\,860 \text{ s}^{-2}} = \boxed{1.21 \times 10^{-4} \text{ kg}\cdot\text{m}^2}$$

The $\boxed{\text{height of the can}}$ is unnecessary data.

P10.58 (a) Energy conservation for the system of the ball and the Earth between the horizontal section and top of loop:

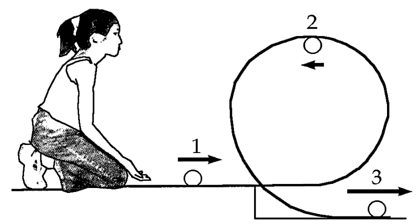

FIG. P10.58

$$\frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2$$

$$\frac{1}{2}mv_2^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_2}{r}\right)^2 + mgy_2$$

$$= \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$\frac{5}{6}v_2^2 + gy_2 = \frac{5}{6}v_1^2$$

$$v_2 = \sqrt{v_1^2 - \frac{6}{5}gy_2} = \sqrt{(4.03 \text{ m/s})^2 - \frac{6}{5}\left(9.80 \text{ m/s}^2\right)(0.900 \text{ m})} = \boxed{2.38 \text{ m/s}}$$

The centripetal acceleration is $\dfrac{v_2^2}{r} = \dfrac{(2.38 \text{ m/s})^2}{0.450 \text{ m}} = 12.6 \text{ m/s}^2 > g$

Thus, the ball must be in contact with the track, with the track pushing downward on it.

(b) $$\frac{1}{2}mv_3^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_3}{r}\right)^2 + mgy_3 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$v_3 = \sqrt{v_1^2 - \frac{6}{5}gy_3} = \sqrt{(4.03 \text{ m/s})^2 - \frac{6}{5}\left(9.80 \text{ m/s}^2\right)(-0.200 \text{ m})} = \boxed{4.31 \text{ m/s}}$$

(c) $$\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_1^2$$

$$v_2 = \sqrt{v_1^2 - 2gy_2} = \sqrt{(4.03 \text{ m/s})^2 - 2\left(9.80 \text{ m/s}^2\right)(0.900 \text{ m})} = \sqrt{-1.40 \text{ m}^2/\text{s}^2}$$

This result is imaginary. In the case where the ball does not roll, the ball starts with less energy than in part (a) and $\boxed{\text{never makes it to the top}}$ of the loop.

Additional Problems

P10.59 $mg\dfrac{\ell}{2}\sin\theta = \dfrac{1}{3}m\ell^2\alpha$

$\alpha = \dfrac{3}{2}\dfrac{g}{\ell}\sin\theta$

$a_t = \left(\dfrac{3}{2}\dfrac{g}{\ell}\sin\theta\right)r$

Then $\left(\dfrac{3}{2}\dfrac{g}{\ell}\right)r > g\sin\theta$

for $r > \dfrac{2}{3}\ell$

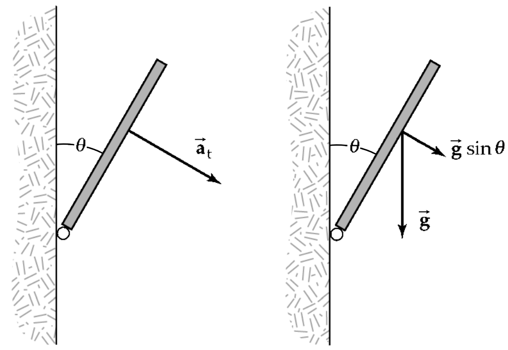

FIG. P10.59

$\therefore$ About $\boxed{\dfrac{1}{3}\text{ the length of the chimney}}$ will have a tangential acceleration greater than $g\sin\theta$.

P10.60 The resistive force on each ball is $R = D\rho A v^2$. Here $v = r\omega$, where r is the radius of each ball's path. The resistive torque on each ball is $\tau = rR$, so the total resistive torque on the three ball system is $\tau_{\text{total}} = 3rR$.

The power required to maintain a constant rotation rate is $\mathcal{P} = \tau_{\text{total}}\omega = 3rR\omega$. This required power may be written as

$$\mathcal{P} = \tau_{\text{total}}\omega = 3r\left[D\rho A(r\omega)^2\right]\omega = \left(3r^3 DA\omega^3\right)\rho$$

With

$$\omega = \frac{2\pi\text{ rad}}{1\text{ rev}}\left(\frac{10^3\text{ rev}}{1\text{ min}}\right)\left(\frac{1\text{ min}}{60.0\text{ s}}\right) = \frac{1\,000\pi}{30.0}\ \text{rad/s}$$

$$\mathcal{P} = 3(0.100\text{ m})^3(0.600)\left(4.00\times10^{-4}\text{ m}^2\right)\left(\frac{1\,000\pi}{30.0\text{ s}}\right)^3\rho$$

or

$\mathcal{P} = \left(0.827\ \text{m}^5/\text{s}^3\right)\rho$, where ρ is the density of the resisting medium.

(a) In air, $\rho = 1.20\ \text{kg/m}^3$,

and $\mathcal{P} = 0.827\ \text{m}^5/\text{s}^3\left(1.20\ \text{kg/m}^3\right) = 0.992\ \text{N}\cdot\text{m/s} = \boxed{0.992\text{ W}}$

(b) In water, $\rho = 1\,000\ \text{kg/m}^3$ and $\mathcal{P} = \boxed{827\text{ W}}$.

P10.61 (a) $W = \Delta K = \dfrac{1}{2}I\omega_f^2 - \dfrac{1}{2}I\omega_i^2 = \dfrac{1}{2}I\left(\omega_f^2 - \omega_i^2\right)$ where $I = \dfrac{1}{2}mR^2$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1.00\text{ kg})(0.500\text{ m})^2\left[(8.00\text{ rad/s})^2 - 0\right] = \boxed{4.00\text{ J}}$$

(b) $t = \dfrac{\omega_f - 0}{\alpha} = \dfrac{\omega r}{a} = \dfrac{(8.00\text{ rad/s})(0.500\text{ m})}{2.50\text{ m/s}^2} = \boxed{1.60\text{ s}}$

(c) $\theta_f = \theta_i + \omega_i t + \dfrac{1}{2}\alpha t^2;\ \theta_i = 0;\ \omega_i = 0$

$$\theta_f = \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(\frac{2.50\text{ m/s}^2}{0.500\text{ m}}\right)(1.60\text{ s})^2 = 6.40\text{ rad}$$

$$s = r\theta = (0.500\text{ m})(6.40\text{ rad}) = \boxed{3.20\text{ m} < 4.00\text{ m Yes.}}$$

***P10.62** (a) We consider the elevator-sheave-counterweight-Earth system, including n passengers, as an isolated system and apply the conservation of mechanical energy. We take the initial configuration, at the moment the drive mechanism switches off, as representing zero gravitational potential energy of the system.
Therefore, the initial mechanical energy of the system is

$$E_i = K_i + U_i = (1/2)\ m_e v^2 + (1/2)\ m_c v^2 + (1/2) I_s \omega^2$$

$$= (1/2)\ m_e v^2 + (1/2)\ m_c v^2 + (1/2)[(1/2) m_s r^2](v/r)^2$$

$$= (1/2)\ [m_e + m_c v^2 + (1/2) m_s]\ v^2$$

The final mechanical energy of the system is entirely gravitational because the system is momentarily at rest:

$$E_f = K_f + U_f = 0 + m_e g d - m_c g d$$

where we have recognized that the elevator car goes up by the same distance d that the counterweight goes down. Setting the initial and final energies of the system equal to each other, we have

$$(1/2)\ [m_e + m_c + (1/2) m_s]\ v^2 = (m_e - m_c)\ g d$$

$$(1/2)\ [800\text{ kg} + n\ 80\text{ kg} + 950\text{ kg} + 140\text{ kg}](3\text{ m/s})^2 = (800\text{ kg} + n\ 80\text{ kg} - 950\text{ kg})(9.8\text{ m/s}^2)\ d$$

$$\boxed{d = [1890 + 80n](0.459\text{ m})/(80n - 150)}$$

(b) $d = [1890 + 80 \times 2](0.459\text{ m})/(80 \times 2 - 150) = \boxed{94.1\text{ m}}$

(c) $d = [1890 + 80 \times 12](0.459\text{ m})/(80 \times 12 - 150) = \boxed{1.62\text{ m}}$

(d) $d = [1890 + 80 \times 0](0.459\text{ m})/(80 \times 0 - 150) = \boxed{-5.79\text{ m}}$

(e) The rising car will coast to a stop only for $n \geq 2$. For $n = 0$ or $n = 1$, the car would accelerate upward if released.

(f) The graph looks roughly like one branch of a hyperbola. It comes down steeply from 94.1 m for $n = 2$, flattens out, and very slowly approaches 0.459 m as n becomes large.

(g) The radius of the sheave is not necessary. It divides out in the expression $(1/2)I\omega^2 = (1/4)m_{\text{sheave}}\ v^2$.

(h) In this problem, as often in everyday life, energy conservation refers to minimizing use of electric energy or fuel. In physical theory, energy conservation refers to the constancy of the total energy of an isolated system, without regard to the different prices of energy in different forms.

(i) The result of applying $\Sigma F = ma$ and $\Sigma\tau = I\alpha$ to elevator car, counterweight, and sheave, and adding up the resulting equations is

$$(800\text{ kg} + n\ 80\text{ kg} - 950\text{ kg})(9.8\text{ m/s}^2) = [800\text{ kg} + n\ 80\text{ kg} + 950\text{ kg} + 140\text{ kg}]a$$

$\boxed{a = (9.80\text{ m/s}^2)(80n - 150)/(1\ 890 + 80n)}$ downward

***P10.63** (a) We model the assembly as a rigid body in equilibrium. Two torques acting on it are the frictional torque and the driving torque due to the emitted water:

$$\Sigma\tau = \tau_{thrust} - \tau_{friction} = 0 \qquad 3F\ell - b\omega = 0 \qquad \boxed{\omega = 3F\ell/b}$$

Notice that we have included a driving torque *only* from the single holes at distance ℓ. Because of the third assumption, the radially-directed water from the ends exerts no torque on the assembly—its thrust force is along the radial direction.

(b) We model the assembly as a rigid body under a net torque. Because the assembly begins from rest, there is no frictional torque at the beginning. Therefore,

$$\Sigma\tau = \tau_{thrust} = I\alpha \qquad 3F\ell = 3[mL^2/3]\alpha \qquad \alpha = \boxed{3F\ell/mL^2}$$

(c) The constant angular speed with which the assembly rotates will be larger. The arms are bent in the same direction as that in which the water is emitted from the holes at distance ℓ. This water will exert a force on the arms like that of a rocket exhaust. The driving torque from the water emitted from the ends will add to that from the single holes and the total driving torque will be larger. This will result in a larger angular speed.

(d) The bending of the arms has two effects on the initial angular acceleration. The driving torque is increased, as discussed in part (c). In addition, because the arms are bent, the moment of inertia of each arm is smaller than that for a straight arm. Looking at the answer to part (b), we see that both of these effects cause an increase in α, so the initial angular acceleration will be larger.

P10.64 $\alpha = -10.0\ \text{rad/s}^2 - (5.00\ \text{rad/s}^3)t = \dfrac{d\omega}{dt}$

$$\int_{65.0}^{\omega} d\omega = \int_0^t [-10.0 - 5.00t]\,dt = -10.0t - 2.50t^2 = \omega - 65.0\ \text{rad/s}$$

$$\omega = \frac{d\theta}{dt} = 65.0\ \text{rad/s} - (10.0\ \text{rad/s}^2)t - (2.50\ \text{rad/s}^3)t^2$$

(a) At $t = 3.00$ s,

$$\omega = 65.0\ \text{rad/s} - (10.0\ \text{rad/s}^2)(3.00\ \text{s}) - (2.50\ \text{rad/s}^3)(9.00\ \text{s}^2) = \boxed{12.5\ \text{rad/s}}$$

(b) $$\int_0^{\theta} d\theta = \int_0^t \omega\,dt = \int_0^t \left[65.0\ \text{rad/s} - (10.0\ \text{rad/s}^2)t - (2.50\ \text{rad/s}^3)t^2\right]dt$$

$$\theta = (65.0\ \text{rad/s})t - (5.00\ \text{rad/s}^2)t^2 - (0.833\ \text{rad/s}^3)t^3$$

At $t = 3.00$ s,

$$\theta = (65.0\ \text{rad/s})(3.00\ \text{s}) - (5.00\ \text{rad/s}^2)9.00\ \text{s}^2 - (0.833\ \text{rad/s}^3)27.0\ \text{s}^3$$

$$\theta = \boxed{128\ \text{rad}}$$

P10.65 (a) Since only conservative forces act within the system of the rod and the Earth,

$$\Delta E = 0 \quad \text{so} \quad K_f + U_f = K_i + U_i$$

$$\frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{L}{2}\right)$$

where

$$I = \frac{1}{3}ML^2$$

Therefore,

$$\omega = \boxed{\sqrt{\frac{3g}{L}}}$$

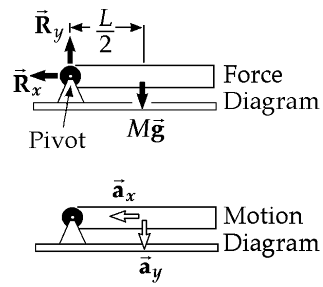

FIG. P10.65

(b) $\sum \tau = I\alpha$, so that in the horizontal orientation,

$$Mg\left(\frac{L}{2}\right) = \frac{ML^2}{3}\alpha$$

$$\alpha = \boxed{\frac{3g}{2L}}$$

(c) $a_x = a_r = -r\omega^2 = -\left(\frac{L}{2}\right)\omega^2 = \boxed{-\frac{3g}{2}}$ $\qquad a_y = -a_t = -r\alpha = -\alpha\left(\frac{L}{2}\right) = \boxed{-\frac{3g}{4}}$

(d) Using Newton's second law, we have

$$R_x = Ma_x = \boxed{-\frac{3Mg}{2}}$$

$$R_y - Mg = Ma_y = -\frac{3Mg}{4} \qquad R_y = \boxed{\frac{Mg}{4}}$$

P10.66 $K_f = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2$: $U_f = Mgh_f = 0$; $K_i = \frac{1}{2}Mv_i^2 + \frac{1}{2}I\omega_i^2 = 0$

$U_i = (Mgh)_i$: $f = \mu N = \mu Mg\cos\theta$; $\omega = \frac{v}{r}$; $h = d\sin\theta$ and $I = \frac{1}{2}mr^2$

(a) $\Delta E = E_f - E_i$ or $-fd = K_f + U_f - K_i - U_i$

$$-fd = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2 - Mgh$$

$$-(\mu Mg\cos\theta)d = \frac{1}{2}Mv^2 + \left(\frac{mr^2}{2}\right)\frac{v^2/r^2}{2} - Mgd\sin\theta$$

$$\frac{1}{2}\left[M + \frac{m}{2}\right]v^2 = Mgd\sin\theta - (\mu Mg\cos\theta)d \text{ or}$$

$$v^2 = 2Mgd\frac{(\sin\theta - \mu\cos\theta)}{m/2 + M}$$

$$v_d = \left[4gd\frac{M}{(m+2M)}(\sin\theta - \mu\cos\theta)\right]^{1/2}$$

(b) $v_f^2 = v_i^2 + 2a\Delta x$, $v_d^2 = 2ad$

$$a = \frac{v_d^2}{2d} = \boxed{2g\left(\frac{M}{m+2M}\right)(\sin\theta - \mu\cos\theta)}$$

P10.67 The first drop has a velocity leaving the wheel given by $\frac{1}{2}mv_i^2 = mgh_1$, so

$$v_1 = \sqrt{2gh_1} = \sqrt{2\left(9.80 \text{ m/s}^2\right)(0.540 \text{ m})} = 3.25 \text{ m/s}$$

The second drop has a velocity given by

$$v_2 = \sqrt{2gh_2} = \sqrt{2\left(9.80 \text{ m/s}^2\right)(0.510 \text{ m})} = 3.16 \text{ m/s}$$

From $\omega = \frac{v}{r}$, we find

$$\omega_1 = \frac{v_1}{r} = \frac{3.25 \text{ m/s}}{0.381 \text{ m}} = 8.53 \text{ rad/s} \text{ and } \omega_2 = \frac{v_2}{r} = \frac{3.16 \text{ m/s}}{0.381 \text{ m}} = 8.29 \text{ rad/s}$$

or

$$\alpha = \frac{\omega_2^2 - \omega_1^2}{2\theta} = \frac{(8.29 \text{ rad/s})^2 - (8.53 \text{ rad/s})^2}{4\pi} = \boxed{-0.322 \text{ rad/s}^2}$$

P10.68 At the instant it comes off the wheel, the first drop has a velocity v_1 directed upward. The magnitude of this velocity is found from

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}mv_1^2 + 0 = 0 + mgh_1 \text{ or } v_1 = \sqrt{2gh_1}$$

and the angular velocity of the wheel at the instant the first drop leaves is

$$\omega_1 = \frac{v_1}{R} = \sqrt{\frac{2gh_1}{R^2}}$$

Similarly for the second drop: $v_2 = \sqrt{2gh_2}$ and $\omega_2 = \frac{v_2}{R} = \sqrt{\frac{2gh_2}{R^2}}$

The angular acceleration of the wheel is then

$$a = \frac{\omega_2^2 - \omega_1^2}{2\theta} = \frac{2gh_2/R^2 - 2gh_1/R^2}{2(2\pi)} = \boxed{\frac{g(h_2 - h_1)}{2\pi R^2}}$$

P10.69 τ_f will oppose the torque due to the hanging object:

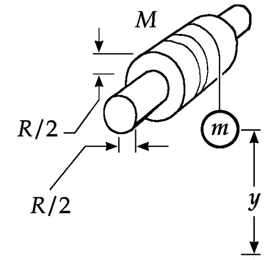

FIG. P10.69

$$\sum \tau = I\alpha = TR - \tau_f: \qquad \tau_f = TR - I\alpha \qquad (1)$$

Now find T, I and α in given or known terms and substitute into equation (1).

$$\sum F_y = T - mg = -ma: \qquad T = m(g-a) \qquad (2)$$

also

$$\Delta y = v_i t + \frac{at^2}{2} \qquad a = \frac{2y}{t^2} \qquad (3)$$

and

$$\alpha = \frac{a}{R} = \frac{2y}{Rt^2} \qquad (4)$$

$$I = \frac{1}{2}M\left[R^2 + \left(\frac{R}{2}\right)^2\right] = \frac{5}{8}MR^2 \qquad (5)$$

Substituting (2), (3), (4), and (5) into (1), we find

$$\tau_f = m\left(g - \frac{2y}{t^2}\right)R - \frac{5}{8}\frac{MR^2(2y)}{Rt^2} = \boxed{R\left[m\left(g - \frac{2y}{t^2}\right) - \frac{5}{4}\frac{My}{t^2}\right]}$$

P10.70 (a) $E = \frac{1}{2}\left(\frac{2}{5}MR^2\right)(\omega^2)$

$$E = \frac{1}{2}\cdot\frac{2}{5}(5.98\times10^{24})(6.37\times10^6)^2\left(\frac{2\pi}{86\,400}\right)^2 = \boxed{2.57\times10^{29}\ \text{J}}$$

(b) $$\frac{dE}{dt} = \frac{d}{dt}\left[\frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{2\pi}{T}\right)^2\right]$$

$$= \frac{1}{5}MR^2(2\pi)^2\left(-2T^{-3}\right)\frac{dT}{dt}$$

$$= \frac{1}{5}MR^2\left(\frac{2\pi}{T}\right)^2\left(\frac{-2}{T}\right)\frac{dT}{dt}$$

$$= \left(2.57\times10^{29}\ \text{J}\right)\left(\frac{-2}{86\,400\ \text{s}}\right)\left(\frac{10\times10^{-6}\ \text{s}}{3.16\times10^7\ \text{s}}\right)(86\,400\ \text{s/day})$$

$$\frac{dE}{dt} = \boxed{-1.63\times10^{17}\ \text{J/day}}$$

P10.71 (a) $m_2 g - T_2 = m_2 a$

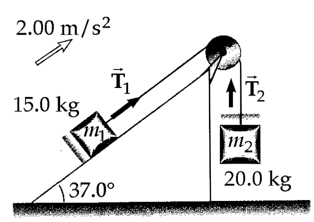

FIG. P10.71

$$T_2 = m_2(g-a) = 20.0\ \text{kg}\left(9.80\ \text{m/s}^2 - 2.00\ \text{m/s}^2\right) = \boxed{156\ \text{N}}$$

$$T_1 - m_1 g\sin 37.0° = m_1 a$$

$$T_1 = (15.0\ \text{kg})(9.80\sin 37.0° + 2.00)\ \text{m/s}^2 = \boxed{118\ \text{N}}$$

(b) $(T_2 - T_1)R = I\alpha = I\left(\frac{a}{R}\right)$

$$I = \frac{(T_2 - T_1)R^2}{a} = \frac{(156\ \text{N} - 118\ \text{N})(0.250\ \text{m})^2}{2.00\ \text{m/s}^2} = \boxed{1.17\ \text{kg}\cdot\text{m}^2}$$

P10.72 (a) $W = \Delta K + \Delta U$

$$W = K_f - K_i + U_f - U_i$$

$$0 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - mgd\sin\theta - \frac{1}{2}kd^2$$

$$\frac{1}{2}\omega^2\left(I + mR^2\right) = mgd\sin\theta + \frac{1}{2}kd^2$$

$$\boxed{\omega = \sqrt{\frac{2mgd\sin\theta + kd^2}{I + mR^2}}}$$

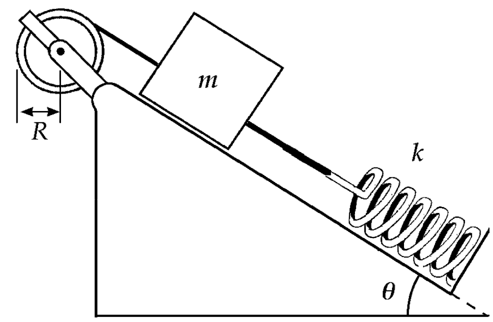

FIG. P10.72

(b) $$\omega = \sqrt{\frac{2(0.500\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})(\sin 37.0°) + 50.0\text{ N/m}(0.200\text{ m})^2}{1.00\text{ kg}\cdot\text{m}^2 + 0.500\text{ kg}(0.300\text{ m})^2}}$$

$$\omega = \sqrt{\frac{1.18 + 2.00}{1.05}} = \sqrt{3.04} = \boxed{1.74\text{ rad/s}}$$

P10.73 At $t = 0, \omega = 3.50\text{ rad/s} = \omega_0 e^0$. Thus, $\omega_0 = 3.50$ rad/s

At $t = 9.30\text{ s}, \omega = 2.00\text{ rad/s} = \omega_0 e^{-\sigma(9.30\text{ s})}$, yielding $\sigma = 6.02\times10^{-2}\text{ s}^{-1}$

(a) $$\alpha = \frac{d\omega}{dt} = \frac{d\left(\omega_0 e^{-\sigma t}\right)}{dt} = \omega_0(-\sigma)e^{-\sigma t}$$

At $t = 3.00$ s,

$$\alpha = (3.50\text{ rad/s})\left(-6.02\times10^{-2}\text{ s}^{-1}\right)e^{-3.00\left(6.02\times10^{-2}\right)} = \boxed{-0.176\text{ rad/s}^2}$$

(b) $$\theta = \int_0^t \omega_0 e^{-\sigma t}dt = \frac{\omega_0}{-\sigma}\left[e^{-\sigma t} - 1\right] = \frac{\omega_0}{\sigma}\left[1 - e^{-\sigma t}\right]$$

At $t = 2.50$ s,

$$\theta = \frac{3.50\text{ rad/s}}{\left(6.02\times10^{-2}\right)1/\text{s}}\left[1 - e^{-\left(6.02\times10^{-2}\right)(2.50)}\right] = 8.12\text{ rad} = \boxed{1.29\text{ rev}}$$

(c) As $t \to \infty, \theta \to \frac{\omega_0}{\sigma}\left(1 - e^{-\infty}\right) = \frac{3.50\text{ rad/s}}{6.02\times10^{-2}\text{ s}^{-1}} = 58.2\text{ rad} = \boxed{9.26\text{ rev}}$

P10.74 For the board just starting to move,

$$\sum\tau = I\alpha: \qquad mg\left(\frac{\ell}{2}\right)\cos\theta = \left(\frac{1}{3}m\ell^2\right)\alpha$$

$$\alpha = \frac{3}{2}\left(\frac{g}{\ell}\right)\cos\theta$$

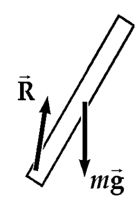

FIG. P10.74

The tangential acceleration of the end is $\quad a_t = \ell\alpha = \frac{3}{2}g\cos\theta$

The vertical component is $\quad a_y = a_t\cos\theta = \frac{3}{2}g\cos^2\theta$

If this is greater than g, the board will pull ahead of the ball falling:

(a) $\frac{3}{2}g\cos^2\theta \ge g$ gives $\cos^2\theta \ge \frac{2}{3}$ so $\cos\theta \ge \sqrt{\frac{2}{3}}$ and $\boxed{\theta \le 35.3°}$

continued on next page

(b) When $\theta = 35.3°$, the cup will land underneath the release-point of the ball if $r_c = \ell \cos\theta$

When $\ell = 1.00$ m, and $\theta = 35.3°$ $\quad r_c = 1.00 \text{ m}\sqrt{\dfrac{2}{3}} = 0.816 \text{ m}$

so the cup should be $(1.00 \text{ m} - 0.816 \text{ m}) = \boxed{0.184 \text{ m from the moving end}}$.

P10.75 (a) Let R_E represent the radius of the Earth. The base of the building moves east at $v_1 = \omega R_E$ where ω is one revolution per day. The top of the building moves east at $v_2 = \omega(R_E + h)$. Its eastward speed relative to the ground is $v_2 - v_1 = \omega h$. The object's time of fall is given by $\Delta y = 0 + \dfrac{1}{2}gt^2$, $t = \sqrt{\dfrac{2h}{g}}$. During its fall the object's eastward motion is unimpeded so its deflection distance is $\Delta x = (v_2 - v_1)t = \omega h\sqrt{\dfrac{2h}{g}} = \boxed{\omega h^{3/2}\left(\dfrac{2}{g}\right)^{1/2}}$.

(b) $\dfrac{2\pi \text{ rad}}{86\,400 \text{ s}}(50 \text{ m})^{3/2}\left(\dfrac{2 \text{ s}^2}{9.8 \text{ m}}\right)^{1/2} = \boxed{1.16 \text{ cm}}$

(c) The deflection is only 0.02% of the original height, so it is negligible in many practical cases.

P10.76 Consider the total weight of each hand to act at the center of gravity (mid-point) of that hand. Then the total torque (taking CCW as positive) of these hands about the center of the clock is given by

$$\tau = -m_h g\left(\frac{L_h}{2}\right)\sin\theta_h - m_m g\left(\frac{L_m}{2}\right)\sin\theta_m = -\frac{g}{2}\left(m_h L_h \sin\theta_h + m_m L_m \sin\theta_m\right)$$

If we take $t = 0$ at 12 o'clock, then the angular positions of the hands at time t are

$$\theta_h = \omega_h t$$

where

$$\omega_h = \frac{\pi}{6} \text{ rad/h}$$

and

$$\theta_m = \omega_m t$$

where

$$\omega_m = 2\pi \text{ rad/h}$$

Therefore,

$$\tau = -4.90 \text{ m/s}^2\left[60.0 \text{ kg}(2.70 \text{ m})\sin\left(\frac{\pi t}{6}\right) + 100 \text{ kg}(4.50 \text{ m})\sin 2\pi t\right]$$

or

$$\tau = -794 \text{ N}\cdot\text{m}\left[\sin\left(\frac{\pi t}{6}\right) + 2.78\sin 2\pi t\right], \text{ where } t \text{ is in hours.}$$

(i) (a) At 3:00, $t = 3.00$ h,

so $\quad \tau = -794 \text{ N}\cdot\text{m}\left[\sin\left(\dfrac{\pi}{2}\right) + 2.78\sin 6\pi\right] = \boxed{-794 \text{ N}\cdot\text{m}}$

(b) At 5:15, $t = 5 \text{ h} + \dfrac{15}{60} \text{ h} = 5.25$ h, and substitution gives:

$$\tau = \boxed{-2\,510 \text{ N}\cdot\text{m}}$$

continued on next page

(c) At 6:00, $\tau = \boxed{0\text{ N}\cdot\text{m}}$

(d) At 8:20, $\tau = \boxed{-1160\text{ N}\cdot\text{m}}$

(e) At 9:45, $\tau = \boxed{-2\,940\text{ N}\cdot\text{m}}$

(ii) The total torque is zero at those times when

$$\sin\left(\frac{\pi t}{6}\right) + 2.78\sin 2\pi t = 0$$

We proceed numerically, to find 0, 0.515 295 5,…, corresponding to the times

12:00:00	12:30:55	12:58:19	1:32:31	1:57:01
2:33:25	2:56:29	3:33:22	3:56:55	4:32:24
4:58:14	5:30:52	6:00:00	6:29:08	7:01:46
7:27:36	8:03:05	8:26:38	9:03:31	9:26:35
10:02:59	10:27:29	11:01:41	11:29:05	

P10.77 $\sum F = T - Mg = -Ma$: $\sum \tau = TR = I\alpha = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$

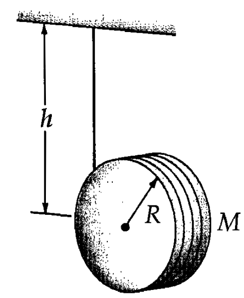

FIG. P10.77

(a) Combining the above two equations we find

$$T = M(g-a)$$

and

$$a = \frac{2T}{M}$$

thus

$$T = \boxed{\frac{Mg}{3}}$$

(b) $a = \frac{2T}{M} = \frac{2}{M}\left(\frac{Mg}{3}\right) = \boxed{\frac{2}{3}g}$

(c) $v_f^2 = v_i^2 + 2a(x_f - x_i)$ $\quad v_f^2 = 0 + 2\left(\frac{2}{3}g\right)(h-0)$

$$v_f = \boxed{\sqrt{\frac{4gh}{3}}}$$

For comparison, from conservation of energy for the system of the disk and the Earth we have

$$U_{gi} + K_{\text{rot}\,i} + K_{\text{trans}\,i} = U_{gf} + K_{\text{rot}\,f} + K_{\text{trans}\,f}: \qquad Mgh + 0 + 0 = 0 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_f}{R}\right)^2 + \frac{1}{2}Mv_f^{\,2}$$

$$v_f = \sqrt{\frac{4gh}{3}}$$

P10.78 Energy is conserved so $\Delta U + \Delta K_{\text{rot}} + \Delta K_{\text{trans}} = 0$

$$mg(R-r)(\cos\theta - 1) + \left[\frac{1}{2}mv^2 - 0\right] + \frac{1}{2}\left[\frac{2}{5}mr^2\right]\omega^2 = 0$$

Since $r\omega = v$, this gives

$$\omega = \sqrt{\frac{10}{7}\frac{(R-r)(1-\cos\theta)g}{r^2}}$$

or

$$\omega = \boxed{\sqrt{\frac{10Rg(1-\cos\theta)}{7r^2}}} \quad \text{since } R >> r$$

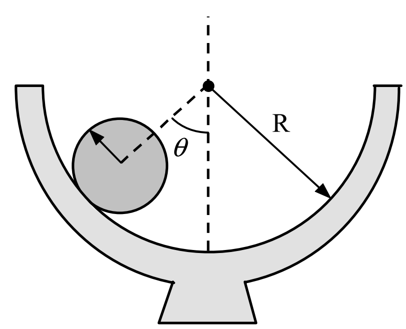

FIG. P10.78

P10.79 (a) $\Delta K_{\text{rot}} + \Delta K_{\text{trans}} + \Delta U = 0$

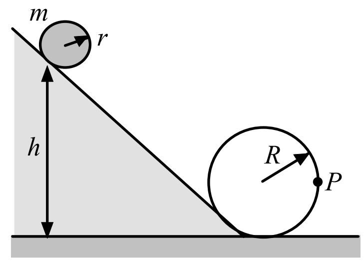

FIG. P10.79

Note that initially the center of mass of the sphere is a distance $h + r$ above the bottom of the loop; and as the mass reaches the top of the loop, this distance above the reference level is $2R - r$. The conservation of energy requirement gives

$$mg(h+r) = mg(2R-r) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

For the sphere $I = \frac{2}{5}mr^2$ and $v = r\omega$ so that the expression becomes

$$gh + 2gr = 2gR + \frac{7}{10}v^2 \qquad (1)$$

Note that $h = h_{\min}$ when the speed of the sphere at the top of the loop satisfies the condition

$$\sum F = mg = \frac{mv^2}{(R-r)} \quad \text{or} \quad v^2 = g(R-r)$$

Substituting this into Equation (1) gives

$$h_{\min} = 2(R-r) + 0.700(R-r) \text{ or } \boxed{h_{\min} = 2.70(R-r) = 2.70R}$$

(b) When the sphere is initially at $h = 3R$ and finally at point P, the conservation of energy equation gives

$$mg(3R+r) = mgR + \frac{1}{2}mv^2 + \frac{1}{5}mv^2, \text{ or } v^2 = \frac{10}{7}(2R+r)g$$

Turning clockwise as it rolls without slipping past point P, the sphere is slowing down with counterclockwise angular acceleration caused by the torque of an upward force f of static friction. We have $\sum F_y = ma_y$ and $\sum \tau = I\alpha$ becoming $f - mg = -m\alpha r$ and $fr = \left(\frac{2}{5}\right)mr^2\alpha.$

Eliminating f by substitution yields $\alpha = \frac{5g}{7r}$ so that $\sum F_y = \boxed{-\frac{5}{7}mg}$

$$\sum F_x = -n = -\frac{mv^2}{R-r} = -\frac{(10/7)(2R+r)}{R-r}mg = \boxed{\frac{-20mg}{7}} \text{ (since } R >> r\text{)}$$

P10.80 Consider the free-body diagram shown. The sum of torques about the chosen pivot is

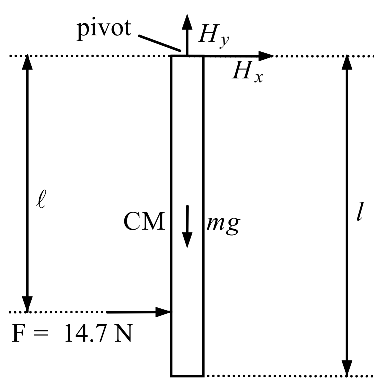

FIG. P10.80

$$\sum \tau = I\alpha \Rightarrow F\ell = \left(\frac{1}{3}ml^2\right)\left(\frac{a_{\text{CM}}}{\frac{l}{2}}\right) = \left(\frac{2}{3}ml\right)a_{\text{CM}} \qquad (1)$$

(a) $\ell = l = 1.24$ m: In this case, Equation (1) becomes

$$a_{\text{CM}} = \frac{3F}{2m} = \frac{3(14.7 \text{ N})}{2(0.630 \text{ kg})} = \boxed{35.0 \text{ m/s}^2}$$

$$\sum F_x = ma_{\text{CM}} \Rightarrow F + H_x = ma_{\text{CM}} \text{ or } H_x = ma_{\text{CM}} - F$$

Thus,

$$H_x = (0.630 \text{ kg})(35.0 \text{ m/s}^2) - 14.7 \text{ N} = +7.35 \text{ N}$$

or

$$\vec{\mathbf{H}}_x = \boxed{7.35\hat{\mathbf{i}} \text{ N}}$$

(b) $\ell = \frac{1}{2} = 0.620$ m: For this situation, Equation (1) yields

$$a_{\mathrm{CM}} = \frac{3F}{4m} = \frac{3(14.7\ \mathrm{N})}{4(0.630\ \mathrm{kg})} = \boxed{17.5\ \mathrm{m/s^2}}$$

Again, $\sum F_x = ma_{\mathrm{CM}} \Rightarrow H_x = ma_{\mathrm{CM}} - F$, so

$$H_x = (0.630\ \mathrm{kg})(17.5\ \mathrm{m/s^2}) - 14.7\ \mathrm{N} = -3.68\ \mathrm{N} \text{ or } \vec{\mathbf{H}}_x = \boxed{-3.68\hat{\mathbf{i}}\ \mathrm{N}}$$

(c) If $H_x = 0$, then $\sum F_x = ma_{\mathrm{CM}} \Rightarrow F = ma_{\mathrm{CM}}$, or $a_{\mathrm{CM}} = \dfrac{F}{m}$.

Thus, Equation (1) becomes

$$F\ell = \left(\frac{2}{3}ml\right)\left(\frac{F}{m}\right) \text{ so } \ell = \frac{2}{3}l = \frac{2}{3}(1.24\ \mathrm{m}) = \boxed{0.827\ \mathrm{m}\ (\text{from the top})}$$

P10.81 (a) There are not any horizontal forces acting on the rod, so the center of mass will not move horizontally. Rather, the center of mass drops straight downward (distance $h/2$) with the rod rotating about the center of mass as it falls. From conservation of energy:

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}Mv_{\mathrm{CM}}^2 + \frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{h}{2}\right) \text{ or}$$

$$\frac{1}{2}Mv_{\mathrm{CM}}^2 + \frac{1}{2}\left(\frac{1}{12}Mh^2\right)\left(\frac{v_{\mathrm{CM}}}{\frac{h}{2}}\right)^2 = Mg\left(\frac{h}{2}\right) \text{ which reduces to}$$

$$v_{\mathrm{CM}} = \boxed{\sqrt{\frac{3gh}{4}}}$$

(b) In this case, the motion is a pure rotation about a fixed pivot point (the lower end of the rod) with the center of mass moving in a circular path of radius $h/2$. From conservation of energy:

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{h}{2}\right) \text{ or}$$

$$\frac{1}{2}\left(\frac{1}{3}Mh^2\right)\left(\frac{v_{\mathrm{CM}}}{\frac{h}{2}}\right)^2 = Mg\left(\frac{h}{2}\right) \text{ which reduces to}$$

$$v_{\mathrm{CM}} = \boxed{\sqrt{\frac{3gh}{4}}}$$

P10.82 Conservation of energy between apex and the point where the grape leaves the surface:

$$mg\Delta y = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

$$mgR(1-\cos\theta) = \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v_f}{R}\right)^2$$

which gives $g(1-\cos\theta) = \frac{7}{10}\left(\frac{v_f^2}{R}\right)$ (1)

Consider the radial forces acting on the grape:

$$mg\cos\theta - n = \frac{mv_f^2}{R}$$

At the point where the grape leaves the surface, $n \to 0$.

Thus, $mg\cos\theta = \frac{mv_f^2}{R}$ or $\frac{v_f^2}{R} = g\cos\theta$

Substituting this into Equation (1) gives

$g - g\cos\theta = \frac{7}{10}g\cos\theta$ or $\cos\theta = \frac{10}{17}$ and $\theta = \boxed{54.0°}$

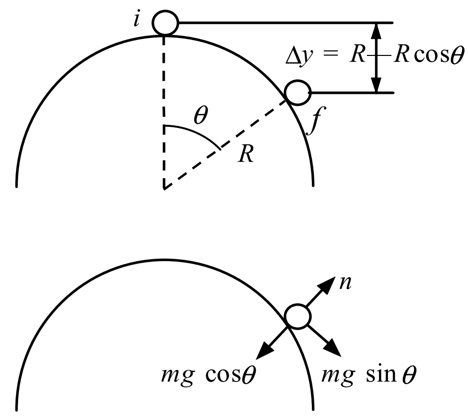

FIG. P10.82

P10.83 (a) $\sum F_x = F + f = Ma_{\text{CM}}$

$$\sum \tau = FR - fR = I\alpha$$

$$FR - (Ma_{\text{CM}} - F)R = \frac{Ia_{\text{CM}}}{R} \qquad \boxed{a_{\text{CM}} = \frac{4F}{3M}}$$

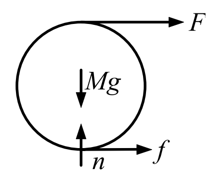

FIG. P10.83

(b) $f = Ma_{\text{CM}} - F = M\left(\frac{4F}{3M}\right) - F = \boxed{\frac{1}{3}F}$

(c) $v_f^2 = v_i^2 + 2a(x_f - x_i)$

$$v_f = \boxed{\sqrt{\frac{8Fd}{3M}}}$$

P10.84 Call f_t the frictional force exerted by each roller backward on the plank. Name as f_b the rolling resistance exerted backward by the ground on each roller. Suppose the rollers are equally far from the ends of the plank.

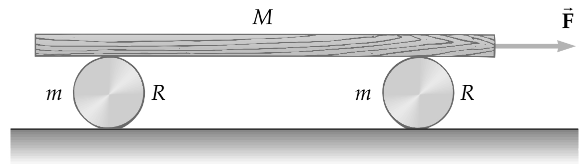

FIG. P10.84

For the plank,

$$\sum F_x = ma_x \qquad 6.00\text{ N} - 2f_t = (6.00\text{ kg})a_p$$

The center of each roller moves forward only half as far as the plank. Each roller has acceleration $\frac{a_p}{2}$ and angular acceleration

$$\frac{a_p/2}{(5.00\text{ cm})} = \frac{a_p}{(0.100\text{ m})}$$

Then for each,

$$\sum F_x = ma_x \qquad +f_t - f_b = (2.00\text{ kg})\frac{a_p}{2}$$

$$\sum \tau = I\alpha \qquad f_t(5.00\text{ cm}) + f_b(5.00\text{ cm}) = \frac{1}{2}(2.00\text{ kg})(5.00\text{ cm})^2\frac{a_p}{10.0\text{ cm}}$$

So

$$f_t + f_b = \left(\frac{1}{2}\text{ kg}\right)a_p$$

Add to eliminate f_b:

$$2f_t = (1.50\text{ kg})a_p$$

(a) And $6.00\text{ N} - (1.50\text{ kg})a_p = (6.00\text{ kg})a_p$

$$a_p = \frac{(6.00\text{ N})}{(7.50\text{ kg})} = \boxed{0.800\ \text{m/s}^2}$$

For each roller, $a = \frac{a_p}{2} = \boxed{0.400\ \text{m/s}^2}$

(b) Substituting back, $2f_t = (1.50\text{ kg})0.800\ \text{m/s}^2$

$$f_t = \boxed{0.600\text{ N}}$$

$$0.600\text{ N} + f_b = \frac{1}{2}\text{ kg}\left(0.800\ \text{m/s}^2\right)$$

$$f_b = -0.200\text{ N}$$

The negative sign means that the horizontal force of ground on each roller is $\boxed{0.200\text{ N forward}}$ rather than backward as we assumed.

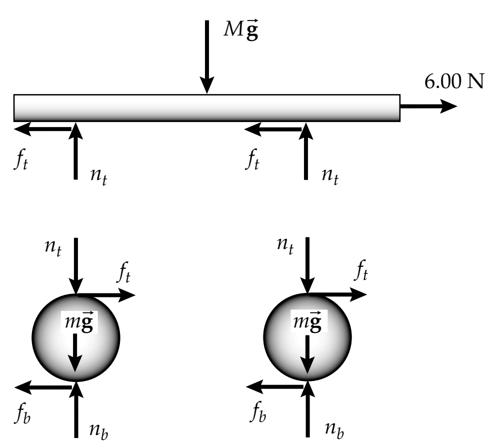

FIG. P10.84(b)

P10.85 $\sum F_x = ma_x$ reads $-f + T = ma$. If we take torques around the center of mass, we can use $\sum \tau = I\alpha$, which reads $+fR_2 - TR_1 = I\alpha$. For rolling without slipping, $\alpha = \dfrac{a}{R_2}$. By substitution,

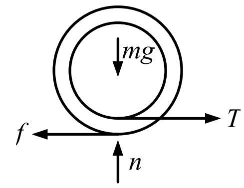

FIG. P10.85

$$fR_2 - TR_1 = \frac{Ia}{R_2} = \frac{I}{R_2 m}(T - f)$$

$$fR_2^2 m - TR_1 R_2 m = IT - If$$

$$f\left(I + mR_2^2\right) = T\left(I + mR_1 R_2\right)$$

$$f = \left(\frac{I + mR_1 R_2}{I + mR_2^2}\right)T$$

Since the answer is positive, the friction force is confirmed to be $\boxed{\text{to the left}}$.

P10.86 (a) The mass of the roll decreases as it unrolls. We have $m = \dfrac{Mr^2}{R^2}$ where M is the initial mass of the roll. Since $\Delta E = 0$, we then have $\Delta U_g + \Delta K_{\text{trans}} + \Delta K_{\text{rot}} = 0$. Thus, when $I = \dfrac{mr^2}{2}$,

$$(mgr - MgR) + \frac{mv^2}{2} + \left[\frac{mr^2}{2}\frac{\omega^2}{2}\right] = 0$$

Since $\omega r = v$, this becomes $v = \boxed{\sqrt{\dfrac{4g\left(R^3 - r^3\right)}{3r^2}}}$

(b) Using the given data, we find $v = \boxed{5.31 \times 10^4 \text{ m/s}}$

(c) We have assumed that $\Delta E = 0$. When the roll gets to the end, we will have an inelastic collision with the surface. $\boxed{\text{The energy goes into internal energy.}}$ With the assumption we made, there are problems with this question. It would take an infinite time to unwrap the tissue since $dr \to 0$. Also, as r approaches zero, the velocity of the center of mass approaches infinity, which is physically impossible.

ANSWERS TO EVEN PROBLEMS

P10.2 144 rad

P10.4 -226 rad/s^2

P10.6 13.7 rad/s^2

P10.8 (a) 2.88 s (b) 12.8 s

P10.10 (a) 0.180 rad/s (b) 8.10 m/s^2 toward the center of the track

P10.12 (a) 0.605 m/s (b) 17.3 rad/s (c) 5.82 m/s (d) the crank length is unnecessary

P10.14 (a) 25.0 rad/s (b) 39.8 rad/s^2 (c) 0.628 s

P10.16 (a) 54.3 rev (b) 12.1 rev/s

P10.18 (a) 5.77 cm (b) Yes. The ladder undergoes pure rotation about its right foot, with its angular displacement given in radians by $\theta = 0.690 \text{ m}/4.90 \text{ m} = t/0.410 \text{ m}$.

P10.20 (c) $\theta = \frac{2\pi r_i}{h}\left(\sqrt{1+\frac{vh}{\pi r_i^2}t}-1\right)$ (d) $\alpha = -\frac{hv^2}{2\pi r_i^3\left(1+\frac{vh}{\pi r_i^2}t\right)^{3/2}}$

P10.22 (a) $92.0\ \text{kg}\cdot\text{m}^2$; 184 J (b) 6.00 m/s; 4.00 m/s; 8.00 m/s; 184 J (c) The kinetic energies computed in parts (a) and (b) are the same. Rotational kinetic energy can be viewed as the total translational kinetic energy of the particles in the rotating object.

P10.24 The flywheel can be shaped like a cup or open barrel, 9.00 cm in outer radius and 7.68 cm in inner radius, with its wall 6 cm high, and with its bottom forming a disk 2.00 cm thick and 9.00 cm in radius. It is mounted to the crankshaft at the center of this disk and turns about its axis of symmetry. Its mass is 7.27 kg. If the disk were made somewhat thinner and the barrel wall thicker, the mass could be smaller.

P10.26 $11mL^2/12$

P10.28 $5.80\ \text{kg}\cdot\text{m}^2$ The height of the door is unnecessary.

P10.30 $23MR^2\omega^2/48$

P10.32 $168\ \text{N}\cdot\text{m}$ clockwise

P10.34 (a) 1.03 s (b) 10.3 rev

P10.36 (a) $21.6\ \text{kg}\cdot\text{m}^2$ (b) $3.60\ \text{N}\cdot\text{m}$ (c) 52.4 rev

P10.38 0.312

P10.40 25.1 N and 1.00 m or 41.8 N and 0.600 m; infinitely many answers exist, such that $TR = 25.1\ \text{N}\cdot\text{m}$

P10.42 1.04×10^{-3} J

P10.44 1.95 s If the pulley were massless, the time would be reduced by 3.64%

P10.46 (a) 6.90 J (b) 8.73 rad/s (c) 2.44 m/s (d) 1.043 2 times larger

P10.48 276 J

P10.50 (a) 74.3 W (b) 401 W

P10.52 (a) $v_f = [10gh/7]^{1/2}$ (b) $v_f = [2gh]^{1/2}$ (c) The time to roll is longer by a factor of 1.18

P10.54 (a) The cylinder (b) $v^2/4g\sin\theta$ (c) The cylinder does not lose mechanical energy because static friction does no work on it. Its rotation means that it has 50% more kinetic energy than the cube at the start, and so it travels 50% farther up the incline.

P10.56 The disk; $\sqrt{\frac{4gh}{3}}$ versus $\sqrt{gh}$

P10.58 (a) 2.38 m/s (b) 4.31 m/s (c) It will not reach the top of the loop.

P10.60 (a) 0.992 W (b) 827 W

P10.62 (a) $(1\,890 + 80n)0.459$ m$/(80n - 150)$ (b) 94.1 m (c) 1.62 m (d) −5.79 m (e) The rising car will coast to a stop only for $n \geq 2$. For $n = 0$ or $n = 1$, the car would accelerate upward if released. (f) The graph looks roughly like one branch of a hyperbola. It comes down steeply from 94.1 m for $n = 2$, flattens out, and very slowly approaches 0.459 m as n becomes large. (g) The radius of the sheave is not necessary. It divides out in the expression $(1/2)I\omega^2 = (1/4)m_{\text{sheave}}v^2$. (h) In this problem, as often in everyday life, energy conservation refers to minimizing use of electric energy or fuel. In physical theory, energy conservation refers to the constancy of the total energy of an isolated system, without regard to the different prices of energy in different forms. (i) $(9.80\text{ m/s}^2)(80n - 150)/(1\,890 + 80n)$

P10.64 (a) 12.5 rad/s (b) 128 rad

P10.66 (a) see the solution (b) $a = 2Mg(\sin\theta - \mu\cos\theta)/(m + 2M)$

P10.68 $\dfrac{g(h_2 - h_1)}{2\pi R^2}$

P10.70 (a) 2.57×10^{29} J (b) -1.63×10^{17} J/day

P10.72 (a) $\sqrt{\dfrac{2mgd\sin\theta + kd^2}{I + mR^2}}$ (b) 1.74 rad/s

P10.74 see the solution

P10.76 (i) -794 N·m; $-2\,510$ N·m; 0; $-1\,160$ N·m; $-2\,940$ N·m (ii) see the solution

P10.78 $\sqrt{\dfrac{10Rg(1-\cos\theta)}{7r^2}}$

P10.80 (a) 35.0 m/s^2; $7.35\hat{\mathbf{i}}$ N (b) 17.5 m/s^2; $-3.68\hat{\mathbf{i}}$ N (c) At 0.827 m from the top.

P10.82 54.0°

P10.84 (a) 0.800 m/s^2; 0.400 m/s^2 (b) 0.600 N between each cylinder and the plank; 0.200 N forward on each cylinder by the ground

P10.86 (a) $\sqrt{\dfrac{4g(R^3 - r^3)}{3r^2}}$ (b) 5.31×10^4 m/s (c) It becomes internal energy.

11

Angular Momentum

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q11.1 No to both questions. An axis of rotation must be defined to calculate the torque acting on an object. The moment arm of each force is measured from the axis, so the value of the torque depends on the location of the axis.

***Q11.2** (i) Down–cross–left is away from you: $-\hat{\mathbf{j}} \times \left(-\hat{\mathbf{i}}\right) = -\hat{\mathbf{k}}$ answer (f), as in the first picture.

(ii) Left–cross–down is toward you: $-\hat{\mathbf{i}} \times \left(-\hat{\mathbf{j}}\right) = \hat{\mathbf{k}}$ answer (e), as in the second picture.

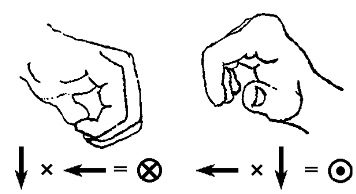

FIG. Q11.2

***Q11.3** (3 m down) × (2 N toward you) = 6 N · m left. The answers are (i) a (ii) a (iii) f

***Q11.4** The unit vectors have magnitude 1, so the magnitude of each cross product is $|1 \cdot 1 \cdot \sin\theta|$ where θ is the angle between the factors. Thus for (a) the magnitude of the cross product is $\sin 0° = 0$. For (b), $|\sin 135°| = 0.707$ (c) $\sin 90° = 1$ (d) $\sin 45° = 0.707$ (e) $\sin 90° = 1$. The assembled answer is c = e > b = d > a = 0.

Q11.5 Its angular momentum about that axis is constant in time. You cannot conclude anything about the magnitude of the angular momentum.

Q11.6 No. The angular momentum about any axis that does not lie along the instantaneous line of motion of the ball is nonzero.

***Q11.7** (a) Yes. Rotational kinetic energy is one contribution to a system's total energy.

(b) No. Pulling down on one side of a steering wheel and pushing up equally hard on the other side causes a total torque on the wheel with zero total force.

(c) No. A top spinning with its center of mass on a fixed axis has angular momentum with no momentum. A car driving straight toward a light pole has momentum but no angular momentum about the axis of the pole.

Q11.8 The long pole has a large moment of inertia about an axis along the rope. An unbalanced torque will then produce only a small angular acceleration of the performer-pole system, to extend the time available for getting back in balance. To keep the center of mass above the rope, the performer can shift the pole left or right, instead of having to bend his body around. The pole sags down at the ends to lower the system center of gravity.

***Q11.9** Her angular momentum stays constant as I is cut in half and ω doubles. Then $(1/2)I\omega^2$ doubles. Answer (b).

Q11.10 Since the source reel stops almost instantly when the tape stops playing, the friction on the source reel axle must be fairly large. Since the source reel appears to us to rotate at almost constant angular velocity, the angular acceleration must be very small. Therefore, the torque on the source reel due to the tension in the tape must almost exactly balance the frictional torque. In turn, the frictional torque is nearly constant because kinetic friction forces don't depend on velocity, and the radius of the axle where the friction is applied is constant. Thus we conclude that the torque exerted by the tape on the source reel is essentially constant in time as the tape plays.

As the source reel radius R shrinks, the reel's angular speed $\omega = \frac{v}{R}$ must increase to keep the tape speed v constant. But the biggest change is to the reel's moment of inertia. We model the reel as a roll of tape, ignoring any spool or platter carrying the tape. If we think of the roll of tape as a uniform disk, then its moment of inertia is $I = \frac{1}{2}MR^2$. But the roll's mass is proportional to its base area πR^2. Thus, on the whole the moment of inertia is proportional to R^4. The moment of inertia decreases very rapidly as the reel shrinks!

The tension in the tape coming into the read-and-write heads is normally dominated by balancing frictional torque on the source reel, according to $TR \approx \tau_{\text{friction}}$. Therefore, as the tape plays the tension is largest when the reel is smallest. However, in the case of a sudden jerk on the tape, the rotational dynamics of the source reel becomes important. If the source reel is full, then the moment of inertia, proportional to R^4, will be so large that higher tension in the tape will be required to give the source reel its angular acceleration. If the reel is nearly empty, then the same tape acceleration will require a smaller tension. Thus, the tape will be more likely to break when the source reel is nearly full. One sees the same effect in the case of paper towels; it is easier to snap a towel free when the roll is new than when it is nearly empty.

***Q11.11** The angular momentum of the mouse-turntable system is initially zero, with both at rest. The frictionless axle isolates the mouse-turntable system from outside torques, so its angular momentum must stay constant with the value zero.

(i) The mouse makes some progress north, or counterclockwise. Answer (a).

(ii) The turntable will rotate clockwise. The turntable rotates in the direction opposite to the motion of the mouse, for the angular momentum of the system to remain zero. Answer (b).

(iii) No, mechanical energy changes as the mouse converts some chemical into mechanical energy, positive for the motions of both the mouse and the turntable.

(iv) No, momentum is not conserved. The turntable has zero momentum while the mouse has a bit of northward momentum. The sheave around the turntable axis exerts a force northward to feed in this momentum.

(v) Yes, angular momentum is constant with the value zero.

***Q11.12** (i) The angular momentum is constant. The moment of inertia decreases, so the angular speed must increase. Answer (a).

(ii) No, mechanical energy increases. The ponies must do work to push themselves inward.

(iii) Yes, momentum stays constant with the value zero.

(iv) Yes, angular momentum is constant with a nonzero value.

***Q11.13** Angular momentum is conserved according to the equation $I_1\omega_0 + 0 = (I_1 + I_2)\omega_f$. Solving for ω_f gives answer (c).

Q11.14 Suppose we look at the motorcycle moving to the right. Its drive wheel is turning clockwise. The wheel speeds up when it leaves the ground. No outside torque about its center of mass acts on the airborne cycle, so its angular momentum is conserved. As the drive wheel's clockwise angular momentum increases, the frame of the cycle acquires counterclockwise angular momentum. The cycle's front end moves up and its back end moves down.

Q11.15 Mass moves away from axis of rotation, so moment of inertia increases, angular speed decreases, and period increases. We would not have more hours in a day, but more nanoseconds.

Q11.16 The suitcase might contain a spinning gyroscope. If the gyroscope is spinning about an axis that is oriented horizontally passing through the bellhop, the force he applies to turn the corner results in a torque that could make the suitcase swing away. If the bellhop turns quickly enough, anything at all could be in the suitcase and need not be rotating. Since the suitcase is massive, it will want to follow an inertial path. This could be perceived as the suitcase swinging away by the bellhop.

SOLUTIONS TO PROBLEMS

Section 11.1 **The Vector Product and Torque**

P11.1 $$\vec{\mathbf{M}}\times\vec{\mathbf{N}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 6 & 2 & -1 \\ 2 & -1 & -3 \end{vmatrix} = \hat{\mathbf{i}}(-6-1) + \hat{\mathbf{j}}(-2+18) + \hat{\mathbf{k}}(-6-4) = \boxed{-7.00\hat{\mathbf{i}} + 16.0\hat{\mathbf{j}} - 10.0\hat{\mathbf{k}}}$$

P11.2 (a) $$\text{area} = \left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = AB\sin\theta = (42.0\text{ cm})(23.0\text{ cm})\sin(65.0° - 15.0°) = \boxed{740\text{ cm}^2}$$

(b) $$\vec{\mathbf{A}}+\vec{\mathbf{B}} = \left[(42.0\text{ cm})\cos 15.0° + (23.0\text{ cm})\cos 65.0°\right]\hat{\mathbf{i}}$$
$$+\left[(42.0\text{ cm})\sin 15.0° + (23.0\text{ cm})\sin 65.0°\right]\hat{\mathbf{j}}$$
$$\vec{\mathbf{A}}+\vec{\mathbf{B}} = (50.3\text{ cm})\hat{\mathbf{i}} + (31.7\text{ cm})\hat{\mathbf{j}}$$
$$\text{length} = \left|\vec{\mathbf{A}}+\vec{\mathbf{B}}\right| = \sqrt{(50.3\text{ cm})^2 + (31.7\text{ cm})^2} = \boxed{59.5\text{ cm}}$$

P11.3 (a) $$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -3 & 4 & 0 \\ 2 & 3 & 0 \end{vmatrix} = \boxed{-17.0\hat{\mathbf{k}}}$$

(b) $$\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = \left|\vec{\mathbf{A}}\right|\left|\vec{\mathbf{B}}\right|\sin\theta$$

$$17 = 5\sqrt{13}\sin\theta$$
$$\theta = \sin^{-1}\left(\frac{17}{5\sqrt{13}}\right) = \boxed{70.6°}$$

P11.4 $\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = -3.00(6.00)+7.00(-10.0)+(-4.00)(9.00) = -124$

$$AB = \sqrt{(-3.00)^2+(7.00)^2+(-4.00)^2}\cdot\sqrt{(6.00)^2+(-10.0)^2+(9.00)^2} = 127$$

(a) $\cos^{-1}\left(\dfrac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB}\right) = \cos^{-1}(-0.979) = \boxed{168°}$

(b) $$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -3.00 & 7.00 & -4.00 \\ 6.00 & -10.0 & 9.00 \end{vmatrix} = 23.0\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}} - 12.0\hat{\mathbf{k}}$$

$$\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = \sqrt{(23.0)^2+(3.00)^2+(-12.0)^2} = 26.1$$

$$\sin^{-1}\left(\frac{\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right|}{AB}\right) = \sin^{-1}(0.206) = \boxed{11.9°} \text{ or } 168°$$

(c) Only $\boxed{\text{the first method}}$ gives the angle between the vectors unambiguously.

P11.5 $\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}}\times\vec{\mathbf{F}}$

$= 0.450\ \text{m}(0.785\ \text{N})\sin(90° - 14°)\,\text{up}\times\text{east}$

$= \boxed{0.343\ \text{N}\cdot\text{m}\quad\text{horizontally north}}$

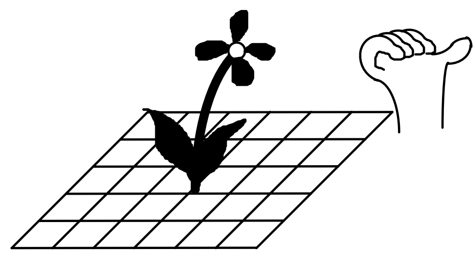

FIG. P11.5

P11.6 The cross-product vector must be perpendicular to both of the factors, so its dot product with either factor must be zero:

Does $(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})\cdot(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}) = 0$?

We have $8 - 9 - 4 = -5 \neq 0$ so the answer is

$\boxed{\text{No. The cross product could not work out that way.}}$

P11.7 $\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = \vec{\mathbf{A}}\cdot\vec{\mathbf{B}} \quad\Rightarrow\quad AB\sin\theta = AB\cos\theta \quad\Rightarrow\quad \tan\theta = 1$ or

$\theta = \boxed{45.0°}$

***P11.8** (a) $\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 4 & 6 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \hat{\mathbf{i}}(0-0) - \hat{\mathbf{j}}(0-0) + \hat{\mathbf{k}}(8-18) = \boxed{(-10.0\ \mathrm{N\cdot m})\hat{\mathbf{k}}}$

(b) Yes. The line of action of the force is the dashed line in the diagram. The point or axis must be on the other side of the line of action, and half as far from this line along which the force acts. Then the lever arm of the force about this new axis will be half as large and the force will produce counterclockwise instead of clockwise torque. There are infinitely many such points, along the dotted line in the diagram. But the locus of these points intersects the *y* axis in only one point, which we now determine.

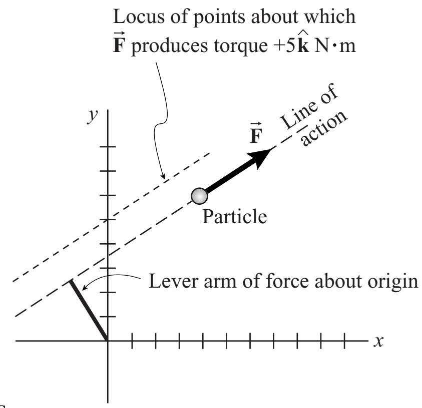

FIG. P11.8(b)

Let (0, *y*) represent the coordinates of the special axis of rotation located on the *y* axis of coordinates. Then the displacement from this point to the particle feeling the force is $\vec{\mathbf{r}}_{new} = 4\hat{\mathbf{i}} + (6-y)\hat{\mathbf{j}}$ meters. The torque of the force about this new axis is

$$\vec{\boldsymbol{\tau}}_{new} = \vec{\mathbf{r}}_{new} \times \vec{\mathbf{F}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 4 & 6-y & 0 \\ 3 & 2 & 0 \end{vmatrix} = \hat{\mathbf{i}}(0-0) - \hat{\mathbf{j}}(0-0) + \hat{\mathbf{k}}(8-18+3y) = (+5\ \mathrm{N\cdot m})\hat{\mathbf{k}}$$

Then we need only $-10 + 3y = 5$ $\quad y = 5$ m. The position vector of the new axis is $\boxed{5.00\hat{\mathbf{j}}\ \mathrm{m}}$.

P11.9 $\boxed{\left|\vec{\mathbf{F}}_3\right| = \left|\vec{\mathbf{F}}_1\right| + \left|\vec{\mathbf{F}}_2\right|}$

The torque produced by $\vec{\mathbf{F}}_3$ depends on the perpendicular distance **OD**, therefore translating the point of application of $\vec{\mathbf{F}}_3$ to any other point along **BC** $\boxed{\text{will not change the net torque}}$.

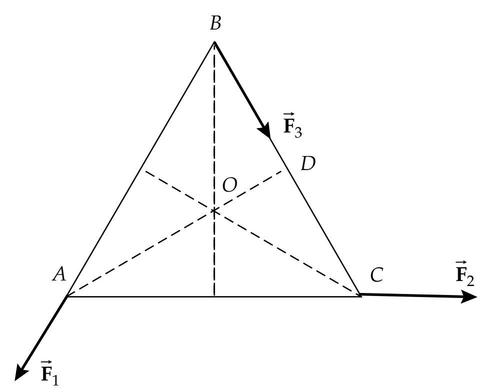

FIG. P11.9

P11.10 $\left|\hat{\mathbf{i}} \times \hat{\mathbf{i}}\right| = 1 \cdot 1 \cdot \sin 0° = 0$

$\hat{\mathbf{j}} \times \hat{\mathbf{j}}$ and $\hat{\mathbf{k}} \times \hat{\mathbf{k}}$ are zero similarly since the vectors being multiplied are parallel.

$\left|\hat{\mathbf{i}} \times \hat{\mathbf{j}}\right| = 1 \cdot 1 \cdot \sin 90° = 1$

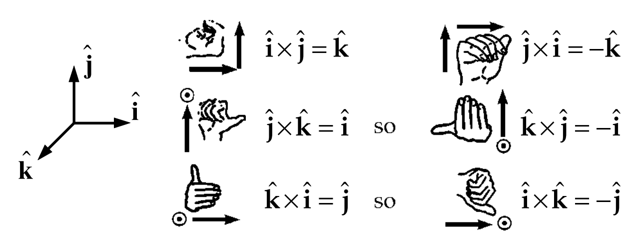

FIG. P11.10

Section 11.2 **Angular Momentum**

P11.11 $L = \sum m_i v_i r_i$

$$= (4.00\text{ kg})(5.00\text{ m/s})(0.500\text{ m}) + (3.00\text{ kg})(5.00\text{ m/s})(0.500\text{ m})$$

$L = 17.5\text{ kg}\cdot\text{m}^2/\text{s}$, and

$$\boxed{\vec{\mathbf{L}} = \left(17.5\text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$$

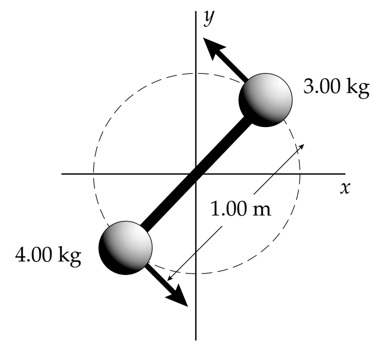

FIG. P11.11

P11.12 $\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$

$$\vec{\mathbf{L}} = \left(1.50\hat{\mathbf{i}} + 2.20\hat{\mathbf{j}}\right)\text{ m} \times (1.50\text{ kg})\left(4.20\hat{\mathbf{i}} - 3.60\hat{\mathbf{j}}\right)\text{ m/s}$$

$$\vec{\mathbf{L}} = \left(-8.10\hat{\mathbf{k}} - 13.9\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m}^2/\text{s} = \boxed{\left(-22.0\text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$$

P11.13 $\vec{\mathbf{r}} = \left(6.00\hat{\mathbf{i}} + 5.00t\hat{\mathbf{j}}\text{ m}\right)$ $\qquad \vec{\mathbf{v}} = \dfrac{d\vec{\mathbf{r}}}{dt} = 5.00\hat{\mathbf{j}}\text{ m/s}$

so

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} = 2.00\text{ kg}\left(5.00\hat{\mathbf{j}}\text{ m/s}\right) = 10.0\hat{\mathbf{j}}\text{ kg}\cdot\text{m/s}$$

and

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 6.00 & 5.00t & 0 \\ 0 & 10.0 & 0 \end{vmatrix} = \boxed{\left(60.0\text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$$

P11.14 $\sum F_x = ma_x \qquad T\sin\theta = \dfrac{mv^2}{r}$

$\sum F_y = ma_y \qquad T\cos\theta = mg$

So

$$\frac{\sin\theta}{\cos\theta} = \frac{v^2}{rg} \qquad v = \sqrt{rg\frac{\sin\theta}{\cos\theta}}$$

$$L = rmv\sin 90.0°$$

$$L = rm\sqrt{rg\frac{\sin\theta}{\cos\theta}}$$

$$L = \sqrt{m^2 g r^3 \frac{\sin\theta}{\cos\theta}}$$

$r = \ell\sin\theta$, so

$$L = \boxed{\sqrt{m^2 g \ell^3 \frac{\sin^4\theta}{\cos\theta}}}$$

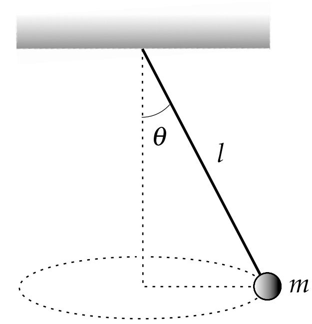

FIG. P11.14

P11.15 The angular displacement of the particle around the circle is $\theta = \omega t = \frac{vt}{R}$.

The vector from the center of the circle to the mass is then $R\cos\theta\,\hat{\mathbf{i}} + R\sin\theta\,\hat{\mathbf{j}}$.

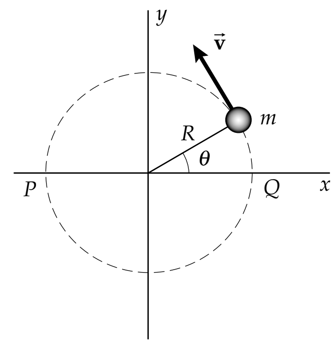

FIG. P11.15

The vector from point P to the mass is

$$\vec{\mathbf{r}} = R\hat{\mathbf{i}} + R\cos\theta\,\hat{\mathbf{i}} + R\sin\theta\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{r}} = R\left[\left(1+\cos\left(\frac{vt}{R}\right)\right)\hat{\mathbf{i}} + \sin\left(\frac{vt}{R}\right)\hat{\mathbf{j}}\right]$$

The velocity is

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = -v\sin\left(\frac{vt}{R}\right)\hat{\mathbf{i}} + v\cos\left(\frac{vt}{R}\right)\hat{\mathbf{j}}$$

So

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times m\vec{\mathbf{v}}$$

$$\vec{\mathbf{L}} = mvR\left[(1+\cos\omega t)\hat{\mathbf{i}} + \sin\omega t\hat{\mathbf{j}}\right] \times \left[-\sin\omega t\hat{\mathbf{i}} + \cos\omega t\hat{\mathbf{j}}\right]$$

$$\vec{\mathbf{L}} = \boxed{mvR\hat{\mathbf{k}}\left[\cos\left(\frac{vt}{R}\right)+1\right]}$$

P11.16 (a) The net torque on the counterweight-cord-spool system is:

$$|\vec{\tau}| = |\vec{\mathbf{r}} \times \vec{\mathbf{F}}| = 8.00\times 10^{-2}\text{ m}(4.00\text{ kg})(9.80\text{ m/s}^2) = \boxed{3.14\text{ N}\cdot\text{m}}$$

(b) $|\vec{\mathbf{L}}| = |\vec{\mathbf{r}} \times m\vec{\mathbf{v}}| + I\omega$ $\qquad$ $|\vec{\mathbf{L}}| = Rmv + \frac{1}{2}MR^2\left(\frac{v}{R}\right) = R\left(m + \frac{M}{2}\right)v = \boxed{(0.400\text{ kg}\cdot\text{m})v}$

(c) $\tau = \frac{dL}{dt} = (0.400\text{ kg}\cdot\text{m})a$ $\qquad$ $a = \frac{3.14\text{ N}\cdot\text{m}}{0.400\text{ kg}\cdot\text{m}} = \boxed{7.85\text{ m/s}^2}$

P11.17 (a) $\boxed{\text{zero}}$

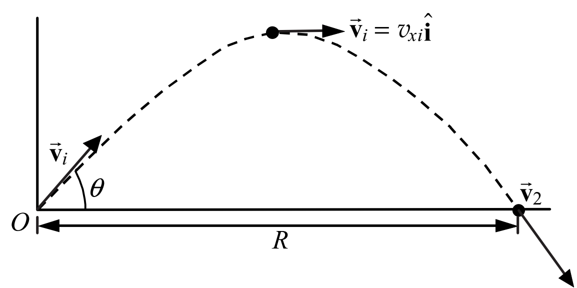

FIG. P11.17

(b) At the highest point of the trajectory,

$$x = \frac{1}{2}R = \frac{v_i^2\sin 2\theta}{2g} \text{ and}$$

$$y = h_{\max} = \frac{(v_i\sin\theta)^2}{2g}$$

$$\vec{\mathbf{L}}_1 = \vec{\mathbf{r}}_1 \times m\vec{\mathbf{v}}_1$$

$$= \left[\frac{v_i^2\sin 2\theta}{2g}\hat{\mathbf{i}} + \frac{(v_i\sin\theta)^2}{2g}\hat{\mathbf{j}}\right] \times mv_{xi}\hat{\mathbf{i}}$$

$$= \boxed{\frac{-m(v_i\sin\theta)^2 v_i\cos\theta}{2g}\hat{\mathbf{k}}}$$

continued on next page

(c) $\vec{\mathbf{L}}_2 = R\hat{\mathbf{i}} \times m\vec{\mathbf{v}}_2$, where $R = \dfrac{v_i^2 \sin 2\theta}{g}$

$$= mR\hat{\mathbf{i}} \times \left(v_i \cos\theta\, \hat{\mathbf{i}} - v_i \sin\theta\, \hat{\mathbf{j}}\right)$$

$$= -mRv_i \sin\theta\, \hat{\mathbf{k}} = \boxed{\frac{-mv_i^3 \sin 2\theta \sin\theta}{g}\hat{\mathbf{k}}}$$

(d) The downward force of gravity exerts a torque in the $-z$ direction.

P11.18 Whether we think of the Earth's surface as curved or flat, we interpret the problem to mean that the plane's line of flight extended is precisely tangent to the mountain at its peak, and nearly parallel to the wheat field. Let the positive x direction be eastward, positive y be northward, and positive z be vertically upward.

(a) $\vec{\mathbf{r}} = (4.30\text{ km})\hat{\mathbf{k}} = \left(4.30\times10^3\text{ m}\right)\hat{\mathbf{k}}$

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} = 12\,000\text{ kg}\left(-175\hat{\mathbf{i}}\text{ m/s}\right) = -2.10\times10^6\,\hat{\mathbf{i}}\text{ kg}\cdot\text{m/s}$$

$$\vec{\mathbf{L}} = \vec{\mathbf{r}}\times\vec{\mathbf{p}} = \left(4.30\times10^3\hat{\mathbf{k}}\text{ m}\right)\times\left(-2.10\times10^6\,\hat{\mathbf{i}}\text{ kg}\cdot\text{m/s}\right) = \boxed{\left(-9.03\times10^9\text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{j}}}$$

(b) $\boxed{\text{No.}}$ $L = |\vec{\mathbf{r}}||\vec{\mathbf{p}}|\sin\theta = mv(r\sin\theta)$, and $r\sin\theta$ is the altitude of the plane. Therefore, $L = $ *constant* as the plane moves in level flight with constant velocity.

(c) $\boxed{\text{Zero.}}$ The position vector from Pike's Peak to the plane is anti-parallel to the velocity of the plane. That is, it is directed along the same line and opposite in direction. Thus, $L = mvr\sin 180° = 0$.

***P11.19** (a) The vector from P to the falling ball is

$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$$

$$\vec{\mathbf{r}} = \left(\ell\cos\theta\,\hat{\mathbf{i}} + \ell\sin\theta\,\hat{\mathbf{j}}\right) + 0 - \left(\frac{1}{2}gt^2\right)\hat{\mathbf{j}}$$

The velocity of the ball is

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = 0 - gt\hat{\mathbf{j}}$$

So

$$\vec{\mathbf{L}} = \vec{\mathbf{r}}\times m\vec{\mathbf{v}}$$

$$\vec{\mathbf{L}} = m\left[\left(\ell\cos\theta\,\hat{\mathbf{i}} + \ell\sin\theta\,\hat{\mathbf{j}}\right) + 0 - \left(\frac{1}{2}gt^2\right)\hat{\mathbf{j}}\right]\times\left(-gt\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{L}} = \boxed{-m\ell gt\cos\theta\,\hat{\mathbf{k}}}$$

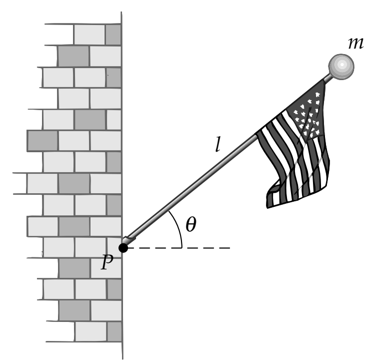

FIG. P11.19

(b) The Earth exerts a gravitational torque on the ball.

(c) Differentiating with respect to time, we have $\boxed{-mg\ell\cos\theta\,\hat{\mathbf{k}}}$ for the rate of change of angular momentum, which is also the torque due to the gravitational force on the ball.

***P11.20** (a) $\int_0^{\vec{\mathbf{r}}} d\vec{\mathbf{r}} = \int_0^t \vec{\mathbf{v}}dt = \vec{\mathbf{r}} - 0 = \int_0^t (6t^2\hat{\mathbf{i}} + 2t\hat{\mathbf{j}})dt = \vec{\mathbf{r}} = \left(6t^3/3\right)\hat{\mathbf{i}} + \left(2t^2/2\right)\hat{\mathbf{j}}$

$= \boxed{2t^3\hat{\mathbf{i}} + t^2\hat{\mathbf{j}}}$ meters, where t is in seconds

(b) The particle starts from rest at the origin, starts moving in the y direction, and gains speed faster and faster while turning to move more and more nearly parallel to the x axis.

(c) $\vec{\mathbf{a}} = (d/dt)(6t^2\hat{\mathbf{i}} + 2t\hat{\mathbf{j}}) = \boxed{(12t\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\ \text{m/s}^2}$

(d) $\vec{\mathbf{F}} = m\vec{\mathbf{a}} = (5\text{ kg})(12t\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\ \text{m/s}^2 = \boxed{(60t\hat{\mathbf{i}} + 10\hat{\mathbf{j}})\text{ N}}$

(e) $\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = (2t^3\hat{\mathbf{i}} + t^2\hat{\mathbf{j}}) \times (60t\hat{\mathbf{i}} + 10\hat{\mathbf{j}}) = 20t^3\hat{\mathbf{k}} - 60t^3\hat{\mathbf{k}} = \boxed{-40t^3\hat{\mathbf{k}}\text{ N}\cdot\text{m}}$

(f) $\vec{\mathbf{L}} = \vec{\mathbf{r}} \times m\vec{\mathbf{v}} = (5\text{ kg})(2t^3\hat{\mathbf{i}} + t^2\hat{\mathbf{j}}) \times (6t^2\hat{\mathbf{i}} + 2t\hat{\mathbf{j}}) = 5(4t^4\hat{\mathbf{k}} - 6t^4\hat{\mathbf{k}}) = \boxed{-10t^4\hat{\mathbf{k}}\text{ kg}\cdot\text{m}^2/\text{s}}$

(g) $K = \frac{1}{2}m\vec{\mathbf{v}}\cdot\vec{\mathbf{v}} = \frac{1}{2}(5\text{ kg})(6t^2\hat{\mathbf{i}} + 2t\hat{\mathbf{j}})\cdot(6t^2\hat{\mathbf{i}} + 2t\hat{\mathbf{j}}) = (2.5)(36t^4 + 4t^2) = \boxed{(90t^4 + 10t^2)\text{ J}}$

(h) $\mathcal{P} = (d/dt)(90t^4 + 10t^2)\text{ J} \boxed{= (360t^3 + 20t)\text{ W}}$, all where t is in seconds.

Section 11.3 **Angular Momentum of a Rotating Rigid Object**

P11.21 $K = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}\dfrac{I^2\omega^2}{I} = \dfrac{L^2}{2I}$

P11.22 The moment of inertia of the sphere about an axis through its center is

$$I = \frac{2}{5}MR^2 = \frac{2}{5}(15.0\text{ kg})(0.500\text{ m})^2 = 1.50\text{ kg}\cdot\text{m}^2$$

Therefore, the magnitude of the angular momentum is

$$L = I\omega = \left(1.50\text{ kg}\cdot\text{m}^2\right)(3.00\text{ rad/s}) = 4.50\text{ kg}\cdot\text{m}^2/\text{s}$$

Since the sphere rotates counterclockwise about the vertical axis, the angular momentum vector is directed upward in the $+z$ direction.

Thus,

$$\boxed{\vec{\mathbf{L}} = \left(4.50\text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$$

P11.23 (a) $L = I\omega = \left(\dfrac{1}{2}MR^2\right)\omega = \dfrac{1}{2}(3.00\text{ kg})(0.200\text{ m})^2(6.00\text{ rad/s}) = \boxed{0.360\text{ kg}\cdot\text{m}^2/\text{s}}$

(b) $L = I\omega = \left[\dfrac{1}{2}MR^2 + M\left(\dfrac{R}{2}\right)^2\right]\omega$

$= \dfrac{3}{4}(3.00\text{ kg})(0.200\text{ m})^2(6.00\text{ rad/s}) = \boxed{0.540\text{ kg}\cdot\text{m}^2/\text{s}}$

***P11.24** (a) $I = (2/5)MR^2 = (2/5)(5.98 \times 10^{24}\text{ kg})(6.37 \times 10^6\text{ m})^2 = 9.71 \times 10^{37}\text{ kg} \cdot \text{m}^2$

$\omega = 1\text{ rev}/24\text{ h} = 2\pi\text{ rad}/86400\text{ s} = 7.27 \times 10^{-5}/\text{s}$

$L = I\omega = (9.71 \times 10^{37}\text{ kg} \cdot \text{m}^2)(7.27 \times 10^{-5}/\text{s}) = \boxed{7.06 \times 10^{33}\ \text{kg} \cdot \text{m}^2/\text{s}}$

The earth turns toward the east, counterclockwise as seen from above north, so the vector angular momentum points north along the earth's axis, $\boxed{\text{toward the north celestial pole}}$ or nearly toward the star Polaris.

(b) $I = MR^2 = (5.98 \times 10^{24}\text{ kg})(1.496 \times 10^{11}\text{ m})^2 = 1.34 \times 10^{47}\text{ kg} \cdot \text{m}^2$

$\omega = 1\text{ rev}/365.25\text{ d} = 2\pi\text{ rad}/(365.25 \times 86400\text{ s}) = 1.99 \times 10^{-7}/\text{s}$

$L = I\omega = (1.34 \times 10^{47}\text{ kg} \cdot \text{m}^2)(1.99 \times 10^{-7}/\text{s}) = \boxed{2.66 \times 10^{40}\ \text{kg} \cdot \text{m}^2/\text{s}}$

The earth plods around the Sun, counterclockwise as seen from above north, so the vector angular momentum points north perpendicular to the plane of the ecliptic, $\boxed{\text{toward the north ecliptic pole}}$ or 23.5° away from Polaris, toward the center of the circle that the north celestial pole moves in as the equinoxes precess. The north ecliptic pole is in the constellation Draco.

(c) The earth is so far from the Sun that the $\boxed{\text{orbital angular momentum is much larger,}}$ $\boxed{\text{by } 3.78 \times 10^6 \text{ times.}}$

P11.25 (a) $I = \frac{1}{12}m_1L^2 + m_2(0.500)^2 = \frac{1}{12}(0.100)(1.00)^2 + 0.400(0.500)^2 = 0.108\,3\text{ kg} \cdot \text{m}^2$

$L = I\omega = 0.108\,3(4.00) = \boxed{0.433\ \text{kg} \cdot \text{m}^2/\text{s}}$

(b) $I = \frac{1}{3}m_1L^2 + m_2R^2 = \frac{1}{3}(0.100)(1.00)^2 + 0.400(1.00)^2 = 0.433$

$L = I\omega = 0.433(4.00) = \boxed{1.73\ \text{kg} \cdot \text{m}^2/\text{s}}$

P11.26 The total angular momentum about the center point is given by $L = I_h\omega_h + I_m\omega_m$

with

$$I_h = \frac{m_hL_h^2}{3} = \frac{60.0\text{ kg}(2.70\text{ m})^2}{3} = 146\text{ kg} \cdot \text{m}^2$$

and

$$I_m = \frac{m_mL_m^2}{3} = \frac{100\text{ kg}(4.50\text{ m})^2}{3} = 675\text{ kg} \cdot \text{m}^2$$

In addition,

$$\omega_h = \frac{2\pi\text{ rad}}{12\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.45 \times 10^{-4}\ \text{rad/s}$$

while

$$\omega_m = \frac{2\pi\text{ rad}}{1\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.75 \times 10^{-3}\ \text{rad/s}$$

Thus,

$$L = 146\text{ kg} \cdot \text{m}^2\left(1.45 \times 10^{-4}\ \text{rad/s}\right) + 675\text{ kg} \cdot \text{m}^2\left(1.75 \times 10^{-3}\ \text{rad/s}\right)$$

or $\boxed{L = 1.20\ \text{kg} \cdot \text{m}^2/\text{s}}$ The hands turn clockwise, so their vector angular momentum is $\boxed{\text{perpendicularly into the clock face}}$.

P11.27 We require $a_c = g = \frac{v^2}{r} = \omega^2 r$

$$\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{(9.80 \text{ m/s}^2)}{100 \text{ m}}} = 0.313 \text{ rad/s}$$
$$I = Mr^2 = 5\times10^4 \text{ kg}(100 \text{ m})^2 = 5\times10^8 \text{ kg}\cdot\text{m}^2$$

(a) $L = I\omega = 5\times10^8 \text{ kg}\cdot\text{m}^2\, 0.313/\text{s} = \boxed{1.57\times10^8 \text{ kg}\cdot\text{m}^2/\text{s}}$

(b) $\sum \tau = I\alpha = \frac{I(\omega_f - \omega_i)}{\Delta t}$

$\sum \tau \Delta t = I\omega_f - I\omega_i = L_f - L_i$

This is the angular impulse-angular momentum theorem.

(c) $\Delta t = \frac{L_f - 0}{\sum \tau} = \frac{1.57\times10^8 \text{ kg}\cdot\text{m}^2/\text{s}}{2(125 \text{ N})(100 \text{ m})} = \boxed{6.26\times10^3 \text{ s}} = 1.74 \text{ h}$

P11.28 $\sum F_x = ma_x$: $\quad +f_s = ma_x$

We must use the center of mass as the axis in

$\sum \tau = I\alpha$: $\quad F_g(0) - n(77.5 \text{ cm}) + f_s(88 \text{ cm}) = 0$

$\sum F_y = ma_y$: $\quad +n - F_g = 0$

We combine the equations by substitution:

$$-mg(77.5 \text{ cm}) + ma_x(88 \text{ cm}) = 0$$
$$a_x = \frac{(9.80 \text{ m/s}^2)77.5 \text{ cm}}{88 \text{ cm}} = \boxed{8.63 \text{ m/s}^2}$$

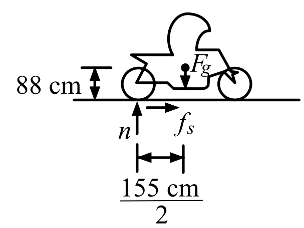

FIG. P11.28

Section 11.4 **Conservation of Angular Momentum**

P11.29 (a) From conservation of angular momentum for the system of two cylinders:

$(I_1 + I_2)\omega_f = I_1\omega_i \quad$ or $\quad \omega_f = \boxed{\frac{I_1}{I_1 + I_2}\omega_i}$

(b) $K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2 \quad$ and $\quad K_i = \frac{1}{2}I_1\omega_i^2$

so

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}(I_1 + I_2)}{\frac{1}{2}I_1\omega_i^2}\left(\frac{I_1}{I_1 + I_2}\omega_i\right)^2 = \boxed{\frac{I_1}{I_1 + I_2} \text{ which is less than 1}}$$

***P11.30** (a) We choose to solve by conservation of angular momentum, because it would be true even if the rod had considerable mass:

$$I\omega_{initial} = I\omega_{final} \qquad mR^2(v/R)_i = (mR^2 + m_pR^2)(v/R)_f$$

$(2.4\text{ kg})(1.5\text{ m})(5\text{ m/s}) = (2.4 + 1.3)\text{kg}(1.5\text{ m})\,v_f \quad v_f = 3.24\text{ m/s} = 2\pi(1.5\text{ m})/T \quad T = \boxed{2.91\text{ s}}$

(b) Angular momentum of the puck-putty system is conserved because the pivot exerts no torque.

(c) If the putty-puck collision lasts so short a time that the puck slides through a negligibly small arc of the circle, then momentum is also conserved. But the pivot pin is always pulling on the rod to change the direction of the momentum.

(d) No. Some mechanical energy is converted into internal energy. The collision is perfectly inelastic.

***P11.31** (a) We solve by using conservation of angular momentum for the turntable-clay system, which is isolated from outside torques:

$$I\omega_{initial} = I\omega_{final} \qquad (1/2)mR^2(\omega)_i = [(1/2)mR^2 + m_cR^2)]\omega_f$$

$(1/2)(30\text{ kg})(1.9\text{ m})^2(4\pi\text{ /s}) = [(1/2)(30\text{ kg})(1.9\text{ m})^2 + (2.25\text{ kg})(1.8\text{ m})^2]\,\omega_f$

$(54.15)(4\pi) = (61.44)\omega_f \quad \omega_f = \boxed{11.1\text{ rad/s counterclockwise}}$

(b) $\boxed{\text{No.}}$ The "angular collision" is completely inelastic, so some mechanical energy is degraded into internal energy. The initial energy is $(1/2)I\omega_i^2 = (1/2)(54.15)(4\pi)^2 = 4276$ J. The final mechanical energy is $(1/2)(61.44)(11.1)^2 = 3768$ J. Thus $\boxed{\text{507 J of extra internal energy appears}}$.

(c) $\boxed{\text{No.}}$ The turntable bearing must exert an impulsive force toward the north.The original horizontal momentum is zero. As soon as the clay has stopped skidding on the turntable, the final momentum is $2.25\text{ kg }(1.8\text{ m})(11.1\text{ /s}) = \boxed{44.9\text{ kg}\cdot\text{m/s north. This is the amount of impulse injected by the bearing.}}$ The bearing thereafter keeps changing the system momentum to change the direction of the motion of the clay.

P11.32 (a) The total angular momentum of the system of the student, the stool, and the weights about the axis of rotation is given by

$$I_{\text{total}} = I_{\text{weights}} + I_{\text{student}} = 2\left(mr^2\right) + 3.00\text{ kg}\cdot\text{m}^2$$

Before:

$$r = 1.00\text{ m}$$

Thus,

$$I_i = 2(3.00\text{ kg})(1.00\text{ m})^2 + 3.00\text{ kg}\cdot\text{m}^2 = 9.00\text{ kg}\cdot\text{m}^2$$

After:

$$r = 0.300\text{ m}$$

Thus,

$$I_f = 2(3.00\text{ kg})(0.300\text{ m})^2 + 3.00\text{ kg}\cdot\text{m}^2 = 3.54\text{ kg}\cdot\text{m}^2$$

We now use conservation of angular momentum.

$$I_f\omega_f = I_i\omega_i$$

or

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{9.00}{3.54}\right)(0.750\text{ rad/s}) = \boxed{1.91\text{ rad/s}}$$

continued on next page

(b) $K_i = \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}(9.00\ \text{kg}\cdot\text{m}^2)(0.750\ \text{rad/s})^2 = \boxed{2.53\ \text{J}}$

$K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(3.54\ \text{kg}\cdot\text{m}^2)(1.91\ \text{rad/s})^2 = \boxed{6.44\ \text{J}}$

P11.33 $I_i\omega_i = I_f\omega_f$: $(250\ \text{kg}\cdot\text{m}^2)(10.0\ \text{rev/min}) = \left[250\ \text{kg}\cdot\text{m}^2 + 25.0\ \text{kg}(2.00\ \text{m})^2\right]\omega_2$

$\omega_2 = \boxed{7.14\ \text{rev/min}}$

***P11.34** (a) Let M = mass of rod and m = mass of each bead. From $I_i\omega_i = I_f\omega_f$ between the moment of release and the moment the beads slide off, we have

$$\left[\frac{1}{12}M\ell^2 + 2mr_1^2\right]\omega_i = \left[\frac{1}{12}M\ell^2 + 2mr_2^2\right]\omega_f$$

When $M = 0.3$ kg, $\ell = 0.500$ m, $r_1 = 0.100$ m, $r_2 = 0.250$ m, $\omega_i = 36$/s, we find

$[0.00625 + 0.02\ m]36 = [0.00625 + 0.125\ m]\ \omega_f$

$\boxed{\omega_f = (36/\text{s})(1 + 3.2\ m)/(1 + 20\ m)}$

(b) The denominator of this fraction always exceeds the numerator, so

ω_f decreases smoothly from a maximum value of 36.0 rad/s for $m = 0$ toward a minimum value of $(36 \times 3.2/20) = 5.76$ rad/s as $m \to \infty$.

As a bonus, we find the work that the bar does on the beads as a function of m. Consider the beads alone. Their kinetic energy increases because of work done on them by the bar.

initial kinetic energy + work = final kinetic energy

$(1/2)(2mr_1^{\,2})(\omega_i)^2 + W_b = (1/2)(2mr_2^{\,2})(\omega_f)^2$

$m\ (0.1)^2(36)^2 + W_b = m\ (0.25)^2[(36/\text{s})(1 + 3.2\ m)/(1 + 20\ m)]^2$

$W_b = m[81(1 + 3.2m)^2 - 12.96(1 + 20m)^2]/\ (1 + 20\ m)^2$
$= (68.04\ m)(1 - 64\ m^2)/(1 + 20\ m)^2$ joules

W_b increases from 0 for $m = 0$ toward a maximum value of about 0.8 J at about $m = 0.035$ kg, and then decreases and goes negative, diverging to $-\infty$ as $m\to\infty$.

***P11.35** (a) Mechanical energy is not conserved; some chemical energy is converted into mechanical energy. Momentum is not conserved. The turntable bearing exerts an external northward force on the axle. Angular momentum is conserved. The bearing isolates the system from outside torques. The table turns opposite to the way the woman walks, so its angular momentum cancels that of the woman.

continued on next page

(b) From conservation of angular momentum for the system of the woman and the turntable, we have $L_f = L_i = 0$

so,

$$L_f = I_{\text{woman}}\omega_{\text{woman}} + I_{\text{table}}\omega_{\text{table}} = 0$$

and

$$\omega_{\text{table}} = \left(-\frac{I_{\text{woman}}}{I_{\text{table}}}\right)\omega_{\text{woman}} = \left(-\frac{m_{\text{woman}}r^2}{I_{\text{table}}}\right)\left(\frac{v_{\text{woman}}}{r}\right) = -\frac{m_{\text{woman}}rv_{\text{woman}}}{I_{\text{table}}}$$

$$\omega_{\text{table}} = -\frac{60.0\text{ kg}(2.00\text{ m})(1.50\text{ m/s})}{500\text{ kg}\cdot\text{m}^2} = -0.360\text{ rad/s}$$

or

$$\omega_{\text{table}} = \boxed{0.360\text{ rad/s (counterclockwise)}}$$

(c) chemical energy converted into mechanical $= \Delta K = K_f - 0 = \frac{1}{2}m_{\text{woman}}v_{\text{woman}}^2 + \frac{1}{2}I\omega_{\text{table}}^2$

$$\Delta K = \frac{1}{2}(60\text{ kg})(1.50\text{ m/s})^2 + \frac{1}{2}(500\text{ kg}\cdot\text{m}^2)(0.360\text{ rad/s})^2 = \boxed{99.9\text{ J}}$$

P11.36 When they touch, the center of mass is distant from the center of the larger puck by

$$y_{\text{CM}} = \frac{0 + 80.0\text{ g}(4.00\text{ cm} + 6.00\text{ cm})}{120\text{ g} + 80.0\text{ g}} = 4.00\text{ cm}$$

(a) $L = r_1m_1v_1 + r_2m_2v_2 = 0 + (6.00\times10^{-2}\text{ m})(80.0\times10^{-3}\text{ kg})(1.50\text{ m/s})$
$= \boxed{7.20\times10^{-3}\text{ kg}\cdot\text{m}^2/\text{s}}$

(b) The moment of inertia about the CM is

$$I = \left(\frac{1}{2}m_1r_1^2 + m_1d_1^2\right) + \left(\frac{1}{2}m_2r_2^2 + m_2d_2^2\right)$$

$$I = \frac{1}{2}(0.120\text{ kg})(6.00\times10^{-2}\text{ m})^2 + (0.120\text{ kg})(4.00\times10^{-2})^2$$
$$+\frac{1}{2}(80.0\times10^{-3}\text{ kg})(4.00\times10^{-2}\text{ m})^2 + (80.0\times10^{-3}\text{ kg})(6.00\times10^{-2}\text{ m})^2$$

$$I = 7.60\times10^{-4}\text{ kg}\cdot\text{m}^2$$

Angular momentum of the two-puck system is conserved: $L = I\omega$

$$\omega = \frac{L}{I} = \frac{7.20\times10^{-3}\text{ kg}\cdot\text{m}^2/\text{s}}{7.60\times10^{-4}\text{ kg}\cdot\text{m}^2} = \boxed{9.47\text{ rad/s}}$$

P11.37 (a) $L_i = mv\ell \quad \sum \tau_{\text{ext}} = 0,$

so

$$L_f = L_i = \boxed{mv\ell}$$

$$L_f = (m+M)v_f\ell$$

$$v_f = \left(\frac{m}{m+M}\right)v$$

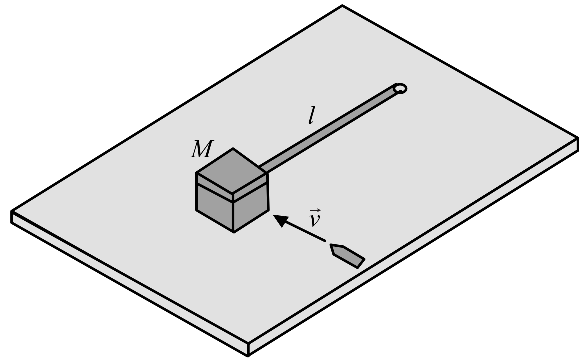

FIG. P11.37

(b) $K_i = \frac{1}{2}mv^2$

$$K_f = \frac{1}{2}(M+m)v_f^2$$

$$v_f = \left(\frac{m}{M+m}\right)v \Rightarrow \text{ velocity of the bullet and block}$$

$$\text{Fraction of } K \text{ lost } = \frac{\frac{1}{2}mv^2 - \frac{1}{2}m^2v^2/(M+m)}{\frac{1}{2}mv^2} = \boxed{\frac{M}{M+m}}$$

P11.38 For one of the crew,

$$\sum F_r = ma_r: \qquad n = \frac{mv^2}{r} = m\omega_i^2 r$$

We require $\qquad n = mg$, so $\omega_i = \sqrt{\frac{g}{r}}$

Now, $\qquad I_i\omega_i = I_f\omega_f$

$$\left[5.00\times10^8\ \text{kg}\cdot\text{m}^2 + 150\times65.0\ \text{kg}\times(100\ \text{m})^2\right]\sqrt{\frac{g}{r}}$$

$$= \left[5.00\times10^8\ \text{kg}\cdot\text{m}^2 + 50\times65.0\ \text{kg}(100\ \text{m})^2\right]\omega_f$$

$$\left(\frac{5.98\times10^8}{5.32\times10^8}\right)\sqrt{\frac{g}{r}} = \omega_f = 1.12\sqrt{\frac{g}{r}}$$

Now,

$$|a_r| = \omega_f^2 r = 1.26g = \boxed{12.3\ \text{m/s}^2}$$

***P11.39** (a) Consider the system to consist of the wad of clay and the cylinder. No external forces acting on this system have a torque about the center of the cylinder. Thus, angular momentum of the system is conserved about the axis of the cylinder.

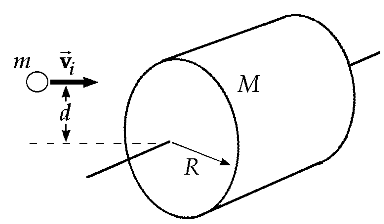

FIG. P11.39

$$L_f = L_i: \qquad I\omega = mv_i d$$

or

$$\left[\frac{1}{2}MR^2 + mR^2\right]\omega = mv_i d$$

Thus,

$$\omega = \boxed{\frac{2mv_i d}{(M+2m)R^2}}$$

continued on next page

(b) $\boxed{\text{No; some mechanical energy changes into internal energy.}}$

(c) $\boxed{\text{Momentum is not conserved. The axle exerts a backward force on the cylinder.}}$

P11.40 (a) Let ω be the angular speed of the signboard when it is vertical.

$$\frac{1}{2}I\omega^2 = Mgh$$

$$\therefore \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 = Mg\frac{1}{2}L(1-\cos\theta)$$

$$\therefore \omega = \sqrt{\frac{3g(1-\cos\theta)}{L}}$$

$$= \sqrt{\frac{3(9.80\ \text{m/s}^2)(1-\cos 25.0°)}{0.50\ \text{m}}}$$

$$= \boxed{2.35\ \text{rad/s}}$$

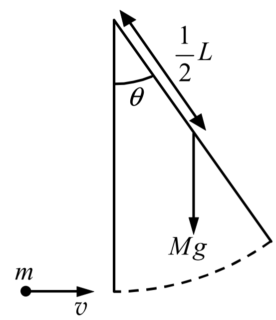

FIG. P11.40

(b) $I_i\omega_i - mvL = I_f\omega_f$ represents angular momentum conservation for the sign-snowball system.

In more detail,

$$\left(\frac{1}{3}ML^2 + mL^2\right)\omega_f = \frac{1}{3}ML^2\omega_i - mvL$$

Solving,

$$\omega_f = \frac{\frac{1}{3}ML\omega_i - mv}{\left(\frac{1}{3}M + m\right)L}$$

$$= \frac{\frac{1}{3}(2.40\ \text{kg})(0.5\ \text{m})(2.347\ \text{rad/s}) - (0.4\ \text{kg})(1.6\ \text{m/s})}{\left[\frac{1}{3}(2.40\ \text{kg}) + 0.4\ \text{kg}\right](0.5\ \text{m})} = \boxed{0.498\ \text{rad/s}}$$

(c) Let h_{CM} = distance of center of mass from the axis of rotation.

$$h_{\text{CM}} = \frac{(2.40\ \text{kg})(0.25\ \text{m}) + (0.4\ \text{kg})(0.50\ \text{m})}{2.40\ \text{kg} + 0.4\ \text{kg}} = 0.285\,7\ \text{m}$$

Apply conservation of mechanical energy:

$$(M+m)gh_{\text{CM}}(1-\cos\theta) = \frac{1}{2}\left(\frac{1}{3}ML^2 + mL^2\right)\omega^2$$

$$\therefore \theta = \cos^{-1}\left[1 - \frac{\left(\frac{1}{3}M + m\right)L^2\omega^2}{2(M+m)gh_{\text{CM}}}\right]$$

$$= \cos^{-1}\left\{1 - \frac{\left[\frac{1}{3}(2.40\ \text{kg}) + 0.4\ \text{kg}\right](0.50\ \text{m})^2(0.498\ \text{rad/s})^2}{2(2.40\ \text{kg} + 0.4\ \text{kg})(9.80\ \text{m/s}^2)(0.285\,7\ \text{m})}\right\}$$

$$= \boxed{5.58°}$$

P11.41 $\boxed{\text{The meteor will slow the rotation of the Earth by the largest amount if its line of motion passes farthest from the Earth's axis. The meteor should be headed west and strike a point on the equator tangentially.}}$

Let the z axis coincide with the axis of the Earth with $+z$ pointing northward. Then, conserving angular momentum about this axis,

$$\sum \vec{\mathbf{L}}_f = \sum \vec{\mathbf{L}}_i \Rightarrow I\omega_f = I\omega_i + m\vec{\mathbf{v}} \times \vec{\mathbf{r}}$$

or

$$\frac{2}{5}MR^2\omega_f\hat{\mathbf{k}} = \frac{2}{5}MR^2\omega_i\hat{\mathbf{k}} - mvR\hat{\mathbf{k}}$$

Thus,

$$\omega_i - \omega_f = \frac{mvR}{\frac{2}{5}MR^2} = \frac{5mv}{2MR}$$

or

$$\omega_i - \omega_f = \frac{5(3.00\times10^{13}\text{ kg})(30.0\times10^3\text{ m/s})}{2(5.98\times10^{24}\text{ kg})(6.37\times10^6\text{ m})} = 5.91\times10^{-14}\text{ rad/s}$$

$$\boxed{|\Delta\omega_{\max}| \sim 10^{-13}\text{ rad/s}}$$

Section 11.5 **The Motion of Gyroscopes and Tops**

P11.42 Angular momentum of the system of the spacecraft and the gyroscope is conserved. The gyroscope and spacecraft turn in opposite directions.

$$0 = I_1\omega_1 + I_2\omega_2: \qquad -I_1\omega_1 = I_2\frac{\theta}{t}$$

$$-20\text{ kg}\cdot\text{m}^2(-100\text{ rad/s}) = 5\times10^5\text{ kg}\cdot\text{m}^2\left(\frac{30°}{t}\right)\left(\frac{\pi\text{ rad}}{180°}\right)$$

$$t = \frac{2.62\times10^5\text{ s}}{2\,000} = \boxed{131\text{ s}}$$

P11.43 $I = \frac{2}{5}MR^2 = \frac{2}{5}(5.98\times10^{24}\text{ kg})(6.37\times10^6\text{ m})^2 = 9.71\times10^{37}\text{ kg}\cdot\text{m}^2$

$$L = I\omega = 9.71\times10^{37}\text{ kg}\cdot\text{m}^2\left(\frac{2\pi\text{ rad}}{86\,400\text{ s}}\right) = 7.06\times10^{33}\text{ kg}\cdot\text{m}^2/\text{s}^2$$

$$\tau = L\omega_p = (7.06\times10^{33}\text{ kg}\cdot\text{m}^2/\text{s})\left(\frac{2\pi\text{ rad}}{2.58\times10^4\text{ yr}}\right)\left(\frac{1\text{ yr}}{365.25\text{ d}}\right)\left(\frac{1\text{ d}}{86\,400\text{ s}}\right) = \boxed{5.45\times10^{22}\text{ N}\cdot\text{m}}$$

Additional Problems

P11.44 First, we define the following symbols:

I_P = moment of inertia due to mass of people on the equator

I_E = moment of inertia of the Earth alone (without people)

ω = angular velocity of the Earth (due to rotation on its axis)

$T = \dfrac{2\pi}{\omega}$ = rotational period of the Earth (length of the day)

R = radius of the Earth

The initial angular momentum of the system (before people start running) is

$$L_i = I_P\omega_i + I_E\omega_i = (I_P + I_E)\omega_i$$

When the Earth has angular speed ω, the tangential speed of a point on the equator is $v_t = R\omega$. Thus, when the people run eastward along the equator at speed v relative to the surface of the Earth, their tangential speed is $v_p = v_t + v = R\omega + v$ and their angular speed is $\omega_P = \dfrac{v_p}{R} = \omega + \dfrac{v}{R}$.

The angular momentum of the system after the people begin to run is

$$L_f = I_P\omega_p + I_E\omega = I_P\left(\omega + \frac{v}{R}\right) + I_E\omega = (I_P + I_E)\omega + \frac{I_P v}{R}$$

Since no external torques have acted on the system, angular momentum is conserved $(L_f = L_i)$, giving $(I_P + I_E)\omega + \dfrac{I_P v}{R} = (I_P + I_E)\omega_i$. Thus, the final angular velocity of the Earth is

$\omega = \omega_i - \dfrac{I_P v}{(I_P + I_E)R} = \omega_i(1 - x) =$, where $x \equiv \dfrac{I_P v}{(I_P + I_E)R\omega_i}$

The new length of the day is $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\omega_i(1-x)} = \dfrac{T_i}{1-x} \approx T_i(1+x)$, so the increase in the length of the day is $\Delta T = T - T_i \approx T_i x = T_i\left[\dfrac{I_P v}{(I_P + I_E)R\omega_i}\right]$. Since $\omega_i = \dfrac{2\pi}{T_i}$, this may be written as

$$\Delta T \approx \frac{T_i^2 I_P v}{2\pi(I_P + I_E)R}$$

To obtain a numeric answer, we compute

$$I_P = m_p R^2 = \left[(7\times10^9)(70\text{ kg})\right](6.37\times10^6\text{ m})^2 = 1.99\times10^{25}\text{ kg}\cdot\text{m}^2$$

and

$$I_E = \frac{2}{5}m_E R^2 = \frac{2}{5}(5.98\times10^{24}\text{ kg})(6.37\times10^6\text{ m})^2 = 9.71\times10^{37}\text{ kg}\cdot\text{m}^2$$

Thus,

$$\Delta T \approx \frac{(8.64\times10^4\text{ s})^2(1.99\times10^{25}\text{ kg}\cdot\text{m}^2)(2.5\text{ m/s})}{2\pi\left[(1.99\times10^{25} + 9.71\times10^{37})\text{ kg}\cdot\text{m}^2\right](6.37\times10^6\text{ m})} = \boxed{9.55\times10^{-11}\text{ s}}$$

***P11.45** (a) Momentum is conserved for the system of two men:
(162 kg)(+ 8 m/s) + (81 kg)(−11 m/s) = (243 kg) $\vec{\mathbf{v}}_f$ $\qquad \vec{\mathbf{v}}_f = \boxed{1.67\ \hat{\mathbf{i}}\ \text{m/s}}$

(b) original mechanical energy = (1/2)(162 kg)(+8 m/s)^2 + (1/2)(81 kg)(−11 m/s)^2 = 10 084 J
final mechanical energy = (1/2)(243 kg)(1.67 m/s)^2 = 338 J

Thus the fraction remaining is 338/10 084 = $\boxed{0.033\ 5}$ = 3.35%

(c) The calculation in part (a) still applies: $\vec{\mathbf{v}}_f = \boxed{1.67\ \hat{\mathbf{i}}\ \text{m/s}}$

(d) With half the mass of Perry, Flutie is distant from the center of mass by (2/3)(1.2 m) = 0.8 m. His angular speed relative to the center of mass just before they link arms is $\omega = v/r$ = (11 + 1.67)(m/s)/0.8 m = 15.8 rad/s. That of Perry is necessarily the same (8 − 1.67)/0.4 m = 15.8 rad/s.

In their linking of arms, angular momentum is conserved. Their total moment of inertia stays constant, so their angular speed also stays constant at $\boxed{15.8\ \text{rad/s}}$.

(e) Only the men's direction of motion is changed by their linking arms. Each keeps constant speed relative to the center of mass and the center of mass keeps constant speed, so all of the kinetic energy is still present. The fraction remaining mechanical is $\boxed{1.00 = 100\%}$.

We can compute this explicitly: the final total kinetic energy is
(1/2)(243 kg)(1.67 m/s)^2 + (1/2)[(81 kg)(0.8 m)^2 + (162 kg)(0.4 m)^2]((15.8 /s)^2 =
338 J + 6498 J + 3249 J = 10 084 J, the same as the original kinetic energy.

P11.46 (a) $(K+U_s)_A = (K+U_s)_B$

$$0 + mgy_A = \frac{1}{2}mv_B^2 + 0$$

$$v_B = \sqrt{2gy_A} = \sqrt{2\left(9.8\ \text{m/s}^2\right)6.30\ \text{m}} = \boxed{11.1\ \text{m/s}}$$

(b) $L = mvr = 76\ \text{kg}\ 11.1\ \text{m/s}\ 6.3\ \text{m} = \boxed{5.32\times10^3\ \text{kg}\cdot\text{m}^2/\text{s}}$ toward you along the axis of the channel.

(c) The wheels on his skateboard prevent any tangential force from acting on him. Then no torque about the axis of the channel acts on him and his angular momentum is constant. His legs convert chemical into mechanical energy. They do work to increase his kinetic energy. The normal force acts forward on his body on its rising trajectory, to increase his linear momentum.

(d) $L = mvr \qquad v = \dfrac{5.32\times10^3\ \text{kg}\cdot\text{m}^2/\text{s}}{76\ \text{kg}\ 5.85\ \text{m}} = \boxed{12.0\ \text{m/s}}$

(e) $(K+U_g)_B + U_{chemical,B} = (K+U_g)_C$

$$\frac{1}{2}76\ \text{kg}(11.1\ \text{m/s})^2 + 0 + U_{chem} = \frac{1}{2}76\ \text{kg}(12.0\ \text{m/s})^2 + 76\ \text{kg}\ 9.8\ \text{m/s}^2\ 0.45\ \text{m}$$

$$U_{chem} = 5.44\ \text{kJ} - 4.69\ \text{kJ} + 335\ \text{J} = \boxed{1.08\ \text{kJ}}$$

(f) $(K+U_g)_C = (K+U_g)_D$

$$\frac{1}{2}76\ \text{kg}(12.0\ \text{m/s})^2 + 0 = \frac{1}{2}76\ \text{kg}v_D^2 + 76\ \text{kg}\ 9.8\ \text{m/s}^2\ 5.85\ \text{m}$$

$$v_D = \boxed{5.34\ \text{m/s}}$$

continued on next page

(g) Let point E be the apex of his flight:

$$(K+U_g)_D = (K+U_g)_E$$

$$\frac{1}{2}76\text{ kg}(5.34\text{ m/s})^2 + 0 = 0 + 76\text{ kg}(9.8\text{ m/s}^2)(y_E - y_D)$$

$$(y_E - y_D) = \boxed{1.46\text{ m}}$$

(h) For the motion between takeoff and touchdown

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$-2.34\text{ m} = 0 + 5.34\text{ m/s}\,t - 4.9\text{ m/s}^2\,t^2$$

$$t = \frac{-5.34 \pm \sqrt{5.34^2 + 4(4.9)(2.34)}}{-9.8} = \boxed{1.43\text{ s}}$$

(i)	This solution is more accurate. In Chapter 8 we modeled the normal force as constant while the skateboarder stands up. Really it increases as the process goes on.

P11.47 (a) $I = \sum m_i r_i^2$

$$= m\left(\frac{4d}{3}\right)^2 + m\left(\frac{d}{3}\right)^2 + m\left(\frac{2d}{3}\right)^2$$

$$= \boxed{7m\frac{d^2}{3}}$$

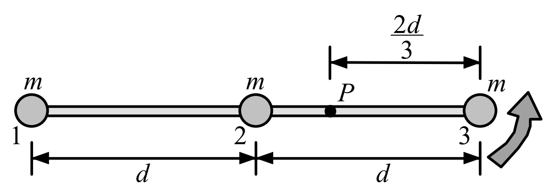

FIG. P11.47

(b) Think of the whole weight, $3mg$, acting at the center of gravity.

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \left(\frac{d}{3}\right)(-\hat{\mathbf{i}}) \times 3mg(-\hat{\mathbf{j}}) = \boxed{(mgd)\hat{\mathbf{k}}}$$

(c) $\alpha = \dfrac{\tau}{I} = \dfrac{3mgd}{7md^2} = \boxed{\dfrac{3g}{7d}\text{ counterclockwise}}$

(d) $a = \alpha r = \left(\dfrac{3g}{7d}\right)\left(\dfrac{2d}{3}\right) = \boxed{\dfrac{2g}{7}\text{ up}}$

The angular acceleration is not constant, but energy is.

$$(K+U)_i + \Delta E = (K+U)_f$$

$$0 + (3m)g\left(\frac{d}{3}\right) + 0 = \frac{1}{2}I\omega_f^2 + 0$$

(e) maximum kinetic energy $= \boxed{mgd}$

(f) $\omega_f = \boxed{\sqrt{\dfrac{6g}{7d}}}$

(g) $L_f = I\omega_f = \dfrac{7md^2}{3}\sqrt{\dfrac{6g}{7d}} = \boxed{\left(\dfrac{14g}{3}\right)^{1/2} md^{3/2}}$

(h) $v_f = \omega_f r = \sqrt{\dfrac{6g}{7d}}\dfrac{d}{3} = \boxed{\sqrt{\dfrac{2gd}{21}}}$

P11.48 (a) $\sum \tau = MgR - MgR = \boxed{0}$

(b) $\sum \tau = \dfrac{dL}{dt}$, and since $\sum \tau = 0$, $L = \text{constant}$.

Since the total angular momentum of the system is zero, the $\boxed{\text{monkey and bananas move upward with the same speed}}$ at any instant, and $\boxed{\text{he will not reach the bananas}}$ (until they get tangled in the pulley). To state the evidence differently, the tension in the rope is the same on both sides. Newton's second law applied to the monkey and bananas give the same acceleration upwards.

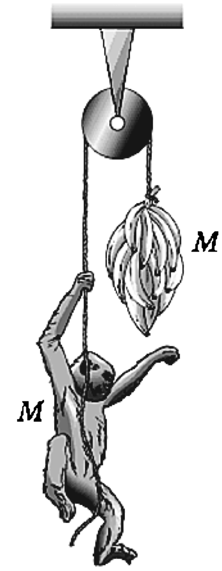

FIG. P11.48

P11.49 Using conservation of angular momentum, we have

$$L_{\text{aphelion}} = L_{\text{perihelion}} \text{ or } \left(mr_a^2\right)\omega_a = \left(mr_p^2\right)\omega_p$$

Thus,

$$\left(mr_a^2\right)\frac{v_a}{r_a} = \left(mr_p^2\right)\frac{v_p}{r_p} \text{ giving}$$

$$r_a v_a = r_p v_p \text{ or } v_a = \frac{r_p}{r_a} v_p = \frac{0.590 \text{ AU}}{35.0 \text{ AU}}(54.0 \text{ km/s}) = \boxed{0.910 \text{ km/s}}$$

P11.50 (a) Angular momentum is conserved:

$$\frac{mv_i d}{2} = \left(\frac{1}{12}Md^2 + m\left(\frac{d}{2}\right)^2\right)\omega$$

$$\omega = \boxed{\frac{6mv_i}{Md + 3md}}$$

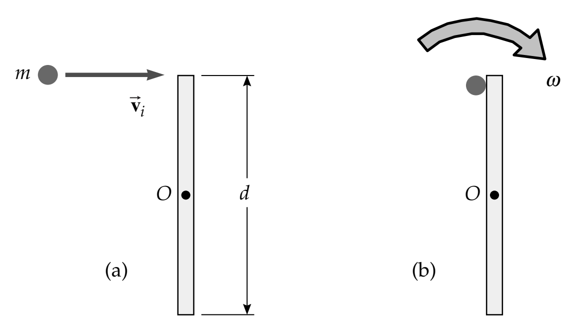

FIG. P11.50

(b) The original energy is $\frac{1}{2}mv_i^2$.

The final energy is

$$\frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{12}Md^2 + \frac{md^2}{4}\right)\frac{36m^2v_i^2}{(Md+3md)^2} = \frac{3m^2v_i^2 d}{2(Md+3md)}$$

The loss of energy is

$$\frac{1}{2}mv_i^2 - \frac{3m^2v_i^2 d}{2(Md+3md)} = \frac{mMv_i^2 d}{2(Md+3md)}$$

and the fractional loss of energy is

$$\frac{mMv_i^2 d2}{2(Md+3md)mv_i^2} = \boxed{\frac{M}{M+3m}}$$

P11.51 (a) $\tau = |\vec{\mathbf{r}} \times \vec{\mathbf{F}}| = |\vec{\mathbf{r}}||\vec{\mathbf{F}}|\sin 180° = 0$

Angular momentum is conserved.

$$L_f = L_i$$
$$mrv = mr_i v_i$$
$$v = \boxed{\frac{r_i v_i}{r}}$$

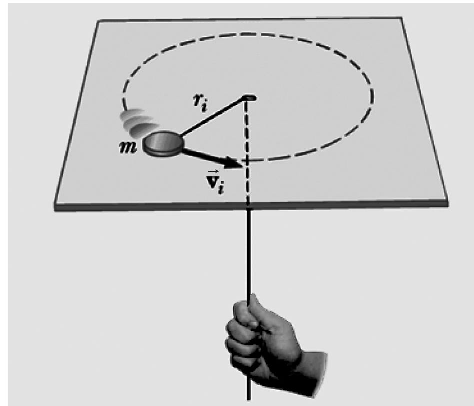

FIG. P11.51

(b) $T = \dfrac{mv^2}{r} = \boxed{\dfrac{m(r_i v_i)^2}{r^3}}$

(c) The work is done by the centripetal force in the *negative-r*, inward direction.

METHOD 1:

$$W = \int F \cdot d\ell = -\int T dr' = -\int_{r_i}^{r} \frac{m(r_i v_i)^2}{(r')^3} dr' = \left.\frac{m(r_i v_i)^2}{2(r')^2}\right|_{r_i}^{r}$$

$$= \frac{m(r_i v_i)^2}{2}\left(\frac{1}{r^2} - \frac{1}{r_i^2}\right) = \boxed{\frac{1}{2} m v_i^2 \left(\frac{r_i^2}{r^2} - 1\right)}$$

METHOD 2:

$$W = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_i^2 = \boxed{\frac{1}{2} m v_i^2 \left(\frac{r_i^2}{r^2} - 1\right)}$$

(d) Using the data given, we find

$$v = \boxed{4.50 \text{ m/s}} \qquad T = \boxed{10.1 \text{ N}} \qquad W = \boxed{0.450 \text{ J}}$$

***P11.52** (a) The equation simplifies to

$$(1.75 \text{ kg} \cdot \text{m}^2/\text{s} - 0.181 \text{ kg} \cdot \text{m}^2/\text{s})\,\hat{\mathbf{j}} = (0.745 \text{ kg} \cdot \text{m}^2)\vec{\omega} \qquad \vec{\omega} = \boxed{2.11\hat{\mathbf{j}} \text{ rad/s}}$$

(b) We take the x axis east, the y axis up, and the z axis south.

The child has moment of inertia $0.730 \text{ kg} \cdot \text{m}^2$ about the axis of the stool and is originally turning counterclockwise at 2.40 rad/s. At a point 0.350 m to the east of the axis, he catches a 0.120 kg ball moving toward the south at 4.30 m/s. He continues to hold the ball in his outstretched arm. Find his final angular velocity.

(c) Yes, with the left-hand side representing the final situation and the right-hand side representing the original situation, the equation describes the throwing process.

P11.53 (a) $L_i = m_1 v_{1i} r_{1i} + m_2 v_{2i} r_{2i} = 2mv\left(\frac{d}{2}\right)$

$L_i = 2(75.0 \text{ kg})(5.00 \text{ m/s})(5.00 \text{ m})$

$L_i = \boxed{3\,750 \text{ kg} \cdot \text{m}^2/\text{s}}$

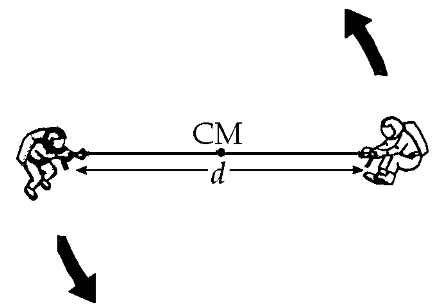

FIG. P11.53

(b) $K_i = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2$

$K_i = 2\left(\frac{1}{2}\right)(75.0 \text{ kg})(5.00 \text{ m/s})^2 = \boxed{1.88 \text{ kJ}}$

(c) Angular momentum is conserved: $L_f = L_i = \boxed{3\,750 \text{ kg} \cdot \text{m}^2/\text{s}}$

(d) $v_f = \frac{L_f}{2(mr_f)} = \frac{3\,750 \text{ kg} \cdot \text{m}^2/\text{s}}{2(75.0 \text{ kg})(2.50 \text{ m})} = \boxed{10.0 \text{ m/s}}$

(e) $K_f = 2\left(\frac{1}{2}\right)(75.0 \text{ kg})(10.0 \text{ m/s})^2 = \boxed{7.50 \text{ kJ}}$

(f) $W = K_f - K_i = \boxed{5.62 \text{ kJ}}$

P11.54 (a) $L_i = 2\left[Mv\left(\frac{d}{2}\right)\right] = \boxed{Mvd}$

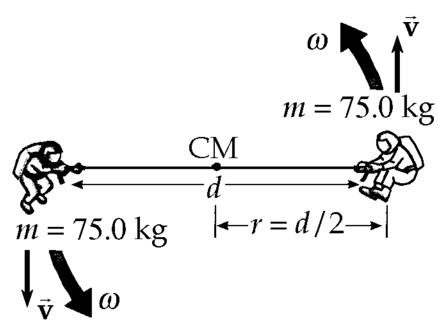

FIG. P11.54

(b) $K = 2\left(\frac{1}{2}Mv^2\right) = \boxed{Mv^2}$

(c) $L_f = L_i = \boxed{Mvd}$

(d) $v_f = \frac{L_f}{2Mr_f} = \frac{Mvd}{2M\left(\frac{d}{4}\right)} = \boxed{2v}$

(e) $K_f = 2\left(\frac{1}{2}Mv_f^2\right) = M(2v)^2 = \boxed{4Mv^2}$

(f) $W = K_f - K_i = \boxed{3Mv^2}$

***P11.55** (a) At the moment of release, two stones are moving with speed v_0. The total momentum has magnitude $\boxed{2mv_0}$. It keeps this same horizontal component of momentum as it flies away.

(b) The center of mass speed relative to the hunter is $2mv_0/3m = \boxed{2v_0/3}$, before the hunter lets go and, as far as horizontal motion is concerned, afterward.

(c) The one ball just being released is at distance $4\ell/3$ from the center of mass and is moving at speed $2v_0/3$ relative to the center of mass. Its angular speed is $\omega = v/r = (2v_0/3)/(4\ell/3) = v_0/2\ell$.

The other two balls are at distance $2\ell/3$ from the center of mass and moving relative to it at speed $v_0/3$. Their angular speed is necessarily the same $\omega = v/r = (v_0/3)/(2\ell/3) = v_0/2\ell$.

The total angular momentum around the center of mass is
$\Sigma mvr = m(2v_0/3)(4\ell/3) + 2m(v_0/3)(2\ell/3) = \boxed{4m\ell v_0/3}$. The angular momentum remains constant with this value as the bola flies away.

(d) As computed in part (c), the angular speed at the moment of release is $v_0/2\ell$. As it moves through the air, the bola keeps constant angular momentum, but its moment of inertia changes to $3m\ell^2$. Then the new angular speed is given by $L = I\omega \quad 4m\ell v_0/3 = 3m\ell^2\omega \quad \omega = \boxed{4v_0/9\ell}$
The angular speed drops as the moment of inertia increases.

(e) At the moment of release, $K = (1/2)m(0)^2 + (1/2)(2m)v_0^2 = \boxed{mv_0^2}$

(f) As it flies off in its horizontal motion it has kinetic energy
$(1/2)(3m)(v_{CM})^2 + (1/2)I\omega^2 = (1/2)(3m)(2v_0/3)^2 + (1/2)(3m\ell^2)(4v_0/9\ell)^2 = \boxed{(26/27)mv_0^2}$

(g) No horizontal forces act on the bola from outside after release, so the horizontal momentum stays constant. Its center of mass moves steadily with the horizontal velocity it had at release. No torques about its axis of rotation act on the bola, so its spin angular momentum stays constant. Internal forces cannot affect momentum conservation and angular momentum conservation, but they can affect mechanical energy. Energy $mv_0^2/27$ changes from mechanical energy into internal energy as the bola takes its stable configuration.

P11.56 For the cube to tip over, the center of mass (CM) must rise so that it is over the axis of rotation AB. To do this, the CM must be raised a distance of $a\left(\sqrt{2}-1\right)$.

For conservation of energy as the cube turns,

$$Mga\left(\sqrt{2}-1\right)=\frac{1}{2}I_{\text{cube}}\omega^2$$

From conservation of angular momentum,

$$\frac{4a}{3}mv=\left(\frac{8Ma^2}{3}\right)\omega$$

$$\omega=\frac{mv}{2Ma}$$

$$\frac{1}{2}\left(\frac{8Ma^2}{3}\right)\frac{m^2v^2}{4M^2a^2}=Mga\left(\sqrt{2}-1\right)$$

$$v=\boxed{\frac{M}{m}\sqrt{3ga\left(\sqrt{2}-1\right)}}$$

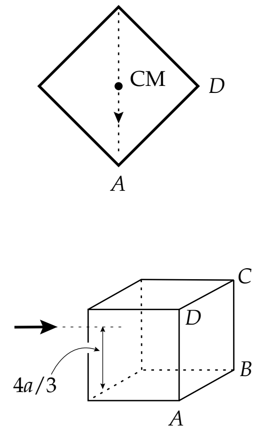

FIG. P11.56

***P11.57** The moment of inertia of the rest of the Earth is

$$I=\frac{2}{5}MR^2=\frac{2}{5}5.98\times10^{24}\text{ kg}\left(6.37\times10^6\text{ m}\right)^2=9.71\times10^{37}\text{ kg}\cdot\text{m}^2$$

For the original ice disks,

$$I=\frac{1}{2}Mr^2=\frac{1}{2}2.30\times10^{19}\text{ kg}\left(6\times10^5\text{ m}\right)^2=4.14\times10^{30}\text{ kg}\cdot\text{m}^2$$

For the final thin shell of water,

$$I=\frac{2}{3}Mr^2=\frac{2}{3}2.30\times10^{19}\text{ kg}\left(6.37\times10^6\text{ m}\right)^2=6.22\times10^{32}\text{ kg}\cdot\text{m}^2$$

Conservation of angular momentum for the spinning planet is expressed by $I_i\omega_i=I_f\omega_f$

$$\left(4.14\times10^{30}+9.71\times10^{37}\right)\frac{2\pi}{86\,400\text{ s}}=\left(6.22\times10^{32}+9.71\times10^{37}\right)\frac{2\pi}{\left(86\,400\text{ s}+\delta\right)}$$

$$\left(1+\frac{\delta}{86\,400\text{ s}}\right)\left(1+\frac{4.14\times10^{30}}{9.71\times10^{37}}\right)=\left(1+\frac{6.22\times10^{32}}{9.71\times10^{37}}\right)$$

$$\frac{\delta}{86\,400\text{ s}}=\frac{6.22\times10^{32}}{9.71\times10^{37}}-\frac{4.14\times10^{30}}{9.71\times10^{37}}$$

$$\boxed{\delta=0.550\text{ s}}$$

It is a measurable change, but not significant for everyday life.

P11.58 (a) The net torque is zero at the point of contact, so the angular momentum before and after the collision must be equal.

$$\left(\frac{1}{2}MR^2\right)\omega_i = \left(\frac{1}{2}MR^2\right)\omega + \left(MR^2\right)\omega \qquad \omega = \boxed{\frac{\omega_i}{3}}$$

(b) $$\frac{\Delta E}{E} = \frac{\frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\omega_i/3\right)^2 + \frac{1}{2}M\left(R\omega_i/3\right)^2 - \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_i^2}{\frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_i^2} = \boxed{-\frac{2}{3}}$$

P11.59 (a) $$\Delta t = \frac{\Delta p}{f} = \frac{Mv}{\mu Mg} = \frac{MR\omega}{\mu Mg} = \boxed{\frac{R\omega_i}{3\mu g}}$$

(b) $$W = \Delta K_{translational} = \frac{1}{2}Mv^2 - 0 = \frac{1}{2}M(\omega R)^2 = \frac{1}{2}M\left(\frac{\omega_i}{3}R\right)^2 = \frac{1}{18}MR^2\omega_i^2$$

(See Problem 11.58)

$$\mu Mgx = \frac{1}{18}MR^2\omega_i^2 \qquad \boxed{x = \frac{R^2\omega_i^2}{18\mu g}}$$

P11.60 Angular momentum is conserved during the inelastic collision.

$$Mva = I\omega$$

$$\omega = \frac{Mva}{I} = \frac{3v}{8a}$$

The condition, that the box falls off the table, is that the center of mass must reach its maximum height as the box rotates, $h_{max} = a\sqrt{2}$. Using conservation of energy:

$$\frac{1}{2}I\omega^2 = Mg\left(a\sqrt{2} - a\right)$$

$$\frac{1}{2}\left(\frac{8Ma^2}{3}\right)\left(\frac{3v}{8a}\right)^2 = Mg\left(a\sqrt{2} - a\right)$$

$$v^2 = \frac{16}{3}ga\left(\sqrt{2} - 1\right)$$

$$v = \boxed{4\left[\frac{ga}{3}\left(\sqrt{2} - 1\right)\right]^{1/2}}$$

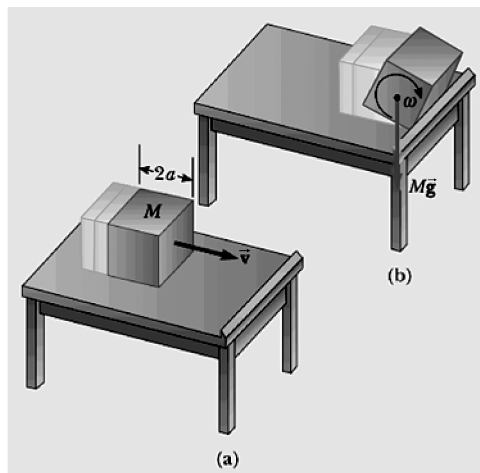

FIG. P11.60

ANSWERS TO EVEN PROBLEMS

P11.2 (a) 740 cm^2 (b) 59.5 cm

P11.4 (a) 168° (b) 11.9° principal value (c) Only the first is unambiguous.

P11.6 No. The cross product must be perpendicular to each factor.

P11.8 (a) $-10.0\ \hat{\mathbf{k}}$ N · m (b) yes; yes, infinitely many; yes; no, only one. $\vec{\mathbf{r}} = 5.00\hat{\mathbf{j}}$ m

P11.10 see the solution

P11.12 $\left(-22.0\ \text{kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}$

P11.14 see the solution

P11.16 (a) $3.14\ \text{N}\cdot\text{m}$ (b) $(0.400\ \text{kg}\cdot\text{m})v$ (c) $7.85\ \text{m/s}^2$

P11.18 (a) $\left(+9.03\times10^{9}\ \text{kg}\cdot\text{m}^2/\text{s}\right)$ south (b) no (c) 0

P11.20 (a) $\vec{\mathbf{r}} = (2t^3\,\hat{\mathbf{i}} + t^2\,\hat{\mathbf{j}})$ m, where t is in s. (b) The particle starts from rest at the origin, starts moving in the y direction, and gains speed faster and faster while turning to move more and more nearly parallel to the x axis. (c) $\vec{\mathbf{a}} = (12t\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}})$ m/s^2 (d) $\vec{\mathbf{F}} = (60t\,\hat{\mathbf{i}} + 10\,\hat{\mathbf{j}})$ N
(e) $\vec{\tau} = (-40t^3\hat{\mathbf{k}})$ N · m (f) $\vec{\mathbf{L}} = -10t^4\,\hat{\mathbf{k}}$ kg · m^2/s (g) $K = (90t^4 + 10t^2)$ J (h) $\mathcal{P} = (360t^3 + 20t)$ W, all where t is in s.

P11.22 $\left(4.50\ \text{kg}\cdot\text{m}^2/\text{s}\right)$ up

P11.24 (a) 7.06×10^{33} kg · m^2/s toward Polaris (b) 2.66×10^{40} kg · m^2/s toward Draco (c) The orbital angular momentum is much larger, by 3.78×10^{6} times.

P11.26 $1.20\ \text{kg}\cdot\text{m}^2/\text{s}$ perpendicularly into the clock face

P11.28 $8.63\ \text{m/s}^2$

P11.30 (a) 2.91 s (b) yes (c) Yes, but the pivot pin is always pulling on the rod to change the direction of the momentum. (d) No. Some mechanical energy is converted into internal energy.

P11.32 (a) $1.91\ \text{rad/s}$ (b) 2.53 J; 6.44 J

P11.34 (a) $\omega_f = (36\ \text{rad/s})(1 + 3.2\,m)/(1 + 20\,m)$ (b) ω_f decreases smoothly from a maximum value of 36.0 rad/s for $m = 0$ toward a minimum value of 5.76 rad/s as $m \to \infty$.

P11.36 (a) $7.20\times10^{-3}\ \text{kg}\cdot\text{m}^2/\text{s}$ (b) $9.47\ \text{rad/s}$

P11.38 $12.3\ \text{m/s}^2$

P11.40 (a) $2.35\ \text{rad/s}$ (b) $0.498\ \text{rad/s}$ (c) 5.58°

P11.42 131 s

P11.44 $9.55\times10^{-11}\ \text{s}$

P11.46 (a) $11.1\ \text{m/s}$ (b) $5.32\times10^{3}\ \text{kg}\cdot\text{m}^2/\text{s}$ (c) The wheels on his skateboard prevent any tangential force from acting on him. Then no torque about the axis of the channel acts on him and his angular momentum is constant. His legs convert chemical into mechanical energy. They do work to increase his kinetic energy. The normal force acts forward on his body on its rising trajectory, to increase his linear momentum. (d) $12.0\ \text{m/s}$ (e) $1.08\ \text{kJ}$ (f) $5.34\ \text{m/s}$ (g) 1.46 m (h) 1.43 s (i) This solution is more accurate. In Chapter 8 we modeled the normal force as constant while the skateboarder stands up. Really it increases as the process goes on.

P11.48 (a) 0 (b) The total angular momentum is constant, zero if the system is initially at rest. The monkey and the bananas move upward with the same speed. He will not reach the bananas.

P11.50 (a) $\dfrac{6mv_i}{Md+3md}$ (b) $\dfrac{M}{M+3m}$

P11.52 (a) $2.11\text{p}\,\hat{\mathbf{j}}$ rad/s (b) The child has moment of inertia $0.730\ \text{kg}\cdot\text{m}^2$ about the axis of the stool, and is originally turning counterclockwise at 2.40 rad/s. At a point 0.350 m to the east of the axis, he catches a 0.120 kg ball moving toward the south at 4.30 m/s. In his outstretched arm he continues to hold the ball. Find his final angular velocity. (c) Yes, with the left-hand side representing the final situation and the right-hand side representing the original situation, the equation describes the throwing process.

P11.54 (a) Mvd (b) Mv^2 (c) Mvd (d) $2v$ (e) $4Mv^2$ (f) $3Mv^2$

P11.56 $\dfrac{M}{m}\sqrt{3ga\left(\sqrt{2}-1\right)}$

P11.58 (a) $\dfrac{\omega_i}{3}$ (b) $\dfrac{\Delta E}{E}=-\dfrac{2}{3}$

P11.60 $4\left[\dfrac{ga}{3}\left(\sqrt{2}-1\right)\right]^{1/2}$

12

Static Equilibrium and Elasticity

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

***Q12.1** The force exerts counterclockwise torque about pivot D. The line of action of the force passes through C, so the torque about this axis is zero. In order of increasing negative (clockwise) values come the torques about F, E and B essentially together, and A. The answer is then $\tau_D > \tau_C > \tau_F > \tau_E = \tau_B > \tau_A$

Q12.2 When you bend over, your center of gravity shifts forward. Once your CG is no longer over your feet, gravity contributes to a nonzero net torque on your body and you begin to rotate.

Q12.3 Yes, it can. Consider an object on a spring oscillating back and forth. In the center of the motion both the sum of the torques and the sum of the forces acting on the object are (separately) zero. Again, a meteoroid flying freely through interstellar space feels essentially no forces and keeps moving with constant velocity.

Q12.4 (a) Consider pushing up with one hand on one side of a steering wheel and pulling down equally hard with the other hand on the other side. A pair of equal-magnitude oppositely-directed forces applied at different points is called a couple.

(b) An object in free fall has a non-zero net force acting on it, but a net torque of zero about its center of mass.

***Q12.5** Answer (a). Our theory of rotational motion does not contradict our previous theory of translational motion. The center of mass of the object moves as if the object were a particle, with all of the forces applied there. This is true whether the object is starting to rotate or not.

Q12.6 A V-shaped boomerang, a barstool, an empty coffee cup, a satellite dish, and a curving plastic slide at the edge of a swimming pool each have a center of mass that is not within the bulk of the object.

Q12.7 Suspend the plywood from the nail, and hang the plumb bob from the nail. Trace on the plywood along the string of the plumb bob. Now suspend the plywood with the nail through a different point on the plywood, not along the first line you drew. Again hang the plumb bob from the nail and trace along the string. The center of gravity is located halfway through the thickness of the plywood under the intersection of the two lines you drew.

***Q12.8** In cases (a) and (c) the center of gravity is above the base by one-half the height of the can. So (b) is the answer. In this case the center of gravity is above the base by only a bit more than one-quarter of the height of the can.

***Q12.9** Answer (b). The skyscraper is about 300 m tall. The gravitational field (acceleration) is weaker at the top by about 900 parts in ten million, by on the order of 10^{-4} times. The top half of the uniform building is lighter than the bottom half by about $(1/2)(10^{-4})$ times. Relative to the center of mass at the geometric center, this effect moves the center of gravity down, by about $(1/2)(10^{-4})(150\text{ m}) \sim 10$ mm.

Q12.10 She can be correct. If the dog stands on a relatively thick scale, the dog's legs on the ground might support more of its weight than its legs on the scale. She can check for and if necessary correct for this error by having the dog stand like a bridge with two legs on the scale and two on a book of equal thickness—a physics textbook is a good choice.

***Q12.11** Answer (b). Visualize the hatchet as like a balanced playground seesaw with one large-mass person on one side, close to the fulcrum, and a small-mass person far from the fulcrum on the other side. Different masses are on the two sides of the center of mass. The mean position of mass is not the median position.

Q12.12 The free body diagram demonstrates that it is necessary to have friction on the ground to counterbalance the normal force of the wall and to keep the base of the ladder from sliding. If there is friction on the floor *and* on the wall, it is not possible to determine whether the ladder will slip, from the equilibrium conditions alone.

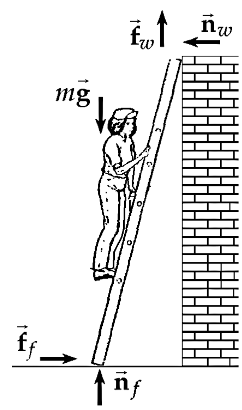

FIG. Q12.12

***Q12.13** Answer (g). In the problems we study, the forces applied to the object lie in a plane, and the axis we choose is a line perpendicular to this plane, so it appears as a point on the free-body diagram. It can be chosen anywhere. The algebra of solving for unknown forces is generally easier if we choose the axis where some unknown forces are acting.

***Q12.14** (i) Answer (b). The extension is directly proportional to the original dimension, according to $F/A = Y\Delta L/L_i$.
(ii) Answer (e). Doubling the diameter quadruples the area to make the extension four times smaller.

Q12.15 Shear deformation.

SOLUTIONS TO PROBLEMS

Section 12.1 The Conditions for Equilibrium

P12.1 Take torques about P.

$$\sum \tau_p = -n_0\left[\frac{\ell}{2}+d\right]+m_1 g\left[\frac{\ell}{2}+d\right]+m_b g d - m_2 g x = 0$$

We want to find x for which $n_0 = 0$.

$$x = \frac{(m_1 g + m_b g)d + m_1 g \frac{\ell}{2}}{m_2 g} = \boxed{\frac{(m_1 + m_b)d + m_1 \frac{\ell}{2}}{m_2}}$$

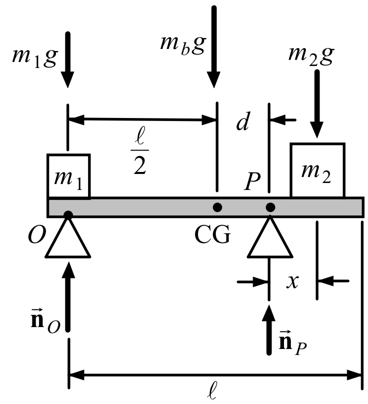

FIG. P12.1

P12.2 Use distances, angles, and forces as shown. The conditions of equilibrium are:

$$\sum F_y = 0 \Rightarrow \boxed{F_y + R_y - F_g = 0}$$

$$\sum F_x = 0 \Rightarrow \boxed{F_x - R_x = 0}$$

$$\sum \tau = 0 \Rightarrow \boxed{F_y \ell \cos\theta - F_g\left(\frac{\ell}{2}\right)\cos\theta - F_x \ell \sin\theta = 0}$$

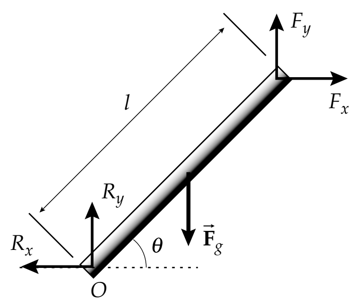

FIG. P12.2

Section 12.2 More on the Center of Gravity

P12.3 The coordinates of the center of gravity of piece 1 are

$$x_1 = 2.00 \text{ cm} \quad \text{and} \quad y_1 = 9.00 \text{ cm}$$

The coordinates for piece 2 are

$$x_2 = 8.00 \text{ cm} \quad \text{and} \quad y_2 = 2.00 \text{ cm}$$

The area of each piece is

$$A_1 = 72.0 \text{ cm}^2 \quad \text{and} \quad A_2 = 32.0 \text{ cm}^2$$

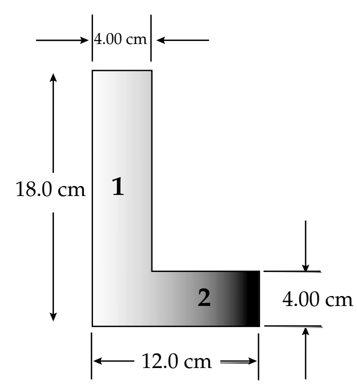

FIG. P12.3

And the mass of each piece is proportional to the area. Thus,

$$x_{CG} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(72.0 \text{ cm}^2)(2.00 \text{ cm}) + (32.0 \text{ cm}^2)(8.00 \text{ cm})}{72.0 \text{ cm}^2 + 32.0 \text{ cm}^2} = \boxed{3.85 \text{ cm}}$$

and

$$y_{CG} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(72.0 \text{ cm}^2)(9.00 \text{ cm}) + (32.0 \text{ cm}^2)(2.00 \text{ cm})}{104 \text{ cm}^2} = \boxed{6.85 \text{ cm}}$$

P12.4 The hole we can count as negative mass

$$x_{CG} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}$$

Call σ the mass of each unit of pizza area.

$$x_{CG} = \frac{\sigma \pi R^2 0 - \sigma \pi \left(\frac{R}{2}\right)^2 \left(-\frac{R}{2}\right)}{\sigma \pi R^2 - \sigma \pi \left(\frac{R}{2}\right)^2}$$

$$x_{CG} = \frac{R/8}{3/4} = \boxed{\frac{R}{6}}$$

P12.5 Let the fourth mass (8.00 kg) be placed at (x, y), then

$$x_{CG} = 0 = \frac{(3.00)(4.00) + m_4(x)}{12.0 + m_4}$$

$$x = -\frac{12.0}{8.00} = \boxed{-1.50\text{ m}}$$

Similarly,

$$y_{CG} = 0 = \frac{(3.00)(4.00) + 8.00(y)}{12.0 + 8.00}$$

$$y = \boxed{-1.50\text{ m}}$$

P12.6 Let σ represent the mass-per-face area. A vertical strip at position x, with width dx and height $\frac{(x-3.00)^2}{9}$ has mass

$$dm = \frac{\sigma (x-3.00)^2\,dx}{9}$$

The total mass is

$$M = \int dm = \int_{x=0}^{3.00} \frac{\sigma (x-3)^2\,dx}{9}$$

$$M = \left(\frac{\sigma}{9}\right) \int_0^{3.00} \left(x^2 - 6x + 9\right) dx$$

$$M = \left(\frac{\sigma}{9}\right) \left[\frac{x^3}{3} - \frac{6x^2}{2} + 9x\right]_0^{3.00} = \sigma$$

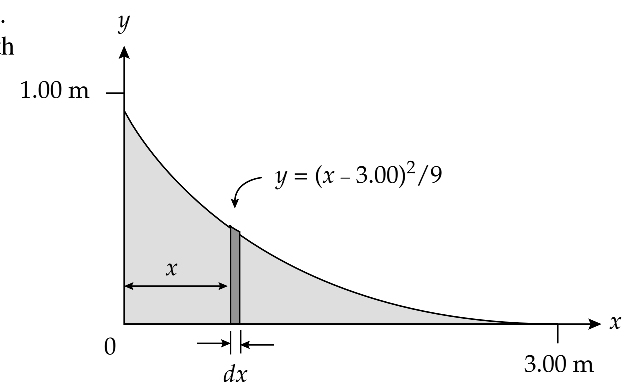

FIG. P12.6

The x-coordinate of the center of gravity is

$$x_{CG} = \frac{\int x\,dm}{M} = \frac{1}{9\sigma} \int_0^{3.00} \sigma x (x-3)^2\,dx = \frac{\sigma}{9\sigma} \int_0^{3.00} \left(x^3 - 6x^2 + 9x\right) dx$$

$$= \frac{1}{9}\left[\frac{x^4}{4} - \frac{6x^3}{3} + \frac{9x^2}{2}\right]_0^{3.00} = \frac{6.75\text{ m}}{9.00} = \boxed{0.750\text{ m}}$$

P12.7 In a uniform gravitational field, the center of mass and center of gravity of an object coincide. Thus, the center of gravity of the triangle is located at $x = 6.67$ m, $y = 2.33$ m (see the Example on the center of mass of a triangle in Chapter 9).

The coordinates of the center of gravity of the three-object system are then:

$$x_{CG} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(6.00\text{ kg})(5.50\text{ m}) + (3.00\text{ kg})(6.67\text{ m}) + (5.00\text{ kg})(-3.50\text{ m})}{(6.00 + 3.00 + 5.00)\text{ kg}}$$

$$x_{CG} = \frac{35.5\text{ kg}\cdot\text{m}}{14.0\text{ kg}} = \boxed{2.54\text{ m}} \text{ and}$$

$$y_{CG} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(6.00\text{ kg})(7.00\text{ m}) + (3.00\text{ kg})(2.33\text{ m}) + (5.00\text{ kg})(+3.50\text{ m})}{14.0\text{ kg}}$$

$$y_{CG} = \frac{66.5\text{ kg}\cdot\text{m}}{14.0\text{ kg}} = \boxed{4.75\text{ m}}$$

Section 12.3 **Examples of Rigid Objects in Static Equilibrium**

P12.8 (a) For rotational equilibrium of the lowest rod about its point of support, $\sum \tau = 0$.

$$+12.0\text{ g } g\text{ 3 cm} - m_1 g 4\text{ cm} \qquad \boxed{m_1 = 9.00\text{ g}}$$

(b) For the middle rod,

$$+m_2 2\text{ cm} - (12.0\text{ g} + 9.0\text{ g})5\text{ cm} = 0 \qquad \boxed{m_2 = 52.5\text{ g}}$$

(c) For the top rod,

$$(52.5\text{ g} + 12.0\text{ g} + 9.0\text{ g})4\text{ cm} - m_3 6\text{ cm} = 0 \qquad \boxed{m_3 = 49.0\text{ g}}$$

P12.9 $\sum \tau = 0 = mg(3r) - Tr$

$2T - Mg\sin 45.0° = 0$

$$T = \frac{Mg\sin 45.0°}{2} = \frac{1\,500\text{ kg}(g)\sin 45.0°}{2}$$

$$= (530)(9.80)\text{ N}$$

$$m = \frac{T}{3g} = \frac{530g}{3g} = \boxed{177\text{ kg}}$$

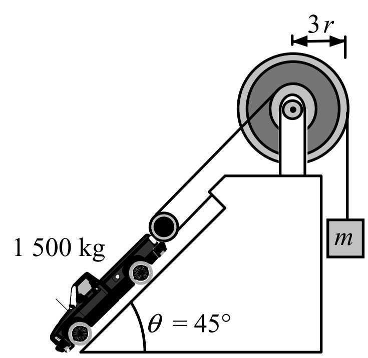

FIG. P12.9

P12.10 (a) Taking moments about P,

$$(R\sin 30.0°)0 + (R\cos 30.0°)(5.00\text{ cm}) - (150\text{ N})(30.0\text{ cm}) = 0$$

$$R = 1\,039.2\text{ N} = 1.04\text{ kN}$$

The force exerted by the hammer on the nail is equal in magnitude and opposite in direction:

$\boxed{\text{1.04 kN at 60° upward and to the right.}}$

(b) $f = R\sin 30.0° - 150\text{ N} = 370\text{ N}$

$$n = R\cos 30.0° = 900\text{ N}$$

$$\boxed{\vec{\mathbf{F}}_{\text{surface}} = (370\text{ N})\hat{\mathbf{i}} + (900\text{ N})\hat{\mathbf{j}}}$$

P12.11 (a) $\sum F_x = f - n_w = 0$

$\sum F_y = n_g - 800\text{ N} - 500\text{ N} = 0$

Taking torques about an axis at the foot of the ladder,

$$(800\text{ N})(4.00\text{ m})\sin 30.0° + (500\text{ N})(7.50\text{ m})\sin 30.0° - n_w(15.0\text{ cm})\cos 30.0° = 0$$

Solving the torque equation,

$$n_w = \frac{[(4.00\text{ m})(800\text{ N}) + (7.50\text{ m})(500\text{ N})]\tan 30.0°}{15.0\text{ m}} = 268\text{ N}$$

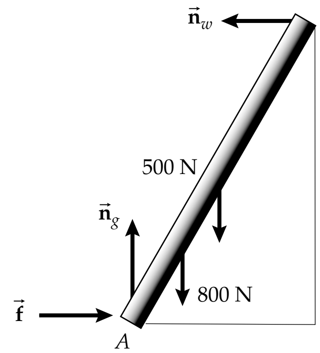

FIG. P12.11

Next substitute this value into the F_x equation to find

$$f = n_w = \boxed{268\text{ N}} \text{ in the positive } x \text{ direction.}$$

Solving the equation $\sum F_y = 0$,

$$n_g = \boxed{1\,300\text{ N}} \text{ in the positive } y \text{ direction.}$$

(b) In this case, the torque equation $\sum \tau_A = 0$ gives:

$$(9.00\text{ m})(800\text{ N})\sin 30.0° + (7.50\text{ m})(500\text{ N})\sin 30.0° - (15.0\text{ m})(n_w)\sin 60.0° = 0$$

or

$$n_w = 421\text{ N}$$

Since $f = n_w = 421\text{ N}$ and $f = f_{\text{max}} = \mu n_g$, we find

$$\mu = \frac{f_{\text{max}}}{n_g} = \frac{421\text{ N}}{1\,300\text{ N}} = \boxed{0.324}$$

P12.12 (a) $\sum F_x = f - n_w = 0$ (1)

$\sum F_y = n_g - m_1 g - m_2 g = 0$ (2)

$\sum \tau_A = -m_1 g\left(\frac{L}{2}\right)\cos\theta - m_2 g x\cos\theta + n_w L\sin\theta = 0$

From the torque equation,

$$n_w = \left[\frac{1}{2}m_1 g + \left(\frac{x}{L}\right)m_2 g\right]\cot\theta$$

Then, from equation (1): $f = n_w = \boxed{\left[\frac{1}{2}m_1 g + \left(\frac{x}{L}\right)m_2 g\right]\cot\theta}$

and from equation (2): $n_g = \boxed{(m_1 + m_2)g}$

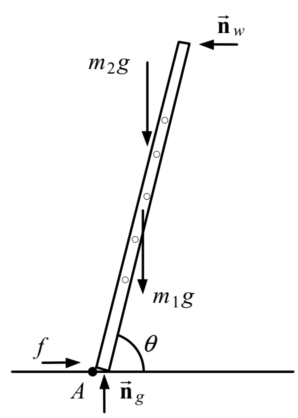

FIG. P12.12

(b) If the ladder is on the verge of slipping when $x = d$,

then

$$\mu = \frac{f|_{x=d}}{n_g} = \boxed{\frac{(m_1/2 + m_2 d/L)\cot\theta}{m_1 + m_2}}$$

P12.13 Torque about the front wheel is zero.

$$0=(1.20\text{ m})(mg)-(3.00\text{ m})(2F_r)$$

Thus, the force at each rear wheel is

$$F_r=0.200mg=\boxed{2.94\text{ kN}}$$

The force at each front wheel is then

$$F_f=\frac{mg-2F_r}{2}=\boxed{4.41\text{ kN}}$$

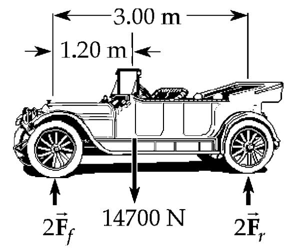

FIG. P12.13

***P12.14** (a) The gravitational force on the floodlight is $(20\text{ kg})(9.8\text{ m/s}^2)=196\text{ N}$
We consider the torques acting on the beam, about an axis perpendicular to the page and through the left end of the horizontal beam.

$$\sum\tau=+(T\sin 30.0°)d-(196\text{ N})d=0$$

giving $T=\boxed{392\text{ N}}$.

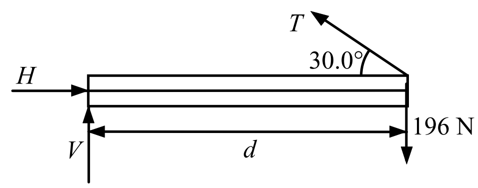

FIG. P12.14

(b) From $\sum F_x=0$, $H-T\cos 30.0°=0$, or $H=(392\text{ N})\cos 30.0°=\boxed{339\text{ N to the right}}$.

(c) From $\sum F_y=0$, $V+T\sin 30.0°-196\text{ N}=0$, or $V=196\text{ N}-(392\text{ N})\sin 30.0°=\boxed{0}$.

(d) From the same free-body diagram with the axis chosen at the right-hand end, we write
$\sum\tau=H(0)-Vd+T(0)+196\text{ N}(0)=0$ so $\boxed{V=0}$

(e) From $\sum F_y=0$, $V+T\sin 30.0°-196\text{ N}=0$, or $T=0+196\text{ N/s}\text{in}\,30.0°=\boxed{392\text{ N}}$.

(f) From $\sum F_x=0$, $H-T\cos 30.0°=0$, or $H=(392\text{ N})\cos 30.0°=\boxed{339\text{ N to the right}}$.

(g) The two solutions agree precisely. They are equally accurate. They are essentially equally simple. But note that many students would make a mistake on the negative (clockwise) sign for the torque of the upward force V in the equation in part (d).

Taking together the equations we have written, we appear to have four equations but we cannot determine four unknowns. Only three of the equations are independent, so we can determine only three unknowns.

P12.15 (a) Vertical forces on one half of the chain: $T_e\sin 42.0°=20.0\text{ N}$ $\boxed{T_e=29.9\text{ N}}$

(b) Horizontal forces on one half of the chain: $T_e\cos 42.0°=T_m$ $\boxed{T_m=22.2\text{ N}}$

P12.16 Relative to the hinge end of the bridge, the cable is attached horizontally out a distance $x = (5.00 \text{ m})\cos 20.0° = 4.70 \text{ m}$ and vertically down a distance $y = (5.00 \text{ m})\sin 20.0° =$ 1.71 m. The cable then makes the following angle with the horizontal:

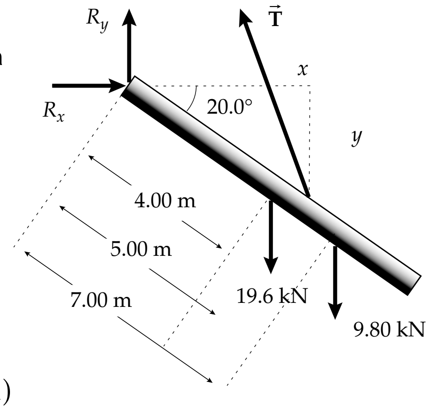

FIG. P12.16

$$\theta = \tan^{-1}\left[\frac{(12.0+1.71) \text{ m}}{4.70 \text{ m}}\right] = 71.1°$$

(a) Take torques about the hinge end of the bridge:

$$R_x(0) + R_y(0) - 19.6 \text{ kN}(4.00 \text{ m})\cos 20.0°$$
$$-T\cos 71.1°(1.71 \text{ m}) + T\sin 71.1°(4.70 \text{ m})$$
$$-9.80 \text{ kN}(7.00 \text{ m})\cos 20.0° = 0$$

which yields $T = \boxed{35.5 \text{ kN}}$

(b) $\sum F_x = 0 \Rightarrow R_x - T\cos 71.1° = 0$

or

$$R_x = (35.5 \text{ kN})\cos 71.1° = \boxed{11.5 \text{ kN (right)}}$$

(c) $\sum F_y = 0 \Rightarrow R_y - 19.6 \text{ kN} + T\sin 71.1° - 9.80 \text{ kN} = 0$

Thus,

$$R_y = 29.4 \text{ kN} - (35.5 \text{ kN})\sin 71.1° = -4.19 \text{ kN}$$
$$= \boxed{4.19 \text{ kN down}}$$

P12.17 (a) We model the horse as a particle. The drawbridge will fall out from under the horse.

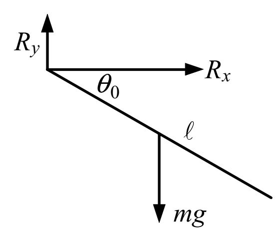

FIG. P12.17(a)

$$\alpha = mg\frac{\frac{1}{2}\ell\cos\theta_0}{\frac{1}{3}m\ell^2} = \frac{3g}{2\ell}\cos\theta_0$$
$$= \frac{3(9.80 \text{ m/s}^2)\cos 20.0°}{2(8.00 \text{ m})} = \boxed{1.73 \text{ rad/s}^2}$$

(b) $\frac{1}{2}I\omega^2 = mgh$

$$\therefore \frac{1}{2}\cdot\frac{1}{3}m\ell^2\omega^2 = mg\cdot\frac{1}{2}\ell(1-\sin\theta_0)$$

Solving,

$$\omega = \sqrt{\frac{3g}{\ell}(1-\sin\theta_0)} = \sqrt{\frac{3(9.80 \text{ m/s}^2)}{8.00 \text{ m}}(1-\sin 20°)} = \boxed{1.56 \text{ rad/s}}$$

continued on next page

(c) The linear acceleration of the center of mass of the bridge is

$$a=\frac{1}{2}\ell\alpha=\frac{1}{2}(8.0\text{ m})\left(1.73\text{ rad/s}^2\right)=6.907\text{ m/s}^2$$

The force at the hinge plus the gravitational force produce the acceleration $a=6.907\text{ m/s}^2$ at right angles to the bridge.

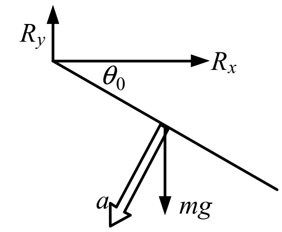

FIG. P12.17(c)

$$R_x=ma_x=(2\,000\text{ kg})\left(6.907\text{ m/s}^2\right)\cos 250°=-4.72\text{ kN}$$

$$R_y-mg=ma_y$$

Solving,

$$R_y=m\left(g+a_y\right)=(2\,000\text{ kg})\left[9.80\text{ m/s}^2+\left(6.907\text{ m/s}^2\right)\sin 250°\right]=6.62\text{ kN}$$

Thus

$$\boxed{\vec{\mathbf{R}}=\left(-4.72\hat{\mathbf{i}}+6.62\hat{\mathbf{j}}\right)\text{ kN}}$$

(d) $R_x=0$

$$a=\omega^2\left(\frac{1}{2}\ell\right)=(1.56\text{ rad/s})^2(4.0\text{ m})=9.67\text{ m/s}^2$$

$$R_y-mg=ma$$

$$\therefore R_y=(2\,000\text{ kg})\left(9.8\text{ m/s}^2+9.67\text{ m/s}^2\right)=38.9\text{ kN}$$

Thus:

$$\boxed{R_y=38.9\hat{\mathbf{j}}\text{ kN}}$$

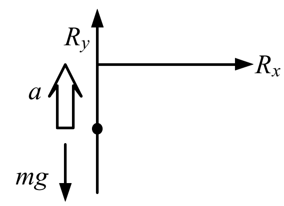

FIG. P12.17(d)

P12.18 Call the required force F, with components $F_x=F\cos 15.0°$ and $F_y=-F\sin 15.0°$, transmitted to the center of the wheel by the handles.

Just as the wheel leaves the ground, the ground exerts no force on it.

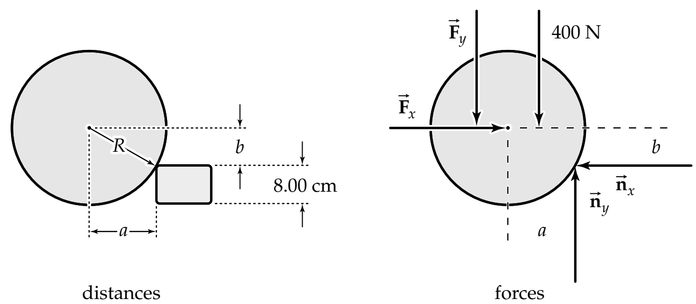

FIG. P12.18

$$\sum F_x=0\text{: } F\cos 15.0°-n_x \qquad (1)$$

$$\sum F_y=0\text{: } -F\sin 15.0°-400\text{ N}+n_y=0 \qquad (2)$$

Take torques about its contact point with the brick. The needed distances are seen to be:

$$b=R-8.00\text{ cm}=(20.0-8.00)\text{ cm}=12.0\text{ cm}$$

$$a=\sqrt{R^2-b^2}=16.0\text{ cm}$$

(a) $\sum\tau=0$: $-F_xb+F_ya+(400\text{ N})a=0$, or

$$F\left[-(12.0\text{ cm})\cos 15.0°+(16.0\text{ cm})\sin 15.0°\right]+(400\text{ N})(16.0\text{ cm})=0$$

so

$$F=\frac{6\,400\text{ N}\cdot\text{cm}}{7.45\text{ cm}}=\boxed{859\text{ N}}$$

continued on next page

(b) Then, using Equations (1) and (2),

$n_x = (859\text{ N})\cos 15.0° = 830\text{ N}$ and

$n_y = 400\text{ N} + (859\text{ N})\sin 15.0° = 622\text{ N}$

$$n = \sqrt{n_x^2 + n_y^2} = \boxed{1.04\text{ kN}}$$

$$\theta = \tan^{-1}\left(\frac{n_y}{n_x}\right) = \tan^{-1}(0.749) = \boxed{36.9° \text{ to the left and upward}}$$

P12.19 When $x = x_{\min}$, the rod is on the verge of slipping, so

$$f = (f_s)_{\max} = \mu_s n = 0.50n$$

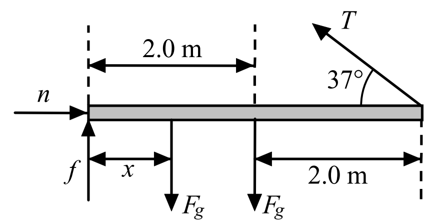

FIG. P12.19

From $\sum F_x = 0$, $n - T\cos 37° = 0$, or $n = 0.799T$.

Thus,

$$f = 0.50(0.799T) = 0.399T$$

From $\sum F_y = 0$, $f + T\sin 37° - 2F_g = 0$,

or

$$0.399T - 0.602T - 2F_g = 0, \text{ giving } T = 2.00F_g$$

Using $\sum \tau = 0$ for an axis perpendicular to the page and through the left end of the beam gives

$$-F_g \cdot x_{\min} - F_g(2.0\text{ m}) + \left[(2F_g)\sin 37°\right](4.0\text{ m}) = 0, \text{ which reduces to } x_{\min} = \boxed{2.82\text{ m}}$$

P12.20 Consider forces and torques on the beam.

$\sum F_x = 0$: $\quad R\cos\theta - T\cos 53° = 0$

$\sum F_y = 0$: $\quad R\sin\theta + T\sin 53° - 800\text{ N} = 0$

$\sum \tau = 0$: $\quad (T\sin 53°)8\text{ m} - (600\text{ N})x - (200\text{ N})4\text{ m} = 0$

(a) Then $T = \dfrac{600\text{ N}x + 800\text{ N}\cdot\text{m}}{8\text{ m}\sin 53°} = (93.9\text{ N/m})x + 125\text{ N}$

As x increases from 2 m, this expression grows larger.

(b) From substituting back,

$$R\cos\theta = [93.9x + 125]\cos 53°$$
$$R\sin\theta = 800\text{ N} - [93.9x + 125]\sin 53°$$

Dividing,

$$\tan\theta = \frac{R\sin\theta}{R\cos\theta} = -\tan 53° + \frac{800\text{ N}}{(93.9x+125)\cos 53°}$$
$$\tan\theta = \tan 53°\left(\frac{32}{3x+4} - 1\right)$$

As x increases the fraction decreases and $\boxed{\theta \text{ decreases}}$.

continued on next page

(c) To find R we can work out $R^2\cos^2\theta + R^2\sin^2\theta = R^2$. From the expressions above for $R\cos\theta$ and $R\sin\theta$,

$$R^2 = T^2\cos^2 53° + T^2\sin^2 53° - 1\,600\text{ N}T\sin 53° + (800\text{ N})^2$$
$$R^2 = T^2 - 1\,600T\sin 53° + 640\,000$$
$$R^2 = (93.9x + 125)^2 - 1\,278(93.9x + 125) + 640\,000$$
$$R = \left(8\,819x^2 - 96\,482x + 495\,678\right)^{1/2}$$

At $x = 0$ this gives $R = 704$ N. At $x = 2$ m, $R = 581$ N. At $x = 8$ m, $R = 537$ N. Over the range of possible values for x, the negative term $-96\,482x$ dominates the positive term $8\,819x^2$, and $\boxed{R \text{ decreases}}$ as x increases.

P12.21 To find U, measure distances and forces from point A. Then, balancing torques,

$$(0.750)U = 29.4(2.25) \qquad \boxed{U = 88.2\text{ N}}$$

To find D, measure distances and forces from point B. Then, balancing torques,

$$(0.750)D = (1.50)(29.4) \qquad \boxed{D = 58.8\text{ N}}$$

Also, notice that $U = D + F_g$, so $\sum F_y = 0$.

Section 12.4 **Elastic Properties of Solids**

P12.22 The definition of $Y = \dfrac{\text{stress}}{\text{strain}}$ means that Y is the slope of the graph:

$$Y = \frac{300\times10^6\text{ N/m}^2}{0.003} = \boxed{1.0\times10^{11}\text{ N/m}^2}$$

P12.23 $\dfrac{F}{A} = Y\dfrac{\Delta L}{L_i}$

$$\Delta L = \frac{FL_i}{AY} = \frac{(200)(9.80)(4.00)}{\left(0.200\times10^{-4}\right)\left(8.00\times10^{10}\right)} = \boxed{4.90\text{ mm}}$$

P12.24 (a) stress $= \dfrac{F}{A} = \dfrac{F}{\pi r^2}$

$$F = (\text{stress})\pi\left(\frac{d}{2}\right)^2$$
$$F = \left(1.50\times10^8\text{ N/m}^2\right)\pi\left(\frac{2.50\times10^{-2}\text{ m}}{2}\right)^2$$
$$F = \boxed{73.6\text{ kN}}$$

(b) stress $= Y(\text{strain}) = \dfrac{Y\Delta L}{L_i}$

$$\Delta L = \frac{(\text{stress})L_i}{Y} = \frac{\left(1.50\times10^8\text{ N/m}^2\right)(0.250\text{ m})}{1.50\times10^{10}\text{ N/m}^2} = \boxed{2.50\text{ mm}}$$

P12.25 From the defining equation for the shear modulus, we find Δx as

$$\Delta x = \frac{hf}{SA} = \frac{\left(5.00\times10^{-3}\ \text{m}\right)(20.0\ \text{N})}{\left(3.0\times10^{6}\ \text{N/m}^2\right)\left(14.0\times10^{-4}\ \text{m}^2\right)} = 2.38\times10^{-5}\ \text{m}$$

or $\Delta x = \boxed{2.38\times10^{-2}\ \text{mm}}$

P12.26 Count the wires. If they are wrapped together so that all support nearly equal stress, the number should be

$$\frac{20.0\ \text{kN}}{0.200\ \text{kN}} = 100$$

Since cross-sectional area is proportional to diameter squared, the diameter of the cable will be

$$(1\ \text{mm})\sqrt{100}\ \boxed{\sim 1\ \text{cm}}$$

P12.27 (a)

$$\begin{aligned} F &= (A)(\text{stress}) \\ &= \pi\left(5.00\times10^{-3}\ \text{m}\right)^2\left(4.00\times10^{8}\ \text{N/m}^2\right) \\ &= \boxed{3.14\times10^{4}\ \text{N}} \end{aligned}$$

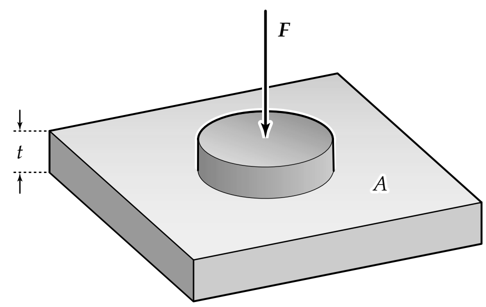

FIG. P12.27(b)

(b) The area over which the shear occurs is equal to the circumference of the hole times its thickness. Thus,

$$\begin{aligned} A &= (2\pi r)t = 2\pi\left(5.00\times10^{-3}\ \text{m}\right)\left(5.00\times10^{-3}\ \text{m}\right) \\ &= 1.57\times10^{-4}\ \text{m}^2 \end{aligned}$$

So,

$$\begin{aligned} F &= (A)\text{Stress} = \left(1.57\times10^{-4}\ \text{m}^2\right)\left(4.00\times10^{8}\ \text{N/m}^2\right) \\ &= \boxed{6.28\times10^{4}\ \text{N}} \end{aligned}$$

P12.28 The force acting on the hammer changes its momentum according to

$$mv_i + \bar{F}(\Delta t) = mv_f \text{ so } \left|\bar{F}\right| = \frac{m\left|v_f - v_i\right|}{\Delta t}$$

Hence,

$$\left|\bar{F}\right| = \frac{30.0\ \text{kg}\left|-10.0\ \text{m/s} - 20.0\ \text{m/s}\right|}{0.110\ \text{s}} = 8.18\times10^{3}\ \text{N}$$

By Newton's third law, this is also the magnitude of the average force exerted on the spike by the hammer during the blow. Thus, the stress in the spike is:

$$\text{stress} = \frac{F}{A} = \frac{8.18\times10^{3}\ \text{N}}{\pi(0.023\,0\ \text{m})^2/4} = 1.97\times10^{7}\ \text{N/m}^2$$

and the strain is: $\text{strain} = \dfrac{\text{stress}}{Y} = \dfrac{1.97\times10^{7}\ \text{N/m}^2}{20.0\times10^{10}\ \text{N/m}^2} = \boxed{9.85\times10^{-5}}$

P12.29 Consider recompressing the ice, which has a volume $1.09V_0$.

$$\Delta P = -B\left(\frac{\Delta V}{V_i}\right) = \frac{-\left(2.00\times10^9 \text{ N/m}^2\right)(-0.090)}{1.09} = \boxed{1.65\times10^8 \text{ N/m}^2}$$

P12.30 Let the 3.00 kg mass be mass #1, with the 5.00 kg mass, mass # 2. Applying Newton's second law to each mass gives:

$$m_1 a = T - m_1 g \qquad (1) \qquad \text{and} \qquad m_2 a = m_2 g - T \qquad (2)$$

where T is the tension in the wire.

Solving equation (1) for the acceleration gives: $a = \dfrac{T}{m_1} - g$

and substituting this into equation (2) yields: $\dfrac{m_2}{m_1}T - m_2 g = m_2 g - T$

Solving for the tension T gives

$$T = \frac{2m_1 m_2 g}{m_2 + m_1} = \frac{2(3.00 \text{ kg})(5.00 \text{ kg})\left(9.80 \text{ m/s}^2\right)}{8.00 \text{ kg}} = 36.8 \text{ N}$$

From the definition of Young's modulus, $Y = \dfrac{FL_i}{A(\Delta L)}$, the elongation of the wire is:

$$\Delta L = \frac{TL_i}{YA} = \frac{(36.8 \text{ N})(2.00 \text{ m})}{\left(2.00\times10^{11} \text{ N/m}^2\right)\pi\left(2.00\times10^{-3} \text{ m}\right)^2} = \boxed{0.029\,3 \text{ mm}}$$

P12.31 Part of the load force extends the cable and part compresses the column by the same distance $\Delta\ell$:

$$F = \frac{Y_A A_A \Delta\ell}{\ell_A} + \frac{Y_s A_s \Delta\ell}{\ell_s}$$

$$\Delta\ell = \frac{F}{Y_A A_A/\ell_A + Y_s A_s/\ell_s}$$

$$= \frac{8\,500 \text{ N}}{7\times10^{10}\pi\left(0.162\,4^2 - 0.161\,4^2\right)/4(3.25) + 20\times10^{10}\pi(0.012\,7)^2/4(5.75)}$$

$$= \boxed{8.60\times10^{-4} \text{ m}}$$

P12.32 $B = -\dfrac{\Delta P}{\Delta V/V_i} = -\dfrac{\Delta P V_i}{\Delta V}$

(a) $\Delta V = -\dfrac{\Delta P V_i}{B} = -\dfrac{\left(1.13\times10^8 \text{ N/m}^2\right)1 \text{ m}^3}{0.21\times10^{10} \text{ N/m}^2} = \boxed{-0.053\,8 \text{ m}^3}$

(b) The quantity of water with mass 1.03×10^3 kg occupies volume at the bottom $1 \text{ m}^3 - 0.053\,8 \text{ m}^3 = 0.946 \text{ m}^3$. So its density is $\dfrac{1.03\times10^3 \text{ kg}}{0.946 \text{ m}^3} = \boxed{1.09\times10^3 \text{ kg/m}^3}$

(c) With only a 5% volume change in this extreme case, liquid water is indeed nearly incompressible.

Additional Problems

P12.33 Let n_A and n_B be the normal forces at the points of support.

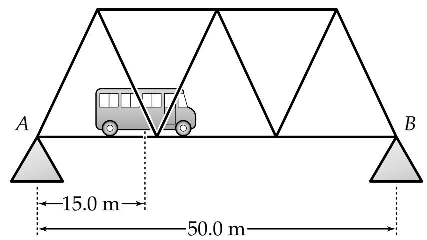

FIG. P12.33

Choosing the origin at point A with $\sum F_y = 0$ and $\sum \tau = 0$, we find:

$$n_A + n_B - (8.00\times10^4)g - (3.00\times10^4)g = 0 \quad \text{and}$$

$$-(3.00\times10^4)(g)15.0 - (8.00\times10^4)(g)25.0 + n_B(50.0) = 0$$

The equations combine to give $n_A = \boxed{5.98\times10^5 \text{ N}}$ and $b_B = \boxed{4.80\times10^5 \text{ N}}$.

***P12.34** Model the stove as a uniform 68 kg box. Its center of mass is at its geometric center, $\frac{28}{2} = 14$ inches behind its feet at the front corners. Assume that the light oven door opens to be horizontal and that a person stands on its outer end, $46.375 - 28 = 18.375$ inches in front of the front feet.

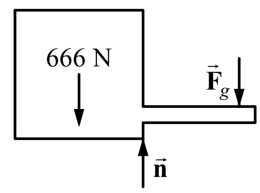

FIG. P12.34

We find the weight F_g of a person standing on the oven door with the stove balanced on its front feet in equilibrium: $\sum \tau = 0$

$$(68 \text{ kg})(9.8 \text{ m/s}^2)(14 \text{ in.}) + n(0) - F_g(18.375 \text{ in.}) = 0$$

$$F_g = \boxed{508 \text{ N}}$$

If the weight of the person is greater than this, the stove can tip forward. This weight corresponds to mass 51.8 kg, so the person could be a child. If the oven door is heavy (compared to the backsplash) or if the front feet are significantly far behind the front corners, the maximum weight will be significantly less than 508 N.

P12.35 With ℓ as large as possible, n_1 and n_2 will both be large. The equality sign in $f_2 \le \mu_s n_2$ will be true, but the less-than sign will apply in $f_1 < \mu_s n_1$. Take torques about the lower end of the pole.

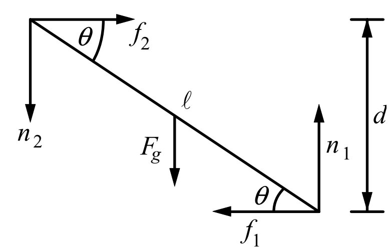

FIG. P12.35

$$n_2\ell\cos\theta + F_g\left(\frac{1}{2}\ell\right)\cos\theta - f_2\ell\sin\theta = 0$$

Setting $f_2 = 0.576n_2$, the torque equation becomes

$$n_2(1 - 0.576\tan\theta) + \frac{1}{2}F_g = 0$$

Since $n_2 > 0$, it is necessary that

$$1 - 0.576\tan\theta < 0$$

$$\therefore \tan\theta > \frac{1}{0.576} = 1.736$$

$$\therefore \theta > 60.1°$$

$$\therefore \ell = \frac{d}{\sin\theta} < \frac{7.80 \text{ ft}}{\sin 60.1°} = \boxed{9.00 \text{ ft}}$$

P12.36 When the concrete has cured and the pre-stressing tension has been released, the rod presses in on the concrete and with equal force, T_2, the concrete produces tension in the rod.

(a) In the concrete: stress $= 8.00\times10^6\ \text{N/m}^2 = Y\cdot(\text{strain}) = Y\left(\dfrac{\Delta L}{L_i}\right)$

Thus,

$$\Delta L = \frac{(\text{stress})L_i}{Y} = \frac{(8.00\times10^6\ \text{N/m}^2)(1.50\ \text{m})}{30.0\times10^9\ \text{N/m}^2}$$

or

$$\Delta L = 4.00\times10^{-4}\ \text{m} = \boxed{0.400\ \text{mm}}$$

(b) In the concrete: stress $= \dfrac{T_2}{A_c} = 8.00\times10^6\ \text{N/m}^2$, so

$$T_2 = (8.00\times10^6\ \text{N/m}^2)(50.0\times10^{-4}\ \text{m}^2) = \boxed{40.0\ \text{kN}}$$

(c) For the rod: $\dfrac{T_2}{A_R} = \left(\dfrac{\Delta L}{L_i}\right)Y_{\text{steel}}$ so $\Delta L = \dfrac{T_2 L_i}{A_R Y_{\text{steel}}}$

$$\Delta L = \frac{(4.00\times10^4\ \text{N})(1.50\ \text{m})}{(1.50\times10^{-4}\ \text{m}^2)(20.0\times10^{10}\ \text{N/m}^2)} = 2.00\times10^{-3}\ \text{m} = \boxed{2.00\ \text{mm}}$$

(d) The rod in the finished concrete is 2.00 mm longer than its unstretched length. To remove stress from the concrete, one must stretch the rod 0.400 mm farther, by a total of $\boxed{2.40\ \text{mm}}$.

(e) For the stretched rod around which the concrete is poured:

$$\frac{T_1}{A_R} = \left(\frac{\Delta L_{\text{total}}}{L_i}\right)Y_{\text{steel}} \quad \text{or} \quad T_1 = \left(\frac{\Delta L_{\text{total}}}{L_i}\right)A_R Y_{\text{steel}}$$

$$T_1 = \left(\frac{2.40\times10^{-3}\ \text{m}}{1.50\ \text{m}}\right)(1.50\times10^{-4}\ \text{m}^2)(20.0\times10^{10}\ \text{N/m}^2) = \boxed{48.0\ \text{kN}}$$

P12.37 (a) See the diagram.

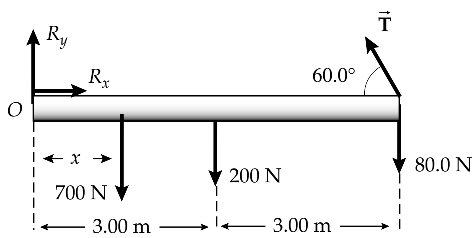

FIG. P12.37

(b) If $x = 1.00$ m, then

$$\sum\tau_O = (-700\ \text{N})(1.00\ \text{m}) - (200\ \text{N})(3.00\ \text{m}) - (80.0\ \text{N})(6.00\ \text{m}) + (T\sin 60.0°)(6.00\ \text{m}) = 0$$

Solving for the tension gives: $T = \boxed{343\ \text{N}}$.

From $\sum F_x = 0$, $R_x = T\cos 60.0° = \boxed{171\ \text{N}}$.

From $\sum F_y = 0$, $R_y = 980\ \text{N} - T\sin 60.0° = \boxed{683\ \text{N}}$.

(c) If $T = 900$ N:

$$\sum\tau_O = (-700\ \text{N})x - (200\ \text{N})(3.00\ \text{m}) - (80.0\ \text{N})(6.00\ \text{m}) + \left[(900\ \text{N})\sin 60.0°\right](6.00\ \text{m}) = 0$$

Solving for x gives: $x = \boxed{5.13\ \text{m}}$.

***P12.38** The 392 N is the weight of the uniform gate, which is 3 m wide. The hinges are 1.8 m apart. They exert horizontal forces A and C. Only one hinge exerts a vertical force. We assume it is the upper hinge.

(a) Free body diagram:

Statement:

A uniform 40.0-kg farm gate, 3.00 m wide and 1.80 m high, supports a 50.0-N bucket of grain hanging from its latch as shown. The gate is supported by hinges at two corners. Find the force each hinge exerts on the gate.

(b) From the torque equation,

$$C = \frac{738 \text{ N}\cdot\text{m}}{1.8 \text{ m}} = 410 \text{ N}$$

Then $A = 410$ N. Also $B = 442$ N.

$\boxed{\begin{array}{l}\text{The upper hinge exerts 410 N to the left and 442 N up.}\\ \text{The lower hinge exerts 410 N to the right.}\end{array}}$

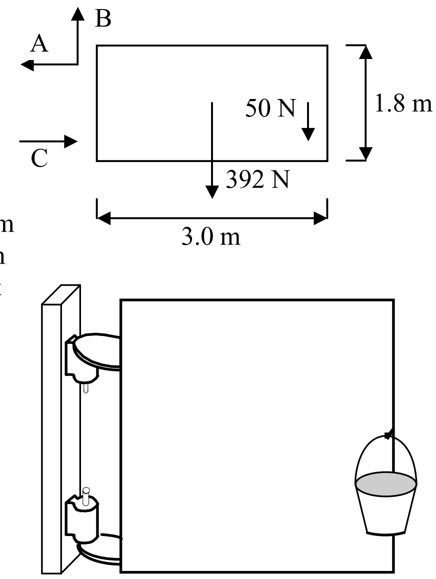

FIG. P12.38

P12.39 Using $\sum F_x = \sum F_y = \sum \tau = 0$, choosing the origin at the left end of the beam, we have (neglecting the weight of the beam)

$$\sum F_x = R_x - T\cos\theta = 0$$
$$\sum F_y = R_y + T\sin\theta - F_g = 0$$

and $\sum \tau = -F_g(L+d) + T\sin\theta(2L+d) = 0$.

Solving these equations, we find:

(a) $T = \boxed{\dfrac{F_g(L+d)}{\sin\theta(2L+d)}}$

(b) $R_x = \boxed{\dfrac{F_g(L+d)\cot\theta}{2L+d}}$ $\quad R_y = \boxed{\dfrac{F_g L}{2L+d}}$

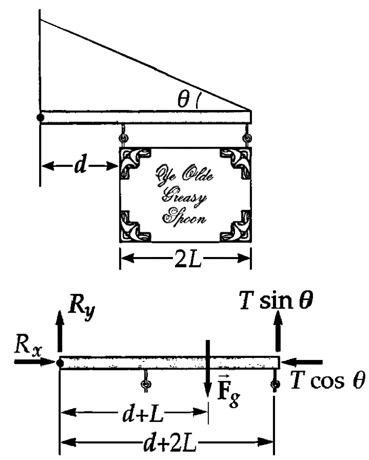

FIG. P12.39

P12.40 $\sum \tau_{\text{point }0} = 0$ gives

$$(T\cos 25.0°)\left(\frac{3\ell}{4}\sin 65.0°\right) + (T\sin 25.0°)\left(\frac{3\ell}{4}\cos 65.0°\right)$$
$$= (2\,000 \text{ N})(\ell\cos 65.0°) + (1\,200 \text{ N})\left(\frac{\ell}{2}\cos 65.0°\right)$$

From which, $T = 1\,465 \text{ N} = \boxed{1.46 \text{ kN}}$

From $\sum F_x = 0$,

$H = T\cos 25.0° = 1\,328 \text{ N}(\text{toward right}) = \boxed{1.33 \text{ kN}}$

From $\sum F_y = 0$,

$V = 3\,200 \text{ N} - T\sin 25.0° = 2\,581 \text{ N}(\text{upward}) = \boxed{2.58 \text{ kN}}$

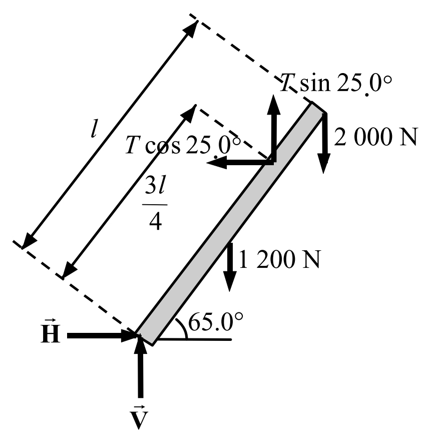

FIG. P12.40

P12.41 We interpret the problem to mean that the support at point B is frictionless. Then the support exerts a force in the x direction and

$$\boxed{F_{By}=0}$$

$$\sum F_x = F_{Bx} - F_{Ax} = 0$$

$$F_{Ay} - (3\,000 + 10\,000)g = 0$$

and

$$\sum \tau = -(3\,000g)(2.00) - (10\,000g)(6.00) + F_{Bx}(1.00) = 0$$

These equations combine to give

$$F_{Ax} = F_{Bx} = \boxed{6.47\times10^5\ \text{N}}$$

$$F_{Ay} = \boxed{1.27\times10^5\ \text{N}}$$

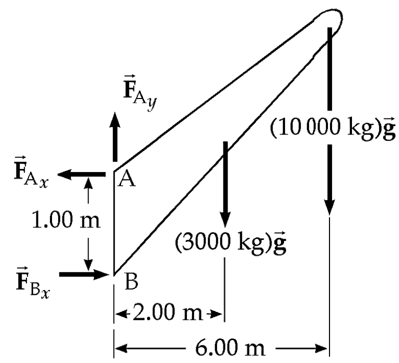

FIG. P12.41

***P12.42** Choosing torques about the hip joint, $\sum \tau = 0$ gives

$$-\frac{L}{2}(350\ \text{N}) + (T\sin 12.0°)\left(\frac{2L}{3}\right) - (200\ \text{N})L = 0$$

From which, $T = \boxed{2.71\ \text{kN}}$.

Let R_x = compression force along spine, and from $\sum F_x = 0$

$$R_x = T_x = T\cos 12.0° = \boxed{2.65\ \text{kN}}$$

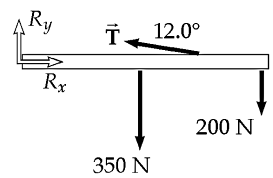

FIG. P12.42

(c)	You should lift "with your knees" rather than "with your back." In this situation, with a load weighing only 200 N, you can make the compressional force in your spine about ten times smaller by bending your knees and lifting with your back as straight as possible.

P12.43 From the free-body diagram, the angle that the string tension makes with the rod is

$$\theta = 60.0° + 20.0° = 80.0°$$

and the perpendicular component of the string tension is $T\sin 80.0°$. Summing torques around the base of the rod gives

$$\sum \tau = 0: \quad -(4.00\ \text{m})(10\,000\ \text{N})\cos 60° + T(4.00\ \text{m})\sin 80° = 0$$

$$T = \frac{(10\,000\ \text{N})\cos 60.0°}{\sin 80.0°} = \boxed{5.08\times10^3\ \text{N}}$$

$$\sum F_x = 0: \quad F_H - T\cos 20.0° = 0$$

$$F_H = T\cos 20.0° = \boxed{4.77\times10^3\ \text{N}}$$

$$\sum F_y = 0: \quad F_V + T\sin 20.0° - 10\,000\ \text{N} = 0$$

and $F_V = (10\,000\ \text{N}) - T\sin 20.0° = \boxed{8.26\times10^3\ \text{N}}$

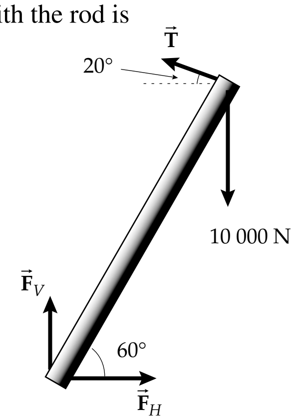

FIG. P12.43

P12.44 (a) Just three forces act on the rod: forces perpendicular to the sides of the trough at A and B, and its weight. The lines of action of A and B will intersect at a point above the rod. They will have no torque about this point. The rod's weight will cause a torque about the point of intersection as in Figure 12.52(a), and the rod will not be in equilibrium unless the center of the rod lies vertically below the intersection point, as in Figure 12.52(b). All three forces must be concurrent. Then the line of action of the weight is a diagonal of the rectangle formed by the trough and the normal forces, and the rod's center of gravity is vertically above the bottom of the trough.

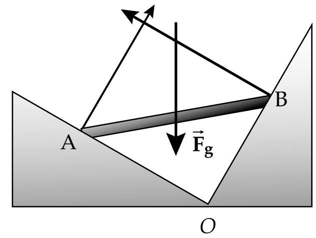

FIG. P12.44(a)

(b) In Figure (b), $\overline{AO}\cos 30.0° = \overline{BO}\cos 60.0°$ and

$$L^2 = \overline{AO}^2 + \overline{BO}^2 = \overline{AO}^2 + \overline{AO}^2\left(\frac{\cos^2 30.0°}{\cos^2 60.0°}\right)$$

$$\overline{AO} = \frac{L}{\sqrt{1+\left(\frac{\cos 30°}{\cos 60°}\right)^2}} = \frac{L}{2}$$

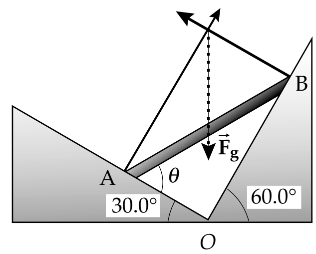

FIG. P12.44(b)

So

$$\cos\theta = \frac{\overline{AO}}{L} = \frac{1}{2} \text{ and } \theta = \boxed{60.0°}$$

P12.45 (a) Locate the origin at the bottom left corner of the cabinet and let x = distance between the *resultant normal force* and the *front of the cabinet.* Then we have

$$\sum F_x = 200\cos 37.0° - \mu n = 0 \qquad (1)$$

$$\sum F_y = 200\sin 37.0° + n - 400 = 0 \qquad (2)$$

$$\sum \tau = n(0.600 - x) - 400(0.300) + 200\sin 37.0°(0.600)$$
$$-200\cos 37.0°(0.400) = 0 \qquad (3)$$

From (2),

$$n = 400 - 200\sin 37.0° = 280 \text{ N}$$

From (3),

$$x = \frac{72.2 - 120 + 280(0.600) - 64.0}{280}$$

$$x = \boxed{20.1 \text{ cm}} \text{ to the left of the front edge}$$

From (1), $\mu_k = \dfrac{200\cos 37.0°}{280} = \boxed{0.571}$

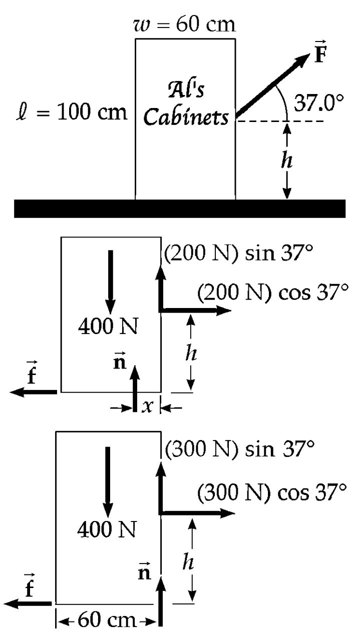

FIG. P12.45

(b) In this case, locate the origin $x = 0$ at the bottom right corner of the cabinet. Since the cabinet is about to tip, we can use $\sum \tau = 0$ to find h:

$$\sum \tau = 400(0.300) - (300\cos 37.0°)h = 0 \qquad h = \frac{120}{300\cos 37.0°} = \boxed{0.501 \text{ m}}$$

P12.46 (a), (b) Use the first diagram and sum the torques about the lower front corner of the cabinet.

$$\sum \tau = 0 \Rightarrow -F(1.00\text{ m}) + (400\text{ N})(0.300\text{ m}) = 0$$

yielding $F = \dfrac{(400\text{ N})(0.300\text{ m})}{1.00\text{ m}} = \boxed{120\text{ N}}$

$$\sum F_x = 0 \Rightarrow -f + 120\text{ N} = 0, \quad \text{or} \quad f = 120\text{ N}$$

$$\sum F_y = 0 \Rightarrow -400\text{ N} + n = 0, \quad \text{so} \quad n = 400\text{ N}$$

Thus,

$$\mu_s = \frac{f}{n} = \frac{120\text{ N}}{400\text{ N}} = \boxed{0.300}$$

(c) Apply F' at the upper rear corner and directed so $\theta + \phi = 90.0°$ to obtain the largest possible lever arm.

$$\theta = \tan^{-1}\left(\frac{1.00\text{ m}}{0.600\text{ m}}\right) = 59.0°$$

Thus,

$$\phi = 90.0° - 59.0° = 31.0°$$

Sum the torques about the lower front corner of the cabinet:

$$-F'\sqrt{(1.00\text{ m})^2 + (0.600\text{ m})^2} + (400\text{ N})(0.300\text{ m}) = 0$$

so

$$F' = \frac{120\text{ N}\cdot\text{m}}{1.17\text{ m}} = 103\text{ N}$$

Therefore, the minimum force required to tip the cabinet is

$\boxed{\text{103 N applied at 31.0° above the horizontal at the upper left corner}}$.

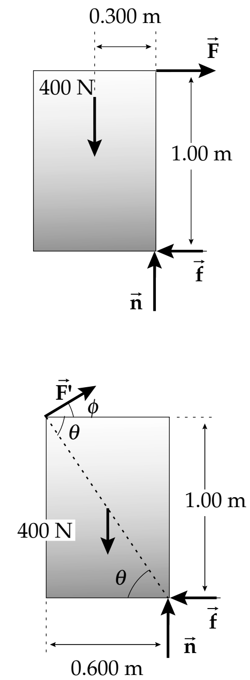

FIG. P12.46

P12.47 (a) We can use $\sum F_x = \sum F_y = 0$ and $\sum \tau = 0$ with pivot point at the contact on the floor.

Then

$$\sum F_x = T - \mu_s n = 0$$

$$\sum F_y = n - Mg - mg = 0, \text{ and}$$

$$\sum \tau = Mg(L\cos\theta) + mg\left(\frac{L}{2}\cos\theta\right) - T(L\sin\theta) = 0$$

Solving the above equations gives

$$M = \boxed{\frac{m}{2}\left(\frac{2\mu_s \sin\theta - \cos\theta}{\cos\theta - \mu_s \sin\theta}\right)}$$

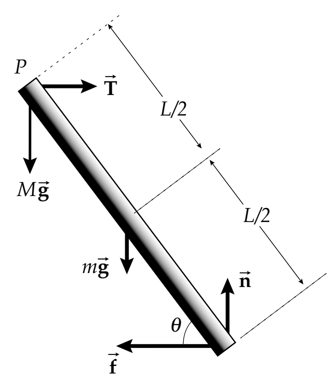

FIG. P12.47

This answer is the maximum value for M if $\mu_s < \cot\theta$. If $\mu_s \geq \cot\theta$, the mass M can increase without limit. It has no maximum value.

(b) At the floor, we have the normal force in the y-direction and frictional force in the x-direction. The reaction force then is

$$R = \sqrt{n^2 + (\mu_s n)^2} = \boxed{(M+m)g\sqrt{1+\mu_s^2}}$$

At point P, the force of the beam on the rope is

$$F = \sqrt{T^2 + (Mg)^2} = \boxed{g\sqrt{M^2 + \mu_s^2(M+m)^2}}$$

***P12.48** Suppose that a bar exerts on a pin a force not along the length of the bar. Then, the pin exerts on the bar a force with a component perpendicular to the bar. The only other force on the bar is the pin force on the other end. For $\sum \vec{\mathbf{F}} = 0$, this force must also have a component perpendicular to the bar. Then each of the forces produces a torque about the center of the bar in the same sense. The total torque on the bar is not zero. The contradiction proves that the bar can only exert forces along its length.

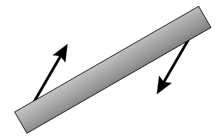

FIG. P12.48

***P12.49** (a) The height of pin B is

$$(10.0\text{ m})\sin 30.0° = 5.00\text{ m}$$

The length of bar BC is then

$$\overline{BC} = \frac{5.00\text{ m}}{\sin 45.0°} = 7.07\text{ m}$$

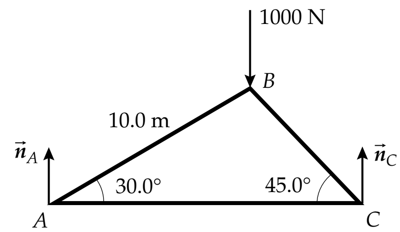

FIG. P12.49(a)

Consider the entire truss:

$$\sum F_y = n_A - 1\,000\text{ N} + n_C = 0$$

$$\sum \tau_A = -(1\,000\text{ N})10.0\cos 30.0° + n_C\left[10.0\cos 30.0° + 7.07\cos 45.0°\right] = 0$$

Which gives $\boxed{n_C = 634\text{ N}}$.

Then,

$$\boxed{n_A = 1\,000\text{ N} - n_C = 366\text{ N}}$$

(b) Joint A:

$$\sum F_y = 0:\ -C_{AB}\sin 30.0° + 366\text{ N} = 0$$

so

$$C_{AB} = \boxed{732\text{ N}}$$

$$\sum F_x = 0:\ -C_{AB}\cos 30.0° + T_{AC} = 0$$

$$T_{AC} = (732\text{ N})\cos 30.0° = \boxed{634\text{ N}}$$

Joint B:

$$\sum F_x = 0:\ (732\text{ N})\cos 30.0° - C_{BC}\cos 45.0° = 0$$

$$C_{BC} = \frac{(732\text{ N})\cos 30.0°}{\cos 45.0°} = \boxed{897\text{ N}}$$

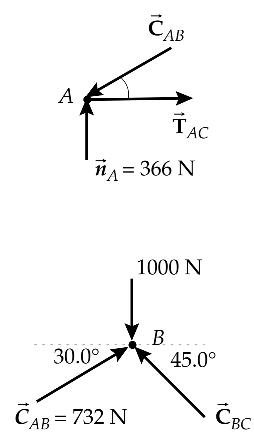

FIG. P12.49(b)

***P12.50** Considering the torques about the point at the bottom of the bracket yields:

$$W(0.050\,0\text{ m}) - F(0.060\,0\text{ m}) = 0 \text{ so } F = 0.833W$$

(a) With $W = 80.0$ N, $F = 0.833(80\text{ N}) = \boxed{66.7\text{ N}}$.

(b) Differentiate with respect to time: $dF/dt = 0.833\ dW/dt$
The force exerted by the screw is increasing at the rate $dF/dt = 0.833(0.15\text{ N/s}) = \boxed{0.125\text{ N/s}}$

P12.51 From geometry, observe that

$$\cos\theta = \frac{1}{4} \qquad \text{and} \qquad \theta = 75.5°$$

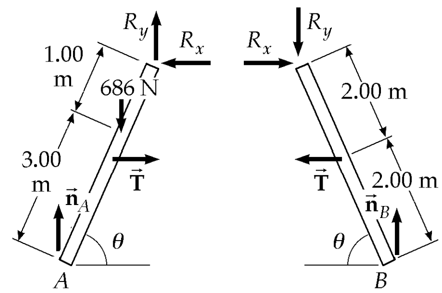

FIG. P12.51

For the left half of the ladder, we have

$$\sum F_x = T - R_x = 0 \qquad (1)$$

$$\sum F_y = R_y + n_A - 686\text{ N} = 0 \qquad (2)$$

$$\sum \tau_{\text{top}} = 686\text{ N}(1.00\cos 75.5°) + T(2.00\sin 75.5°)$$
$$-n_A(4.00\cos 75.5°) = 0 \qquad (3)$$

For the right half of the ladder we have

$$\sum F_x = R_x - T = 0$$

$$\sum F_y = n_B - R_y = 0 \qquad (4)$$

$$\sum \tau_{\text{top}} = n_B(4.00\cos 75.5°) - T(2.00\sin 75.5°) = 0 \qquad (5)$$

Solving equations 1 through 5 simultaneously yields:

(a) $\boxed{T = 133\text{ N}}$

(b) $\boxed{n_A = 429\text{ N}}$ and $\boxed{n_B = 257\text{ N}}$

(c) $\boxed{R_x = 133\text{ N}}$ and $\boxed{R_y = 257\text{ N}}$

The force exerted by the left half of the ladder on the right half is to the right and downward.

P12.52 Imagine gradually increasing the force *P*. This will make the force of static friction at the bottom increase, so that the normal force at the wall increases and the friction force at the wall *can* increase. As *P* reaches its maximum value, the cylinder will turn clockwise microscopically to stress the welds at both contact points and make both forces of friction increase to their maximum values. A comparison: To make a four-legged table start to slide across the floor, you must push on it hard enough to counterbalance the maximum static friction forces on all four legs together.

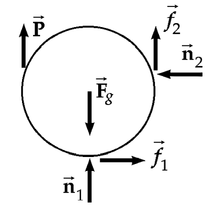

FIG. P12.52

When it is on the verge of slipping, the cylinder is in equilibrium.

$\sum F_x = 0$: $\quad f_1 = n_2 = \mu_s n_1 \quad$ and $\quad f_2 = \mu_s n_2$

$\sum F_y = 0$: $\quad P + n_1 + f_2 = F_g$

$\sum \tau = 0$: $\quad P = f_1 + f_2$

As P grows so do f_1 and f_2.

Therefore, since $\mu_s = \dfrac{1}{2}$, $\quad f_1 = \dfrac{n_1}{2} \quad$ and $\quad f_2 = \dfrac{n_2}{2} = \dfrac{n_1}{4}$

then $\quad P + n_1 + \dfrac{n_1}{4} = F_g \quad (1) \quad$ and $\quad P = \dfrac{n_1}{2} + \dfrac{n_1}{4} = \dfrac{3}{4}n_1 \quad (2)$

So $\quad P + \dfrac{5}{4}n_1 = F_g \quad$ becomes $\quad P + \dfrac{5}{4}\left(\dfrac{4}{3}P\right) = F_g \quad$ or $\quad \dfrac{8}{3}P = F_g$

Therefore,

$$P = \boxed{\frac{3}{8}F_g}$$

P12.53 (a) $|F| = k(\Delta L)$, Young's modulus is $Y = \dfrac{F/A}{\Delta L/L_i} = \dfrac{FL_i}{A(\Delta L)}$

Thus,

$$Y = \frac{kL_i}{A} \quad \text{and} \quad k = \boxed{\frac{YA}{L_i}}$$

(b) $$W = -\int_0^{\Delta L} F\,dx = -\int_0^{\Delta L} (-kx)\,dx = \frac{YA}{L_i}\int_0^{\Delta L} x\,dx = \boxed{YA\frac{(\Delta L)^2}{2L_i}}$$

P12.54 (a) Take both balls together. Their weight is 3.33 N and their CG is at their contact point.

$$\sum F_x = 0:\ +P_3 - P_1 = 0$$

$$\sum F_y = 0:\ +P_2 - 3.33\text{ N} = 0 \qquad P_2 = \boxed{3.33\text{ N}}$$

$$\sum \tau_A = 0:\ -P_3 R + P_2 R - 3.33\text{ N}\,(R + R\cos 45.0°)$$

$$+P_1(R + 2R\cos 45.0°) = 0$$

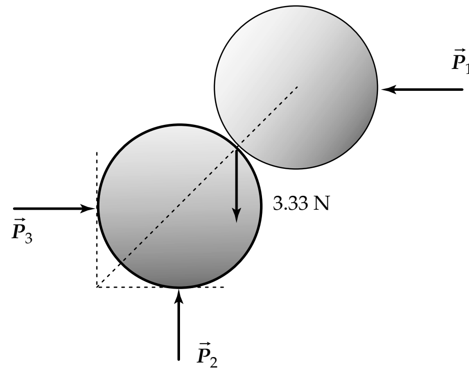

FIG. P12.54(a)

Substituting,

$$-P_1 R + (3.33\text{ N})R - (3.33\text{ N})R(1+\cos 45.0°)$$
$$+P_1 R(1 + 2\cos 45.0°) = 0$$
$$(3.33\text{ N})\cos 45.0° = 2P_1\cos 45.0°$$
$$P_1 = \boxed{1.67\text{ N}} \text{ so } P_3 = \boxed{1.67\text{ N}}$$

(b) Take the upper ball. The lines of action of its weight, of P_1, and of the normal force n exerted by the lower ball all go through its center, so for rotational equilibrium there can be no frictional force.

$$\sum F_x = 0:\ n\cos 45.0° - P_1 = 0$$

$$n = \frac{1.67\text{ N}}{\cos 45.0°} = \boxed{2.36\text{ N}}$$

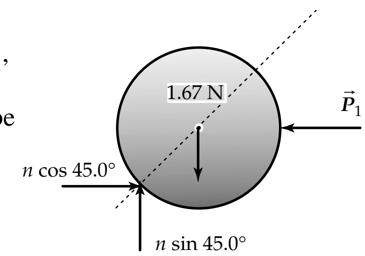

FIG. P12.54(b)

$\sum F_y = 0:\ n\sin 45.0° - 1.67\text{ N} = 0$ gives the same result.

P12.55 $\sum F_y = 0: \quad +380\text{ N} - F_g + 320\text{ N} = 0$

$$F_g = 700\text{ N}$$

Take torques about her feet:

$$\sum \tau = 0: \quad -380\text{ N}(2.00\text{ m}) + (700\text{ N})x + (320\text{ N})0 = 0$$

$$x = \boxed{1.09\text{ m}}$$

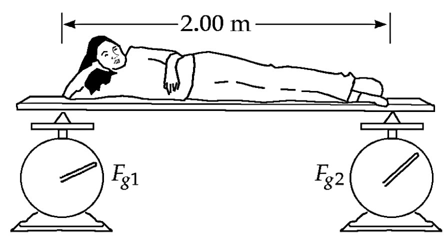

FIG. P12.55

P12.56 The tension in this cable is not uniform, so this becomes a fairly difficult problem.

$$\frac{dL}{L} = \frac{F}{YA}$$

At any point in the cable, F is the weight of cable below that point. Thus, $F = \mu g y$ where μ is the mass per unit length of the cable.

Then,

$$\Delta y = \int_0^{L_i}\left(\frac{dL}{L}\right)dy = \frac{\mu g}{YA}\int_0^{L_i} y\,dy = \frac{1}{2}\frac{\mu g L_i^2}{YA}$$

$$\Delta y = \frac{1}{2}\frac{(2.40)(9.80)(500)^2}{(2.00\times 10^{11})(3.00\times 10^{-4})} = 0.049\,0\text{ m} = \boxed{4.90\text{ cm}}$$

P12.57 (a) $F = m\left(\frac{\Delta v}{\Delta t}\right) = (1.00 \text{ kg})\frac{(10.0-1.00) \text{ m/s}}{0.002 \text{ s}} = \boxed{4\,500 \text{ N}}$

(b) $\text{stress} = \frac{F}{A} = \frac{4\,500 \text{ N}}{(0.010 \text{ m})(0.100 \text{ m})} = \boxed{4.50\times 10^6 \text{ N/m}^2}$

(c) $\boxed{\text{Yes.}}$ This is more than sufficient to break the board.

P12.58 Let θ represent the angle of the wire with the vertical. The radius of the circle of motion is $r = (0.850 \text{ m})\sin\theta$.
For the mass:

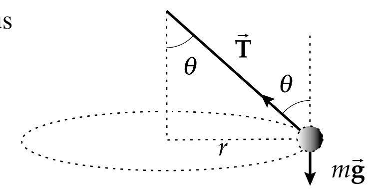

FIG. P12.58

$$\sum F_r = ma_r = m\frac{v^2}{r} = mr\omega^2$$

$$T\sin\theta = m[(0.850 \text{ m})\sin\theta]\omega^2$$

Further, $\frac{T}{A} = Y\cdot(\text{strain})$ or $T = AY\cdot(\text{strain})$

Thus, $AY\cdot(\text{strain}) = m(0.850 \text{ m})\omega^2$, giving

$$\omega = \sqrt{\frac{AY\cdot(\text{strain})}{m(0.850 \text{ m})}} = \sqrt{\frac{\pi(3.90\times 10^{-4} \text{ m})^2(7.00\times 10^{10} \text{ N/m}^2)(1.00\times 10^{-3})}{(1.20 \text{ kg})(0.850 \text{ m})}}$$

or $\omega = \boxed{5.73 \text{ rad/s}}$

P12.59 (a) If the acceleration is a, we have $P_x = ma$ and $P_y + n - F_g = 0$. Taking the origin at the center of gravity, the torque equation gives

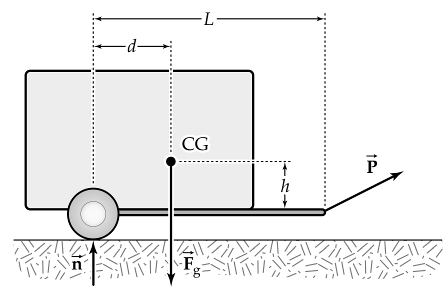

FIG. P12.59

$$P_y(L-d) + P_x h - nd = 0$$

Solving these equations, we find

$$P_y = \boxed{\frac{F_g}{L}\left(d - \frac{ah}{g}\right)}$$

(b) If $P_y = 0$, then $d = \frac{ah}{g} = \frac{(2.00 \text{ m/s}^2)(1.50 \text{ m})}{9.80 \text{ m/s}^2} = \boxed{0.306 \text{ m}}$.

(c) Using the given data, $P_x = -306$ N and $P_y = 553$ N.

Thus, $\boxed{\vec{\mathbf{P}} = (-306\hat{\mathbf{i}} + 553\hat{\mathbf{j}}) \text{ N}}$.

P12.60 When the car is on the point of rolling over, the normal force on its inside wheels is zero.

$$\sum F_y = ma_y: \qquad n - mg = 0$$

$$\sum F_x = ma_x: \qquad f = \frac{mv^2}{R}$$

Take torque about the center of mass: $fh - n\frac{d}{2} = 0$.

Then by substitution $\frac{mv_{\max}^2}{R}h - \frac{mgd}{2} = 0 \qquad v_{\max} = \sqrt{\frac{gdR}{2h}}$

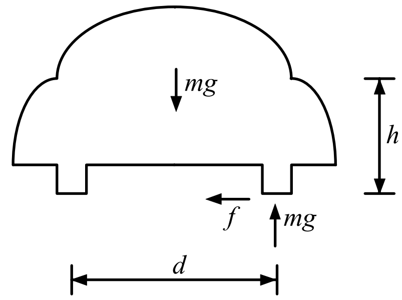

FIG. P12.60

A wider wheelbase (larger d) and a lower center of mass (smaller h) will reduce the risk of rollover.

ANSWERS TO EVEN PROBLEMS

P12.2 $F_y + R_y - F_g = 0; \quad F_x - R_x = 0; \quad F_y \ell\cos\theta - F_g\left(\frac{\ell}{2}\right)\cos\theta - F_x \ell\sin\theta = 0$

P12.4 see the solution

P12.6 0.750 m

P12.8 (a) 9.00 g (b) 52.5 g (c) 49.0 g

P12.10 (a) 1.04 kN at 60° upward and to the right. (b) $\vec{\mathbf{F}}_{\text{surface}} = (370\text{ N})\hat{\mathbf{i}} + (900\text{ N})\hat{\mathbf{j}}$

P12.12 (a) $f = \left[\frac{m_1 g}{2} + \frac{m_2 g x}{L}\right]\cot\theta;\ n_g = (m_1 + m_2)g$ (b) $\mu = \frac{(m_1/2 + m_2 d/L)\cot\theta}{m_1 + m_2}$

P12.14 (a) 392 N (b) 339 N to the right (c) 0 (d) 0 (e) 392 N (f) 339 N to the right (g) Both are equally accurate and essentially equally simple. We appear to have four equations, but only three are independent. We can determine three unknowns.

P12.16 (a) 35.5 kN (b) 11.5 kN to the right (c) 4.19 kN down

P12.18 (a) 859 N (b) 104 kN at 36.9° above the horizontal to the left

P12.20 (a) see the solution (b) θ decreases (c) R decreases

P12.22 1.0×10^{11} N/m^2

P12.24 (a) 73.6 kN (b) 2.50 mm

P12.26 ~ 1 cm

P12.28 9.85×10^{-5}

P12.30 0.029 3 mm

P12.32 (a) $-0.053\,8$ m^3 (b) 1.09×10^3 kg/m^3 (c) With only a 5% change in volume in this extreme case, liquid water is indeed nearly incompressible in biological and student-laboratory situations.

P12.34 The weight must be 508 N or more. The person could be a child. We assume the stove is a uniform box with feet at its corners. We ignore the masses of the backsplash and the oven door. If the oven door is heavy, the minimum weight for the person might be somewhat less than 508 N.

P12.36 (a) 0.400 mm (b) 40.0 kN (c) 2.00 mm (d) 2.40 mm (e) 48.0 kN

P12.38 (a) See the solution. The weight of the uniform gate is 392 N. It is 3.00 m wide. The hinges are separated vertically by 1.80 m. The bucket of grain weighs 50.0 N. One of the hinges, which we suppose is the upper one, supports the whole weight of the gate. Find the components of the forces that both hinges exert on the gate. (b) The upper hinge exerts $A = 410$ N to the left and $B = 442$ N up. The lower hinge exerts $C = 410$ N to the right.

P12.40 1.46 kN; $\left(1.33\hat{\mathbf{i}} + 2.58\hat{\mathbf{j}}\right)$ kN

P12.42 (a) 2.71 kN (b) 2.65 kN (c) You should lift "with your knees" rather than "with your back." In this situation, you can make the compressional force in your spine about ten times smaller by bending your knees and lifting with your back as straight as possible.

P12.44 (a) see the solution (b) 60.0°

P12.46 (a) 120 N (b) 0.300 (c) 103 N at 31.0° above the horizontal to the right

P12.48 Assume a strut exerts on a pin a force with a component perpendicular to the length of the strut. Then the pin must exert a force on the strut with a perpendicular component of this size. For translational equilibrium, the pin at the other end of the strut must also exert the same size force on the strut in the opposite direction. Then the strut will feel two torques about its center in the same sense. It will not be in equilibrium, but will start to rotate. The contradiction proves that we were wrong to assume the existence of the perpendicular force. The strut can exert on the pin only a force parallel to its length.

P12.50 (a) 66.7 N (b) increasing at 0.125 N/s

P12.52 If either static friction force were at less than its maximum value, the cylinder would rotate by a microscopic amount to put more stress on some welds and to bring that friction force to its maximum value. $P = 3F_g/8$

P12.54 (a) $P_1 = 1.67$ N; $P_2 = 3.33$ N; $P_3 = 1.67$ N (b) 2.36 N

P12.56 4.90 cm

P12.58 5.73 rad/s

P12.60 See the solution. A wider wheelbase (larger d) and a lower center of mass (smaller h) will reduce the risk of rollover.

13

Universal Gravitation

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

***Q13.1** The force is proportional to the product of the masses and inversely proportional to the square of the separation distance, so we compute m_1m_2/r^2 for each case:
(a) $2\cdot3/1^2 = 6$ (b) 18 (c) $18/4 = 4.5$ (d) 4.5 (e) $16/4 = 4$.
The ranking is then b > a > c = d > e.

***Q13.2** Answer (d). The International Space Station orbits just above the atmosphere, only a few hundred kilometers above the ground. This distance is small compared to the radius of the Earth, so the gravitational force on the astronaut is only slightly less than on the ground. We think of it as having a very different effect than it does on the ground, just because the normal force on the orbiting astronaut is zero.

***Q13.3** Answer (b). Switching off gravity would let the atmosphere evaporate away, but switching off the atmosphere has no effect on the planet's gravitational field.

Q13.4 To a good first approximation, your bathroom scale reading is unaffected because you, the Earth, and the scale are all in free fall in the Sun's gravitational field, in orbit around the Sun. To a precise second approximation, you weigh slightly less at noon and at midnight than you do at sunrise or sunset. The Sun's gravitational field is a little weaker at the center of the Earth than at the surface subsolar point, and a little weaker still on the far side of the planet. When the Sun is high in your sky, its gravity pulls up on you a little more strongly than on the Earth as a whole. At midnight the Sun pulls down on you a little less strongly than it does on the Earth below you. So you can have another doughnut with lunch, and your bedsprings will still last a little longer.

***Q13.5** Having twice the mass would make the surface gravitational field two times larger. But the inverse square law says that having twice the radius would make the surface acceleration due to gravitation four times smaller. Altogether, g at the surface of B becomes $(2\text{ m/s}^2)(2)/4 = 1\text{ m/s}^2$, answer (e).

***Q13.6** (i) $4^2 = 16$ times smaller: Answer (i), according to the inverse square law.

(ii) $mv^2/r = GMm/r^2$ predicts that v is proportional to $(1/r)^{1/2}$, so it becomes $(1/4)^{1/2} = 1/2$ as large: Answer (f).

(iii) $(4^3)^{1/2} = 8$ times larger: Answer (b), according to Kepler's third law.

***Q13.7** Answer (b). The Earth is farthest from the sun around July 4 every year, when it is summer in the northern hemisphere and winter in the southern hemisphere. As described by Kepler's second law, this is when the planet is moving slowest in its orbit. Thus it takes more time for the planet to plod around the 180° span containing the minimum-speed point.

Q13.8 Air resistance causes a decrease in the energy of the satellite-Earth system. This reduces the diameter of the orbit, bringing the satellite closer to the surface of the Earth. A satellite in a smaller orbit, however, must travel faster. Thus, the effect of air resistance is to speed up the satellite!

***Q13.9** Answer (c). Ten terms are needed in the potential energy:

$$U = U_{12} + U_{13} + U_{14} + U_{15} + U_{23} + U_{24} + U_{25} + U_{34} + U_{35} + U_{45}$$

Q13.10 The escape speed from the Earth is 11.2 km/s and that from the Moon is 2.3 km/s, smaller by a factor of 5. The energy required—and fuel—would be proportional to v^2, or 25 times more fuel is required to leave the Earth versus leaving the Moon.

***Q13.11** The gravitational potential energy of the Earth-Sun system is negative and twice as large in magnitude as the kinetic energy of the Earth relative to the Sun. Then the total energy is negative and equal in absolute value to the kinetic energy. The ranking is a > b = c.

Q13.12 For a satellite in orbit, one focus of an elliptical orbit, or the center of a circular orbit, must be located at the center of the Earth. If the satellite is over the northern hemisphere for half of its orbit, it must be over the southern hemisphere for the other half. We could share with Easter Island a satellite that would look straight down on Arizona each morning and vertically down on Easter Island each evening.

Q13.13 Every point q on the sphere that does not lie along the axis connecting the center of the sphere and the particle will have companion point q' for which the components of the gravitational force perpendicular to the axis will cancel. Point q' can be found by rotating the sphere through 180° about the axis. The forces will not necessarily cancel if the mass is not uniformly distributed, unless the center of mass of the non-uniform sphere still lies along the axis.

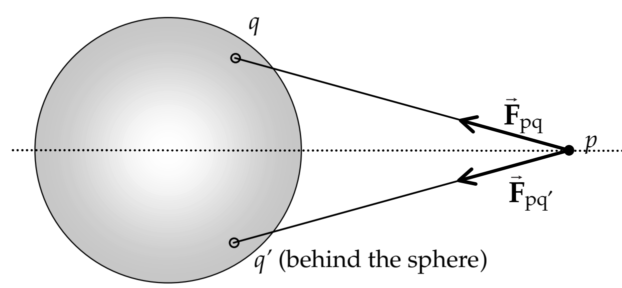

FIG. Q13.13

Q13.14 Speed is maximum at closest approach. Speed is minimum at farthest distance. These two points, perihelion and aphelion respectively, are 180° apart, at opposite ends of the major axis of the orbit.

Q13.15 Set the universal description of the gravitational force, $F_g = \frac{GM_X m}{R_X^2}$, equal to the local description, $F_g = ma_{\text{gravitational}}$, where M_X and R_X are the mass and radius of planet X, respectively, and m is the mass of a "test particle." Divide both sides by m.

Q13.16 The gravitational force of the Earth on an extra particle at its center must be zero, not infinite as one interpretation of Equation 13.1 would suggest. All the bits of matter that make up the Earth will pull in different outward directions on the extra particle.

Q13.17 Cavendish determined G. Then from $g = \frac{GM}{R^2}$, one may determine the mass of the Earth.

***Q13.18** The gravitational force is conservative. An encounter with a stationary mass cannot permanently speed up a spacecraft. But Jupiter is moving. A spacecraft flying across its orbit just behind the planet will gain kinetic energy as the planet's gravity does net positive work on it. This is a collision because the spacecraft and planet exert forces on each other while they are isolated from outside forces. It is an elastic collision. The planet loses kinetic energy as the spacecraft gains it.

SOLUTIONS TO PROBLEMS

Section 13.1 **Newton's Law of Universal Gravitation**

P13.1 For two 70-kg persons, modeled as spheres,

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)(70\ \text{kg})(70\ \text{kg})}{(2\ \text{m})^2}\ \boxed{\sim 10^{-7}\ \text{N}}$$

P13.2 $F = m_1 g = \dfrac{Gm_1m_2}{r^2}$

$$g = \frac{Gm_2}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(4.00\times10^4\times10^3\ \text{kg}\right)}{(100\ \text{m})^2} = \boxed{2.67\times10^{-7}\ \text{m/s}^2}$$

P13.3 (a) At the midpoint between the two objects, the forces exerted by the 200-kg and 500-kg objects are oppositely directed, and from

$$F_g = \frac{Gm_1m_2}{r^2}$$

we have $\sum F = \dfrac{G(50.0\ \text{kg})(500\ \text{kg}-200\ \text{kg})}{(0.200\ \text{m})^2} = \boxed{2.50\times10^{-5}\ \text{N}}$ toward the 500-kg object.

(b) At a point between the two objects at a distance d from the 500-kg objects, the net force on the 50.0-kg object will be zero when

$$\frac{G(50.0\ \text{kg})(200\ \text{kg})}{(0.400\ \text{m}-d)^2} = \frac{G(50.0\ \text{kg})(500\ \text{kg})}{d^2}$$

To solve, cross-multiply to clear of fractions and take the square root of both sides. The result is $d = \boxed{0.245\ \text{m from the 500-kg object toward the smaller object}}$.

P13.4 $m_1 + m_2 = 5.00\ \text{kg} \qquad m_2 = 5.00\ \text{kg} - m_1$

$$F = G\frac{m_1m_2}{r^2} \Rightarrow 1.00\times10^{-8}\ \text{N} = \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{m_1(5.00\ \text{kg}-m_1)}{(0.200\ \text{m})^2}$$

$$(5.00\ \text{kg})m_1 - m_1^2 = \frac{\left(1.00\times10^{-8}\ \text{N}\right)\left(0.040\,0\ \text{m}^2\right)}{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = 6.00\ \text{kg}^2$$

Thus,

$$m_1^2 - (5.00\ \text{kg})m_1 + 6.00\ \text{kg} = 0$$

or

$$(m_1 - 3.00\ \text{kg})(m_1 - 2.00\ \text{kg}) = 0$$

giving $\boxed{m_1 = 3.00\ \text{kg, so}\ m_2 = 2.00\ \text{kg}}$. The answer $m_1 = 2.00$ kg and $m_2 = 3.00$ kg is physically equivalent.

P13.5 The force exerted on the 4.00-kg mass by the 2.00-kg mass is directed upward and given by

$$\vec{\mathbf{F}}_{24} = G\frac{m_4 m_2}{r_{24}^2}\hat{\mathbf{j}}$$

$$= \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{(4.00\ \text{kg})(2.00\ \text{kg})}{(3.00\ \text{m})^2}\hat{\mathbf{j}}$$

$$= 5.93\times10^{-11}\,\hat{\mathbf{j}}\text{N}$$

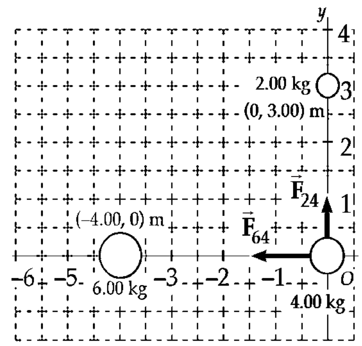

FIG. P13.5

The force exerted on the 4.00-kg mass by the 6.00-kg mass is directed to the left

$$\vec{\mathbf{F}}_{64} = G\frac{m_4 m_6}{r_{64}^2}\left(-\hat{\mathbf{i}}\right) = \left(-6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{(4.00\ \text{kg})(6.00\ \text{kg})}{(4.00\ \text{m})^2}\hat{\mathbf{i}}$$

$$= -10.0\times10^{-11}\,\hat{\mathbf{i}}\text{N}$$

Therefore, the resultant force on the 4.00-kg mass is $\vec{\mathbf{F}}_4 = \vec{\mathbf{F}}_{24} + \vec{\mathbf{F}}_{64} = \boxed{\left(-10.0\hat{\mathbf{i}} + 5.93\hat{\mathbf{j}}\right)\times10^{-11}\ \text{N}}$.

***P13.6** (a) The Sun-Earth distance is 1.496×10^{11} m and the Earth-Moon distance is 3.84×10^{8} m, so the distance from the Sun to the Moon during a solar eclipse is

$$1.496\times10^{11}\ \text{m} - 3.84\times10^{8}\ \text{m} = 1.492\times10^{11}\ \text{m}$$

The mass of the Sun, Earth, and Moon are $M_S = 1.99\times10^{30}$ kg

$M_E = 5.98\times10^{24}$ kg

and $M_M = 7.36\times10^{22}$ kg

We have $F_{SM} = \dfrac{Gm_1m_2}{r^2} = \dfrac{\left(6.67\times10^{-11}\right)\left(1.99\times10^{30}\right)\left(7.36\times10^{22}\right)}{\left(1.492\times10^{11}\right)^2} = \boxed{4.39\times10^{20}\ \text{N}}$

(b) $$F_{EM} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\right)\left(7.36\times10^{22}\right)}{\left(3.84\times10^{8}\right)^2} = \boxed{1.99\times10^{20}\ \text{N}}$$

(c) $$F_{SE} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\right)\left(5.98\times10^{24}\right)}{\left(1.496\times10^{11}\right)^2} = \boxed{3.55\times10^{22}\ \text{N}}$$

(d) The force exerted by the Sun on the Moon is much stronger than the force of the Earth on the Moon. In a sense, the Moon orbits the Sun more than it orbits the Earth. The Moon's path is everywhere concave toward the Sun. Only by subtracting out the solar orbital motion of the Earth-Moon system do we see the Moon orbiting the center of mass of this system.

P13.7 $$F = \frac{GMm}{r^2} = \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{(1.50\ \text{kg})\left(15.0\times10^{-3}\ \text{kg}\right)}{\left(4.50\times10^{-2}\ \text{m}\right)^2} = \boxed{7.41\times10^{-10}\ \text{N}}$$

P13.8 Let θ represent the angle each cable makes with the vertical, L the cable length, x the distance each ball scrunches in, and $d = 1$ m the original distance between them. Then $r = d - 2x$ is the separation of the balls. We have

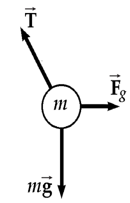

FIG. P13.8

$$\sum F_y = 0: \qquad T\cos\theta - mg = 0$$

$$\sum F_x = 0: \qquad T\sin\theta - \frac{Gmm}{r^2} = 0$$

Then

$$\tan\theta = \frac{Gmm}{r^2 mg} \qquad \frac{x}{\sqrt{L^2 - x^2}} = \frac{Gm}{g(d-2x)^2} \qquad x(d-2x)^2 = \frac{Gm}{g}\sqrt{L^2 - x^2}$$

The factor $\frac{Gm}{g}$ is numerically small. There are two possibilities: either x is small or else $d - 2x$ is small.

Possibility one: We can ignore x in comparison to d and L, obtaining

$$x(1\text{ m})^2 = \frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)(100\text{ kg})}{\left(9.8\text{ m/s}^2\right)}45\text{ m} \qquad x = 3.06\times10^{-8}\text{ m}$$

The separation distance is $r = 1\text{ m} - 2\left(3.06\times10^{-8}\text{ m}\right) = \boxed{1.000\text{ m} - 61.3\text{ nm}}$. This equilibrium is stable.

Possibility two: If $d - 2x$ is small, $x \approx 0.5$ m and the equation becomes

$$(0.5\text{ m})r^2 = \frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)(100\text{ kg})}{(9.8\text{ N/kg})}\sqrt{(45\text{ m})^2 - (0.5\text{ m})^2} \qquad r = \boxed{2.74\times10^{-4}\text{ m}}$$

For this answer to apply, the spheres would have to be compressed to a density like that of the nucleus of atom. This equilibrium is unstable.

Section 13.2 **Free-Fall Acceleration and the Gravitational Force**

P13.9 $a = \frac{MG}{(4R_E)^2} = \frac{9.80\text{ m/s}^2}{16.0} = \boxed{0.613\text{ m/s}^2}$ toward the Earth.

***P13.10** (a) For the gravitational force on an object in the neighborhood of Miranda we have

$$m_{\text{obj}}g = \frac{Gm_{\text{obj}}m_{\text{Miranda}}}{r^2_{\text{Miranda}}}$$

$$g = \frac{Gm_{\text{Miranda}}}{r^2_{\text{Miranda}}} = \frac{6.67\times10^{-11}\text{ N}\cdot\text{m}^2\left(6.68\times10^{19}\text{ kg}\right)}{\text{kg}^2\left(242\times10^3\text{ m}\right)^2} = \boxed{0.0761\text{ m/s}^2}$$

continued on next page

(b) We ignore the difference (of about 4%) in g between the lip and the base of the cliff. For the vertical motion of the athlete we have

$$y_f = y_i + v_{yi} + \frac{1}{2}a_y t^2$$

$$-5\,000 \text{ m} = 0 + 0 + \frac{1}{2}\left(-0.076\,1 \text{ m/s}^2\right)t^2$$

$$t = \left(\frac{2(5\,000 \text{ m})\text{s}^2}{0.076\,1 \text{ m}}\right)^{1/2} = \boxed{363 \text{ s}}$$

(c) $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = 0 + (8.5 \text{ m/s})(363 \text{ s}) + 0 = \boxed{3.08 \times 10^3 \text{ m}}$

We ignore the curvature of the surface (of about 0.7°) over the athlete's trajectory.

(d) $v_{xf} = v_{xi} = 8.50 \text{ m/s}$

$v_{yf} = v_{yi} + a_y t = 0 - \left(0.076\,1 \text{ m/s}^2\right)(363 \text{ s}) = -27.6 \text{ m/s}$

Thus $\vec{\mathbf{v}}_f = \left(8.50\hat{\mathbf{i}} - 27.6\hat{\mathbf{j}}\right) \text{ m/s} = \sqrt{8.5^2 + 27.6^2} \text{ m/s at } \tan^{-1}\frac{27.6}{8.5}$ below the x axis.

$\boxed{\vec{\mathbf{v}}_f = 28.9 \text{ m/s at } 72.9° \text{ below the horizontal}}$

P13.11 $$g = \frac{GM}{R^2} = \frac{G\rho\left(4\pi R^3/3\right)}{R^2} = \frac{4}{3}\pi G\rho R$$

If

$$\frac{g_M}{g_E} = \frac{1}{6} = \frac{4\pi G\rho_M R_M/3}{4\pi G\rho_E R_E/3}$$

then

$$\frac{\rho_M}{\rho_E} = \left(\frac{g_M}{g_E}\right)\left(\frac{R_E}{R_M}\right) = \left(\frac{1}{6}\right)(4) = \boxed{\frac{2}{3}}$$

Section 13.3 **Kepler's Laws and the Motion of Planets**

***P13.12** | The particle does possess angular momentum, because it is not headed straight for the origin. Its angular momentum is constant because the object is free of outside influences. |

Since speed is constant, the distance traveled between t_1 and t_2 is equal to the distance traveled between t_3 and t_4. The area of a triangle is equal to one-half its (base) width across one side times its (height) dimension perpendicular to that side.

So

$$\frac{1}{2}bv\left(t_2 - t_1\right) = \frac{1}{2}bv\left(t_4 - t_3\right)$$

states that the particle's radius vector sweeps out equal areas in equal times.

P13.13 Applying Newton's 2nd Law, $\sum F = ma$ yields $F_g = ma_c$ for each star:

$$\frac{GMM}{(2r)^2} = \frac{Mv^2}{r} \qquad \text{or} \qquad M = \frac{4v^2 r}{G}$$

We can write r in terms of the period, T, by considering the time and distance of one complete cycle. The distance traveled in one orbit is the circumference of the stars' common orbit, so $2\pi r = vT$. Therefore

$$M = \frac{4v^2 r}{G} = \frac{4v^2}{G}\left(\frac{vT}{2\pi}\right)$$

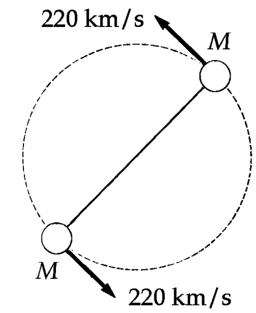

FIG. P13.13

so,

$$M = \frac{2v^3 T}{\pi G} = \frac{2\left(220\times10^3\ \text{m/s}\right)^3 (14.4\ \text{d})(86\,400\ \text{s/d})}{\pi\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)} = \boxed{1.26\times10^{32}\ \text{kg} = 63.3\ \text{solar masses}}$$

P13.14 By Kepler's Third Law, $T^2 = ka^3$ (a = semi-major axis)
For any object orbiting the Sun, with T in years and a in A.U., $k = 1.00$. Therefore, for Comet Halley

$$(75.6)^2 = (1.00)\left(\frac{0.570 + y}{2}\right)^3$$

The farthest distance the comet gets from the Sun is
$y = 2(75.6)^{2/3} - 0.570 = \boxed{35.2\ \text{A.U.}}$ (out around the orbit of Pluto).

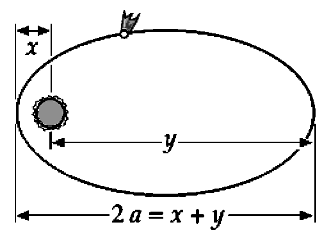

FIG. P13.14

P13.15 $T^2 = \dfrac{4\pi^2 a^3}{GM}$ (Kepler's third law with $m << M$)

$$M = \frac{4\pi^2 a^3}{GT^2} = \frac{4\pi^2\left(4.22\times10^8\ \text{m}\right)^3}{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)(1.77\times86\,400\ \text{s})^2} = \boxed{1.90\times10^{27}\ \text{kg}}$$

(approximately 316 Earth masses)

P13.16 $\sum F = ma$: $\dfrac{Gm_{\text{planet}}M_{\text{star}}}{r^2} = \dfrac{m_{\text{planet}}v^2}{r}$

$$\frac{GM_{\text{star}}}{r} = v^2 = r^2\omega^2$$

$$GM_{\text{star}} = r^3\omega^3 = r_x^3\omega_x^2 = r_y^3\omega_y^2$$

$$\omega_y = \omega_x\left(\frac{r_x}{r_y}\right)^{3/2} \qquad \omega_y = \left(\frac{90.0°}{5.00\ \text{yr}}\right)3^{3/2} = \frac{468°}{5.00\ \text{yr}}$$

So $\boxed{\text{planet } Y \text{ has turned through 1.30 revolutions}}$.

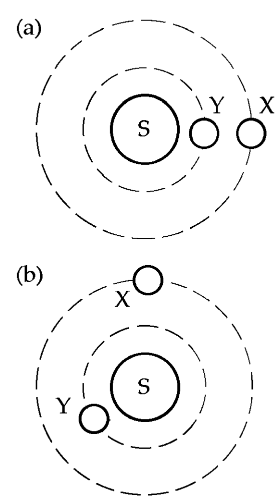

FIG. P13.16

P13.17 $\dfrac{GM_J}{(R_J+d)^2}=\dfrac{4\pi^2(R_J+d)}{T^2}$

$$GM_J T^2=4\pi^2(R_J+d)^3$$

$$(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.90\times10^{27}\ \text{kg})(9.84\times3\,600)^2=4\pi^2(6.99\times10^7+d)^3$$

$d=\boxed{8.92\times10^7\ \text{m}}=\boxed{89\,200\ \text{km}}$ above the planet

P13.18 The gravitational force on a small parcel of material at the star's equator supplies the necessary centripetal acceleration:

$$\frac{GM_s m}{R_s^2}=\frac{mv^2}{R_s}=mR_s\omega^2$$

so

$$\omega=\sqrt{\frac{GM_s}{R_s^3}}=\sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\left[2(1.99\times10^{30}\ \text{kg})\right]}{(10.0\times10^3\ \text{m})^3}}$$

$$\omega=\boxed{1.63\times10^4\ \text{rad/s}}$$

P13.19 The speed of a planet in a circular orbit is given by

$$\sum F=ma:\ \frac{GM_{\text{sun}}m}{r^2}=\frac{mv^2}{r}\qquad v=\sqrt{\frac{GM_{\text{sun}}}{r}}$$

For Mercury the speed is $v_M=\sqrt{\dfrac{(6.67\times10^{-11})(1.99\times10^{30})\ \text{m}^2}{(5.79\times10^{10})\ \text{s}^2}}=4.79\times10^4\ \text{m/s}$

and for Pluto, $v_P=\sqrt{\dfrac{(6.67\times10^{-11})(1.99\times10^{30})\ \text{m}^2}{(5.91\times10^{12})\ \text{s}^2}}=4.74\times10^3\ \text{m/s}$

With greater speed, Mercury will eventually move farther from the Sun than Pluto. With original distances r_P and r_M perpendicular to their lines of motion, they will be equally far from the Sun after time t where

$$\sqrt{r_P^2+v_P^2t^2}=\sqrt{r_M^2+v_M^2t^2}$$

$$r_P^2-r_M^2=(v_M^2-v_P^2)t^2$$

$$t=\sqrt{\frac{(5.91\times10^{12}\ \text{m})^2-(5.79\times10^{10}\ \text{m})^2}{(4.79\times10^4\ \text{m/s})^2-(4.74\times10^3\ \text{m/s})^2}}=\sqrt{\frac{3.49\times10^{25}\ \text{m}^2}{2.27\times10^9\ \text{m}^2/\text{s}^2}}=1.24\times10^8\ \text{s}=\boxed{3.93\ \text{yr}}$$

***P13.20** In $T^2=4\pi^2a^3/GM_{central}$ we take $a=3.84\times10^8$ m.

$$M_{central}=4\pi^2a^3/GT^2=\frac{4\pi^2(3.84\times10^8\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(27.3\times86400\ \text{s})^2}=\boxed{6.02\times10^{24}\ \text{kg}}$$

This is a little larger than 5.98×10^{24} kg.

The Earth wobbles a bit as the Moon orbits it, so both objects move nearly in circles about their center of mass, staying on opposite sides of it. The radius of the Moon's orbit is therefore a bit less than the Earth–Moon distance.

Section 13.4 **The Gravitational Field**

P13.21 $\vec{\mathbf{g}} = \dfrac{Gm}{l^2}\hat{\mathbf{i}} + \dfrac{Gm}{l^2}\hat{\mathbf{j}} + \dfrac{Gm}{2l^2}\left(\cos 45.0°\hat{\mathbf{i}} + \sin 45.0\hat{\mathbf{j}}\right)$

so

$$\vec{\mathbf{g}} = \frac{GM}{l^2}\left(1 + \frac{1}{2\sqrt{2}}\right)\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$$

or

$$\vec{\mathbf{g}} = \boxed{\frac{Gm}{l^2}\left(\sqrt{2} + \frac{1}{2}\right) \text{ toward the opposite corner}}$$

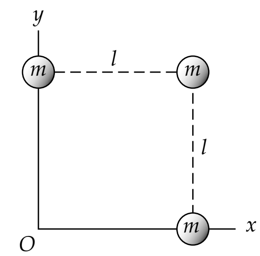

FIG. P13.21

P13.22 (a) $F = \dfrac{GMm}{r^2} = \dfrac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left[100\left(1.99\times10^{30}\ \text{kg}\right)\left(10^3\ \text{kg}\right)\right]}{\left(1.00\times10^4\ \text{m} + 50.0\ \text{m}\right)^2} = \boxed{1.31\times10^{17}\ \text{N}}$

(b) $\Delta F = \dfrac{GMm}{r_{\text{front}}^2} - \dfrac{GMm}{r_{\text{back}}^2}$

$$\Delta g = \frac{\Delta F}{m} = \frac{GM\left(r_{\text{back}}^2 - r_{\text{front}}^2\right)}{r_{\text{front}}^2 r_{\text{back}}^2}$$

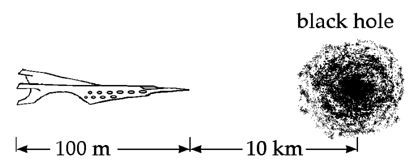

FIG. P13.22

$$\Delta g = \frac{\left(6.67\times10^{-11}\right)\left[100\left(1.99\times10^{30}\right)\right]\left[\left(1.01\times10^4\ \text{m}\right)^2 - \left(1.00\times10^4\ \text{m}\right)^2\right]}{\left(1.00\times10^4\ \text{m}\right)^2\left(1.01\times10^4\ \text{m}\right)^2}$$

$$\Delta g = \boxed{2.62\times10^{12}\ \text{N/kg}}$$

***P13.23** (a) $g_1 = g_2 = \dfrac{MG}{r^2 + a^2}$

$g_{1y} = -g_{2y}$ $\qquad g_y = g_{1y} + g_{2y} = 0$

$g_{1x} = g_{2x} = g_2\cos\theta$ $\qquad \cos\theta = \dfrac{r}{\left(a^2 + r^2\right)^{1/2}}$

$$\vec{\mathbf{g}} = 2g_{2x}\left(-\hat{\mathbf{i}}\right)$$

or

$$\vec{\mathbf{g}} = \boxed{\frac{2MGr}{\left(r^2 + a^2\right)^{3/2}} \text{ toward the center of mass}}$$

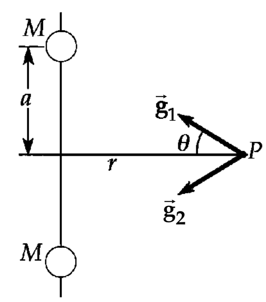

FIG. P13.23

(b) As r goes to zero, we approach the point halfway between the masses. Here the fields of the two are equally strong and in opposite directions so they add to zero.

(c) As $r \to 0$, $2MGr(r^2 + a^2)^{-3/2}$ approaches $2MG(0)/a^3 = 0$

(d) Standing far away from the masses, their separateness makes no difference. They produce equal fields in the same direction to behave like a single object of mass $2M$.

(e) As r becomes much larger than a, the expression approaches $2MGr(r^2 + 0^2)^{-3/2} = 2MGr/r^3 = 2MG/r^2$ as required.

Section 13.5 **Gravitational Potential Energy**

P13.24 (a) $$U=-\frac{GM_Em}{r}=-\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)(100\ \text{kg})}{(6.37+2.00)\times10^6\ \text{m}}=\boxed{-4.77\times10^9\ \text{J}}$$

(b), (c) Planet and satellite exert forces of equal magnitude on each other, directed $\boxed{\text{downward on the satellite and upward on the planet}}$.

$$F=\frac{GM_Em}{r^2}=\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)(100\ \text{kg})}{\left(8.37\times10^6\ \text{m}\right)^2}=\boxed{569\ \text{N}}$$

P13.25 (a) $$\rho=\frac{M_S}{\frac{4}{3}\pi r_E^2}=\frac{3\left(1.99\times10^{30}\ \text{kg}\right)}{4\pi\left(6.37\times10^6\ \text{m}\right)^3}=\boxed{1.84\times10^9\ \text{kg/m}^3}$$

(b) $$g=\frac{GM_S}{r_E^2}=\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)}{\left(6.37\times10^6\ \text{m}\right)^2}=\boxed{3.27\times10^6\ \text{m/s}^2}$$

(c) $$U_g=-\frac{GM_Sm}{r_E}=-\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)(1.00\ \text{kg})}{6.37\times10^6\ \text{m}}=\boxed{-2.08\times10^{13}\ \text{J}}$$

P13.26 The height attained is not small compared to the radius of the Earth, so $U=mgy$ does not apply; $U=-\frac{GM_1M_2}{r}$ does. From launch to apogee at height h,

$$K_i+U_i+\Delta E_{\text{mch}}=K_f+U_f:\qquad \frac{1}{2}M_pv_i^2-\frac{GM_EM_p}{R_E}+0=0-\frac{GM_EM_p}{R_E+h}$$

$$\frac{1}{2}\left(10.0\times10^3\ \text{m/s}\right)^2-\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(\frac{5.98\times10^{24}\ \text{kg}}{6.37\times10^6\ \text{m}}\right)$$

$$=-\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(\frac{5.98\times10^{24}\ \text{kg}}{6.37\times10^6\ \text{m}+h}\right)$$

$$\left(5.00\times10^7\ \text{m}^2/\text{s}^2\right)-\left(6.26\times10^7\ \text{m}^2/\text{s}^2\right)=\frac{-3.99\times10^{14}\ \text{m}^3/\text{s}^2}{6.37\times10^6\ \text{m}+h}$$

$$6.37\times10^6\ \text{m}+h=\frac{3.99\times10^{14}\ \text{m}^3/\text{s}^2}{1.26\times10^7\ \text{m}^2/\text{s}^2}=3.16\times10^7\ \text{m}$$

$$\boxed{h=2.52\times10^7\ \text{m}}$$

***P13.27** (a) $$U_{\text{Tot}}=U_{12}+U_{13}+U_{23}=3U_{12}=3\left(-\frac{Gm_1m_2}{r_{12}}\right)$$

$$U_{\text{Tot}}=-\frac{3\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.00\times10^{-3}\ \text{kg}\right)^2}{0.300\ \text{m}}=\boxed{-1.67\times10^{-14}\ \text{J}}$$

(b) Each particle feels a net force of attraction toward the midpoint between the other two. Each moves toward the center of the triangle with the same acceleration. They collide simultaneously at the center of the triangle.

P13.28 $$W=-\Delta U=-\left(\frac{-Gm_1m_2}{r}-0\right)$$

$$W=\frac{\left(+6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.36\times10^{22}\ \text{kg}\right)\left(1.00\times10^3\ \text{kg}\right)}{1.74\times10^6\ \text{m}}=\boxed{2.82\times10^9\ \text{J}}$$

P13.29 (a) Energy conservation of the object-Earth system from release to radius r:

$$\left(K+U_g\right)_{\text{altitude } h}=\left(K+U_g\right)_{\text{radius } r}$$

$$0-\frac{GM_E m}{R_E+h}=\frac{1}{2}mv^2-\frac{GM_E m}{r}$$

$$v=\left(2GM_E\left(\frac{1}{r}-\frac{1}{R_E+h}\right)\right)^{1/2}=-\frac{dr}{dt}$$

(b) $\int_i^f dt=\int_i^f -\frac{dr}{v}=\int_f^i \frac{dr}{v}$. The time of fall is

$$\Delta t=\int_{R_E}^{R_E+h}\left(2GM_E\left(\frac{1}{r}-\frac{1}{R_E+h}\right)\right)^{-1/2}dr$$

$$\Delta t=\int_{6.37\times10^6\text{ m}}^{6.87\times10^6\text{ m}}\left[2\times6.67\times10^{-11}\times5.98\times10^{24}\left(\frac{1}{r}-\frac{1}{6.87\times10^6\text{ m}}\right)\right]^{-1/2}dr$$

We can enter this expression directly into a mathematical calculation program. Alternatively, to save typing we can change variables to $u=\frac{r}{10^6}$. Then

$$\Delta t=\left(7.977\times10^{14}\right)^{-1/2}\int_{6.37}^{6.87}\left(\frac{1}{10^6u}-\frac{1}{6.87\times10^6}\right)^{-1/2}10^6\,du$$

$$=3.541\times10^{-8}\frac{10^6}{\left(10^6\right)^{-1/2}}\int_{6.37}^{6.87}\left(\frac{1}{u}-\frac{1}{6.87}\right)^{-1/2}du$$

A mathematics program returns the value 9.596 for this integral, giving for the time of fall

$$\Delta t=3.541\times10^{-8}\times10^9\times9.596=339.8=\boxed{340\text{ s}}$$

Section 13.6 **Energy Considerations in Planetary and Satellite Motion**

P13.30 (a) $v_{\text{solar escape}}=\sqrt{\frac{2M_{\text{Sun}}G}{R_{E\cdot\text{Sun}}}}=\boxed{42.1\text{ km/s}}$

(b) Let $r=R_{E\cdot S}x$ represent variable distance from the Sun, with x in astronomical units.

$$v=\sqrt{\frac{2M_{\text{Sun}}G}{R_{E\cdot S}x}}=\frac{42.1}{\sqrt{x}}$$

If $v=\frac{125\,000\text{ km}}{3\,600\text{ s}}$, then $x=1.47\text{ A.U.}=\boxed{2.20\times10^{11}\text{ m}}$

(at or beyond the orbit of Mars, 125 000 km/h is sufficient for escape).

P13.31 $\frac{1}{2}mv_i^2+GM_E m\left(\frac{1}{r_f}-\frac{1}{r_i}\right)=\frac{1}{2}mv_f^2$ $\qquad$ $\frac{1}{2}v_i^2+GM_E\left(0-\frac{1}{R_E}\right)=\frac{1}{2}v_f^2$

or

$$v_f^2=v_1^2-\frac{2GM_E}{R_E}$$

and

$$v_f=\left(v_1^2-\frac{2GM_E}{R_E}\right)^{1/2}$$

$$v_f=\left[\left(2.00\times10^4\right)^2-1.25\times10^8\right]^{1/2}=\boxed{1.66\times10^4\text{ m/s}}$$

***P13.32** $E_{\text{tot}} = -\dfrac{GMm}{2r}$

$$\Delta E = \frac{GMm}{2}\left(\frac{1}{r_i} - \frac{1}{r_f}\right) = \frac{(6.67\times10^{-11})(5.98\times10^{24})}{2}\frac{10^3\text{ kg}}{10^3\text{ m}}\left(\frac{1}{6\,370+100} - \frac{1}{6\,370+200}\right)$$

$$\Delta E = 4.69\times10^{8}\text{ J} = \boxed{469\text{ MJ}}$$

> Both in the original orbit and in the final orbit, the total energy is negative, with an absolute value equal to the positive kinetic energy. The potential energy is negative and twice as large as the total energy. As the satellite is lifted from the lower to the higher orbit, the gravitational energy increases, the kinetic energy decreases, and the total energy increases. The value of each becomes closer to zero. Numerically, the gravitational energy increases by 938 MJ, the kinetic energy decreases by 469 MJ, and the total energy increases by 469 MJ.

P13.33 To obtain the orbital velocity, we use $\sum F = \dfrac{mMG}{R^2} = \dfrac{mv^2}{R}$

or $v = \sqrt{\dfrac{MG}{R}}$

We can obtain the escape velocity from $\dfrac{1}{2}mv_{\text{esc}}^2 = \dfrac{mMG}{R}$

or $v_{\text{esc}} = \sqrt{\dfrac{2MG}{R}} = \boxed{\sqrt{2}v}$

***P13.34** > Gravitational screening does not exist. The presence of the satellite has no effect on the force the planet exerts on the rocket.

The rocket is in a potential well at Ganymede's surface with energy

$$U_1 = -\frac{Gm_1m_2}{r} = -\frac{6.67\times10^{-11}\text{ N}\cdot\text{m}^2 m_2(1.495\times10^{23}\text{ kg})}{\text{kg}^2(2.64\times10^{6}\text{ m})}$$

$$U_1 = -3.78\times10^{6}m_2\text{ m}^2/\text{s}^2$$

The potential well from Jupiter at the distance of Ganymede is

$$U_2 = -\frac{Gm_1m_2}{r} = -\frac{6.67\times10^{-11}\text{ N}\cdot\text{m}^2 m_2(1.90\times10^{27}\text{ kg})}{\text{kg}^2(1.071\times10^{9}\text{ m})}$$

$$U_2 = -1.18\times10^{8}m_2\text{ m}^2/\text{s}^2$$

To escape from both requires

$$\frac{1}{2}m_2v_{\text{esc}}^2 = +(3.78\times10^{6} + 1.18\times10^{8})m_2\text{ m}^2/\text{s}^2$$

$$v_{\text{esc}} = \sqrt{2\times1.22\times10^{8}\text{ m}^2/\text{s}^2} = \boxed{15.6\text{ km/s}}$$

P13.35 $F_c = F_G$ gives $\frac{mv^2}{r} = \frac{GmM_E}{r^2}$

which reduces to $v = \sqrt{\frac{GM_E}{r}}$

and period $= \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM_E}}$

(a) $r = R_E + 200\text{ km} = 6\,370\text{ km} + 200\text{ km} = 6\,570\text{ km}$

Thus,

$$\text{period} = 2\pi\left(6.57\times10^6\text{ m}\right)\sqrt{\frac{\left(6.57\times10^6\text{ m}\right)}{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\text{ kg}\right)}}$$

$$T = 5.30\times10^3\text{ s} = 88.3\text{ min} = \boxed{1.47\text{ h}}$$

(b) $$v = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\text{ kg}\right)}{\left(6.57\times10^6\text{ m}\right)}} = \boxed{7.79\text{ km/s}}$$

(c) $K_f + U_f = K_i + U_i +$ energy input, gives

$$\text{input} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + \left(\frac{-GM_Em}{r_f}\right) - \left(\frac{-GM_Em}{r_i}\right) \qquad (1)$$

$$r_i = R_E = 6.37\times10^6\text{ m}$$

$$v_i = \frac{2\pi R_E}{86\,400\text{ s}} = 4.63\times10^2\text{ m/s}$$

Substituting the appropriate values into (1) yields the

minimum energy input $= \boxed{6.43\times10^9\text{ J}}$

P13.36 The gravitational force supplies the needed centripetal acceleration.

Thus,

$$\frac{GM_Em}{\left(R_E+h\right)^2} = \frac{mv^2}{\left(R_E+h\right)} \qquad \text{or} \qquad v^2 = \frac{GM_E}{R_E+h}$$

(a) $$T = \frac{2\pi r}{v} = \frac{2\pi\left(R_E+h\right)}{\sqrt{\frac{GM_E}{\left(R_E+h\right)}}} \qquad T = \boxed{2\pi\sqrt{\frac{\left(R_E+h\right)^3}{GM_E}}}$$

(b) $$v = \boxed{\sqrt{\frac{GM_E}{R_E+h}}}$$

continued on next page

(c) Minimum energy input is $\Delta E_{\min} = \left(K_f + U_{gf}\right) - \left(K_i - U_{gi}\right)$

It is simplest to $\boxed{\text{launch the satellite from a location on the equator, and launch it toward the east.}}$

This choice has the object starting with energy $K_i = \frac{1}{2}mv_i^2$

with

$$v_i = \frac{2\pi R_E}{1.00\text{ day}} = \frac{2\pi R_E}{86\,400\text{ s}} \qquad \text{and} \qquad U_{gi} = -\frac{GM_E m}{R_E}$$

Thus,

$$\Delta E_{\min} = \frac{1}{2}m\left(\frac{GM_E}{R_E + h}\right) - \frac{GM_E m}{R_E + h} - \frac{1}{2}m\left[\frac{4\pi^2 R_E^2}{(86\,400\text{ s})^2}\right] + \frac{GM_E m}{R_E}$$

or

$$\Delta E_{\min} = \boxed{GM_E m\left[\frac{R_E + 2h}{2R_E\left(R_E + h\right)}\right] - \frac{2\pi^2 R_E^2 m}{(86\,400\text{ s})^2}}$$

P13.37 (a) Energy conservation for the object-Earth system from firing to apex:

$$\left(K + U_g\right)_i = \left(K + U_g\right)_f$$

$$\frac{1}{2}mv_i^2 - \frac{GmM_E}{R_E} = 0 - \frac{GmM_E}{R_E + h}$$

where $\frac{1}{2}mv_{\text{esc}}^2 = \frac{GmM_E}{R_E}$. Then

$$\frac{1}{2}v_i^2 - \frac{1}{2}v_{\text{esc}}^2 = -\frac{1}{2}v_{\text{esc}}^2\frac{R_E}{R_E + h}$$

$$v_{\text{esc}}^2 - v_i^2 = \frac{v_{\text{esc}}^2 R_E}{R_E + h}$$

$$\frac{1}{v_{\text{esc}}^2 - v_i^2} = \frac{R_E + h}{v_{\text{esc}}^2 R_E}$$

$$h = \frac{v_{\text{esc}}^2 R_E}{v_{\text{esc}}^2 - v_i^2} - R_E = \frac{v_{\text{esc}}^2 R_E - v_{\text{esc}}^2 R_E + v_i^2 R_E}{v_{\text{esc}}^2 - v_i^2}$$

$$h = \frac{R_E v_i^2}{v_{\text{esc}}^2 - v_i^2}$$

(b) $h = \dfrac{6.37\times10^6\text{ m }(8.76)^2}{(11.2)^2 - (8.76)^2} = \boxed{1.00\times10^7\text{ m}}$

(c) The fall of the meteorite is the time-reversal of the upward flight of the projectile, so it is described by the same energy equation

$$v_i^2 = v_{\text{esc}}^2\left(1 - \frac{R_E}{R_E + h}\right) = v_{\text{esc}}^2\left(\frac{h}{R_E + h}\right) = \left(11.2\times10^3\text{ m/s}\right)^2\left(\frac{2.51\times10^7\text{ m}}{6.37\times10^6\text{ m} + 2.51\times10^7\text{ m}}\right)$$

$$= 1.00\times10^8\text{ m}^2/\text{s}^2$$

$$v_i = \boxed{1.00\times10^4\text{ m/s}}$$

(d) With $v_i << v_{\text{esc}}$, $h \approx \dfrac{R_E v_i^2}{v_{\text{esc}}^2} = \dfrac{R_E v_i^2 R_E}{2GM_E}$. But $g = \dfrac{GM_E}{R_E^2}$, so $h = \dfrac{v_i^2}{2g}$, in agreement with $0^2 = v_i^2 + 2(-g)(h - 0)$

P13.38 (a) For the satellite $\sum F = ma$ $\quad \dfrac{GmM_E}{r^2} = \dfrac{mv_0^2}{r}$

$$\boxed{v_0 = \left(\frac{GM_E}{r}\right)^{1/2}}$$

(b) Conservation of momentum in the forward direction for the exploding satellite:

$$\left(\sum mv\right)_i = \left(\sum mv\right)_f$$
$$5mv_0 = 4mv_i + m0$$
$$v_i = \frac{5}{4}v_0 = \boxed{\frac{5}{4}\left(\frac{GM_E}{r}\right)^{1/2}}$$

(c) With velocity perpendicular to radius, the orbiting fragment is at perigee. Its apogee distance and speed are related to r and v_i by $4mrv_i = 4mr_f v_f$ and

$\dfrac{1}{2}4mv_i^2 - \dfrac{GM_E 4m}{r} = \dfrac{1}{2}4mv_f^2 - \dfrac{GM_E 4m}{r_f}$. Substituting $v_f = \dfrac{v_i r}{r_f}$ we have

$\dfrac{1}{2}v_i^2 - \dfrac{GM_E}{r} = \dfrac{1}{2}\dfrac{v_i^2 r^2}{r_f^2} - \dfrac{GM_E}{r_f}$. Further, substituting $v_i^2 = \dfrac{25}{16}\dfrac{GM_E}{r}$ gives

$$\frac{25}{32}\frac{GM_E}{r} - \frac{GM_E}{r} = \frac{25}{32}\frac{GM_E r}{r_f^2} - \frac{GM_E}{r_f}$$
$$\frac{-7}{32r} = \frac{25r}{32r_f^2} - \frac{1}{r_f}$$

Clearing of fractions, $-7r_f^2 = 25r^2 - 32rr_f$ or $7\left(\dfrac{r_f}{r}\right)^2 - 32\left(\dfrac{r_f}{r}\right) + 25 = 0$ giving

$\dfrac{r_f}{r} = \dfrac{+32 \pm \sqrt{32^2 - 4(7)(25)}}{14} = \dfrac{50}{14}$ or $\dfrac{14}{14}$. The latter root describes the starting point. The outer end of the orbit has $\dfrac{r_f}{r} = \dfrac{25}{7}$; $\boxed{r_f = \dfrac{25r}{7}}$

P13.39 (a) The major axis of the orbit is $2a = 50.5\text{ AU}$ so $a = 25.25\text{ AU}$
Further, in the textbook's diagram of an ellipse, $a + c = 50\text{ AU}$ so $c = 24.75\text{ AU}$

Then

$$e = \frac{c}{a} = \frac{24.75}{25.25} = \boxed{0.980}$$

(b) In $T^2 = K_s a^3$ for objects in solar orbit, the Earth gives us

$$(1\text{ yr})^2 = K_s(1\text{ AU})^3 \qquad K_s = \frac{(1\text{ yr})^2}{(1\text{ AU})^3}$$

Then

$$T^2 = \frac{(1\text{ yr})^2}{(1\text{ AU})^3}(25.25\text{ AU})^3 \qquad T = \boxed{127\text{ yr}}$$

(c) $$U = -\frac{GMm}{r} = -\frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991\times10^{30}\text{ kg}\right)\left(1.2\times10^{10}\text{ kg}\right)}{50\left(1.496\times10^{11}\text{ m}\right)} = \boxed{-2.13\times10^{17}\text{ J}}$$

Additional Problems

***P13.40** (a) Let R represent the radius of the asteroid. Then its volume is $\frac{4}{3}\pi R^3$ and its mass is $\rho\frac{4}{3}\pi R^3$.

For your orbital motion, $\sum F = ma$, $\quad \frac{Gm_1m_2}{R^2} = \frac{m_2v^2}{R}$, $\quad \frac{G\rho 4\pi R^3}{3R^2} = \frac{v^2}{R}$

$$R = \left(\frac{3v^2}{G\rho 4\pi}\right)^{1/2} = \left(\frac{3(8.5\ \text{m/s})^2\ \text{kg}^2\ \text{m}^3}{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2(1\,100\ \text{kg})4\pi}\right)^{1/2} = \boxed{1.53\times10^4\ \text{m}}$$

(b) $\rho\frac{4}{3}\pi R^3 = (1\,100\ \text{kg/m}^3)\frac{4}{3}\pi(1.53\times10^4\ \text{m})^3 = \boxed{1.66\times10^{16}\ \text{kg}}$

(c) $v = \frac{2\pi R}{T}$ $\qquad T = \frac{2\pi R}{v} = \frac{2\pi(1.53\times10^4\ \text{m})}{8.5\ \text{m/s}} = \boxed{1.13\times10^4\ \text{s}} = 3.15\ \text{h}$

(d) For an illustrative model, we take your mass as 90 kg and assume the asteroid is originally at rest. Angular momentum is conserved for the asteroid-you system:

$$\sum L_i = \sum L_f$$
$$0 = m_2vR - I\omega$$
$$0 = m_2vR - \frac{2}{5}m_1R^2\frac{2\pi}{T_{\text{asteroid}}}$$
$$m_2v = \frac{4\pi}{5}\frac{m_1R}{T_{\text{asteroid}}}$$
$$T_{\text{asteroid}} = \frac{4\pi m_1R}{5m_2v} = \frac{4\pi(1.66\times10^{16}\ \text{kg})(1.53\times10^4\ \text{m})}{5(90\ \text{kg})(8.5\ \text{m/s})} = 8.37\times10^{17}\ \text{s} = 26.5\ \text{billion years}$$

Thus your running does not produce significant rotation of the asteroid if it is originally stationary, and does not significantly affect any rotation it does have.

This problem is realistic. Many asteroids, such as Ida and Eros, are roughly 30 km in diameter. They are typically irregular in shape and not spherical. Satellites such as Phobos (of Mars), Adrastea (of Jupiter), Calypso (of Saturn), and Ophelia (of Uranus) would allow a visitor the same experience of easy orbital motion. So would many Kuiper-belt objects.

P13.41 Let m represent the mass of the spacecraft, r_E the radius of the Earth's orbit, and x the distance from Earth to the spacecraft.

The Sun exerts on the spacecraft a radial inward force of $F_s = \dfrac{GM_s m}{(r_E - x)^2}$

while the Earth exerts on it a radial outward force of $F_E = \dfrac{GM_E m}{x^2}$

The net force on the spacecraft must produce the correct centripetal acceleration for it to have an orbital period of 1.000 year.

Thus,

$$F_S - F_E = \frac{GM_S m}{(r_E - x)^2} - \frac{GM_E m}{x^2} = \frac{mv^2}{(r_E - x)} = \frac{m}{(r_E - x)}\left[\frac{2\pi(r_E - x)}{T}\right]^2$$

which reduces to

$$\frac{GM_S}{(r_E - x)^2} - \frac{GM_E}{x^2} = \frac{4\pi^2(r_E - x)}{T^2} \qquad (1)$$

Cleared of fractions, this equation would contain powers of x ranging from the fifth to the zeroth. We do not solve it algebraically. We may test the assertion that x is between 1.47×10^9 m and 1.48×10^9 m by substituting both of these as trial solutions, along with the following data: $M_S = 1.991 \times 10^{30}$ kg, $M_E = 5.983 \times 10^{24}$ kg, $r_E = 1.496 \times 10^{11}$ m, and $T = 1.000$ yr $= 3.156 \times 10^7$ s.

With $x = 1.47 \times 10^9$ m substituted into equation (1), we obtain

$$6.052 \times 10^{-3} \text{ m/s}^2 - 1.85 \times 10^{-3} \text{ m/s}^2 \approx 5.871 \times 10^{-3} \text{ m/s}^2$$

or

$$5.868 \times 10^{-3} \text{ m/s}^2 \approx 5.871 \times 10^{-3} \text{ m/s}^2$$

With $x = 1.48 \times 10^9$ m substituted into the same equation, the result is

$$6.053 \times 10^{-3} \text{ m/s}^2 - 1.82 \times 10^{-3} \text{ m/s}^2 \approx 5.870\,8 \times 10^{-3} \text{ m/s}^2$$

or

$$5.870\,9 \times 10^{-3} \text{ m/s}^2 \approx 5.870\,8 \times 10^{-3} \text{ m/s}^2$$

Since the first trial solution makes the left-hand side of equation (1) slightly less than the right hand side, and the second trial solution does the opposite, the true solution is determined as between the trial values. To three-digit precision, it is 1.48×10^9 m.

As an equation of fifth degree, equation (1) has five roots. The Sun-Earth system has five Lagrange points, all revolving around the Sun synchronously with the Earth. The SOHO and ACE satellites are at one. Another is beyond the far side of the Sun. Another is beyond the night side of the Earth. Two more are on the Earth's orbit, ahead of the planet and behind it by 60°. Plans are under way to gain perspective on the Sun by placing a spacecraft at one of these two co-orbital Lagrange points. The Greek and Trojan asteroids are at the co-orbital Lagrange points of the Jupiter-Sun system.

P13.42 The acceleration of an object at the center of the Earth due to the gravitational force of the Moon is given by

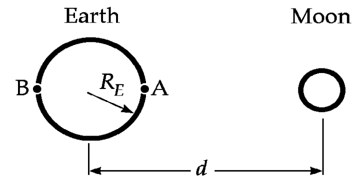

FIG. P13.42

$$a = G\frac{M_{\text{Moon}}}{d^2}$$

At the point A nearest the Moon, $a_+ = G\dfrac{M_M}{(d-r)^2}$

At the point B farthest from the Moon, $a_- = G\dfrac{M_M}{(d+r)^2}$

$$\Delta a = a_+ - a = GM_M\left[\frac{1}{(d-r)^2} - \frac{1}{d^2}\right]$$

For $d >> r$, $\Delta a = \dfrac{2GM_M r}{d^3} = 1.11\times10^{-6}\ \text{m/s}^2$

Across the planet, $\dfrac{\Delta g}{g} = \dfrac{2\Delta a}{g} = \dfrac{2.22\times10^{-6}\ \text{m/s}^2}{9.80\ \text{m/s}^2} = \boxed{2.26\times10^{-7}}$

P13.43 Energy conservation for the two-sphere system from release to contact:

$$-\frac{Gmm}{R} = -\frac{Gmm}{2r} + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$Gm\left(\frac{1}{2r} - \frac{1}{R}\right) = v^2 \qquad v = \left(Gm\left[\frac{1}{2r} - \frac{1}{R}\right]\right)^{1/2}$$

(a) The injected impulse is the final momentum of each sphere,

$$mv = m^{2/2}\left(Gm\left[\frac{1}{2r} - \frac{1}{R}\right]\right)^{1/2} = \left[Gm^3\left(\frac{1}{2r} - \frac{1}{R}\right)\right]^{1/2}$$

(b) If they now collide elastically each sphere reverses its velocity to receive impulse

$$mv - (-mv) = 2mv = \boxed{2\left[Gm^3\left(\frac{1}{2r} - \frac{1}{R}\right)\right]^{1/2}}$$

P13.44 Momentum is conserved:

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

$$0 = M\vec{\mathbf{v}}_{1f} + 2M\vec{\mathbf{v}}_{2f}$$

$$\vec{\mathbf{v}}_{2f} = -\frac{1}{2}\vec{\mathbf{v}}_{1f}$$

Energy is conserved:

$$(K+U)_i + \Delta E = (K+U)_f$$

$$0 - \frac{Gm_1m_2}{r_i} + 0 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 - \frac{Gm_1m_2}{r_f}$$

$$-\frac{GM(2M)}{12R} = \frac{1}{2}Mv_{1f}^2 + \frac{1}{2}(2M)\left(\frac{1}{2}v_{1f}\right)^2 - \frac{GM(2M)}{4R}$$

$$v_{1f} = \boxed{\frac{2}{3}\sqrt{\frac{GM}{R}}} \qquad v_{2f} = \frac{1}{2}v_{1f} = \boxed{\frac{1}{3}\sqrt{\frac{GM}{R}}}$$

P13.45 (a) Each bit of mass dm in the ring is at the same distance from the object at A. The separate contributions $-\frac{Gmdm}{r}$ to the system energy add up to $-\frac{GmM_{\text{ring}}}{r}$. When the object is at A, this is

$$\frac{-6.67\times10^{-11}\ \text{N}\cdot\text{m}^2\,1\,000\ \text{kg}\,2.36\times10^{20}\ \text{kg}}{\text{kg}^2\sqrt{\left(1\times10^8\ \text{m}\right)^2+\left(2\times10^8\ \text{m}\right)^2}}=\boxed{-7.04\times10^4\ \text{J}}$$

(b) When the object is at the center of the ring, the potential energy is

$$-\frac{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2\ 1\,000\ \text{kg}\,2.36\times10^{20}\ \text{kg}}{\text{kg}^2 1\times10^8\ \text{m}}=\boxed{-1.57\times10^5\ \text{J}}$$

(c) Total energy of the object-ring system is conserved:

$$\left(K+U_g\right)_A=\left(K+U_g\right)_B$$

$$0-7.04\times10^4\ \text{J}=\frac{1}{2}1\,000\ \text{kg}v_B^2-1.57\times10^5\ \text{J}$$

$$v_B=\left(\frac{2\times8.70\times10^4\ \text{J}}{1\,000\ \text{kg}}\right)^{1/2}=\boxed{13.2\ \text{m/s}}$$

P13.46 (a) The free-fall acceleration produced by the Earth is $g=\frac{GM_E}{r^2}=GM_Er^{-2}$ (directed downward)

Its rate of change is

$$\frac{dg}{dr}=GM_E(-2)r^{-3}=-2GM_Er^{-3}$$

The minus sign indicates that g decreases with increasing height.

At the Earth's surface,

$$\boxed{\frac{dg}{dr}=-\frac{2GM_E}{R_E^3}}$$

(b) For small differences,

$$\frac{|\Delta g|}{\Delta r}=\frac{|\Delta g|}{h}=\frac{2GM_E}{R_E^3}$$

Thus,

$$\boxed{|\Delta g|=\frac{2GM_Eh}{R_E^3}}$$

(c) $$|\Delta g|=\frac{2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)(6.00\ \text{m})}{\left(6.37\times10^6\ \text{m}\right)^3}=\boxed{1.85\times10^{-5}\ \text{m/s}^2}$$

P13.47 From the walk, $2\pi r=25\,000$ m. Thus, the radius of the planet is $r=\frac{25\,000\ \text{m}}{2\pi}=3.98\times10^3$ m

From the drop: $\Delta y=\frac{1}{2}gt^2=\frac{1}{2}g(29.2\ \text{s})^2=1.40$ m

so,

$$g=\frac{2(1.40\ \text{m})}{(29.2\ \text{s})^2}=3.28\times10^{-3}\ \text{m/s}^2=\frac{MG}{r^2}\qquad\therefore M=\boxed{7.79\times10^{14}\ \text{kg}}$$

P13.48 The distance between the orbiting stars is $d = 2r\cos 30° = \sqrt{3}r$ since $\cos 30° = \frac{\sqrt{3}}{2}$. The net inward force on one orbiting star is

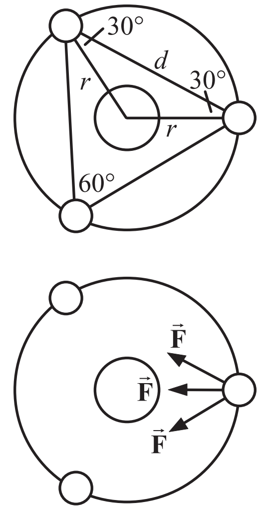

FIG. P13.48

$$\frac{Gmm}{d^2}\cos 30° + \frac{GMm}{r^2} + \frac{Gmm}{d^2}\cos 30° = \frac{mv^2}{r}$$

$$\frac{Gm2\cos 30°}{3r^2} + \frac{GM}{r^2} = \frac{4\pi^2 r^2}{rT^2}$$

$$G\left(\frac{m}{\sqrt{3}} + M\right) = \frac{4\pi^2 r^3}{T^2}$$

$$T^2 = \frac{4\pi^2 r^3}{G\left(M + m/\sqrt{3}\right)}$$

$$T = 2\pi\left(\frac{r^3}{G\left(M + m/\sqrt{3}\right)}\right)^{1/2}$$

P13.49 For a 6.00 km diameter cylinder, $r = 3\ 000$ m and to simulate $1g = 9.80\ \text{m/s}^2$

$$g = \frac{v^2}{r} = \omega^2 r$$

$$\omega = \sqrt{\frac{g}{r}} = \boxed{0.057\ 2\ \text{rad/s}}$$

The required rotation rate of the cylinder is $\boxed{\frac{1\ \text{rev}}{110\ \text{s}}}$

(For a description of proposed cities in space, see Gerard K. O'Neill in *Physics Today*, Sept. 1974.)

P13.50 For both circular orbits,

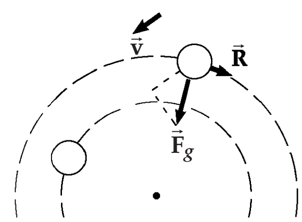

FIG. P13.50

$$\sum F = ma: \qquad \frac{GM_E m}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM_E}{r}}$$

(a) The original speed is $v_i = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(6.37\times10^6\ \text{m} + 2\times10^5\ \text{m}\right)}} = \boxed{7.79\times10^3\ \text{m/s}}$

(b) The final speed is $v_i = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(6.47\times10^6\ \text{m}\right)}} = \boxed{7.85\times10^3\ \text{m/s}}$

The energy of the satellite-Earth system is

$$K + U_g = \frac{1}{2}mv^2 - \frac{GM_E m}{r} = \frac{1}{2}m\frac{GM_E}{r} - \frac{GM_E}{r} = -\frac{GM_E m}{2r}$$

(c) Originally $E_i = -\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)(100\ \text{kg})}{2\left(6.57\times10^6\ \text{m}\right)} = \boxed{-3.04\times10^9\ \text{J}}$

continued on next page

(d) Finally $$E_f = -\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)(100\ \text{kg})}{2\left(6.47\times10^{6}\ \text{m}\right)} = \boxed{-3.08\times10^{9}\ \text{J}}$$

(e) Thus the object speeds up as it spirals down to the planet. The loss of gravitational energy is so large that the total energy decreases by

$$E_i - E_f = -3.04\times10^{9}\ \text{J} - \left(-3.08\times10^{9}\ \text{J}\right) = \boxed{4.69\times10^{7}\ \text{J}}$$

(f) The only forces on the object are the backward force of air resistance R, comparatively very small in magnitude, and the force of gravity. Because the spiral path of the satellite is not perpendicular to the gravitational force, $\boxed{\text{one component of the gravitational force}}$ $\boxed{\text{pulls forward on the satellite}}$ to do positive work and make its speed increase.

P13.51 (a) At infinite separation $U = 0$ and at rest $K = 0$. Since energy of the two-planet system is conserved we have,

$$0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{d} \qquad (1)$$

The initial momentum of the system is zero and momentum is conserved.

Therefore,

$$0 = m_1v_1 - m_2v_2 \qquad (2)$$

Combine equations (1) and (2):

$$\boxed{v_1 = m_2\sqrt{\frac{2G}{d\left(m_1+m_2\right)}}} \qquad \text{and} \qquad \boxed{v_2 = m_1\sqrt{\frac{2G}{d\left(m_1+m_2\right)}}}$$

Relative velocity

$$v_r = v_1 - \left(-v_2\right) = \boxed{\sqrt{\frac{2G\left(m_1+m_2\right)}{d}}}$$

(b) Substitute given numerical values into the equation found for v_1 and v_2 in part (a) to find

$$v_1 = 1.03\times10^{4}\ \text{m/s} \qquad \text{and} \qquad v_2 = 2.58\times10^{3}\ \text{m/s}$$

Therefore,

$$K_1 = \frac{1}{2}m_1v_1^2 = \boxed{1.07\times10^{32}\ \text{J}} \qquad \text{and} \qquad K_2 = \frac{1}{2}m_2v_2^2 = \boxed{2.67\times10^{31}\ \text{J}}$$

P13.52 (a) The net torque exerted on the Earth is zero. Therefore, the angular momentum of the Earth is conserved;

$$mr_av_a = mr_pv_p \text{ and } v_a = v_p\left(\frac{r_p}{r_a}\right) = \left(3.027\times10^{4}\ \text{m/s}\right)\left(\frac{1.471}{1.521}\right) = \boxed{2.93\times10^{4}\ \text{m/s}}$$

(b) $$K_p = \frac{1}{2}mv_p^2 = \frac{1}{2}\left(5.98\times10^{24}\right)\left(3.027\times10^{4}\right)^2 = \boxed{2.74\times10^{33}\ \text{J}}$$

$$U_p = -\frac{GmM}{r_p} = -\frac{\left(6.673\times10^{-11}\right)\left(5.98\times10^{24}\right)\left(1.99\times10^{30}\right)}{1.471\times10^{11}} = \boxed{-5.40\times10^{33}\ \text{J}}$$

(c) Using the same form as in part (b), $K_a = \boxed{2.57\times10^{33}\ \text{J}}$ and $U_a = \boxed{-5.22\times10^{33}\ \text{J}}$.

Compare to find that $K_p + U_p = \boxed{-2.66\times10^{33}\ \text{J}}$ and $K_a + U_a = \boxed{-2.65\times10^{33}\ \text{J}}$. $\boxed{\text{They agree.}}$

P13.53 (a) $T = \dfrac{2\pi r}{v} = \dfrac{2\pi\left(30\,000 \times 9.46 \times 10^{15}\text{ m}\right)}{2.50 \times 10^{5}\text{ m/s}} = 7 \times 10^{15}\text{ s} = \boxed{2 \times 10^{8}\text{ yr}}$

(b) $M = \dfrac{4\pi^2 a^3}{GT^2} = \dfrac{4\pi^2\left(30\,000 \times 9.46 \times 10^{15}\text{ m}\right)^3}{\left(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.13 \times 10^{15}\text{ s}\right)^2} = 2.66 \times 10^{41}\text{ kg}$

$M = 1.34 \times 10^{11}$ solar masses $\boxed{\sim 10^{11}\text{ solar masses}}$

The number of stars is $\boxed{\text{on the order of } 10^{11}}$.

P13.54 Centripetal acceleration comes from gravitational acceleration.

$$\frac{v^2}{r} = \frac{M_c G}{r^2} = \frac{4\pi^2 r^2}{T^2 r}$$

$$GM_c T^2 = 4\pi^2 r^3$$

$$\left(6.67 \times 10^{-11}\right)(20)\left(1.99 \times 10^{30}\right)\left(5.00 \times 10^{-3}\right)^2 = 4\pi^2 r^3$$

$$r_{\text{orbit}} = \boxed{119\text{ km}}$$

P13.55 Let m represent the mass of the meteoroid and v_i its speed when far away. No torque acts on the meteoroid, so its angular momentum is conserved as it moves between the distant point and the point where it grazes the Earth, moving perpendicular to the radius:

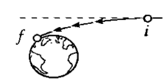

FIG. P13.55

$L_i = L_f$: $\quad m\vec{\mathbf{r}}_i \times \vec{\mathbf{v}}_i = m\vec{\mathbf{r}}_f \times \vec{\mathbf{v}}_f$

$$m\left(3R_E v_i\right) = mR_E v_f$$

$$v_f = 3v_i$$

Now energy of the meteoroid-Earth system is also conserved:

$\left(K + U_g\right)_i = \left(K + U_g\right)_f$: $\quad \dfrac{1}{2}mv_i^2 + 0 = \dfrac{1}{2}mv_f^2 - \dfrac{GM_E m}{R_E}$

$$\frac{1}{2}v_i^2 = \frac{1}{2}\left(9v_i^2\right) - \frac{GM_E}{R_E}$$

$\dfrac{GM_E}{R_E} = 4v_i^2$: $\quad \boxed{v_i = \sqrt{\dfrac{GM_E}{4R_E}}}$

P13.56 (a) From the data about perigee, the energy of the satellite-Earth system is

$$E = \frac{1}{2}mv_p^2 - \frac{GM_E m}{r_p} = \frac{1}{2}(1.60)\left(8.23 \times 10^3\right)^2 - \frac{\left(6.67 \times 10^{-11}\right)\left(5.98 \times 10^{24}\right)(1.60)}{7.02 \times 10^6}$$

or

$$E = \boxed{-3.67 \times 10^7\text{ J}}$$

(b) $L = mvr\sin\theta = mv_p r_p \sin 90.0° = (1.60\text{ kg})\left(8.23 \times 10^3\text{ m/s}\right)\left(7.02 \times 10^6\text{ m}\right)$

$= \boxed{9.24 \times 10^{10}\text{ kg}\cdot\text{m}^2/\text{s}}$

continued on next page

(c) Since both the energy of the satellite-Earth system and the angular momentum of the Earth are conserved,

at apogee we must have $\frac{1}{2}mv_a^2 - \frac{GMm}{r_a} = E$

and $mv_a r_a \sin 90.0° = L$

Thus, $\frac{1}{2}(1.60)v_a^2 - \frac{(6.67\times10^{-11})(5.98\times10^{24})(1.60)}{r_a} = -3.67\times10^7 \text{ J}$

and $(1.60 \text{ kg})v_a r_a = 9.24\times10^{10} \text{ kg}\cdot\text{m}^2/\text{s}$

Solving simultaneously, $\frac{1}{2}(1.60)v_a^2 - \frac{(6.67\times10^{-11})(5.98\times10^{24})(1.60)(1.60)v_a}{9.24\times10^{10}}$

$= -3.67\times10^7$

which reduces to $0.800v_a^2 - 11\,046v_a + 3.672\,3\times10^7 = 0$

so $v_a = \frac{11\,046 \pm \sqrt{(11\,046)^2 - 4(0.800)(3.672\,3\times10^7)}}{2(0.800)}$

This gives $v_a = 8\,230 \text{ m/s}$ or $\boxed{5\,580 \text{ m/s}}$. The smaller answer refers to the velocity at the apogee while the larger refers to perigee.

Thus,

$$r_a = \frac{L}{mv_a} = \frac{9.24\times10^{10} \text{ kg}\cdot\text{m}^2/\text{s}}{(1.60 \text{ kg})(5.58\times10^3 \text{ m/s})} = \boxed{1.04\times10^7 \text{ m}}$$

(d) The major axis is $2a = r_p + r_a$, so the semi-major axis is

$$a = \frac{1}{2}(7.02\times10^6 \text{ m} + 1.04\times10^7 \text{ m}) = \boxed{8.69\times10^6 \text{ m}}$$

(e) $T = \sqrt{\frac{4\pi^2 a^3}{GM_E}} = \sqrt{\frac{4\pi^2(8.69\times10^6 \text{ m})^3}{(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24} \text{ kg})}}$

$T = 8\,060 \text{ s} = \boxed{134 \text{ min}}$

P13.57 If we choose the coordinate of the center of mass at the origin, then

$$0 = \frac{(Mr_2 - mr_1)}{M+m} \qquad \text{and} \qquad Mr_2 = mr_1$$

(Note: this is equivalent to saying that the net torque must be zero and the two experience no angular acceleration.) For each mass $F = ma$ so

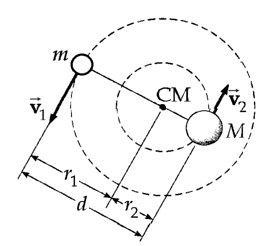

FIG. P13.57

$$mr_1\omega_1^2 = \frac{MGm}{d^2} \qquad \text{and} \qquad Mr_2\omega_2^2 = \frac{MGm}{d^2}$$

Combining these two equations and using $d = r_1 + r_2$ gives $(r_1 + r_2)\omega^2 = \frac{(M+m)G}{d^2}$ with

$$\omega_1 = \omega_2 = \omega$$

and

$$T = \frac{2\pi}{\omega}$$

we find

$$\boxed{T^2 = \frac{4\pi^2 d^3}{G(M+m)}}$$

P13.58 From Kepler's third law, minimum period means minimum orbit size. The "treetop satellite" in Problem 33 has minimum period. The radius of the satellite's circular orbit is essentially equal to the radius R of the planet.

$$\sum F = ma: \qquad \frac{GMm}{R^2} = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2$$

$$G\rho V = \frac{R^2\left(4\pi^2 R^2\right)}{RT^2}$$

$$G\rho\left(\frac{4}{3}\pi R^3\right) = \frac{4\pi^2 R^3}{T^2}$$

The radius divides out: $T^2 G\rho = 3\pi$ $\boxed{T = \sqrt{\frac{3\pi}{G\rho}}}$

***P13.59** The gravitational forces the particles exert on each other are in the x direction. They do not affect the velocity of the center of mass. Energy is conserved for the pair of particles in a reference frame coasting along with their center of mass, and momentum conservation means that the identical particles move toward each other with equal speeds in this frame:

$$U_{gi} + K_i + K_i = U_{gf} + K_f + K_f$$

$$-\frac{Gm_1 m_2}{r_i} + 0 = -\frac{Gm_1 m_2}{r_f} + \tfrac{1}{2}m_1 v^2 + \tfrac{1}{2}m_2 v^2$$

$$-\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1000\ \text{kg})^2}{20\ \text{m}} = -\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1000\ \text{kg})^2}{2\ \text{m}} + 2\left(\tfrac{1}{2}\right)(1000\ \text{kg})v^2$$

$$\left(\frac{3.00\times10^{-5}\ \text{J}}{1000\ \text{kg}}\right)^{1/2} = v = 1.73\times10^{-4}\ \text{m/s}$$

Then their vector velocities are $\boxed{(800 + 1.73\times10^{-4})\,\hat{\mathbf{i}}\ \text{m/s and } (800 - 1.73\times10^{-4})\,\hat{\mathbf{i}}\ \text{m/s}}$ for the trailing particle and the leading particle, respectively.

***P13.60** (a) The gravitational force exerted on m by the Earth (mass ME) accelerates m according to

$mg_2 = \frac{GmM_E}{r^2}$. The equal magnitude force exerted on the Earth by m produces

acceleration of the Earth given by $g_1 = \frac{Gm}{r^2}$. The acceleration of relative approach is then

$$g_2 + g_1 = \frac{Gm}{r^2} + \frac{GM_E}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg} + m\right)}{\left(1.20\times10^7\ \text{m}\right)^2}$$

$$= \boxed{\left(2.77\ \text{m/s}^2\right)\left(1 + \frac{m}{5.98\times10^{24}\ \text{kg}}\right)}$$

(b) and (c) Here $m = 5$ kg and $m = 2000$ kg are both negligible compared to the mass of the Earth, so the acceleration of relative approach is just $\boxed{2.77\ \text{m/s}^2}$.

continued on next page

(d) Again, m accelerates toward the center of mass with $g_2 = 2.77\ \text{m/s}^2$. Now the Earth accelerates toward m with an acceleration given as

$$M_E g_1 = \frac{GM_E m}{r^2}$$

$$g_1 = \frac{Gm}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.00\times10^{24}\ \text{kg}\right)}{\left(1.20\times10^7\ \text{m}\right)^2} = 0.926\ \text{m/s}^2$$

The distance between the masses closes with relative acceleration of

$$g_{\text{rel}} = g_1 + g_2 = 0.926\ \text{m/s}^2 + 2.77\ \text{m/s}^2 = \boxed{3.70\ \text{m/s}^2}$$

(e) Any object with mass small compared to the Earth starts to fall with acceleration 2.77 m/s^2. As m increases to become comparable to the mass of the Earth, the acceleration increases, and can become arbitrarily large. It approaches a direct proportionality to m.

P13.61 For the Earth, $\sum F = ma$: $\dfrac{GM_s m}{r^2} = \dfrac{mv^2}{r} = \dfrac{m}{r}\left(\dfrac{2\pi r}{T}\right)^2$

Then $GM_s T^2 = 4\pi^2 r^3$

Also the angular momentum $L = mvr = m\dfrac{2\pi r}{T} r$ is a constant for the Earth.

We eliminate $r = \sqrt{\dfrac{LT}{2\pi m}}$ between the equations:

$$GM_s T^2 = 4\pi^2\left(\frac{LT}{2\pi m}\right)^{3/2} \qquad GM_s T^{1/2} = 4\pi^2\left(\frac{L}{2\pi m}\right)^{3/2}$$

Now the rates of change with time t are described by

$$GM_s\left(\frac{1}{2}T^{-1/2}\frac{dT}{dt}\right) + G\left(1\frac{dM_s}{dt}T^{1/2}\right) = 0 \qquad \frac{dT}{dt} = -\frac{dM_s}{dt}\left(2\frac{T}{M_s}\right) \approx \frac{\Delta T}{\Delta t}$$

$$\Delta T \approx -\Delta t\frac{dM_s}{dt}\left(2\frac{T}{M_s}\right) = -5\,000\ \text{yr}\left(\frac{3.16\times10^7\ \text{s}}{1\ \text{yr}}\right)\left(-3.64\times10^9\ \text{kg/s}\right)\left(2\frac{1\ \text{yr}}{1.991\times10^{30}\ \text{kg}}\right)$$

$$\Delta T = \boxed{1.82\times10^{-2}\ \text{s}}$$

ANSWERS TO EVEN PROBLEMS

P13.2 $2.67\times10^{-7}\ \text{m/s}^2$

P13.4 3.00 kg and 2.00 kg

P13.6 (a) 4.39×10^{20} N toward the Sun (b) 1.99×10^{20} N toward the Earth (c) 3.55×10^{22} N toward the Sun (d) Note that the force exerted by the Sun on the Moon is much stronger than the force of the Earth on the Moon. In a sense, the Moon orbits the Sun more than it orbits the Earth. The Moon's path is everywhere concave toward the Sun. Only by subtracting out the solar orbital motion of the Earth-Moon system do we see the Moon orbiting the center of mass of this system.

P13.8 There are two possibilities: either 1 m − 61.3 nm or 2.74×10^{-4} m

P13.10 (a) 7.61 cm/s^2 (b) 363 s (c) 3.08 km (d) 28.9 m/s at 72.9° below the horizontal

P13.12 The particle does possess angular momentum, because it is not headed straight for the origin. Its angular momentum is constant because the object is free of outside influences. See the solution.

P13.14 35.2 AU

P13.16 Planet *Y* has turned through 1.30 revolutions.

P13.18 1.63×10^4 rad/s

P13.20 6.02×10^{24} kg. The Earth wobbles a bit as the Moon orbits it, so both objects move nearly in circles about their center of mass, staying on opposite sides of it. The radius of the Moon's orbit is therefore a bit less than the Earth–Moon distance.

P13.22 (a) 1.31×10^{17} N toward the center (b) 2.62×10^{12} N/kg

P13.24 (a) -4.77×10^9 J (b) 569 N down (c) 569 N up

P13.26 2.52×10^7 m

P13.28 2.82×10^9 J

P13.30 (a) 42.1 km/s (b) 2.20×10^{11} m

P13.32 469 MJ. Both in the original orbit and in the final orbit, the total energy is negative, with an absolute value equal to the positive kinetic energy. The potential energy is negative and twice as large as the total energy. As the satellite is lifted from the lower to the higher orbit, the gravitational energy increases, the kinetic energy decreases, and the total energy increases. The value of each becomes closer to zero. Numerically, the gravitational energy increases by 938 MJ, the kinetic energy decreases by 469 MJ, and the total energy increases by 469 MJ.

P13.34 Gravitational screening does not exist. The presence of the satellite has no effect on the force the planet exerts on the rocket. 15.6 km/s

P13.36 (a) $2\pi(R_E+h)^{3/2}(GM_E)^{-1/2}$ (b) $(GM_E)^{1/2}(R_E+h)^{-1/2}$ (c) $GM_Em\left[\dfrac{R_E+2h}{2R_E(R_E+h)}\right]-\dfrac{2\pi^2R_E^2m}{(86\,400\text{ s})^2}$

The satellite should be launched from the Earth's equator toward the east.

P13.38 (a) $v_0=\left(\dfrac{GM_E}{r}\right)^{1/2}$ (b) $v_i=\dfrac{5(GM_E/r)^{1/2}}{4}$ (c) $r_f=\dfrac{25r}{7}$

P13.40 (a) 15.3 km (b) 1.66×10^{16} kg (c) 1.13×10^4 s (d) No. Its mass is so large compared with mine that I would have negligible effect on its rotation.

P13.42 2.26×10^{-7}

P13.44 $\dfrac{2}{3}\sqrt{\dfrac{GM}{R}}$; $\dfrac{1}{3}\sqrt{\dfrac{GM}{R}}$

P13.46 (a), (b) see the solution (c) 1.85×10^{-5} m/s^2

P13.48 see the solution

P13.50 (a) 7.79 km/s (b) 7.85 km/s (c) −3.04 GJ (d) −3.08 GJ (e) loss = 46.9 MJ
(f) A component of the Earth's gravity pulls forward on the satellite in its downward banking trajectory.

P13.52 (a) 29.3 km/s (b) $K_p = 2.74 \times 10^{33}$ J; $U_p = -5.40 \times 10^{33}$ J
(c) $K_a = 2.57 \times 10^{33}$ J; $U_a = -5.22 \times 10^{33}$ J; yes

P13.54 119 km

P13.56 (a) −36.7 MJ (b) 9.24×10^{10} kg·m^2/s (c) 5.58 km/s; 10.4 Mm (d) 8.69 Mm (e) 134 min

P13.58 see the solution

P13.60 (a) $(2.77\ \text{m/s}^2)(1 + m/5.98 \times 10^{24}\ \text{kg})$ (b) 2.77 m/s^2 (c) 2.77 m/s^2 (d) 3.70 m/s^2
(e) Any object with mass small compared with the mass of the Earth starts to fall with acceleration 2.77 m/s^2. As *m* increases to become comparable to the mass of the Earth, the acceleration increases and can become arbitrarily large. It approaches a direct proportionality to *m*.

14

Fluid Mechanics

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

***Q14.1** Answer (c). Both must be built the same. The force on the back of each dam is the average pressure of the water times the area of the dam. If both reservoirs are equally deep, the force is the same.

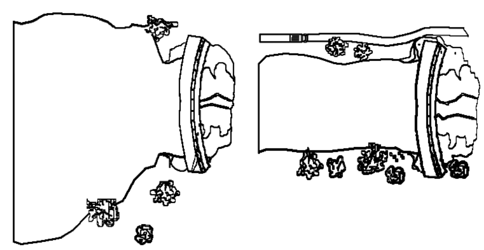

FIG. Q14.1

Q14.2 The weight depends upon the total volume of water in the glass. The pressure at the bottom depends only on the depth. With a cylindrical glass, the water pushes only horizontally on the side walls and does not contribute to an extra downward force above that felt by the base. On the other hand, if the glass is wide at the top with a conical shape, the water pushes outward and downward on each bit of side wall. The downward components add up to an extra downward force, more than that exerted on the small base area.

Q14.3 The air in your lungs, the blood in your arteries and veins, and the protoplasm in each cell exert nearly the same pressure, so that the wall of your chest can be in equilibrium.

Q14.4 Yes. The propulsive force of the fish on the water causes the scale reading to fluctuate. Its average value will still be equal to the total weight of bucket, water, and fish.

Q14.5 Clap your shoe or wallet over the hole, or a seat cushion, or your hand. Anything that can sustain a force on the order of 100 N is strong enough to cover the hole and greatly slow down the escape of the cabin air. You need not worry about the air rushing out instantly, or about your body being "sucked" through the hole, or about your blood boiling or your body exploding. If the cabin pressure drops a lot, your ears will pop and the saliva in your mouth may boil—at body temperature—but you will still have a couple of minutes to plug the hole and put on your emergency oxygen mask. Passengers who have been drinking carbonated beverages may find that the carbon dioxide suddenly comes out of solution in their stomachs, distending their vests, making them belch, and all but frothing from their ears; so you might warn them of this effect.

Q14.6 The boat floats higher in the ocean than in the inland lake. According to Archimedes's principle, the magnitude of buoyant force on the ship is equal to the weight of the water displaced by the ship. Because the density of salty ocean water is greater than fresh lake water, less ocean water needs to be displaced to enable the ship to float.

***Q14.7** Answer (b). The apple does not change volume appreciably in a dunking bucket, and the water also keeps constant density. Then the buoyant force is constant at all depths.

Q14.8 The horizontal force exerted by the outside fluid, on an area element of the object's side wall, has equal magnitude and opposite direction to the horizontal force the fluid exerts on another element diametrically opposite the first.

Q14.9 No. The somewhat lighter barge will float higher in the water.

Q14.10 The metal is more dense than water. If the metal is sufficiently thin, it can float like a ship, with the lip of the dish above the water line. Most of the volume below the water line is filled with air. The mass of the dish divided by the volume of the part below the water line is just equal to the density of water. Placing a bar of soap into this space to replace the air raises the average density of the compound object and the density can become greater than that of water. The dish sinks with its cargo.

***Q14.11** Answer (c). The water keeps nearly constant density as it increases in pressure with depth. The beach ball is compressed to smaller volume as you take it deeper, so the buoyant force decreases.

Q14.12 Like the ball, the balloon will remain in front of you. It will not bob up to the ceiling. Air pressure will be no higher at the floor of the sealed car than at the ceiling. The balloon will experience no buoyant force. You might equally well switch off gravity.

Q14.13 (i) b (ii) c. In both orientations the compound floating object displaces its own weight of water, so it displaces equal volumes of water. The water level in the tub will be unchanged when the object is turned over. Now the steel is underwater and the water exerts on the steel a buoyant force that was not present when the steel was on top surrounded by air. Thus, slightly less wood will be below the water line on the wooden block. It will appear to float higher.

***Q14.14** Use a balance to determine its mass. Then partially fill a graduated cylinder with water. Immerse the rock in the water and determine the volume of water displaced. Divide the mass by the volume and you have the density. It may be more precise to hang the rock from a string, measure the force required to support it under water, and subtract to find the buoyant force. The buoyant force can be thought of as the weight of so many grams of water, which is that number of cubic centimeters of water, which is the volume of the submerged rock. This volume with the actual rock mass tells you its density.

***Q14.15** Objects a and c float, and e barely floats. On them the buoyant forces are equal to the gravitational forces exerted on them, so the ranking is e greater than a by perhaps 1.5 times and e greater than c by perhaps 500 times. Objects b and d sink, and have volumes equal to e, so they feel equal-size buoyant forces: e = b = d. Now f has smaller volume than e and g still smaller volume, so they feel smaller buoyant forces: e is greater than f by 2.7 times and e is greater than g by 7.9 times. We have altogether e = b = d > a > f > g > c.

***Q14.16** Answer (b). The level of the pond falls. This is because the anchor displaces more water while in the boat. A floating object displaces a volume of water whose weight is equal to the weight of the object. A submerged object displaces a volume of water equal to the volume of the object. Because the density of the anchor is greater than that of water, a volume of water that weighs the same as the anchor will be greater than the volume of the anchor.

***Q14.17** The buoyant force is a conservative force. It does positive work on an object moving upward in a fluid and an equal amount of negative work on the object moving down between the same two elevations. Potential energy is not associated with the object on which the buoyant force acts, but with the set of objects interacting by the buoyant force. This system (set) is the immersed object and the fluid. The potential energy then is the gravitational potential energy we have already studied. The higher potential energy associated with a basketball at the bottom of a swimming pool is equally well or more clearly associated with the extra basketball-volume of water that is at the top of the pool, displaced there by the ball.

Q14.18 Regular cola contains a considerable mass of dissolved sugar. Its density is higher than that of water. Diet cola contains a very small mass of artificial sweetener and has nearly the same density as water. The low–density air in the can has a bigger effect than the thin aluminum shell, so the can of diet cola floats.

***Q14.19** The excess pressure is transmitted undiminished throughout the container. It will compress air inside the wood. The water driven into the pores of the wood raises the block's average density and makes if float lower in the water. The answer is (b). Add some thumbtacks to reach neutral buoyancy and you can make the wood sink or rise at will by subtly squeezing a large clear–plastic soft–drink bottle. Bored with graph paper and proving his own existence, René Descartes invented this toy or trick, called a Cartesian diver.

Q14.20 At lower elevation the water pressure is greater because pressure increases with increasing depth below the water surface in the reservoir (or water tower). The penthouse apartment is not so far below the water surface. The pressure behind a closed faucet is weaker there and the flow weaker from an open faucet. Your fire department likely has a record of the precise elevation of every fire hydrant.

Q14.21 The rapidly moving air above the ball exerts less pressure than the atmospheric pressure below the ball. This can give substantial lift to balance the weight of the ball.

Q14.22 The ski–jumper gives her body the shape of an airfoil. She deflects downward the air stream as it rushes past and it deflects her upward by Newton's third law. The air exerts on her a lift force, giving her a higher and longer trajectory. To say it in different words, the pressure on her back is less than the pressure on her front.

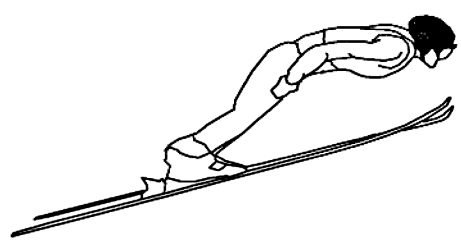

FIG. Q14.22

Q14.23 When taking off into the wind, the increased airspeed over the wings gives a larger lifting force, enabling the pilot to take off in a shorter length of runway.

***Q14.24** You want a water drop to have four times the gravitational energy as it turns around at the top of the fountain. You want it to start out with four times the kinetic energy, which means with twice the speed at the nozzles. Given the constant volume flow rate Av, you want the area to be two times smaller, answer (d). If the nozzle has a circular opening, you need decrease its radius only by the square root of two times.

Q14.25 A breeze from any direction speeds up to go over the mound and the air pressure drops. Air then flows through the burrow from the lower entrance to the upper entrance.

Q14.26 (a) Since the velocity of the air in the right-hand section of the pipe is lower than that in the middle, the pressure is higher.

(b) The equation that predicts the same pressure in the far right and left-hand sections of the tube assumes laminar flow without viscosity. Internal friction will cause some loss of mechanical energy and turbulence will also progressively reduce the pressure. If the pressure at the left were not higher than at the right, the flow would stop.

***Q14.27** (i) Answer (c). The water level stays the same. The solid ice displaced its own mass of liquid water. The meltwater does the same. You can accurately measure the quantity of H_2O going into a recipe, even if some of it is frozen, either by using a kitchen scale or by letting the ice float in liquid water in a measuring cup and looking at the liquid water level.

(ii) Answer (b). Ice on the continent of Antarctica is above sea level.

SOLUTIONS TO PROBLEMS

Section 14.1 **Pressure**

P14.1 $M = \rho_{\text{iron}} V = \left(7\,860 \text{ kg/m}^3\right)\left[\frac{4}{3}\pi\left(0.015\,0 \text{ m}\right)^3\right]$

$M = \boxed{0.111 \text{ kg}}$

P14.2 The density of the nucleus is of the same order of magnitude as that of one proton, according to the assumption of close packing:

$$\rho = \frac{m}{V} \sim \frac{1.67\times10^{-27} \text{ kg}}{\frac{4}{3}\pi\left(10^{-15} \text{ m}\right)^3} \boxed{\sim 10^{18} \text{ kg/m}^3}$$

With vastly smaller average density, a macroscopic chunk of matter or an atom must be mostly empty space.

P14.3 $P = \frac{F}{A} = \frac{50.0(9.80)}{\pi\left(0.500\times10^{-2}\right)^2} = \boxed{6.24\times10^6 \text{ N/m}^2}$

P14.4 The Earth's surface area is $4\pi R^2$. The force pushing inward over this area amounts to

$$F = P_0 A = P_0\left(4\pi R^2\right)$$

This force is the weight of the air:

$$F_g = mg = P_0\left(4\pi R^2\right)$$

so the mass of the air is

$$m = \frac{P_0\left(4\pi R^2\right)}{g} = \frac{\left(1.013\times10^5 \text{ N/m}^2\right)\left[4\pi\left(6.37\times10^6 \text{ m}\right)^2\right]}{9.80 \text{ m/s}^2} = \boxed{5.27\times10^{18} \text{ kg}}$$

Section 14.2 **Variation of Pressure with Depth**

P14.5 $F_{el} = F_{\text{fluid}}$ or $kx = \rho g h A$

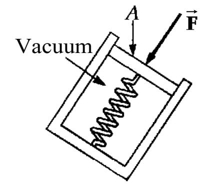

FIG. P14.5

and $h = \dfrac{kx}{\rho g A}$

$$h = \frac{\left(1\,000\ \text{N/m}^2\right)\left(5.00\times10^{-3}\ \text{m}\right)}{\left(10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left[\pi\left(1.00\times10^{-2}\ \text{m}\right)^2\right]} = \boxed{1.62\ \text{m}}$$

P14.6 (a) $P = P_0 + \rho g h = 1.013\times10^5\ \text{Pa} + \left(1\,024\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(1\,000\ \text{m}\right)$

$P = \boxed{1.01\times10^7\ \text{Pa}}$

(b) The gauge pressure is the difference in pressure between the water outside and the air inside the submarine, which we suppose is at 1.00 atmosphere.

$$P_{\text{gauge}} = P - P_0 = \rho g h = 1.00\times10^7\ \text{Pa}$$

The resultant inward force on the porthole is then

$$F = P_{\text{gauge}}A = 1.00\times10^7\ \text{Pa}\left[\pi(0.150\ \text{m})^2\right] = \boxed{7.09\times10^5\ \text{N}}$$

P14.7 $F_g = 80.0\ \text{kg}\left(9.80\ \text{m/s}^2\right) = 784\ \text{N}$

When the cup barely supports the student, the normal force of the ceiling is zero and the cup is in equilibrium.

$$F_g = F = PA = \left(1.013\times10^5\ \text{Pa}\right)A$$

$$A = \frac{F_g}{P} = \frac{784}{1.013\times10^5} = \boxed{7.74\times10^{-3}\ \text{m}^2}$$

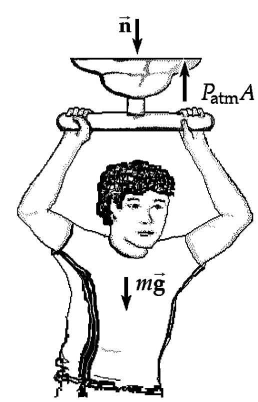

FIG. P14.7

P14.8 Since the pressure is the same on both sides, $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$

In this case, $\dfrac{15\,000}{200} = \dfrac{F_2}{3.00}$ or $F_2 = \boxed{225\ \text{N}}$

P14.9 The excess water pressure (over air pressure) halfway down is

$$P_{\text{gauge}} = \rho g h = \left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(1.20\ \text{m}) = 1.18\times10^4\ \text{Pa}$$

The force on the wall due to the water is

$$F = P_{\text{gauge}}A = \left(1.18\times10^4\ \text{Pa}\right)(2.40\ \text{m})(9.60\ \text{m}) = \boxed{2.71\times10^5\ \text{N}}$$

$\boxed{\text{horizontally toward the back of the hole}}$. Russell Shadle suggested the idea for this problem.

P14.10 (a) Suppose the "vacuum cleaner" functions as a high–vacuum pump. The air below the brick will exert on it a lifting force

$$F = PA = 1.013\times10^5\ \text{Pa}\left[\pi\left(1.43\times10^{-2}\ \text{m}\right)^2\right] = \boxed{65.1\ \text{N}}$$

(b) The octopus can pull the bottom away from the top shell with a force that could be no larger than

$$F = PA = \left(P_0 + \rho g h\right)A$$

$$= \left[1.013\times10^5\ \text{Pa} + \left(1\,030\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(32.3\ \text{m})\right]\left[\pi\left(1.43\times10^{-2}\ \text{m}\right)^2\right]$$

$$F = \boxed{275\ \text{N}}$$

P14.11 The pressure on the bottom due to the water is $P_b = \rho g z = 1.96\times10^4\ \text{Pa}$

So, $$F_b = P_b A = \boxed{5.88\times10^6\ \text{N down}}$$

On each end, $$F = P_{average}A = 9.80\times10^3\ \text{Pa}\left(20.0\ \text{m}^2\right) = \boxed{196\ \text{kN outward}}$$

On the side, $$F = P_{average}A = 9.80\times10^3\ \text{Pa}\left(60.0\ \text{m}^2\right) = \boxed{588\ \text{kN outward}}$$

P14.12 The air outside and water inside both exert atmospheric pressure, so only the excess water pressure $\rho g h$ counts for the net force. Take a strip of hatch between depth h and $h + dh$. It feels force

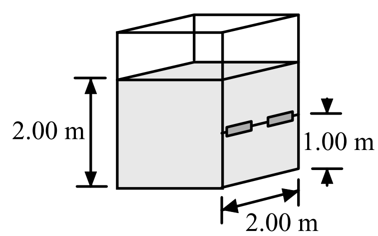

FIG. P14.12

$$dF = P\,dA = \rho g h (2.00\ \text{m})\,dh$$

(a) The total force is

$$F = \int dF = \int_{h=1.00\ \text{m}}^{2.00\ \text{m}} \rho g h (2.00\ \text{m})\,dh$$

$$F = \rho g (2.00\ \text{m})\left.\frac{h^2}{2}\right|_{1.00\ \text{m}}^{2.00\ \text{m}} = \left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\frac{(2.00\ \text{m})}{2}\left[(2.00\ \text{m})^2 - (1.00\ \text{m})^2\right]$$

$$F = \boxed{29.4\ \text{kN (to the right)}}$$

(b) The lever arm of dF is the distance $(h - 1.00\ \text{m})$ from hinge to strip:

$$\tau = \int d\tau = \int_{h=1.00\ \text{m}}^{2.00\ \text{m}} \rho g h (2.00\ \text{m})(h - 1.00\ \text{m})\,dh$$

$$\tau = \rho g (2.00\ \text{m})\left[\frac{h^3}{3} - (1.00\ \text{m})\frac{h^2}{2}\right]_{1.00\ \text{m}}^{2.00\ \text{m}}$$

$$\tau = \left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(2.00\ \text{m})\left(\frac{7.00\ \text{m}^3}{3} - \frac{3.00\ \text{m}^3}{2}\right)$$

$$\tau = \boxed{16.3\ \text{kN}\cdot\text{m counterclockwise}}$$

P14.13 The bell is uniformly compressed, so we can model it with any shape. We choose a sphere of diameter 3.00 m.

The pressure on the ball is given by: $P = P_{atm} + \rho_w gh$ so the change in pressure on the ball from when it is on the surface of the ocean to when it is at the bottom of the ocean is $\Delta P = \rho_w gh$. In addition:

$$\Delta V = \frac{-V\Delta P}{B} = -\frac{\rho_w ghV}{B} = -\frac{4\pi\rho_w ghr^3}{3B}, \text{ where } B \text{ is the Bulk Modulus.}$$

$$\Delta V = -\frac{4\pi(1\,030 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(10\,000 \text{ m})(1.50 \text{ m})^3}{(3)(14.0\times 10^{10} \text{ Pa})} = -0.010\,2 \text{ m}^3$$

Therefore, the volume of the ball at the bottom of the ocean is

$$V - \Delta V = \frac{4}{3}\pi(1.50 \text{ m})^3 - 0.010\,2 \text{ m}^3 = 14.137 \text{ m}^3 - 0.010\,2 \text{ m}^3 = 14.127 \text{ m}^3$$

This gives a radius of 1.499 64 m and a new diameter of 2.999 3 m. Therefore the diameter decreases by $\boxed{0.722 \text{ mm}}$.

Section 14.3 **Pressure Measurements**

P14.14 (a) We imagine the superhero to produce a perfect vacuum in the straw. Take point 1 at the water surface in the basin and point 2 at the water surface in the straw:

$$P_1 + \rho g y_1 = P_2 + \rho g y_2$$

$$1.013\times 10^5 \text{ N/m}^2 + 0 = 0 + (1\,000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)y_2 \qquad y_2 = \boxed{10.3 \text{ m}}$$

(b) No atmosphere can lift the water in the straw through $\boxed{\text{zero}}$ height difference.

P14.15 $P_0 = \rho gh$

$$h = \frac{P_0}{\rho g} = \frac{1.013\times 10^5 \text{ Pa}}{(0.984\times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = \boxed{10.5 \text{ m}}$$

$\boxed{\text{No. The "Torricellian vacuum" is not so good. Some alcohol and water will evaporate.}}$

The equilibrium vapor pressures of alcohol and water are higher than the vapor pressure of mercury.

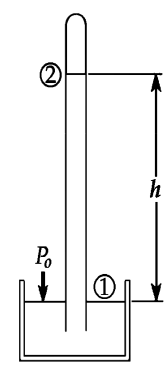

FIG. P14.15

P14.16 (a) Using the definition of density, we have

$$h_w = \frac{m_{\text{water}}}{A_2\rho_{\text{water}}} = \frac{100\ \text{g}}{5.00\ \text{cm}^2\left(1.00\ \text{g/cm}^3\right)}$$

$$= \boxed{20.0\ \text{cm}}$$

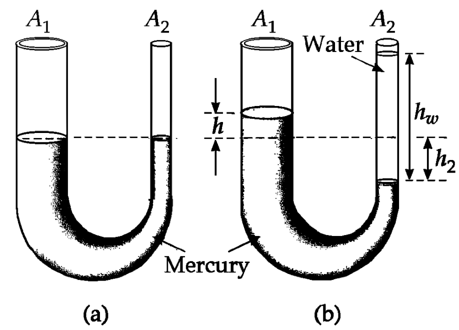

FIG. P14.16

(b) Sketch (b) at the right represents the situation after the water is added. A volume $\left(A_2h_2\right)$ of mercury has been displaced by water in the right tube. The additional volume of mercury now in the left tube is A_1h. Since the total volume of mercury has not changed,

$$A_2h_2 = A_1h \qquad \text{or} \qquad h_2 = \frac{A_1}{A_2}h \tag{1}$$

At the level of the mercury–water interface in the right tube, we may write the absolute pressure as:

$$P = P_0 + \rho_{\text{water}}gh_w$$

The pressure at this same level in the left tube is given by

$$P = P_0 + \rho_{\text{Hg}}g\left(h+h_2\right) = P_0 + \rho_{\text{water}}gh_w$$

which, using equation (1) above, reduces to

$$\rho_{\text{Hg}}h\left[1+\frac{A_1}{A_2}\right] = \rho_{\text{water}}h_w$$

or

$$h = \frac{\rho_{\text{water}}h_w}{\rho_{\text{Hg}}\left(1+A_1/A_2\right)}$$

Thus, the level of mercury has risen a distance of

$$h = \frac{\left(1.00\ \text{g/cm}^3\right)(20.0\ \text{cm})}{\left(13.6\ \text{g/cm}^3\right)(1+10.0/50.0)} = \boxed{0.490\ \text{cm}} \quad \text{above the original level.}$$

P14.17 $\Delta P_0 = \rho g\Delta h = -2.66\times10^3\ \text{Pa}$: $\quad P = P_0 + \Delta P_0 = \left(1.013-0.026\,6\right)\times10^5\ \text{Pa} = \boxed{0.986\times10^5\ \text{Pa}}$

***P14.18** (a) We can directly write the bottom pressure as $P = P_0 + \rho gh$, or we can say that the bottom of the tank must support the weight of the water:

$PA - P_0A = m_{\text{water}}g = \rho Vg = \rho Ahg$ which gives again

$P = P_0 + \rho gh = 101.3\ \text{kPa} + (1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)h = \boxed{101.3\ \text{kPa} + (9.8\ \text{kPa/m})h}$

(b) Now the bottom of the tank must support the weight of the whole contents:

$P_bA - P_0A = m_{\text{water}}g + Mg = \rho Vg + Mg = \rho Ahg + Mg$ so

$P_b = P_0 + \rho hg + Mg/A$ Then $\Delta P = P_b - P = \boxed{Mg/A}$

(c) Before the people enter, $P = 101.3\ \text{kPa} + (9.8\ \text{kPa/m})(1.5\ \text{m}) = \boxed{116\ \text{kPa}}$

afterwards, $\Delta P = Mg/A = (150\ \text{kg})(9.8\ \text{m/s}^2)/\pi(3\ \text{m})^2 = \boxed{52.0\ \text{Pa}}$

P14.19 (a) $P = P_0 + \rho g h$

The gauge pressure is

$$P - P_0 = \rho g h = 1\,000 \text{ kg}\left(9.8 \text{ m/s}^2\right)\left(0.160 \text{ m}\right) = \boxed{1.57 \text{ kPa}}$$

$$= 1.57 \times 10^3 \text{ Pa}\left(\frac{1 \text{ atm}}{1.013 \times 10^5 \text{ Pa}}\right)$$

$$= \boxed{0.015\,5 \text{ atm}}$$

It would lift a mercury column to height

$$h = \frac{P - P_0}{\rho g} = \frac{1\,568 \text{ Pa}}{\left(13\,600 \text{ kg/m}^3\right)\left(9.8 \text{ m/s}^2\right)} = \boxed{11.8 \text{ mm}}$$

(b) Increased pressure of the cerebrospinal fluid will raise the level of the fluid in the spinal tap.

(c) Blockage of the fluid within the spinal column or between the skull and the spinal column would prevent the fluid level from rising.

Section 14.4 **Buoyant Forces and Archimedes's Principle**

P14.20 (a) The balloon is nearly in equilibrium:

$$\sum F_y = ma_y \Rightarrow B - \left(F_g\right)_{\text{helium}} - \left(F_g\right)_{\text{payload}} = 0$$

or

$$\rho_{\text{air}} g V - \rho_{\text{helium}} g V - m_{\text{payload}} g = 0$$

This reduces to

$$m_{\text{payload}} = \left(\rho_{\text{air}} - \rho_{\text{helium}}\right) V = \left(1.29 \text{ kg/m}^3 - 0.179 \text{ kg/m}^3\right)\left(400 \text{ m}^3\right)$$

$$m_{\text{payload}} = \boxed{444 \text{ kg}}$$

(b) Similarly,

$$m_{\text{payload}} = \left(\rho_{\text{air}} - \rho_{\text{hydrogen}}\right) V = \left(1.29 \text{ kg/m}^3 - 0.089\,9 \text{ kg/m}^3\right)\left(400 \text{ m}^3\right)$$

$$m_{\text{payload}} = \boxed{480 \text{ kg}}$$

The surrounding air does the lifting, nearly the same for the two balloons.

P14.21 At equilibrium $\sum F = 0$ or $F_{app} + mg = B$

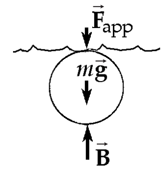

FIG. P14.21

where B is the buoyant force.

The applied force, $F_{app} = B - mg$

where $B = \text{Vol}\left(\rho_{\text{water}}\right)g$

and $m = (\text{Vol})\,\rho_{\text{ball}}$

So, $F_{app} = (\text{Vol})\,g\left(\rho_{\text{water}} - \rho_{\text{ball}}\right) = \frac{4}{3}\pi r^3 g\left(\rho_{\text{water}} - \rho_{\text{ball}}\right)$

$$F_{app} = \frac{4}{3}\pi\left(1.90\times10^{-2}\text{ m}\right)^3\left(9.80\text{ m/s}^2\right)\left(10^3\text{ kg/m}^3 - 84.0\text{ kg/m}^3\right) = \boxed{0.258\text{ N down}}$$

***P14.22** For the submerged object $\Sigma F_y = 0$ $\quad +B - F_g + T = 0 \quad +B = F_g - T = 5\text{ N} - 3.5\text{ N} = 1.5\text{ N}$

This is the weight of the water displaced. Its volume is the same as the volume V of the object:

$B = m_{\text{water}}g = \rho_{\text{w}}V_{\text{object}}g = 1.5\text{ N}$: $\quad V_{\text{object}} = 1.5\text{ N}/\rho_{\text{w}}g$

Now the density of the object is

$$\rho_{\text{object}} = m_{\text{object}}/V_{\text{object}} = \frac{m_{object}\rho_w g}{1.5\text{ N}} = \frac{F_g\rho_w}{1.5\text{ N}} = \frac{5\text{ N }(1000\text{ kg/m}^3)}{1.5\text{ N}} = \boxed{3.33\times10^3\text{ kg/m}^3}$$

P14.23 (a) $P = P_0 + \rho g h$

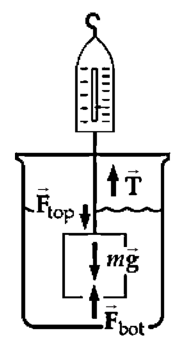

FIG. P14.23

Taking $P_0 = 1.013\times10^5\text{ N/m}^2$ and $h = 5.00$ cm

we find $P_{\text{top}} = 1.017\,9\times10^5\text{ N/m}^2$

For $h = 17.0$ cm, we get $P_{\text{bot}} = 1.029\,7\times10^5\text{ N/m}^2$

Since the areas of the top and bottom are $A = (0.100\text{ m})^2 = 10^{-2}\text{ m}^2$

we find $F_{\text{top}} = P_{\text{top}}A = \boxed{1.017\,9\times10^3\text{ N}}$

and $F_{\text{bot}} = \boxed{1.029\,7\times10^3\text{ N}}$

(b) $T + B - Mg = 0$

where $B = \rho_w Vg = \left(10^3\text{ kg/m}^3\right)\left(1.20\times10^{-3}\text{ m}^3\right)\left(9.80\text{ m/s}^2\right) = 11.8\text{ N}$

and $Mg = 10.0(9.80) = 98.0\text{ N}$

Therefore, $T = Mg - B = 98.0 - 11.8 = \boxed{86.2\text{ N}}$

(c) $F_{\text{bot}} - F_{\text{top}} = (1.029\,7 - 1.017\,9)\times10^3\text{ N} = \boxed{11.8\text{ N}}$

which is equal to B found in part (b).

P14.24 (a)

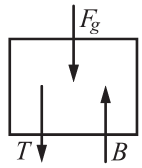

FIG. P14.24(a)

(b) $\sum F_y = 0: \quad -15\text{ N} - 10\text{ N} + B = 0$

$\boxed{B = 25.0\text{ N}}$

(c) The oil pushes $\boxed{\text{horizontally inward}}$ on each side of the block.

(d) String tension increases. The oil causes the water below to be under greater pressure, and the water pushes up more strongly on the bottom of the block.

(e) Consider the equilibrium just before the string breaks:

$$-15\text{ N} - 60\text{ N} + 25\text{ N} + B_{\text{oil}} = 0$$
$$B_{\text{oil}} = 50\text{ N}$$

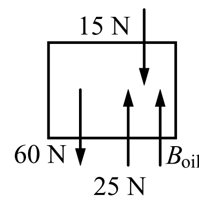

FIG. P14.24(e)

For the buoyant force of the water we have

$$B = \rho V g \qquad 25\text{ N} = \left(1\,000\text{ kg/m}^3\right)\left(0.25V_{\text{block}}\right)9.8\text{ m/s}^2$$
$$V_{\text{block}} = 1.02 \times 10^{-2}\text{ m}^3$$

For the buoyant force of the oil

$$50\text{ N} = \left(800\text{ kg/m}^3\right) f_e \left(1.02 \times 10^{-2}\text{ m}^3\right)9.8\text{ m/s}^2$$
$$f_e = 0.625 = \boxed{62.5\%}$$

(f) $-15\text{ N} + \left(800\text{ kg/m}^3\right) f_f \left(1.02 \times 10^{-2}\text{ m}^3\right)9.8\text{ m/s}^2 = 0$

$f_f = 0.187 = \boxed{18.7\%}$

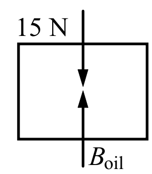

FIG. P14.24(f)

***P14.25** (a) Let P represent the pressure at the center of one face, of edge ℓ. $\quad P = P_0 + \rho g h$

The force on the face is $F = PA = P_0 A + \rho g \ell^2 h$

It increases in time at the rate
$dF/dt = 0 + \rho g \ell^2\, dh/dt = (1030\text{ kg/m}^3)(9.8\text{ m/s}^2)(0.25\text{ m})^2(1.9\text{ m/s}) = \boxed{1.20 \times 10^3\text{ N/s}}$

(b) $B = \rho V g$ is constant as both the force on the top and the bottom of the block increase together. The rate of change is $\boxed{\text{zero}}$.

P14.26 Consider spherical balloons of radius 12.5 cm containing helium at STP and immersed in air at 0°C and 1 atm. If the rubber envelope has mass 5.00 g, the upward force on each is

$$B - F_{g,\mathrm{He}} - F_{g,env} = \rho_{\mathrm{air}} V g - \rho_{\mathrm{He}} V g - m_{env} g$$

$$F_{up} = \left(\rho_{\mathrm{air}} - \rho_{\mathrm{He}}\right)\left(\frac{4}{3}\pi r^3\right) g - m_{env} g$$

$$F_{up} = \left[(1.29 - 0.179)\ \mathrm{kg/m^3}\right]\left[\frac{4}{3}\pi (0.125\ \mathrm{m})^3\right]\left(9.80\ \mathrm{m/s^2}\right) - 5.00\times 10^{-3}\ \mathrm{kg}\left(9.80\ \mathrm{m/s^2}\right)$$

$$= 0.040\,1\ \mathrm{N}$$

If your weight (including harness, strings, and submarine sandwich) is

$$70.0\ \mathrm{kg}\left(9.80\ \mathrm{m/s^2}\right) = 686\ \mathrm{N}$$

you need this many balloons: $\dfrac{686\ \mathrm{N}}{0.040\,1\ \mathrm{N}} = 17\,000\ \boxed{\sim 10^4}$

P14.27 (a) According to Archimedes, $B = \rho_{\mathrm{water}} V_{\mathrm{water}} g = \left(1.00\ \mathrm{g/cm^3}\right)\left[20.0\times 20.0\times(20.0-h)\right] g$

But $B = \text{Weight of block} = mg = \rho_{\mathrm{wood}} V_{\mathrm{wood}} g = \left(0.650\ \mathrm{g/cm^3}\right)(20.0\ \mathrm{cm})^3\, g$

$$0.650(20.0)^3\, g = 1.00(20.0)(20.0)(20.0-h)\, g$$

$$20.0 - h = 20.0(0.650) \text{ so } h = 20.0(1-0.650) = \boxed{7.00\ \mathrm{cm}}$$

(b) $B = F_g + Mg$ where M = mass of lead

$$1.00(20.0)^3\, g = 0.650(20.0)^3\, g + Mg$$

$$M = (1.00-0.650)(20.0)^3 = 0.350(20.0)^3 = 2\,800\ \mathrm{g} = \boxed{2.80\ \mathrm{kg}}$$

P14.28 (a) The weight of the ball must be equal to the buoyant force of the water:

$$1.26\ \mathrm{kg}\ g = \rho_{\mathrm{water}} \frac{4}{3}\pi r_{\mathrm{outer}}^3 g$$

$$r_{\mathrm{outer}} = \left(\frac{3\times 1.26\ \mathrm{kg}}{4\pi\ 1\,000\ \mathrm{kg/m^3}}\right)^{1/3} = \boxed{6.70\ \mathrm{cm}}$$

(b) The mass of the ball is determined by the density of aluminum:

$$m = \rho_{\mathrm{Al}} V = \rho_{\mathrm{Al}}\left(\frac{4}{3}\pi r_0^3 - \frac{4}{3}\pi r_i^3\right)$$

$$1.26\ \mathrm{kg} = 2\,700\ \mathrm{kg/m^3}\left(\frac{4}{3}\pi\right)\left((0.067\ \mathrm{m})^3 - r_i^3\right)$$

$$1.11\times 10^{-4}\ \mathrm{m^3} = 3.01\times 10^{-4}\ \mathrm{m^3} - r_i^3$$

$$r_i = \left(1.89\times 10^{-4}\ \mathrm{m^3}\right)^{1/3} = \boxed{5.74\ \mathrm{cm}}$$

P14.29 Let A represent the horizontal cross-sectional area of the rod, which we presume to be constant. The rod is in equilibrium:

$$\sum F_y = 0: \qquad -mg + B = 0 = -\rho_0 V_{\text{whole rod}} g + \rho_{\text{fluid}} V_{\text{immersed}} g$$

$$\rho_0 ALg = \rho A(L-h)g$$

The density of the liquid is $\rho = \dfrac{\rho_0 L}{L-h}$

P14.30 We use the result of Problem 14.29. For the rod floating in a liquid of density 0.98 g/cm^3,

$$\rho = \rho_0 \frac{L}{L-h}$$

$$0.98 \text{ g/cm}^3 = \frac{\rho_0 L}{(L - 0.2 \text{ cm})}$$

$$0.98 \text{ g/cm}^3\, L - \left(0.98 \text{ g/cm}^3\right)0.2 \text{ cm} = \rho_0 L$$

For floating in the dense liquid,

$$1.14 \text{ g/cm}^3 = \frac{\rho_0 L}{(L - 1.8 \text{ cm})}$$

$$1.14 \text{ g/cm}^3 - \left(1.14 \text{ g/cm}^3\right)1.8 \text{ cm} = \rho_0 L$$

(a) By substitution,

$$1.14L - 1.14(1.8 \text{ cm}) = 0.98L - 0.2(0.98)$$

$$0.16L = 1.856 \text{ cm}$$

$$L = \boxed{11.6 \text{ cm}}$$

(b) Substituting back,

$$0.98 \text{ g/cm}^3 (11.6 \text{ cm} - 0.2 \text{ cm}) = \rho_0 11.6 \text{ cm}$$

$$\rho_0 = \boxed{0.963 \text{ g/cm}^3}$$

(c) The marks are not equally spaced. Because $\rho = \dfrac{\rho_0 L}{L-h}$ is not of the form $\rho = a + bh$, equal-size steps of ρ do not correspond to equal-size steps of h. The number 1.06 is halfway between 0.98 and 1.14 but the mark for that density is 0.0604 cm below the geometric halfway point between the ends of the scale. The marks get closer together as you go down.

P14.31 The balloon stops rising when $\left(\rho_{\text{air}} - \rho_{\text{He}}\right)gV = Mg$ and $\left(\rho_{\text{air}} - \rho_{\text{He}}\right)V = M$

Therefore,

$$V = \frac{M}{\rho_{\text{air}} - \rho_{\text{He}}} = \frac{400}{1.25e^{-1} - 0.180} \qquad V = \boxed{1\,430 \text{ m}^3}$$

P14.32 Constant velocity implies zero acceleration, which means that the submersible is in equilibrium under the gravitational force, the upward buoyant force, and the upward resistance force:

$$\sum F_y = ma_y = 0 \qquad -(1.20\times10^4\ \text{kg}+m)g+\rho_w gV+1\,100\ \text{N}=0$$

where m is the mass of the added water and V is the sphere's volume.

$$1.20\times10^4\ \text{kg}+m=1.03\times10^3\left[\frac{4}{3}\pi(1.50)^3\right]+\frac{1\,100\ \text{N}}{9.8\ \text{m/s}^2}$$

so

$$m=\boxed{2.67\times10^3\ \text{kg}}$$

P14.33 $B=F_g$

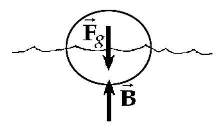

FIG. P14.33

$$\rho_{H_2O}g\frac{V}{2}=\rho_{\text{sphere}}gV$$

$$\rho_{\text{sphere}}=\frac{1}{2}\rho_{H_2O}=\boxed{500\ \text{kg/m}^3}$$

$$\rho_{\text{glycerin}}g\left(\frac{4}{10}V\right)-\rho_{\text{sphere}}gV=0$$

$$\rho_{\text{glycerin}}=\frac{10}{4}(500\ \text{kg/m}^3)=\boxed{1\,250\ \text{kg/m}^3}$$

P14.34 By Archimedes's principle, the weight of the fifty planes is equal to the weight of a horizontal slice of water 11.0 cm thick and circumscribed by the water line:

$$\Delta B=\rho_{\text{water}}g(\Delta V)$$
$$50(2.90\times10^4\ \text{kg})g=(1\,030\ \text{kg/m}^3)g(0.110\ \text{m})A$$

giving $A=\boxed{1.28\times10^4\ \text{m}^2}$. The acceleration of gravity does not affect the answer.

Section 14.5 **Fluid Dynamics**

Section 14.6 **Bernoulli's Equation**

P14.35 Assuming the top is open to the atmosphere, then

$$P_1=P_0$$

Note $P_2=P_0$. The water pushes on the air just as hard as the air pushes on the water.

Flow rate $=2.50\times10^{-3}\ \text{m}^3/\text{min}=4.17\times10^{-5}\ \text{m}^3/\text{s}$.

(a) $A_1 >> A_2$ so $v_1 << v_2$

Assuming $v_1=0$,

$$P_1+\frac{\rho v_1^2}{2}+\rho g y_1=P_2+\frac{\rho v_2^2}{2}+\rho g y_2$$

$$v_2=(2gy_1)^{1/2}=[2(9.80)(16.0)]^{1/2}=\boxed{17.7\ \text{m/s}}$$

(b) Flow rate $=A_2v_2=\left(\dfrac{\pi d^2}{4}\right)(17.7)=4.17\times10^{-5}\ \text{m}^3/\text{s}$

$$d=\boxed{1.73\times10^{-3}\ \text{m}}=1.73\ \text{mm}$$

P14.36 Take point ① at the free surface of the water in the tank and ② inside the nozzle.

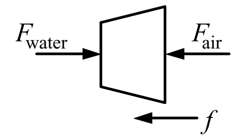

FIG. P14.36

(a) With the cork in place $P_1+\rho g y_1+\frac{1}{2}\rho v_1^2=P_2+\rho g y_2+\frac{1}{2}\rho v_2^2$ becomes

$$P_0+1\,000\ \text{kg/m}^3\ 9.8\ \text{m/s}^2\ 7.5\ \text{m}+0=P_2+0+0\,;\ P_2-P_0=7.35\times10^4\ \text{Pa}$$

For the stopper $\sum F_x=0$

$$F_{\text{water}}-F_{\text{air}}-f=0$$
$$P_2A-P_0A=f$$
$$f=7.35\times10^4\ \text{Pa}\pi(0.011\ \text{m})^2=\boxed{27.9\ \text{N}}$$

(b) Now Bernoulli's equation gives

$$P_0+7.35\times10^4\ \text{Pa}+0=P_0+0+\frac{1}{2}\left(1\,000\ \text{kg/m}^3\right)v_2^2$$
$$v_2=12.1\ \text{m/s}$$

The quantity leaving the nozzle in 2 h is

$$\rho V=\rho A v_2 t=\left(1\,000\ \text{kg/m}^3\right)\pi(0.011\ \text{m})^2(12.1\ \text{m/s})7\,200\ \text{s}=\boxed{3.32\times10^4\ \text{kg}}$$

(c) Take point 1 in the wide hose and 2 just outside the nozzle. Continuity:

$$A_1v_1=A_2v_2$$
$$\pi\left(\frac{6.6\ \text{cm}}{2}\right)^2v_1=\pi\left(\frac{2.2\ \text{cm}}{2}\right)^2 12.1\ \text{m/s}$$
$$v_1=\frac{12.1\ \text{m/s}}{9}=1.35\ \text{m/s}$$
$$P_1+\rho g y_1+\frac{1}{2}\rho v_1^2=P_2+\rho g y_2+\frac{1}{2}\rho v_2^2$$
$$P_1+0+\frac{1}{2}\left(1\,000\ \text{kg/m}^3\right)(1.35\ \text{m/s})^2=P_0+0+\frac{1}{2}\left(1\,000\ \text{kg/m}^3\right)(12.1\ \text{m/s})^2$$
$$P_1-P_0=7.35\times10^4\ \text{Pa}-9.07\times10^2\ \text{Pa}=\boxed{7.26\times10^4\ \text{Pa}}$$

P14.37 Flow rate $Q=0.012\,0\ \text{m}^3/\text{s}=v_2A_2$

$$v_2=\frac{Q}{A_2}=\frac{0.012\,0\ \text{m}^3/\text{s}}{\pi(0.011\ \text{m})^2}=\boxed{31.6\ \text{m/s}}$$

***P14.38** (a) The mass flow rate and the volume flow rate are constant:

$$\rho A_1 v_1 = \rho A_2 v_2 \qquad \pi r_1^2 v_1 = \pi r_2^2 v_2 \qquad (4\text{ cm})^2 v_1 = (2\text{ cm})^2 v_2 \qquad v_2 = 4v_1$$

For ideal flow

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$2.5\times10^4\text{ Pa} + 0 + \frac{1}{2}\left(1\,000\text{ kg/m}^3\right)\left(v_1\right)^2$$

$$= 1.5\times10^4\text{ Pa} + (1000)(9.8)(0.5)\text{ Pa} + \frac{1}{2}\left(1\,000\text{ kg/m}^3\right)(4v_1)^2$$

$$v_1 = \sqrt{\frac{5100\text{ Pa}}{7500\text{ kg/m}^3}} = \boxed{0.825\text{ m/s}}$$

(b) $v_2 = 4v_1 = \boxed{3.30\text{ m/s}}$

(c) $\pi r_1^2 v_1 = \pi(0.04\text{ m})^2(0.825\text{ m/s}) = \boxed{4.14\times10^{-3}\text{ m}^3/\text{s}}$

P14.39 The volume flow rate is

$$\frac{125\text{ cm}^3}{16.3\text{ s}} = Av_1 = \pi\left(\frac{0.96\text{ cm}}{2}\right)^2 v_1$$

The speed at the top of the falling column is

$$v_1 = \frac{7.67\text{ cm}^3/\text{s}}{0.724\text{ cm}^2} = 10.6\text{ cm/s}$$

Take point 2 at 13 cm below:

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$P_0 + \left(1\,000\text{ kg/m}^3\right)\left(9.8\text{ m/s}^2\right)0.13\text{ m} + \frac{1}{2}\left(1\,000\text{ kg/m}^3\right)\left(0.106\text{ m/s}\right)^2$$

$$= P_0 + 0 + \frac{1}{2}\left(1\,000\text{ kg/m}^3\right)v_2^2$$

$$v_2 = \sqrt{2\left(9.8\text{ m/s}^2\right)0.13\text{ m} + \left(0.106\text{ m/s}\right)^2} = 1.60\text{ m/s}$$

The volume flow rate is constant:

$$7.67\text{ cm}^3/\text{s} = \pi\left(\frac{d}{2}\right)^2 160\text{ cm/s}$$

$$d = \boxed{0.247\text{ cm}}$$

P14.40 (a) $\mathcal{P} = \dfrac{\Delta E}{\Delta t} = \dfrac{\Delta mgh}{\Delta t} = \left(\dfrac{\Delta m}{\Delta t}\right)gh = Rgh$

(b) $\mathcal{P}_{\text{EL}} = 0.85\left(8.5\times10^5\right)(9.8)(87) = \boxed{616\text{ MW}}$

P14.41 (a) Between sea surface and clogged hole: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$

$1\text{ atm} + 0 + (1\,030\text{ kg/m}^3)(9.8\text{ m/s}^2)(2\text{ m}) = P_2 + 0 + 0 \quad P_2 = 1\text{ atm} + 20.2\text{ kPa}$

The air on the back of his hand pushes opposite the water, so the net force on his hand is

$F = PA = (20.2\times10^3\text{ N/m}^2)\left(\frac{\pi}{4}\right)(1.2\times10^{-2}\text{ m})^2 \qquad F = \boxed{2.28\text{ N}}$ toward Holland

(b) Now, Bernoulli's theorem is

$1\text{ atm} + 0 + 20.2\text{ kPa} = 1\text{ atm} + \frac{1}{2}(1\,030\text{ kg/m}^3)v_2^2 + 0 \qquad v_2 = 6.26\text{ m/s}$

The volume rate of flow is $A_2 v_2 = \frac{\pi}{4}(1.2\times10^{-2}\text{ m})^2(6.26\text{ m/s}) = 7.08\times10^{-4}\text{ m}^3\text{/s}$

One acre–foot is $4\,047\text{ m}^2 \times 0.304\,8\text{ m} = 1\,234\text{ m}^3$

Requiring $\frac{1\,234\text{ m}^3}{7.08\times10^{-4}\text{ m}^3\text{/s}} = \boxed{1.74\times10^6\text{ s}} = 20.2\text{ days}$

***P14.42** (a) The volume flow rate is the same at the two points: $A_1 v_1 = A_2 v_2$

$\pi(1\text{ cm})^2 v_1 = \pi(0.5\text{ cm})^2 v_2 \qquad v_2 = 4v_1$

We assume the tubes are at the same elevation:

$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$

$P_1 - P_2 = \Delta P = \frac{1}{2}\rho(4v_1)^2 + 0 - \frac{1}{2}\rho v_1^2$

$\Delta P = \frac{1}{2}(850\text{ kg/m}^3)\,15v_1^2$

$v_1 = (0.0125\text{ m/s})\sqrt{\Delta P}$ where the pressure is in Pascals

The volume flow rate is $\pi(0.01\text{ m})^2(0.0125\text{ m/s})\sqrt{\Delta P}$

$= \boxed{(3.93\times10^{-6}\text{ m}^3\text{/s})\sqrt{\Delta P}\quad \text{where } \Delta P \text{ is in pascals}}$

(b) $(3.93\times10^{-6}\text{ m}^3\text{/s})\sqrt{6000} = \boxed{0.305\text{ L/s}}$

(c) With pressure difference 2 times larger, the flow rate is larger by the square root of 2 times: $(2)^{1/2}(0.305\text{ L/s}) = \boxed{0.431\text{ L/s}}$

(d) The flow rate is proportional to the square root of the pressure difference.

P14.43 (a) Suppose the flow is very slow: $\left(P+\frac{1}{2}\rho v^2+\rho g y\right)_{\text{river}}=\left(P+\frac{1}{2}\rho v^2+\rho g y\right)_{\text{rim}}$

$$P+0+\rho g(564\text{ m})=1\text{ atm}+0+\rho g(2\,096\text{ m})$$

$$P=1\text{ atm}+\left(1\,000\text{ kg/m}^3\right)\left(9.8\text{ m/s}^2\right)(1\,532\text{ m})=\boxed{1\text{ atm}+15.0\text{ MPa}}$$

(b) The volume flow rate is $4\,500\text{ m}^3/\text{d}=Av=\dfrac{\pi d^2 v}{4}$

$$v=\left(4\,500\text{ m}^3/\text{d}\right)\left(\frac{1\text{ d}}{86\,400\text{ s}}\right)\left(\frac{4}{\pi(0.150\text{ m})^2}\right)=\boxed{2.95\text{ m/s}}$$

(c) Imagine the pressure as applied to stationary water at the bottom of the pipe:

$$\left(P+\frac{1}{2}\rho v^2+\rho g y\right)_{\text{bottom}}=\left(P+\frac{1}{2}\rho v^2+\rho g y\right)_{\text{top}}$$

$$P+0=1\text{ atm}+\frac{1}{2}\left(1\,000\text{ kg/m}^3\right)(2.95\text{ m/s})^2+1\,000\text{ kg}\left(9.8\text{ m/s}^2\right)(1\,532\text{ m})$$

$$P=1\text{ atm}+15.0\text{ MPa}+4.34\text{ kPa}$$

The additional pressure is $\boxed{4.34\text{ kPa}}$.

***P14.44** (a) For upward flight of a water-drop projectile from geyser vent to fountain-top, $v_{yf}^2=v_{yi}^2+2a_y\Delta y$

Then $0=v_i^2+2\left(-9.80\text{ m/s}^2\right)(+40.0\text{ m})$ and $v_i=\boxed{28.0\text{ m/s}}$

(b) Between geyser vent and fountain-top: $P_1+\frac{1}{2}\rho v_1^2+\rho g y_1=P_2+\frac{1}{2}\rho v_2^2+\rho g y_2$

Air is so low in density that very nearly $P_1=P_2=1\text{ atm}$

Then, $\frac{1}{2}v_i^2+0=0+\left(9.80\text{ m/s}^2\right)(40.0\text{ m})$

$v_1=\boxed{28.0\text{ m/s}}$

(c) The answers agree precisely. The models are consistent with each other.

(d) Between the chamber and the fountain-top: $P_1+\frac{1}{2}\rho v_1^2+\rho g y_1=P_2+\frac{1}{2}\rho v_2^2+\rho g y_2$

$$P_1+0+\left(1\,000\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)(-175\text{ m})$$

$$=P_0+0+\left(1\,000\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)(+40.0\text{ m})$$

$$P_1-P_0=\left(1\,000\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)(215\text{ m})=\boxed{2.11\text{ MPa}}$$

P14.45 $P_1+\dfrac{\rho v_1^2}{2}=P_2+\dfrac{\rho_2^2}{2}$ (Bernoulli equation), $v_1A_1=v_2A_2$ where $\dfrac{A_1}{A_2}=4$

$$\Delta P=P_1-P_2=\frac{\rho}{2}\left(v_2^2-v_1^2\right)=\frac{\rho}{2}v_1^2\left(\frac{A_1^2}{A_2^2}-1\right)\text{ and }\Delta P=\frac{\rho v_1^2}{2}15=21\,000\text{ Pa}$$

$v_1=2.00\text{ m/s};\ v_2=4v_1=8.00\text{ m/s}:$

The volume flow rate is $v_1A_1=\boxed{2.51\times10^{-3}\text{ m}^3/\text{s}}$

Section 14.7 **Other Applications of Fluid Dynamics**

P14.46 $Mg = (P_1 - P_2)A$ for a balanced condition $\dfrac{16\,000(9.80)}{A} = 7.00\times10^4 - P_2$

where $A = 80.0\text{ m}^2$ $\therefore P_2 = 7.0\times10^4 - 0.196\times10^4 = \boxed{6.80\times10^4\text{ Pa}}$

P14.47 (a) $P_0 + \rho g h + 0 = P_0 + 0 + \dfrac{1}{2}\rho v_3^2$ $v_3 = \sqrt{2gh}$

If $h = 1.00$ m $v_3 = \boxed{4.43\text{ m/s}}$

(b) $P + \rho g y + \dfrac{1}{2}\rho v_2^2 = P_0 + 0 + \dfrac{1}{2}\rho v_3^2$

Since $v_2 = v_3$ $P = P_0 - \rho g y$

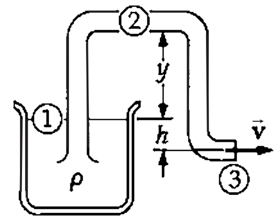

FIG. P14.47

Since $P \ge 0$, the greatest possible siphon height is given by

$$y \le \frac{P_0}{\rho g} = \frac{1.013\times10^5\text{ Pa}}{(10^3\text{ kg/m}^3)(9.8\text{ m/s}^2)} = \boxed{10.3\text{ m}}$$

P14.48 The assumption of incompressibility is surely unrealistic, but allows an estimate of the speed:

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$1.00\text{ atm} + 0 + 0 = 0.287\text{ atm} + 0 + \frac{1}{2}(1.20\text{ kg/m}^3)v_2^2$$

$$v_2 = \sqrt{\frac{2(1.00-0.287)(1.013\times10^5\text{ N/m}^2)}{1.20\text{ kg/m}^3}} = \boxed{347\text{ m/s}}$$

P14.49 In the reservoir, the gauge pressure is $\Delta P = \dfrac{2.00\text{ N}}{2.50\times10^{-5}\text{ m}^2} = 8.00\times10^4\text{ Pa}$

From the equation of continuity: $A_1 v_1 = A_2 v_2$

$(2.50\times10^{-5}\text{ m}^2)v_1 = (1.00\times10^{-8}\text{ m}^2)v_2$ $v_1 = (4.00\times10^{-4})v_2$

Thus, v_1^2 is negligible in comparison to v_2^2.

Then, from Bernoulli's equation: $(P_1 - P_2) + \dfrac{1}{2}\rho v_1^2 + \rho g y_1 = \dfrac{1}{2}\rho v_2^2 + \rho g y_2$

$$8.00\times10^4\text{ Pa} + 0 + 0 = 0 + \frac{1}{2}(1\,000\text{ kg/m}^3)v_2^2$$

$$v_2 = \sqrt{\frac{2(8.00\times10^4\text{ Pa})}{1\,000\text{ kg/m}^3}} = \boxed{12.6\text{ m/s}}$$

P14.50 Take points 1 and 2 in the air just inside and outside the window pane.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$P_0 + 0 = P_2 + \frac{1}{2}\left(1.30\ \text{kg/m}^3\right)\left(11.2\ \text{m/s}\right)^2 \qquad P_2 = P_0 - 81.5\ \text{Pa}$$

(a) The total force exerted by the air is outward,

$$P_1A - P_2A = P_0A - P_0A + \left(81.5\ \text{N/m}^2\right)(4\ \text{m})(1.5\ \text{m}) = \boxed{489\ \text{N outward}}$$

(b) $$P_1A - P_2A = \frac{1}{2}\rho v_2^2 A = \frac{1}{2}\left(1.30\ \text{kg/m}^3\right)\left(22.4\ \text{m/s}\right)^2(4\ \text{m})(1.5\ \text{m}) = \boxed{1.96\ \text{kN outward}}$$

Additional Problems

P14.51 When the balloon comes into equilibrium, we must have

$$\sum F_y = B - F_{g,\text{ balloon}} - F_{g,\text{ He}} - F_{g,\text{ string}} = 0$$

$F_{g,\text{ string}}$ is the weight of the string above the ground, and B is the buoyant force. Now

$$F_{g,\text{ balloon}} = m_{\text{balloon}}g$$
$$F_{g,\text{ He}} = \rho_{\text{He}}Vg$$
$$B = \rho_{\text{air}}Vg$$

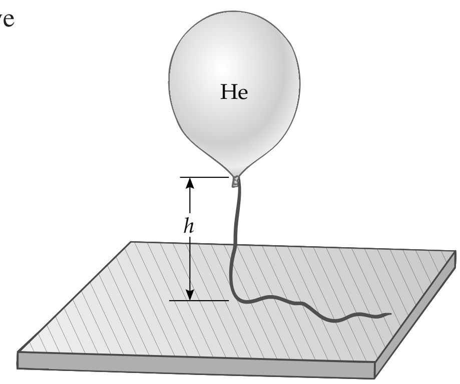

FIG. P14.51

and

$$F_{g,\text{ string}} = m_{\text{string}}\frac{h}{L}g$$

Therefore, we have

$$\rho_{\text{air}}Vg - m_{\text{balloon}}g - \rho_{\text{He}}Vg - m_{\text{string}}\frac{h}{L}g = 0$$

or

$$h = \frac{\left(\rho_{\text{air}} - \rho_{\text{He}}\right)V - m_{\text{balloon}}}{m_{\text{string}}}L$$

giving

$$h = \frac{(1.29 - 0.179)\left(\text{kg/m}^3\right)\left(4\pi(0.400\ \text{m})^3/3\right) - 0.250\ \text{kg}}{0.050\,0\ \text{kg}}(2.00\ \text{m}) = \boxed{1.91\ \text{m}}$$

P14.52 Consider the diagram and apply Bernoulli's equation to points A and B, taking $y = 0$ at the level of point B, and recognizing that v_A is approximately zero. This gives:

$$P_A + \frac{1}{2}\rho_w(0)^2 + \rho_w g(h - L\sin\theta)$$
$$= P_B + \frac{1}{2}\rho_w v_B^2 + \rho_w g(0)$$

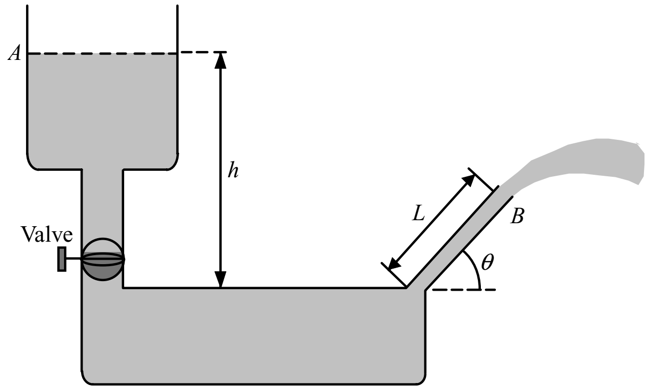

FIG. P14.52

Now, recognize that $P_A = P_B = P_{\text{atmosphere}}$ since both points are open to the atmosphere (neglecting variation of atmospheric pressure with altitude). Thus, we obtain

$$v_B = \sqrt{2g(h - L\sin\theta)} = \sqrt{2\left(9.80\ \text{m/s}^2\right)\left[10.0\ \text{m} - (2.00\ \text{m})\sin 30.0°\right]}$$
$$v_B = 13.3\ \text{m/s}$$

Now the problem reduces to one of projectile motion with $v_{yi} = v_B \sin 30.0° = 6.64\ \text{m/s}$. Then, $v_{yf}^2 = v_{yi}^2 + 2a(\Delta y)$ gives at the top of the arc (where $y = y_{\max}$ and $v_{yf} = 0$)

$$0 = (6.64\ \text{m/s})^2 + 2\left(-9.80\ \text{m/s}^2\right)(y_{\max} - 0)$$

or $y_{\max} = \boxed{2.25\ \text{m (above the level where the water emerges)}}$.

P14.53 The "balanced" condition is one in which the apparent weight of the body equals the apparent weight of the weights. This condition can be written as:

$$F_g - B = F_g' - B'$$

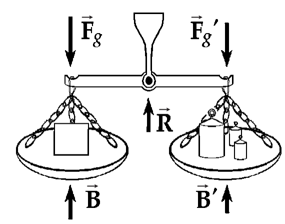

FIG. P14.53

where B and B' are the buoyant forces on the body and weights respectively. The buoyant force experienced by an object of volume V in air equals:

$$\text{Buoyant force} = (\text{Volume of object})\rho_{\text{air}} g$$

so we have $\quad B = V\rho_{\text{air}} g \quad$ and $\quad B' = \left(\frac{F_g'}{\rho g}\right)\rho_{\text{air}} g$

Therefore,

$$F_g = F_g' + \left(V - \frac{F_g'}{\rho g}\right)\rho_{\text{air}} g$$

P14.54 Assume $v_{\text{inside}} \approx 0$ From Bernoulli's equation,

$$P + 0 + 0 = 1\ \text{atm} + \frac{1}{2}(1\,000)(30.0)^2 + 1\,000(9.80)(0.500)$$
$$P_{\text{gauge}} = P - 1\ \text{atm} = 4.50\times10^5 + 4.90\times10^3 = \boxed{455\ \text{kPa}}$$

P14.55 At equilibrium, $\sum F_y = 0$: $B - F_{\text{spring}} - F_{g,\,\text{He}} - F_{g,\,\text{balloon}} = 0$

giving $F_{\text{spring}} = kL = B - (m_{\text{He}} + m_{\text{balloon}})g$

But $B = \text{weight of displaced air} = \rho_{\text{air}} Vg$

and $m_{\text{He}} = \rho_{\text{He}} V$

Therefore, we have: $kL = \rho_{\text{air}} Vg - \rho_{\text{He}} Vg - m_{\text{balloon}} g$

or $$L = \frac{(\rho_{\text{air}} - \rho_{\text{He}})V - m_{\text{balloon}}}{k} g$$

From the data given, $$L = \frac{(1.29\ \text{kg/m}^3 - 0.180\ \text{kg/m}^3)5.00\ \text{m}^3 - 2.00 \times 10^{-3}\ \text{kg}}{90.0\ \text{N/m}}(9.80\ \text{m/s}^2)$$

Thus, this gives $L = \boxed{0.604\ \text{m}}$

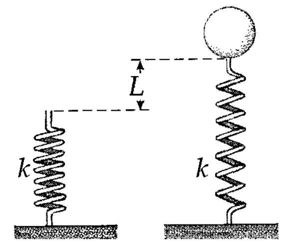

FIG. P14.55

***P14.56** Let the ball be released at point 1, enter the liquid at point 2, attain maximum depth at point 3, and pop through the surface on the way up at point 4.

(a) Energy conservation for the fall through the air:

$0 + mgy_1 = (1/2)mv_2^2$
$v_2 = (2gy_1)^{1/2} = [2(9.8)(3.3)]^{1/2} = \boxed{8.04\ \text{m/s}}$

(b) $\boxed{\text{The gravitational force and the buoyant force.}}$
The gravitational force is $mg = (2.1\ \text{kg})(9.8\ \text{N/kg}) = 20.6\ \text{N}$ down and the buoyant force is

$m_{fluid}g = \rho_{fluid}V_{object}g = \rho_{\text{fluid}}(4/3)\pi r^3 g = (1230\ \text{kg/m}^3)(4\pi/3)(0.09\ \text{m})^3(9.8\ \text{m/s}^2) = 36.8\ \text{N up}.$

(c) The buoyant force is greater than the gravitational force. $\boxed{\text{The net upward force on the ball brings its downward motion to a stop.}}$
We choose to use the work-kinetic energy theorem.

$(1/2)mv_2^2 + F_{net} \cdot \Delta y = (1/2)mv_3^2$
$(1/2)(2.1\ \text{kg})(8.04\ \text{m/s})^2 + (36.8\ \text{N} - 20.6\ \text{N})(-\Delta y) = 0$
$\Delta y = 67.9\ \text{J}/16.2\ \text{N} = \boxed{4.18\ \text{m}}$

(d) The same net force acts on the ball over the same distance as it moves down and as it moves up, to produce the same speed change. Thus $v_4 = \boxed{8.04\ \text{m/s}}$.

(e) $\boxed{\text{The time intervals are equal}}$, because the ball moves with the same range of speeds over equal distance intervals.

(f) With friction present, Δt_{down} is less than Δt_{up}. The magnitude of the ball's acceleration on the way down is greater than its acceleration on the way up. The two motions cover equal distances and both have zero speed at one end point, so the downward trip with larger-magnitude acceleration must take less time.

***P14.57** First consider a hovering rocket that creates the gas it blows downward. The impulse-momentum theorem is

$$F_g \Delta t = \Delta(mv) \qquad F_g = v\, dm/dt \qquad dm/dt = 950 \text{ kg}(9.8 \text{ N/kg})/(40 \text{ m/s}) = 233 \text{ kg/s}$$

If the helicopter could create the air it expels downward, the mass flow rate of the air would have to be at least 233 kg/s. Really the rotor takes in air from above, moving over a larger area with lower speed, and blows it downward at higher speed. The incoming air from above brings momentum with it, so the mass flow rate must be a few times larger than 233 kg every second, or more.

P14.58 $P = \rho g h \qquad 1.013\times10^5 = 1.29(9.80)h$

$h = \boxed{8.01 \text{ km}}$ For Mt. Everest, $29\,300 \text{ ft} = 8.88 \text{ km}$ $\boxed{\text{Yes}}$

P14.59 The torque is $\tau = \int d\tau = \int r dF$

From the figure $\tau = \int_0^H y\left[\rho g(H-y) w dy\right] = \boxed{\frac{1}{6}\rho g w H^3}$

The total force is given as $\frac{1}{2}\rho g w H^2$

If this were applied at a height y_{eff} such that the torque remains unchanged, we have

$$\frac{1}{6}\rho g w H^3 = y_{eff}\left[\frac{1}{2}\rho g w H^2\right] \quad \text{and} \quad y_{eff} = \boxed{\frac{1}{3}H}$$

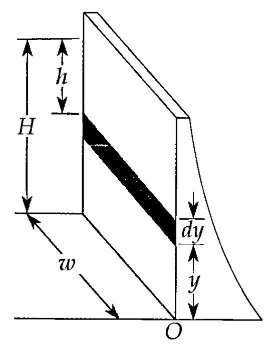

FIG. P14.59

P14.60 (a) The pressure on the surface of the two hemispheres is constant at all points, and the force on each element of surface area is directed along the radius of the hemispheres. The applied force along the axis must balance the force on the "effective" area, which is the projection of the actual surface onto a plane perpendicular to the *x* axis,

$$A = \pi R^2$$

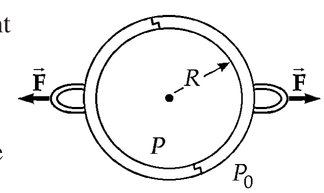

FIG. P14.60

Therefore,

$$F = \boxed{(P_0 - P)\pi R^2}$$

(b) For the values given $F = (P_0 - 0.100P_0)\left[\pi(0.300 \text{ m})^2\right] = 0.254P_0 = \boxed{2.58\times10^4 \text{ N}}$

P14.61 Looking first at the top scale and the iron block, we have:

$$T_1 + B = F_{g,\text{ iron}}$$

where T_1 is the tension in the spring scale, B is the buoyant force, and $F_{g,\text{ iron}}$ is the weight of the iron block. Now if m_{iron} is the mass of the iron block, we have

$$m_{\text{iron}} = \rho_{\text{iron}} V \qquad \text{so} \qquad V = \frac{m_{\text{iron}}}{\rho_{\text{iron}}} = V_{\text{displaced oil}}$$

Then,

$$B = \rho_{\text{oil}} V_{\text{iron}} g$$

Therefore,

$$T_1 = F_{g,\text{ iron}} - \rho_{\text{oil}} V_{\text{iron}} g = m_{\text{iron}} g - \rho_{\text{oil}} \frac{m_{\text{iron}}}{\rho_{\text{iron}}} g$$

or

$$T_1 = \left(1 - \frac{\rho_{\text{oil}}}{\rho_{\text{iron}}}\right) m_{\text{iron}} g = \left(1 - \frac{916}{7\,860}\right)(2.00)(9.80) = \boxed{17.3 \text{ N}}$$

Next, we look at the bottom scale which reads T_2 (i.e., exerts an upward force T_2 on the system). Consider the external vertical forces acting on the beaker–oil–iron combination.

$\sum F_y = 0$ gives

$T_1 + T_2 - F_{g,\text{ beaker}} - F_{g,\text{ oil}} - F_{g,\text{ iron}} = 0$

or

$$T_2 = (m_{\text{beaker}} + m_{\text{oil}} + m_{\text{iron}}) g - T_1 = (5.00 \text{ kg})(9.80 \text{ m/s}^2) - 17.3 \text{ N}$$

Thus, $T_2 = \boxed{31.7 \text{ N}}$ is the lower scale reading.

P14.62 Looking at the top scale and the iron block:

$$T_1 + B = F_{g,\text{ Fe}} \qquad \text{where} \qquad B = \rho_0 V_{\text{Fe}} g = \rho_0 \left(\frac{m_{\text{Fe}}}{\rho_{\text{Fe}}}\right) g$$

is the buoyant force exerted on the iron block by the oil.

Thus, $$T_1 = F_{g,\text{ Fe}} - B = m_{\text{Fe}} g - \rho_0 \left(\frac{m_{\text{Fe}}}{\rho_{\text{Fe}}}\right) g$$

or $$T_1 = \boxed{\left(1 - \frac{\rho_0}{\rho_{\text{Fe}}}\right) m_{\text{Fe}} g}$$ is the reading on the top scale.

Now, consider the bottom scale, which exerts an upward force of T_2 on the beaker–oil–iron combination.

$$\sum F_y = 0: \qquad T_1 + T_2 - F_{g,\text{ beaker}} - F_{g,\text{ oil}} - F_{g,\text{ Fe}} = 0$$

$$T_2 = F_{g,\text{ beaker}} + F_{g,\text{ oil}} + F_{g,\text{ Fe}} - T_1 = (m_b + m_0 + m_{\text{Fe}}) g - \left(1 - \frac{\rho_0}{\rho_{\text{Fe}}}\right) m_{\text{Fe}} g$$

or $$T_2 = \boxed{\left[m_b + m_0 + \left(\frac{\rho_0}{\rho_{\text{Fe}}}\right) m_{\text{Fe}}\right] g}$$ is the reading on the bottom scale.

P14.63 $\rho_{Cu}V = 3.083$ g

$\rho_{Zn}(xV) + \rho_{Cu}(1-x)V = 2.517$ g

$$\rho_{Zn}\left(\frac{3.083}{\rho_{Cu}}\right)x + 3.083(1-x) = 2.517$$

$$\left(1-\frac{7.133}{8.960}\right)x = \left(1-\frac{2.517}{3.083}\right)$$

$x = 0.900\,4$

$\%\text{Zn} = \boxed{90.04\%}$

P14.64 The incremental version of $P - P_0 = \rho g y$ is $\quad dP = -\rho g dy$

We assume that the density of air is proportional to pressure, or $\quad \dfrac{P}{\rho} = \dfrac{P_0}{\rho_0}$

Combining these two equations we have $\quad dP = -P\dfrac{\rho_0}{P_0}gdy$

$$\int_{P_0}^{P}\frac{dP}{P} = -g\frac{\rho_0}{P_0}\int_0^h dy$$

and integrating gives $\quad \ln\left(\dfrac{P}{P_0}\right) = -\dfrac{\rho_0 g h}{P_0}$

so where $\alpha = \dfrac{\rho_0 g}{P_0}$ $\quad P = P_0 e^{-\alpha h}$

P14.65 Inertia of the disk: $I = \frac{1}{2}MR^2 = \frac{1}{2}(10.0\text{ kg})(0.250\text{ m})^2 = 0.312\text{ kg}\cdot\text{m}^2$

Angular acceleration: $\quad \omega_f = \omega_i + \alpha t$

$$\alpha = \left(\frac{0-300\text{ rev/min}}{60.0\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60.0\text{ s}}\right) = -0.524\text{ rad/s}^2$$

Braking torque: $\quad \sum\tau = I\alpha \Rightarrow -fd = I\alpha, \quad \text{so} \quad f = \dfrac{-I\alpha}{d}$

Friction force: $\quad f = \dfrac{(0.312\text{ kg}\cdot\text{m}^2)(0.524\text{ rad/s}^2)}{0.220\text{ m}} = 0.744\text{ N}$

Normal force: $\quad f = \mu_k n \Rightarrow n = \dfrac{f}{\mu_k} = \dfrac{0.744\text{ N}}{0.500} = 1.49\text{ N}$

Gauge pressure: $\quad P = \dfrac{n}{A} = \dfrac{1.49\text{ N}}{\pi(2.50\times10^{-2}\text{ m})^2} = \boxed{758\text{ Pa}}$

P14.66 Let s stand for the edge of the cube, h for the depth of immersion, ρ_{ice} stand for the density of the ice, ρ_w stand for density of water, and ρ_a stand for density of the alcohol.

(a) According to Archimedes's principle, at equilibrium we have

$$\rho_{ice}gs^3 = \rho_w ghs^2 \Rightarrow h = s\frac{\rho_{ice}}{\rho_w}$$

With $\quad \rho_{ice} = 0.917\times10^3\text{ kg/m}^3$

$\quad \rho_w = 1.00\times10^3\text{ kg/m}^3$

and $\quad s = 20.0$ mm

we get $\quad h = 20.0(0.917) = 18.34\text{ mm} \approx \boxed{18.3\text{ mm}}$

continued on next page

(b) We assume that the top of the cube is still above the alcohol surface. Letting h_a stand for the thickness of the alcohol layer, we have

$$\rho_a g s^2 h_a + \rho_w g s^2 h_w = \rho_{\text{ice}} g s^3 \qquad \text{so} \qquad h_w = \left(\frac{\rho_{\text{ice}}}{\rho_w}\right)s - \left(\frac{\rho_a}{\rho_w}\right)h_a$$

With $\rho_a = 0.806 \times 10^3 \text{ kg/m}^3$

and $h_a = 5.00 \text{ mm}$

we obtain $h_w = 18.34 - 0.806(5.00) = 14.31 \text{ mm} \approx \boxed{14.3 \text{ mm}}$

(c) Here $h'_w = s - h'_a$, so Archimedes's principle gives

$$\rho_a g s^2 h'_a + \rho_w g s^2 (s - h'_a) = \rho_{\text{ice}} g s^3 \Rightarrow \rho_a h'_a + \rho_w (s - h'_a) = \rho_{\text{ice}} s$$

$$h'_a = s\frac{(\rho_w - \rho_{\text{ice}})}{(\rho_w - \rho_a)} = 20.0\frac{(1.000 - 0.917)}{(1.000 - 0.806)} = 8.557 \approx \boxed{8.56 \text{ mm}}$$

P14.67 Energy for the fluid-Earth system is conserved.

$$(K+U)_i + \Delta E_{\text{mech}} = (K+U)_f \qquad 0 + \frac{mgL}{2} + 0 = \frac{1}{2}mv^2 + 0$$

$$v = \sqrt{gL} = \sqrt{2.00 \text{ m}(9.8 \text{ m/s}^2)} = \boxed{4.43 \text{ m/s}}$$

P14.68 (a) The flow rate, Av, as given may be expressed as follows:

$$\frac{25.0 \text{ liters}}{30.0 \text{ s}} = 0.833 \text{ liters/s} = 833 \text{ cm}^3/\text{s}$$

The area of the faucet tap is π cm^2, so we can find the velocity as

$$v = \frac{\text{flow rate}}{A} = \frac{833 \text{ cm}^3/\text{s}}{\pi \text{ cm}^2} = 265 \text{ cm/s} = \boxed{2.65 \text{ m/s}}$$

(b) We choose point 1 to be in the entrance pipe and point 2 to be at the faucet tap. $A_1 v_1 = A_2 v_2$ gives $v_1 = 0.295$ m/s. Bernoulli's equation is:

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) + \rho g\left(y_2 - y_1\right)$$

and gives

$$P_1 - P_2 = \frac{1}{2}\left(10^3 \text{ kg/m}^3\right)\left[(2.65 \text{ m/s})^2 - (0.295 \text{ m/s})^2\right] + \left(10^3 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)(2.00 \text{ m})$$

or

$$P_{\text{gauge}} = P_1 - P_2 = \boxed{2.31 \times 10^4 \text{ Pa}}$$

P14.69 **Note:** Variation of atmospheric pressure with altitude is included in this solution. Because of the small distances involved, this effect is unimportant in the final answers.

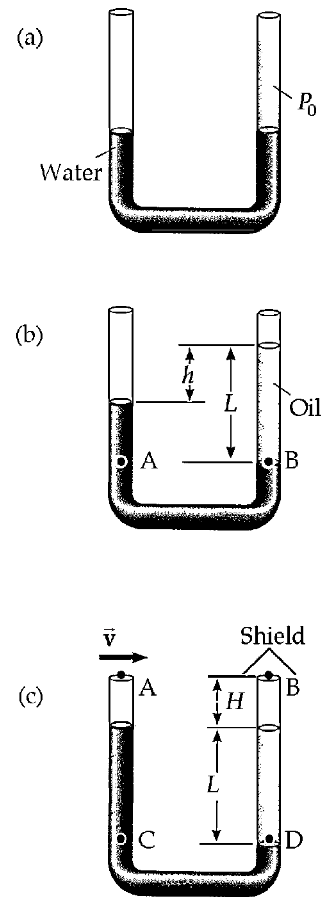

FIG. P14.69

(a) Consider the pressure at points A and B in part (b) of the figure:

Using the left tube: $P_A = P_{\text{atm}} + \rho_a gh + \rho_w g(L-h)$
where the second term is due to the variation of air pressure with altitude.

Using the right tube: $P_B = P_{\text{atm}} + \rho_0 gL$

But Pascal's principle says that $P_A = P_B$.

Therefore, $P_{\text{atm}} + \rho_0 gL = P_{\text{atm}} + \rho_a gh + \rho_w g(L-h)$

or $(\rho_w - \rho_a)h = (\rho_w - \rho_0)L$, giving

$$h = \left(\frac{\rho_w - \rho_0}{\rho_w - \rho_a}\right)L = \left(\frac{1\,000-750}{1\,000-1.29}\right)5.00\text{ cm} = \boxed{1.25\text{ cm}}$$

(b) Consider part (c) of the diagram showing the situation when the air flow over the left tube equalizes the fluid levels in the two tubes. First, apply Bernoulli's equation to points A and B $(y_A = y_B,\ v_A = v,\text{ and } v_B = 0)$

This gives: $P_A + \frac{1}{2}\rho_a v^2 + \rho_a g y_A = P_B + \frac{1}{2}\rho_a(0)^2 + \rho_a g y_B$

and since $y_A = y_B$, this reduces to: $P_B - P_A = \frac{1}{2}\rho_a v^2$ (1)

Now consider points C and D, both at the level of the oil–water interface in the right tube. Using the variation of pressure with depth in static fluids, we have:

$P_C = P_A + \rho_a gH + \rho_w gL$ and $P_D = P_B + \rho_a gH + \rho_0 gL$

But Pascal's principle says that $P_C = P_D$. Equating these two gives:

$P_B + \rho_a gH + \rho_0 gL = P_A + \rho_a gH + \rho_w gL$ or $P_B - P_A = (\rho_w - \rho_0)gL$ (2)

Substitute equation (1) for $P_B - P_A$ into (2) to obtain $\frac{1}{2}\rho_a v^2 = (\rho_w - \rho_0)gL$

or

$$v = \sqrt{\frac{2gL(\rho_w - \rho_0)}{\rho_a}} = \sqrt{2(9.80\text{ m/s}^2)(0.050\,0\text{ m})\left(\frac{1\,000-750}{1.29}\right)}$$

$$v = \boxed{13.8\text{ m/s}}$$

P14.70 (a) Take point ① at the free water surface in the tank and point ② at the bottom end of the tube:

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$P_0 + \rho g d + 0 = P_0 + 0 + \frac{1}{2}\rho v_2^2$$

$$v_2 = \sqrt{2gd}$$

The volume flow rate is $\frac{V}{t} = \frac{Ah}{t} = v_2 A'$. Then $t = \frac{Ah}{v_2 A'} = \frac{Ah}{A'\sqrt{2gd}}$

(b) $t = \dfrac{(0.5\text{ m})^2\, 0.5\text{ m}}{2\times 10^{-4}\text{ m}^2\sqrt{2(9.8\text{ m/s}^2)10\text{ m}}} = \boxed{44.6\text{ s}}$

P14.71 (a) For diverging stream lines that pass just above and just below the hydrofoil we have

$$P_t + \rho g y_t + \frac{1}{2}\rho v_t^2 = P_b + \rho g y_b + \frac{1}{2}\rho v_b^2$$

Ignoring the buoyant force means taking $y_t \approx y_b$

$$P_t + \frac{1}{2}\rho (n v_b)^2 = P_b + \frac{1}{2}\rho v_b^2$$

$$P_b - P_t = \frac{1}{2}\rho v_b^2 (n^2 - 1)$$

The lift force is $(P_b - P_t)A = \frac{1}{2}\rho v_b^2 (n^2 - 1)A$

(b) For liftoff,

$$\frac{1}{2}\rho v_b^2 (n^2 - 1)A = Mg$$

$$v_b = \left(\frac{2Mg}{\rho(n^2 - 1)A}\right)^{1/2}$$

The speed of the boat relative to the shore must be nearly equal to this speed of the water below the hydrofoil relative to the boat.

(c) $v^2 (n^2 - 1)A\rho = 2Mg$

$$A = \frac{2(800\text{ kg})9.8\text{ m/s}^2}{(9.5\text{ m/s})^2 (1.05^2 - 1)1\,000\text{ kg/m}^3} = \boxed{1.70\text{ m}^2}$$

ANSWERS TO EVEN PROBLEMS

P14.2 $\sim 10^{18}$ kg/m^3. An atom is mostly empty space, so the matter we perceive is mostly empty space.

P14.4 5.27×10^{18} kg

P14.6 (a) 1.01×10^7 Pa (b) 7.09×10^5 N outward

P14.8 255 N

P14.10 (a) 65.1 N (b) 275 N

P14.12 (a) 29.4 kN to the right (b) 16.3 kN·m counterclockwise

P14.14 (a) 10.3 m (b) zero

P14.16 (a) 20.0 cm (b) 0.490 cm

P14.18 (a) 101.3 kPa + (9.80 kPa/m)h (b) Mg/A (c) 116 kPa; 52.0 Pa

P14.20 (a) 444 kg (b) 480 kg

P14.22 3.33×10^3 kg/m^3

P14.24 (a) see the solution (b) 25.0 N up (c) horizontally inward (d) tension increases; see the solution (e) 62.5% (f) 18.7%

P14.26 ~10^4 balloons of 25-cm diameter

P14.28 (a) 6.70 cm (b) 5.74 cm

P14.30 (a) 11.6 cm (b) 0.963 g/cm^3 (c) Not quite. The number 1.06 is halfway between 0.98 and 1.14 but the mark for that density is 0.0604 cm below the geometric halfway point between the ends of the scale. The marks get closer together as you go down.

P14.32 2.67×10^3 kg

P14.34 1.28×10^4 m^2

P14.36 (a) 27.9 N (b) 3.32×10^4 kg (c) 7.26×10^4 Pa

P14.38 (a) 0.825 m/s (b) 3.30 m/s (c) 4.15 L/s

P14.40 (a) see the solution (b) 616 MW

P14.42 (a) $(3.93 \times 10^{-6}\ \text{m}^3/\text{s})\sqrt{\Delta P}$ where ΔP is in pascals. (b) 0.305 L/s (c) 0.431 L/s (d) The flow rate is proportional to the square root of the pressure difference.

P14.44 (a), (b) 28.0 m/s (c) The answers agree. (d) 2.11 MPa

P14.46 6.80×10^4 Pa

P14.48 347 m/s

P14.50 (a) 489 N outward (b) 1.96 kN outward

P14.52 2.25 m above the level where the water emerges

P14.54 455 kPa

P14.56 (a) 8.04 m/s (b) The gravitational force 20.6 N down and the buoyant force 36.8 N up. (c) The net upward force on the ball brings its downward motion to a stop over 4.18 m (d) 8.04 m/s (e) The time intervals are equal. (f) With friction present, Δt_{down} is less than Δt_{up}. The magnitude of the ball's acceleration on the way down is greater than its acceleration on the way up. The two motions cover equal distances and both have zero speed at one end point, so the downward trip with larger-magnitude acceleration must take less time.

P14.58 8.01 km; yes

P14.60 (a) see the solution (b) 2.58×10^4 N

P14.62 top scale: $\left(1-\dfrac{\rho_0}{\rho_{\mathrm{Fe}}}\right)m_{\mathrm{Fe}}g$ bottom scale: $\left(m_b+m_0+\dfrac{\rho_0 m_{\mathrm{Fe}}}{\rho_{\mathrm{Fe}}}\right)g$

P14.64 see the solution

P14.66 (a) 18.3 mm (b) 14.3 mm (c) 8.56 mm

P14.68 (a) 2.65 m/s (b) 2.31×10^4 Pa

P14.70 (a) see the solution (b) 44.6 s

15

Oscillatory Motion

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q15.1 Neither are examples of simple harmonic motion, although they are both periodic motion. In neither case is the acceleration proportional to the position. Neither motion is so smooth as SHM. The ball's acceleration is very large when it is in contact with the floor, and the student's when the dismissal bell rings.

***Q15.2** (i) Answer (c). At 120 cm we have the midpoint between the turning points, so it is the equilibrium position and the point of maximum speed.

(ii) Answer (a). In simple harmonic motion the acceleration is a maximum when the excursion from equilibrium is a maximum.

(iii) Answer (a), by the same logic as in part (ii).

(iv) Answer (c), by the same logic as in part (i).

(v) Answer (c), by the same logic as in part (i).

(vi) Answer (e). The total energy is a constant.

Q15.3 You can take $\phi = \pi$, or equally well, $\phi = -\pi$. At $t = 0$, the particle is at its turning point on the negative side of equilibrium, at $x = -A$.

***Q15.4** The amplitude does not affect the period in simple harmonic motion; neither do constant forces that offset the equilibrium position. Thus a, b, e, and f all have equal periods. The period is proportional to the square root of mass divided by spring constant. So c, with larger mass, has larger period than a. And d with greater stiffness has smaller period. In situation g the motion is not quite simple harmonic, but has slightly smaller angular frequency and so slightly longer period. Thus the ranking is c > g > a = b = e = f > d.

***Q15.5** (a) Yes. In simple harmonic motion, one-half of the time, the velocity is in the same direction as the displacement away from equilibrium.

(b) Yes. Velocity and acceleration are in the same direction half the time.

(c) No. Acceleration is always opposite to the position vector, and never in the same direction.

***Q15.6** Answer (e). We assume that the coils of the spring do not hit one another. The frequency will be higher than f by the factor $\sqrt{2}$. When the spring with two blocks is set into oscillation in space, the coil in the center of the spring does not move. We can imagine clamping the center coil in place without affecting the motion. We can effectively duplicate the motion of each individual block in space by hanging a single block on a half-spring here on Earth. The half-spring with its center coil clamped—or its other half cut off—has twice the spring constant as the original uncut spring, because an applied force of the same size would produce only one-half the extension distance. Thus the oscillation frequency in space is $\left(\frac{1}{2\pi}\right)\left(\frac{2k}{m}\right)^{1/2} = \sqrt{2}f$. The absence of a force required to support the vibrating system in orbital free fall has no effect on the frequency of its vibration.

***Q15.7** Answer (c). The equilibrium position is 15 cm below the starting point. The motion is symmetric about the equilibrium position, so the two turning points are 30 cm apart.

Q15.8 Since the acceleration is not constant in simple harmonic motion, none of the equations in Table 2.2 are valid.

Equation	*Information given by equation*
$x(t) = A\cos(\omega t + \phi)$	position as a function of time
$v(t) = -\omega A\sin(\omega t + \phi)$	velocity as a function of time
$v(x) = \pm\omega\left(A^2 - x^2\right)^{1/2}$	velocity as a function of position
$a(t) = -\omega^2 A\cos(\omega t + \phi)$	acceleration as a function of time
$a(t) = -\omega^2 x(t)$	acceleration as a function of position

The angular frequency ω appears in every equation. It is a good idea to figure out the value of angular frequency early in the solution to a problem about vibration, and to store it in calculator memory.

***Q15.9** (i) Answer (e). We have $T_i = \sqrt{\frac{L_i}{g}}$ and $T_f = \sqrt{\frac{L_f}{g}} = \sqrt{\frac{4L_i}{g}} = 2T_i$. The period gets larger by 2 times, to become 5 s.

(ii) Answer (c). Changing the mass has no effect on the period of a simple pendulum.

***Q15.10** (i) Answer (b). The upward acceleration has the same effect as an increased gravitational field.

(ii) Answer (a). The restoring force is smaller for the same displacement.

(iii) Answer (c).

Q15.11 (a) No force is exerted on the particle. The particle moves with constant velocity.

(b) The particle feels a constant force toward the left. It moves with constant acceleration toward the left. If its initial push is toward the right, it will slow down, turn around, and speed up in motion toward the left. If its initial push is toward the left, it will just speed up.

(c) A constant force towards the right acts on the particle to produce constant acceleration toward the right.

(d) The particle moves in simple harmonic motion about the lowest point of the potential energy curve.

Q15.12 The motion will be periodic—that is, it will repeat. The period is nearly constant as the angular amplitude increases through small values; then the period becomes noticeably larger as θ increases farther.

***Q15.13** The mechanical energy of a damped oscillator changes back and forth between kinetic and potential while it gradually and permanently changes into internal energy.

***Q15.14** The oscillation of an atom in a crystal at constant temperature is not damped but keeps constant amplitude forever.

Q15.15 No. If the resistive force is greater than the restoring force of the spring (in particular, if $b^2 > 4mk$), the system will be overdamped and will not oscillate.

Q15.16 Yes. An oscillator with damping can vibrate at resonance with amplitude that remains constant in time. Without damping, the amplitude would increase without limit at resonance.

Q15.17 Higher frequency. When it supports your weight, the center of the diving board flexes down less than the end does when it supports your weight. Thus the stiffness constant describing the center of the board is greater than the stiffness constant describing the end. And then $f = \left(\frac{1}{2\pi}\right)\sqrt{\frac{k}{m}}$ is greater for you bouncing on the center of the board.

Q15.18 An imperceptibly slight breeze may be blowing past the leaves in tiny puffs. As a leaf twists in the wind, the fibers in its stem provide a restoring torque. If the frequency of the breeze matches the natural frequency of vibration of one particular leaf as a torsional pendulum, that leaf can be driven into a large-amplitude resonance vibration. Note that it is not the *size* of the driving force that sets the leaf into resonance, but the *frequency* of the driving force. If the frequency changes, another leaf will be set into resonant oscillation.

Q15.19 We assume the diameter of the bob is not very small compared to the length of the cord supporting it. As the water leaks out, the center of mass of the bob moves down, increasing the effective length of the pendulum and slightly lowering its frequency. As the last drops of water dribble out, the center of mass of the bob hops back up to the center of the sphere, and the pendulum frequency quickly increases to its original value.

SOLUTIONS TO PROBLEMS

Section 15.1 **Motion of an Object Attached to a Spring**

P15.1 (a) Since the collision is perfectly elastic, the ball will rebound to the height of 4.00 m and then repeat the motion over and over again. Thus, the $\boxed{\text{motion is periodic}}$.

(b) To determine the period, we use: $x = \frac{1}{2}gt^2$.

The time for the ball to hit the ground is $t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(4.00\text{ m})}{9.80\text{ m/s}^2}} = 0.904\text{ s}$.

This equals one-half the period, so $T = 2(0.904\text{ s}) = \boxed{1.81\text{ s}}$.

(c) The motion is not simple harmonic. The net force acting on the ball is a constant given by $F = -mg$ (except when it is in contact with the ground), which is not in the form of Hooke's law.

Section 15.2 The Particle in Simple Harmonic Motion

P15.2 (a) $x = (5.00 \text{ cm})\cos\left(2t + \dfrac{\pi}{6}\right)$ At $t = 0$, $x = (5.00 \text{ cm})\cos\left(\dfrac{\pi}{6}\right) = \boxed{4.33 \text{ cm}}$

(b) $v = \dfrac{dx}{dt} = -(10.0 \text{ cm/s})\sin\left(2t + \dfrac{\pi}{6}\right)$ At $t = 0$, $v = \boxed{-5.00 \text{ cm/s}}$

(c) $a = \dfrac{dv}{dt} = -(20.0 \text{ cm/s}^2)\cos\left(2t + \dfrac{\pi}{6}\right)$ At $t = 0$, $a = \boxed{-17.3 \text{ cm/s}^2}$

(d) $A = \boxed{5.00 \text{ cm}}$ and $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{2} = \boxed{3.14 \text{ s}}$

P15.3 $x = (4.00 \text{ m})\cos(3.00\pi t + \pi)$ Compare this with $x = A\cos(\omega t + \phi)$ to find

(a) $\omega = 2\pi f = 3.00\pi$

or $\boxed{f = 1.50 \text{ Hz}}$ $T = \dfrac{1}{f} = \boxed{0.667 \text{ s}}$

(b) $A = \boxed{4.00 \text{ m}}$

(c) $\phi = \boxed{\pi \text{ rad}}$

(d) $x(t = 0.250 \text{ s}) = (4.00 \text{ m})\cos(1.75\pi) = \boxed{2.83 \text{ m}}$

P15.4 (a) The spring constant of this spring is

$$k = \frac{F}{x} = \frac{0.45 \text{ kg } 9.8 \text{ m/s}^2}{0.35 \text{ m}} = 12.6 \text{ N/m}$$

we take the x-axis pointing downward, so $\phi = 0$

$$x = A\cos\omega t = 18.0 \text{ cm}\cos\sqrt{\frac{12.6 \text{ kg}}{0.45 \text{ kg}\cdot\text{s}^2}}84.4 \text{ s} = 18.0 \text{ cm}\cos 446.6 \text{ rad} = \boxed{15.8 \text{ cm}}$$

We choose to solve the parts in a different order.

(d) Now $446.6 \text{ rad} = 71 \times 2\pi + 0.497 \text{ rad}$. In each cycle the object moves $4(18) = 72$ cm, so it has moved $71(72 \text{ cm}) + (18 - 15.8) \text{ cm} = \boxed{51.1 \text{ m}}$.

(b) By the same steps, $k = \dfrac{0.44 \text{ kg } 9.8 \text{ m/s}^2}{0.355 \text{ m}} = 12.1 \text{ N/m}$

$$x = A\cos\sqrt{\frac{k}{m}}t = 18.0 \text{ cm}\cos\sqrt{\frac{12.1}{0.44}}84.4 = 18.0 \text{ cm } \cos 443.5 \text{ rad} = \boxed{-15.9 \text{ cm}}$$

(e) $443.5 \text{ rad} = 70(2\pi) + 3.62 \text{ rad}$

$$\text{Distance moved} = 70(72 \text{ cm}) + 18 + 15.9 \text{ cm} = \boxed{50.7 \text{ m}}$$

(c) The answers to (d) and (e) are not very different given the difference in the data about the two vibrating systems. But when we ask about details of the future, the imprecision in our knowledge about the present makes it impossible to make precise predictions. The two oscillations start out in phase but get completely out of phase.

P15.5 (a) At $t = 0$, $x = 0$ and v is positive (to the right). Therefore, this situation corresponds to $x = A\sin\omega t$

and $\quad v = v_i\cos\omega t$

Since $f = 1.50\text{ Hz}$, $\quad \omega = 2\pi f = 3.00\pi$

Also, $A = 2.00\text{ cm}$, so that $\quad \boxed{x = (2.00\text{ cm})\sin 3.00\pi t}$

(b) $v_{\max} = v_i = A\omega = 2.00(3.00\pi) = 6.00\pi\ \text{cm/s} = \boxed{18.8\ \text{cm/s}}$

The particle has this speed at $t = 0$ and next at $\quad t = \dfrac{T}{2} = \boxed{\dfrac{1}{3}\text{ s}}$

(c) $a_{\max} = A\omega^2 = 2.00(3.00\pi)^2 = 18.0\pi^2\ \text{cm/s}^2 = \boxed{178\ \text{cm/s}^2}$

This positive value of acceleration first occurs at $\quad t = \dfrac{3}{4}T = \boxed{0.500\text{ s}}$

(d) Since $T = \dfrac{2}{3}$ s and $A = 2.00\text{ cm}$, the particle will travel 8.00 cm in this time.

Hence, in $1.00\text{ s}\left(= \dfrac{3}{2}T\right)$, the particle will travel $\quad 8.00\text{ cm} + 4.00\text{ cm} = \boxed{12.0\text{ cm}}$.

P15.6 (a) $T = \dfrac{12.0\text{ s}}{5} = \boxed{2.40\text{ s}}$

(b) $f = \dfrac{1}{T} = \dfrac{1}{2.40} = \boxed{0.417\text{ Hz}}$

(c) $\omega = 2\pi f = 2\pi(0.417) = \boxed{2.62\ \text{rad/s}}$

P15.7 $\quad f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}} \quad$ or $\quad T = \dfrac{1}{f} = 2\pi\sqrt{\dfrac{m}{k}}$

Solving for k, $\quad k = \dfrac{4\pi^2 m}{T^2} = \dfrac{4\pi^2(7.00\text{ kg})}{(2.60\text{ s})^2} = \boxed{40.9\ \text{N/m}}$

***P15.8** (a) For constant acceleration position is given as a function of time by

$$x = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$= 0.27\text{ m} + (0.14\ \text{m/s})(4.5\text{ s}) + \frac{1}{2}\left(-0.32\ \text{m/s}^2\right)(4.5\text{ s})^2$$

$$= \boxed{-2.34\text{ m}}$$

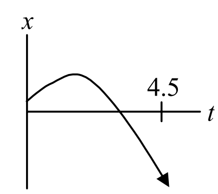

FIG. P15.8(a)

(b) $v_x = v_{xi} + a_x t = 0.14\ \text{m/s} - \left(0.32\ \text{m/s}^2\right)(4.5\text{ s}) = \boxed{-1.30\ \text{m/s}}$

(c) For simple harmonic motion we have instead $x = A\cos(\omega t + \phi)$ and $v = -A\omega\sin(\omega t + \phi)$ where $a = -\omega^2 x$, so that $-0.32\ \text{m/s}^2 = -\omega^2(0.27\text{ m})$, $\omega = 1.09\ \text{rad/s}$. At $t = 0$, $0.27\text{ m} = A\cos\phi$ and $0.14\ \text{m/s} = -A(1.09/\text{s})\sin\phi$. Dividing gives $\dfrac{0.14\ \text{m/s}}{0.27\text{ m}} = -(1.09/\text{s})\tan\phi$, $\tan\phi = -0.476$, $\phi = -25.5°$. Still at $t = 0$, $0.27\text{ m} = A\cos(-25.5°)$, $A = 0.299\text{ m}$. Now at $t = 4.5$ s,

$$x = (0.299\text{ m})\cos\left[(1.09\ \text{rad/s})(4.5\text{ s}) - 25.5°\right] = (0.299\text{ m})\cos(4.90\text{ rad} - 25.5°)$$

$$= (0.299\text{ m})\cos 255° = \boxed{-0.076\,3\text{ m}}$$

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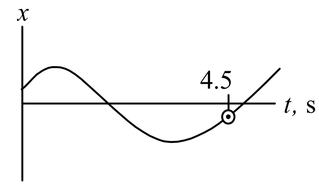
(d) $v = -(0.299\text{ m})(1.09/\text{s})\sin 255° = \boxed{+0.315\text{ m/s}}$



P15.9 $x = A\cos\omega t$ $\quad A = 0.05\text{ m}$ $\quad v = -A\omega\sin\omega t$ $\quad a = -A\omega^2\cos\omega t$

If $f = 3\,600\text{ rev/min} = 60\text{ Hz}$, then $\omega = 120\pi\text{ s}^{-1}$

$v_{\max} = 0.05(120\pi)\text{ m/s} = \boxed{18.8\text{ m/s}}$ $\quad a_{\max} = 0.05(120\pi)^2\text{ m/s}^2 = \boxed{7.11\text{ km/s}^2}$

P15.10 $m = 1.00\text{ kg}$, $k = 25.0\text{ N/m}$, and $A = 3.00\text{ cm}$. At $t = 0$, $x = -3.00\text{ cm}$

(a) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{25.0}{1.00}} = 5.00\text{ rad/s}$

so that, $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{5.00} = \boxed{1.26\text{ s}}$

(b) $v_{\max} = A\omega = 3.00\times 10^{-2}\text{ m}(5.00\text{ rad/s}) = \boxed{0.150\text{ m/s}}$

$a_{\max} = A\omega^2 = 3.00\times 10^{-2}\text{ m}(5.00\text{ rad/s})^2 = \boxed{0.750\text{ m/s}^2}$

(c) Because $x = -3.00\text{ cm}$ and $v = 0$ at $t = 0$, the required solution is $x = -A\cos\omega t$

or $\boxed{x = -3.00\cos(5.00t)\text{ cm}}$

$v = \dfrac{dx}{dt} = \boxed{15.0\sin(5.00t)\text{ cm/s}}$

$a = \dfrac{dv}{dt} = \boxed{75.0\cos(5.00t)\text{ cm/s}^2}$

P15.11 (a) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{8.00\text{ N/m}}{0.500\text{ kg}}} = 4.00\text{ s}^{-1}$ so position is given by $x = 10.0\sin(4.00t)\text{ cm}$

From this we find that $v = 40.0\cos(4.00t)\text{ cm/s}$ $\quad v_{\max} = \boxed{40.0\text{ cm/s}}$

$a = -160\sin(4.00t)\text{ cm/s}^2$ $\quad a_{\max} = \boxed{160\text{ cm/s}^2}$

(b) $t = \left(\dfrac{1}{4.00}\right)\sin^{-1}\left(\dfrac{x}{10.0}\right)$ and when $x = 6.00\text{ cm}$, $t = 0.161\text{ s}$.

We find $v = 40.0\cos[4.00(0.161)] = \boxed{32.0\text{ cm/s}}$

$a = -160\sin[4.00(0.161)] = \boxed{-96.0\text{ cm/s}^2}$

(c) Using $t = \left(\dfrac{1}{4.00}\right)\sin^{-1}\left(\dfrac{x}{10.0}\right)$

when $x = 0$, $t = 0$ and when $x = 8.00\text{ cm}$, $t = 0.232\text{ s}$

Therefore, $\Delta t = \boxed{0.232\text{ s}}$

***P15.12** We assume that the mass of the spring is negligible and that we are on Earth. Let m represent the mass of the object. Its hanging at rest is described by

$$\Sigma F_y = 0 \qquad -F_g + kx = 0 \qquad mg = k(0.183\ \text{m}) \qquad m/k = (0.183\ \text{m})/(9.8\ \text{N/kg})$$

The object's bouncing is described by $T = 2\pi(m/k)^{1/2} = 2\pi[(0.183\ \text{m})/(9.8\ \text{m/s}^2)]^{1/2} = \boxed{0.859\ \text{s}}$

We do have enough information to find the period. Whether the object has small or large mass, the ratio m/k must be equal to $0.183\ \text{m}/(9.80\ \text{m/s}^2)$. The period is 0.859 s.

P15.13 The 0.500 s must elapse between one turning point and the other. Thus the period is 1.00 s.

$$\omega = \frac{2\pi}{T} = 6.28/\text{s}$$

and $v_{\max} = \omega A = (6.28/\text{s})(0.100\ \text{m}) = \boxed{0.628\ \text{m/s}}$.

Section 15.3 Energy of the Simple Harmonic Oscillator

P15.14 $m = 200\ \text{g},\ T = 0.250\ \text{s},\ E = 2.00\ \text{J};\ \omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.250} = 25.1\ \text{rad/s}$

(a) $k = m\omega^2 = 0.200\ \text{kg}(25.1\ \text{rad/s})^2 = \boxed{126\ \text{N/m}}$

(b) $E = \dfrac{kA^2}{2} \Rightarrow A = \sqrt{\dfrac{2E}{k}} = \sqrt{\dfrac{2(2.00)}{126}} = \boxed{0.178\ \text{m}}$

P15.15 Choose the car with its shock-absorbing bumper as the system; by conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2: \qquad v = x\sqrt{\frac{k}{m}} = \left(3.16\times10^{-2}\ \text{m}\right)\sqrt{\frac{5.00\times10^6}{10^3}} = \boxed{2.23\ \text{m/s}}$$

P15.16 (a) $E = \dfrac{kA^2}{2} = \dfrac{250\ \text{N/m}\left(3.50\times10^{-2}\ \text{m}\right)^2}{2} = \boxed{0.153\ \text{J}}$

(b) $v_{\max} = A\omega$ where $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{250}{0.500}} = 22.4\ \text{s}^{-1}$ $\qquad v_{\max} = \boxed{0.784\ \text{m/s}}$

(c) $a_{\max} = A\omega^2 = 3.50\times10^{-2}\ \text{m}\left(22.4\ \text{s}^{-1}\right)^2 = \boxed{17.5\ \text{m/s}^2}$

P15.17 (a) $E = \dfrac{1}{2}kA^2 = \dfrac{1}{2}(35.0\ \text{N/m})\left(4.00\times10^{-2}\ \text{m}\right)^2 = \boxed{28.0\ \text{mJ}}$

(b) $|v| = \omega\sqrt{A^2 - x^2} = \sqrt{\dfrac{k}{m}}\sqrt{A^2 - x^2}$

$$|v| = \sqrt{\frac{35.0}{50.0\times10^{-3}}}\sqrt{\left(4.00\times10^{-2}\right)^2 - \left(1.00\times10^{-2}\right)^2} = \boxed{1.02\ \text{m/s}}$$

(c) $\dfrac{1}{2}mv^2 = \dfrac{1}{2}kA^2 - \dfrac{1}{2}kx^2 = \dfrac{1}{2}(35.0)\left[\left(4.00\times10^{-2}\right)^2 - \left(3.00\times10^{-2}\right)^2\right] = \boxed{12.2\ \text{mJ}}$

(d) $\dfrac{1}{2}kx^2 = E - \dfrac{1}{2}mv^2 = \boxed{15.8\ \text{mJ}}$

P15.18 (a) $k = \dfrac{|F|}{x} = \dfrac{20.0\text{ N}}{0.200\text{ m}} = \boxed{100\text{ N/m}}$

(b) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{50.0}\text{ rad/s}$ so $f = \dfrac{\omega}{2\pi} = \boxed{1.13\text{ Hz}}$

(c) $v_{\max} = \omega A = \sqrt{50.0}(0.200) = \boxed{1.41\text{ m/s}}$ at $x = 0$

(d) $a_{\max} = \omega^2 A = 50.0(0.200) = \boxed{10.0\text{ m/s}^2}$ at $x = \pm A$

(e) $E = \dfrac{1}{2}kA^2 = \dfrac{1}{2}(100)(0.200)^2 = \boxed{2.00\text{ J}}$

(f) $|v| = \omega\sqrt{A^2 - x^2} = \sqrt{50.0}\sqrt{\dfrac{8}{9}(0.200)^2} = \boxed{1.33\text{ m/s}}$

(g) $|a| = \omega^2 x = 50.0\left(\dfrac{0.200}{3}\right) = \boxed{3.33\text{ m/s}^2}$

P15.19 Model the oscillator as a block-spring system.

From energy considerations, $v^2 + \omega^2 x^2 = \omega^2 A^2$

$v_{\max} = \omega A$ and $v = \dfrac{\omega A}{2}$ so $\left(\dfrac{\omega A}{2}\right)^2 + \omega^2 x^2 = \omega^2 A^2$

From this we find $x^2 = \dfrac{3}{4}A^2$ and $x = \dfrac{\sqrt{3}}{2}A = \boxed{\pm 2.60\text{ cm}}$ where $A = 3.00\text{ cm}$

P15.20 (a) $y_f = y_i + v_{yi}t + \dfrac{1}{2}a_y t^2$

$$-11\text{ m} = 0 + 0 + \frac{1}{2}\left(-9.8\text{ m/s}^2\right)t^2$$

$$t = \sqrt{\frac{22\text{ m}\cdot\text{s}^2}{9.8\text{ m}}} = \boxed{1.50\text{ s}}$$

(b) Take the initial point where she steps off the bridge and the final point at the bottom of her motion.

$$\left(K + U_g + U_s\right)_i = \left(K + U_g + U_s\right)_f$$

$$0 + mgy + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$65\text{ kg }9.8\text{ m/s}^2\text{ }36\text{ m} = \frac{1}{2}k(25\text{ m})^2$$

$$k = \boxed{73.4\text{ N/m}}$$

(c) The spring extension at equilibrium is $x = \dfrac{F}{k} = \dfrac{65\text{ kg }9.8\text{ m/s}^2}{73.4\text{ N/m}} = 8.68\text{ m}$, so this point is $11 + 8.68\text{ m} = \boxed{19.7\text{ m below the bridge}}$ and the amplitude of her oscillation is $36 - 19.7 = 16.3\text{ m}$.

(d) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{73.4\text{ N/m}}{65\text{ kg}}} = \boxed{1.06\text{ rad/s}}$

continued on next page

(e) Take the phase as zero at maximum downward extension. We find what the phase was 25 m higher, where $x = -8.68$ m:

In $x = A\cos\omega t$ $\qquad$ $16.3 \text{ m} = 16.3 \text{ m}\cos 0$

$$-8.68 \text{ m} = 16.3 \text{ m}\cos\left(1.06\ \frac{t}{\text{s}}\right) \qquad 1.06\frac{t}{\text{s}} = -122° = -2.13 \text{ rad}$$

$t = -2.01$ s

Then $\boxed{+2.01 \text{ s}}$ is the time over which the spring stretches.

(f) total time $= 1.50 \text{ s} + 2.01 \text{ s} = \boxed{3.50 \text{ s}}$

P15.21 The potential energy is

$$U_s = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t)$$

The rate of change of potential energy is

$$\frac{dU_s}{dt} = \frac{1}{2}kA^2 2\cos(\omega t)\left[-\omega\sin(\omega t)\right] = -\frac{1}{2}kA^2\omega\sin 2\omega t$$

(a) This rate of change is maximal and negative at

$$2\omega t = \frac{\pi}{2},\ 2\omega t = 2\pi + \frac{\pi}{2}, \text{ or in general, } 2\omega t = 2n\pi + \frac{\pi}{2} \text{ for integer } n$$

Then,

$$t = \frac{\pi}{4\omega}(4n+1) = \frac{\pi(4n+1)}{4\left(3.60 \text{ s}^{-1}\right)}$$

For $n = 0$, this gives $t = \boxed{0.218 \text{ s}}$ while $n = 1$ gives $t = \boxed{1.09 \text{ s}}$.

All other values of n yield times outside the specified range.

(b) $$\left|\frac{dU_s}{dt}\right|_{\max} = \frac{1}{2}kA^2\omega = \frac{1}{2}(3.24 \text{ N/m})\left(5.00\times 10^{-2} \text{ m}\right)^2\left(3.60 \text{ s}^{-1}\right) = \boxed{14.6 \text{ mW}}$$

Section 15.4 **Comparing Simple Harmonic Motion with Uniform Circular Motion**

P15.22 The angle of the crank pin is $\theta = \omega t$. Its x-coordinate is

$$x = A\cos\theta = A\cos\omega t$$

where A is the distance from the center of the wheel to the crank pin. This is of the form $x = A\cos(\omega t + \phi)$, so the yoke and piston rod move with simple harmonic motion.

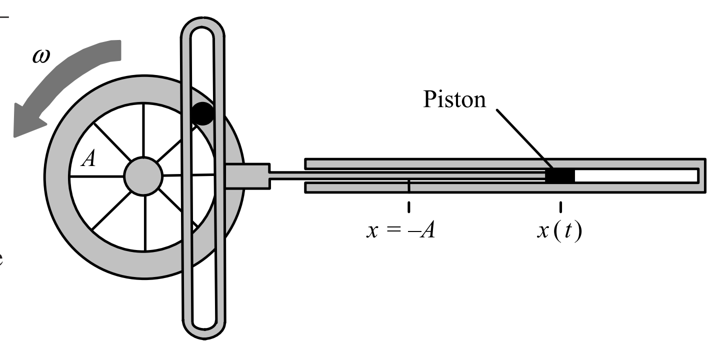


FIG. P15.22

P15.23 (a) The motion is simple harmonic because the tire is rotating with constant velocity and you are looking at the motion of the bump projected in a plane perpendicular to the tire.

(b) Since the car is moving with speed $v = 3.00\ \text{m/s}$, and its radius is 0.300 m, we have

$$\omega = \frac{3.00\ \text{m/s}}{0.300\ \text{m}} = 10.0\ \text{rad/s}$$

Therefore, the period of the motion is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{(10.0\ \text{rad/s})} = \boxed{0.628\ \text{s}}$$

Section 15.5 **The Pendulum**

P15.24 The period in Tokyo is $T_T = 2\pi\sqrt{\dfrac{L_T}{g_T}}$

and the period in Cambridge is $T_C = 2\pi\sqrt{\dfrac{L_C}{g_C}}$

We know $T_T = T_C = 2.00\ \text{s}$

For which, we see $\dfrac{L_T}{g_T} = \dfrac{L_C}{g_C}$

or $\dfrac{g_C}{g_T} = \dfrac{L_C}{L_T} = \dfrac{0.994\,2}{0.992\,7} = \boxed{1.001\,5}$

P15.25 Using the simple harmonic motion model:

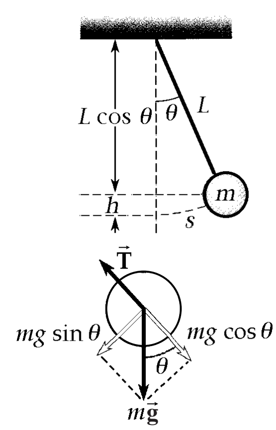


FIG. P15.25

$$A = r\theta = 1\ \text{m}\ 15°\frac{\pi}{180°} = 0.262\ \text{m}$$

$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8\ \text{m/s}^2}{1\ \text{m}}} = 3.13\ \text{rad/s}$$

(a) $v_{\max} = A\omega = 0.262\ \text{m}\ 3.13/\text{s} = \boxed{0.820\ \text{m/s}}$

(b) $a_{\max} = A\omega^2 = 0.262\ \text{m}(3.13/\text{s})^2 = 2.57\ \text{m/s}^2$

$a_{\tan} = r\alpha \qquad \alpha = \dfrac{a_{\tan}}{r} = \dfrac{2.57\ \text{m/s}^2}{1\ \text{m}} = \boxed{2.57\ \text{rad/s}^2}$

(c) $F = ma = 0.25\ \text{kg}\ 2.57\ \text{m/s}^2 = \boxed{0.641\ \text{N}}$

More precisely,

(a) $mgh = \dfrac{1}{2}mv^2$ and $h = L(1 - \cos\theta)$

$\therefore v_{\max} = \sqrt{2gL(1-\cos\theta)} = \boxed{0.817\ \text{m/s}}$

continued on next page

(b) $I\alpha = mgL\sin\theta$

$$\alpha_{\max} = \frac{mgL\sin\theta}{mL^2} = \frac{g}{L}\sin\theta_i = \boxed{2.54\ \text{rad/s}^2}$$

(c) $F_{\max} = mg\sin\theta_i = 0.250(9.80)(\sin 15.0°) = \boxed{0.634\ \text{N}}$

The answers agree to two digits. The answers computed from conservation of energy and from Newton's second law are more precisely correct. With this amplitude the motion of the pendulum is approximately simple harmonic.

***P15.26** Note that the angular amplitude 0.032 rad = 1.83 degree is small, as required for the SHM model of a pendulum.

$$\omega = \frac{2\pi}{T}: \qquad T = \frac{2\pi}{\omega} = \frac{2\pi}{4.43} = \boxed{1.42\ \text{s}}$$

$$\omega = \sqrt{\frac{g}{L}}: \qquad L = \frac{g}{\omega^2} = \frac{9.80}{(4.43)^2} = \boxed{0.499\ \text{m}}$$

P15.27 Referring to the sketch we have

$F = -mg\sin\theta$ and $\tan\theta = \dfrac{x}{R}$

For small displacements, $\tan\theta \approx \sin\theta$

and $F = -\dfrac{mg}{R}x = -kx$

Since the restoring force is proportional to the displacement from equilibrium, the motion is simple harmonic motion.

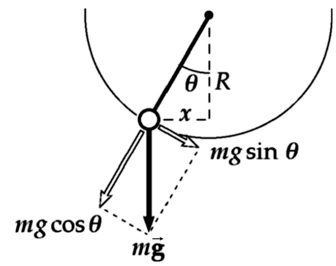


FIG. P15.27

Comparing to $F = -m\omega^2 x$ shows $\boxed{\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{g}{R}}}$.

P15.28 (a) The string tension must support the weight of the bob, accelerate it upward, and also provide the restoring force, just as if the elevator were at rest in a gravity field of $(9.80 + 5.00)\ \text{m/s}^2$. Thus the period is

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{5.00\ \text{m}}{14.8\ \text{m/s}^2}}$$

$$T = \boxed{3.65\ \text{s}}$$

(b) $$T = 2\pi\sqrt{\frac{5.00\ \text{m}}{(9.80\ \text{m/s}^2 - 5.00\ \text{m/s}^2)}} = \boxed{6.41\ \text{s}}$$

(c) $$g_{eff} = \sqrt{(9.80\ \text{m/s}^2)^2 + (5.00\ \text{m/s}^2)^2} = 11.0\ \text{m/s}^2$$

$$T = 2\pi = \sqrt{\frac{5.00\ \text{m}}{11.0\ \text{m/s}^2}} = \boxed{4.24\ \text{s}}$$

P15.29 $f = 0.450$ Hz, $d = 0.350$ m, and $m = 2.20$ kg

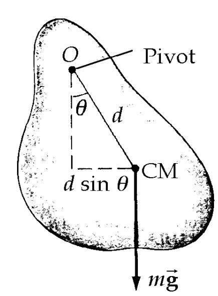


FIG. P15.29

$$T = \frac{1}{f};$$

$$T = 2\pi\sqrt{\frac{I}{mgd}}; \qquad T^2 = \frac{4\pi^2 I}{mgd}$$

$$I = T^2\frac{mgd}{4\pi^2} = \left(\frac{1}{f}\right)^2\frac{mgd}{4\pi^2} = \frac{2.20(9.80)(0.350)}{4\pi^2(0.450\ \text{s}^{-1})^2} = \boxed{0.944\ \text{kg}\cdot\text{m}^2}$$

P15.30 (a) From $T = \dfrac{\text{total measured time}}{50}$

the measured periods are:

Length, L (m)	1.000	0.750	0.500
Period, T (s)	1.996	1.732	1.422

(b) $T = 2\pi\sqrt{\dfrac{L}{g}}$ so $g = \dfrac{4\pi^2 L}{T^2}$

The calculated values for g are:

Period, T (s)	1.996	1.732	1.422
g (m/s^2)	9.91	9.87	9.76

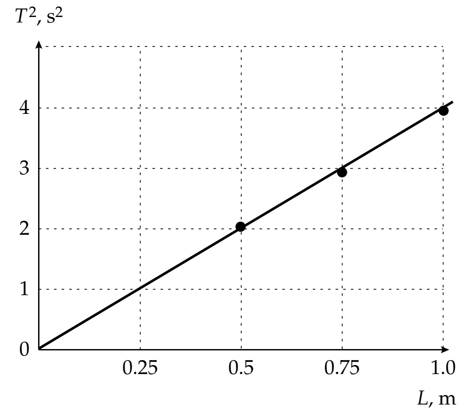


FIG. P15.30

Thus, $g_{\text{ave}} = 9.85\ \text{m/s}^2$ $\boxed{\text{This agrees with the accepted value of } g = 9.80\ \text{m/s}^2 \text{ within } 0.5\%.}$

(c) From $T^2 = \left(\dfrac{4\pi^2}{g}\right)L$, the slope of T^2 versus L graph $= \dfrac{4\pi^2}{g} = 4.01\ \text{s}^2/\text{m}$.

Thus, $g = \dfrac{4\pi^2}{\text{slope}} = \boxed{9.85\ \text{m/s}^2}$. This is the same as the value in (b).

P15.31 (a) The parallel axis theorem says directly $I = I_{\text{CM}} + md^2$

so $T = 2\pi\sqrt{\dfrac{I}{mgd}} = \boxed{2\pi\sqrt{\dfrac{\left(I_{\text{CM}} + md^2\right)}{mgd}}}$

(b) When d is very large $T \to 2\pi\sqrt{\dfrac{d}{g}}$ gets large.

When d is very small $T \to 2\pi\sqrt{\dfrac{I_{\text{CM}}}{mgd}}$ gets large.

So there must be a minimum, found by

$$\frac{dT}{dd} = 0 = \frac{d}{dd}2\pi\left(I_{\text{CM}} + md^2\right)^{1/2}(mgd)^{-1/2}$$

$$= 2\pi\left(I_{\text{CM}} + md^2\right)^{1/2}\left(-\frac{1}{2}\right)(mgd)^{-3/2}mg + 2\pi(mgd)^{-1/2}\left(\frac{1}{2}\right)\left(I_{\text{CM}} + md^2\right)^{-1/2}2md$$

$$= \frac{-\pi\left(I_{\text{CM}} + md^2\right)mg}{\left(I_{\text{CM}} + md^2\right)^{1/2}(mgd)^{3/2}} + \frac{2\pi\, md\, mgd}{\left(I_{\text{CM}} + md^2\right)^{1/2}(mgd)^{3/2}} = 0$$

This requires

$$-I_{\text{CM}} - md^2 + 2md^2 = 0$$

or $\boxed{I_{\text{CM}} = md^2}$.

P15.32 (a) The parallel-axis theorem:

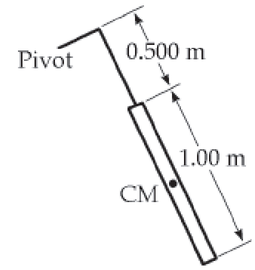


FIG. P15.32

$$I = I_{\text{CM}} + Md^2 = \frac{1}{12}ML^2 + Md^2$$

$$= \frac{1}{12}M\left(1.00\text{ m}\right)^2 + M\left(1.00\text{ m}\right)^2 = M\left(\frac{13}{12}\text{ m}^2\right)$$

$$T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{M\left(13\text{ m}^2\right)}{12Mg\left(1.00\text{ m}\right)}}$$

$$= 2\pi\sqrt{\frac{13\text{ m}}{12\left(9.80\text{ m/s}^2\right)}} = \boxed{2.09\text{ s}}$$

(b) For the simple pendulum

$$T = 2\pi\sqrt{\frac{1.00\text{ m}}{9.80\text{ m/s}^2}} = 2.01\text{ s} \qquad \text{difference} = \frac{2.09\text{ s} - 2.01\text{ s}}{2.01\text{ s}} = \boxed{4.08\%}$$

P15.33 $T = 0.250\text{ s},\ I = mr^2 = \left(20.0\times10^{-3}\text{ kg}\right)\left(5.00\times10^{-3}\text{ m}\right)^2$

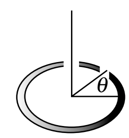


FIG. P15.33

(a) $I = \boxed{5.00\times10^{-7}\text{ kg}\cdot\text{m}^2}$

(b) $I\dfrac{d^2\theta}{dt^2} = -\kappa\theta;\ \sqrt{\dfrac{\kappa}{I}} = \omega = \dfrac{2\pi}{T}$

$$\kappa = I\omega^2 = \left(5.00\times10^{-7}\right)\left(\frac{2\pi}{0.250}\right)^2 = \boxed{3.16\times10^{-4}\ \frac{\text{N}\cdot\text{m}}{\text{rad}}}$$

Section 15.6 **Damped Oscillations**

P15.34 The total energy is $\qquad E = \dfrac{1}{2}mv^2 + \dfrac{1}{2}kx^2$

Taking the time derivative, $\qquad \dfrac{dE}{dt} = mv\dfrac{d^2x}{dt^2} + kxv$

Use Equation 15.31: $\qquad \dfrac{md^2x}{dt^2} = -kx - bv$

$$\frac{dE}{dt} = v\left(-kx - bv\right) + kvx$$

Thus, $\qquad \boxed{\dfrac{dE}{dt} = -bv^2 < 0}$

We have proved that the mechanical energy of a damped oscillator is always decreasing.

P15.35 $\theta_i = 15.0°$ $\qquad \theta\left(t = 1\,000\right) = 5.50°$

$x = Ae^{-bt/2m}$ $\qquad \dfrac{x_{1\,000}}{x_i} = \dfrac{Ae^{-bt/2m}}{A} = \dfrac{5.50}{15.0} = e^{-b(1\,000)/2m}$

$$\ln\left(\frac{5.50}{15.0}\right) = -1.00 = \frac{-b\left(1\,000\right)}{2m}$$

$$\therefore \frac{b}{2m} = \boxed{1.00\times10^{-3}\text{ s}^{-1}}$$

P15.36 To show that $x = Ae^{-bt/2m}\cos(\omega t + \phi)$

is a solution of $-kx - b\dfrac{dx}{dt} = m\dfrac{d^2x}{dt^2}$ (1)

where $\omega = \sqrt{\dfrac{k}{m} - \left(\dfrac{b}{2m}\right)^2}$ (2)

We take $x = Ae^{-bt/2m}\cos(\omega t + \phi)$ and compute (3)

$$\frac{dx}{dt} = Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t + \phi) - Ae^{-bt/2m}\omega\sin(\omega t + \phi) \quad (4)$$

$$\frac{d^2x}{dt^2} = -\frac{b}{2m}\left[Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t + \phi) - Ae^{-bt/2m}\omega\sin(\omega t + \phi)\right]$$
$$-\left[Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\omega\sin(\omega t + \phi) + Ae^{-bt/2m}\omega^2\cos(\omega t + \phi)\right] \quad (5)$$

We substitute (3), (4) into the left side of (1) and (5) into the right side of (1);

$$-kAe^{-bt/2m}\cos(\omega t + \phi) + \frac{b^2}{2m}Ae^{-bt/2m}\cos(\omega t + \phi) + b\omega Ae^{-bt/2m}\sin(\omega t + \phi)$$
$$= -\frac{b}{2}\left[Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t + \phi) - Ae^{-bt/2m}\omega\sin(\omega t + \phi)\right]$$
$$+\frac{b}{2}Ae^{-bt/2m}\omega\sin(\omega t + \phi) - m\omega^2 Ae^{-bt/2m}\cos(\omega t + \phi)$$

Compare the coefficients of $Ae^{-bt/2m}\cos(\omega t + \phi)$ and $Ae^{-bt/2m}\sin(\omega t + \phi)$:

cosine-term: $-k + \dfrac{b^2}{2m} = -\dfrac{b}{2}\left(-\dfrac{b}{2m}\right) - m\omega^2 = \dfrac{b^2}{4m} - m\left(\dfrac{k}{m} - \dfrac{b^2}{4m^2}\right) = -k + \dfrac{b^2}{2m}$

sine-term: $b\omega = +\dfrac{b}{2}(\omega) + \dfrac{b}{2}(\omega) = b\omega$

Since the coefficients are equal, $x = Ae^{-bt/2m}\cos(\omega t + \phi)$ is a solution of the equation.

P15.37 The frequency if undamped would be $\omega_0 = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{2.05\times10^4\ \text{N/m}}{10.6\ \text{kg}}} = 44.0/\text{s}$.

(a) With damping

$$\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2} = \sqrt{\left(44\ \frac{1}{\text{s}}\right)^2 - \left(\frac{3\ \text{kg}}{\text{s}\ 2\ 10.6\ \text{kg}}\right)}$$
$$= \sqrt{1\,933.96 - 0.02} = 44.0\ \frac{1}{\text{s}}$$
$$f = \frac{\omega}{2\pi} = \frac{44.0}{2\pi\ \text{s}} = \boxed{7.00\ \text{Hz}}$$

(b) In $x = A_0e^{-bt/2m}\cos(\omega t + \phi)$ over one cycle, a time $T = \dfrac{2\pi}{\omega}$, the amplitude changes from A_0 to $A_0e^{-b2\pi/2m\omega}$ for a fractional decrease of

$$\frac{A_0 - A_0e^{-\pi b/m\omega}}{A_0} = 1 - e^{-\pi 3/(10.6\cdot 44.0)} = 1 - e^{-0.020\,2} = 1 - 0.979\,98 = 0.020\,0 = \boxed{2.00\%}$$

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(c) The energy is proportional to the square of the amplitude, so its fractional rate of decrease is twice as fast:

$$E = \frac{1}{2}kA^2 = \frac{1}{2}kA_0^2 e^{-2bt/2m} = E_0 e^{-bt/m}$$

We specify

$$0.05E_0 = E_0 e^{-3t/10.6}$$
$$0.05 = e^{-3t/10.6}$$
$$e^{+3t/10.6} = 20$$
$$\frac{3t}{10.6} = \ln 20 = 3.00$$
$$t = \boxed{10.6 \text{ s}}$$

Section 15.7 **Forced Oscillations**

P15.38 (a) For resonance, her frequency must match

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.30\times10^3 \text{ N/m}}{12.5 \text{ kg}}} = \boxed{2.95 \text{ Hz}}$$

(b) From $x = A\cos\omega t$, $v = \frac{dx}{dt} = -A\omega\sin\omega t$, and $a = \frac{dv}{dt} = -A\omega^2\cos\omega t$, the maximum acceleration is $A\omega^2$. When this becomes equal to the acceleration due to gravity, the normal force exerted on her by the mattress will drop to zero at one point in the cycle:

$$A\omega^2 = g \quad \text{or} \quad A = \frac{g}{\omega^2} = \frac{g}{k/m} = \frac{gm}{k} \qquad A = \frac{(9.80 \text{ m/s}^2)(12.5 \text{ kg})}{4.30\times10^3 \text{ N/m}} = \boxed{2.85 \text{ cm}}$$

P15.39 $F = 3.00\sin(2\pi t)$ N and $k = 20.0$ N/m

(a) $\omega = \frac{2\pi}{T} = 2\pi$ rad/s so $T = \boxed{1.00 \text{ s}}$

(b) In this case, $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{20.0}{2.00}} = 3.16$ rad/s

The equation for the amplitude of a driven oscillator,

with $b = 0$, gives $A = \left(\frac{F_0}{m}\right)(\omega^2 - \omega_0^2)^{-1} = \frac{3}{2}\left[4\pi^2 - (3.16)^2\right]^{-1}$

Thus, $A = 0.050\,9 \text{ m} = \boxed{5.09 \text{ cm}}$

P15.40 $F_0 \sin \omega t - kx = m\dfrac{d^2x}{dt^2} \qquad \omega_0 = \sqrt{\dfrac{k}{m}}$ (1)

$x = A\cos(\omega t + \phi)$ (2)

$\dfrac{dx}{dt} = -A\omega \sin(\omega t + \phi)$ (3)

$\dfrac{d^2x}{dt^2} = -A\omega^2 \cos(\omega t + \phi)$ (4)

Substitute (2) and (4) into (1): $F_0 \sin \omega t - kA\cos(\omega t + \phi) = m(-A\omega^2)\cos(\omega t + \phi)$

Solve for the amplitude: $(kA - mA\omega^2)\cos(\omega t + \phi) = F_0 \sin \omega t = F_0 \cos(\omega t - 90°)$

These will be equal, provided only that ϕ must be $-90°$ and $kA - mA\omega^2 = F_0$

Thus,

$$A = \frac{F_0/m}{(k/m) - \omega^2}$$

P15.41 From the equation for the amplitude of a driven oscillator with no damping,

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2}} = \frac{F_0/m}{\omega^2 - \omega_0^2}$$

$$\omega = 2\pi f = (20.0\pi \text{ s}^{-1}) \qquad \omega_0^2 = \frac{k}{m} = \frac{200}{(40.0/9.80)} = 49.0 \text{ s}^{-2}$$

$$F_0 = mA(\omega^2 - \omega_0^2)$$

$$F_0 = \left(\frac{40.0}{9.80}\right)(2.00 \times 10^{-2})(3\,950 - 49.0) = \boxed{318 \text{ N}}$$

P15.42 $A = \dfrac{F_{ext}/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (b\omega/m)^2}}$

With $b = 0$, $A = \dfrac{F_{ext}/m}{\sqrt{(\omega^2 - \omega_0^2)^2}} = \dfrac{F_{ext}/m}{\pm(\omega^2 - \omega_0^2)} = \pm\dfrac{F_{ext}/m}{\omega^2 - \omega_0^2}$

Thus, $\omega^2 = \omega_0^2 \pm \dfrac{F_{ext}/m}{A} = \dfrac{k}{m} \pm \dfrac{F_{ext}}{mA} = \dfrac{6.30 \text{ N/m}}{0.150 \text{ kg}} \pm \dfrac{1.70 \text{ N}}{(0.150 \text{ kg})(0.440 \text{ m})}$

This yields $\omega = 8.23$ rad/s or $\omega = 4.03$ rad/s

Then, $f = \dfrac{\omega}{2\pi}$ gives either $f = \boxed{1.31 \text{ Hz}}$ or $f = \boxed{0.641 \text{ Hz}}$

P15.43 The beeper must resonate at the frequency of a simple pendulum of length 8.21 cm:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.80 \text{ m/s}^2}{0.082\,1 \text{ m}}} = \boxed{1.74 \text{ Hz}}$$

Additional Problems

***P15.44** (a) Consider the first process of spring compression. It continues as long as glider 1 is moving faster than glider 2. The spring instantaneously has maximum compression when both gliders are moving with the same speed v_a.

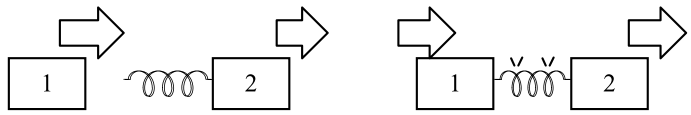


FIG. P15.44(a)

Momentum conservation:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$
$$(0.24 \text{ kg})(0.74 \text{ m/s}) + (0.36 \text{ kg})(0.12 \text{ m/s}) = (0.24 \text{ kg})v_a + (0.36 \text{ kg})v_a$$
$$\frac{0.220\,8 \text{ kg}\cdot\text{m/s}}{0.60 \text{ kg}} = v_a \qquad \vec{\mathbf{v}}_a = \boxed{0.368 \text{ m/s}\,\hat{\mathbf{i}}}$$

(b) Energy conservation:

$$(K_1 + K_2 + U_s)_i = (K_1 + K_2 + U_s)_f$$
$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 + 0 = \frac{1}{2}(m_1 + m_2)v_a^2 + \frac{1}{2}kx^2$$
$$\frac{1}{2}(0.24 \text{ kg})(0.74 \text{ m/s})^2 + \frac{1}{2}(0.36 \text{ kg})(0.12 \text{ m/s})^2$$
$$= \frac{1}{2}(0.60 \text{ kg})(0.368 \text{ m/s})^2 + \frac{1}{2}(45 \text{ N/m})x^2$$
$$0.068\,3 \text{ J} = 0.040\,6 \text{ J} + \frac{1}{2}(45 \text{ N/m})x^2$$
$$x = \left(\frac{2(0.027\,7 \text{ J})}{45 \text{ N/m}}\right)^{1/2} = \boxed{0.035\,1 \text{ m}}$$

(c) Conservation of momentum guarantees that the center of mass moves with constant velocity. Imagine viewing the gliders from a reference frame moving with the center of mass. We see the two gliders approach each other with momenta in opposite directions of equal magnitude. Upon colliding they compress the ideal spring and then together bounce, extending and compressing it cyclically.

(d) $\frac{1}{2}m_{\text{tot}}v_{\text{CM}}^2 = \frac{1}{2}(0.60 \text{ kg})(0.368 \text{ m/s})^2 = \boxed{0.040\,6 \text{ J}}$

(e) $\frac{1}{2}kA^2 = \frac{1}{2}(45 \text{ N/m})(0.035\,1 \text{ m})^2 = \boxed{0.027\,7 \text{ J}}$

***P15.45** From $a = -\omega^2 x$, the maximum acceleration is given by $a_{max} = \omega^2 A$. Then $108 \text{ cm/s}^2 = \omega^2(12 \text{ cm})$ $\omega = 3.00/\text{s}$.

(a) $T = 1/f = 2\pi/\omega = 2\pi/(3/\text{s}) = \boxed{2.09 \text{ s}}$

(b) $f = \omega/2\pi = (3/\text{s})/2\omega = \boxed{0.477 \text{ Hz}}$

(c) $v_{max} = \omega A = (3/\text{s})(12 \text{ cm}) = \boxed{36.0 \text{ cm/s}}$

(d) $E = (1/2)\, m\, v_{max}^{\ 2} = (1/2)\, m\, (0.36 \text{ m/s})^2 = \boxed{(0.0648 \text{ m}^2/\text{s}^2)m}$

(e) $\omega^2 = k/m \qquad k = \omega^2 m = (3/\text{s})^2 m = \boxed{(9.00/\text{s}^2)m}$

(f) Period, frequency, and maximum speed are all independent of mass in this situation. The energy and the force constant are directly proportional to mass.

***P15.46** (a) From $a = -\omega^2 x$, the maximum acceleration is given by $a_{max} = \omega^2 A$. As A increases, the maximum acceleration increases. When it becomes greater than the acceleration due to gravity, the rock will no longer stay in contact with the vibrating ground, but lag behind as the ground moves down with greater acceleration. We have then
$A = g/\omega^2 = g/(2\pi f)^2 = g/4\pi^2 f^2 = (9.8 \text{ m/s}^2)/4\pi^2(2.4/\text{s})^2 = \boxed{4.31 \text{ cm}}$

(b) When the rock is on the point of lifting off, the surrounding water is also barely in free fall. No pressure gradient exists in the water, so no buoyant force acts on the rock.

P15.47 Let F represent the tension in the rod.

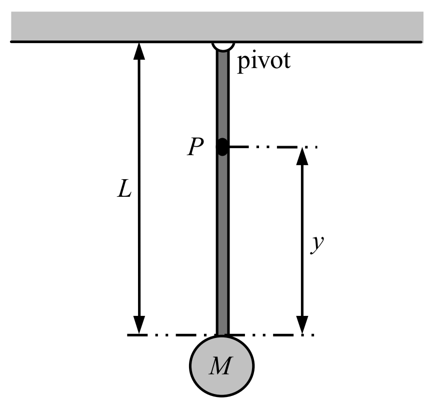


FIG. P15.47

(a) At the pivot, $F = Mg + Mg = \boxed{2Mg}$

A fraction of the rod's weight $Mg\left(\dfrac{y}{L}\right)$ as well as the weight of the ball pulls down on point P. Thus, the tension in the rod at point P is

$$F = Mg\left(\frac{y}{L}\right) + Mg = \boxed{Mg\left(1 + \frac{y}{L}\right)}$$

(b) Relative to the pivot, $I = I_{\text{rod}} + I_{\text{ball}} = \dfrac{1}{3}ML^2 + ML^2 = \dfrac{4}{3}ML^2$

For the physical pendulum, $T = 2\pi\sqrt{\dfrac{I}{mgd}}$ where $m = 2M$ and d is the distance from the pivot to the center of mass of the rod and ball combination. Therefore,

$$d = \frac{M(L/2) + ML}{M + M} = \frac{3L}{4} \qquad \text{and} \qquad T = 2\pi\sqrt{\frac{(4/3)ML^2}{(2M)g(3L/4)}} = \boxed{\frac{4\pi}{3}\sqrt{\frac{2L}{g}}}$$

For $L = 2.00$ m, $T = \dfrac{4\pi}{3}\sqrt{\dfrac{2(2.00 \text{ m})}{9.80 \text{ m/s}^2}} = \boxed{2.68 \text{ s}}$.

P15.48 (a) Total energy $= \frac{1}{2}kA^2 = \frac{1}{2}(100\ \text{N/m})(0.200\ \text{m})^2 = 2.00\ \text{J}$

At equilibrium, the total energy is:

$$\frac{1}{2}(m_1+m_2)v^2 = \frac{1}{2}(16.0\ \text{kg})v^2 = (8.00\ \text{kg})v^2$$

Therefore,

$$(8.00\ \text{kg})v^2 = 2.00\ \text{J, and } v = \boxed{0.500\ \text{m/s}}$$

This is the speed of m_1 and m_2 at the equilibrium point. Beyond this point, the mass m_2 moves with the constant speed of 0.500 m/s while mass m_1 starts to slow down due to the restoring force of the spring.

(b) The energy of the m_1-spring system at equilibrium is:

$$\frac{1}{2}m_1v^2 = \frac{1}{2}(9.00\ \text{kg})(0.500\ \text{m/s})^2 = 1.125\ \text{J}$$

This is also equal to $\frac{1}{2}k(A')^2$, where A' is the amplitude of the m_1-spring system.

Therefore,

$$\frac{1}{2}(100)(A')^2 = 1.125 \qquad \text{or} \qquad A' = 0.150\ \text{m}$$

The period of the m_1-spring system is $T = 2\pi\sqrt{\frac{m_1}{k}} = 1.885\ \text{s}$

and it takes $\frac{1}{4}T = 0.471\ \text{s}$ after it passes the equilibrium point for the spring to become fully stretched the first time. The distance separating m_1 and m_2 at this time is:

$$D = v\left(\frac{T}{4}\right) - A' = 0.500\ \text{m/s}(0.471\ \text{s}) - 0.150\ \text{m} = 0.085\,6 = \boxed{8.56\ \text{cm}}$$

P15.49 The maximum acceleration of the oscillating system is $a_{\max} = A\omega^2 = 4\pi^2Af^2$. The friction force exerted between the two blocks must be capable of accelerating block B at this rate. Thus, if Block B is about to slip,

$$f = f_{\max} = \mu_s n = \mu_s mg = m\left(4\pi^2Af^2\right)$$

$$A = \frac{\mu_s g}{4\pi^2 f^2} = \frac{0.6(980\ \text{cm/s}^2)}{4\pi^2(1.5/\text{s})^2} = \boxed{6.62\ \text{cm}}$$

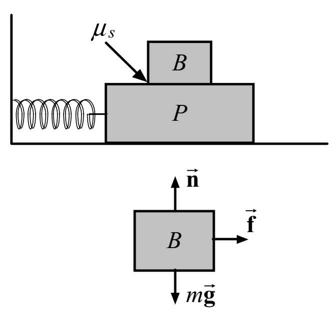


FIG. P15.49

P15.50 Refer to the diagram in the previous problem. The maximum acceleration of the oscillating system is $a_{\max} = A\omega^2 = 4\pi^2Af^2$. The friction force exerted between the two blocks must be capable of accelerating block B at this rate. Thus, if Block B is about to slip,

$$f = f_{\max} = \mu_s n = \mu_s mg = m\left(4\pi^2Af^2\right) \qquad \text{or} \qquad A = \boxed{\frac{\mu_s g}{4\pi^2 f^2}}$$

P15.51 Deuterium is the isotope of the element hydrogen with atoms having nuclei consisting of one proton and one neutron. For brevity we refer to the molecule formed by two deuterium atoms as *D* and to the diatomic molecule of hydrogen-1 as *H*.

$$M_D = 2M_H \qquad \frac{\omega_D}{\omega_H} = \frac{\sqrt{k/M_D}}{\sqrt{k/M_H}} = \sqrt{\frac{M_H}{M_D}} = \sqrt{\frac{1}{2}} \qquad f_D = \frac{f_H}{\sqrt{2}} = \boxed{0.919\times10^{14}\ \text{Hz}}$$

***P15.52** (a) A time interval. If the interaction occupied no time, each ball would move with infinite acceleration. The force exerted by each ball on the other would be infinite, and that cannot happen.

(b) $k = |F|/|x| = 16\ 000\ \text{N}/0.000\ 2\ \text{m} = \boxed{80\ \text{MN/m}}$

(c) We assume that steel has the density of its main constituent, iron, shown in Table 14.1.
Then its mass is $\rho V = \rho\,(4/3)\pi r^3 = (4\pi/3)(7860\ \text{kg/m}^3)(0.0254\ \text{m}/2)^3 = 0.0674\ \text{kg}$
and $K = (1/2)\,mv^2 = (1/2)\,(0.0674\ \text{kg})(5\ \text{m/s})^2 = \boxed{0.843\ \text{J}}$

(d) Imagine one ball running into an infinitely hard wall and bouncing off elastically. The original kinetic energy becomes elastic potential energy

$$0.843\ \text{J} = (1/2)\,(8\times10^7\ \text{N/m})x^2 \qquad x = \boxed{0.145\ \text{mm}}$$

(e) The half-cycle is from the equilibrium position of the model spring to maximum compression and back to equilibrium again. The time is one-half the period,

$$(1/2)T = (1/2)2\pi(m/k)^{1/2} = \pi(0.0674\ \text{kg}/80\times10^6\ \text{kg/s}^2)^{1/2} = \boxed{9.12\times10^{-5}\,\text{s}}$$

P15.53 (a)

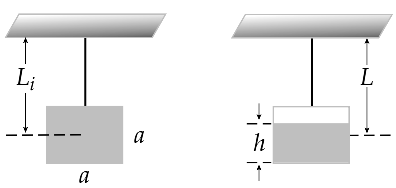


FIG. P15.53(a)

(b) $$T = 2\pi\sqrt{\frac{L}{g}} \qquad \frac{dT}{dt} = \frac{\pi}{\sqrt{g}}\frac{1}{\sqrt{L}}\frac{dL}{dt} \tag{1}$$

We need to find $L(t)$ and $\frac{dL}{dt}$. From the diagram in (a),

$$L = L_i + \frac{a}{2} - \frac{h}{2} \qquad \text{and} \qquad \frac{dL}{dt} = -\left(\frac{1}{2}\right)\frac{dh}{dt}$$

But $\frac{dM}{dt} = \rho\frac{dV}{dt} = -\rho A\frac{dh}{dt}$. Therefore,

$$\frac{dh}{dt} = -\frac{1}{\rho A}\frac{dM}{dt} \qquad \frac{dL}{dt} = \left(\frac{1}{2\rho A}\right)\frac{dM}{dt} \tag{2}$$

Also,

$$\int_{L_i}^{L} dL = \left(\frac{1}{2\rho A}\right)\left(\frac{dM}{dt}\right)t = L - L_i \tag{3}$$

Substituting Equation (2) and Equation (3) into Equation (1):

$$\frac{dT}{dt} = \boxed{\frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\frac{1}{\sqrt{L_i + \left(t/2\rho a^2\right)\left(dM/dt\right)}}}$$

continued on next page

(c) Substitute Equation (3) into the equation for the period.

$$T = \boxed{\frac{2\pi}{\sqrt{g}}\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}}$$

Or one can obtain T by integrating (b):

$$\int_T^{T_i} dT = \frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\int_0^t \frac{dt}{\sqrt{L_i + \left(1/2\rho a^2\right)\left(dM/dt\right)t}}$$

$$T - T_i = \frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\left[\frac{2}{\left(1/2\rho a^2\right)\left(dM/dt\right)}\right]\left[\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t} - \sqrt{L_i}\right]$$

But $T_i = 2\pi\sqrt{\frac{L_i}{g}}$, so $T = \frac{2\pi}{\sqrt{g}}\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}$

P15.54 $\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$

(a) $k = \omega^2 m = \boxed{\frac{4\pi^2 m}{T^2}}$ (b) $m' = \frac{k(T')^2}{4\pi^2} = \boxed{m\left(\frac{T'}{T}\right)^2}$

P15.55 We draw a free-body diagram of the pendulum. The force **H** exerted by the hinge causes no torque about the axis of rotation.

$\tau = I\alpha$ and $\frac{d^2\theta}{dt^2} = -\alpha$

$$\tau = MgL\sin\theta + kxh\cos\theta = -I\frac{d^2\theta}{dt^2}$$

For small amplitude vibrations, use the approximations: $\sin\theta \approx \theta$, $\cos\theta \approx 1$, and $x \approx s = h\theta$.

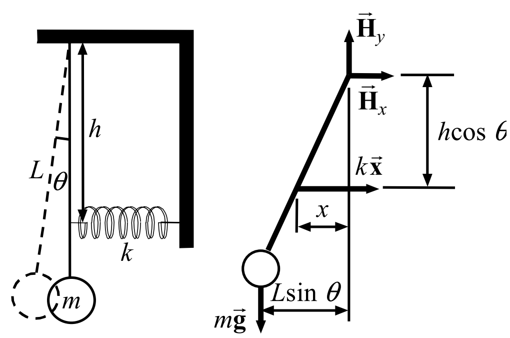


FIG. P15.55

Therefore,

$$\frac{d^2\theta}{dt^2} = -\left(\frac{MgL + kh^2}{I}\right)\theta = -\omega^2\theta \qquad \omega = \sqrt{\frac{MgL + kh^2}{ML^2}} = 2\pi f$$

$$f = \boxed{\frac{1}{2\pi}\sqrt{\frac{MgL + kh^2}{ML^2}}}$$

P15.56 (a) In $x = A\cos(\omega t + \phi)$, $v = -\omega A\sin(\omega t + \phi)$

we have at $t = 0$ $v = -\omega A\sin\phi = -v_{max}$

This requires $\phi = 90°$, so $x = A\cos(\omega t + 90°)$

And this is equivalent to $x = -A\sin\omega t$

Numerically we have $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50\ \text{N/m}}{0.5\ \text{kg}}} = 10\ \text{s}^{-1}$

and $v_{max} = \omega A$ $20\ \text{m/s} = (10\ \text{s}^{-1})A$ $A = 2\ \text{m}$

So $\boxed{x = (-2\ \text{m})\sin\left[(10\ \text{s}^{-1})t\right]}$

(b) In $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$, $\frac{1}{2}kx^2 = 3\left(\frac{1}{2}mv^2\right)$

implies $\frac{1}{3}\frac{1}{2}kx^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ $\frac{4}{3}x^2 = A^2$

$$x = \pm\sqrt{\frac{3}{4}}A = \pm 0.866A = \boxed{\pm 1.73\ \text{m}}$$

(c) $\omega = \sqrt{\frac{g}{L}}$ $L = \frac{g}{\omega^2} = \frac{9.8\ \text{m/s}^2}{(10\ \text{s}^{-1})^2} = \boxed{0.098\,0\ \text{m}}$

(d) In $x = (-2\ \text{m})\sin\left[(10\ \text{s}^{-1})t\right]$

the particle is at $x = 0$ at $t = 0$, at $10t = \pi$ s, and so on.

The particle is at $x = 1\ \text{m}$

when $-\frac{1}{2} = \sin\left[(10\ \text{s}^{-1})t\right]$

with solutions $(10\ \text{s}^{-1})t = -\frac{\pi}{6}$

$(10\ \text{s}^{-1})t = \pi + \frac{\pi}{6}$, and so on.

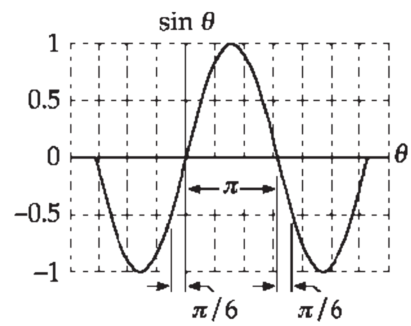


FIG. P15.56(d)

The minimum time for the motion is Δt in $10\Delta t = \left(\frac{\pi}{6}\right)\text{s}$

$$\Delta t = \left(\frac{\pi}{60}\right)\text{s} = \boxed{0.052\,4\ \text{s}}$$

P15.57 (a) At equilibrium, we have

$$\sum \tau = 0 - mg\left(\frac{L}{2}\right) + kx_0 L$$

where x_0 is the equilibrium compression.

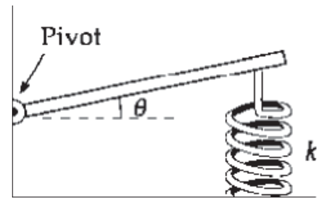


FIG. P15.57

After displacement by a small angle,

$$\sum \tau = -mg\left(\frac{L}{2}\right) + kxL = -mg\left(\frac{L}{2}\right) + k\left(x_0 - L\theta\right)L = -k\theta L^2$$

But,

$$\sum \tau = I\alpha = \frac{1}{3}mL^2\frac{d^2\theta}{dt^2}$$

So

$$\frac{d^2\theta}{dt^2} = -\frac{3k}{m}\theta$$

The angular acceleration is opposite in direction and proportional to the displacement, so we have simple harmonic motion with $\boxed{\omega^2 = \frac{3k}{m}}$.

(b) $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{3k}{m}} = \frac{1}{2\pi}\sqrt{\frac{3(100\ \text{N/m})}{5.00\ \text{kg}}} = \boxed{1.23\ \text{Hz}}$

P15.58 As it passes through equilibrium, the 4-kg object has speed

$$v_{\max} = \omega A = \sqrt{\frac{k}{m}}A = \sqrt{\frac{100\ \text{N/m}}{4\ \text{kg}}}2\ \text{m} = 10.0\ \text{m/s}$$

In the completely inelastic collision momentum of the two-object system is conserved. So the new 10-kg object starts its oscillation with speed given by

$$4\ \text{kg}(10\ \text{m/s}) + (6\ \text{kg})0 = (10\ \text{kg})v_{\max}$$
$$v_{\max} = 4.00\ \text{m/s}$$

(a) The new amplitude is given by $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$

$$10\ \text{kg}(4\ \text{m/s})^2 = (100\ \text{N/m})A^2$$
$$A = 1.26\ \text{m}$$

Thus the amplitude has $\boxed{\text{decreased by}}$ $2.00\ \text{m} - 1.26\ \text{m} = \boxed{0.735\ \text{m}}$

(b) The old period was $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4\ \text{kg}}{100\ \text{N/m}}} = 1.26\ \text{s}$

The new period is $T = 2\pi\sqrt{\frac{10}{100}\ \text{s}^2} = 1.99\ \text{s}$

The period has $\boxed{\text{increased by}}$ $1.99\ \text{m} - 1.26\ \text{m} = \boxed{0.730\ \text{s}}$

continued on next page

(c) The old energy was $\frac{1}{2}mv_{max}^2 = \frac{1}{2}(4\text{ kg})(10\text{ m/s})^2 = 200\text{ J}$

The new mechanical energy is $\frac{1}{2}(10\text{ kg})(4\text{ m/s})^2 = 80\text{ J}$

The energy has $\boxed{\text{decreased by 120 J}}$.

(d) The missing mechanical energy has turned into internal energy in the completely inelastic collision.

P15.59 (a) $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} = \boxed{3.00\text{ s}}$

(b) $E = \frac{1}{2}mv^2 = \frac{1}{2}(6.74)(2.06)^2 = \boxed{14.3\text{ J}}$

(c) At maximum angular displacement $mgh = \frac{1}{2}mv^2$ $h = \frac{v^2}{2g} = 0.217\text{ m}$

$$h = L - L\cos\theta = L(1-\cos\theta) \qquad \cos\theta = 1 - \frac{h}{L} \qquad \boxed{\theta = 25.5°}$$

P15.60 One can write the following equations of motion:

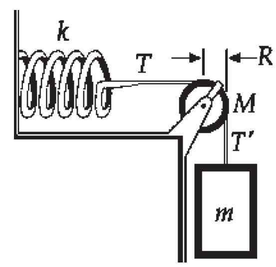


FIG. P15.60

$T - kx = 0$ (describes the spring)

$mg - T' = ma = m\frac{d^2x}{dt^2}$ (for the hanging object)

$R(T' - T) = I\frac{d^2\theta}{dt^2} = \frac{I}{R}\frac{d^2x}{dt^2}$ (for the pulley)

with $I = \frac{1}{2}MR^2$

Combining these equations gives the equation of motion

$$\left(m + \frac{1}{2}M\right)\frac{d^2x}{dt^2} + kx = mg$$

The solution is $x(t) = A\sin\omega t + \frac{mg}{k}$ (where $\frac{mg}{k}$ arises because of the extension of the spring due to the weight of the hanging object), with frequency

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m+\frac{1}{2}M}} = \frac{1}{2\pi}\sqrt{\frac{100\text{ N/m}}{0.200\text{ kg}+\frac{1}{2}M}}$$

(a) For $M = 0$ $f = \boxed{3.56\text{ Hz}}$

(b) For $M = 0.250\text{ kg}$ $f = \boxed{2.79\text{ Hz}}$

(c) For $M = 0.750\text{ kg}$ $f = \boxed{2.10\text{ Hz}}$

P15.61 Suppose a 100-kg biker compresses the suspension 2.00 cm.

Then,

$$k = \frac{F}{x} = \frac{980 \text{ N}}{2.00 \times 10^{-2} \text{ m}} = 4.90 \times 10^{4} \text{ N/m}$$

If total mass of motorcycle and biker is 500 kg, the frequency of free vibration is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.90 \times 10^{4} \text{ N/m}}{500 \text{ kg}}} = 1.58 \text{ Hz}$$

> If he encounters washboard bumps at the same frequency as the free vibration, resonance will make the motorcycle bounce a lot. It may bounce so much as to interfere with the rider's control of the machine.

Assuming a speed of 20.0 m/s, we find these ridges are separated by

$$\frac{20.0 \text{ m/s}}{1.58 \text{ s}^{-1}} = 12.7 \text{ m} \boxed{\sim 10^{1} \text{ m}}$$

In addition to this vibration mode of bouncing up and down as one unit, the motorcycle can also vibrate at higher frequencies by rocking back and forth between front and rear wheels, by having just the front wheel bounce inside its fork, or by doing other things. Other spacing of bumps will excite all of these other resonances.

P15.62 (a) For each segment of the spring

$$dK = \frac{1}{2}(dm)v_x^2$$

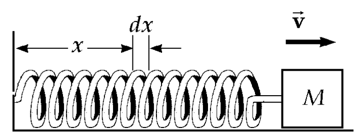


FIG. P15.62

Also,

$$v_x = \frac{x}{\ell}v \qquad \text{and} \qquad dm = \frac{m}{\ell}dx$$

Therefore, the total kinetic energy of the block-spring system is

$$K = \frac{1}{2}Mv^2 + \frac{1}{2}\int_0^{\ell}\left(\frac{x^2v^2}{\ell^2}\right)\frac{m}{\ell}dx = \boxed{\frac{1}{2}\left(M + \frac{m}{3}\right)v^2}$$

(b) $\omega = \sqrt{\dfrac{k}{m_{eff}}}$ and $\dfrac{1}{2}m_{eff}v^2 = \dfrac{1}{2}\left(M + \dfrac{m}{3}\right)v^2$

Therefore,

$$T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{M + m/3}{k}}}$$

P15.63 (a) $\sum \vec{\mathbf{F}} = -2T\sin\theta\,\hat{\mathbf{j}}$ where $\theta = \tan^{-1}\left(\dfrac{y}{L}\right)$

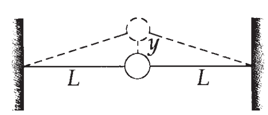


FIG. P15.63

Therefore, for a small displacement

$$\sin\theta \approx \tan\theta = \frac{y}{L} \qquad \text{and} \qquad \boxed{\sum \vec{\mathbf{F}} = \frac{-2Ty}{L}\hat{\mathbf{j}}}$$

(b) The total force exerted on the ball is opposite in direction and proportional to its displacement from equilibrium, so the ball moves with simple harmonic motion. For a spring system,

$$\sum \vec{\mathbf{F}} = -k\vec{\mathbf{x}} \qquad \text{becomes here} \qquad \sum \vec{\mathbf{F}} = -\frac{2T}{L}\vec{\mathbf{y}}$$

Therefore, the effective spring constant is $\dfrac{2T}{L}$ and $\boxed{\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{2T}{mL}}}$.

P15.64 (a) Assuming a Hooke's Law type spring,

$$F = Mg = kx$$

and empirically

$$Mg = 1.74x - 0.113$$

so

$$k = \boxed{1.74\ \text{N/m} \pm 6\%}$$

M, kg	x, m	Mg, N
0.020 0	0.17	0.196
0.040 0	0.293	0.392
0.050 0	0.353	0.49
0.060 0	0.413	0.588
0.070 0	0.471	0.686
0.080 0	0.493	0.784

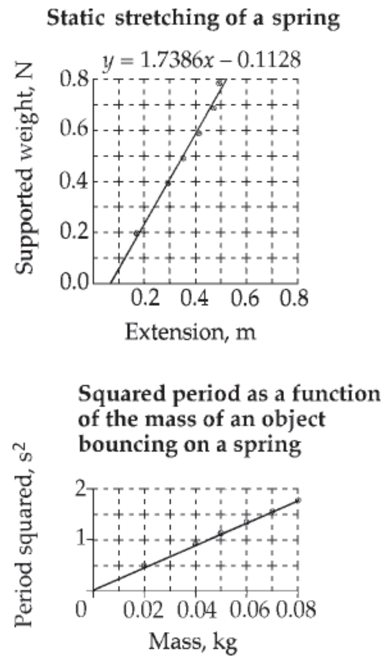


FIG. P15.64

(b) We may write the equation as theoretically

$$T^2 = \frac{4\pi^2}{k}M + \frac{4\pi^2}{3k}m_s$$

and empirically

$$T^2 = 21.7M + 0.058\,9$$

so

$$k = \frac{4\pi^2}{21.7} = \boxed{1.82\ \text{N/m} \pm 3\%}$$

Time, s	T, s	M, kg	T^2, s^2
7.03	0.703	0.020 0	0.494
9.62	0.962	0.040 0	0.925
10.67	1.067	0.050 0	1.138
11.67	1.167	0.060 0	1.362
12.52	1.252	0.070 0	1.568
13.41	1.341	0.080 0	1.798

The k values $1.74\ \text{N/m} \pm 6\%$

and $1.82\ \text{N/m} \pm 3\%$ differ by 4% so $\boxed{\text{they agree}}$.

(c) Utilizing the axis-crossing point, $m_s = 3\left(\dfrac{0.058\,9}{21.7}\right)\text{kg} = \boxed{8\ \text{grams} \pm 12\%}$

$\boxed{\text{in agreement}}$ with 7.4 grams.

P15.65 (a) $\Delta K + \Delta U = 0$

Thus, $K_{\text{top}} + U_{\text{top}} = K_{\text{bot}} + U_{\text{bot}}$

where $K_{\text{top}} = U_{\text{bot}} = 0$

Therefore, $mgh = \frac{1}{2} I\omega^2$, but

$$h = R - R\cos\theta = R(1-\cos\theta)$$

$$\omega = \frac{v}{R}$$

and $I = \frac{MR^2}{2} + \frac{mr^2}{2} + mR^2$

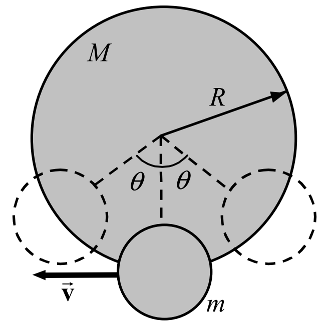


FIG. P15.65

Substituting we find

$$mgR(1-\cos\theta) = \frac{1}{2}\left(\frac{MR^2}{2} + \frac{mr^2}{2} + mR^2\right)\frac{v^2}{R^2}$$

$$mgR(1-\cos\theta) = \left[\frac{M}{4} + \frac{mr^2}{4R^2} + \frac{m}{2}\right]v^2$$

and $v^2 = 4gR\dfrac{(1-\cos\theta)}{\left(M/m + r^2/R^2 + 2\right)}$

so $\boxed{v = 2\sqrt{\dfrac{Rg(1-\cos\theta)}{M/m + r^2/R^2 + 2}}}$

(b) $T = 2\pi\sqrt{\dfrac{I}{m_T g d_{\text{CM}}}}$

$m_T = m + M \qquad d_{\text{CM}} = \dfrac{mR + M(0)}{m+M}$

$$T = \boxed{2\pi\sqrt{\frac{\frac{1}{2}MR^2 + \frac{1}{2}mr^2 + mR^2}{mgR}}}$$

P15.66 (a) We require $Ae^{-bt/2m} = \dfrac{A}{2} \qquad e^{+bt/2m} = 2$

or $\dfrac{bt}{2m} = \ln 2$ or $\dfrac{0.100\ \text{kg/s}}{2(0.375\ \text{kg})}t = 0.693 \qquad \therefore t = \boxed{5.20\ \text{s}}$

The spring constant is irrelevant.

(b) We can evaluate the energy at successive turning points, where

$\cos(\omega t + \phi) = \pm 1$ and the energy is $\frac{1}{2}kx^2 = \frac{1}{2}kA^2e^{-bt/m}$. We require $\frac{1}{2}kA^2e^{-bt/m} = \frac{1}{2}\left(\frac{1}{2}kA^2\right)$

or $e^{+bt/m} = 2 \qquad \therefore t = \dfrac{m\ln 2}{b} = \dfrac{0.375\ \text{kg}(0.693)}{0.100\ \text{kg/s}} = \boxed{2.60\ \text{s}}$

(c) From $E = \frac{1}{2}kA^2$, the fractional rate of change of energy over time is

$$\frac{dE/dt}{E} = \frac{(d/dt)\left(\frac{1}{2}kA^2\right)}{\frac{1}{2}kA^2} = \frac{\frac{1}{2}k(2A)(dA/dt)}{\frac{1}{2}kA^2} = 2\frac{dA/dt}{A}$$

two times faster than the fractional rate of change in amplitude.

P15.67 (a) When the mass is displaced a distance x from equilibrium, spring 1 is stretched a distance x_1 and spring 2 is stretched a distance x_2.

By Newton's third law, we expect

$$k_1x_1 = k_2x_2.$$

When this is combined with the requirement that

$$x = x_1 + x_2,$$

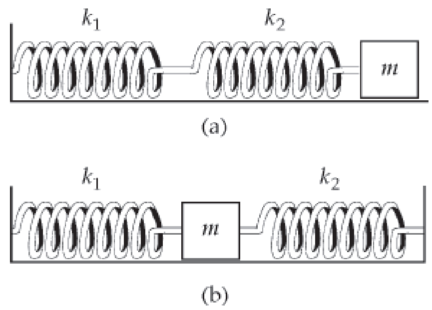


FIG. P15.67

we find $x_1 = \left[\dfrac{k_2}{k_1 + k_2}\right]x$

The force on either spring is given by $F_1 = \left[\dfrac{k_1k_2}{k_1 + k_2}\right]x = ma$

where a is the acceleration of the mass m.

This is in the form $F = k_{eff}x = ma$

and $T = 2\pi\sqrt{\dfrac{m}{k_{eff}}} = \boxed{2\pi\sqrt{\dfrac{m(k_1 + k_2)}{k_1k_2}}}$

(b) In this case each spring is distorted by the distance x which the mass is displaced. Therefore, the restoring force is

$$F = -(k_1 + k_2)x \qquad \text{and} \qquad k_{eff} = k_1 + k_2$$

so that $T = \boxed{2\pi\sqrt{\dfrac{m}{(k_1 + k_2)}}}$

P15.68 Let ℓ represent the length below water at equilibrium and M the tube's mass:

$$\sum F_y = 0 \Rightarrow -Mg + \rho\pi r^2\ell g = 0$$

Now with any excursion x from equilibrium

$$-Mg + \rho\pi r^2(\ell - x)g = Ma$$

Subtracting the equilibrium equation gives

$$-\rho\pi r^2 gx = Ma$$

$$a = -\left(\frac{\rho\pi r^2 g}{M}\right)x = -\omega^2 x$$

$\boxed{\text{The opposite direction and direct proportionality of } a \text{ to } x \text{ imply SHM}}$ with angular frequency

$$\omega = \sqrt{\frac{\rho\pi r^2 g}{M}}$$

$$T = \frac{2\pi}{\omega} = \boxed{\left(\frac{2}{r}\right)\sqrt{\frac{\pi M}{\rho g}}}$$

The acceleration $a = -\rho\pi r^2 gx/M$ is a negative constant times the displacement from equilibrium.

$$T = \frac{2}{r}\sqrt{\frac{\pi M}{\rho g}}$$

P15.69 (a) Newton's law of universal gravitation is $F = -\frac{GMm}{r^2} = -\frac{Gm}{r^2}\left(\frac{4}{3}\pi r^3\right)\rho$

Thus, $F = -\left(\frac{4}{3}\pi\rho Gm\right)r$

Which is of Hooke's law form with $k = \frac{4}{3}\pi\rho Gm$

(b) The sack of mail moves without friction according to $-\left(\frac{4}{3}\right)\pi\rho Gmr = ma$

$$a = -\left(\frac{4}{3}\right)\pi\rho Gr = -\omega^2 r$$

Since acceleration is a negative constant times excursion from equilibrium, it executes SHM with

$\omega = \sqrt{\frac{4\pi\rho G}{3}}$ and period $T = \frac{2\pi}{\omega} = \sqrt{\frac{3\pi}{\rho G}}$

The time for a one-way trip through the earth is $\frac{T}{2} = \sqrt{\frac{3\pi}{4\rho G}}$

We have also $g = \frac{GM_e}{R_e^2} = \frac{G4\pi R_e^3\rho}{3R_e^2} = \frac{4}{3}\pi\rho GR_e$

so

$$\frac{4\rho G}{3} = \frac{g}{(\pi R_e)}$$

and

$$\frac{T}{2} = \pi\sqrt{\frac{R_e}{g}} = \pi\sqrt{\frac{6.37\times10^6\text{ m}}{9.8\text{ m/s}^2}} = 2.53\times10^3\text{ s} = \boxed{42.2\text{ min}}$$

P15.70 (a) The block moves with the board in what we take as the positive x direction, stretching the spring until the spring force $-kx$ is equal in magnitude to the maximum force of static friction

$\mu_s n = \mu_s mg$. This occurs at $x = \frac{\mu_s mg}{k}$

(b) Since v is small, the block is nearly at the rest at this break point. It starts almost immediately to move back to the left, the forces on it being $-kx$ and $+\mu_k mg$. While it is sliding the net force exerted on it can be written as

$$-kx + \mu_k mg = -kx + \frac{k\mu_k mg}{k} = -k\left(x - \frac{\mu_k mg}{k}\right) = -kx_{rel}$$

where x_{rel} is the excursion of the block away from the point $\frac{\mu_k mg}{k}$.

Conclusion: the block goes into simple harmonic motion centered about the equilibrium position where the spring is stretched by $\frac{\mu_k mg}{k}$.

(d) The amplitude of its motion is its original displacement, $A = \frac{\mu_s mg}{k} - \frac{\mu_k mg}{k}$. It first comes to rest at spring extension $\frac{\mu_k mg}{k} - A = \frac{(2\mu_k - \mu_s)mg}{k}$. Almost immediately at this point it latches onto the slowly-moving board to move with the board. The board exerts a force of static friction on the block, and the cycle continues.

continued on next page

(c) The graph of the motion looks like this:

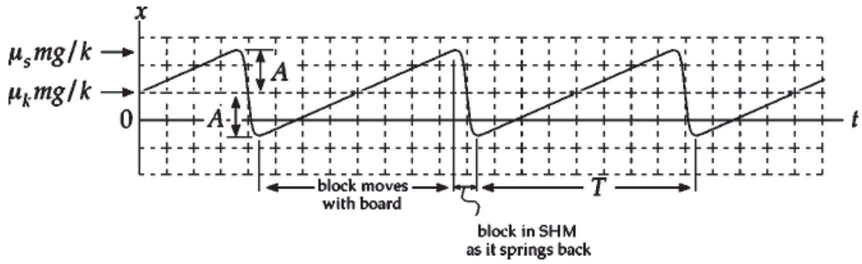


FIG. P15.70(c)

(e) The time during each cycle when the block is moving with the board is $\dfrac{2A}{v}=\dfrac{2(\mu_s-\mu_k)mg}{kv}$.
The time for which the block is springing back is one half a cycle of simple harmonic motion, $\dfrac{1}{2}\left(2\pi\sqrt{\dfrac{m}{k}}\right)=\pi\sqrt{\dfrac{m}{k}}$. We ignore the times at the end points of the motion when the speed of the block changes from v to 0 and from 0 to v. Since v is small compared to $\dfrac{2A}{\pi\sqrt{m/k}}$, these times are negligible. Then the period is

$$T=\frac{2(\mu_s-\mu_k)mg}{kv}+\pi\sqrt{\frac{m}{k}}$$

(f) $$T=\frac{2(0.4-0.25)(0.3\text{ kg})(9.8\text{ m/s}^2)}{(0.024\text{ m/s})(12\text{ N/m})}+\pi\sqrt{\frac{0.3\text{ kg}}{12\text{ N/m}}}=3.06\text{ s}+0.497\text{ s}=3.56\text{ s}$$

Then

$$f=\frac{1}{T}=\boxed{0.281\text{ Hz}}$$

(g) $T=\dfrac{2(\mu_s-\mu_k)mg}{kv}+\pi\sqrt{\dfrac{m}{k}}$ increases as m increases, so the frequency $\boxed{\text{decreases}}$.

(h) As k increases, T decreases and f $\boxed{\text{increases}}$.

(i) As v increases, T decreases and f $\boxed{\text{increases}}$.

(j) As $(\mu_s-\mu_k)$ increases, T increases and f $\boxed{\text{decreases}}$.

ANSWERS TO EVEN PROBLEMS

P15.2 (a) 4.33 cm (b) −5.00 cm/s (c) −17.3 cm/s^2 (d) 3.14 s; 5.00 cm

P15.4 (a) 15.8 cm (b) −15.9 cm (c) The patterns of oscillation diverge from each other, starting out in phase but becoming completely out of phase. To calculate the future we would need exact knowledge of the present, an impossibility. (d) 51.1 m (e) 50.7 m

P15.6 (a) 2.40 s (b) 0.417 Hz (c) 2.62 rad/s

P15.8 (a) −2.34 m (b) −1.30 m/s (c) −0.076 3 m (d) 0.315 m/s

P15.10 (a) 1.26 s (b) 0.150 m/s; 0.750 $\mathrm{m/s^2}$ (c) $x = -3$ cm cos $5t$; $v = \left(\dfrac{15\text{ cm}}{\text{s}}\right)\sin 5t$; $a = \left(\dfrac{75\text{ cm}}{\text{s}^2}\right)\cos 5t$

P15.12 Yes. Whether the object has small or large mass, the ratio *m*/*k* must be equal to 0.183 m/(9.80 $\mathrm{m/s^2}$). The period is 0.859 s.

P15.14 (a) 126 N/m (b) 0.178 m

P15.16 (a) 0.153 J (b) 0.784 m/s (c) 17.5 $\mathrm{m/s^2}$

P15.18 (a) 100 N/m (b) 1.13 Hz (c) 1.41 m/s at $x = 0$ (d) 10.0 $\mathrm{m/s^2}$ at $x = \pm A$ (e) 2.00 J (f) 1.33 m/s (g) 3.33 $\mathrm{m/s^2}$

P15.20 (a) 1.50 s (b) 73.4 N/m (c) 19.7 m below the bridge (d) 1.06 rad/s (e) 2.01 s (f) 3.50 s

P15.22 The position of the piston is given by $x = A\cos\omega t$.

P15.24 $\dfrac{g_C}{g_T} = 1.0015$

P15.26 1.42 s; 0.499 m

P15.28 (a) 3.65 s (b) 6.41 s (c) 4.24 s

P15.30 (a) see the solution (b), (c) 9.85 $\mathrm{m/s^2}$, agreeing with the accepted value within 0.5%

P15.32 (a) 2.09 s (b) 4.08%

P15.34 see the solution

P15.36 see the solution

P15.38 (a) 2.95 Hz (b) 2.85 cm

P15.40 see the solution

P15.42 either 1.31 Hz or 0.641 Hz

P15.44 (a) 0.368 $\hat{\mathbf{i}}$ m/s (b) 3.51 cm (c) Conservation of momentum for the glider-spring-glider system requires that the center of mass move with constant velocity. Conservation of mechanical energy for the system implies that in the center-of-mass reference frame the gliders both oscillate after they couple together. (d) 40.6 mJ (e) 27.7 mJ

P15.46 (a) 4.31 cm (b) When the rock is on the point of lifting off, the surrounding water is also barely in free fall. No pressure gradient exists in the water, so no buoyant force acts on the rock.

P15.48 (a) 0.500 m/s (b) 8.56 cm

P15.50 $A = \dfrac{\mu_s g}{4\pi^2 f^2}$

P15.52 (a) A time interval. If the interaction occupied no time, the force exerted by each ball on the other would be infinite, and that cannot happen. (b) 80.0 MN/m (c) 0.843 J. (d) 0.145 mm (e) 9.12×10^{-5} s

P15.54 (a) $k = \dfrac{4\pi^2 m}{T^2}$ (b) $m' = m\left(\dfrac{T'}{T}\right)^2$

P15.56 (a) $x = -(2\text{ m})\sin(10\,t)$ (b) at $x = \pm 1.73$ m (c) 98.0 mm (d) 52.4 ms

P15.58 (a) The amplitude is reduced by 0.735 m (b) The period increases by 0.730 s (c) The energy decreases by 120 J (d) Mechanical energy is converted into internal energy in the perfectly inelastic collision.

P15.60 (a) 3.56 Hz (b) 2.79 Hz (c) 2.10 Hz

P15.62 (a) $\dfrac{1}{2}\left(M + \dfrac{m}{3}\right)v^2$ (b) $T = 2\pi\sqrt{\dfrac{M + m/3}{k}}$

P15.64 see the solution (a) $k = 1.74\ \text{N/m} \pm 6\%$ (b) $1.82\ \text{N/m} \pm 3\%$; they agree (c) $8\ \text{g} \pm 12\%$; it agrees

P15.66 (a) 5.20 s (b) 2.60 s (c) see the solution

P15.68 The acceleration $a = -\rho\pi r^2 gx/M$ is a negative constant times the displacement from equilibrium.

$$T = \frac{2}{r}\sqrt{\frac{\pi M}{\rho g}}$$

P15.70 see the solution (f) 0.281 Hz (g) decreases (h) increases (i) increases (j) decreases

16

Wave Motion

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q16.1 As the pulse moves down the string, the particles of the string itself move side to side. Since the medium—here, the string—moves perpendicular to the direction of wave propagation, the wave is transverse by definition.

Q16.2 To use a slinky to create a longitudinal wave, pull a few coils back and release. For a transverse wave, jostle the end coil side to side.

***Q16.3** (i) Look at the coefficients of the sine and cosine functions: 2, 4, 6, 8, 8, 7. The ranking is d = e > f > c > b > a.

(ii) Look at the coefficients of x. Each is the wave number, $2\pi/\lambda$, so the smallest k goes with the largest wavelength. The ranking is d > a = b = c > e > f.

(iii) Look at the coefficients of t. The absolute value of each is the angular frequency $\omega = 2\pi f$. The ranking is f > e > a = b = c = d.

(iv) Period is the reciprocal of frequency, so the ranking is the reverse of that in part iii: d = c = b = a > e > f.

(v) From $v = f\lambda = \omega/k$, we compute the absolute value of the ratio of the coefficient of t to the coefficient of x in each case. From a to f respectively the numerical speeds are 5, 5, 5, 7.5, 5, 4. The ranking is d > a = b = c = e > f.

***Q16.4** From $v = \sqrt{\dfrac{T}{\mu}}$, we must increase the tension by a factor of 4 to make v double. Answer (b).

***Q16.5** Answer (b). Wave speed is inversely proportional to the square root of linear density.

***Q16.6** (i) Answer (a). Higher tension makes wave speed higher.

(ii) Answer (b). Greater linear density makes the wave move more slowly.

Q16.7 It depends on from what the wave reflects. If reflecting from a less dense string, the reflected part of the wave will be right side up.

Q16.8 Yes, among other things it depends on. The particle speed is described by $v_{y,\max} = \omega A = 2\pi f A = \dfrac{2\pi v A}{\lambda}$. Here v is the speed of the wave.

***Q16.9** (a) through (d): Yes to all. The maximum particle speed and the wave speed are related by $v_{y,\max} = \omega A = 2\pi fA = \dfrac{2\pi vA}{\lambda}$. Thus the amplitude or the wavelength of the wave can be adjusted to make either $v_{y,\max}$ or v larger.

Q16.10 Since the frequency is 3 cycles per second, the period is $\dfrac{1}{3}$ second = 333 ms.

Q16.11 Each element of the rope must support the weight of the rope below it. The tension increases with height. (It increases linearly, if the rope does not stretch.) Then the wave speed $v = \sqrt{\dfrac{T}{\mu}}$ increases with height.

***Q16.12** Answer (c). If the frequency does not change, the amplitude is increased by a factor of $\sqrt{2}$. The wave speed does not change.

***Q16.13** (i) Answer a. As the wave passes from the massive string to the less massive string, the wave speed will increase according to $v = \sqrt{\dfrac{T}{\mu}}$.

(ii) Answer c. The frequency will remain unchanged. However often crests come up to the boundary they leave the boundary.

(iii) Answer a. Since $v = f\lambda$, the wavelength must increase.

Q16.14 Longitudinal waves depend on the compressibility of the fluid for their propagation. Transverse waves require a restoring force in response to shear strain. Fluids do not have the underlying structure to supply such a force. A fluid cannot support static shear. A viscous fluid can temporarily be put under shear, but the higher its viscosity the more quickly it converts input work into internal energy. A local vibration imposed on it is strongly damped, and not a source of wave propagation.

Q16.15 Let $\Delta t = t_s - t_p$ represent the difference in arrival times of the two waves at a station at distance $d = v_s t_s = v_p t_p$ from the hypocenter. Then $d = \Delta t\left(\dfrac{1}{v_s} - \dfrac{1}{v_p}\right)^{-1}$. Knowing the distance from the first station places the hypocenter on a sphere around it. A measurement from a second station limits it to another sphere, which intersects with the first in a circle. Data from a third non-collinear station will generally limit the possibilities to a point.

Q16.16 The speed of a wave on a "massless" string would be infinite!

SOLUTIONS TO PROBLEMS

Section 16.1 Propagation of a Disturbance

P16.1 Replace x by $x - vt = x - 4.5t$

to get $\boxed{y = \dfrac{6}{\left[(x-4.5t)^2 + 3\right]}}$

***P16.2**

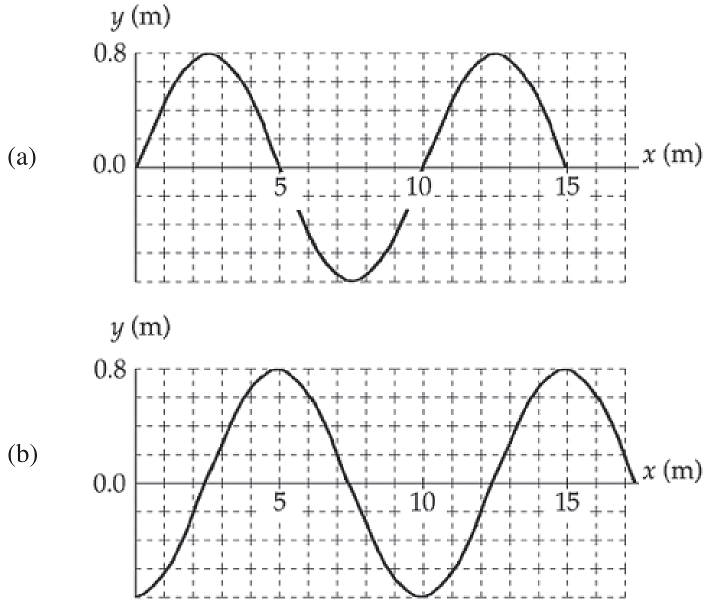


FIG. P16.2

The graph (b) has the same amplitude and wavelength as graph (a). it differs just by being shifted toward larger x by 2.40 m. The wave has traveled 2.40 m to the right.

P16.3 (a) The $\boxed{\text{longitudinal P wave}}$ travels a shorter distance and is moving faster, so it will arrive at point B first.

(b) The wave that travels through the Earth must travel

a distance of $2R\sin 30.0° = 2(6.37\times10^6\text{ m})\sin 30.0° = 6.37\times10^6\text{ m}$

at a speed of 7 800 m/s

Therefore, it takes $\dfrac{6.37\times10^6\text{ m}}{7\,800\text{ m/s}} = 817\text{ s}$

The wave that travels along the Earth's surface must travel

a distance of $s = R\theta = R\left(\dfrac{\pi}{3}\text{ rad}\right) = 6.67\times10^6\text{ m}$

at a speed of 4 500 m/s

Therefore, it takes $\dfrac{6.67\times10^6}{4\,500} = 1\,482\text{ s}$

The time difference is $\boxed{665\text{ s}} = 11.1\text{ min}$

P16.4 The distance the waves have traveled is $d = (7.80\text{ km/s})t = (4.50\text{ km/s})(t + 17.3\text{ s})$ where t is the travel time for the faster wave.

Then, $(7.80 - 4.50)(\text{km/s})t = (4.50\text{ km/s})(17.3\text{ s})$

or $t = \dfrac{(4.50\text{ km/s})(17.3\text{ s})}{(7.80-4.50)\text{ km/s}} = 23.6\text{ s}$

and the distance is $d = (7.80\text{ km/s})(23.6\text{ s}) = \boxed{184\text{ km}}$.

P16.5 (a) Let $u = 10\pi t - 3\pi x + \dfrac{\pi}{4}$ $\qquad \dfrac{du}{dt} = 10\pi - 3\pi\dfrac{dx}{dt} = 0$ at a point of constant phase

$$\frac{dx}{dt} = \frac{10}{3} = \boxed{3.33\ \text{m/s}}$$

The velocity is in the $\boxed{\text{positive } x\text{-direction}}$.

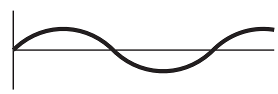
(b) $y(0.100,\ 0) = (0.350\ \text{m})\sin\left(-0.300\pi + \dfrac{\pi}{4}\right) = -0.054\,8\ \text{m} = \boxed{-5.48\ \text{cm}}$

(c) $k = \dfrac{2\pi}{\lambda} = 3\pi$: $\qquad \lambda = \boxed{0.667\ \text{m}}$ $\qquad \omega = 2\pi f = 10\pi$: $\qquad f = \boxed{5.00\ \text{Hz}}$

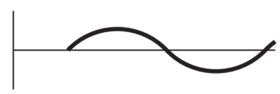
(d) $v_y = \dfrac{\partial y}{\partial t} = (0.350)(10\pi)\cos\left(10\pi t - 3\pi x + \dfrac{\pi}{4}\right)$ $\qquad v_{y,\,\max} = (10\pi)(0.350) = \boxed{11.0\ \text{m/s}}$

Section 16.2 **The Traveling Wave Model**

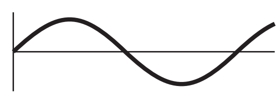
***P16.6** (a) a wave



(b) later by $T/4$

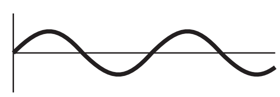


(c) A is 1.5 times larger



(d) λ is 1.5 times larger

(e) λ is 2/3 as large



P16.7 $f = \dfrac{40.0\ \text{vibrations}}{30.0\ \text{s}} = \dfrac{4}{3}\ \text{Hz}$ $\qquad v = \dfrac{425\ \text{cm}}{10.0\ \text{s}} = 42.5\ \text{cm/s}$

$$\lambda = \frac{v}{f} = \frac{42.5\ \text{cm/s}}{\frac{4}{3}\ \text{Hz}} = 31.9\ \text{cm} = \boxed{0.319\ \text{m}}$$

P16.8 Using data from the observations, we have $\lambda = 1.20\ \text{m}$ and $f = \dfrac{8.00}{12.0\ \text{s}}$

Therefore, $\qquad v = \lambda f = (1.20\ \text{m})\left(\dfrac{8.00}{12.0\ \text{s}}\right) = \boxed{0.800\ \text{m/s}}$

P16.9 $y = (0.020\,0\ \text{m})\sin(2.11x - 3.62t)$ in SI units $\qquad A = \boxed{2.00\ \text{cm}}$

$k = 2.11\ \text{rad/m}$ $\qquad \lambda = \dfrac{2\pi}{k} = \boxed{2.98\ \text{m}}$

$\omega = 3.62\ \text{rad/s}$ $\qquad f = \dfrac{\omega}{2\pi} = \boxed{0.576\ \text{Hz}}$

$$v = f\lambda = \frac{\omega}{2\pi}\frac{2\pi}{k} = \frac{3.62}{2.11} = \boxed{1.72\ \text{m/s}}$$

P16.10 $v = f\lambda = (4.00\text{ Hz})(60.0\text{ cm}) = 240\text{ cm/s} = \boxed{2.40\text{ m/s}}$

***P16.11** (a) $\omega = 2\pi f = 2\pi\left(5\text{ s}^{-1}\right) = \boxed{31.4\text{ rad/s}}$

(b) $\lambda = \dfrac{v}{f} = \dfrac{20\text{ m/s}}{5\text{ s}^{-1}} = 4.00\text{ m}$

$k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{4\text{ m}} = \boxed{1.57\text{ rad/m}}$

(c) In $y = A\sin(kx - \omega t + \phi)$ we take $A = 12$ cm. At $x = 0$ and $t = 0$ we have $y = (12\text{ cm})\sin\phi$. To make this fit $y = 0$, we take $\phi = 0$. Then

$\boxed{y = (12.0\text{ cm})\sin\left((1.57\text{ rad/m})x - (31.4\text{ rad/s})t\right)}$

(d) The transverse velocity is $\dfrac{\partial y}{\partial t} = -A\omega\cos(kx - \omega t)$

Its maximum magnitude is $A\omega = 12\text{ cm}(31.4\text{ rad/s}) = \boxed{3.77\text{ m/s}}$

(e) $a_y = \dfrac{\partial v_y}{\partial t} = \dfrac{\partial}{\partial t}\left(-A\omega\cos(kx - \omega t)\right) = -A\omega^2\sin(kx - \omega t)$

The maximum value is $A\omega^2 = (0.12\text{ m})\left(31.4\text{ s}^{-1}\right)^2 = \boxed{118\text{ m/s}^2}$

P16.12 At time t, the phase of $y = (15.0\text{ cm})\cos(0.157x - 50.3t)$ at coordinate x is $\phi = (0.157\text{ rad/cm})x - (50.3\text{ rad/s})t$. Since $60.0° = \dfrac{\pi}{3}$ rad, the requirement for point B is that $\phi_B = \phi_A \pm \dfrac{\pi}{3}$ rad, or (since $x_A = 0$),

$$(0.157\text{ rad/cm})x_B - (50.3\text{ rad/s})t = 0 - (50.3\text{ rad/s})t \pm \frac{\pi}{3}\text{ rad}$$

This reduces to $x_B = \dfrac{\pm\pi\text{ rad}}{3(0.157\text{ rad/cm})} = \boxed{\pm 6.67\text{ cm}}$.

P16.13 $y = 0.250\sin(0.300x - 40.0t)$ m

Compare this with the general expression $y = A\sin(kx - \omega t)$

(a) $A = \boxed{0.250\text{ m}}$

(b) $\omega = \boxed{40.0\text{ rad/s}}$

(c) $k = \boxed{0.300\text{ rad/m}}$

(d) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{0.300\text{ rad/m}} = \boxed{20.9\text{ m}}$

(e) $v = f\lambda = \left(\dfrac{\omega}{2\pi}\right)\lambda = \left(\dfrac{40.0\text{ rad/s}}{2\pi}\right)(20.9\text{ m}) = \boxed{133\text{ m/s}}$

(f) The wave moves to the right, $\boxed{\text{in } +x \text{ direction}}$.

***P16.14** (a) See figure at right.

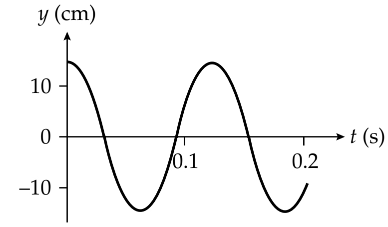


FIG. P16.14

(b) $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{50.3} = \boxed{0.125\text{ s}}$ is the time from one peak to the next one.

$\boxed{\text{This agrees with the period found in the example in the text.}}$

P16.15 (a) $A = y_{\max} = 8.00\text{ cm} = 0.080\,0\text{ m}$ $\qquad k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{(0.800\text{ m})} = 7.85\text{ m}^{-1}$

$$\omega = 2\pi f = 2\pi(3.00) = 6.00\pi \text{ rad/s}$$

Therefore, $y = A\sin(kx + \omega t)$

Or (where $y(0,\ t) = 0$ at $t = 0$) $\quad \boxed{y = (0.080\,0)\sin(7.85x + 6\pi t)\text{ m}}$

(b) In general, $y = 0.080\,0\sin(7.85x + 6\pi t + \phi)$

Assuming $y(x,\ 0) = 0$ at $x = 0.100$ m

then we require that $0 = 0.080\,0\sin(0.785 + \phi)$

or $\phi = -0.785$

Therefore, $\boxed{y = 0.080\,0\sin(7.85x + 6\pi t - 0.785)\text{ m}}$

P16.16 (a)

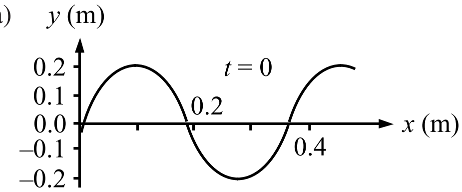


FIG. P16.16(a)

(b) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{0.350\text{ m}} = \boxed{18.0\text{ rad/m}}$

$T = \dfrac{1}{f} = \dfrac{1}{12.0/\text{s}} = \boxed{0.083\,3\text{ s}}$

$\omega = 2\pi f = 2\pi\, 12.0/\text{s} = \boxed{75.4\text{ rad/s}}$

$|v| = f\lambda = (12.0/\text{s})(0.350\text{ m}) = \boxed{4.20\text{ m/s}}$

(c) $y = A\sin(kx + \omega t + \phi)$ specializes to

$$y = 0.200\text{ m}\sin(18.0x/\text{m} + 75.4t/\text{s} + \phi)$$

at $x = 0$, $t = 0$ we require

$$-3.00\times10^{-2}\text{ m} = 0.200\text{ m}\sin(+\phi)$$

$$\phi = -8.63° = -0.151\text{ rad}$$

so $\quad y(x,\ t) = \boxed{(0.200\text{ m})\sin(18.0x/\text{m} + 75.4t/\text{s} - 0.151\text{ rad})}$

P16.17 $y = (0.120 \text{ m})\sin\left(\frac{\pi}{8}x + 4\pi t\right)$

(a) $v = \frac{dy}{dt}$: $\quad v = (0.120)(4\pi)\cos\left(\frac{\pi}{8}x + 4\pi t\right)$

$v(0.200 \text{ s}, 1.60 \text{ m}) = \boxed{-1.51 \text{ m/s}}$

$a = \frac{dv}{dt}$: $\quad a = (-0.120 \text{ m})(4\pi)^2 \sin\left(\frac{\pi}{8}x + 4\pi t\right)$

$a(0.200 \text{ s}, 1.60 \text{ m}) = \boxed{0}$

(b) $k = \frac{\pi}{8} = \frac{2\pi}{\lambda}$: $\quad \lambda = \boxed{16.0 \text{ m}}$

$\omega = 4\pi = \frac{2\pi}{T}$: $\quad T = \boxed{0.500 \text{ s}}$

$v = \frac{\lambda}{T} = \frac{16.0 \text{ m}}{0.500 \text{ s}} = \boxed{32.0 \text{ m/s}}$

P16.18 (a) Let us write the wave function as $\quad y(x, t) = A\sin(kx + \omega t + \phi)$

$y(0, 0) = A\sin\phi = 0.020\,0 \text{ m}$

$\left.\frac{dy}{dt}\right|_{0,\,0} = A\omega\cos\phi = -2.00 \text{ m/s}$

Also, $\quad \omega = \frac{2\pi}{T} = \frac{2\pi}{0.025\,0 \text{ s}} = 80.0\,\pi/\text{s}$

$A^2 = x_i^2 + \left(\frac{v_i}{\omega}\right)^2 = (0.020\,0 \text{ m})^2 + \left(\frac{2.00 \text{ m/s}}{80.0\,\pi/\text{s}}\right)^2$

$A = \boxed{0.021\,5 \text{ m}}$

(b) $\frac{A\sin\phi}{A\cos\phi} = \frac{0.020\,0}{-2/80.0\pi} = -2.51 = \tan\phi$

Your calculator's answer $\tan^{-1}(-2.51) = -1.19$ rad has a negative sine and positive cosine, just the reverse of what is required. You must look beyond your calculator to find

$\phi = \pi - 1.19 \text{ rad} = \boxed{1.95 \text{ rad}}$

(c) $v_{y,\,\text{max}} = A\omega = 0.021\,5 \text{ m}(80.0\pi/\text{s}) = \boxed{5.41 \text{ m/s}}$

(d) $\lambda = v_x T = (30.0 \text{ m/s})(0.025\,0 \text{ s}) = 0.750 \text{ m}$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.750 \text{ m}} = 8.38/\text{m} \qquad \omega = 80.0\pi/\text{s}$

$$\boxed{y(x, t) = (0.021\,5 \text{ m})\sin(8.38x \text{ rad/m} + 80.0\pi t \text{ rad/s} + 1.95 \text{ rad})}$$

P16.19 (a) $f = \dfrac{v}{\lambda} = \dfrac{(1.00\ \text{m/s})}{2.00\ \text{m}} = \boxed{0.500\ \text{Hz}}$

$\omega = 2\pi f = 2\pi(0.500/\text{s}) = \boxed{3.14\ \text{rad/s}}$

(b) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{2.00\ \text{m}} = \boxed{3.14\ \text{rad/m}}$

(c) $y = A\sin(kx - \omega t + \phi)$ becomes

$y = \boxed{(0.100\ \text{m})\sin(3.14x/\text{m} - 3.14t/\text{s} + 0)}$

(d) For $x = 0$ the wave function requires

$\boxed{y = (0.100\ \text{m})\sin(-3.14t/\text{s})}$

(e) $\boxed{y = (0.100\ \text{m})\sin(4.71\ \text{rad} - 3.14t/\text{s})}$

(f) $v_y = \dfrac{\partial y}{\partial t} = 0.100\ \text{m}(-3.14/\text{s})\cos(3.14x/\text{m} - 3.14t/\text{s})$

The cosine varies between +1 and −1, so

$v_y \le \boxed{0.314\ \text{m/s}}$

P16.20 (a) At $x = 2.00$ m, $y = \boxed{(0.100\ \text{m})\sin(1.00\ \text{rad} - 20.0t)}$ Because this disturbance varies sinusoidally in time, it describes simple harmonic motion.

(b) $y = (0.100\ \text{m})\sin(0.500x - 20.0t) = A\sin(kx - \omega t)$

so $\omega = 20.0\ \text{rad/s}$ and $f = \dfrac{\omega}{2\pi} = \boxed{3.18\ \text{Hz}}$

Section 16.3 **The Speed of Waves on Strings**

P16.21 The down and back distance is $4.00\ \text{m} + 4.00\ \text{m} = 8.00\ \text{m}$.

The speed is then $v = \dfrac{d_{\text{total}}}{t} = \dfrac{4(8.00\ \text{m})}{0.800\ \text{s}} = 40.0\ \text{m/s} = \sqrt{\dfrac{T}{\mu}}$

Now, $\mu = \dfrac{0.200\ \text{kg}}{4.00\ \text{m}} = 5.00 \times 10^{-2}\ \text{kg/m}$

So $T = \mu v^2 = (5.00 \times 10^{-2}\ \text{kg/m})(40.0\ \text{m/s})^2 = \boxed{80.0\ \text{N}}$

P16.22 (a) $\omega = 2\pi f = 2\pi(500) = 3\,140\ \text{rad/s},\ k = \dfrac{\omega}{v} = \dfrac{3\,140}{196} = 16.0\ \text{rad/m}$

$\boxed{y = (2.00 \times 10^{-4}\ \text{m})\sin(16.0x - 3\,140t)}$

(b) $v = 196\ \text{m/s} = \sqrt{\dfrac{T}{4.10 \times 10^{-3}\ \text{kg/m}}}$

$T = \boxed{158\ \text{N}}$

P16.23 $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{1\,350\ \text{kg}\cdot\text{m/s}^2}{5.00\times10^{-3}\ \text{kg/m}}} = \boxed{520\ \text{m/s}}$

P16.24 $v = \sqrt{\dfrac{T}{\mu}}$

$$T = \mu v^2 = \rho A v^2 = \rho\pi r^2 v^2$$

$$T = \left(8\,920\ \text{kg/m}^3\right)(\pi)\left(7.50\times10^{-4}\ \text{m}\right)^2(200\ \text{m/s})^2$$

$$T = \boxed{631\ \text{N}}$$

P16.25 $T = Mg$ is the tension; $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{Mg}{m/L}} = \sqrt{\dfrac{MgL}{m}} = \dfrac{L}{t}$ is the wave speed.

Then, $\dfrac{MgL}{m} = \dfrac{L^2}{t^2}$

and $g = \dfrac{Lm}{Mt^2} = \dfrac{1.60\ \text{m}\left(4.00\times10^{-3}\ \text{kg}\right)}{3.00\ \text{kg}\left(3.61\times10^{-2}\ \text{s}\right)^2} = \boxed{1.64\ \text{m/s}^2}$

P16.26 The period of the pendulum is $T = 2\pi\sqrt{\dfrac{L}{g}}$

Let F represent the tension in the string (to avoid confusion with the period) when the pendulum is vertical and stationary. The speed of waves in the string is then:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{Mg}{m/L}} = \sqrt{\frac{MgL}{m}}$$

Since it might be difficult to measure L precisely, we eliminate $\sqrt{L} = \dfrac{T\sqrt{g}}{2\pi}$

so

$$v = \sqrt{\frac{Mg}{m}}\frac{T\sqrt{g}}{2\pi} = \boxed{\frac{Tg}{2\pi}\sqrt{\frac{M}{m}}}$$

P16.27 Since μ is constant, $\mu = \dfrac{T_2}{v_2^2} = \dfrac{T_1}{v_1^2}$ and

$$T_2 = \left(\frac{v_2}{v_1}\right)^2 T_1 = \left(\frac{30.0\ \text{m/s}}{20.0\ \text{m/s}}\right)^2(6.00\ \text{N}) = \boxed{13.5\ \text{N}}$$

P16.28 From the free-body diagram $mg = 2T\sin\theta$

$$T = \frac{mg}{2\sin\theta}$$

The angle θ is found from $\cos\theta = \dfrac{3L/8}{L/2} = \dfrac{3}{4}$

$$\therefore \theta = 41.4°$$

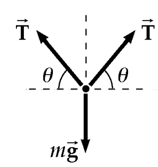


FIG. P16.28

(a) $v = \sqrt{\dfrac{T}{\mu}}$ $\qquad v = \sqrt{\dfrac{mg}{2\mu\sin 41.4°}} = \left(\sqrt{\dfrac{9.80\ \text{m/s}^2}{2\left(8.00\times10^{-3}\ \text{kg/m}\right)\sin 41.4°}}\right)\sqrt{m}$

or $\qquad v = \boxed{\left(30.4\ \dfrac{\text{m/s}}{\sqrt{\text{kg}}}\right)\sqrt{m}}$

(b) $v = 60.0 = 30.4\sqrt{m}$ and $\boxed{m = 3.89\ \text{kg}}$

P16.29 If the tension in the wire is T, the tensile stress is

$$\text{Stress} = \frac{T}{A} \qquad \text{so} \qquad T = A(\text{stress})$$

The speed of transverse waves in the wire is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{A(\text{Stress})}{m/L}} = \sqrt{\frac{\text{Stress}}{m/AL}} = \sqrt{\frac{\text{Stress}}{m/\text{Volume}}} = \sqrt{\frac{\text{Stress}}{\rho}}$$

where ρ is the density. The maximum velocity occurs when the stress is a maximum:

$$v_{\text{max}} = \sqrt{\frac{2.70 \times 10^8 \text{ Pa}}{7\,860 \text{ kg/m}^3}} = \boxed{185 \text{ m/s}}$$

***P16.30** (a) f has units Hz = 1/s, so $T = \frac{1}{f}$ has units of seconds, $\boxed{\text{s}}$. For the other T we have $T = \mu v^2$, with units $\frac{\text{kg}}{\text{m}}\frac{\text{m}^2}{\text{s}^2} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} = \boxed{\text{N}}$.

(b) $\boxed{\text{The first } T \text{ is period of time; the second is force of tension.}}$

P16.31 The total time is the sum of the two times.

In each wire $t = \frac{L}{v} = L\sqrt{\frac{\mu}{T}}$

Let A represent the cross-sectional area of one wire. The mass of one wire can be written both as $m = \rho V = \rho AL$ and also as $m = \mu L$.

Then we have $\mu = \rho A = \frac{\pi \rho d^2}{4}$

Thus, $t = L\left(\frac{\pi \rho d^2}{4T}\right)^{1/2}$

For copper, $t = (20.0)\left[\frac{(\pi)(8\,920)(1.00 \times 10^{-3})^2}{(4)(150)}\right]^{1/2} = 0.137 \text{ s}$

For steel, $t = (30.0)\left[\frac{(\pi)(7\,860)(1.00 \times 10^{-3})^2}{(4)(150)}\right]^{1/2} = 0.192 \text{ s}$

The total time is $0.137 + 0.192 = \boxed{0.329 \text{ s}}$

Section 16.5 Rate of Energy Transfer by Sinusoidal Waves on Strings

P16.32 $f = \frac{v}{\lambda} = \frac{30.0}{0.500} = 60.0 \text{ Hz} \qquad \omega = 2\pi f = 120\pi \text{ rad/s}$

$$\mathcal{P} = \frac{1}{2}\mu\omega^2 A^2 v = \frac{1}{2}\left(\frac{0.180}{3.60}\right)(120\pi)^2 (0.100)^2 (30.0) = \boxed{1.07 \text{ kW}}$$

P16.33 Suppose that no energy is absorbed or carried down into the water. Then a fixed amount of power is spread thinner farther away from the source. It is spread over the circumference $2\pi r$ of an expanding circle. The power-per-width across the wave front

$$\frac{\mathcal{P}}{2\pi r}$$

is proportional to amplitude squared so amplitude is proportional to

$$\sqrt{\frac{\mathcal{P}}{2\pi r}}$$

P16.34 T = constant; $v=\sqrt{\frac{T}{\mu}}$; $\mathcal{P}=\frac{1}{2}\mu\omega^2A^2v$

(a) If L is doubled, v remains constant and $\boxed{\mathcal{P}\text{ is constant}}$.

(b) If A is doubled and ω is halved, $\mathcal{P}\propto\omega^2A^2$ $\boxed{\text{remains constant}}$.

(c) If λ and A are doubled, the product $\omega^2A^2\propto\frac{A^2}{\lambda^2}$ remains constant, so $\boxed{\mathcal{P}\text{ remains constant}}$.

(d) If L and λ are halved, then $\omega^2\propto\frac{1}{\lambda^2}$ is quadrupled, so $\boxed{\mathcal{P}\text{ is quadrupled}}$.

(Changing L doesn't affect $\mathcal{P}$.)

P16.35 $A=5.00\times10^{-2}\text{ m}$ $\quad\mu=4.00\times10^{-2}\text{ kg/m}$ $\quad\mathcal{P}=300\text{ W}$ $\quad T=100\text{ N}$

Therefore, $\quad v=\sqrt{\frac{T}{\mu}}=50.0\text{ m/s}$

$\mathcal{P}=\frac{1}{2}\mu\omega^2A^2v$: $\quad\omega^2=\frac{2\mathcal{P}}{\mu A^2v}=\frac{2(300)}{(4.00\times10^{-2})(5.00\times10^{-2})^2(50.0)}$

$$\omega=346\text{ rad/s}$$

$$f=\frac{\omega}{2\pi}=\boxed{55.1\text{ Hz}}$$

P16.36 $\mu=30.0\text{ g/m}=30.0\times10^{-3}\text{ kg/m}$

$\lambda=1.50\text{ m}$

$f=50.0\text{ Hz}$: $\quad\omega=2\pi f=314\text{ s}^{-1}$

$2A=0.150\text{ m}$: $\quad A=7.50\times10^{-2}\text{ m}$

(a) $y=A\sin\left(\frac{2\pi}{\lambda}x-\omega t\right)$

$$\boxed{y=(7.50\times10^{-2})\sin(4.19x-314t)}$$

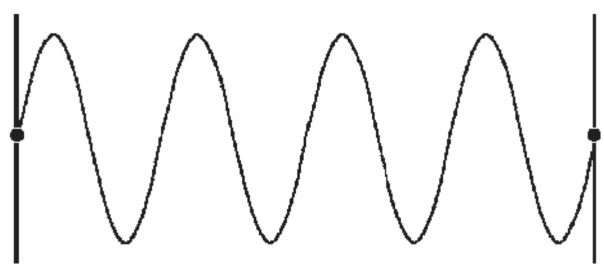


FIG. P16.36

(b) $\mathcal{P}=\frac{1}{2}\mu\omega^2A^2v=\frac{1}{2}(30.0\times10^{-3})(314)^2(7.50\times10^{-2})^2\left(\frac{314}{4.19}\right)\text{ W}$ $\quad\boxed{\mathcal{P}=625\text{ W}}$

P16.37 (a) $v = f\lambda = \frac{\omega}{2\pi}\frac{2\pi}{k} = \frac{\omega}{k} = \frac{50.0}{0.800}\ \text{m/s} = \boxed{62.5\ \text{m/s}}$

(b) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.800}\ \text{m} = \boxed{7.85\ \text{m}}$

(c) $f = \frac{50.0}{2\pi} = \boxed{7.96\ \text{Hz}}$

(d) $\mathcal{P} = \frac{1}{2}\mu\omega^2A^2v = \frac{1}{2}\left(12.0\times10^{-3}\right)(50.0)^2(0.150)^2(62.5)\ \text{W} = \boxed{21.1\ \text{W}}$

P16.38 Comparing $y = 0.35\sin\left(10\pi t - 3\pi x + \frac{\pi}{4}\right)$ with $y = A\sin(kx - \omega t + \phi) = A\sin(\omega t - kx - \phi + \pi)$

we have $k = \frac{3\pi}{m}$, $\omega = 10\pi/\text{s}$, $A = 0.35$ m. Then $v = f\lambda = 2\pi f\frac{\lambda}{2\pi} = \frac{\omega}{k} = \frac{10\pi/\text{s}}{3\pi/\text{m}} = 3.33\ \text{m/s}$.

(a) The rate of energy transport is

$$\mathcal{P} = \frac{1}{2}\mu\omega^2A^2v = \frac{1}{2}\left(75\times10^{-3}\ \text{kg/m}\right)(10\pi/\text{s})^2(0.35\ \text{m})^2\,3.33\ \text{m/s} = \boxed{15.1\ \text{W}}$$

(b) The energy per cycle is

$$E_\lambda = \mathcal{P}T = \frac{1}{2}\mu\omega^2A^2\lambda = \frac{1}{2}\left(75\times10^{-3}\ \text{kg/m}\right)(10\pi/\text{s})^2(0.35\ \text{m})^2\frac{2\pi\ \text{m}}{3\pi} = \boxed{3.02\ \text{J}}$$

P16.39 Originally,

$$\mathcal{P}_0 = \frac{1}{2}\mu\omega^2A^2v$$
$$\mathcal{P}_0 = \frac{1}{2}\mu\omega^2A^2\sqrt{\frac{T}{\mu}}$$
$$\mathcal{P}_0 = \frac{1}{2}\omega^2A^2\sqrt{T\mu}$$

The doubled string will have doubled mass-per-length. Presuming that we hold tension constant, it can carry power larger by $\sqrt{2}$ times.

$$\boxed{\sqrt{2}\mathcal{P}_0} = \frac{1}{2}\omega^2A^2\sqrt{T2\mu}$$

***P16.40** $\boxed{\text{As for a string wave, the rate of energy transfer is proportional to the square of the amplitude and to the speed. The rate of energy transfer stays constant because each wavefront carries constant energy and the frequency stays constant. As the speed drops the amplitude must increase.}}$

We write $\mathcal{P} = FvA^2$ where F is some constant. With no absorption of energy,

$$Fv_{\text{bedrock}}A^2_{\text{bedrock}} = Fv_{\text{mudfill}}A^2_{\text{mudfill}}$$
$$\sqrt{\frac{v_{\text{bedrock}}}{v_{\text{mudfill}}}} = \frac{A_{\text{mudfill}}}{A_{\text{bedrock}}} = \sqrt{\frac{25v_{\text{mudfill}}}{v_{\text{mudfill}}}} = 5$$

$\boxed{\text{The amplitude increases by 5.00 times.}}$

Section 16.6 **The Linear Wave Equation**

P16.41 (a) $A = (7.00 + 3.00)4.00$ yields $\boxed{A = 40.0}$

(b) In order for two vectors to be equal, they must have the same magnitude and the same direction in three-dimensional space. All of their components must be equal.

Thus, $7.00\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 3.00\hat{\mathbf{k}} = A\hat{\mathbf{i}} + B\hat{\mathbf{j}} + C\hat{\mathbf{k}}$ requires $\boxed{A = 7.00,\ B = 0,\ \text{and}\ C = 3.00}$.

(c) In order for two functions to be identically equal, they must be equal for every value of every variable. They must have the same graphs.

In

$$A + B\cos(Cx + Dt + E) = 0 + 7.00\ \text{mm}\cos(3.00x + 4.00t + 2.00)$$

the equality of average values requires that $\boxed{A = 0}$. The equality of maximum values requires $\boxed{B = 7.00\ \text{mm}}$. The equality for the wavelength or periodicity as a function of x requires $\boxed{C = 3.00\ \text{rad/m}}$. The equality of period requires $\boxed{D = 4.00\ \text{rad/s}}$, and the equality of zero-crossings requires $\boxed{E = 2.00\ \text{rad}}$.

P16.42 The linear wave equation is $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$

If $y = e^{b(x-vt)}$

then $\dfrac{\partial y}{\partial t} = -bve^{b(x-vt)}$ and $\dfrac{\partial y}{\partial x} = be^{b(x-vt)}$

$\dfrac{\partial^2 y}{\partial t^2} = b^2v^2e^{b(x-vt)}$ and $\dfrac{\partial^2 y}{\partial x^2} = b^2e^{b(x-vt)}$

Therefore, $\dfrac{\partial^2 y}{\partial t^2} = v^2\dfrac{\partial^2 y}{\partial x^2}$, demonstrating that $e^{b(x-vt)}$ is a solution.

P16.43 The linear wave equation is $\dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2} = \dfrac{\partial^2 y}{\partial x^2}$

To show that $y = \ln[b(x - vt)]$ is a solution, we find its first and second derivatives with respect to x and t and substitute into the equation.

$$\frac{\partial y}{\partial t} = \frac{1}{b(x-vt)}(-bv) \qquad \frac{\partial^2 y}{\partial t^2} = \frac{-1(-bv)^2}{b^2(x-vt)^2} = -\frac{v^2}{(x-vt)^2}$$

$$\frac{\partial y}{\partial x} = [b(x-vt)]^{-1}b \qquad \frac{\partial^2 y}{\partial x^2} = -\frac{b}{b}(x-vt)^{-2} = -\frac{1}{(x-vt)^2}$$

Then $\dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2} = \dfrac{1}{v^2}\dfrac{(-v^2)}{(x-vt)^2} = -\dfrac{1}{(x-vt)^2} = \dfrac{\partial^2 y}{\partial x^2}$ so the given wave function is a solution.

P16.44 (a) From $y = x^2 + v^2t^2$,

evaluate $\dfrac{\partial y}{\partial x} = 2x \qquad \dfrac{\partial^2 y}{\partial x^2} = 2$

$$\frac{\partial y}{\partial t} = v^2 2t \qquad \frac{\partial^2 y}{\partial t^2} = 2v^2$$

Does $\dfrac{\partial^2 y}{\partial t^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$?

By substitution, we must test $2 = \dfrac{1}{v^2}2v^2$ and this is true, so the wave function does satisfy the wave equation.

(b) Note $\dfrac{1}{2}(x+vt)^2 + \dfrac{1}{2}(x-vt)^2 = \dfrac{1}{2}x^2 + xvt + \dfrac{1}{2}v^2t^2 + \dfrac{1}{2}x^2 - xvt + \dfrac{1}{2}v^2t^2$

$$= x^2 + v^2t^2 \text{ as required.}$$

So

$$\boxed{f(x+vt) = \frac{1}{2}(x+vt)^2} \quad \text{and} \quad \boxed{g(x-vt) = \frac{1}{2}(x-vt)^2}$$

(c) $y = \sin x \cos vt$ makes

$$\frac{\partial y}{\partial x} = \cos x \cos vt \qquad \frac{\partial^2 y}{\partial x^2} = -\sin x \cos vt$$

$$\frac{\partial y}{\partial t} = -v \sin x \sin vt \qquad \frac{\partial^2 y}{\partial t^2} = -v^2 \sin x \cos vt$$

Then

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

becomes $-\sin x \cos vt = \dfrac{-1}{v^2}v^2 \sin x \cos vt$ which is true as required.

Note $\sin(x+vt) = \sin x \cos vt + \cos x \sin vt$

$\sin(x-vt) = \sin x \cos vt - \cos x \sin vt$

So $\sin x \cos vt = f(x+vt) + g(x-vt)$ with

$$\boxed{f(x+vt) = \frac{1}{2}\sin(x+vt)} \quad \text{and} \quad \boxed{g(x-vt) = \frac{1}{2}\sin(x-vt)}$$

Additional Problems

P16.45 Assume a typical distance between adjacent people ~1 m.

Then the wave speed is $v = \dfrac{\Delta x}{\Delta t} \sim \dfrac{1\text{ m}}{0.1\text{ s}} \sim 10\text{ m/s}$

Model the stadium as a circle with a radius of order 100 m. Then, the time for one circuit around the stadium is

$$T = \frac{2\pi r}{v} \sim \frac{2\pi(10^2)}{10\text{ m/s}} = 63\text{ s}\ \boxed{\sim 1\text{ min}}$$

***P16.46** (a) From $y = 0.150\text{ m}\sin(0.8x - 50t)$

we compute $dy/dt = 0.150\text{ m}(-50)\cos(0.8x - 50t)$

and $a = d^2y/dt^2 = -0.150\text{ m}(-50/\text{s})^2\sin(0.8x - 50t)$

Then $a_{max} = \boxed{375\text{ m/s}^2}$

(b) For the 1-cm segment with maximum force acting on it, $\Sigma F = ma = [12\text{ g}/(100\text{ cm})]$ $1\text{ cm } 375\text{ m/s}^2 = \boxed{0.045\,0\text{ N}}$

We find the tension in the string from $v = f\lambda = \omega/k = (50/\text{s})/(0.8/\text{m}) = 62.5\text{ m/s} = (T/\mu)^{1/2}$ $T = v^2\mu = (62.5\text{ m/s})^2(0.012\text{ kg/m}) = 46.9\text{ N}$.

The maximum transverse force is very small compared to the tension, more than a thousand times smaller.

P16.47 The equation $v = \lambda f$ is a special case of

speed = (cycle length)(repetition rate)

Thus,

$$v = \left(19.0\times10^{-3}\text{ m/frame}\right)\left(24.0\text{ frames/s}\right) = \boxed{0.456\text{ m/s}}$$

P16.48 (a) $0.175\text{ m} = (0.350\text{ m})\sin\left[(99.6\text{ rad/s})t\right]$

$\therefore \sin\left[(99.6\text{ rad/s})t\right] = 0.5$

The smallest two angles for which the sine function is 0.5 are 30° and 150°, i.e., 0.523 6 rad and 2.618 rad.

$(99.6\text{ rad/s})t_1 = 0.523\,6\text{ rad}$, thus $t_1 = 5.26\text{ ms}$

$(99.6\text{ rad/s})t_2 = 2.618\text{ rad}$, thus $t_2 = 26.3\text{ ms}$

$\Delta t \equiv t_2 - t_1 = 26.3\text{ ms} - 5.26\text{ ms} = \boxed{21.0\text{ ms}}$

(b) Distance traveled by the wave $= \left(\dfrac{\omega}{k}\right)\Delta t = \left(\dfrac{99.6\text{ rad/s}}{1.25\text{ rad/m}}\right)\left(21.0\times10^{-3}\text{ s}\right) = \boxed{1.68\text{ m}}$.

P16.49 Energy is conserved as the block moves down distance x:

$$\left(K + U_g + U_s\right)_{\text{top}} + \Delta E = \left(K + U_g + U_s\right)_{\text{bottom}}$$

$$0 + Mgx + 0 + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$x = \frac{2Mg}{k}$$

(a) $T = kx = 2Mg = 2(2.00\text{ kg})(9.80\text{ m/s}^2) = \boxed{39.2\text{ N}}$

(b) $L = L_0 + x = L_0 + \dfrac{2Mg}{k}$

$$L = 0.500\text{ m} + \frac{39.2\text{ N}}{100\text{ N/m}} = \boxed{0.892\text{ m}}$$

(c) $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{TL}{m}}$

$$v = \sqrt{\frac{39.2\text{ N}\times0.892\text{ m}}{5.0\times10^{-3}\text{ kg}}}$$

$$v = \boxed{83.6\text{ m/s}}$$

P16.50 $Mgx = \frac{1}{2}kx^2$

(a) $T = kx = \boxed{2Mg}$

(b) $L = L_0 + x = \boxed{L_0 + \frac{2Mg}{k}}$

(c) $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{TL}{m}} = \boxed{\sqrt{\frac{2Mg}{m}\left(L_0 + \frac{2Mg}{k}\right)}}$

***P16.51** (a) The energy a wave crest carries is constant in the absence of absorption. Then the rate at which energy passes a stationary point, which is the power of the wave, is constant. The power is proportional to the square of the amplitude and to the wave speed. The speed decreases as the wave moves into shallower water near shore, so the amplitude must increase.

(b) For the wave described, with a single direction of energy transport, the intensity is the same at the deep-water location ① and at the place ② with depth 9 m. To express the constant intensity we write

$$A_1^2 v_1 = A_2^2 v_2 = A_2^2\sqrt{gd_2}$$

$$(1.8\text{ m})^2\ 200\text{ m/s} = A_2^2\sqrt{(9.8\text{ m/s}^2)9\text{ m}}$$

$$= A_2^2 9.39\text{ m/s}$$

$$A_2 = 1.8\left(\frac{200\text{ m/s}}{9.39\text{ m/s}}\right)^{1/2}$$

$$= \boxed{8.31\text{ m}}$$

(c) As the water depth goes to zero, our model would predict zero speed and infinite amplitude. In fact the amplitude must be finite as the wave comes ashore. As the speed decreases the wavelength also decreases. When it becomes comparable to the water depth, or smaller, our formula $\sqrt{gd}$ for wave speed no longer applies.

P16.52 Assuming the incline to be frictionless and taking the positive x-direction to be up the incline:

$\sum F_x = T - Mg\sin\theta = 0$ or the tension in the string is $T = Mg\sin\theta$

The speed of transverse waves in the string is then $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg\sin\theta}{m/L}} = \sqrt{\frac{MgL\sin\theta}{m}}$

The time interval for a pulse to travel the string's length is $\Delta t = \frac{L}{v} = L\sqrt{\frac{m}{MgL\sin\theta}} = \boxed{\sqrt{\frac{mL}{Mg\sin\theta}}}$

***P16.53** (a) In $\mathcal{P} = \frac{1}{2}\mu\omega^2 A^2 v$ where v is the wave speed, the quantity ωA is the maximum particle speed v_{ymax}. We have $\mu = 0.5 \times 10^{-3}$ kg/m and $v = (T/\mu)^{1/2} = (20\text{ N}/0.5 \times 10^{-3}\text{ kg/m})^{1/2} =$ 200 m/s

Then $\mathcal{P} = \frac{1}{2}(0.5 \times 10^{-3}\text{ kg/m})\, v^2_{ymax}\,(200\text{ m/s}) = \boxed{(0.050\,0\text{ kg/s})v^2_{y,max}}$

(b) $\boxed{\text{The power is proportional to the square of the maximum particle speed.}}$

(c) In time $t = (3\text{ m})/v = (3\text{ m})/(200\text{ m/s}) = 1.5 \times 10^{-2}$ s, all the energy in a 3-m length of string goes past a point. Therefore the amount of this energy is

$E = \mathcal{P}t = (0.05\text{ kg/s})\, v^2_{y,\max}\,(0.015\text{ s}) = 7.5 \times 10^{-4}\text{ kg } v^2_{y,\max}$

The mass of this section is $m_3 = (0.5 \times 10^{-3}\text{ kg/m})3\text{ m} = 1.5 \times 10^{-3}$ kg so $(1/2)m_3 = 7.5 \times 10^{-4}$ kg and $\boxed{E = (1/2)\, m_3 v^2_{y,\max}} = K_{max}$. The string also contains potential energy. We could write its energy as U_{max} or as $U_{avg} + K_{avg}$

(d) $E = \mathcal{P}t = (0.05\text{ kg/s})\, v^2_{y,max}\,(6\text{ s}) = \boxed{0.300\text{ kg } v^2_{y,\max}}$

P16.54 $v = \sqrt{\dfrac{T}{\mu}}$ and in this case $T = mg$; therefore, $m = \dfrac{\mu v^2}{g}$

Now $v = f\lambda$ implies $v = \dfrac{\omega}{k}$ so that

$$m = \frac{\mu}{g}\left(\frac{\omega}{k}\right)^2 = \frac{0.250\ \text{kg/m}}{9.80\ \text{m/s}^2}\left[\frac{18\pi\ \text{s}^{-1}}{0.750\pi\ \text{m}^{-1}}\right]^2 = \boxed{14.7\text{ kg}}$$

P16.55 Let M = mass of block, m = mass of string. For the block, $\sum F = ma$ implies $T = \dfrac{mv_b^2}{r} = m\omega^2 r$

The speed of a wave on the string is then

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{M\omega^2 r}{m/r}} = r\omega\sqrt{\frac{M}{m}}$$

$$t = \frac{r}{v} = \frac{1}{\omega}\sqrt{\frac{m}{M}}$$

$$\theta = \omega t = \sqrt{\frac{m}{M}} = \sqrt{\frac{0.003\,2\text{ kg}}{0.450\text{ kg}}} = \boxed{0.084\,3\text{ rad}}$$

P16.56 (a) $\mu = \dfrac{dm}{dL} = \rho A\dfrac{dx}{dx} = \rho A$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{T}{[\rho(ax+b)]}} = \sqrt{\frac{T}{\left[\rho\left(10^{-3}x + 10^{-2}\right)\text{cm}^2\right]}}$$

With all SI units,

$$\boxed{v = \sqrt{\frac{T}{\left[\rho\left(10^{-3}x + 10^{-2}\right)10^{-4}\right]}}\ \text{m/s}}$$

(b) $$v\big|_{x=0} = \sqrt{\frac{24.0}{\left[(2\,700)\left(0 + 10^{-2}\right)\left(10^{-4}\right)\right]}} = \boxed{94.3\ \text{m/s}}$$

$$v\big|_{x=10.0} = \sqrt{\frac{24.0}{\left[(2\,700)\left(10^{-2} + 10^{-2}\right)\left(10^{-4}\right)\right]}} = \boxed{66.7\ \text{m/s}}$$

P16.57 $v=\sqrt{\dfrac{T}{\mu}}$ where $T=\mu xg$, to support the weight of a length x, of rope.

Therefore, $v=\sqrt{gx}$

But $v=\dfrac{dx}{dt}$, so that $dt=\dfrac{dx}{\sqrt{gx}}$

and $t=\displaystyle\int_0^L \frac{dx}{\sqrt{gx}}=\frac{1}{\sqrt{g}}\frac{\sqrt{x}}{\frac{1}{2}}\Bigg|_0^L=\boxed{2\sqrt{\frac{L}{g}}}$

P16.58 At distance x from the bottom, the tension is $T=\left(\dfrac{mxg}{L}\right)+Mg$, so the wave speed is:

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{TL}{m}}=\sqrt{xg+\left(\frac{MgL}{m}\right)}=\frac{dx}{dt}$$

(a) Then $t=\displaystyle\int_0^t dt=\int_0^L\left[xg+\left(\frac{MgL}{m}\right)\right]^{-1/2}dx$ $\qquad t=\dfrac{1}{g}\dfrac{\left[xg+(MgL/m)\right]^{1/2}}{\frac{1}{2}}\Bigg|_{x=0}^{x=L}$

$$t=\frac{2}{g}\left[\left(Lg+\frac{MgL}{m}\right)^{1/2}-\left(\frac{MgL}{m}\right)^{1/2}\right] \qquad \boxed{t=2\sqrt{\frac{L}{g}}\left(\frac{\sqrt{m+M}-\sqrt{M}}{\sqrt{m}}\right)}$$

(b) When $M=0$, as in the previous problem, $t=2\sqrt{\dfrac{L}{g}}\left(\dfrac{\sqrt{m}-0}{\sqrt{m}}\right)=\boxed{2\sqrt{\dfrac{L}{g}}}$

(c) As $m\to 0$ we expand $\sqrt{m+M}=\sqrt{M}\left(1+\dfrac{m}{M}\right)^{1/2}=\sqrt{M}\left(1+\dfrac{1}{2}\dfrac{m}{M}-\dfrac{1}{8}\dfrac{m^2}{M^2}+\cdots\right)$

to obtain $t=2\sqrt{\dfrac{L}{g}}\left(\dfrac{\sqrt{M}+\frac{1}{2}\left(m/\sqrt{M}\right)-\frac{1}{8}\left(m^2/M^{3/2}\right)+\cdots-\sqrt{M}}{\sqrt{m}}\right)$

$$t\approx 2\sqrt{\frac{L}{g}}\left(\frac{1}{2}\sqrt{\frac{m}{M}}\right)=\boxed{\sqrt{\frac{mL}{Mg}}}$$

P16.59 (a) The speed in the lower half of a rope of length L is the same function of distance (from the bottom end) as the speed along the entire length of a rope of length $\left(\dfrac{L}{2}\right)$.

Thus, the time required $=2\sqrt{\dfrac{L'}{g}}$ with $L'=\dfrac{L}{2}$

and the time required $=2\sqrt{\dfrac{L}{2g}}=\boxed{0.707\left(2\sqrt{\dfrac{L}{g}}\right)}$

It takes the pulse more that 70% of the total time to cover 50% of the distance.

(b) By the same reasoning applied in part (a), the distance climbed in τ is given by $d=\dfrac{g\tau^2}{4}$

For $\tau=\dfrac{t}{2}=\sqrt{\dfrac{L}{g}}$, we find the *distance climbed* $=\boxed{\dfrac{L}{4}}$.

In half the total trip time, the pulse has climbed $\dfrac{1}{4}$ of the total length.

P16.60 (a) Consider a short section of chain at the top of the loop. A free-body diagram is shown. Its length is $s = R(2\theta)$ and its mass is $\mu R2\theta$. In the frame of reference of the center of the loop, Newton's second law is

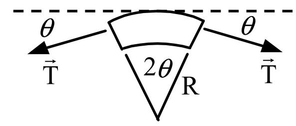


FIG. P16.60(a)

$$\sum F_y = ma_y \qquad 2T\sin\theta \text{ down} = \frac{mv_0^2}{R} \text{ down} = \frac{\mu R2\theta v_0^2}{R}$$

For a very short section, $\sin\theta = \theta$ and $\boxed{T = \mu v_0^2}$

(b) The wave speed is $v = \sqrt{\dfrac{T}{\mu}} = \boxed{v_0}$

(c) In the frame of reference of the center of the loop, each pulse moves with equal speed clockwise and counterclockwise.

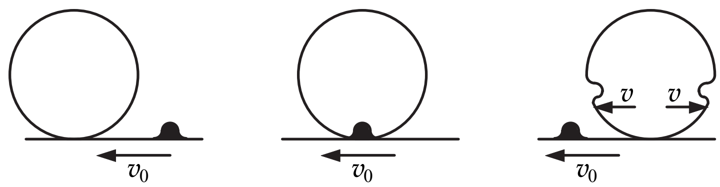


FIG. P16.60(c1)

In the frame of reference of the ground, once pulse moves backward at speed $v_0 + v = 2v_0$ and the other forward at $v_0 - v = 0$.

The one pulse makes two revolutions while the loop makes one revolution and the other pulse does not move around the loop. If it is generated at the six-o'clock position, it will stay at the six-o'clock position.

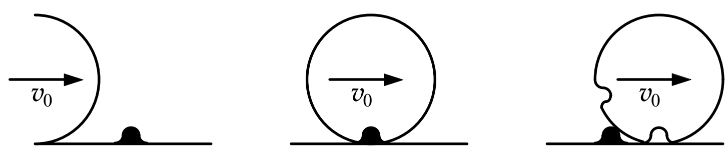


FIG. P16.60(c2)

P16.61 Young's modulus for the wire may be written as $Y = \dfrac{T/A}{\Delta L/L}$, where T is the tension maintained in the wire and ΔL is the elongation produced by this tension. Also, the mass density of the wire may be expressed as $\rho = \dfrac{\mu}{A}$

The speed of transverse waves in the wire is then

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T/A}{\mu/A}} = \sqrt{\frac{Y(\Delta L/L)}{\rho}}$$

and the strain in the wire is $\dfrac{\Delta L}{L} = \dfrac{\rho v^2}{Y}$

If the wire is aluminum and $v = 100$ m/s, the strain is

$$\frac{\Delta L}{L} = \frac{(2.70\times10^3 \text{ kg/m}^3)(100 \text{ m/s})^2}{7.00\times10^{10} \text{ N/m}^2} = \boxed{3.86\times10^{-4}}$$

P16.62 (a) Assume the spring is originally stationary throughout, extended to have a length L much greater than its equilibrium length. We start moving one end forward with the speed v at which a wave propagates on the spring. In this way we create a single pulse of compression that moves down the length of the spring. For an increment of spring with length dx and mass dm, just as the pulse swallows it up, $\sum F = ma$

becomes $\quad kdx = adm \quad$ or $\quad \dfrac{k}{dm/dx} = a$

But $\quad \dfrac{dm}{dx} = \mu \quad$ so $\quad a = \dfrac{k}{\mu}$

Also, $\quad a = \dfrac{dv}{dt} = \dfrac{v}{t} \quad$ when $\quad v_i = 0$

But $\quad L = vt, \quad$ so $\quad a = \dfrac{v^2}{L}$

Equating the two expressions for a, we have $\quad \dfrac{k}{\mu} = \dfrac{v^2}{L} \quad$ or $\quad \boxed{v = \sqrt{\dfrac{kL}{\mu}}}$

(b) Using the expression from part (a) $v = \sqrt{\dfrac{kL}{\mu}} = \sqrt{\dfrac{kL^2}{m}} = \sqrt{\dfrac{(100\ \text{N/m})(2.00\ \text{m})^2}{0.400\ \text{kg}}} = \boxed{31.6\ \text{m/s}}$

P16.63 (a) $\mathcal{P}(x) = \dfrac{1}{2}\mu\omega^2 A^2 v = \dfrac{1}{2}\mu\omega^2 A_0^2 e^{-2bx}\left(\dfrac{\omega}{k}\right) = \boxed{\dfrac{\mu\omega^3}{2k} A_0^2 e^{-2bx}}$

(b) $\mathcal{P}(0) = \boxed{\dfrac{\mu\omega^3}{2k} A_0^2}$

(c) $\dfrac{\mathcal{P}(x)}{\mathcal{P}(0)} = \boxed{e^{-2bx}}$

P16.64 $v = \dfrac{4\,450\ \text{km}}{9.50\ \text{h}} = 468\ \text{km/h} = \boxed{130\ \text{m/s}}$

$$\bar{d} = \frac{v^2}{g} = \frac{(130\ \text{m/s})^2}{(9.80\ \text{m/s}^2)} = \boxed{1\,730\ \text{m}}$$

P16.65 (a) $\mu(x)$ is a linear function, so it is of the form $\mu(x) = mx + b$

To have $\mu(0) = \mu_0$ we require $b = \mu_0$. Then $\mu(L) = \mu_L = mL + \mu_0$

so $m = \frac{\mu_L - \mu_0}{L}$

Then $\boxed{\mu(x) = \frac{(\mu_L - \mu_0)x}{L} + \mu_0}$

(b) From $v = \frac{dx}{dt}$, the time required to move from x to $x + dx$ is $\frac{dx}{v}$. The time required to move from 0 to L is

$$\Delta t = \int_0^L \frac{dx}{v} = \int_0^L \frac{dx}{\sqrt{T/\mu}} = \frac{1}{\sqrt{T}} \int_0^L \sqrt{\mu(x)}dx$$

$$\Delta t = \frac{1}{\sqrt{T}} \int_0^L \left(\frac{(\mu_L - \mu_0)x}{L} + \mu_0 \right)^{1/2} \left(\frac{\mu_L - \mu_0}{L} \right) dx \left(\frac{L}{\mu_L - \mu_0} \right)$$

$$\Delta t = \frac{1}{\sqrt{T}} \left(\frac{L}{\mu_L - \mu_0} \right) \left(\frac{(\mu_L - \mu_0)x}{L} + \mu_0 \right)^{3/2} \frac{1}{\frac{3}{2}} \Bigg|_0^L$$

$$\Delta t = \frac{2L}{3\sqrt{T}(\mu_L - \mu_0)} \left(\mu_L^{3/2} - \mu_0^{3/2} \right)$$

$$\Delta t = \frac{2L\left(\sqrt{\mu_L} - \sqrt{\mu_0}\right)\left(\mu_L + \sqrt{\mu_L \mu_0} + \mu_0\right)}{3\sqrt{T}\left(\sqrt{\mu_L} - \sqrt{\mu_0}\right)\left(\sqrt{\mu_L} + \sqrt{\mu_0}\right)}$$

$$\Delta t = \frac{2L}{3\sqrt{T}} \left(\frac{\mu_L + \sqrt{\mu_L \mu_0} + \mu_0}{\sqrt{\mu_L} + \sqrt{\mu_0}} \right)$$

ANSWERS TO EVEN PROBLEMS

P16.2 See the solution. The graph (b) has the same amplitude and wavelength as graph (a). It differs just by being shifted toward larger x by 2.40 m. The wave has traveled 2.40 m to the right.

P16.4 184 km

P16.6 See the solution

P16.8 0.800 m/s

P16.10 2.40 m/s

P16.12 ±6.67 cm

P16.14 (a) see the solution (b) 0.125 s, in agreement with the example

P16.16 (a) see the solution (b) 18.0/m; 83.3 ms; 75.4 rad/s; 4.20 m/s
(c) $(0.2 \text{ m})\sin(18x + 75.4t - 0.151)$

P16.18 (a) 0.021 5 m (b) 1.95 rad (c) 5.41 m/s (d) $y(x, t) = (0.021\,5 \text{ m})\sin(8.38x + 80.0\pi t + 1.95)$

P16.20 (a) see the solution (b) 3.18 Hz

P16.22 (a) $y = (0.2 \text{ mm})\sin(16x - 3\,140t)$ (b) 158 N

P16.24 631 N

P16.26 $v = \dfrac{Tg}{2\pi}\sqrt{\dfrac{M}{m}}$

P16.28 (a) $v = \left(30.4 \dfrac{\text{m}}{\text{s}\cdot\sqrt{\text{kg}}}\right)\sqrt{m}$ (b) 3.89 kg

P16.30 (a) s and N (b) The first T is period of time; the second is force of tension.

P16.32 1.07 kW

P16.34 (a), (b), (c) $\mathcal{P}$ is a constant (d) $\mathcal{P}$ is quadrupled

P16.36 (a) $y = (0.075\,0)\sin(4.19x - 314t)$ (b) 625 W

P16.38 (a) 15.1 W (b) 3.02 J

P16.40 As for a string wave, the rate of energy transfer is proportional to the square of the amplitude and to the speed. The rate of energy transfer stays constant because each wavefront carries constant energy and the frequency stays constant. As the speed drops the amplitude must increase. It increases by 5.00 times.

P16.42 see the solution

P16.44 (a) see the solution (b) $\dfrac{1}{2}(x+vt)^2 + \dfrac{1}{2}(x-vt)^2$ (c) $\dfrac{1}{2}\sin(x+vt) + \dfrac{1}{2}\sin(x-vt)$

P16.46 (a) 375 m/s^2 (b) 0.0450 N. This force is very small compared to the 46.9-N tension, more than a thousand times smaller.

P16.48 (a) 21.0 ms (b) 1.68 m

P16.50 (a) $2Mg$ (b) $L_0 + \dfrac{2Mg}{k}$ (c) $\sqrt{\dfrac{2Mg}{m}\left(L_0 + \dfrac{2Mg}{k}\right)}$

P16.52 $\Delta t = \sqrt{\dfrac{mL}{Mg\sin\theta}}$

P16.54 14.7 Kg

P16.56 (a) $v = \sqrt{\dfrac{T}{\rho\left(10^{-7}x + 10^{-6}\right)}}$ in SI units (b) 94.3 m/s; 66.7 m/s

P16.58 See the solution.

P16.60 (a) μv_0^2 (b) v_0 (c) One travels 2 rev and the other does not move around the loop.

P16.62 (a) see the solution (b) 31.6 m/s

P16.64 130 m/s; 1.73 km

17

Sound Waves

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

***Q17.1** Answer (b). The typically higher density would by itself make the speed of sound lower in a solid compared to a gas.

Q17.2 We assume that a perfect vacuum surrounds the clock. The sound waves require a medium for them to travel to your ear. The hammer on the alarm will strike the bell, and the vibration will spread as sound waves through the body of the clock. If a bone of your skull were in contact with the clock, you would hear the bell. However, in the absence of a surrounding medium like air or water, no sound can be radiated away. A larger-scale example of the same effect: Colossal storms raging on the Sun are deathly still for us.

What happens to the sound energy within the clock? Here is the answer: As the sound wave travels through the steel and plastic, traversing joints and going around corners, its energy is converted into additional internal energy, raising the temperature of the materials. After the sound has died away, the clock will glow very slightly brighter in the infrared portion of the electromagnetic spectrum.

Q17.3 If an object is $\frac{1}{2}$ meter from the sonic ranger, then the sensor would have to measure how long it would take for a sound pulse to travel one meter. Since sound of any frequency moves at about 343 m/s, then the sonic ranger would have to be able to measure a time difference of under 0.003 seconds. This small time measurement is possible with modern electronics. But it would be more expensive to outfit sonic rangers with the more sensitive equipment than it is to print "do not use to measure distances less than $\frac{1}{2}$ meter" in the users' manual.

Q17.4 The speed of sound to two significant figures is 340 m/s. Let's assume that you can measure time to $\frac{1}{10}$ second by using a stopwatch. To get a speed to two significant figures, you need to measure a time of at least 1.0 seconds. Since $d = vt$, the minimum distance is 340 meters.

***Q17.5** (i) Answer (b). The frequency increases by a factor of 2 because the wave speed, which is dependent only on the medium through which the wave travels, remains constant.

(ii) Answer (c).

***Q17.6** (i) Answer (c). Every crest in air produces one crest in water immediately as it reaches the interface, so there must be 500 in every second.

(ii) Answer (a). The speed increases greatly so the wavelength must increase.

Q17.7 When listening, you are approximately the same distance from all of the members of the group. If different frequencies traveled at different speeds, then you might hear the higher pitched frequencies before you heard the lower ones produced at the same time. Although it might be interesting to think that each listener heard his or her own personal performance depending on where they were seated, a time lag like this could make a Beethoven sonata sound as if it were written by Charles Ives.

***Q17.8** Answer (a). We suppose that a point source has no structure, and radiates sound equally in all directions (isotropically). The sound wavefronts are expanding spheres, so the area over which the sound energy spreads increases according to $A = 4\pi r^2$. Thus, if the distance is tripled, the area increases by a factor of nine, and the new intensity will be one-ninth of the old intensity. This answer according to the inverse-square law applies if the medium is uniform and unbounded.

For contrast, suppose that the sound is confined to move in a horizontal layer. (Thermal stratification in an ocean can have this effect on sonar "pings.") Then the area over which the sound energy is dispersed will only increase according to the circumference of an expanding circle: $A = 2\pi rh$, and so three times the distance will result in one third the intensity.

In the case of an entirely enclosed speaking tube (such as a ship's telephone), the area perpendicular to the energy flow stays the same, and increasing the distance will not change the intensity appreciably.

***Q17.9** Answer (d). The drop in intensity is what we should expect according to the inverse-square law: $4\pi r_1^2 \mathcal{P}_1$ and $4\pi r_2^2 \mathcal{P}_2$ should agree. $(300\text{ m})^2(2\ \mu\text{W/m}^2)$ and $(950\text{ m})^2(0.2\ \mu\text{W/m}^2)$ are 0.18 W and 0.18 W, agreeing with each other.

***Q17.10** Answer (c). Normal conversation has an intensity level of about 60 dB.

***Q17.11** Answer (c). The intensity is about 10^{-13} W/m^2.

Q17.12 Our brave Siberian saw the first wave he encountered, light traveling at 3.00×10^8 m/s. At the same moment, infrared as well as visible light began warming his skin, but some time was required to raise the temperature of the outer skin layers before he noticed it. The meteor produced compressional waves in the air and in the ground. The wave in the ground, which can be called either sound or a seismic wave, traveled much faster than the wave in air, since the ground is much stiffer against compression. Our witness received it next and noticed it as a little earthquake. He was no doubt unable to distinguish the P and S waves from each other. The first air-compression wave he received was a shock wave with an amplitude on the order of meters. It transported him off his doorstep. Then he could hear some additional direct sound, reflected sound, and perhaps the sound of the falling trees.

Q17.13 As you move towards the canyon wall, the echo of your car horn would be shifted up in frequency; as you move away, the echo would be shifted down in frequency.

***Q17.14** In $f' = (v + v_o)f/(v - v_s)$ we can consider the size of the fraction $(v + v_o)/(v - v_s)$ in each case. The positive direction is defined to run from the observer toward the source.

In (a), $340/340 = 1$ In (b), $340/(340 - 25) = 1.08$ In (c), $340/(340 + 25) = 0.932$ In (d), $(340 + 25)/340 = 1.07$ In (e), $(340 - 25)/340 = 0.926$ In (f), $(340 + 25)/(340 + 25) = 1$ In (g), $(340 - 25)/(340 - 25) = 1$. In order of decreasing size we have b > d > a = f = g > c > e.

***Q17.15** (i) Answer (c). Both observer and source have equal speeds in opposite directions relative to the medium, so in $f' = (v + v_o)f/(v - v_s)$ we would have something like $(340 - 25)f/(340 - 25) = f$.

(ii) Answer (a). The speed of the medium adds to the speed of sound as far as the observer is concerned, to cause an increase in $\lambda = v/f$.

(iii) Answer (a).

Q17.16 For the sound from a source not to shift in frequency, the radial velocity of the source relative to the observer must be zero; that is, the source must not be moving toward or away from the observer. The source can be moving in a plane perpendicular to the line between it and the observer. Other possibilities: The source and observer might both have zero velocity. They might have equal velocities relative to the medium. The source might be moving around the observer on a sphere of constant radius. Even if the source speeds up on the sphere, slows down, or stops, the frequency heard will be equal to the frequency emitted by the source.

SOLUTIONS TO PROBLEMS

Section 17.1 **Speed of Sound Waves**

***P17.1** Since $v_{\text{light}} >> v_{\text{sound}}$ we have $d \approx (343\ \text{m/s})(16.2\ \text{s}) = \boxed{5.56\ \text{km}}$

We do not need to know the value of the speed of light. As long as it is very large, the travel time for the light is negligible compared to that for the sound.

P17.2 $v = \sqrt{\dfrac{B}{\rho}} = \sqrt{\dfrac{2.80 \times 10^{10}}{13.6 \times 10^{3}}} = \boxed{1.43\ \text{km/s}}$

***P17.3** The sound pulse must travel 150 m before reflection and 150 m after reflection. We have $d = vt$

$$t = \frac{d}{v} = \frac{300\ \text{m}}{1\,533\ \text{m/s}} = \boxed{0.196\ \text{s}}$$

P17.4 (a) At 9 000 m, $\Delta T = \left(\dfrac{9\,000}{150}\right)(-1.00°\text{C}) = -60.0°\text{C}$ so $T = -30.0°\text{C}$

Using the chain rule:

$$\frac{dv}{dt} = \frac{dv}{dT}\frac{dT}{dx}\frac{dx}{dt} = v\frac{dv}{dT}\frac{dT}{dx} = v(0.607)\left(\frac{1}{150}\right) = \frac{v}{247}, \quad \text{so} \quad dt = (247\ \text{s})\frac{dv}{v}$$

$$\int_0^t dt = (247\ \text{s})\int_{v_i}^{v_f}\frac{dv}{v}$$

$$t = (247\ \text{s})\ln\left(\frac{v_f}{v_i}\right) = (247\ \text{s})\ln\left[\frac{331.5 + 0.607(30.0)}{331.5 + 0.607(-30.0)}\right]$$

$t = \boxed{27.2\ \text{s}}$ for sound to reach ground.

(b) $t = \dfrac{h}{v} = \dfrac{9\,000}{[331.5 + 0.607(30.0)]} = \boxed{25.7\ \text{s}}$

It takes longer when the air cools off than if it were at a uniform temperature.

P17.5 Sound takes this time to reach the man: $\dfrac{(20.0\text{ m}-1.75\text{ m})}{343\text{ m/s}} = 5.32\times10^{-2}\text{ s}$

so the warning should be shouted no later than before the pot strikes. $0.300\text{ s}+5.32\times10^{-2}\text{ s}=0.353\text{ s}$

Since the whole time of fall is given by $y=\dfrac{1}{2}gt^2$: $18.25\text{ m}=\dfrac{1}{2}\left(9.80\text{ m/s}^2\right)t^2$

$t=1.93\text{ s}$

the warning needs to come $1.93\text{ s}-0.353\text{ s}=1.58\text{ s}$

into the fall, when the pot has fallen $\dfrac{1}{2}\left(9.80\text{ m/s}^2\right)(1.58\text{ s})^2=12.2\text{ m}$

to be above the ground by $20.0\text{ m}-12.2\text{ m}=\boxed{7.82\text{ m}}$

P17.6 It is easiest to solve part (b) first:

(b) The distance the sound travels to the plane is $d_s=\sqrt{h^2+\left(\dfrac{h}{2}\right)^2}=\dfrac{h\sqrt{5}}{2}$

The sound travels this distance in 2.00 s, so

$$d_s=\frac{h\sqrt{5}}{2}=(343\text{ m/s})(2.00\text{ s})=686\text{ m}$$

giving the altitude of the plane as $h=\dfrac{2(686\text{ m})}{\sqrt{5}}=\boxed{614\text{ m}}$

(a) The distance the plane has traveled in 2.00 s is $v(2.00\text{ s})=\dfrac{h}{2}=307\text{ m}$

Thus, the speed of the plane is: $v=\dfrac{307\text{ m}}{2.00\text{ s}}=\boxed{153\text{ m/s}}$

P17.7 Let x_1 represent the cowboy's distance from the nearer canyon wall and x_2 his distance from the farther cliff. The sound for the first echo travels distance $2x_1$. For the second, $2x_2$. For the third, $2x_1+2x_2$. For the fourth echo, $2x_1+2x_2+2x_1$.

Then

$$\frac{2x_2-2x_1}{340\text{ m/s}}=1.92\text{ s} \qquad \text{and} \qquad \frac{2x_1+2x_2-2x_2}{340\text{ m/s}}=1.47\text{ s}$$

Thus

$$x_1=\frac{1}{2}340\text{ m/s }1.47\text{ s}=250\text{ m} \qquad \text{and} \qquad \frac{2x_2}{340\text{ m/s}}=1.92\text{ s}+1.47\text{ s};\ x_2=576\text{ m}$$

(a) So $x_1+x_2=\boxed{826\text{ m}}$

(b) $\dfrac{2x_1+2x_2+2x_1-(2x_1+2x_2)}{340\text{ m/s}}=\boxed{1.47\text{ s}}$

Section 17.2 **Periodic Sound Waves**

***P17.8** (a) The speed gradually changes from $v = (331\text{ m/s})(1 + 27°\text{C}/273°\text{C})^{1/2} = 347$ m/s to $(331\text{ m/s})(1 + 0/273°\text{C})^{1/2} = 331$ m/s, a 4.6% decrease. The cooler air at the same pressure is more dense.

(b) The frequency is unchanged, because every wave crest in the hot air becomes one crest without delay in the cold air.

(c) The wavelength decreases by 4.6%, from $v/f = (347\text{ m/s})/(4\,000/\text{s}) = 86.7$ mm to $(331\text{ m/s})/(4\,000/\text{s}) = 82.8$ mm. The crests are more crowded together when they move slower.

***P17.9** (a) If $f = 2.4$ MHz, $\lambda = \dfrac{v}{f} = \dfrac{1\,500\text{ m/s}}{2.4\times10^{6}/\text{s}} = \boxed{0.625\text{ mm}}$

(b) If $f = 1$ MHz, $\lambda = \dfrac{v}{f} = \dfrac{1\,500\text{ m/s}}{10^{6}/\text{s}} = \boxed{1.50\text{ mm}}$

If $f = 20$ MHz, $\lambda = \dfrac{1\,500\text{ m/s}}{2\times10^{7}/\text{s}} = \boxed{75.0\ \mu\text{m}}$

P17.10 $\Delta P_{\max} = \rho v \omega s_{\max}$

$$s_{\max} = \frac{\Delta P_{\max}}{\rho v \omega} = \frac{\left(4.00\times10^{-3}\text{ N/m}^2\right)}{\left(1.20\text{ kg/m}^3\right)(343\text{ m/s})(2\pi)\left(10.0\times10^{3}\text{ s}^{-1}\right)} = \boxed{1.55\times10^{-10}\text{ m}}$$

P17.11 (a) $A = \boxed{2.00\ \mu\text{m}}$

$$\lambda = \frac{2\pi}{15.7} = 0.400\text{ m} = \boxed{40.0\text{ cm}}$$

$$v = \frac{\omega}{k} = \frac{858}{15.7} = \boxed{54.6\text{ m/s}}$$

(b) $s = 2.00\cos\left[(15.7)(0.050\,0) - (858)\left(3.00\times10^{-3}\right)\right] = \boxed{-0.433\ \mu\text{m}}$

(c) $v_{\max} = A\omega = (2.00\ \mu\text{m})\left(858\text{ s}^{-1}\right) = \boxed{1.72\text{ mm/s}}$

P17.12 (a) $\Delta P = (1.27\text{ Pa})\sin\left(\dfrac{\pi x}{\text{m}} - \dfrac{340\pi t}{\text{s}}\right)$ (SI units)

The pressure amplitude is: $\Delta P_{\max} = \boxed{1.27\text{ Pa}}$

(b) $\omega = 2\pi f = 340\pi/\text{s}$, so $f = \boxed{170\text{ Hz}}$

(c) $k = \dfrac{2\pi}{\lambda} = \pi/\text{m}$, giving $\lambda = \boxed{2.00\text{ m}}$

(d) $v = \lambda f = (2.00\text{ m})(170\text{ Hz}) = \boxed{340\text{ m/s}}$

P17.13 $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{(0.100\text{ m})} = 62.8\text{ m}^{-1}$

$$\omega = \frac{2\pi v}{\lambda} = \frac{2\pi(343\text{ m/s})}{(0.100\text{ m})} = 2.16\times10^4\text{ s}^{-1}$$

Therefore,

$$\boxed{\Delta P = (0.200\text{ Pa})\sin\left[62.8\,x/\text{m} - 2.16\times10^4\,t/\text{s}\right]}$$

P17.14 (a) The sound "pressure" is extra tensile stress for one-half of each cycle. When it becomes $(0.500\%)(13.0\times10^{10}\text{ Pa}) = 6.50\times10^8\text{ Pa}$, the rod will break. Then, $\Delta P_{\max} = \rho v\omega s_{\max}$

$$s_{\max} = \frac{\Delta P_{\max}}{\rho v\omega} = \frac{6.50\times10^8\text{ N/m}^2}{(8.92\times10^3\text{ kg/m}^3)(5\,010\text{ m/s})(2\pi500/\text{s})} = \boxed{4.63\text{ mm}}$$

(b) From $s = s_{\max}\cos(kx - \omega t)$

$$v = \frac{\partial s}{\partial t} = -\omega s_{\max}\sin(kx - \omega t)$$

$$v_{\max} = \omega s_{\max} = (2\pi500/\text{s})(4.63\text{ mm}) = \boxed{14.5\text{ m/s}}$$

(c) $I = \dfrac{1}{2}\rho v(\omega s_{\max})^2 = \dfrac{1}{2}\rho v v_{\max}^2 = \dfrac{1}{2}(8.92\times10^3\text{ kg/m}^3)(5\,010\text{ m/s})(14.5\text{ m/s})^2$

$$= \boxed{4.73\times10^9\text{ W/m}^2}$$

P17.15 $\Delta P_{\max} = \rho v\omega s_{\max} = \rho v\left(\dfrac{2\pi v}{\lambda}\right)s_{\max}$

$$\lambda = \frac{2\pi\rho v^2 s_{\max}}{\Delta P_{\max}} = \frac{2\pi(1.20)(343)^2(5.50\times10^{-6})}{0.840} = \boxed{5.81\text{ m}}$$

Section 17.3 **Intensity of Periodic Sound Waves**

P17.16 The sound power incident on the eardrum is $\mathcal{P} = IA$ where I is the intensity of the sound and $A = 5.0\times10^{-5}\text{ m}^2$ is the area of the eardrum.

(a) At the threshold of hearing, $I = 1.0\times10^{-12}\text{ W/m}^2$, and

$$\mathcal{P} = (1.0\times10^{-12}\text{ W/m}^2)(5.0\times10^{-5}\text{ m}^2) = \boxed{5.00\times10^{-17}\text{ W}}$$

(b) At the threshold of pain, $I = 1.0\text{ W/m}^2$, and

$$\mathcal{P} = (1.0\text{ W/m}^2)(5.0\times10^{-5}\text{ m}^2) = \boxed{5.00\times10^{-5}\text{ W}}$$

P17.17 $\beta = 10\log\left(\dfrac{I}{I_0}\right) = 10\log\left(\dfrac{4.00\times10^{-6}}{1.00\times10^{-12}}\right) = \boxed{66.0\text{ dB}}$

P17.18 The power necessarily supplied to the speaker is the power carried away by the sound wave:

$$\mathcal{P} = \frac{1}{2}\rho A v\left(\omega s_{\max}\right)^2 = 2\pi^2 \rho A v f^2 s_{\max}^2$$

$$= 2\pi^2\left(1.20 \text{ kg/m}^3\right)\pi\left(\frac{0.08 \text{ m}}{2}\right)^2 (343 \text{ m/s})(600 \text{ 1/s})^2\left(0.12\times10^{-2} \text{ m}\right)^2 = \boxed{21.2 \text{ W}}$$

P17.19 $I = \dfrac{1}{2}\rho\omega^2 s_{\max}^2 v$

(a) At $f = 2\,500$ Hz, the frequency is increased by a factor of 2.50, so the intensity (at constant $s_{\max}$) increases by $(2.50)^2 = 6.25$.

Therefore, $6.25(0.600) = \boxed{3.75 \text{ W/m}^2}$

(b) $\boxed{0.600 \text{ W/m}^2}$

P17.20 The original intensity is $I_1 = \dfrac{1}{2}\rho\omega^2 s_{\max}^2 v = 2\pi^2 \rho v f^2 s_{\max}^2$

(a) If the frequency is increased to f' while a constant displacement amplitude is maintained, the new intensity is

$$I_2 = 2\pi^2\rho v\left(f'\right)^2 s_{\max}^2 \quad \text{so} \quad \frac{I_2}{I_1} = \frac{2\pi^2\rho v\left(f'\right)s_{\max}^2}{2\pi^2\rho v f^2 s_{\max}^2} = \left(\frac{f'}{f}\right)^2 \quad \text{or} \quad \boxed{I_2 = \left(\frac{f'}{f}\right)^2 I_1}$$

(b) If the frequency is reduced to $f' = \dfrac{f}{2}$ while the displacement amplitude is doubled, the new intensity is

$$I_2 = 2\pi^2\rho v\left(\frac{f}{2}\right)^2\left(2s_{\max}\right)^2 = 2\pi^2\rho v f^2 s_{\max}^2 = I_1$$

or the $\boxed{\text{intensity is unchanged}}$.

P17.21 (a) For the low note the wavelength is $\lambda = \dfrac{v}{f} = \dfrac{343 \text{ m/s}}{146.8/\text{s}} = \boxed{2.34 \text{ m}}$

For the high note $\lambda = \dfrac{343 \text{ m/s}}{880/\text{s}} = \boxed{0.390 \text{ m}}$

We observe that the ratio of the frequencies of these two notes is $\dfrac{880 \text{ Hz}}{146.8 \text{ Hz}} = 5.99$ nearly equal to a small integer. This fact is associated with the consonance of the notes D and A.

(b) $\beta = 10 \text{ dB} \log\left(\dfrac{I}{10^{-12} \text{ W/m}^2}\right) = 75$ dB gives $I = 3.16\times10^{-5} \text{ W/m}^2$

$$I = \frac{\Delta P_{\max}^2}{2\rho v}$$

$$\Delta P_{\max} = \sqrt{3.16\times10^{-5} \text{ W/m}^2\, 2\left(1.20 \text{ kg/m}^3\right)(343 \text{ m/s})} = \boxed{0.161 \text{ Pa}}$$

for both low and high notes.

continued on next page

(c) $I = \frac{1}{2}\rho v\left(\omega s_{\max}\right)^2 = \frac{1}{2}\rho v 4\pi^2 f^2 s_{\max}^2$

$$s_{\max} = \sqrt{\frac{I}{2\pi^2 \rho v f^2}}$$

for the low note, $s_{\max} = \sqrt{\frac{3.16\times10^{-5}\ \text{W/m}^2}{2\pi^2 1.20\ \text{kg/m}^3\ 343\ \text{m/s}}}\frac{1}{146.8/\text{s}}$

$$= \frac{6.24\times10^{-5}}{146.8}\ \text{m} = \boxed{4.25\times10^{-7}\ \text{m}}$$

for the high note, $s_{\max} = \frac{6.24\times10^{-5}}{880}\ \text{m} = \boxed{7.09\times10^{-8}\ \text{m}}$

(d) With both frequencies lower (numerically smaller) by the factor $\frac{146.8}{134.3} = \frac{880}{804.9} = 1.093$, the wavelengths and displacement amplitudes are made 1.093 times larger, and the pressure amplitudes are unchanged.

P17.22 We begin with $\beta_2 = 10\log\left(\frac{I_2}{I_0}\right)$ and $\beta_1 = 10\log\left(\frac{I_1}{I_0}\right)$ so $\beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_1}\right)$

Also, $I_2 = \frac{\mathcal{P}}{4\pi r_2^2}$ and $I_1 = \frac{\mathcal{P}}{4\pi r_1^2}$ giving $\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2$

Then, $\beta_2 - \beta_1 = 10\log\left(\frac{r_1}{r_2}\right)^2 = \boxed{20\log\left(\frac{r_1}{r_2}\right)}$

P17.23 (a) $I_1 = \left(1.00\times10^{-12}\ \text{W/m}^2\right)10^{(\beta_1/10)} = \left(1.00\times10^{-12}\ \text{W/m}^2\right)10^{80.0/10}$

or $I_1 = 1.00\times10^{-4}\ \text{W/m}^2$

$I_2 = \left(1.00\times10^{-12}\ \text{W/m}^2\right)10^{(\beta_2/10)} = \left(1.00\times10^{-12}\ \text{W/m}^2\right)10^{75.0/10}$

or $I_2 = 1.00\times10^{-4.5}\ \text{W/m}^2 = 3.16\times10^{-5}\ \text{W/m}^2$

When both sounds are present, the total intensity is

$$I = I_1 + I_2 = 1.00\times10^{-4}\ \text{W/m}^2 + 3.16\times10^{-5}\ \text{W/m}^2 = \boxed{1.32\times10^{-4}\ \text{W/m}^2}$$

(b) The decibel level for the combined sounds is

$$\beta = 10\log\left(\frac{1.32\times10^{-4}\ \text{W/m}^2}{1.00\times10^{-12}\ \text{W/m}^2}\right) = 10\log\left(1.32\times10^{8}\right) = \boxed{81.2\ \text{dB}}$$

P17.24 In $I = \dfrac{\mathcal{P}}{4\pi r^2}$, intensity I is proportional to $\dfrac{1}{r^2}$, so between locations 1 and 2: $\dfrac{I_2}{I_1} = \dfrac{r_1^2}{r_2^2}$

In $I = \dfrac{1}{2}\rho v(\omega s_{\max})^2$, intensity is proportional to $s_{\max}^2$, so $\dfrac{I_2}{I_1} = \dfrac{s_2^2}{s_1^2}$

Then, $\left(\dfrac{s_2}{s_1}\right)^2 = \left(\dfrac{r_1}{r_2}\right)^2$ or $\left(\dfrac{1}{2}\right)^2 = \left(\dfrac{r_1}{r_2}\right)^2$ giving $r_2 = 2r_1 = 2(50.0 \text{ m}) = 100 \text{ m}$

But, $r_2 = \sqrt{(50.0 \text{ m})^2 + d^2}$ yields $d = \boxed{86.6 \text{ m}}$

P17.25 (a) $120 \text{ dB} = 10 \text{ dB} \log\left[\dfrac{I}{10^{-12} \text{ W/m}^2}\right]$

$$I = 1.00 \text{ W/m}^2 = \frac{\mathcal{P}}{4\pi r^2}$$

$$r = \sqrt{\frac{\mathcal{P}}{4\pi I}} = \sqrt{\frac{6.00 \text{ W}}{4\pi(1.00 \text{ W/m}^2)}} = \boxed{0.691 \text{ m}}$$

We have assumed the speaker is an isotropic point source.

(b) $0 \text{ dB} = 10 \text{ dB} \log\left(\dfrac{I}{10^{-12} \text{ W/m}^2}\right)$

$$I = 1.00 \times 10^{-12} \text{ W/m}^2$$

$$r = \sqrt{\frac{\mathcal{P}}{4\pi I}} = \sqrt{\frac{6.00 \text{ W}}{4\pi(1.00 \times 10^{-12} \text{ W/m}^2)}} = \boxed{691 \text{ km}}$$

We have assumed a uniform medium that absorbs no energy.

P17.26 We presume the speakers broadcast equally in all directions.

(a) $r_{AC} = \sqrt{3.00^2 + 4.00^2} \text{ m} = 5.00 \text{ m}$

$$I = \frac{\mathcal{P}}{4\pi r^2} = \frac{1.00 \times 10^{-3} \text{ W}}{4\pi(5.00 \text{ m})^2} = 3.18 \times 10^{-6} \text{ W/m}^2$$

$$\beta = 10 \text{ dB} \log\left(\frac{3.18 \times 10^{-6} \text{ W/m}^2}{10^{-12} \text{ W/m}^2}\right)$$

$$\beta = 10 \text{ dB } 6.50 = \boxed{65.0 \text{ dB}}$$

(b) $r_{BC} = 4.47 \text{ m}$

$$I = \frac{1.50 \times 10^{-3} \text{ W}}{4\pi(4.47 \text{ m})^2} = 5.97 \times 10^{-6} \text{ W/m}^2$$

$$\beta = 10 \text{ dB} \log\left(\frac{5.97 \times 10^{-6}}{10^{-12}}\right)$$

$$\beta = \boxed{67.8 \text{ dB}}$$

(c) $I = 3.18 \ \mu\text{W/m}^2 + 5.97 \ \mu\text{W/m}^2$

$$\beta = 10 \text{ dB} \log\left(\frac{9.15 \times 10^{-6}}{10^{-12}}\right) = \boxed{69.6 \text{ dB}}$$

P17.27 Since intensity is inversely proportional to the square of the distance,

$$I_4 = \frac{1}{100} I_{0.4} \text{ and } I_{0.4} = \frac{\Delta P_{\max}^2}{2\rho v} = \frac{(10.0)^2}{2(1.20)(343)} = 0.121 \text{ W/m}^2$$

The difference in sound intensity level is

$$\Delta\beta = 10 \log\left(\frac{I_{4\text{ km}}}{I_{0.4\text{ km}}}\right) = 10(-2.00) = -20.0 \text{ dB}$$

At 0.400 km,

$$\beta_{0.4} = 10 \log\left(\frac{0.121 \text{ W/m}^2}{10^{-12} \text{ W/m}^2}\right) = 110.8 \text{ dB}$$

At 4.00 km,

$$\beta_4 = \beta_{0.4} + \Delta\beta = (110.8 - 20.0) \text{ dB} = 90.8 \text{ dB}$$

Allowing for absorption of the wave over the distance traveled,

$$\beta_4' = \beta_4 - (7.00 \text{ dB/km})(3.60 \text{ km}) = \boxed{65.6 \text{ dB}}$$

This is equivalent to the sound intensity level of heavy traffic.

P17.28 (a) $E = \mathcal{P}t = 4\pi r^2 It = 4\pi(100 \text{ m})^2(7.00\times10^{-2} \text{ W/m}^2)(0.200 \text{ s}) = \boxed{1.76 \text{ kJ}}$

(b) $\beta = 10\log\left(\frac{7.00\times10^{-2}}{1.00\times10^{-12}}\right) = \boxed{108 \text{ dB}}$

P17.29 $\beta = 10\log\left(\frac{I}{10^{-12}}\right) \qquad I = \left[10^{(\beta/10)}\right]\left(10^{-12}\right) \text{ W/m}^2$

$I_{(120\text{ dB})} = 1.00 \text{ W/m}^2$; $\quad I_{(100\text{ dB})} = 1.00\times10^{-2} \text{ W/m}^2$; $\quad I_{(10\text{ dB})} = 1.00\times10^{-11} \text{ W/m}^2$

(a) $\mathcal{P} = 4\pi r^2 I \quad$ so that $\quad r_1^2 I_1 = r_2^2 I_2$

$$r_2 = r_1\left(\frac{I_1}{I_2}\right)^{1/2} = (3.00 \text{ m})\sqrt{\frac{1.00}{1.00\times10^{-2}}} = \boxed{30.0 \text{ m}}$$

(b) $r_2 = r_1\left(\frac{I_1}{I_2}\right)^{1/2} = (3.00 \text{ m})\sqrt{\frac{1.00}{1.00\times10^{-11}}} = \boxed{9.49\times10^5 \text{ m}}$

P17.30 Assume you are 1 m away from your lawnmower and receiving 100 dB sound from it. The intensity of this sound is given by $100\text{ dB} = 10\text{ dB}\log\dfrac{I}{10^{-12}\text{ W/m}^2}$; $I = 10^{-2}\text{ W/m}^2$. If the lawnmower radiates as a point source, its sound power is given by $I = \dfrac{\mathcal{P}}{4\pi r^2}$

$$\mathcal{P} = 4\pi(1\text{ m})^2 10^{-2}\text{ W/m}^2 = 0.126\text{ W}$$

Now let your neighbor have an identical lawnmower 20 m away. You receive from it sound with intensity $I = \dfrac{0.126\text{ W}}{4\pi(20\text{ m})^2} = 2.5\times10^{-5}\text{ W/m}^2$. The total sound intensity impinging on you is $10^{-2}\text{ W/m}^2 + 2.5\times10^{-5}\text{ W/m}^2 = 1.002\,5\times10^{-2}\text{ W/m}^2$. So its level is

$$10\text{ dB}\log\frac{1.002\,5\times10^{-2}}{10^{-12}} = 100.01\text{ dB}$$

If the smallest noticeable difference is between 100 dB and 101 dB, this $\boxed{\text{cannot}}$ be heard as a change from 100 dB.

P17.31 (a) The sound intensity inside the church is given by

$$\beta = 10\ln\left(\frac{I}{I_0}\right)$$
$$101\text{ dB} = (10\text{ dB})\ln\left(\frac{I}{10^{-12}\text{ W/m}^2}\right)$$
$$I = 10^{10.1}\left(10^{-12}\text{ W/m}^2\right) = 10^{-1.90}\text{ W/m}^2 = 0.012\,6\text{ W/m}^2$$

We suppose that sound comes perpendicularly out through the windows and doors. Then, the radiated power is

$$\mathcal{P} = IA = \left(0.012\,6\text{ W/m}^2\right)\left(22.0\text{ m}^2\right) = 0.277\text{ W}$$

Are you surprised by how small this is? The energy radiated in 20.0 minutes is

$$E = \mathcal{P}t = (0.277\text{ J/s})(20.0\text{ min})\left(\frac{60.0\text{ s}}{1.00\text{ min}}\right) = \boxed{332\text{ J}}$$

(b) If the ground reflects all sound energy headed downward, the sound power, $\mathcal{P} = 0.277$ W, covers the area of a hemisphere. One kilometer away, this area is

$$A = 2\pi r^2 = 2\pi(1\,000\text{ m})^2 = 2\pi\times10^6\text{ m}^2$$

The intensity at this distance is

$$I = \frac{\mathcal{P}}{A} = \frac{0.277\text{ W}}{2\pi\times10^6\text{ m}^2} = 4.41\times10^{-8}\text{ W/m}^2$$

and the sound intensity level is

$$\beta = (10\text{ dB})\ln\left(\frac{4.41\times10^{-8}\text{ W/m}^2}{1.00\times10^{-12}\text{ W/m}^2}\right) = \boxed{46.4\text{ dB}}$$

Section 17.4 **The Doppler Effect**

P17.32 (a) $\omega = 2\pi f = 2\pi\left(\dfrac{115/\text{min}}{60.0 \text{ s/min}}\right) = 12.0 \text{ rad/s}$

$$v_{\max} = \omega A = (12.0 \text{ rad/s})(1.80\times10^{-3} \text{ m}) = \boxed{0.021\,7 \text{ m/s}}$$

(b) The heart wall is a moving observer.

$$f' = f\left(\frac{v+v_O}{v}\right) = (2\,000\,000 \text{ Hz})\left(\frac{1\,500+0.021\,7}{1\,500}\right) = \boxed{2\,000\,028.9 \text{ Hz}}$$

(c) Now the heart wall is a moving source.

$$f'' = f'\left(\frac{v}{v-v_s}\right) = (2\,000\,029 \text{ Hz})\left(\frac{1\,500}{1\,500-0.021\,7}\right) = \boxed{2\,000\,057.8 \text{ Hz}}$$

***P17.33** (a) $f' = \dfrac{f(v+v_o)}{(v-v_s)}$

$$f' = 2\,500\frac{(343+25.0)}{(343-40.0)} = \boxed{3.04 \text{ kHz}}$$

(b) $f' = 2\,500\left(\dfrac{343+(-25.0)}{343-(-40.0)}\right) = \boxed{2.08 \text{ kHz}}$

(c) $f' = 2\,500\left(\dfrac{343+(-25.0)}{343-40.0}\right) = \boxed{2.62 \text{ kHz}}$ while police car overtakes

$f' = 2\,500\left(\dfrac{343+25.0}{343-(-40.0)}\right) = \boxed{2.40 \text{ kHz}}$ after police car passes

P17.34 (a) The maximum speed of the speaker is described by

$$\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$$

$$v_{\max} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{20.0 \text{ N/m}}{5.00 \text{ kg}}}(0.500 \text{ m}) = 1.00 \text{ m/s}$$

The frequencies heard by the stationary observer range from

$$f'_{\min} = f\left(\frac{v}{v+v_{\max}}\right) \text{ to } f'_{\max} = f\left(\frac{v}{v-v_{\max}}\right)$$

where v is the speed of sound.

$$f'_{\min} = 440 \text{ Hz}\left(\frac{343 \text{ m/s}}{343 \text{ m/s}+1.00 \text{ m/s}}\right) = \boxed{439 \text{ Hz}}$$

$$f'_{\max} = 440 \text{ Hz}\left(\frac{343 \text{ m/s}}{343 \text{ m/s}-1.00 \text{ m/s}}\right) = \boxed{441 \text{ Hz}}$$

continued on next page

(b) $\beta = 10\text{ dB}\log\left(\frac{I}{I_0}\right) = 10\text{ dB}\log\left(\frac{\mathcal{P}/4\pi r^2}{I_0}\right)$

The maximum intensity level (of 60.0 dB) occurs at $r = r_{\min} = 1.00$ m. The minimum intensity level occurs when the speaker is farthest from the listener (i.e., when $r = r_{\max} = r_{\min} + 2A = 2.00$ m).

Thus,

$$\beta_{\max} - \beta_{\min} = 10\text{ dB}\log\left(\frac{\mathcal{P}}{4\pi I_0 r_{\min}^2}\right) - 10\text{ dB}\log\left(\frac{\mathcal{P}}{4\pi I_0 r_{\max}^2}\right)$$

or

$$\beta_{\max} - \beta_{\min} = 10\text{ dB}\log\left(\frac{\mathcal{P}}{4\pi I_0 r_{\min}^2}\frac{4\pi I_0 r_{\max}^2}{\mathcal{P}}\right) = 10\text{ dB}\log\left(\frac{r_{\max}^2}{r_{\min}^2}\right)$$

This gives:

$$60.0\text{ dB} - \beta_{\min} = 10\text{ dB}\log(4.00) = 6.02\text{ dB} \quad \text{and} \quad \beta_{\min} = \boxed{54.0\text{ dB}}$$

P17.35 Approaching ambulance: $f' = \frac{f}{(1 - v_S/v)}$

Departing ambulance: $f'' = \frac{f}{(1 - (-v_S/v))}$

Since $f' = 560$ Hz and $f'' = 480$ Hz $\quad 560\left(1 - \frac{v_S}{v}\right) = 480\left(1 + \frac{v_S}{v}\right)$

$$1\,040\frac{v_S}{v} = 80.0$$

$$v_S = \frac{80.0(343)}{1\,040}\text{ m/s} = \boxed{26.4\text{ m/s}}$$

P17.36 (a) $v = (331\text{ m/s}) + 0.6\frac{\text{m}}{\text{s}\cdot{}^\circ\text{C}}(-10^\circ\text{C}) = \boxed{325\text{ m/s}}$

(b) Approaching the bell, the athlete hears a frequency of $f' = f\left(\frac{v + v_O}{v}\right)$

After passing the bell, she hears a lower frequency of $f'' = f\left(\frac{v + (-v_O)}{v}\right)$

The ratio is $\frac{f''}{f'} = \frac{v - v_O}{v + v_O} = \frac{5}{6}$

which gives $6v - 6v_o = 5v + 5v_o$ or $v_O = \frac{v}{11} = \frac{325\text{ m/s}}{11} = \boxed{29.5\text{ m/s}}$

P17.37 $f' = f\left(\frac{v}{v - v_s}\right)$ $\quad 485 = 512\left(\frac{340}{340 - (-9.80t_{\text{fall}})}\right)$

$$485(340) + (485)(9.80t_f) = (512)(340)$$

$$t_f = \left(\frac{512 - 485}{485}\right)\frac{340}{9.80} = 1.93\text{ s}$$

$d_1 = \frac{1}{2}gt_f^2 = 18.3$ m: $\quad t_{\text{return}} = \frac{18.3}{340} = 0.053\,8$ s

The fork continues to fall while the sound returns.

$$t_{\text{total fall}} = t_f + t_{\text{return}} = 1.93\text{ s} + 0.053\,8\text{ s} = 1.985\text{ s}$$

$$d_{\text{total}} = \frac{1}{2}gt_{\text{total fall}}^2 = \boxed{19.3\text{ m}}$$

P17.38 (a) Sound moves upwind with speed $(343-15)$ m/s. Crests pass a stationary upwind point at frequency 900 Hz.

Then $$\lambda = \frac{v}{f} = \frac{328\ \text{m/s}}{900/\text{s}} = \boxed{0.364\ \text{m}}$$

(b) By similar logic, $$\lambda = \frac{v}{f} = \frac{(343+15)\ \text{m/s}}{900/\text{s}} = \boxed{0.398\ \text{m}}$$

(c) The source is moving through the air at 15 m/s toward the observer. The observer is stationary relative to the air.

$$f' = f\left(\frac{v+v_o}{v-v_s}\right) = 900\ \text{Hz}\left(\frac{343+0}{343-15}\right) = \boxed{941\ \text{Hz}}$$

(d) The source is moving through the air at 15 m/s away from the downwind firefighter. Her speed relative to the air is 30 m/s toward the source.

$$f' = f\left(\frac{v+v_o}{v-v_s}\right) = 900\ \text{Hz}\left(\frac{343+30}{343-(-15)}\right) = 900\ \text{Hz}\left(\frac{373}{358}\right) = \boxed{938\ \text{Hz}}$$

P17.39 (b) $\sin\theta = \dfrac{v}{v_S} = \dfrac{1}{3.00}$; $\theta = 19.5°$

$\tan\theta = \dfrac{h}{x}$; $x = \dfrac{h}{\tan\theta}$

$$x = \frac{20\ 000\ \text{m}}{\tan 19.5°} = 5.66\times10^4\ \text{m} = \boxed{56.6\ \text{km}}$$

(a) It takes the plane $t = \dfrac{x}{v_S} = \dfrac{5.66\times10^4\ \text{m}}{3.00(335\ \text{m/s})} = \boxed{56.3\ \text{s}}$ to travel this distance.

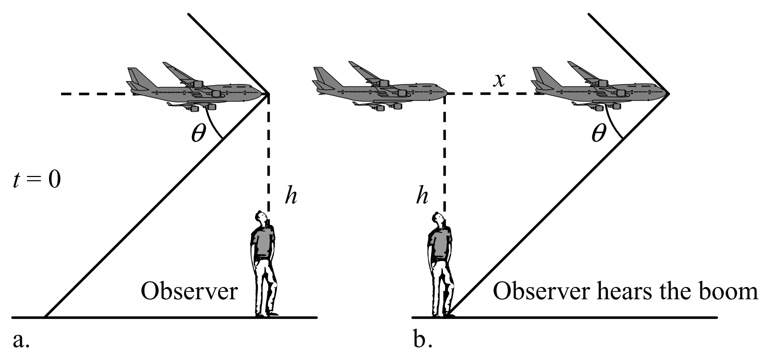


FIG. P17.39(a)

P17.40 $$\theta = \sin^{-1}\frac{v}{v_S} = \sin^{-1}\frac{1}{1.38} = \boxed{46.4°}$$

P17.41 The *half angle* of the shock wave cone is given by $\sin\theta = \dfrac{v_{\text{light}}}{v_S}$

$$v_S = \frac{v_{\text{light}}}{\sin\theta} = \frac{2.25\times10^8\ \text{m/s}}{\sin(53.0°)} = \boxed{2.82\times10^8\ \text{m/s}}$$

Section 17.5 **Digital Sound Recording**

Section 17.6 **Motion Picture Sound**

P17.42 For a 40-dB sound,

$$40\ \text{dB} = 10\ \text{dB}\log\left[\frac{I}{10^{-12}\ \text{W/m}^2}\right]$$

$$I = 10^{-8}\ \text{W/m}^2 = \frac{\Delta P_{\max}^2}{2\rho v}$$

$$\Delta P_{\max} = \sqrt{2\rho v I} = \sqrt{2\left(1.20\ \text{kg/m}^2\right)\left(343\ \text{m/s}\right)10^{-8}\ \text{W/m}^2} = 2.87\times 10^{-3}\ \text{N/m}^2$$

(a) $\text{code} = \dfrac{2.87\times 10^{-3}\ \text{N/m}^2}{28.7\ \text{N/m}^2}65\,536 = \boxed{7}$

(b) For sounds of 40 dB or softer, too few digital words are available to represent the wave form with good fidelity.

(c) In a sound wave ΔP is negative half of the time but this coding scheme has no words available for negative pressure variations.

Additional Problems

***P17.43** The gliders stick together and move with final speed given by momentum conservation for the two-glider system:

$$0.15\ \text{kg}\ 2.3\ \text{m/s} + 0 = (0.15 + 0.2)\ \text{kg}\ v \qquad v = 0.986\ \text{m/s}$$

The missing mechanical energy is

$$(1/2)(0.15\ \text{kg})(2.3\ \text{m/s})^2 - (1/2)(0.35\ \text{kg})(0.986\ \text{m/s})^2 = 0.397\ \text{J} - 0.170\ \text{J} = 0.227\ \text{J}$$

We imagine one-half of 227 mJ going into internal energy and half into sound radiated isotropically in 7 ms. Its intensity 0.8 m away is

$$I = E/At = 0.5(0.227\ \text{J})/[4\pi(0.8\ \text{m})^2\ 0.007\ \text{s}] = 2.01\ \text{W/m}^2$$

Its intensity level is $\beta = 10\log(2.01/10^{-12}) = 123\ \text{dB}$

The sound of air track gliders latching together is many orders of magnitude less intense. The idea is unreasonable. Nearly all of the missing mechanical energy becomes internal energy in the latch.

***P17.44** The wave moves outward equally in all directions. (We can tell it is outward because of the negative sign in 1.36 r – 2030 t.) Its amplitude is inversely proportional to its distance from the center. Its intensity is proportional to the square of the amplitude, so the intensity follows the inverse-square law, with no absorption of energy by the medium. Its speed is constant at $v = f\lambda = \omega/k = (2030/\text{s})/(1.36/\text{m}) = 1.49$ km/s. By comparison to the table in the chapter, it can be moving through water at 25°C, and we assume that it is. Its frequency is constant at $(2030/\text{s})/2\pi = 323$ Hz. Its wavelength is constant at $2\pi/k = 2\pi/(1.36/\text{m}) = 4.62$ m. Its pressure amplitude is 25.0 Pa at radius 1 m. Its intensity at this distance is

$$I = \frac{\Delta P_{\max}^2}{2\rho v} = \frac{\left(25\ \text{N/m}^2\right)^2}{2(1000\ \text{kg/m}^3)(1490\ \text{m/s})} = 209\ \mu\text{W/m}^2$$

so the power of the source and the net power of the wave at all distances is $\mathcal{P} = I4\pi r^2 = \left(2.09\times10^{-4}\ \text{W/m}^2\right)4\pi(1\ \text{m})^2 = 2.63$ mW.

***P17.45** Model your loud, sharp sound impulse as a single narrow peak in a graph of air pressure versus time. It is a noise with no pitch, no frequency, wavelength, or period. It radiates away from you in all directions and some of it is incident on each one of the solid vertical risers of the bleachers. Suppose that, at the ambient temperature, sound moves at 340 m/s; and suppose that the horizontal width of each row of seats is 60 cm. Then there is a time delay of

$$\frac{0.6\ \text{m}}{(340\ \text{m/s})} = 0.002\ \text{s}$$

between your sound impulse reaching each riser and the next. Whatever its material, each will reflect much of the sound that reaches it. The reflected wave sounds very different from the sharp pop you made. If there are twenty rows of seats, you hear from the bleachers a tone with twenty crests, each separated from the next in time by

$$\frac{2(0.6\ \text{m})}{(340\ \text{m/s})} = 0.004\ \text{s}$$

This is the extra time for it to cross the width of one seat twice, once as an incident pulse and once again after its reflection. Thus, you hear a sound of definite pitch, with period about 0.004 s, frequency

$$\frac{1}{0.003\,5\ \text{s}} = 300\ \text{Hz}\ \boxed{\sim \text{a few hundred Hz}}$$

wavelength

$$\lambda = \frac{v}{f} = \frac{(340\ \text{m/s})}{(300/\text{s})} = 1.2\ \text{m} \sim \boxed{10^0\ \text{m}}$$

and duration

$$20(0.004\ \text{s}) \sim \boxed{10^{-1}\ \text{s}}$$

(b) Yes. With the steps narrower, the frequency can be close to 1000 Hz. If the person clapping his hands is at the base of the pyramid, the echo can drop somewhat in frequency and in loudness as sound returns, with the later cycles coming from the smaller and more distant upper risers. The sound could imitate some particular bird, and could in fact constitute a recording of the call.

***P17.46** (a) The distance is larger by 240/60 = 4 times. The intensity is 16 times smaller at the larger distance, because the sound power is spread over a 4^2 times larger area.

(b) The amplitude is 4 times smaller at the larger distance, because intensity is proportional to the square of amplitude.

(c) The extra distance is (240 – 60)/45 = 4 wavelengths. The phase is the same at both points, because they are separated by an integer number of wavelengths.

P17.47 Since $\cos^2\theta + \sin^2\theta = 1$, $\sin\theta = \pm\sqrt{1-\cos^2\theta}$ (each sign applying half the time)

$$\Delta P = \Delta P_{\max}\sin(kx-\omega t) = \pm\rho v\omega s_{\max}\sqrt{1-\cos^2(kx-\omega t)}$$

Therefore $$\Delta P = \pm\rho v\omega\sqrt{s_{\max}^2 - s_{\max}^2\cos^2(kx-\omega t)} = \pm\rho v\omega\sqrt{s_{\max}^2 - s^2}$$

P17.48 (a) $\lambda = \dfrac{v}{f} = \dfrac{343\ \text{m/s}}{1\,480\ \text{s}^{-1}} = \boxed{0.232\ \text{m}}$

(b) $\beta = 81.0\ \text{dB} = 10\ \text{dB}\log\left[\dfrac{I}{10^{-12}\ \text{W/m}^2}\right]$

$$I = \left(10^{-12}\ \text{W/m}^2\right)10^{8.10} = 10^{-3.90}\ \text{W/m}^2 = 1.26\times10^{-4}\ \text{W/m}^2 = \frac{1}{2}\rho v\omega^2 s_{\max}^2$$

$$s_{\max} = \sqrt{\frac{2I}{\rho v\omega^2}} = \sqrt{\frac{2\left(1.26\times10^{-4}\ \text{W/m}^2\right)}{\left(1.20\ \text{kg/m}^3\right)(343\ \text{m/s})4\pi^2\left(1\,480\ \text{s}^{-1}\right)^2}} = \boxed{8.41\times10^{-8}\ \text{m}}$$

(c) $\lambda' = \dfrac{v}{f'} = \dfrac{343\ \text{m/s}}{1\,397\ \text{s}^{-1}} = 0.246\ \text{m}$ $\qquad \Delta\lambda = \lambda' - \lambda = \boxed{13.8\ \text{mm}}$

P17.49 The trucks form a train analogous to a wave train of crests with speed $v = 19.7\ \text{m/s}$ and unshifted frequency $f = \dfrac{2}{3.00\ \text{min}} = 0.667\ \text{min}^{-1}$

(a) The cyclist as observer measures a lower Doppler-shifted frequency:

$$f' = f\left(\frac{v+v_o}{v}\right) = \left(0.667\ \text{min}^{-1}\right)\left(\frac{19.7+(-4.47)}{19.7}\right) = \boxed{0.515/\text{min}}$$

(b) $$f'' = f\left(\frac{v+v_o'}{v}\right) = \left(0.667\ \text{min}^{-1}\right)\left(\frac{19.7+(-1.56)}{19.7}\right) = \boxed{0.614/\text{min}}$$

The cyclist's speed has decreased very significantly, but there is only a modest increase in the frequency of trucks passing him.

P17.50 (a) The speed of a compression wave in a bar is

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{20.0\times10^{10}\ \text{N/m}^2}{7\,860\ \text{kg/m}^3}} = \boxed{5.04\times10^3\ \text{m/s}}$$

(b) The signal to stop passes between layers of atoms as a sound wave, reaching the back end of the bar in time

$$t = \frac{L}{v} = \frac{0.800\ \text{m}}{5.04\times10^3\ \text{m/s}} = \boxed{1.59\times10^{-4}\ \text{s}}$$

continued on next page

(c) As described by Newton's first law, the rearmost layer of steel has continued to move forward with its original speed v_i for this time, compressing the bar by

$$\Delta L = v_i t = (12.0\ \text{m/s})(1.59\times10^{-4}\ \text{s}) = 1.90\times10^{-3}\ \text{m} = \boxed{1.90\ \text{mm}}$$

(d) The strain in the rod is: $\dfrac{\Delta L}{L} = \dfrac{1.90\times10^{-3}\ \text{m}}{0.800\ \text{m}} = \boxed{2.38\times10^{-3}}$

(e) The stress in the rod is: $\sigma = Y\left(\dfrac{\Delta L}{L}\right) = (20.0\times10^{10}\ \text{N/m}^2)(2.38\times10^{-3}) = \boxed{476\ \text{MPa}}$

Since $\sigma > 400$ MPa, the rod will be permanently distorted.

(f) We go through the same steps as in parts (a) through (e), but use algebraic expressions rather than numbers:

The speed of sound in the rod is $v = \sqrt{\dfrac{Y}{\rho}}$

The back end of the rod continues to move forward at speed v_i for a time of $t = \dfrac{L}{v} = L\sqrt{\dfrac{\rho}{Y}}$, traveling distance $\Delta L = v_i t$ after the front end hits the wall.

The strain in the rod is: $\dfrac{\Delta L}{L} = \dfrac{v_i t}{L} = v_i\sqrt{\dfrac{\rho}{Y}}$

The stress is then: $\sigma = Y\left(\dfrac{\Delta L}{L}\right) = Yv_i\sqrt{\dfrac{\rho}{Y}} = v_i\sqrt{\rho Y}$

For this to be less than the yield stress, σ_y, it is necessary that

$$v_i\sqrt{\rho Y} < \sigma_y \qquad \text{or} \qquad \boxed{v_i < \frac{\sigma_y}{\sqrt{\rho Y}}}$$

With the given numbers, this speed is 10.1 m/s. The fact that the length of the rod divides out means that the steel will start to bend right away at the front end of the rod. There it will yield enough so that eventually the remainder of the rod will experience only stress within the elastic range. You can see this effect when sledgehammer blows give a mushroom top to a rod used as a tent stake.

P17.51 (a) $f' = f\dfrac{v}{(v - v_{\text{diver}})}$

so $\quad 1 - \dfrac{v_{\text{diver}}}{v} = \dfrac{f}{f'} \Rightarrow v_{\text{diver}} = v\left(1 - \dfrac{f}{f'}\right)$

with $\quad v = 343\ \text{m/s},\ f = 1\,800\ \text{Hz} \quad$ and $\quad f' = 2\,150\ \text{Hz}$

we find $\quad v_{\text{diver}} = 343\left(1 - \dfrac{1\,800}{2\,150}\right) = \boxed{55.8\ \text{m/s}}$

(b) If the waves are reflected, and the skydiver is moving into them, we have

$$f'' = f'\frac{(v + v_{\text{diver}})}{v} \Rightarrow f'' = f\left[\frac{v}{(v - v_{\text{diver}})}\right]\frac{(v + v_{\text{diver}})}{v}$$

so $\quad f'' = 1\,800\dfrac{(343 + 55.8)}{(343 - 55.8)} = \boxed{2\,500\ \text{Hz}}$

P17.52 Let $P(x)$ represent absolute pressure as a function of x. The net force to the right on the chunk of air is $+P(x)A - P(x+\Delta x)A$. Atmospheric pressure subtracts out, leaving $\left[-\Delta P(x+\Delta x)+\Delta P(x)\right]A = -\dfrac{\partial \Delta P}{\partial x}\Delta x A$. The mass of the air is $\Delta m = \rho \Delta V = \rho A \Delta x$ and its acceleration is $\dfrac{\partial^2 s}{\partial t^2}$. So Newton's second law becomes

$$-\frac{\partial \Delta P}{\partial x}\Delta x A = \rho A \Delta x \frac{\partial^2 s}{\partial t^2}$$

$$-\frac{\partial}{\partial x}\left(-B\frac{\partial s}{\partial x}\right) = \rho \frac{\partial^2 s}{\partial t^2}$$

$$\frac{B}{\rho}\frac{\partial^2 s}{\partial x^2} = \frac{\partial^2 s}{\partial t^2}$$

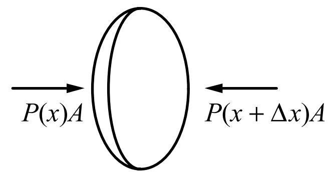


FIG. P17.52

Into this wave equation as a trial solution we substitute the wave function $s(x,\ t) = s_{\max}\cos(kx - \omega t)$ We find

$$\frac{\partial s}{\partial x} = -k s_{\max}\sin(kx-\omega t)$$

$$\frac{\partial^2 s}{\partial x^2} = -k^2 s_{\max}\cos(kx-\omega t)$$

$$\frac{\partial s}{\partial t} = +\omega s_{\max}\sin(kx-\omega t)$$

$$\frac{\partial^2 s}{\partial t^2} = -\omega^2 s_{\max}\cos(kx-\omega t)$$

$\dfrac{B}{\rho}\dfrac{\partial^2 s}{\partial x^2} = \dfrac{\partial^2 s}{\partial t^2}$ becomes $-\dfrac{B}{\rho}k^2 s_{\max}\cos(kx-\omega t) = -\omega^2 s_{\max}\cos(kx-\omega t)$

This is true provided $\dfrac{B}{\rho}\dfrac{4\pi^2}{\lambda^2} = 4\pi^2 f^2$

The sound wave can propagate provided it has $\lambda^2 f^2 = v^2 = \dfrac{B}{\rho}$; that is, provided it propagates with speed $v = \sqrt{\dfrac{B}{\rho}}$

P17.53 When observer is moving in front of and in the same direction as the source, $f' = f\dfrac{v - v_O}{v - v_S}$ where v_O and v_S are measured relative to the medium in which the sound is propagated. In this case the ocean current is opposite the direction of travel of the ships and

$$v_O = 45.0\ \text{km/h} - (-10.0\ \text{km/h}) = 55.0\ \text{km/h} = 15.3\ \text{m/s}, \text{ and}$$

$$v_S = 64.0\ \text{km/h} - (-10.0\ \text{km/h}) = 74.0\ \text{km/h} = 20.55\ \text{m/s}$$

Therefore,

$$f' = (1\,200.0\ \text{Hz})\frac{1\,520\ \text{m/s} - 15.3\ \text{m/s}}{1\,520\ \text{m/s} - 20.55\ \text{m/s}} = \boxed{1\,204.2\ \text{Hz}}$$

P17.54 Use the Doppler formula, and remember that the bat is a moving source. If the velocity of the insect is v_x,

$$40.4 = 40.0\frac{(340+5.00)(340-v_x)}{(340-5.00)(340+v_x)}$$

Solving,

$$v_x = 3.31\ \text{m/s}$$

Therefore, $\boxed{\text{the bat is gaining on its prey at } 1.69\ \text{m/s}}$.

P17.55 $103\text{ dB} = 10\text{ dB}\log\left[\dfrac{I}{10^{-12}\text{ W/m}^2}\right]$

(a) $I = 2.00\times 10^{-2}\text{ W/m}^2 = \dfrac{\mathcal{P}}{4\pi r^2} = \dfrac{\mathcal{P}}{4\pi(1.6\text{ m})^2}$

$\mathcal{P} = \boxed{0.642\text{ W}}$

(b) $\text{efficiency} = \dfrac{\text{sound output power}}{\text{total input power}} = \dfrac{0.642\text{ W}}{150\text{ W}} = \boxed{0.004\ 28}$

P17.56 (a)

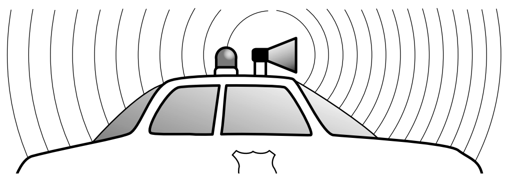


FIG. P17.56(a)

(b) $\lambda = \dfrac{v}{f} = \dfrac{343\text{ m/s}}{1\ 000\text{ s}^{-1}} = \boxed{0.343\text{ m}}$

(c) $\lambda' = \dfrac{v}{f'} = \dfrac{v}{f}\left(\dfrac{v - v_S}{v}\right) = \dfrac{(343-40.0)\text{ m/s}}{1\ 000\text{ s}^{-1}} = \boxed{0.303\text{ m}}$

(d) $\lambda'' = \dfrac{v}{f''} = \dfrac{v}{f}\left(\dfrac{v + v_S}{v}\right) = \dfrac{(343+40.0)\text{ m/s}}{1\ 000\text{ s}^{-1}} = \boxed{0.383\text{ m}}$

(e) $f' = f\left(\dfrac{v - v_O}{v - v_S}\right) = (1\ 000\text{ Hz})\dfrac{(343-30.0)\text{ m/s}}{(343-40.0)\text{ m/s}} = \boxed{1.03\text{ kHz}}$

***P17.57** (a) $\boxed{\text{The sound through the metal arrives first, because it moves faster than sound in air.}}$

(b) Each travel time is individually given by $\Delta t = L/v$. Then the delay between the pulses' arrivals is

$$\Delta t = L\left(\frac{1}{v_{\text{air}}} - \frac{1}{v_{\text{cu}}}\right) = L\frac{v_{\text{cu}} - v_{\text{air}}}{v_{\text{air}}v_{\text{cu}}}$$

and the length of the bar is $L = \dfrac{v_{\text{air}}v_{\text{cu}}}{v_{\text{cu}} - v_{\text{air}}}\Delta t = \dfrac{(331\text{ m/s})(3.56\times 10^3\text{ m/s})}{(3\ 560 - 331)\text{ m/s}}\Delta t = \boxed{(365\text{ m/s})\Delta t}$

(c) $L = (365\text{ m/s})(0.127\text{ s}) = \boxed{46.3\text{ m}}$

(d) The answer becomes $L = \dfrac{\Delta t}{\dfrac{1}{331\text{ m/s}} - \dfrac{1}{v_r}}$ where v_r is the speed of sound in the rod. As v_r goes to infinity, the travel time in the rod becomes negligible. The answer approaches $(331\text{ m/s})\Delta t$, which is just the distance that the sound travels in air during the delay time.

P17.58 $\mathcal{P}_2 = \dfrac{1}{20.0}\mathcal{P}_1 \quad \beta_1 - \beta_2 = 10\log\dfrac{\mathcal{P}_1}{\mathcal{P}_2}$

$80.0 - \beta_2 = 10\log 20.0 = +13.0$

$\beta_2 = \boxed{67.0\text{ dB}}$

P17.59 (a) $\theta = \sin^{-1}\left(\dfrac{v_{\text{sound}}}{v_{\text{obj}}}\right) = \sin^{-1}\left(\dfrac{331}{20.0\times 10^3}\right) = \boxed{0.948°}$

(b) $\theta' = \sin^{-1}\left(\dfrac{1\,533}{20.0\times 10^3}\right) = \boxed{4.40°}$

P17.60 Let T represent the period of the source vibration, and E be the energy put into each wavefront. Then $\mathcal{P}_{\text{av}} = \dfrac{E}{T}$. When the observer is at distance r in front of the source, he is receiving a spherical wavefront of radius vt, where t is the time since this energy was radiated, given by $vt - v_S t = r$. Then,

$$t = \frac{r}{v - v_S}$$

The area of the sphere is $4\pi(vt)^2 = \dfrac{4\pi v^2 r^2}{(v-v_S)^2}$. The energy per unit area over the spherical wavefront is uniform with the value $\dfrac{E}{A} = \dfrac{\mathcal{P}_{\text{av}} T(v-v_S)^2}{4\pi v^2 r^2}$. The observer receives parcels of energy with the Doppler shifted frequency $f' = f\left(\dfrac{v}{v-v_S}\right) = \dfrac{v}{T(v-v_S)}$, so the observer receives a wave with intensity

$$I = \left(\frac{E}{A}\right) f' = \left(\frac{\mathcal{P}_{\text{av}} T(v-v_S)^2}{4\pi v^2 r^2}\right)\left(\frac{v}{T(v-v_S)}\right) = \boxed{\frac{\mathcal{P}_{\text{av}}}{4\pi r^2}\left(\frac{v-v_S}{v}\right)}$$

P17.61 For the longitudinal wave $v_L = \left(\dfrac{Y}{\rho}\right)^{1/2}$

For the transverse wave $v_T = \left(\dfrac{T}{\mu}\right)^{1/2}$

If we require $\dfrac{v_L}{v_T} = 8.00$, we have $T = \dfrac{\mu Y}{64.0\rho}$ where $\mu = \dfrac{m}{L}$ and

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{\pi r^2 L}$$

This gives

$$T = \frac{\pi r^2 Y}{64.0} = \frac{\pi\left(2.00\times 10^{-3}\text{ m}\right)^2\left(6.80\times 10^{10}\text{ N/m}^2\right)}{64.0} = \boxed{1.34\times 10^4\text{ N}}$$

P17.62 (a) If the source and the observer are moving away from each other, we have: $\theta_S = \theta_0 = 180°$, and since $\cos 180° = -1$, we get Equation (17.13) with negative values for both v_O and v_S.

(b) If $v_O = 0$ m/s then $f' = \dfrac{v}{v - v_S \cos\theta_S} f$

Also, when the train is 40.0 m from the intersection, and the car is 30.0 m from the intersection,

$$\cos\theta_S = \frac{4}{5}$$

so $\quad f' = \dfrac{343 \text{ m/s}}{343 \text{ m/s} - 0.800(25.0 \text{ m/s})}(500 \text{ Hz})$

or $\quad f' = \boxed{531 \text{ Hz}}$

Note that as the train approaches, passes, and departs from the intersection, θ_S varies from 0° to 180° and the frequency heard by the observer varies between the limits

$$f'_{\max} = \frac{v}{v - v_S \cos 0°} f = \frac{343 \text{ m/s}}{343 \text{ m/s} - 25.0 \text{ m/s}}(500 \text{ Hz}) = 539 \text{ Hz}$$

$$f'_{\min} = \frac{v}{v - v_S \cos 180°} f = \frac{343 \text{ m/s}}{343 \text{ m/s} + 25.0 \text{ m/s}}(500 \text{ Hz}) = 466 \text{ Hz}$$

P17.63 (a) The time required for a sound pulse to travel distance L at speed v is given by $t = \dfrac{L}{v} = \dfrac{L}{\sqrt{Y/\rho}}$

Using this expression we find

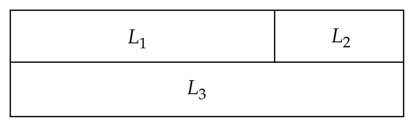


FIG. P17.63

$$t_1 = \frac{L_1}{\sqrt{Y_1/\rho_1}} = \frac{L_1}{\sqrt{(7.00 \times 10^{10} \text{ N/m}^2)/(2\,700 \text{ kg/m}^3)}} = (1.96 \times 10^{-4} L_1) \text{ s}$$

$$t_2 = \frac{1.50 \text{ m} - L_1}{\sqrt{Y_2/\rho_2}} = \frac{1.50 \text{ m} - L_1}{\sqrt{(1.60 \times 10^{10} \text{ N/m}^2)/(11.3 \times 10^3 \text{ kg/m}^3)}}$$

or $\quad t_2 = (1.26 \times 10^{-3} - 8.40 \times 10^{-4} L_1) \text{ s}$

$$t_3 = \frac{1.50 \text{ m}}{\sqrt{(11.0 \times 10^{10} \text{ N/m}^3)/(8\,800 \text{ kg/m}^3)}}$$

$$t_3 = 4.24 \times 10^{-4} \text{ s}$$

We require $t_1 + t_2 = t_3$, or

$$1.96 \times 10^{-4} L_1 + 1.26 \times 10^{-3} - 8.40 \times 10^{-4} L_1 = 4.24 \times 10^{-4}$$

This gives $\quad L_1 = 1.30$ m $\quad$ and $\quad L_2 = 1.50 - 1.30 = 0.201$ m

The ratio of lengths is then $\dfrac{L_1}{L_2} = \boxed{6.45}$

(b) The ratio of lengths $\dfrac{L_1}{L_2}$ is adjusted in part (a) so that $t_1 + t_2 = t_3$. Sound travels the two paths in equal time and the phase difference $\boxed{\Delta\phi = 0}$.

ANSWERS TO EVEN PROBLEMS

P17.2 1.43 km/s

P17.4 (a) 27.2 s (b) longer than 25.7 s, because the air is cooler

P17.6 (a) 153 m/s (b) 614 m

P14.8 (a) The speed decreases by 4.6%, from 347 m/s to 331 m/s. (b) The frequency is unchanged, because every wave crest in the hot air becomes one crest without delay in the cold air. (c) The wavelength decreases by 4.6%, from 86.7 mm to 82.8 mm. The crests are more crowded together when they move slower.

P17.10 1.55×10^{-10} m

P17.12 (a) 1.27 Pa (b) 170 Hz (c) 2.00 m (d) 340 m/s

P17.14 (a) 4.63 mm (b) 14.5 m/s (c) $4.73 \times 10^9\ \text{W/m}^2$

P17.16 (a) 5.00×10^{-17} W (b) 5.00×10^{-5} W

P17.18 21.2 W

P17.20 (a) $I_2 = \left(\frac{f'}{f}\right)^2 I_1$ (b) $I_2 = I_1$

P17.22 see the solution

P17.24 86.6 m

P17.26 (a) 65.0 dB (b) 67.8 dB (c) 69.6 dB

P17.28 (a) 1.76 kJ (b) 108 dB

P17.30 no

P17.32 (a) 2.17 cm/s (b) 2 000 028.9 Hz (c) 2 000 057.8 Hz

P17.34 (a) 441 Hz; 439 Hz (b) 54.0 dB

P17.36 (a) 325 m/s (b) 29.5 m/s

P17.38 (a) 0.364 m (b) 0.398 m (c) 941 Hz (d) 938 Hz

P17.40 46.4°

P17.42 (a) 7 (b) For sounds of 40 dB or softer, too few digital words are available to represent the wave form with good fidelity. (c) In a sound wave ΔP is negative half of the time but this coding scheme has no words available for negative pressure variations.

P17.44 The wave moves outward equally in all directions. Its amplitude is inversely proportional to its distance from the center so that its intensity follows the inverse-square law, with no absorption of energy by the medium. Its speed is constant at 1.49 km/s, so it can be moving through water at 25°C, and we assume that it is. Its frequency is constant at 323 Hz. Its wavelength is constant at 4.62 m. Its pressure amplitude is 25.0 Pa at radius 1 m. Its intensity at this distance is 209 $\mu\text{W/m}^2$, so the power of the source and the net power of the wave at all distances is 2.63 mW.

P17.46 (a) The intensity is 16 times smaller at the larger distance, because the sound power is spread over a 4^2 times larger area. (b) The amplitude is 4 times smaller at the larger distance, because intensity is proportional to the square of amplitude. (c) The phase is the same at both points, because they are separated by an integer number of wavelengths.

P17.48 (a) 0.232 m (b) 84.1 nm (c) 13.8 mm

P17.50 (a) 5.04 km/s (b) 159 μs (c) 1.90 mm (d) 0.002 38 (e) 476 MPa (f) see the solution

P17.52 see the solution

P17.54 The gap between bat and insect is closing at 1.69 m/s.

P17.56 (a) see the solution (b) 0.343 m (c) 0.303 m (d) 0.383 m (e) 1.03 kHz

P17.58 67.0 dB

P17.60 see the solution

P17.62 (a) see the solution (b) 531 Hz

18

Superposition and Standing Waves

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q18.1 No. Waves with all waveforms interfere. Waves with other wave shapes are also trains of disturbance that add together when waves from different sources move through the same medium at the same time.

***Q18.2** (i) If the end is fixed, there is inversion of the pulse upon reflection. Thus, when they meet, they cancel and the amplitude is zero. Answer (d).

(ii) If the end is free, there is no inversion on reflection. When they meet, the amplitude is $2A = 2(0.1\text{ m}) = 0.2\text{ m}$. Answer (b).

***Q18.3** In the starting situation, the waves interfere constructively. When the sliding section is moved out by 0.1 m, the wave going through it has an extra path length of 0.2 m = $\lambda/4$, to show partial interference. When the slide has come out 0.2 m from the starting configuration, the extra path length is 0.4 m = $\lambda/2$, for destructive interference. Another 0.1 m and we are at $r_2 - r_1 = 3\lambda/4$ for partial interference as before. At last, another equal step of sliding and one wave travels one wavelength farther to interfere constructively. The ranking is then d > a = c > b.

Q18.4 No. The total energy of the pair of waves remains the same. Energy missing from zones of destructive interference appears in zones of constructive interference.

***Q18.5** Answer (c). The two waves must have slightly different amplitudes at *P* because of their different distances, so they cannot cancel each other exactly.

Q18.6 Damping, and non–linear effects in the vibration turn the energy of vibration into internal energy.

***Q18.7** The strings have different linear densities and are stretched to different tensions, so they carry string waves with different speeds and vibrate with different fundamental frequencies. They are all equally long, so the string waves have equal wavelengths. They all radiate sound into air, where the sound moves with the same speed for different sound wavelengths. The answer is (b) and (e).

***Q18.8** The fundamental frequency is described by $f_1 = \dfrac{v}{2L}$, where $v = \left(\dfrac{T}{\mu}\right)^{1/2}$

(i) If L is doubled, then $f_1 \propto L^{-1}$ will be reduced by a factor $\dfrac{1}{2}$. Answer (f).

(ii) If μ is doubled, then $f_1 \propto \mu^{-1/2}$ will be reduced by a factor $\dfrac{1}{\sqrt{2}}$. Answer (e).

(iii) If T is doubled, then $f_1 \propto \sqrt{T}$ will increase by a factor of $\sqrt{2}$. Answer (c).

***Q18.9** Answer (d). The energy has not disappeared, but is still carried by the wave pulses. Each particle of the string still has kinetic energy. This is similar to the motion of a simple pendulum. The pendulum does not stop at its equilibrium position during oscillation—likewise the particles of the string do not stop at the equilibrium position of the string when these two waves superimpose.

***Q18.10** The resultant amplitude is greater than either individual amplitude, wherever the two waves are nearly enough in phase that $2A\cos(\phi/2)$ is greater than A. This condition is satisfied whenever the absolute value of the phase difference ϕ between the two waves is less than 120°. Answer (d).

Q18.11 What is needed is a tuning fork—or other pure-tone generator—of the desired frequency. Strike the tuning fork and pluck the corresponding string on the piano at the same time. If they are precisely in tune, you will hear a single pitch with no amplitude modulation. If the two pitches are a bit off, you will hear beats. As they vibrate, retune the piano string until the beat frequency goes to zero.

***Q18.12** The bow string is pulled away from equilibrium and released, similar to the way that a guitar string is pulled and released when it is plucked. Thus, standing waves will be excited in the bow string. If the arrow leaves from the exact center of the string, then a series of odd harmonics will be excited. Even harmonies will not be excited because they have a node at the point where the string exhibits its maximum displacement. Answer (c).

***Q18.13** (a) The tuning fork hits the paper repetitively to make a sound like a buzzer, and the paper efficiently moves the surrounding air. The tuning fork will vibrate audibly for a shorter time.

(b) Instead of just radiating sound very softly into the surrounding air, the tuning fork makes the chalkboard vibrate. With its large area this stiff sounding board radiates sound into the air with higher power. So it drains away the fork's energy of vibration faster and the fork stops vibrating sooner.

(c) The tuning fork in resonance makes the column of air vibrate, especially at the antinode of displacement at the top of the tube. Its area is larger than that of the fork tines, so it radiates louder sound into the environment. The tuning fork will not vibrate for so long.

(d) The tuning fork ordinarily pushes air to the right on one side and simultaneously pushes air to the left a couple of centimeters away, on the far side of its other time. Its net disturbance for sound radiation is small. The slot in the cardboard admits the 'back wave' from the far side of the fork and keeps much of it from interfering destructively with the sound radiated by the tine in front. Thus the sound radiated in front of the screen can become noticeably louder. The fork will vibrate for a shorter time.

All four of these processes exemplify conservation of energy, as the energy of vibration of the fork is transferred faster into energy of vibration of the air. The reduction in the time of audible fork vibration is easy to observe in case (a), but may be challenging to observe in the other cases.

Q18.14 Walking makes the person's hand vibrate a little. If the frequency of this motion is equal to the natural frequency of coffee sloshing from side to side in the cup, then a large–amplitude vibration of the coffee will build up in resonance. To get off resonance and back to the normal case of a small-amplitude disturbance producing a small–amplitude result, the person can walk faster, walk slower, or get a larger or smaller cup. Alternatively, even at resonance he can reduce the amplitude by adding damping, as by stirring high–fiber quick–cooking oatmeal into the hot coffee. You do not need a cover on your cup.

***Q18.15** The tape will reduce the frequency of the fork, leaving the string frequency unchanged. If the bit of tape is small, the fork must have started with a frequency 4 Hz below that of the string, to end up with a frequency 5 Hz below that of the string. The string frequency is $262 + 4 = 266$ Hz, answer (d).

Q18.16 Beats. The propellers are rotating at slightly different frequencies.

SOLUTIONS TO PROBLEMS

Section 18.1 **Superposition and Interference**

P18.1 $y = y_1 + y_2 = 3.00\cos(4.00x - 1.60t) + 4.00\sin(5.0x - 2.00t)$ evaluated at the given x values.

(a) $x = 1.00,\ t = 1.00$ $\quad y = 3.00\cos(2.40\text{ rad}) + 4.00\sin(+3.00\text{ rad}) = \boxed{-1.65\text{ cm}}$

(b) $x = 1.00,\ t = 0.500$ $\quad y = 3.00\cos(+3.20\text{ rad}) + 4.00\sin(+4.00\text{ rad}) = \boxed{-6.02\text{ cm}}$

(c) $x = 0.500,\ t = 0$ $\quad y = 3.00\cos(+2.00\text{ rad}) + 4.00\sin(+2.50\text{ rad}) = \boxed{1.15\text{ cm}}$

P18.2

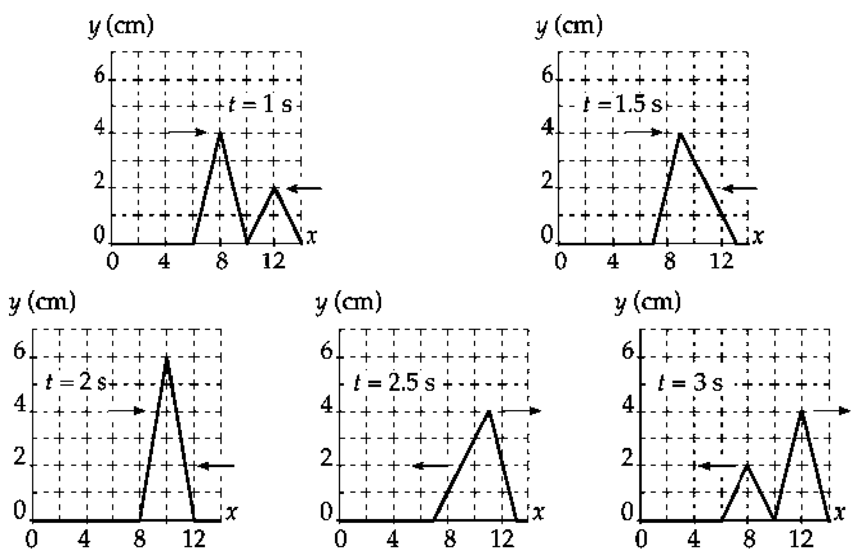


FIG. P18.2

P18.3 (a) $y_1 = f(x - vt)$, so wave 1 travels in the $\boxed{+x\text{ direction}}$

$y_2 = f(x + vt)$, so wave 2 travels in the $\boxed{-x\text{ direction}}$

(b) To cancel, $y_1 + y_2 = 0$:

$$\frac{5}{(3x-4t)^2+2} = \frac{+5}{(3x+4t-6)^2+2}$$

$$(3x-4t)^2 = (3x+4t-6)^2$$

$$3x - 4t = \pm(3x + 4t - 6)$$

from the positive root, $\quad 8t = 6 \quad \boxed{t = 0.750\text{ s}}$

(at $t = 0.750$ s, the waves cancel everywhere)

(c) from the negative root, $\quad 6x = 6 \quad \boxed{x = 1.00\text{ m}}$

(at $x = 1.00$ m, the waves cancel always)

P18.4 Suppose the waves are sinusoidal.

The sum is $(4.00\text{ cm})\sin(kx-\omega t)+(4.00\text{ cm})\sin(kx-\omega t+90.0°)$

$2(4.00\text{ cm})\sin(kx-\omega t+45.0°)\cos 45.0°$

So the amplitude is $(8.00\text{ cm})\cos 45.0° = \boxed{5.66\text{ cm}}$.

P18.5 The resultant wave function has the form

$$y=2A_0\cos\left(\frac{\phi}{2}\right)\sin\left(kx-\omega t+\frac{\phi}{2}\right)$$

(a) $A=2A_0\cos\left(\frac{\phi}{2}\right)=2(5.00)\cos\left[\frac{-\pi/4}{2}\right]=\boxed{9.24\text{ m}}$

(b) $f=\frac{\omega}{2\pi}=\frac{1\,200\pi}{2\pi}=\boxed{600\text{ Hz}}$

P18.6 (a) $\Delta x=\sqrt{9.00+4.00}-3.00=\sqrt{13}-3.00=0.606\text{ m}$

The wavelength is $\lambda=\frac{v}{f}=\frac{343\text{ m/s}}{300\text{ Hz}}=1.14\text{ m}$

Thus, $\frac{\Delta x}{\lambda}=\frac{0.606}{1.14}=0.530$ of a wave,

or $\Delta\phi=2\pi(0.530)=\boxed{3.33\text{ rad}}$

(b) For destructive interference, we want $\frac{\Delta x}{\lambda}=0.500=f\frac{\Delta x}{v}$

where Δx is a constant in this set up. $f=\frac{v}{2\Delta x}=\frac{343}{2(0.606)}=\boxed{283\text{ Hz}}$

P18.7 We suppose the man's ears are at the same level as the lower speaker. Sound from the upper speaker is delayed by traveling the extra distance $\sqrt{L^2+d^2}-L$.

He hears a minimum when this is $\frac{(2n-1)\lambda}{2}$ with $n=1, 2, 3, \ldots$

Then,

$$\sqrt{L^2+d^2}-L=\frac{(n-1/2)v}{f}$$

$$\sqrt{L^2+d^2}=\frac{(n-1/2)v}{f}+L$$

$$L^2+d^2=\frac{(n-1/2)^2v^2}{f^2}+L^2+\frac{2(n-1/2)vL}{f}$$

$$L=\frac{d^2-(n-1/2)^2v^2/f^2}{2(n-1/2)v/f}\quad n=1, 2, 3,\ldots$$

This will give us the answer to (b). The path difference starts from nearly zero when the man is very far away and increases to d when $L=0$. The number of minima he hears is the greatest integer solution to $d\ge\frac{(n-1/2)v}{f}$

$$n=\text{greatest integer}\le\frac{df}{v}+\frac{1}{2}$$

continued on next page

(a) $\dfrac{df}{v}+\dfrac{1}{2}=\dfrac{(4.00\text{ m})(200/\text{s})}{330\text{ m/s}}+\dfrac{1}{2}=2.92$

He hears $\boxed{\text{two}}$ minima.

(b) With $n=1$,

$$L=\frac{d^2-(1/2)^2v^2/f^2}{2(1/2)v/f}=\frac{(4.00\text{ m})^2-(330\text{ m/s})^2/4(200/\text{s})^2}{(330\text{ m/s})/200/\text{s}}$$

$$L=\boxed{9.28\text{ m}}$$

With $n=2$,

$$L=\frac{d^2-(3/2)^2v^2/f^2}{2(3/2)v/f}=\boxed{1.99\text{ m}}$$

P18.8 Suppose the man's ears are at the same level as the lower speaker. Sound from the upper speaker is delayed by traveling the extra distance $\Delta r=\sqrt{L^2+d^2}-L$.

He hears a minimum when $\Delta r=(2n-1)\left(\dfrac{\lambda}{2}\right)$ with $n=1, 2, 3, \ldots$

Then,

$$\sqrt{L^2+d^2}-L=\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)$$

$$\sqrt{L^2+d^2}=\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)+L$$

$$L^2+d^2=\left(n-\frac{1}{2}\right)^2\left(\frac{v}{f}\right)^2+2\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)L+L^2$$

$$d^2-\left(n-\frac{1}{2}\right)^2\left(\frac{v}{f}\right)^2=2\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)L \qquad (1)$$

Equation 1 gives the distances from the lower speaker at which the man will hear a minimum. The path difference Δr starts from nearly zero when the man is very far away and increases to d when $L=0$.

(a) The number of minima he hears is the greatest integer value for which $L\geq 0$. This is the same as the greatest integer solution to $d\geq\left(n-\dfrac{1}{2}\right)\left(\dfrac{v}{f}\right)$, or

$$\boxed{\text{number of minima heard} = n_{\max} = \text{greatest integer} \leq d\left(\frac{f}{v}\right)+\frac{1}{2}}$$

(b) From equation 1, the distances at which minima occur are given by

$$\boxed{L_n=\frac{d^2-(n-1/2)^2(v/f)^2}{2(n-1/2)(v/f)}\ \text{ where } n=1, 2, \ldots, n_{\max}}$$

P18.9 (a) $\phi_1 = (20.0 \text{ rad/cm})(5.00 \text{ cm}) - (32.0 \text{ rad/s})(2.00 \text{ s}) = 36.0 \text{ rad}$

$\phi_1 = (25.0 \text{ rad/cm})(5.00 \text{ cm}) - (40.0 \text{ rad/s})(2.00 \text{ s}) = 45.0 \text{ rad}$

$\Delta\phi = 9.00 \text{ radians} = 516° = \boxed{156°}$

(b) $\Delta\phi = |20.0x - 32.0t - [25.0x - 40.0t]| = |-5.00x + 8.00t|$

At $t = 2.00$ s, the requirement is

$$\Delta\phi = |-5.00x + 8.00(2.00)| = (2n+1)\pi \text{ for any integer } n.$$

For $x < 3.20$, $-5.00x + 16.0$ is positive, so we have

$$-5.00x + 16.0 = (2n+1)\pi, \text{ or}$$

$$x = 3.20 - \frac{(2n+1)\pi}{5.00}$$

The smallest positive value of x occurs for $n = 2$ and is

$$x = 3.20 - \frac{(4+1)\pi}{5.00} = 3.20 - \pi = \boxed{0.058\,4 \text{ cm}}$$

***P18.10** (a) First we calculate the wavelength: $\lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{21.5 \text{ Hz}} = 16.0 \text{ m}$

Then we note that the path difference equals $9.00 \text{ m} - 1.00 \text{ m} = \frac{1}{2}\lambda$

Point A is one-half wavelength farther from one speaker than from the other. The waves from the two sources interfere destructively, so the receiver records a minimum in sound intensity.

(b) We choose the origin at the midpoint between the speakers. If the receiver is located at point (x, y), then we must solve:

$$\sqrt{(x+5.00)^2 + y^2} - \sqrt{(x-5.00)^2 + y^2} = \frac{1}{2}\lambda$$

Then, $\sqrt{(x+5.00)^2 + y^2} = \sqrt{(x-5.00)^2 + y^2} + \frac{1}{2}\lambda$

Square both sides and simplify to get: $20.0x - \frac{\lambda^2}{4} = \lambda\sqrt{(x-5.00)^2 + y^2}$

Upon squaring again, this reduces to: $400x^2 - 10.0\lambda^2 x + \frac{\lambda^4}{16.0} = \lambda^2(x-5.00)^2 + \lambda^2 y^2$

Substituting $\lambda = 16.0$ m, and reducing, $9.00x^2 - 16.0y^2 = 144$

or $\frac{x^2}{16.0} - \frac{y^2}{9.00} = 1$

The point should move along the hyperbola $9x^2 - 16y^2 = 144$.

(c) Yes. Far from the origin the equation might as well be $9x^2 - 16y^2 = 0$, so the point can move along the straight line through the origin with slope 0.75 or the straight line through the origin with slope -0.75.

Section 18.2 **Standing Waves**

P18.11 $y = (1.50\text{ m})\sin(0.400x)\cos(200t) = 2A_0 \sin kx \cos \omega t$

Therefore, $k = \dfrac{2\pi}{\lambda} = 0.400\text{ rad/m}$ $\quad \lambda = \dfrac{2\pi}{0.400\text{ rad/m}} = \boxed{15.7\text{ m}}$

and $\omega = 2\pi f$ so $\quad f = \dfrac{\omega}{2\pi} = \dfrac{200\text{ rad/s}}{2\pi\text{ rad}} = \boxed{31.8\text{ Hz}}$

The speed of waves in the medium is $\quad v = \lambda f = \dfrac{\lambda}{2\pi} 2\pi f = \dfrac{\omega}{k} = \dfrac{200\text{ rad/s}}{0.400\text{ rad/m}} = \boxed{500\text{ m/s}}$

P18.12 From $y = 2A_0 \sin kx \cos \omega t$ we find

$$\frac{\partial y}{\partial x} = 2A_0 k \cos kx \cos \omega t \qquad \frac{\partial y}{\partial t} = -2A_0 \omega \sin kx \sin \omega t$$

$$\frac{\partial^2 y}{\partial x^2} = -2A_0 k^2 \sin kx \cos \omega t \qquad \frac{\partial^2 y}{\partial t^2} = -2A_0 \omega^2 \sin kx \cos \omega t$$

Substitution into the wave equation gives $\quad -2A_0 k^2 \sin kx \cos \omega t = \left(\dfrac{1}{v^2}\right)\left(-2A_0 \omega^2 \sin kx \cos \omega t\right)$

This is satisfied, provided that $v = \dfrac{\omega}{k}$. But this is true, because $v = \lambda f = \dfrac{\lambda}{2\pi} 2\pi f = \dfrac{\omega}{k}$

P18.13 The facing speakers produce a standing wave in the space between them, with the spacing between nodes being

$$d_{\text{NN}} = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343\text{ m/s}}{2\left(800\text{ s}^{-1}\right)} = 0.214\text{ m}$$

If the speakers vibrate in phase, the point halfway between them is an antinode of pressure at a distance from either speaker of

$$\frac{1.25\text{ m}}{2} = 0.625$$

Then there is a node at $\quad 0.625 - \dfrac{0.214}{2} = \boxed{0.518\text{ m}}$

a node at $\quad 0.518\text{ m} - 0.214\text{ m} = \boxed{0.303\text{ m}}$

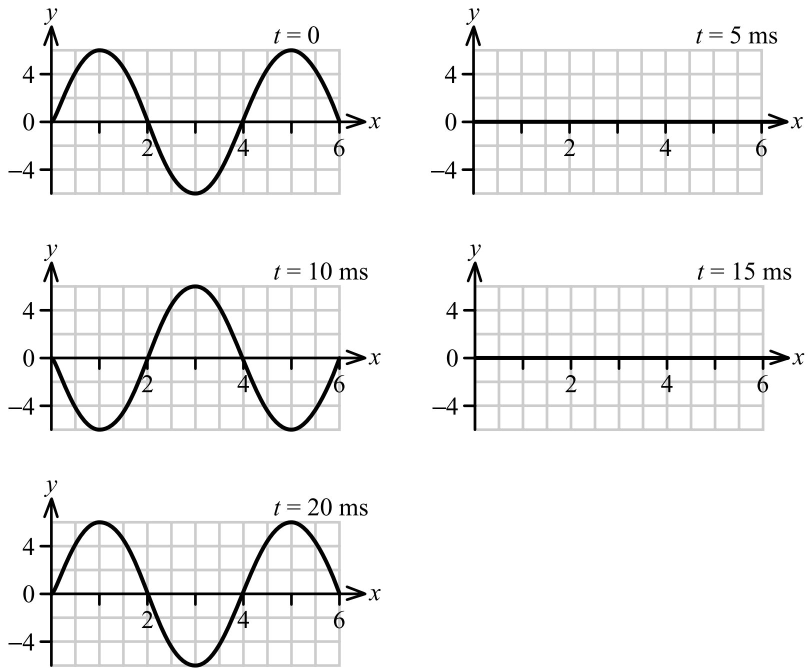
a node at $\quad 0.303\text{ m} - 0.214\text{ m} = \boxed{0.089\,1\text{ m}}$

a node at $\quad 0.518\text{ m} + 0.214\text{ m} = \boxed{0.732\text{ m}}$

a node at $\quad 0.732\text{ m} + 0.214\text{ m} = \boxed{0.947\text{ m}}$

and a node at $\quad 0.947\text{ m} + 0.214\text{ m} = \boxed{1.16\text{ m}}$ from either speaker.

***P18.14** (a)



(b) $\boxed{\text{In any one picture, the distance from one positive-going zero crossing to the next is } \lambda = 4 \text{ m.}}$

(c) $\boxed{f = 50 \text{ Hz. The oscillation at any point starts to repeat after a period of 20 ms, and } f = 1/T.}$

(d) 4 m. By comparison with the wave function $y = (2A \sin kx)\cos \omega t$, we identify $k = \pi/2$, and then compute $\lambda = 2\pi/k$.

(e) 50 Hz. By comparison with the wave function $y = (2A \sin kx)\cos \omega t$, we identify $\omega = 2\pi f = 100\pi$.

P18.15 $y_1 = 3.00\sin\left[\pi\left(x + 0.600t\right)\right]$ cm; $y_2 = 3.00\sin\left[\pi\left(x - 0.600t\right)\right]$ cm

$$y = y_1 + y_2 = \left[3.00\sin(\pi x)\cos(0.600\pi t) + 3.00\sin(\pi x)\cos(0.600\pi t)\right] \text{ cm}$$

$$y = (6.00 \text{ cm})\sin(\pi x)\cos(0.600\pi t)$$

(a) We can take $\cos(0.600\pi t) = 1$ to get the maximum y.

At $x = 0.250$ cm, $\quad y_{max} = (6.00 \text{ cm})\sin(0.250\pi) = \boxed{4.24 \text{ cm}}$

(b) At $x = 0.500$ cm, $\quad y_{max} = (6.00 \text{ cm})\sin(0.500\pi) = \boxed{6.00 \text{ cm}}$

(c) Now take $\cos(0.600\pi t) = -1$ to get y_{max}:

At $x = 1.50$ cm, $\quad y_{max} = (6.00 \text{ cm})\sin(1.50\pi)(-1) = \boxed{6.00 \text{ cm}}$

continued on next page

(d) The antinodes occur when $x = \frac{n\lambda}{4}$ $(n = 1, 3, 5, \ldots)$

But $k = \frac{2\pi}{\lambda} = \pi$ so $\lambda = 2.00\text{ cm}$

and $x_1 = \frac{\lambda}{4} = \boxed{0.500\text{ cm}}$ as in (b)

$x_2 = \frac{3\lambda}{4} = \boxed{1.50\text{ cm}}$ as in (c)

$x_3 = \frac{5\lambda}{4} = \boxed{2.50\text{ cm}}$

***P18.16** (a) The resultant wave is $y = 2A\sin\left(kx + \frac{\phi}{2}\right)\cos\left(\omega t - \frac{\phi}{2}\right)$

The oscillation of the $\sin(kx + \phi/2)$ factor means that this wave shows alternating nodes and antinodes. It is a standing wave.

The nodes are located at $kx + \frac{\phi}{2} = n\pi$ so $x = \frac{n\pi}{k} - \frac{\phi}{2k}$

which means that each node is shifted $\frac{\phi}{2k}$ to the left by the phase difference between the traveling waves.

(b) The separation of nodes is $\Delta x = \left[(n+1)\frac{\pi}{k} - \frac{\phi}{2k}\right] - \left[\frac{n\pi}{k} - \frac{\phi}{2k}\right]\Delta x = \frac{\pi}{k} = \frac{\lambda}{2}$

The nodes are still separated by half a wavelength.

(c) As noted in part (a), the nodes are all shifted by the distance $\phi/2k$ to the left.

Section 18.3 Standing Waves in a String Fixed at Both Ends

P18.17 $L = 30.0\text{ m}$; $\mu = 9.00 \times 10^{-3}\text{ kg/m}$; $T = 20.0\text{ N}$; $f_1 = \frac{v}{2L}$

where $v = \left(\frac{T}{\mu}\right)^{1/2} = 47.1\text{ m/s}$

so $f_1 = \frac{47.1}{60.0} = \boxed{0.786\text{ Hz}}$ $f_2 = 2f_1 = \boxed{1.57\text{ Hz}}$

$f_3 = 3f_1 = \boxed{2.36\text{ Hz}}$ $f_4 = 4f_1 = \boxed{3.14\text{ Hz}}$

P18.18 The tension in the string is $T = (4\text{ kg})(9.8\text{ m/s}^2) = 39.2\text{ N}$

Its linear density is $\mu = \frac{m}{L} = \frac{8 \times 10^{-3}\text{ kg}}{5\text{ m}} = 1.6 \times 10^{-3}\text{ kg/m}$

and the wave speed on the string is $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{39.2\text{ N}}{1.6 \times 10^{-3}\text{ kg/m}}} = 156.5\text{ m/s}$

In its fundamental mode of vibration, we have $\lambda = 2L = 2(5\text{ m}) = 10\text{ m}$

Thus, $f = \frac{v}{\lambda} = \frac{156.5\text{ m/s}}{10\text{ m}} = \boxed{15.7\text{ Hz}}$

P18.19 (a) Let n be the number of nodes in the standing wave resulting from the 25.0-kg mass. Then $n+1$ is the number of nodes for the standing wave resulting from the 16.0-kg mass. For standing waves, $\lambda = \frac{2L}{n}$, and the frequency is $f = \frac{v}{\lambda}$

Thus, $$f = \frac{n}{2L}\sqrt{\frac{T_n}{\mu}}$$

and also $$f = \frac{n+1}{2L}\sqrt{\frac{T_{n+1}}{\mu}}$$

Thus, $$\frac{n+1}{n} = \sqrt{\frac{T_n}{T_{n+1}}} = \sqrt{\frac{(25.0\text{ kg})g}{(16.0\text{ kg})g}} = \frac{5}{4}$$

Therefore, $4n + 4 = 5n$, or $n = 4$

Then, $$f = \frac{4}{2(2.00\text{ m})}\sqrt{\frac{(25.0\text{ kg})(9.80\text{ m/s}^2)}{0.002\,00\text{ kg/m}}} = \boxed{350\text{ Hz}}$$

(b) The largest mass will correspond to a standing wave of 1 loop

$(n = 1)$ so $$350\text{ Hz} = \frac{1}{2(2.00\text{ m})}\sqrt{\frac{m(9.80\text{ m/s}^2)}{0.002\,00\text{ kg/m}}}$$

yielding $m = \boxed{400\text{ kg}}$

P18.20 For the whole string vibrating, $d_{NN} = 0.64\text{ m} = \frac{\lambda}{2}$; $\lambda = 1.28\text{ m}$

The speed of a pulse on the string is

$v = f\lambda = 330\frac{1}{s}1.28\text{ m} = 422\text{ m/s}$

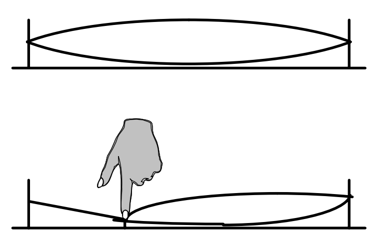


FIG. P18.20(a)

(a) When the string is stopped at the fret,

$d_{NN} = \frac{2}{3}0.64\text{ m} = \frac{\lambda}{2}$; $\lambda = 0.853\text{ m}$

$$f = \frac{v}{\lambda} = \frac{422\text{ m/s}}{0.853\text{ m}} = \boxed{495\text{ Hz}}$$

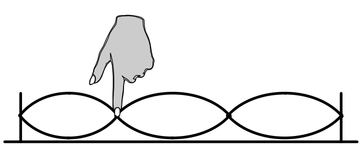


FIG. P18.20(b)

(b) The light touch at a point one third of the way along the string damps out vibration in the two lowest vibration states of the string as a whole. The whole string vibrates in its third resonance possibility: $3d_{NN} = 0.64\text{ m} = 3\frac{\lambda}{2}$

$\lambda = 0.427\text{ m}$

$$f = \frac{v}{\lambda} = \frac{422\text{ m/s}}{0.427\text{ m}} = \boxed{990\text{ Hz}}$$

P18.21 $d_{NN} = 0.700\text{ m} = \lambda/2$

$\lambda = 1.40\text{ m}$

$$f\lambda = v = 308\text{ m/s} = \sqrt{\frac{T}{(1.20\times10^{-3})/(0.700)}}$$

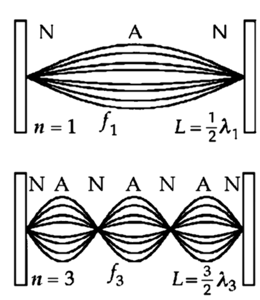


FIG. P18.21

(a) $T = \boxed{163\text{ N}}$

(b) With one-third the distance between nodes, the frequency is $f_3 = 3\cdot220\text{ Hz} = \boxed{660\text{ Hz}}$

P18.22 $\lambda_G = 2(0.350 \text{ m}) = \dfrac{v}{f_G}$; $\lambda_A = 2L_A = \dfrac{v}{f_A}$

$$L_G - L_A = L_G - \left(\frac{f_G}{f_A}\right)L_G = L_G\left(1 - \frac{f_G}{f_A}\right) = (0.350 \text{ m})\left(1 - \frac{392}{440}\right) = 0.038\,2 \text{ m}$$

Thus, $L_A = L_G - 0.038\,2 \text{ m} = 0.350 \text{ m} - 0.038\,2 \text{ m} = 0.312 \text{ m}$, or the finger should be placed $\boxed{\text{31.2 cm from the bridge}}$.

$$L_A = \frac{v}{2f_A} = \frac{1}{2f_A}\sqrt{\frac{T}{\mu}};\ dL_A = \frac{dT}{4f_A\sqrt{T\mu}};\ \frac{dL_A}{L_A} = \frac{1}{2}\frac{dT}{T}$$

$$\frac{dT}{T} = 2\frac{dL_A}{L_A} = 2\frac{0.600 \text{ cm}}{(35.0 - 3.82) \text{ cm}} = \boxed{3.84\%}$$

P18.23 In the fundamental mode, the string above the rod has only two nodes, at A and B, with an anti-node halfway between A and B. Thus,

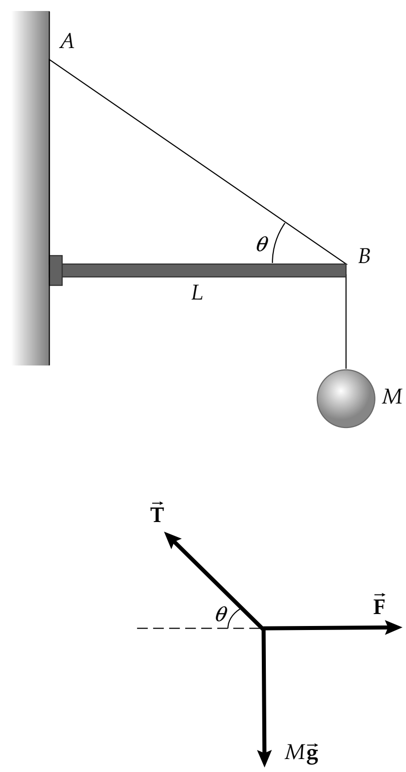


FIG. P18.23

$$\frac{\lambda}{2} = \overline{AB} = \frac{L}{\cos\theta} \text{ or } \lambda = \frac{2L}{\cos\theta}$$

Since the fundamental frequency is f, the wave speed in this segment of string is

$$v = \lambda f = \frac{2Lf}{\cos\theta}$$

Also,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{m/\overline{AB}}} = \sqrt{\frac{TL}{m\cos\theta}}$$

where T is the tension in this part of the string. Thus,

$$\frac{2Lf}{\cos\theta} = \sqrt{\frac{TL}{m\cos\theta}} \quad \text{or} \quad \frac{4L^2f^2}{\cos^2\theta} = \frac{TL}{m\cos\theta}$$

and the mass of string above the rod is:

$$m = \frac{T\cos\theta}{4Lf^2} \qquad [1]$$

Now, consider the tension in the string. The light rod would rotate about point P if the string exerted any vertical force on it. Therefore, recalling Newton's third law, the rod must exert only a horizontal force on the string. Consider a free-body diagram of the string segment in contact with the end of the rod.

$$\sum F_y = T\sin\theta - Mg = 0 \Rightarrow T = \frac{Mg}{\sin\theta}$$

Then, from Equation [1], the mass of string above the rod is

$$m = \left(\frac{Mg}{\sin\theta}\right)\frac{\cos\theta}{4Lf^2} = \boxed{\frac{Mg}{4Lf^2\tan\theta}}$$

P18.24 Let $m = \rho V$ represent the mass of the copper cylinder. The original tension in the wire is $T_1 = mg = \rho V g$. The water exerts a buoyant force $\rho_{\text{water}}\left(\frac{V}{2}\right)g$ on the cylinder, to reduce the tension to

$$T_2 = \rho V g - \rho_{\text{water}}\left(\frac{V}{2}\right)g = \left(\rho - \frac{\rho_{\text{water}}}{2}\right)Vg$$

The speed of a wave on the string changes from $\sqrt{\frac{T_1}{\mu}}$ to $\sqrt{\frac{T_2}{\mu}}$. The frequency changes from

$$f_1 = \frac{v_1}{\lambda} = \sqrt{\frac{T_1}{\mu}}\frac{1}{\lambda} \quad \text{to} \quad f_2 = \sqrt{\frac{T_2}{\mu}}\frac{1}{\lambda}$$

where we assume $\lambda = 2L$ is constant.

Then

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{\rho - \rho_{\text{water}}/2}{\rho}} = \sqrt{\frac{8.92 - 1.00/2}{8.92}}$$

$$f_2 = 300\text{ Hz}\sqrt{\frac{8.42}{8.92}} = \boxed{291\text{ Hz}}$$

P18.25 Comparing $y = (0.002\text{ m})\sin\left((\pi\text{ rad/m})x\right)\cos\left((100\,\pi\text{ rad/s})t\right)$

with $y = 2A\sin kx\cos\omega t$

we find $k = \frac{2\pi}{\lambda} = \pi\text{ m}^{-1}$, $\lambda = 2.00\text{ m}$, and $\omega = 2\pi f = 100\pi\text{ s}^{-1}$: $f = 50.0\text{ Hz}$

(a) Then the distance between adjacent nodes is $d_{\text{NN}} = \frac{\lambda}{2} = 1.00\text{ m}$

and on the string are $\frac{L}{d_{\text{NN}}} = \frac{3.00\text{ m}}{1.00\text{ m}} = \boxed{3\text{ loops}}$

For the speed we have $v = f\lambda = \left(50\text{ s}^{-1}\right)2\text{ m} = 100\text{ m/s}$

(b) In the simplest standing wave vibration, $d_{\text{NN}} = 3.00\text{ m} = \frac{\lambda_b}{2}$, $\lambda_b = 6.00\text{ m}$

and

$$f_b = \frac{v_a}{\lambda_b} = \frac{100\text{ m/s}}{6.00\text{ m}} = \boxed{16.7\text{ Hz}}$$

(c) In $v_0 = \sqrt{\frac{T_0}{\mu}}$, if the tension increases to $T_c = 9T_0$ and the string does not stretch, the speed increases to

$$v_c = \sqrt{\frac{9T_0}{\mu}} = 3\sqrt{\frac{T_0}{\mu}} = 3v_0 = 3(100\text{ m/s}) = 300\text{ m/s}$$

Then

$$\lambda_c = \frac{v_c}{f_a} = \frac{300\text{ m/s}}{50\text{ s}^{-1}} = 6.00\text{ m} \qquad d_{\text{NN}} = \frac{\lambda_c}{2} = 3.00\text{ m}$$

and $\boxed{\text{one}}$ loop fits onto the string.

Section 18.4 **Resonance**

P18.26 The wave speed is $v = \sqrt{gd} = \sqrt{(9.80 \text{ m/s}^2)(36.1 \text{ m})} = 18.8 \text{ m/s}$

The bay has one end open and one closed. Its simplest resonance is with a node of horizontal velocity, which is also an antinode of vertical displacement, at the head of the bay and an antinode of velocity, which is a node of displacement, at the mouth. The vibration of the water in the bay is like that in one half of the pond shown in Figure P18.27.

Then, $d_{NA} = 210 \times 10^3 \text{ m} = \dfrac{\lambda}{4}$

and $\lambda = 840 \times 10^3 \text{ m}$

Therefore, the period is $T = \dfrac{1}{f} = \dfrac{\lambda}{v} = \dfrac{840 \times 10^3 \text{ m}}{18.8 \text{ m/s}} = 4.47 \times 10^4 \text{ s} = \boxed{12 \text{ h } 24 \text{ min}}$

The natural frequency of the water sloshing in the bay agrees precisely with that of lunar excitation, so we identify the extra-high tides as amplified by resonance.

P18.27 (a) The wave speed is $v = \dfrac{9.15 \text{ m}}{2.50 \text{ s}} = \boxed{3.66 \text{ m/s}}$

(b) From the figure, there are antinodes at both ends of the pond, so the distance between adjacent antinodes

is $d_{AA} = \dfrac{\lambda}{2} = 9.15 \text{ m}$

and the wavelength is $\lambda = 18.3 \text{ m}$

The frequency is then $f = \dfrac{v}{\lambda} = \dfrac{3.66 \text{ m/s}}{18.3 \text{ m}} = \boxed{0.200 \text{ Hz}}$

We have assumed the wave speed is the same for all wavelengths.

P18.28 The distance between adjacent nodes is one-quarter of the circumference.

$$d_{NN} = d_{AA} = \frac{\lambda}{2} = \frac{20.0 \text{ cm}}{4} = 5.00 \text{ cm}$$

so

$$\lambda = 10.0 \text{ cm}$$

and

$$f = \frac{v}{\lambda} = \frac{900 \text{ m/s}}{0.100 \text{ m}} = 9\,000 \text{ Hz} = \boxed{9.00 \text{ kHz}}$$

The singer must match this frequency quite precisely for some interval of time to feed enough energy into the glass to crack it.

Section 18.5 **Standing Waves in Air Columns**

P18.29 (a) For the fundamental mode in a closed pipe, $\lambda = 4L$, as in the diagram.

But $v = f\lambda$, therefore $L = \dfrac{v}{4f}$

So,

$$L = \frac{343\ \text{m/s}}{4\left(240\ \text{s}^{-1}\right)} = \boxed{0.357\ \text{m}}$$

(b) For an open pipe, $\lambda = 2L$, as in the diagram.

So,

$$L = \frac{v}{2f} = \frac{343\ \text{m/s}}{2\left(240\ \text{s}^{-1}\right)} = \boxed{0.715\ \text{m}}$$

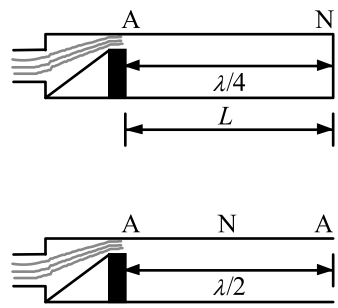


FIG. P18.29

P18.30 $d_{\text{AA}} = 0.320$ m; $\lambda = 0.640$ m

(a) $f = \dfrac{v}{\lambda} = \boxed{531\ \text{Hz}}$

(b) $\lambda = v/f = 0.085\,0$ m; $d_{\text{AA}} = \boxed{42.5\ \text{mm}}$

P18.31 The wavelength is $\lambda = \dfrac{v}{f} = \dfrac{343\ \text{m/s}}{261.6/\text{s}} = 1.31$ m

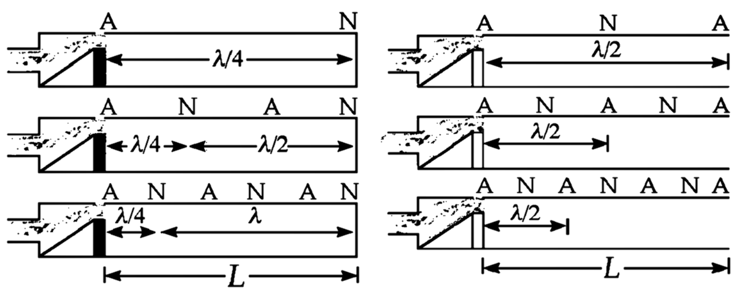


FIG. P18.31

so the length of the open pipe vibrating in its simplest (A-N-A) mode is

$$d_{\text{A to A}} = \frac{1}{2}\lambda = \boxed{0.656\ \text{m}}$$

A closed pipe has (N-A) for its simplest resonance,
(N-A-N-A) for the second,
and (N-A-N-A-N-A) for the third.

Here, the pipe length is $5d_{\text{N to A}} = \dfrac{5\lambda}{4} = \dfrac{5}{4}(1.31\ \text{m}) = \boxed{1.64\ \text{m}}$

P18.32 The air in the auditory canal, about 3 cm long, can vibrate with a node at the closed end and antinode at the open end,

with $d_{\text{N to A}} = 3\ \text{cm} = \dfrac{\lambda}{4}$

so $\lambda = 0.12$ m

and $f = \dfrac{v}{\lambda} = \dfrac{343\ \text{m/s}}{0.12\ \text{m}} \approx \boxed{3\ \text{kHz}}$

> A small-amplitude external excitation at this frequency can, over time, feed energy into a larger-amplitude resonance vibration of the air in the canal, making it audible.

P18.33 For a closed box, the resonant frequencies will have nodes at both sides, so the permitted wavelengths will be $L = \frac{1}{2}n\lambda$, $(n = 1, 2, 3, \ldots)$.

i.e.,

$$L = \frac{n\lambda}{2} = \frac{nv}{2f} \qquad \text{and} \qquad f = \frac{nv}{2L}$$

Therefore, with $L = 0.860$ m and $L' = 2.10$ m, the resonant frequencies are $f_n = \boxed{n(206\text{ Hz})}$ for $L = 0.860$ m for each n from 1 to 9 and $f_n' = \boxed{n(84.5\text{ Hz})}$ for $L' = 2.10$ m for each n from 2 to 23.

P18.34 The wavelength of sound is $\lambda = \frac{v}{f}$

The distance between water levels at resonance is $d = \frac{v}{2f}$ $\quad \therefore Rt = \pi r^2 d = \frac{\pi r^2 v}{2f}$

and $t = \boxed{\frac{\pi r^2 v}{2Rf}}$

P18.35 For both open and closed pipes, resonant frequencies are equally spaced as numbers. The set of resonant frequencies then must be 650 Hz, 550 Hz, 450 Hz, 350 Hz, 250 Hz, 150 Hz, 50 Hz. These are odd-integer multipliers of the fundamental frequency of $\boxed{50.0\text{ Hz}}$. Then the pipe length is $d_{NA} = \frac{\lambda}{4} = \frac{v}{4f} = \frac{340\text{ m/s}}{4(50/\text{s})} = \boxed{1.70\text{ m}}$.

***P18.36** (a) The open ends of the tunnel are antinodes, so $d_{AA} = 2000$ m/n with $n = 1, 2, 3, \ldots$. Then $\lambda = 2d_{AA} = 4000$ m/n. And $f = v/\lambda = (343\text{ m/s})/(4000\text{ m}/n) =$ $\boxed{0.0858\, n\text{ Hz, with } n = 1, 2, 3, \ldots}$

(b) It is a good rule. Any car horn would produce several or many of the closely-spaced resonance frequencies of the air in the tunnel, so it would be greatly amplified. Other drivers might be frightened directly into dangerous behavior, or might blow their horns also.

P18.37 For resonance in a narrow tube open at one end,

$$f = n\frac{v}{4L} \quad (n = 1, 3, 5, \ldots)$$

(a) Assuming $n = 1$ and $n = 3$,

$$384 = \frac{v}{4(0.228)} \qquad \text{and} \qquad 384 = \frac{3v}{4(0.683)}$$

In either case, $v = \boxed{350\text{ m/s}}$.

(b) For the next resonance $n = 5$, and

$$L = \frac{5v}{4f} = \frac{5(350\text{ m/s})}{4(384\text{ s}^{-1})} = \boxed{1.14\text{ m}}$$

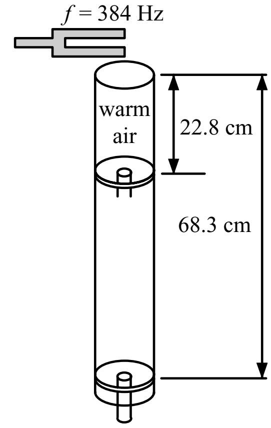


FIG. P18.37

P18.38 The length corresponding to the fundamental satisfies $f = \dfrac{v}{4L}$: $L_1 = \dfrac{v}{4f} = \dfrac{34}{4(512)} = 0.167\text{ m}$.

Since $L > 20.0$ cm, the *next* two modes will be observed, corresponding to

$$f = \frac{3v}{4L_2} \text{ and } f = \frac{5v}{4L_3}$$

or

$$L_2 = \frac{3v}{4f} = \boxed{0.502\text{ m}} \qquad \text{and} \qquad L_3 = \frac{5v}{4f} = \boxed{0.837\text{ m}}$$

***P18.39** Call L the depth of the well and v the speed of sound.

Then for some integer n $\quad L = (2n-1)\dfrac{\lambda_1}{4} = (2n-1)\dfrac{v}{4f_1} = \dfrac{(2n-1)(343\text{ m/s})}{4(51.5\text{ s}^{-1})}$

and for the next resonance $\quad L = [2(n+1)-1]\dfrac{\lambda_2}{4} = (2n+1)\dfrac{v}{4f_2} = \dfrac{(2n+1)(343\text{ m/s})}{4(60.0\text{ s}^{-1})}$

Thus, $\quad \dfrac{(2n-1)(343\text{ m/s})}{4(51.5\text{ s}^{-1})} = \dfrac{(2n+1)(343\text{ m/s})}{4(60.0\text{ s}^{-1})}$

and we require an *integer* solution to $\quad \dfrac{2n+1}{60.0} = \dfrac{2n-1}{51.5}$

The equation gives $n = \dfrac{111.5}{17} = 6.56$, so the best fitting integer is $n = 7$.

Then the results $\quad L = \dfrac{[2(7)-1](343\text{ m/s})}{4(51.5\text{ s}^{-1})} = 21.6\text{ m}$

and $\quad L = \dfrac{[2(7)+1](343\text{ m/s})}{4(60.0\text{ s}^{-1})} = 21.4\text{ m}$

suggest that we can say

$\boxed{\text{the depth of the well is } (21.5 \pm 0.1)\text{ m. The data suggest 0.6-Hz uncertainty in the frequency measurements, which is only a little more than 1\%.}}$

P18.40 (a) For the fundamental mode of an open tube,

$$L = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343\text{ m/s}}{2(880\text{ s}^{-1})} = \boxed{0.195\text{ m}}$$

(b) $v = 331\text{ m/s}\sqrt{1 + \dfrac{(-5.00)}{273}} = 328\text{ m/s}$

We ignore the thermal expansion of the metal.

$$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{328\text{ m/s}}{2(0.195\text{ m})} = \boxed{841\text{ Hz}}$$

The flute is flat by a semitone.

Section 18.6 Standing Waves in Rod and Membranes

P18.41 (a) $f = \frac{v}{2L} = \frac{5\,100}{(2)(1.60)} = \boxed{1.59\text{ kHz}}$

(b) Since it is held in the center, there must be a node in the center as well as antinodes at the ends. The even harmonics have an antinode at the center so only $\boxed{\text{the odd harmonics}}$ are present.

(c) $f = \frac{v'}{2L} = \frac{3\,560}{(2)(1.60)} = \boxed{1.11\text{ kHz}}$

P18.42 When the rod is clamped at one-quarter of its length, the vibration pattern reads ANANA and the rod length is $L = 2d_{AA} = \lambda$.

Therefore,

$$L = \frac{v}{f} = \frac{5\,100\text{ m/s}}{4\,400\text{ Hz}} = \boxed{1.16\text{ m}}$$

Section 18.7 Beats: Interference in Time

P18.43 $f \propto v \propto \sqrt{T}$ $\quad f_{\text{new}} = 110\sqrt{\frac{540}{600}} = 104.4\text{ Hz}$

$\Delta f = 110/\text{s} - 104.4/\text{s} = \boxed{5.64\text{ beats/s}}$

P18.44 (a) The string could be tuned to either $\boxed{521\text{ Hz or }525\text{ Hz}}$ from this evidence.

(b) Tightening the string raises the wave speed and frequency. If the frequency were originally 521 Hz, the beats would slow down.

Instead, the frequency must have started at 525 Hz to become $\boxed{526\text{ Hz}}$.

(c) From $f = \frac{v}{\lambda} = \frac{\sqrt{T/\mu}}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} \quad \text{and} \quad T_2 = \left(\frac{f_2}{f_1}\right)^2 T_1 = \left(\frac{523\text{ Hz}}{526\text{ Hz}}\right)^2 T_1 = 0.989T_1$$

The fractional change that should be made in the tension is then

$$\text{fractional change} = \frac{T_1 - T_2}{T_1} = 1 - 0.989 = 0.011\,4 = 1.14\%\text{ lower}$$

The tension should be $\boxed{\text{reduced by }1.14\%}$.

P18.45 For an echo $f' = f\frac{(v+v_s)}{(v-v_s)}$ the beat frequency is $f_b = |f' - f|$.

Solving for f_b, gives $f_b = f\frac{(2v_s)}{(v-v_s)}$ when approaching wall.

(a) $f_b = (256)\frac{2(1.33)}{(343-1.33)} = \boxed{1.99\text{ Hz}}$ beat frequency

(b) When he is moving away from the wall, v_s changes sign. Solving for v_s gives

$$v_s = \frac{f_b v}{2f - f_b} = \frac{(5)(343)}{(2)(256)-5} = \boxed{3.38\text{ m/s}}$$

P18.46 Using the $\boxed{4 \text{ and } 2\frac{2}{3}\text{-foot pipes}}$ produces actual frequencies of 131 Hz and 196 Hz and a combination tone at $(196-131)\,\text{Hz} = 65.4$ Hz, so this pair supplies the so-called missing fundamental. The 4 and 2-foot pipes produce a combination tone $(262-131)\,\text{Hz} = 131$ Hz, so this does not work.

The $\boxed{2\frac{2}{3} \text{ and 2-foot pipes}}$ produce a combination tone at $(262-196)\,\text{Hz} = 65.4$ Hz, so this works.

Also, $\boxed{4,\ 2\frac{2}{3}, \text{ and 2-foot pipes}}$ all playing together produce the 65.4-Hz combination tone.

Section 18.8 **Nonsinusoidal Wave Patterns**

P18.47 We list the frequencies of the harmonics of each note in Hz:

	Harmonic				
Note	1	2	3	4	5
A	440.00	880.00	1 320.0	1 760.0	2 200.0
C#	554.37	1 108.7	1 663.1	2 217.5	2 771.9
E	659.26	1 318.5	1 977.8	2 637.0	3 296.3

$\boxed{\text{The second harmonic of E is close the the third harmonic of A, and the fourth harmonic of C\# is close to the fifth harmonic of A.}}$

P18.48 We evaluate

$$s = 100\sin\theta + 157\sin 2\theta + 62.9\sin 3\theta + 105\sin 4\theta$$
$$+ 51.9\sin 5\theta + 29.5\sin 6\theta + 25.3\sin 7\theta$$

where s represents particle displacement in nanometers and θ represents the phase of the wave in radians. As θ advances by 2π, time advances by (1/523) s. Here is the result:

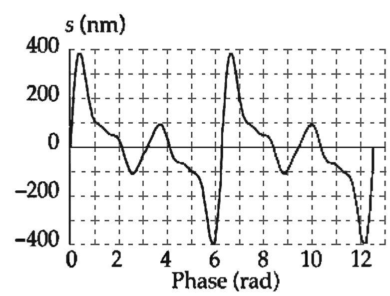


FIG. P18.48

Additional Problems

***P18.49** (a) The yo-yo's downward speed is $dL/dt = 0 + (0.8\text{ m/s}^2)(1.2\text{ s}) = 0.960$ m/s. The instantaneous wavelength of the fundamental string wave is given by $d_{NN} = \lambda/2 = L$ so $\lambda = 2L$ and $d\lambda/dt = 2\,dL/dt = 2(0.96\text{ m/s}) = 1.92$ m/s.

(b) For the second harmonic, the wavelength is equal to the length of the string. Then the rate of change of wavelength is equal to $dL/dt = \boxed{0.960\text{ m/s, half as much as for the first harmonic}}$.

(c) A yo-yo of different mass will hold the string under different tension to make each string wave vibrate with a different frequency, but the geometrical argument given in parts (a) and (b) still applies to the wavelength. $\boxed{\text{The answers are unchanged}}$: $d\lambda_1/dt = 1.92$ m/s and $d\lambda_2/dt = 0.960$ m/s.

***P18.50** (a) Use the Doppler formula

$$f' = f\frac{(v \pm v_0)}{(v \mp v_s)}$$

With f_1' = frequency of the speaker in front of student and

f_2' = frequency of the speaker behind the student.

$$f_1' = (456\text{ Hz})\frac{(343\text{ m/s} + 1.50\text{ m/s})}{(343\text{ m/s} - 0)} = 458\text{ Hz}$$

$$f_2' = (456\text{ Hz})\frac{(343\text{ m/s} - 1.50\text{ m/s})}{(343\text{ m/s} + 0)} = 454\text{ Hz}$$

Therefore, $f_b = f_1' - f_2' = \boxed{3.99\text{ Hz}}$

(b) The waves broadcast by both speakers have $\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{456/\text{s}} = 0.752\text{ m}$. The standing wave between them has $d_{AA} = \frac{\lambda}{2} = 0.376\text{ m}$. The student walks from one maximum to the next in time $\Delta t = \frac{0.376\text{ m}}{1.50\text{ m/s}} = 0.251\text{ s}$, so the frequency at which she hears maxima is $f = \frac{1}{T} = \boxed{3.99\text{ Hz}}$

(c) The answers are identical. The models are equally valid. We may think of the interference of the two waves as interference in space or in time, linked to space by the steady motion of the student.

P18.51 $f = 87.0\text{ Hz}$

speed of sound in air: $v_a = 340\text{ m/s}$

(a) $\lambda_b = \ell$ $\qquad v = f\lambda_b = (87.0\text{ s}^{-1})(0.400\text{ m})$

$v = \boxed{34.8\text{ m/s}}$

(b) $\left.\begin{array}{l}\lambda_a = 4L\\ v_a = \lambda_a f\end{array}\right\}$ $\qquad L = \frac{v_a}{4f} = \frac{340\text{ m/s}}{4(87.0\text{ s}^{-1})} = \boxed{0.977\text{ m}}$

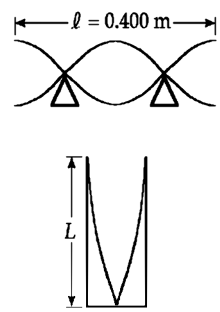


FIG. P18.51

P18.52

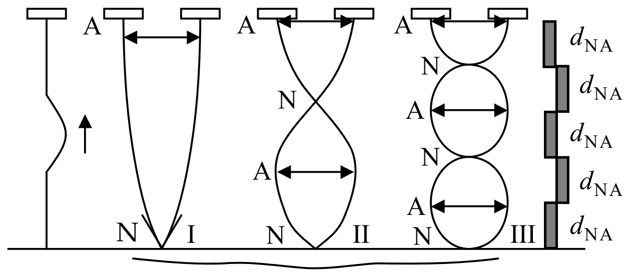


FIG. P18.52

(a) $\mu = \dfrac{5.5\times10^{-3}\text{ kg}}{0.86\text{ m}} = 6.40\times10^{-3}\text{ kg/m}$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{1.30\text{ kg}\cdot\text{m/s}^2}{6.40\times10^{-3}\text{ kg/m}}} = \boxed{14.3\text{ m/s}}$$

(b) In state I, $d_{NA} = \boxed{0.860\text{ m}} = \dfrac{\lambda}{4}$

(c) $\lambda = 3.44\text{ m}$ $f = \dfrac{v}{\lambda} = \dfrac{14.3\text{ m/s}}{3.44\text{ m}} = \boxed{4.14\text{ Hz}}$

In state II, $d_{NA} = \dfrac{1}{3}(0.86\text{ m}) = \boxed{0.287\text{ m}}$

$$\lambda = 4(0.287\text{ m}) = 1.15\text{ m} \qquad f = \frac{v}{\lambda} = \frac{14.3\text{ m/s}}{1.15\text{ m}} = \boxed{12.4\text{ Hz}}$$

In state III, $d_{NA} = \dfrac{1}{5}(0.86\text{ m}) = \boxed{0.172\text{ m}}$

$$f = \frac{v}{\lambda} = \frac{14.3\text{ m/s}}{4(0.172\text{ m})} = \boxed{20.7\text{ Hz}}$$

P18.53 If the train is moving away from station, its frequency is depressed:

$$f' = 180 - 2.00 = 178\text{ Hz:} \qquad 178 = 180\frac{343}{343-(-v)}$$

Solving for v gives $v = \dfrac{(2.00)(343)}{178}$

Therefore, $v = \boxed{3.85\text{ m/s away from station}}$

If it is moving toward the station, the frequency is enhanced:

$f' = 180 + 2.00 = 182\text{ Hz:}$ $182 = 180\dfrac{343}{343-v}$

Solving for v gives $4 = \dfrac{(2.00)(343)}{182}$

Therefore, $v = \boxed{3.77\text{ m/s toward the station}}$

P18.54 $v = \sqrt{\dfrac{(48.0)(2.00)}{4.80\times10^{-3}}} = 141\text{ m/s}$

$d_{NN} = 1.00\text{ m}; \lambda = 2.00\text{ m}; f = \dfrac{v}{\lambda} = 70.7\text{ Hz}$

$\lambda_a = \dfrac{v_a}{f} = \dfrac{343\text{ m/s}}{70.7\text{ Hz}} = \boxed{4.85\text{ m}}$

P18.55 (a) Since the first node is at the weld, the wavelength in the thin wire is $2L$ or 80.0 cm. The frequency and tension are the same in both sections, so

$$f=\frac{1}{2L}\sqrt{\frac{T}{\mu}}=\frac{1}{2(0.400)}\sqrt{\frac{4.60}{2.00\times10^{-3}}}=\boxed{59.9\text{ Hz}}$$

(b) As the thick wire is twice the diameter, the linear density is 4 times that of the thin wire. $\mu'=8.00\ \text{g/m}$

so $$L'=\frac{1}{2f}\sqrt{\frac{T}{\mu'}}$$

$$L'=\left[\frac{1}{(2)(59.9)}\right]\sqrt{\frac{4.60}{8.00\times10^{-3}}}=\boxed{20.0\text{ cm}}$$ half the length of the thin wire.

***P18.56** The wavelength stays constant at 0.96 m while the wavespeed rises according to $v=(T/\mu)^{1/2}=[(15+2.86t)/0.0016]^{1/2}=[9375+1786t]^{1/2}$ so the frequency rises as $f=v/\lambda=[9375+1786t]^{1/2}/0.96=[10\,173+1938t]^{1/2}$ The number of cycles is $f\,dt$ in each incremental bit of time, or altogether

$$\int_0^{3.5}(10173+1938t)^{1/2}\,dt=\frac{1}{1938}\int_0^{3.5}(10173+1938t)^{1/2}\,1938\,dt$$

$$=\frac{1}{1938}\left.\frac{[10173+1938t]^{3/2}}{3/2}\right|_0^{3.5}=\frac{(16954)^{3/2}-(10173)^{3/2}}{2906}=\boxed{407\text{ cycles}}$$

P18.57 (a) $$f=\frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

so $$\frac{f'}{f}=\frac{L}{L'}=\frac{L}{2L}=\frac{1}{2}$$

The $\boxed{\text{frequency should be halved}}$ to get the same number of antinodes for twice the length.

(b) $$\frac{n'}{n}=\sqrt{\frac{T}{T'}}\qquad\text{so}\qquad\frac{T'}{T}=\left(\frac{n}{n'}\right)^2=\left[\frac{n}{n+1}\right]^2$$

The tension must be $$T'=\boxed{\left[\frac{n}{n+1}\right]^2T}$$

(c) $$\frac{f'}{f}=\frac{n'L}{nL'}\sqrt{\frac{T'}{T}}\qquad\text{so}\qquad\frac{T'}{T}=\left(\frac{nf'L'}{n'fL}\right)^2$$

$$\frac{T'}{T}=\left(\frac{3}{2\cdot2}\right)^2\qquad\boxed{\frac{T'}{T}=\frac{9}{16}}$$ to get twice as many antinodes.

P18.58 (a) For the block:

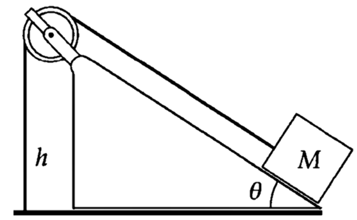


FIG. P18.58

$$\sum F_x = T - Mg\sin 30.0° = 0$$

so $\quad T = Mg\sin 30.0° = \boxed{\frac{1}{2}Mg}$

(b) The length of the section of string parallel to the incline is $\frac{h}{\sin 30.0°} = 2h$. The total length of the string is then $\boxed{3h}$.

(c) The mass per unit length of the string is $\quad \mu = \boxed{\frac{m}{3h}}$

(d) The speed of waves in the string is $\quad v = \sqrt{\frac{T}{\mu}} = \sqrt{\left(\frac{Mg}{2}\right)\left(\frac{3h}{m}\right)} = \boxed{\sqrt{\frac{3Mgh}{2m}}}$

(e) In the fundamental mode, the segment of length h vibrates as one loop. The distance between adjacent nodes is then $d_{NN} = \frac{\lambda}{2} = h$, so the wavelength is $\lambda = 2h$.

The frequency is $\quad f = \frac{v}{\lambda} = \frac{1}{2h}\sqrt{\frac{3Mgh}{2m}} = \boxed{\sqrt{\frac{3Mg}{8mh}}}$

(g) When the vertical segment of string vibrates with 2 loops (i.e., 3 nodes), then $h = 2\left(\frac{\lambda}{2}\right)$ and the wavelength is $\quad \lambda = \boxed{h}$

(f) The period of the standing wave of 3 nodes (or two loops) is

$$T = \frac{1}{f} = \frac{\lambda}{v} = h\sqrt{\frac{2m}{3Mgh}} = \boxed{\sqrt{\frac{2mh}{3Mg}}}$$

(h) $\quad f_b = 1.02f - f = \left(2.00\times10^{-2}\right)f = \boxed{\left(2.00\times10^{-2}\right)\sqrt{\frac{3Mg}{8mh}}}$

P18.59 We look for a solution of the form

$$5.00\sin(2.00x - 10.0t) + 10.0\cos(2.00x - 10.0t)$$
$$= A\sin(2.00x - 10.0t + \phi)$$
$$= A\sin(2.00x - 10.0t)\cos\phi + A\cos(2.00x - 10.0t)\sin\phi$$

This will be true if both $\quad 5.00 = A\cos\phi$ and $10.0 = A\sin\phi$,

requiring $\quad (5.00)^2 + (10.0)^2 = A^2$

$A = 11.2$ and $\phi = 63.4°$

The resultant wave $\boxed{11.2\sin(2.00x - 10.0t + 63.4°)}$ is sinusoidal.

P18.60 $d_{AA} = \frac{\lambda}{2} = 7.05 \times 10^{-3}$ m is the distance between antinodes.

Then $\lambda = 14.1 \times 10^{-3}$ m

and $f = \frac{v}{\lambda} = \frac{3.70 \times 10^3 \text{ m/s}}{14.1 \times 10^{-3} \text{ m}} = \boxed{2.62 \times 10^5 \text{ Hz}}$

The crystal can be tuned to vibrate at 2^{18} Hz, so that binary counters can derive from it a signal at precisely 1 Hz.

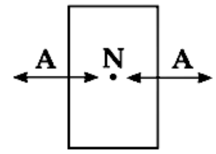


FIG. P18.60

P18.61 (a) Let θ represent the angle each slanted rope makes with the vertical.

In the diagram, observe that:

$$\sin\theta = \frac{1.00 \text{ m}}{1.50 \text{ m}} = \frac{2}{3} \quad \text{or} \quad \theta = 41.8°$$

Considering the mass,

$$\sum F_y = 0: \quad 2T\cos\theta = mg$$

or $$T = \frac{(12.0 \text{ kg})(9.80 \text{ m/s}^2)}{2\cos 41.8°} = \boxed{78.9 \text{ N}}$$

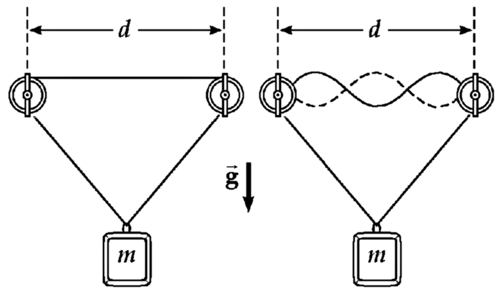


FIG. P18.61

(b) The speed of transverse waves in the string is $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{78.9 \text{ N}}{0.001\,00 \text{ kg/m}}} = 281$ m/s

For the standing wave pattern shown (3 loops), $d = \frac{3}{2}\lambda$

or $\lambda = \frac{2(2.00 \text{ m})}{3} = 1.33$ m

Thus, the required frequency is $f = \frac{v}{\lambda} = \frac{281 \text{ m/s}}{1.33 \text{ m}} = \boxed{211 \text{ Hz}}$

ANSWERS TO EVEN PROBLEMS

P18.2 see the solution

P18.4 5.66 cm

P18.6 (a) 3.33 rad (b) 283 Hz

P18.8 (a) The number is the greatest integer $\le d\left(\frac{f}{v}\right) + \frac{1}{2}$ (b) $L_n = \frac{d^2 - (n - 1/2)^2 (v/f)^2}{2(n-1/2)(v/f)}$ where $n = 1, 2, \ldots, n_{max}$

P18.10 (a) Point A is one half wavelength farther from one speaker than from the other. The waves it receives interfere destructively. (b) Along the hyperbola $9x^2 - 16y^2 = 144$. (c) Yes; along the straight line through the origin with slope 0.75 or the straight line through the origin with slope –0.75.

P18.12 see the solution

P18.14 (a) see the solution (b) 4 m is the distance between crests. (c) 50 Hz. The oscillation at any point starts to repeat after a period of 20 ms, and $f = 1/T$. (d) 4 m. By comparison with equation 18.3, $k = \pi/2$, and $\lambda = 2\pi/k$. (e) 50 Hz. By comparison with equation 18.3, $\omega = 2\pi f = 100\pi$.

P18.16 (a) Yes. The resultant wave contains points of no motion. (b) and (c) The nodes are still separated by $\lambda/2$. They are all shifted by the distance $\phi/2k$ to the left.

P18.18 15.7 Hz

P18.20 (a) 495 Hz (b) 990 Hz

P18.22 31.2 cm from the bridge; 3.84%

P18.24 291 Hz

P18.26 The natural frequency of the water sloshing in the bay agrees precisely with that of lunar excitation, so we identify the extra-high tides as amplified by resonance.

P18.28 9.00 kHz

P18.30 (a) 531 Hz (b) 42.5 mm

P18.32 3 kHz; a small-amplitude external excitation at this frequency can, over times, feed energy into a larger-amplitude resonance vibration of the air in the canal, making it audible.

P18.34 $\Delta t = \dfrac{\pi r^2 v}{2Rf}$

P18.36 (a) 0.0858 n Hz, with $n = 1, 2, 3, \ldots$ (b) It is a good rule. Any car horn would produce several or many of the closely-spaced resonance frequencies of the air in the tunnel, so it would be greatly amplified. Other drivers might be frightened directly into dangerous behavior, or might blow their horns also.

P18.38 0.502 m; 0.837 m

P18.40 (a) 0.195 m (b) 841 m

P18.42 1.16 m

P18.44 (a) 521 Hz or 525 Hz (b) 526 Hz (c) reduce by 1.14%

P18.46 4-foot and $2\frac{2}{3}$-foot; $2\frac{2}{3}$ and 2-foot; and all three together

P18.48 see the solution

P18.50 (a) 3.99 beats/s (b) 3.99 beats/s (c) The answers are identical. The models are equally valid. We may think of the interference of the two waves as interference in space or in time, linked to space by the steady motion of the student.

P18.52 (a) 14.3 m/s (b) 86.0 cm, 28.7 cm, 17.2 cm (c) 4.14 Hz, 12.4 Hz, 20.7 Hz

P18.54 4.85 m

P18.56 407 cycles

P18.58 (a) $\frac{1}{2}Mg$ (b) $3h$ (c) $\dfrac{m}{3h}$ (d) $\sqrt{\dfrac{3Mgh}{2m}}$ (e) $\sqrt{\dfrac{3Mg}{8mh}}$ (f) $\sqrt{\dfrac{2mh}{3Mg}}$ (g) h (h) $\left(2.00\times10^{-2}\right)\sqrt{\dfrac{3Mg}{8mh}}$

P18.60 262 kHz

19

Temperature

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q19.1 Two objects in thermal equilibrium need not be in contact. Consider the two objects that are in thermal equilibrium in Figure 19.1(c). The act of separating them by a small distance does not affect how the molecules are moving inside either object, so they will still be in thermal equilibrium.

Q19.2 The copper's temperature drops and the water temperature rises until both temperatures are the same. Then the metal and the water are in thermal equilibrium.

Q19.3 The astronaut is referring to the temperature of the lunar surface, specifically a 400 °F difference. A thermometer would register the temperature of the thermometer liquid. Since there is no atmosphere in the moon, the thermometer will not read the temperature of some other object unless it is placed into the lunar soil.

***Q19.4** Answer (e). The thermometer works by differential expansion. As the thermometer is warmed the liquid level falls relative to the tube wall. If the liquid and the tube material were to expand by equal amounts, the thermometer could not be used.

***Q19.5** Answer (b). Around atmospheric pressure, 0 °C is the only temperature at which liquid water and solid water can both exist.

***Q19.6** Mentally multiply 93 m and 17 and 1/(1 000 000 °C) and say 5 °C for the temperature increase. To simplify, multiply 100 and 100 and 1/1 000 000 for an answer in meters: it is on the order of 1 cm, answer (c).

Q19.7 The measurements made with the heated steel tape will be too short—but only by a factor of 5×10^{-5} of the measured length.

Q19.8 (a) One mole of H_2 has a mass of 2.016 0 g.

(b) One mole of He has a mass of 4.002 6 g.

(c) One mole of CO has a mass of 28.010 g.

Q19.9 $PV = nRT$ predicts V going to zero as T goes to zero. The ideal-gas model does not apply when the material gets close to liquefaction and then turns into a liquid or solid. The molecules start to interact all the time, not just in brief collisions. The molecules start to take up a significant portion of the volume of the container.

***Q19.10** Call the process isobaric cooling or isobaric contraction. The rubber wall is easy to stretch. The air inside is nearly at atmospheric pressure originally and stays at atmospheric pressure as the wall moves in, just maintaining equality of pressure outside and inside. The air is nearly an ideal gas to start with, and stays fairly ideal—fairly far from liquefaction—even at 100 K. The water vapor liquefies and then freezes, and the carbon dioxide turns to snow, but these are minor constituents of the air. Thus as the absolute temperature drops to 1/3 of its original value the volume (i) will drop to 1/3 of what it was: answer (b). (ii) As noted above, the pressure stays nearly constant at 1 atm. Answer (d).

***Q19.11** Cylinder A must be at lower pressure. If the gas is thin, $PV = nRT$ applies to both with the same value of nRT for both. Then A will be at one-third the absolute pressure of B. Answer (e).

***Q19.12** Most definitively, we should say that pressure is proportional to absolute temperature. Pressure is a linear function of Celsius temperature, but this relationship is not a proportionality because pressure does not go to zero at 0 °C. Pressure is a linear function of Kelvin temperature, on its way to being a linear function with a graph going through the origin. Statement (c) is ambiguous. The rate of increase in pressure might refer to a time rate, with units of pascals per second, which could not describe a temperature increase. Statement (d) is a way of saying that the graph has constant slope, so it is a correct statement, if uncommunicative. Thus (b) and (d) are correct.

***Q19.13** We think about nRT/V in each case. Since R is constant, we need only think about nT/V, and units of mmol · K/cm^3 are as convenient as any. In case a, we have $2 \cdot 3/1 = 6$. In b we have 6. In c we have 4. In d we have 6. In e we have 5. Then the ranking is a = b = d > e > c

Q19.14 As the temperature increases, the brass expands. This would effectively increase the distance d from the pivot point to the center of mass of the pendulum, and also increase the moment of inertia of the pendulum. Since the moment of inertia is proportional to d^2, and the period of a physical pendulum is $T = 2\pi\sqrt{\dfrac{I}{mgd}}$, the period would increase, and the clock would run slow.

Q19.15 As the water rises in temperature, it expands or rises in pressure or both. The excess volume would spill out of the cooling system, or else the pressure would rise very high indeed. Modern cooling systems have an overflow reservoir to accept the excess volume when the coolant heats up and expands.

Q19.16 The coefficient of expansion of metal is larger than that of glass. When hot water is run over the jar, both the glass and the lid expand, but at different rates. Since *all* dimensions expand the inner diameter of the lid expands more than the top of the jar, and the lid will be easier to remove.

Q19.17 The sphere expands when heated, so that it no longer fits through the ring. With the sphere still hot, you can separate the sphere and ring by heating the ring. This more surprising result occurs because the thermal expansion of the ring is not like the inflation of a blood-pressure cuff. Rather, it is like a photographic enlargement; every linear dimension, including the hole diameter, increases by the same factor. The reason for this is that the atoms everywhere, including those around the inner circumference, push away from each other. The only way that the atoms can accommodate the greater distances is for the circumference—and corresponding diameter—to grow. This property was once used to fit metal rims to wooden wagon wheels. If the ring is heated and the sphere left at room temperature, the sphere would pass through the ring with more space to spare.

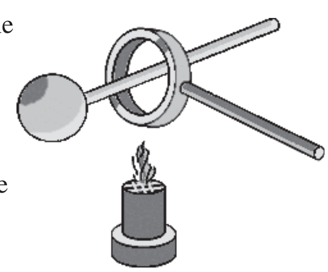


FIG. Q19.17

SOLUTIONS TO PROBLEMS

Section 19.2 **Thermometers and the Celsius Temperature Scale**

Section 19.3 **The Constant-Volume Gas Thermometer and the Absolute Temperature Scale**

P19.1 Since we have a linear graph, the pressure is related to the temperature as $P = A + BT$, where A and B are constants. To find A and B, we use the data

$$0.900 \text{ atm} = A + (-80.0°\text{C})B \qquad (1)$$

$$1.635 \text{ atm} = A + (78.0°\text{C})B \qquad (2)$$

Solving (1) and (2) simultaneously, $1.635 - 0.900 = 78\,B + 80\,B$

we find $B = 4.652 \times 10^{-3}\ \text{atm/°C}$

and $A = 1.272\ \text{atm}$

Therefore, $P = 1.272\ \text{atm} + \left(4.652 \times 10^{-3}\ \text{atm/°C}\right)T$

(a) At absolute zero $P = 0 = 1.272\ \text{atm} + \left(4.652 \times 10^{-3}\ \text{atm/°C}\right)T$

which gives $\boxed{T = -274°\text{C}}$

(b) At the freezing point of water $P = 1.272\ \text{atm} + 0 = \boxed{1.27\ \text{atm}}$.

(c) And at the boiling point $P = 1.272\ \text{atm} + \left(4.652 \times 10^{-3}\ \text{atm/°C}\right)(100°\text{C}) = \boxed{1.74\ \text{atm}}$.

P19.2 (a) $\Delta T = 450°\text{C} = 450°\text{C}\left(\dfrac{212°\text{F} - 32.0°\text{F}}{100°\text{C} - 0.00°\text{C}}\right) = \boxed{810°\text{F}}$

(b) $\Delta T = 450°\text{C} = \boxed{450\ \text{K}}$ A Celsius degree and a kelvin of temperature difference are the same space on a thermometer.

P19.3 (a) $T_F = \dfrac{9}{5}T_C + 32.0°\text{F} = \dfrac{9}{5}(-195.81) + 32.0 = \boxed{-320°\text{F}}$

(b) $T = T_C + 273.15 = -195.81 + 273.15 = \boxed{77.3\ \text{K}}$

P19.4 (a) $T = 1\,064 + 273 = \boxed{1\,337\ \text{K}}$ melting point

$T = 2\,660 + 273 = \boxed{2\,933\ \text{K}}$ boiling point

(b) $\Delta T = \boxed{1\,596°\text{C}} = \boxed{1\,596\ \text{K}}$ The differences are the same.

Section 19.4 Thermal Expansion of Solids and Liquids

P19.5 The wire is 35.0 m long when $T_C = -20.0°\text{C}$.

$$\Delta L = L_i \bar{\alpha}(T - T_i)$$

$\bar{\alpha} = \alpha(20.0°\text{C}) = 1.70 \times 10^{-5}\ (\text{C}°)^{-1}$ for Cu.

$$\Delta L = (35.0\text{ m})\left(1.70 \times 10^{-5}\ (\text{C}°)^{-1}\right)\left(35.0°\text{C} - (-20.0°\text{C})\right) = \boxed{+3.27\text{ cm}}$$

P19.6 Each section can expand into the joint space to the north of it. We need think of only one section expanding. $\Delta L = L_i \alpha\, \Delta T = (25.0\text{ m})\left(12.0 \times 10^{-6}/\text{C}°\right)(40.0°\text{C}) = \boxed{1.20\text{ cm}}$

P19.7 (a) $\Delta L = \alpha L_i \Delta T = 9.00 \times 10^{-6}\,°\text{C}^{-1}(30.0\text{ cm})(65.0°\text{C}) = \boxed{0.176\text{ mm}}$

(b) L stands for any linear dimension.

$$\Delta L = \alpha L_i \Delta T = 9.00 \times 10^{-6}\,°\text{C}^{-1}(1.50\text{ cm})(65.0°\text{C}) = \boxed{8.78 \times 10^{-4}\text{ cm}}$$

(c) $\Delta V = 3\alpha V_i \Delta T = 3\left(9.00 \times 10^{-6}\,°\text{C}^{-1}\right)\left(\dfrac{30.0(\pi)(1.50)^2}{4}\text{ cm}^3\right)(65.0°\text{C}) = \boxed{0.093\,0\text{ cm}^3}$

P19.8 The horizontal section expands according to $\Delta L = \alpha L_i \Delta T$.

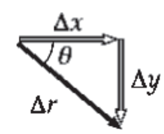


FIG. P19.8

$$\Delta x = \left(17 \times 10^{-6}\,°\text{C}^{-1}\right)(28.0\text{ cm})(46.5°\text{C} - 18.0°\text{C}) = 1.36 \times 10^{-2}\text{ cm}$$

The vertical section expands similarly by

$$\Delta y = \left(17 \times 10^{-6}\,°\text{C}^{-1}\right)(134\text{ cm})(28.5°\text{C}) = 6.49 \times 10^{-2}\text{ cm}$$

The vector displacement of the pipe elbow has magnitude

$$\Delta r = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(0.136\text{ mm})^2 + (0.649\text{ mm})^2} = 0.663\text{ mm}$$

and is directed to the right below the horizontal at angle

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{0.649\text{ mm}}{0.136\text{ mm}}\right) = 78.2°$$

$$\boxed{\Delta r = 0.663\text{ mm to the right at } 78.2° \text{ below the horizontal}}$$

***P19.9** (a) $L_{\text{Al}}\left(1 + \alpha_{\text{Al}} \Delta T\right) = L_{\text{Brass}}\left(1 + \alpha_{\text{Brass}} \Delta T\right)$

$$\Delta T = \frac{L_{\text{Al}} - L_{\text{Brass}}}{L_{\text{Brass}}\alpha_{\text{Brass}} - L_{\text{Al}}\alpha_{\text{Al}}}$$

$$\Delta T = \frac{(10.01 - 10.00)}{(10.00)\left(19.0 \times 10^{-6}\right) - (10.01)\left(24.0 \times 10^{-6}\right)}$$

$\Delta T = -199°\text{C}$ so $T = \boxed{\begin{array}{l} -179°\text{C} \\ \text{This is attainable, because it is above absolute zero.} \end{array}}$

(b) $\Delta T = \dfrac{(10.02 - 10.00)}{(10.00)\left(19.0 \times 10^{-6}\right) - (10.02)\left(24.0 \times 10^{-6}\right)}$

$\Delta T = -396°\text{C}$ so

$T = \boxed{\begin{array}{l} -376°\text{C, which is below 0 K so it cannot be reached.} \\ \text{The rod and ring cannot be separated by changing their temperatures together.} \end{array}}$

***P19.10** (a) $L = L_i(1+\alpha\Delta T)$: $\quad 5.050\text{ cm} = 5.000\text{ cm}\left[1+24.0\times10^{-6}\,{}^\circ\text{C}^{-1}(T-20.0^\circ\text{C})\right]$

$$T = \boxed{437^\circ\text{C}}$$

(b) We must get $\quad L_{\text{Al}} = L_{\text{Brass}}$ for some ΔT, or

$$L_{i,\text{ Al}}\left(1+\alpha_{\text{Al}}\Delta T\right) = L_{i,\text{ Brass}}\left(1+\alpha_{\text{Brass}}\Delta T\right)$$

$$5.000\text{ cm}\left[1+\left(24.0\times10^{-6}\,{}^\circ\text{C}^{-1}\right)\Delta T\right] = 5.050\text{ cm}\left[1+\left(19.0\times10^{-6}\,{}^\circ\text{C}^{-1}\right)\Delta T\right]$$

Solving for ΔT, $\quad \Delta T = 2\,080^\circ\text{C}$,

so $\quad \boxed{T = 2\,100^\circ\text{C}}$

$\boxed{\text{This will not work because aluminum melts at } 660^\circ\text{C.}}$

P19.11 (a) $V_f = V_i(1+\beta\Delta T) = 100\left[1+1.50\times10^{-4}(-15.0)\right] = \boxed{99.8\text{ mL}}$

(b) $\Delta V_{\text{acetone}} = \left(\beta V_i \Delta T\right)_{\text{acetone}}$

$\Delta V_{\text{flask}} = \left(\beta V_i \Delta T\right)_{\text{Pyrex}} = \left(3\alpha V_i \Delta T\right)_{\text{Pyrex}}$

for the same V_i and ΔT,

$$\frac{\Delta V_{\text{acetone}}}{\Delta V_{\text{flask}}} = \frac{\beta_{\text{acetone}}}{\beta_{\text{flask}}} = \frac{1.50\times10^{-4}}{3\left(3.20\times10^{-6}\right)} = \frac{1}{6.40\times10^{-2}}$$

$\boxed{\text{The volume change of flask is about 6\% of the change in the volume of the acetone.}}$

P19.12 (a), (b) The material would expand by $\Delta L = \alpha L_i \Delta T$,

$\dfrac{\Delta L}{L_i} = \alpha\Delta T$, but instead feels stress

$$\frac{F}{A} = \frac{Y\Delta L}{L_i} = Y\alpha\Delta T = \left(7.00\times10^9\ \text{N/m}^2\right)12.0\times10^{-6}\,(\text{C}^\circ)^{-1}(30.0^\circ\text{C})$$

$= \boxed{2.52\times10^6\ \text{N/m}^2}$. This will $\boxed{\text{not break}}$ concrete.

P19.13 (a) $\Delta V = V_t\beta_t\Delta T - V_{\text{Al}}\beta_{\text{Al}}\Delta T = \left(\beta_t - 3\alpha_{\text{Al}}\right)V_i\Delta T$

$$= \left(9.00\times10^{-4} - 0.720\times10^{-4}\right){}^\circ\text{C}^{-1}\left(2\,000\text{ cm}^3\right)(60.0^\circ\text{C})$$

$\Delta V = \boxed{99.4\text{ cm}^3}$ overflows.

(b) The whole new volume of turpentine is

$$2\,000\text{ cm}^3 + 9.00\times10^{-4}\,{}^\circ\text{C}^{-1}\left(2\,000\text{ cm}^3\right)(60.0^\circ\text{C}) = 2\,108\text{ cm}^3$$

so the fraction lost is

$$\frac{99.4\text{ cm}^3}{2\,108\text{ cm}^3} = 4.71\times10^{-2}$$

and this fraction of the cylinder's depth will be empty upon cooling:

$$4.71\times10^{-2}(20.0\text{ cm}) = \boxed{0.943\text{ cm}}$$

***P19.14** Model the wire as contracting according to $\Delta L = \alpha L_i \Delta T$ and then stretching according to $\text{stress} = \frac{F}{A} = Y\frac{\Delta L}{L_i} = \frac{Y}{L_i}\alpha L_i \Delta T = Y\alpha\Delta T$

(a) $F = YA\alpha\Delta T = \left(20\times10^{10}\ \text{N/m}^2\right)4\times10^{-6}\ \text{m}^2\ 11\times10^{-6}\ \frac{1}{\text{C}^\circ}45°\text{C} = \boxed{396\ \text{N}}$

(b) $\Delta T = \frac{\text{stress}}{Y\alpha} = \frac{3\times10^{8}\ \text{N/m}^2}{\left(20\times10^{10}\ \text{N/m}^2\right)11\times10^{-6}/\text{C}^\circ} = 136°\text{C}$

To increase the stress the temperature must decrease to $35°\text{C} - 136°\text{C} = \boxed{-101°\text{C}}$.

(c) $\boxed{\text{The original length divides out, so the answers would not change.}}$

P19.15 The area of the chip decreases according to

$$\Delta A = \gamma A_1 \Delta T = A_f - A_i$$
$$A_f = A_i\left(1+\gamma\Delta T\right) = A_i\left(1+2\alpha\Delta T\right)$$

The star images are scattered uniformly, so the number N of stars that fit is proportional to the area.

Then $N_f = N_i\left(1+2\alpha\Delta T\right) = 5\,342\left[1+2\left(4.68\times10^{-6}\ °\text{C}^{-1}\right)\left(-100°\text{C}-20°\text{C}\right)\right] = \boxed{5\,336\ \text{star images}}$.

Section 19.5 Macroscopic Description of an Ideal Gas

P19.16 Mass of gold abraded: $|\Delta m| = 3.80\ \text{g} - 3.35\ \text{g} = 0.45\ \text{g} = \left(0.45\ \text{g}\right)\left(\frac{1\ \text{kg}}{10^3\ \text{g}}\right) = 4.5\times10^{-4}\ \text{kg}$

Each atom has mass $m_0 = 197\ \text{u} = 197\ \text{u}\left(\frac{1.66\times10^{-27}\ \text{kg}}{1\ \text{u}}\right) = 3.27\times10^{-25}\ \text{kg}$

Now, $|\Delta m| = |\Delta N|\,n$, and the number of atoms missing is

$$|\Delta N| = \frac{|\Delta m|}{m_0} = \frac{4.5\times10^{-4}\ \text{kg}}{3.27\times10^{-25}\ \text{kg}} = 1.38\times10^{21}\ \text{atoms}$$

The rate of loss is

$$\frac{|\Delta N|}{\Delta t} = \frac{1.38\times10^{21}\ \text{atoms}}{50\ \text{yr}}\left(\frac{1\ \text{yr}}{365.25\ \text{d}}\right)\left(\frac{1\ \text{d}}{24\ \text{h}}\right)\left(\frac{1\ \text{h}}{60\ \text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$
$$\frac{|\Delta N|}{\Delta t} = \boxed{8.72\times10^{11}\ \text{atoms/s}}$$

P19.17 (a) Initially, $P_iV_i = n_iRT_i$ $\quad (1.00\ \text{atm})V_i = n_iR(10.0+273.15)\ \text{K}$

Finally, $P_fV_f = n_fRT_f$ $\quad P_f\left(0.280V_i\right) = n_iR(40.0+273.15)\ \text{K}$

Dividing these equations, $\quad \frac{0.280P_f}{1.00\ \text{atm}} = \frac{313.15\ \text{K}}{283.15\ \text{K}}$

giving $\quad P_f = 3.95\ \text{atm}$

or $\quad P_f = \boxed{4.00\times10^5\ \text{Pa(abs.)}}$

(b) After being driven $\quad P_d(1.02)\left(0.280V_i\right) = n_iR(85.0+273.15)\ \text{K}$

$$P_d = 1.121P_f = \boxed{4.49\times10^5\ \text{Pa}}$$

P19.18 (a) $n = \dfrac{PV}{RT} = \dfrac{(9.00\text{ atm})\left(1.013\times10^5\ \text{Pa/atm}\right)\left(8.00\times10^{-3}\ \text{m}^3\right)}{(8.314\ \text{N}\cdot\text{mol K})(293\ \text{K})} = \boxed{2.99\ \text{mol}}$

(b) $N = nN_\text{A} = (2.99\ \text{mol})\left(6.02\times10^{23}\ \text{molecules/mol}\right) = \boxed{1.80\times10^{24}\ \text{molecules}}$

P19.19 The equation of state of an ideal gas is $PV = nRT$ so we need to solve for the number of moles to find N.

$$n = \frac{PV}{RT} = \frac{\left(1.01\times10^5\ \text{N/m}^2\right)\left[(10.0\ \text{m})(20.0\ \text{m})(30.0\ \text{m})\right]}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})} = 2.49\times10^5\ \text{mol}$$

$$N = nN_A = 2.49\times10^5\ \text{mol}\left(6.022\times10^{23}\ \text{molecules/mol}\right) = \boxed{1.50\times10^{29}\ \text{molecules}}$$

P19.20 $P = \dfrac{nRT}{V} = \left(\dfrac{9.00\ \text{g}}{18.0\ \text{g/mol}}\right)\left(\dfrac{8.314\ \text{J}}{\text{mol K}}\right)\left(\dfrac{773\ \text{K}}{2.00\times10^{-3}\ \text{m}^3}\right) = \boxed{1.61\ \text{MPa}} = 15.9\ \text{atm}$

P19.21

$$\sum F_y = 0: \qquad \rho_\text{out} gV - \rho_\text{in} gV - (200\ \text{kg})g = 0$$

$$\left(\rho_\text{out} - \rho_\text{in}\right)\left(400\ \text{m}^3\right) = 200\ \text{kg}$$

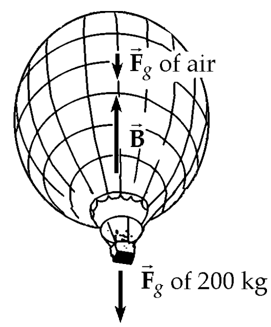


FIG. P19.21

The density of the air outside is $1.25\ \text{kg/m}^3$.

From $PV = nRT$, $\dfrac{n}{V} = \dfrac{P}{RT}$ This equation means that at constant pressure the density is inversely proportional to the temperature. Then the density of the hot air is

$$\rho_\text{in} = \left(1.25\ \text{kg/m}^3\right)\left(\frac{283\ \text{K}}{T_\text{in}}\right)$$

Then

$$\left(1.25\ \text{kg/m}^3\right)\left(1 - \frac{283\ \text{K}}{T_\text{in}}\right)\left(400\ \text{m}^3\right) = 200\ \text{kg}$$

$$1 - \frac{283\ \text{K}}{T_\text{in}} = 0.400$$

$$0.600 = \frac{283\ \text{K}}{T_\text{in}} \qquad T_\text{in} = \boxed{472\ \text{K}}$$

P19.22 Consider the air in the tank during one discharge process. We suppose that the process is slow enough that the temperature remains constant. Then as the pressure drops from 2.40 atm to 1.20 atm, the volume of the air doubles. During the first discharge, the air volume changes from 1 L to 2 L. Just 1 L of water is expelled and 3 L remains. In the second discharge, the air volume changes from 2 L to 4 L and 2 L of water is sprayed out. In the third discharge, only the last 1 L of water comes out. Each person could more efficiently use his device by starting with the tank half full of water.

***P19.23** (a) $PV = nRT$

$$n = \frac{PV}{RT} = \frac{(1.013\times10^5\ \text{Pa})(1.00\ \text{m}^3)}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})} = \boxed{41.6\ \text{mol}}$$

(b) $m = nM = (41.6\ \text{mol})(28.9\ \text{g/mol}) = \boxed{1.20\ \text{kg}}$ $\boxed{\text{This value agrees with the tabulated density}}$ of $1.20\ \text{kg/m}^3$ at 20.0°C.

P19.24 At depth, $P = P_0 + \rho g h$ and $PV_i = nRT_i$

At the surface, $P_0V_f = nRT_f$: $\dfrac{P_0V_f}{(P_0+\rho g h)V_i} = \dfrac{T_f}{T_i}$

Therefore $V_f = V_i\left(\dfrac{T_f}{T_i}\right)\left(\dfrac{P_0+\rho g h}{P_0}\right)$

$$V_f = 1.00\ \text{cm}^3\left(\frac{293\ \text{K}}{278\ \text{K}}\right)\left(\frac{1.013\times10^5\ \text{Pa} + (1\,025\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(25.0\ \text{m})}{1.013\times10^5\ \text{Pa}}\right)$$

$$V_f = \boxed{3.67\ \text{cm}^3}$$

P19.25 (a) $PV = nRT$ $\quad n = \dfrac{PV}{RT}$

$$m = nM = \frac{PVM}{RT} = \frac{1.013\times10^5\ \text{Pa}(0.100\ \text{m})^3(28.9\times10^{-3}\ \text{kg/mol})}{(8.314\ \text{J/mol}\cdot\text{K})(300\ \text{K})}$$

$$m = \boxed{1.17\times10^{-3}\ \text{kg}}$$

(b) $F_g = mg = 1.17\times10^{-3}\ \text{kg}(9.80\ \text{m/s}^2) = \boxed{11.5\ \text{mN}}$

(c) $F = PA = (1.013\times10^5\ \text{N/m}^2)(0.100\ \text{m})^2 = \boxed{1.01\ \text{kN}}$

(d) The $\boxed{\text{molecules must be moving very fast}}$ to hit the walls hard.

P19.26 My bedroom is 4 m long, 4 m wide, and 2.4 m high, enclosing air at 100 kPa and 20°C = 293 K. Think of the air as 80.0% N_2 and 20.0% O_2.

Avogadro's number of molecules has mass

$$(0.800)(28.0\ \text{g/mol}) + (0.200)(32.0\ \text{g/mol}) = 0.028\,8\ \text{kg/mol}$$

Then $PV = nRT = \left(\dfrac{m}{M}\right)RT$

gives $m = \dfrac{PVM}{RT} = \dfrac{(1.00\times10^5\ \text{N/m}^2)(38.4\ \text{m}^3)(0.028\,8\ \text{kg/mol})}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})} = 45.4\ \text{kg}\ \boxed{\sim10^2\ \text{kg}}$

P19.27 $PV = nRT$: $$\frac{m_f}{m_i} = \frac{n_f}{n_i} = \frac{P_f V_f}{RT_f}\frac{RT_i}{P_i V_i} = \frac{P_f}{P_i}$$

so $$m_f = m_i\left(\frac{P_f}{P_i}\right)$$

$$|\Delta m| = m_i - m_f = m_i\left(\frac{P_i - P_f}{P_i}\right) = 12.0\text{ kg}\left(\frac{41.0\text{ atm} - 26.0\text{ atm}}{41.0\text{ atm}}\right) = \boxed{4.39\text{ kg}}$$

P19.28 $$N = \frac{PVN_A}{RT} = \frac{\left(10^{-9}\text{ Pa}\right)\left(1.00\text{ m}^3\right)\left(6.02\times 10^{23}\text{ molecules/mol}\right)}{(8.314\text{ J/K}\cdot\text{mol})(300\text{ K})} = \boxed{2.41\times 10^{11}\text{ molecules}}$$

***P19.29** (a) The air in the tube is far from liquefaction, so it behaves as an ideal gas. At the ocean surface it is described by $P_t V_t = nRT$ where $P_t = 1$ atm, $V_t = A(6.50\text{ cm})$, and A is the cross-sectional area of the interior of the tube. At the bottom of the dive, $P_b V_b = nRT = P_b A(6.50\text{ cm} - 2.70\text{ cm})$. By division,

$$\frac{P_b(3.8\text{ cm})}{(1\text{ atm})(6.5\text{ cm})} = 1$$

$$P_b = 1.013\times 10^5\text{ N/m}^2\,\frac{6.5}{3.8} = 1.73\times 10^5\text{ N/m}^2$$

The salt water enters the tube until the air pressure is equal to the water pressure at depth, which is described by

$$P_b = P_t + \rho g h$$

$$1.73\times 10^5\text{ N/m}^2 = 1.013\times 10^5\text{ N/m}^2 + \left(1\,030\text{ kg/m}^3\right)\left(9.8\text{ m/s}^2\right)h$$

$$h = \frac{7.20\times 10^4\text{ kg}\cdot\text{m}\cdot\text{m}^2\cdot\text{s}^2}{1.01\times 10^4\text{ s}^2\cdot\text{m}^2\cdot\text{kg}} = \boxed{7.13\text{ m}}$$

(b) With a very thin tube, air does not bubble out. At the bottom of the dive, the tube gives a valid reading in any orientation. The open end of the tube should be at the bottom after the bird surfaces, so that the water will drain away as the expanding air pushes it out.

Students can make the tubes and dive with them in a swimming pool, to observe how dependably they work and how accurate they are.

P19.30 $$P_0 V = n_1 R T_1 = \left(\frac{m_1}{M}\right)RT_1$$

$$P_0 V = n_2 R T_2 = \left(\frac{m_2}{M}\right)RT_2$$

$$\boxed{m_1 - m_2 = \frac{P_0 V M}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}$$

Additional Problems

P19.31 The excess expansion of the brass is

$$\Delta L_{\text{rod}} - \Delta L_{\text{tape}} = (\alpha_{\text{brass}} - \alpha_{\text{steel}}) L_i \Delta T$$
$$\Delta(\Delta L) = (19.0 - 11.0)\times 10^{-6}\ (^\circ\text{C})^{-1}(0.950\ \text{m})(35.0^\circ\text{C})$$
$$\Delta(\Delta L) = 2.66\times 10^{-4}\ \text{m}$$

(a) The rod contracts more than tape to a length reading

$$0.950\,0\ \text{m} - 0.000\,266\ \text{m} = \boxed{0.949\,7\ \text{m}}$$

(b) $0.950\,0\ \text{m} + 0.000\,266\ \text{m} = \boxed{0.950\,3\ \text{m}}$

P19.32 At 0°C, 10.0 gallons of gasoline has mass,

from $\rho = \dfrac{m}{V}$

$$m = \rho V = (730\ \text{kg/m}^3)(10.0\ \text{gal})\left(\frac{0.003\,80\ \text{m}^3}{1.00\ \text{gal}}\right) = 27.7\ \text{kg}$$

The gasoline will expand in volume by

$$\Delta V = \beta V_i \Delta T = 9.60\times 10^{-4}\ {}^\circ\text{C}^{-1}(10.0\ \text{gal})(20.0^\circ\text{C} - 0.0^\circ\text{C}) = 0.192\ \text{gal}$$

At 20.0°C, $10.192\ \text{gal} = 27.7\ \text{kg}$

$$10.0\ \text{gal} = 27.7\ \text{kg}\left(\frac{10.0\ \text{gal}}{10.192\ \text{gal}}\right) = 27.2\ \text{kg}$$

The extra mass contained in 10.0 gallons at 0.0°C is

$$27.7\ \text{kg} - 27.2\ \text{kg} = \boxed{0.523\ \text{kg}}$$

P19.33 Neglecting the expansion of the glass,

$$\Delta h = \frac{V}{A}\beta\Delta T$$
$$\Delta h = \frac{\frac{4}{3}\pi(0.250\ \text{cm}/2)^3}{\pi(2.00\times 10^{-3}\ \text{cm})^2}(1.82\times 10^{-4}\ {}^\circ\text{C}^{-1})(30.0^\circ\text{C}) = \boxed{3.55\ \text{cm}}$$

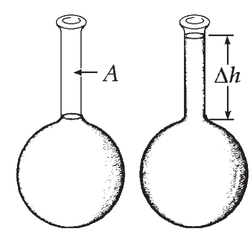


FIG. P19.33

P19.34 (a) The volume of the liquid increases as $\Delta V_\ell = V_i \beta \Delta T$. The volume of the flask increases as $\Delta V_g = 3\alpha V_i \Delta T$. Therefore, the overflow in the capillary is $V_c = V_i \Delta T(\beta - 3\alpha)$; and in the capillary $V_c = A\Delta h$.

Therefore,

$$\boxed{\Delta h = \frac{V_i}{A}(\beta - 3\alpha)\Delta T}$$

(b) For a mercury thermometer $\beta(\text{Hg}) = 1.82\times 10^{-4}\ {}^\circ\text{C}^{-1}$

and for glass, $3\alpha = 3\times 3.20\times 10^{-6}\ {}^\circ\text{C}^{-1}$

Thus $\beta - 3\alpha \approx \beta$ within better than 6%. The value of α is typically so small compared to β that it can be ignored in the equation for a good approximation.

P19.35 The frequency played by the cold-walled flute is

$$f_i = \frac{v}{\lambda_i} = \frac{v}{2L_i}$$

When the instrument warms up

$$f_f = \frac{v}{\lambda_f} = \frac{v}{2L_f} = \frac{v}{2L_i(1+\alpha\Delta T)} = \frac{f_i}{1+\alpha\Delta T}$$

The final frequency is lower. The change in frequency is

$$\Delta f = f_i - f_f = f_i\left(1 - \frac{1}{1+\alpha\Delta T}\right)$$

$$\Delta f = \frac{v}{2L_i}\left(\frac{\alpha\Delta T}{1+\alpha\Delta T}\right) \approx \frac{v}{2L_i}(\alpha\Delta T)$$

$$\Delta f \approx \frac{(343\text{ m/s})(24.0\times 10^{-6}/\text{C}^\circ)(15.0^\circ\text{C})}{2(0.655\text{ m})} = \boxed{0.094\,3\text{ Hz}}$$

This change in frequency is imperceptibly small.

***P19.36** Let L_0 represent the length of each bar at 0°C.

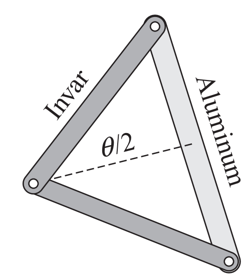


FIG. P19.36

(a) In the diagram consider the right triangle that each invar bar makes with one half of the aluminum bar. We have

$$\sin\left(\frac{\theta}{2}\right) = \frac{L_0(1+\alpha_{Al}\Delta T)}{2L_0}$$

solving gives directly

$$\boxed{\theta = 2\sin^{-1}\left(\frac{1+\alpha_{\text{Al}}T_C}{2}\right)}$$

where T_C is the Celsius temperature.

(b) If the temperature drops, the negative value of Celsius temperature describes the contraction. So $\boxed{\text{the answer is accurate}}$. At $T_C = 0$ we have $\theta = 2\sin^{-1}(1/2) = 60.0°$, and $\boxed{\text{this is accurate}}$.

(c) From the same triangle we have

$$\sin\left(\frac{\theta}{2}\right) = \frac{L_0(1+\alpha_{Al}\Delta T)}{2L_0(1+\alpha_{inv}\Delta T)} \quad \text{giving} \quad \boxed{\theta = 2\sin^{-1}\left(\frac{1+\alpha_{\text{Al}}T_C}{2(1+\alpha_{\text{invar}}T_C)}\right)}$$

(d) The greatest angle is at 660°C,

$$2\sin^{-1}\left(\frac{1+\alpha_{\text{Al}}T_C}{2(1+\alpha_{\text{invar}}T_C)}\right) = 2\sin^{-1}\left(\frac{1+(24\times 10^{-6})660}{2(1+[0.9\times 10^{-6}]660)}\right)$$

$$= 2\sin^{-1}\left(\frac{1.01584}{2.001188}\right) = 2\sin^{-1}0.508 = \boxed{61.0°}$$

The smallest angle is at −273°C,

$$2\sin^{-1}\left(\frac{1+(24\times 10^{-6})(-273)}{2(1+[0.9\times 10^{-6}][-273])}\right) = 2\sin^{-1}\left(\frac{0.9934}{1.9995}\right) = 2\sin^{-1}0.497 = \boxed{59.6°}$$

P19.37 (a) $\rho = \dfrac{m}{V}$ and $d\rho = -\dfrac{m}{V^2}dV$

For very small changes in V and ρ, this can be expressed as

$$\Delta\rho = -\frac{m}{V}\frac{\Delta V}{V} = -\rho\beta\Delta T$$

$$\boxed{\text{The negative sign means that any increase in temperature causes the density to decrease and vice versa.}}$$

(b) For water we have $\beta = \left|\dfrac{\Delta\rho}{\rho\Delta T}\right| = \left|\dfrac{1.000\,0\ \text{g/cm}^3 - 0.999\,7\ \text{g/cm}^3}{\left(1.000\,0\ \text{g/cm}^3\right)\left(10.0°\text{C} - 4.0°\text{C}\right)}\right| = \boxed{5\times10^{-5}\,°\text{C}^{-1}}$

P19.38 (a) $\dfrac{P_0V}{T} = \dfrac{P'V'}{T'}$

$V' = V + Ah$

$P' = P_0 + \dfrac{kh}{A}$

$\left(P_0 + \dfrac{kh}{A}\right)(V + Ah) = P_0V\left(\dfrac{T'}{T}\right)$

$$\left(1.013\times10^5\ \text{N/m}^2 + 2.00\times10^5\ \text{N/m}^3\,h\right)\left(5.00\times10^{-3}\ \text{m}^3 + \left(0.010\,0\ \text{m}^2\right)h\right)$$
$$= \left(1.013\times10^5\ \text{N/m}^2\right)\left(5.00\times10^{-3}\ \text{m}^3\right)\left(\frac{523\ \text{K}}{293\ \text{K}}\right)$$

$2\,000h^2 + 2\,013h - 397 = 0$

$h = \dfrac{-2\,013 \pm 2\,689}{4\,000} = \boxed{0.169\ \text{m}}$

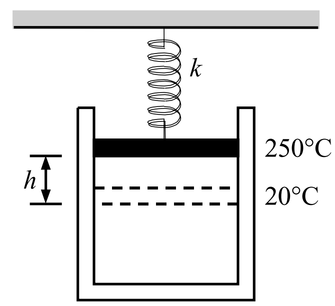


FIG. P19.38

(b) $P' = P + \dfrac{kh}{A} = 1.013\times10^5\ \text{Pa} + \dfrac{\left(2.00\times10^3\ \text{N/m}\right)(0.169)}{0.010\,0\ \text{m}^2}$

$P' = \boxed{1.35\times10^5\ \text{Pa}}$

P19.39 (a) We assume that air at atmospheric pressure is above the piston.

In equilibrium $\quad P_{\text{gas}} = \dfrac{mg}{A} + P_0$

Therefore, $\quad \dfrac{nRT}{hA} = \dfrac{mg}{A} + P_0$

or $\quad \boxed{h = \dfrac{nRT}{mg + P_0A}}$

where we have used $V = hA$ as the volume of the gas.

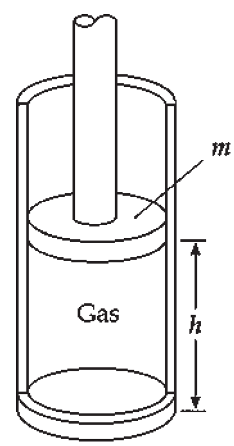


FIG. P19.39

(b) From the data given,

$$h = \frac{0.200\ \text{mol}\left(8.314\ \text{J/K}\cdot\text{mol}\right)(400\ \text{K})}{20.0\ \text{kg}\left(9.80\ \text{m/s}^2\right) + \left(1.013\times10^5\ \text{N/m}^2\right)\left(0.008\,00\ \text{m}^2\right)}$$

$= \boxed{0.661\ \text{m}}$

P19.40 The angle of bending θ, between tangents to the two ends of the strip, is equal to the angle the strip subtends at its center of curvature. (The angles are equal because their sides are perpendicular, right side to the right side and left side to left side.)

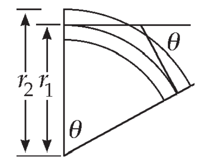


FIG. P19.40

(a) The definition of radian measure gives $L_i + \Delta L_1 = \theta r_1$

and $L_i + \Delta L_2 = \theta r_2$

By subtraction, $\Delta L_2 - \Delta L_1 = \theta(r_2 - r_1)$

$$\alpha_2 L_i \Delta T - \alpha_1 L_i \Delta T = \theta \Delta r$$

$$\boxed{\theta = \frac{(\alpha_2 - \alpha_1) L_i \Delta T}{\Delta r}}$$

(b) In the expression from part (a), θ is directly proportional to ΔT and also to $(\alpha_2 - \alpha_1)$. Therefore θ is zero when either of these quantities becomes zero.

(c) The material that expands more when heated contracts more when cooled, so $\boxed{\text{the bimetallic strip bends the other way}}$. It is fun to demonstrate this with liquid nitrogen.

(d) $$\theta = \frac{2(\alpha_2 - \alpha_1) L_i \Delta T}{2\Delta r} = \frac{2\left(\left(19\times10^{-6} - 0.9\times10^{-6}\right)°\text{C}^{-1}\right)(200 \text{ mm})(1°\text{C})}{0.500 \text{ mm}}$$

$$= 1.45\times10^{-2} = 1.45\times10^{-2} \text{ rad}\left(\frac{180°}{\pi \text{ rad}}\right) = \boxed{0.830°}$$

P19.41 From the diagram we see that the change in area is

$$\Delta A = \ell \Delta w + w \Delta \ell + \Delta w \Delta \ell$$

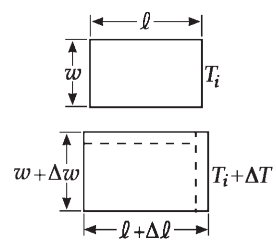


FIG. P19.41

Since $\Delta\ell$ and Δw are each small quantities, the product $\Delta w \Delta \ell$ will be very small. Therefore, we assume $\Delta w \Delta \ell \approx 0$.

Since $\Delta w = w\alpha\Delta T$ and $\Delta \ell = \ell \alpha \Delta T$,

we then have $\Delta A = \ell w \alpha \Delta T + \ell w \alpha \Delta T$

and since $A = \ell w$, $\boxed{\Delta A = 2\alpha A \Delta T}$

The approximation assumes $\Delta w \Delta \ell \approx 0$, or $\alpha \Delta T \approx 0$. Another way of stating this is $\boxed{\alpha \Delta T << 1}$.

***P19.42** (a) $\boxed{\text{The different diameters of the arms of the U-tube do not affect the pressures exerted by the liquids of different density on the liquid in the base. Because the base of the U-tube is horizontal, the pattern of temperature change in the base does not affect the equilibrium heights.}}$

(b) Let ρ_0 represent the density of the liquid at 0°C. At temperature T_C, the volume of a sample has changed according to $\Delta V = \beta V \Delta T = \beta V T_C$,

so the density has become

$$\rho = \frac{m}{V + \beta V T_C} = \rho_0 \frac{1}{1+\beta T_C} \qquad \text{so} \qquad \rho(1 + \beta T_C) = \rho_0$$

Now the pressure at the bottom of the U tube is equal, whichever column it supports:

$$P_0 + \rho_0 g h_0 = P_0 + \rho g h_t$$

simplifying, $\rho_0 h_0 = \rho h_t$

and substituting, $\rho(1 + \beta T_C)h_0 = \rho h_t$

$$(1 + \beta T_C)h_0 = h_t \qquad \beta = \frac{1}{T_C}\left(\frac{h_t}{h_0} - 1\right)$$

***P19.43** (a) The copper rod has a greater coefficient of linear expansion, so it should start with a smaller length. $\boxed{\text{The steel rod is longer.}}$ With $L_C + 5\text{ cm} = L_S$ at 0°C we want also $L_C(1 + 17 \times 10^{-6}\,T_C) + 5\text{ cm} = L_S(1 + 11 \times 10^{-6}\,T_C)$ or by subtraction $17\,L_C = 11\,L_S$. So

$\boxed{\text{yes, this pair of equations can be satisfied as long as the coefficients of expansion remain constant.}}$

By substitution,

$$L_C + 5\text{ cm} = (17/11)L_C \qquad \boxed{L_C = (11/6)\,5\text{ cm} = 9.17\text{ cm} \quad \text{so} \quad L_S = 14.2\text{ cm}}$$

P19.44 (a) $T_i = 2\pi\sqrt{\dfrac{L_i}{g}}$ so

$$L_i = \frac{T_i^2 g}{4\pi^2} = \frac{(1.000\text{ s})^2(9.80\text{ m/s}^2)}{4\pi^2} = 0.248\,2\text{ m}$$

$$\Delta L = \alpha L_i \Delta T = 19.0\times10^{-6}\,°\text{C}^{-1}(0.248\,2\text{ m})(10.0°\text{C}) = 4.72\times10^{-5}\text{ m}$$

$$T_f = 2\pi\sqrt{\frac{L_i + \Delta L}{g}} = 2\pi\sqrt{\frac{0.248\,3\text{ m}}{9.80\text{ m/s}^2}} = 1.000\,095\,0\text{ s}$$

$$\Delta T = \boxed{9.50\times10^{-5}\text{ s}}$$

(b) In one week, the time lost $= 1$ week$\left(9.50\times10^{-5}\text{ s lost per second}\right)$

$$\text{time lost} = (7.00\text{ d/week})\left(\frac{86\,400\text{ s}}{1.00\text{ d}}\right)\left(9.50\times10^{-5}\ \frac{\text{s lost}}{\text{s}}\right)$$

$$\text{time lost} = \boxed{57.5\text{ s lost}}$$

P19.45 $I = \int r^2 dm$ and since $r(T) = r(T_i)(1+\alpha\Delta T)$

for $\alpha\Delta T << 1$ we find $\dfrac{I(T)}{I(T_i)} = (1+\alpha\Delta T)^2$

thus $\dfrac{I(T) - I(T_i)}{I(T_i)} \approx 2\alpha\Delta T$

(a) With $\alpha = 17.0\times 10^{-6}\ ^\circ\text{C}^{-1}$ and $\Delta T = 100^\circ\text{C}$

we find for Cu: $\dfrac{\Delta I}{I} = 2\left(17.0\times 10^{-6}\ ^\circ\text{C}^{-1}\right)(100^\circ\text{C}) = \boxed{0.340\%}$

(b) With $\alpha = 24.0\times 10^{-6}\ ^\circ\text{C}^{-1}$

and $\Delta T = 100^\circ\text{C}$

we find for Al: $\dfrac{\Delta I}{I} = 2\left(24.0\times 10^{-6}\ ^\circ\text{C}^{-1}\right)(100^\circ\text{C}) = \boxed{0.480\%}$

P19.46 (a) Let V' represent the compressed volume at depth

$B = \rho g V' \qquad P' = P_0 + \rho g d \qquad P'V' = P_0 V_i$

$$B = \frac{\rho g P_0 V_i}{P'} = \boxed{\frac{\rho g P_0 V_i}{(P_0 + \rho g d)}}$$

(b) Since d is in the denominator, B must $\boxed{\text{decrease}}$ as the depth increases.

(The volume of the balloon becomes smaller with increasing pressure.)

(c) $$\frac{1}{2} = \frac{B(d)}{B(0)} = \frac{\rho g P_0 V_i/(P_0 + \rho g d)}{\rho g P_0 V_i / P_0} = \frac{P_0}{P_0 + \rho g d}$$

$P_0 + \rho g d = 2P_0$

$$d = \frac{P_0}{\rho g} = \frac{1.013\times 10^5\ \text{N/m}^2}{\left(1.00\times 10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)} = \boxed{10.3\ \text{m}}$$

P19.47 After expansion, the length of one of the spans is

$$L_f = L_i(1+\alpha\Delta T) = 125\ \text{m}\left[1 + 12\times 10^{-6}\ ^\circ\text{C}^{-1}(20.0^\circ\text{C})\right] = 125.03\ \text{m}$$

L_f, y, and the original 125 m length of this span form a right triangle with y as the altitude. Using the Pythagorean theorem gives:

$$(125.03\ \text{m})^2 = y^2 + (125\ \text{m})^2 \qquad \text{yielding} \qquad y = \boxed{2.74\ \text{m}}$$

P19.48 Let $\ell = L/2$ represent the original length of one of the concrete slabs. After expansion, the length of each one of the spans is $\ell_f = \ell(1+\alpha\Delta T)$. Now ℓ_f, y, and the original length ℓ of this span form a right triangle with y as the altitude. Using the Pythagorean theorem gives

$$\ell_f^2 = \ell^2 + y^2, \qquad \text{or} \qquad y = \sqrt{\ell_f^2 - \ell^2} = \ell\sqrt{(1+\alpha\Delta T)^2 - 1} = (L/2)\sqrt{2\alpha\Delta T + (\alpha\Delta T)^2}$$

Since $\alpha\Delta T << 1$, we have $\boxed{y \approx L\sqrt{\alpha\Delta T/2}}$

The height of the center of the buckling bridge is directly proportional to the bridge length. A small bridge is geometrically similar to a large one. The height is proportional to the square root of the temperature increase. Doubling ΔT makes y increase by only 41%. A small value of ΔT can have a surprisingly large effect. In units, the equation reads m = m(°C/°C)$^{1/2}$, so it is dimensionally correct.

P19.49 (a) Let m represent the sample mass. The number of moles is $n = \frac{m}{M}$ and the density is $\rho = \frac{m}{V}$.

So $PV = nRT$ becomes $PV = \frac{m}{M}RT$ or $PM = \frac{m}{V}RT$

Then,

$$\rho = \frac{m}{V} = \boxed{\frac{PM}{RT}}$$

(b) $$\rho = \frac{PM}{RT} = \frac{(1.013\times10^5\ \text{N/m}^2)(0.032\,0\ \text{kg/mol})}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})} = \boxed{1.33\ \text{kg/m}^3}$$

***P19.50** (a) From $PV = nRT$, the volume is: $V = \left(\frac{nR}{P}\right)T$

Therefore, when pressure is held constant, $\frac{dV}{dT} = \frac{nR}{P} = \frac{V}{T}$

Thus, $\beta \equiv \left(\frac{1}{V}\right)\frac{dV}{dT} = \left(\frac{1}{V}\right)\frac{V}{T}$, or $\beta = \boxed{\frac{1}{T}}$

(b) At $T = 0°\text{C} = 273\ \text{K}$, this predicts $\beta = \frac{1}{273\ \text{K}} = \boxed{3.66\times10^{-3}\ \text{K}^{-1}}$

Experimental values are: $\beta_{\text{He}} = 3.665\times10^{-3}\ \text{K}^{-1}$ and $\beta_{\text{air}} = 3.67\times10^{-3}\ \text{K}^{-1}$

> Our single theoretical value agrees within 0.06% and 0.2%, respectively, with the tabulated values for helium and air.

P19.51 Visualize the molecules of various species all moving randomly. The net force on any section of wall is the sum of the forces of all of the molecules pounding on it.

For each gas alone, $P_1 = \frac{N_1kT}{V}$ and $P_2 = \frac{N_2kT}{V}$ and $P_3 = \frac{N_3kT}{V}$, etc.

For all gases $P_1V_1 + P_2V_2 + P_3V_3 \ldots = (N_1 + N_2 + N_3 \ldots)kT$ and

$(N_1 + N_2 + N_3 \ldots)kT = PV$

Also, $V_1 = V_2 = V_3 = \ldots = V$ therefore $\boxed{P = P_1 + P_2 + P_3 \ldots}$

P19.52

> (a) No torque acts on the disk so its angular momentum is constant. Its moment of inertia decreases as it contracts so its angular speed must increase.

(b) $$I_i\omega_i = I_f\omega_f = \frac{1}{2}MR_i^2\omega_i = \frac{1}{2}MR_f^2\omega_f = \frac{1}{2}M\left[R_i + R_i\alpha\Delta T\right]^2\omega_f = \frac{1}{2}MR_i^2\left[1-\alpha|\Delta T|\right]^2\omega_f$$

$$\omega_f = \omega_i\left[1-\alpha|\Delta T|\right]^{-2} = \frac{25.0\ \text{rad/s}}{\left(1-\left(17\times10^{-6}\ 1/\text{C}°\right)830°\text{C}\right)^2} = \frac{25.0\ \text{rad/s}}{0.972} = \boxed{25.7\ \text{rad/s}}$$

P19.53 Consider a spherical steel shell of inner radius r and much smaller thickness t, containing helium at pressure P. When it contains so much helium that it is on the point of bursting into two hemispheres, we have $P\pi r^2 = \left(5\times 10^8 \text{ N/m}^2\right)2\pi rt$. The mass of the steel is $\rho_s V = \rho_s 4\pi r^2 t = \rho_s 4\pi r^2 \dfrac{Pr}{10^9 \text{ Pa}}$

For the helium in the tank,

$$PV = nRT \qquad \text{becomes} \qquad P\frac{4}{3}\pi r^3 = nRT = \frac{m_{\text{He}}}{M_{\text{He}}}RT = 1\text{ atm}V_{\text{balloon}}$$

The buoyant force on the balloon is the weight of the air it displaces, which is described by $1\text{ atm}V_{\text{balloon}} = \dfrac{m_{\text{air}}}{M_{\text{air}}}RT = P\dfrac{4}{3}\pi r^3$. The net upward force on the balloon with the steel tank hanging from it is

$$+m_{\text{air}}g - m_{\text{He}}g - m_s g = \frac{M_{\text{air}}P4\pi r^3 g}{3RT} - \frac{M_{\text{He}}P4\pi r^3 g}{3RT} - \frac{\rho_s P4\pi r^3 g}{10^9 \text{ Pa}}$$

The balloon will or will not lift the tank depending on whether this quantity is positive or negative, which depends on the sign of $\dfrac{\left(M_{\text{air}} - M_{\text{He}}\right)}{3RT} - \dfrac{\rho_s}{10^9 \text{ Pa}}$. At 20°C this quantity is

$$= \frac{(28.9 - 4.00)\times 10^{-3} \text{ kg/mol}}{3(8.314 \text{ J/mol}\cdot\text{K})293 \text{ K}} - \frac{7\,860 \text{ kg/m}^3}{10^9 \text{ N/m}^2}$$

$$= 3.41\times 10^{-6} \text{ s}^2/\text{m}^2 - 7.86\times 10^{-6} \text{ s}^2/\text{m}^2$$

where we have used the density of iron. The net force on the balloon is downward so the helium balloon is $\boxed{\text{not able to lift}}$ its tank. Steel would need to be 2.30 times stronger to contain enough helium to lift the steel tank.

P19.54 With piston alone: $T = \text{constant}$, so $PV = P_0 V_0$

or $P\left(Ah_i\right) = P_0\left(Ah_0\right)$

With A = constant, $P = P_0\left(\dfrac{h_0}{h_i}\right)$

But, $P = P_0 + \dfrac{m_p g}{A}$

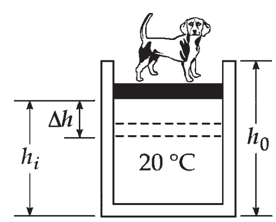


FIG. P19.54

where m_p is the mass of the piston.

Thus, $P_0 + \dfrac{m_p g}{A} = P_0\left(\dfrac{h_0}{h_i}\right)$

which reduces to

$$h_i = \frac{h_0}{1 + m_p g / P_0 A} = \frac{50.0 \text{ cm}}{1 + 20.0 \text{ kg}\left(9.80 \text{ m/s}^2\right)/\left[1.013\times 10^5 \text{ Pa }\pi(0.400 \text{ m})^2\right]} = 49.81 \text{ cm}$$

With the dog of mass M on the piston, a very similar calculation (replacing m_p by $m_p + M$) gives:

$$h' = \frac{h_0}{1 + \left(m_p + M\right)g / P_0 A} = \frac{50.0 \text{ cm}}{1 + 95.0 \text{ kg}\left(9.80 \text{ m/s}^2\right)/\left[1.013\times 10^5 \text{ Pa }\pi(0.400 \text{ m})^2\right]} = 49.10 \text{ cm}$$

Thus, when the dog steps on the piston, it moves downward by

$$\Delta h = h_i - h' = 49.81 \text{ cm} - 49.10 \text{ cm} = 0.706 \text{ cm} = \boxed{7.06 \text{ mm}}$$

(b) $P = \text{const}$, so $\dfrac{V}{T} = \dfrac{V'}{T_i}$ or $\dfrac{Ah_i}{T} = \dfrac{Ah'}{T_i}$

giving $T = T_i\left(\dfrac{h_i}{h'}\right) = 293 \text{ K}\left(\dfrac{49.81}{49.10}\right) = \boxed{297 \text{ K}}$ (or 24°C)

P19.55 (a) $\frac{dL}{L} = \alpha dT$: $\int_{T_i}^{T_f} \alpha dT = \int_{L_i}^{L_f} \frac{dL}{L} \Rightarrow \ln\left(\frac{L_f}{L_i}\right) = \alpha \Delta T \Rightarrow \boxed{L_f = L_i e^{\alpha \Delta T}}$

(b) $L_f = (1.00\text{ m})e^{\left[2.00\times10^{-5}\,°\text{C}^{-1}(100°\text{C})\right]} = 1.002\,002\text{ m}$

$L_f' = 1.00\text{ m}\left[1 + 2.00\times10^{-5}\,°\text{C}^{-1}(100°\text{C})\right] = 1.002\,000\text{ m}$: $\frac{L_f - L_f'}{L_f} = 2.00\times10^{-6}$

$= \boxed{2.00\times10^{-4}\%}$

$L_f = (1.00\text{ m})e^{\left[2.00\times10^{-2}\,°\text{C}^{-1}(100°\text{C})\right]} = 7.389\text{ m}$

$L_f' = 1.00\text{ m}\left[1 + 0.020\,0°\text{C}^{-1}(100°\text{C})\right] = 3.000\text{ m}$: $\frac{L_f - L_f'}{L_f} = \boxed{59.4\%}$

P19.56 At 20.0°C, the unstretched lengths of the steel and copper wires are

$$L_s(20.0°\text{C}) = (2.000\text{ m})\left[1 + 11.0\times10^{-6}(\text{C}°)^{-1}(-20.0°\text{C})\right] = 1.999\,56\text{ m}$$

$$L_c(20.0°\text{C}) = (2.000\text{ m})\left[1 + 17.0\times10^{-6}(\text{C}°)^{-1}(-20.0°\text{C})\right] = 1.999\,32\text{ m}$$

Under a tension F, the length of the steel and copper wires are

$$L_s' = L_s\left[1 + \frac{F}{YA}\right]_s \qquad L_c' = L_c\left[1 + \frac{F}{YA}\right]_c \qquad \text{where} \quad L_s' + L_c' = 4.000\text{ m}$$

Since the tension F must be the same in each wire, we solve for F:

$$F = \frac{(L_s' + L_c') - (L_s + L_c)}{L_s/Y_sA_s + L_c/Y_cA_c}$$

When the wires are stretched, their areas become

$$A_s = \pi\left(1.000\times10^{-3}\text{ m}\right)^2\left[1 + \left(11.0\times10^{-6}\right)(-20.0)\right]^2 = 3.140\times10^{-6}\text{ m}^2$$

$$A_c = \pi\left(1.000\times10^{-3}\text{ m}\right)^2\left[1 + \left(17.0\times10^{-6}\right)(-20.0)\right]^2 = 3.139\times10^{-6}\text{ m}^2$$

Recall $Y_s = 20.0\times10^{10}$ Pa and $Y_c = 11.0\times10^{10}$ Pa. Substituting into the equation for F, we obtain

$$F = \frac{4.000\text{ m} - (1.999\,56\text{ m} + 1.999\,32\text{ m})}{[1.999\,56\text{ m}]/\left[\left(20.0\times10^{10}\text{ Pa}\right)\left(3.140\times10^{-6}\right)\text{ m}^2\right] + [1.999\,32\text{ m}]/\left[\left(11.0\times10^{10}\text{ Pa}\right)\left(3.139\times10^{-6}\right)\text{ m}^2\right]}$$

$$F = \boxed{125\text{ N}}$$

To find the x-coordinate of the junction,

$$L_s' = (1.999\,56\text{ m})\left[1 + \frac{125\text{ N}}{\left(20.0\times10^{10}\text{ N/m}^2\right)\left(3.140\times10^{-6}\text{ m}^2\right)}\right] = 1.999\,958\text{ m}$$

Thus the x-coordinate is $-2.000 + 1.999\,958 = \boxed{-4.20\times10^{-5}\text{ m}}$

P19.57 (a) $\mu = \pi r^2 \rho = \pi\left(5.00\times10^{-4}\text{ m}\right)^2\left(7.86\times10^3\text{ kg/m}^3\right) = \boxed{6.17\times10^{-3}\text{ kg/m}}$

(b) $f_1 = \dfrac{v}{2L}$ and $v = \sqrt{\dfrac{T}{\mu}}$ so $f_1 = \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$

Therefore,

$$T = \mu\left(2Lf_1\right)^2 = \left(6.17\times10^{-3}\right)(2\times0.800\times200)^2 = \boxed{632\text{ N}}$$

(c) First find the unstressed length of the string at 0°C:

$$L = L_{\text{natural}}\left(1+\frac{T}{AY}\right) \quad \text{so} \quad L_{\text{natural}} = \frac{L}{1+T/AY}$$

$$A = \pi\left(5.00\times10^{-4}\text{ m}\right)^2 = 7.854\times10^{-7}\text{ m}^2 \quad \text{and} \quad Y = 20.0\times10^{10}\text{ Pa}$$

Therefore,

$$\frac{T}{AY} = \frac{632}{\left(7.854\times10^{-7}\right)\left(20.0\times10^{10}\right)} = 4.02\times10^{-3}\text{, and}$$

$$L_{\text{natural}} = \frac{(0.800\text{ m})}{\left(1+4.02\times10^{-3}\right)} = 0.796\,8\text{ m}$$

The unstressed length at 30.0°C is

$$L_{30^\circ\text{C}} = L_{\text{natural}}\left[1+\alpha(30.0^\circ\text{C}-0.0^\circ\text{C})\right]\text{, or}$$

$$L_{30^\circ\text{C}} = (0.796\,8\text{ m})\left[1+\left(11.0\times10^{-6}\right)(30.0)\right] = 0.797\,06\text{ m}$$

Since $L = L_{30^\circ\text{C}}\left[1+\dfrac{T'}{AY}\right]$, where T' is the tension in the string at 30.0°C,

$$T' = AY\left[\frac{L}{L_{30^\circ\text{C}}}-1\right] = \left(7.854\times10^{-7}\right)\left(20.0\times10^{10}\right)\left[\frac{0.800}{0.797\,06}-1\right] = \boxed{580\text{ N}}$$

To find the frequency at 30.0°C, realize that

$$\frac{f_1'}{f_1} = \sqrt{\frac{T'}{T}} \quad \text{so} \quad f_1' = (200\text{ Hz})\sqrt{\frac{580\text{ N}}{632\text{ N}}} = \boxed{192\text{ Hz}}$$

P19.58 Some gas will pass through the porous plug from the reaction chamber 1 to the reservoir 2 as the reaction chamber is heated, but the net quantity of gas stays constant according to

$$n_{i1}+n_{i2} = n_{f1}+n_{f2}$$

Assuming the gas is ideal, we apply $n = \dfrac{PV}{RT}$ to each term:

$$\frac{P_iV_0}{(300\text{ K})R}+\frac{P_i\left(4V_0\right)}{(300\text{ K})R} = \frac{P_fV_0}{(673\text{ K})R}+\frac{P_f\left(4V_0\right)}{(300\text{ K})R}$$

$$1\text{ atm}\left(\frac{5}{300\text{ K}}\right) = P_f\left(\frac{1}{673\text{ K}}+\frac{4}{300\text{ K}}\right) \qquad \boxed{P_f = 1.12\text{ atm}}$$

P19.59 Let 2θ represent the angle the curved rail subtends. We have

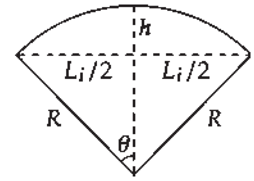


FIG. P19.59

$$L_i + \Delta L = 2\theta R = L_i\left(1+\alpha\Delta T\right) \qquad \text{and} \qquad \sin\theta = \frac{\frac{L_i}{2}}{R} = \frac{L_i}{2R}$$

Thus,

$$\theta = \frac{L_i}{2R}\left(1+\alpha\Delta T\right) = \left(1+\alpha\Delta T\right)\sin\theta$$

and we must solve the transcendental equation $\theta = \left(1+\alpha\Delta T\right)\sin\theta = \left(1.000\,005\,5\right)\sin\theta$
Your calculator is likely to want to find the zero solution.

Homing in on the nonzero solution gives, to four digits, $\theta = 0.018\,16\text{ rad} = 1.040\,5°$

Now, $$h = R - R\cos\theta = \frac{L_i\left(1-\cos\theta\right)}{2\sin\theta}$$

This yields $\boxed{h = 4.54\text{ m}}$, a remarkably large value compared to $\Delta L = 5.50$ cm.

P19.60 (a) Let xL represent the distance of the stationary line below the top edge of the plate. The normal force on the lower part of the plate is $mg\left(1-x\right)\cos\theta$ and the force of kinetic friction on it is $\mu_k mg\left(1-x\right)\cos\theta$ up the roof. Again, $\mu_k mgx\cos\theta$ acts down the roof on the upper part of the plate. The near-equilibrium of the plate requires $\sum F_x = 0$

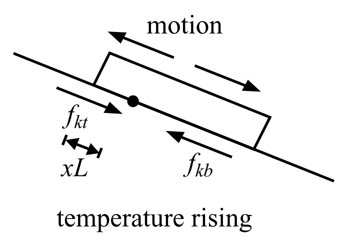


FIG. P19.60(a)

$$-\mu_k mgx\cos\theta + \mu_k mg\left(1-x\right)\cos\theta - mg\sin\theta = 0$$
$$-2\mu_k mgx\cos\theta = mg\sin\theta - \mu_k mg\cos\theta$$
$$2\mu_k x = \mu_k - \tan\theta$$
$$x = \frac{1}{2} - \frac{\tan\theta}{2\mu_k}$$

and the stationary line is indeed below the top edge by $xL = \dfrac{L}{2}\left(1-\dfrac{\tan\theta}{\mu_k}\right)$

(b) With the temperature falling, the plate contracts faster than the roof. The upper part slides down and feels an upward frictional force $\mu_k mg\left(1-x\right)\cos\theta$. The lower part slides up and feels downward frictional force $\mu_k mgx\cos\theta$. The equation $\sum F_x = 0$ is then the same as in part (a) and the stationary line is above the bottom edge by $xL = \dfrac{L}{2}\left(1-\dfrac{\tan\theta}{\mu_k}\right)$

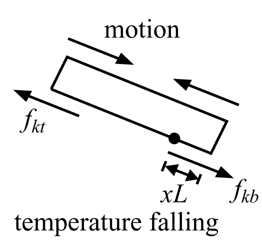


FIG. P19.60(b)

continued on next page

(c) Start thinking about the plate at dawn, as the temperature starts to rise. As in part (a), a line at distance xL below the top edge of the plate stays stationary relative to the roof as long as the temperature rises. The point P on the plate at distance xL above the bottom edge is destined to become the fixed point when the temperature starts falling. As the temperature rises, this point moves down the roof because of the expansion of the central part of the plate. Its displacement for the day is

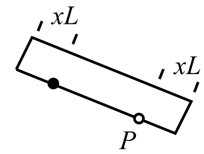


FIG. P19.60(c)

$$\Delta L = (\alpha_2 - \alpha_1)(L - xL - xL)\Delta T$$
$$= (\alpha_2 - \alpha_1)\left[L - 2\frac{L}{2}\left(1 - \frac{\tan\theta}{\mu_k}\right)\right](T_h - T_c)$$
$$= (\alpha_2 - \alpha_1)\left(\frac{L\tan\theta}{\mu_k}\right)(T_h - T_c)$$

At dawn the next day the point P is farther down the roof by the distance ΔL. It represents the displacement of every other point on the plate.

(d) $$(\alpha_2 - \alpha_1)\left(\frac{L\tan\theta}{\mu_k}\right)(T_h - T_c) = \left(24\times10^{-6}\frac{1}{\text{C}^\circ} - 15\times10^{-6}\frac{1}{\text{C}^\circ}\right)\frac{1.20\text{ m}\tan 18.5^\circ}{0.42}32^\circ\text{C}$$
$$= \boxed{0.275\text{ mm}}$$

(e) If $\alpha_2 < \alpha_1$, the diagram in part (a) applies to temperature falling and the diagram in part (b) applies to temperature rising. The weight of the plate still pulls it step by step down the roof. The same expression describes how far it moves each day.

ANSWERS TO EVEN PROBLEMS

P19.2 (a) 810°F (b) 450 K

P19.4 (a) 1 337 K and 2 993 K (b) 1 596°C = 1 596 K

P19.6 1.20 cm

P19.8 0.663 mm to the right at 78.2° below the horizontal

P19.10 (a) 437°C (b) 2 100°C This will not work because aluminum melts at 660°C.

P19.12 (a) $2.52\times10^6\ \text{N/m}^2$ (b) no

P19.14 (a) 396 N (b) –101°C (c) The original length divides out, so the answers would not change.

P19.16 8.72×10^{11} atoms/s

P19.18 (a) 2.99 mol (b) 1.80×10^{24} molecules

P19.20 1.61 MPa

P19.22 In each pump-up-and-discharge cycle, the volume of air in the tank doubles. Thus 1.00 L of water is driven out by the air injected at the first pumping, 2.00 L by the second, and only the remaining 1.00 L by the third. Each person could more efficiently use his device by starting with the tank half full of water, instead of 80% full.

P19.24 3.67 cm^3

P19.26 between 10^1 kg and 10^2 kg

P19.28 2.41×10^{11} molecules

P19.30 $m_1 - m_2 = \dfrac{P_0 VM}{R}\left(\dfrac{1}{T_1} - \dfrac{1}{T_2}\right)$

P19.32 0.523 kg

P19.34 (a) see the solution (b) We have $\beta - 3\alpha \approx \beta$ within better than 6%. The value of α is typically so small compared to β that it can be ignored in the equation for a good approximation.

P19.36 (a) $2\sin^{-1}\left(\dfrac{1+\alpha_{\text{Al}}T_C}{2}\right)$ (b) Yes; yes. (c) $2\sin^{-1}\left(\dfrac{1+\alpha_{\text{Al}}T_C}{2(1+\alpha_{\text{invar}}T_C)}\right)$ (d) 61.0° and 59.6°

P19.38 (a) 0.169 m (b) 1.35×10^5 Pa

P19.40 (a) $\theta = \dfrac{(\alpha_2 - \alpha_1)L_i \Delta T}{\Delta r}$ (b) see the solution (c) it bends the other way (d) 0.830°

P19.42 (a) The different diameters of the arms of the U-tube do not affect the pressures exerted by the liquids of different density on the liquid in the base. Because the base of the U-tube is horizontal, the pattern of temperature change in the base does not affect the equilibrium heights.

(b) $\beta = \dfrac{1}{T_C}\left(\dfrac{h_t}{h_0} - 1\right)$

P19.44 (a) increase by 95.0 μs (b) loses 57.5 s

P19.46 (a) $B = \rho g P_0 V_i \left(P_0 + \rho g d\right)^{-1}$ up (b) decrease (c) 10.3 m

P19.48 $y \approx L(\alpha \Delta T/2)^{1/2}$

P19.50 (a) see the solution (b) $3.66 \times 10^{-3}\ \text{K}^{-1}$, within 0.06% and 0.2% of the experimental values

P19.52 (a) Yes: it increases. As the disk cools, its radius, and hence its moment of inertia, decreases. Conservation of angular momentum then requires that its angular speed increase. (b) 25.7 rad/s

P19.54 (a) 7.06 mm (b) 297 K

P19.56 125 N; –42.0 μm

P19.58 1.12 atm

P19.60 (a), (b), (c) see the solution (d) 0.275 mm (e) The plate creeps down the roof each day by an amount given by the same expression.

20

Heat and the First Law of Thermodynamics

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q20.1 Temperature is a measure of molecular motion. Heat is energy in the process of being transferred between objects by random molecular collisions. Internal energy is an object's energy of random molecular motion and molecular interaction.

***Q20.2** With a specific heat half as large, the ΔT is twice as great in the ethyl alcohol. Answer (c).

Q20.3 Heat is energy being transferred, not energy contained in an object. Further, a low-temperature object with large mass, or an object made of a material with high specific heat, can contain more internal energy than a higher-temperature object.

***Q20.4** We think of the product $mc\Delta T$ in each case, with $c = 1$ for water and about 0.5 for beryllium. For (a) we have $1 \cdot 1 \cdot 6 = 6$. For (b), $2 \cdot 1 \cdot 3 = 6$. For (c), $2 \cdot 1 \cdot 3 = 6$. For (d), $2(0.5)3 = 3$. For (e), a large quantity of energy input is required to melt the ice. Then we have e > a = b = c > d.

Q20.5 There are three properties to consider here: thermal conductivity, specific heat, and mass. With dry aluminum, the thermal conductivity of aluminum is much greater than that of (dry) skin. This means that the internal energy in the aluminum can more readily be transferred to the atmosphere than to your fingers. In essence, your skin acts as a thermal insulator to some degree (pun intended). If the aluminum is wet, it can wet the outer layer of your skin to make it into a good conductor of heat; then more internal energy from the aluminum can get into you. Further, the water itself, with additional mass and with a relatively large specific heat compared to aluminum, can be a significant source of extra energy to burn you. In practical terms, when you let go of a hot, dry piece of aluminum foil, the heat transfer immediately ends. When you let go of a hot *and* wet piece of aluminum foil, the hot water sticks to your skin, continuing the heat transfer, and resulting in more energy transfer to you!

Q20.6 Write $1\,000\ \text{kg}(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(1^\circ\text{C}) = V\left(1.3\ \text{kg/m}^3\right)(1\,000\ \text{J/kg}\cdot{}^\circ\text{C})(1^\circ\text{C})$ to find $V = 3.2\times10^3\ \text{m}^3$.

***Q20.7** Answer (a). Do a few trials with water at different original temperatures and choose the one where room temperature is halfway between the original and the final temperature of the water. Then you can reasonably assume that the contents of the calorimeter gained and lost equal quantities of heat to the surroundings, for net transfer zero. James Joule did it like this in his basement in London.

Q20.8 If the system is isolated, no energy enters or leaves the system by heat, work, or other transfer processes. Within the system energy can change from one form to another, but since energy is conserved these transformations cannot affect the total amount of energy. The total energy is constant.

***Q20.9** (i) Answer (d). (ii) Answer (d). Internal energy and temperature both increase by minuscule amounts due to the work input.

Q20.10 The steam locomotive engine is one perfect example of turning internal energy into mechanical energy. Liquid water is heated past the point of vaporization. Through a controlled mechanical process, the expanding water vapor is allowed to push a piston. The translational kinetic energy of the piston is usually turned into rotational kinetic energy of the drive wheel.

Q20.11 The tile is a better thermal conductor than carpet. Thus, energy is conducted away from your feet more rapidly by the tile than by the carpeted floor.

***Q20.12** Yes, wrap the blanket around the ice chest. The insulation will slow the transfer of heat from the exterior to the interior. Explain to your little sister that her winter coat helps to keep the outdoors cold to the same extent that it helps to keep her warm. If that is too advanced, promise her a *really cold* can of Dr. Pepper at the picnic.

Q20.13 The sunlight hitting the peaks warms the air immediately around them. This air, which is slightly warmer and less dense than the surrounding air, rises, as it is buoyed up by cooler air from the valley below. The air from the valley flows up toward the sunny peaks, creating the morning breeze.

***Q20.14** Answer (d). The high specific heat will keep the end in the fire from warming up very fast. The low conductivity will make your end warm up only very slowly.

***Q20.15** Twice the radius means four times the surface area. Twice the absolute temperature makes T^4 sixteen times larger in Stefan's law. We multiply 4 times 16 to get answer (e).

Q20.16 The bit of water immediately over the flame warms up and expands. It is buoyed up and rises through the rest of the water. Colder, more dense water flows in to take its place. Convection currents are set up. They effectively warm the bulk of the water all at once, much more rapidly than it would be warmed by heat being conducted through the water from the flame.

Q20.17 Keep them dry. The air pockets in the pad conduct energy by heat, but only slowly. Wet pads would absorb some energy in warming up themselves, but the pot would still be hot and the water would quickly conduct and convect a lot of energy right into you.

Q20.18 The person should add the cream immediately when the coffee is poured. Then the smaller temperature difference between coffee and environment will reduce the rate of energy loss during the several minutes.

***Q20.19** Convection: answer (b). The bridge deck loses energy rapidly to the air both above it and below it.

Q20.20 The marshmallow has very small mass compared to the saliva in the teacher's mouth and the surrounding tissues. Mostly air and sugar, the marshmallow also has a low specific heat compared to living matter. Then the marshmallow can zoom up through a large temperature change while causing only a small temperature drop of the teacher's mouth. The marshmallow is a foam with closed cells and it carries very little liquid nitrogen into the mouth. (Note that microwaving the marshmallow beforehand might change it into an open-cell sponge, with disasterous effects.) The liquid nitrogen still on the undamaged marshmallow comes in contact with the much hotter saliva and immediately boils into cold gaseous nitrogen. This nitrogen gas has very low thermal conductivity. It creates an insulating thermal barrier between the marshmallow and the teacher's mouth (the Leydenfrost effect). A similar effect can be seen when water droplets are put on a hot skillet. Each one dances around as it slowly shrinks, because it is levitated on a thin film of steam. Upon application to the author of this manual, a teacher who does this demonstration for a class using the Serway-Jewett textbook may have a button reading "I am a professional. Do not try this at home."

The most extreme demonstration of this effect is pouring liquid nitrogen into one's mouth and blowing out a plume of nitrogen gas. We strongly recommended that you read of Jearl Walker's adventures with this demonstration rather than trying it.

Q20.21 (a) Warm a pot of coffee on a hot stove.

(b) Place an ice cube at 0°C in warm water—the ice will absorb energy while melting, but not increase in temperature.

(c) Let a high-pressure gas at room temperature slowly expand by pushing on a piston. Work comes out of the gas in a constant-temperature expansion as the same quantity of heat flows in from the surroundings.

(d) Warm your hands by rubbing them together. Heat your tepid coffee in a microwave oven. Energy input by work, by electromagnetic radiation, or by other means, can all alike produce a temperature increase.

(e) Davy's experiment is an example of this process.

(f) This is not necessarily true. Consider some supercooled liquid water, unstable but with temperature below 0°C. Drop in a snowflake or a grain of dust to trigger its freezing into ice, and the loss of internal energy measured by its latent heat of fusion can actually push its temperature up.

Q20.22 Heat is conducted from the warm oil to the pipe that carries it. That heat is then conducted to the cooling fins and up through the solid material of the fins. The energy then radiates off in all directions and is efficiently carried away by convection into the air. The ground below is left frozen.

SOLUTIONS TO PROBLEMS

Section 20.1 Heat and Internal Energy

P20.1 Taking $m = 1.00$ kg, we have

$$\Delta U_g = mgh = (1.00 \text{ kg})(9.80 \text{ m/s}^2)(50.0 \text{ m}) = 490 \text{ J}$$

But $\Delta U_g = Q = mc\Delta T = (1.00 \text{ kg})(4\,186 \text{ J/kg}\cdot{}^\circ\text{C})\Delta T = 490 \text{ J}$ so $\Delta T = 0.117°\text{C}$

$$T_f = T_i + \Delta T = \boxed{(10.0 + 0.117)°\text{C}}$$

P20.2 The container is thermally insulated, so no energy flows by heat:

$$Q = 0 \quad \text{and} \quad \Delta E_{\text{int}} = Q + W_{\text{input}} = 0 + W_{\text{input}} = 2mgh$$

The work on the falling weights is equal to the work done on the water in the container by the rotating blades. This work results in an increase in internal energy of the water:

$$2mgh = \Delta E_{\text{int}} = m_{\text{water}} c \Delta T$$

$$\Delta T = \frac{2mgh}{m_{\text{water}} c} = \frac{2 \times 1.50 \text{ kg}(9.80 \text{ m/s}^2)(3.00 \text{ m})}{0.200 \text{ kg}(4\,186 \text{ J/kg}\cdot{}^\circ\text{C})} = \frac{88.2 \text{ J}}{837 \text{ J/}^\circ\text{C}}$$

$$= \boxed{0.105°\text{C}}$$

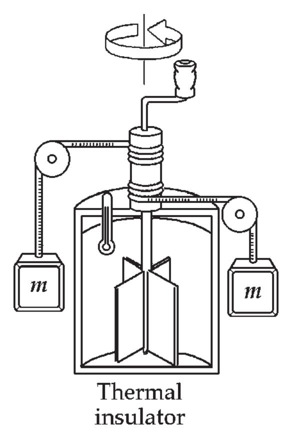


FIG. P20.2

Section 20.2 **Specific Heat and Calorimetry**

P20.3 $\Delta Q = mc_{\text{silver}}\Delta T$

$$1.23\ \text{kJ} = (0.525\ \text{kg})c_{\text{silver}}(10.0°\text{C})$$

$$c_{\text{silver}} = \boxed{0.234\ \text{kJ/kg}\cdot°\text{C}}$$

P20.4 The laser energy output:

$$\mathcal{P}\Delta t = (1.60\times10^{13}\ \text{J/s})2.50\times10^{-9}\ \text{s} = 4.00\times10^{4}\ \text{J}$$

The teakettle input:

$$Q = mc\Delta T = 0.800\ \text{kg}(4\,186\ \text{J/kg}\cdot°\text{C})80°\text{C} = 2.68\times10^{5}\ \text{J}$$

$\boxed{\text{The energy input to the water is 6.70 times larger than the laser output of 40 kJ.}}$

P20.5 We imagine the stone energy reservoir has a large area in contact with air and is always at nearly the same temperature as the air. Its overnight loss of energy is described by

$$\mathcal{P} = \frac{Q}{\Delta t} = \frac{mc\Delta T}{\Delta t}$$

$$m = \frac{\mathcal{P}\Delta t}{c\Delta T} = \frac{(-6\,000\ \text{J/s})(14\ \text{h})(3\,600\ \text{s/h})}{(850\ \text{J/kg}\cdot°\text{C})(18°\text{C}-38°\text{C})} = \frac{3.02\times10^{8}\ \text{J}\cdot\text{kg}\cdot°\text{C}}{850\ \text{J}(20°\text{C})} = \boxed{1.78\times10^{4}\ \text{kg}}$$

P20.6 Let us find the energy transferred in one minute.

$$Q = \left[m_{\text{cup}}c_{\text{cup}} + m_{\text{water}}c_{\text{water}}\right]\Delta T$$

$$Q = \left[(0.200\ \text{kg})(900\ \text{J/kg}\cdot°\text{C}) + (0.800\ \text{kg})(4\,186\ \text{J/kg}\cdot°\text{C})\right](-1.50°\text{C}) = -5\,290\ \text{J}$$

If this much energy is removed from the system each minute, the rate of removal is

$$\mathcal{P} = \frac{|Q|}{\Delta t} = \frac{5\,290\ \text{J}}{60.0\ \text{s}} = 88.2\ \text{J/s} = \boxed{88.2\ \text{W}}$$

P20.7 $Q_{\text{cold}} = -Q_{\text{hot}}$

$$(mc\Delta T)_{\text{water}} = -(mc\Delta T)_{\text{iron}}$$

$$20.0\ \text{kg}(4\,186\ \text{J/kg}\cdot°\text{C})(T_f - 25.0°\text{C}) = -(1.50\ \text{kg})(448\ \text{J/kg}\cdot°\text{C})(T_f - 600°\text{C})$$

$$T_f = \boxed{29.6°\text{C}}$$

***P20.8** (a) Work that the bit does in deforming the block, breaking chips off, and giving them kinetic energy is not a final destination for energy. All of this work turns entirely into internal energy as soon as the chips stop their macroscopic motion. The amount of energy input to the steel is the work done by the bit:

$$W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}} = (3.2\ \text{N})(40\ \text{m/s})(15\ \text{s})\cos 0° = 1\,920\ \text{J}$$

To evaluate the temperature change produced by this energy we imagine injecting the same quantity of energy as heat from a stove. The bit, chips, and block all undergo the same temperature change. Any difference in temperature between one bit of steel and another would erase itself by causing a heat transfer from the temporarily hotter to the colder region.

$$Q = mc\Delta T$$

$$\Delta T = \frac{Q}{mc} = \frac{1\,920\ \text{J}\cdot\text{kg}\cdot°\text{C}}{(0.267\ \text{kg})(448\ \text{J})} = \boxed{16.1°\text{C}}$$

(b) See part (a). $\boxed{16.1°\text{C}}$

(c) It makes no difference whether the drill bit is dull or sharp, how far into the block it cuts, or what its diameter is. The answers to (a) and (b) are the same because work (or 'work to produce deformation') cannot be a final form of energy: all of the work done by the bit constitutes energy being transferred into the internal energy of the steel.

***P20.9** (a) $Q_{\text{cold}} = -Q_{\text{hot}}$

$$(m_w c_w + m_c c_c)(T_f - T_c) = -m_{\text{Cu}} c_{\text{Cu}}(T_f - T_{\text{Cu}}) - m_{unk} c_{unk}(T_f - T_{unk})$$

where w is for water, c the calorimeter, Cu the copper sample, and *unk* the unknown.

$$[250\ \text{g}(1.00\ \text{cal/g}\cdot°\text{C}) + 100\ \text{g}(0.215\ \text{cal/g}\cdot°\text{C})](20.0 - 10.0)°\text{C}$$
$$= -(50.0\ \text{g})(0.092\,4\ \text{cal/g}\cdot°\text{C})(20.0 - 80.0)°\text{C} - (70.0\ \text{g})c_{unk}(20.0 - 100)°\text{C}$$
$$2.44\times 10^3\ \text{cal} = (5.60\times 10^3\ \text{g}\cdot°\text{C})c_{unk}$$

or

$$c_{unk} = \boxed{0.435\ \text{cal/g}\cdot°\text{C}}$$

(b) We cannot make a definite identification. The material might be beryllium. It might be some alloy or a material not listed in the table.

P20.10 (a) $(f)(mgh) = mc\Delta T$

$$\frac{(0.600)(3.00\times 10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)(50.0\ \text{m})}{4.186\ \text{J/cal}} = (3.00\ \text{g})(0.092\,4\ \text{cal/g}\cdot°\text{C})(\Delta T)$$

$\Delta T = 0.760°\text{C}$; $\boxed{T = 25.8°\text{C}}$

(b) The final temperature does not depend on the mass. Both the change in potential energy, and the heat that would be required from a stove to produce the temperature change, are proportional to the mass; hence, the mass divides out in the energy relation.

P20.11 We do not know whether the aluminum will rise or drop in temperature. The energy the water can absorb in rising to 26°C is $mc\Delta T = 0.25\text{ kg}\ 4\,186\dfrac{\text{J}}{\text{kg °C}}6\text{°C} = 6\,279\text{ J}$. The energy the copper can put out in dropping to 26°C is $mc\Delta T = 0.1\text{ kg}\ 387\dfrac{\text{J}}{\text{kg °C}}74\text{°C} = 2\,864\text{ J}$. Since 6 279 J > 2 864 J, the final temperature is less than 26°C. We can write $Q_h = -Q_c$ as

$$Q_{\text{water}} + Q_{\text{Al}} + Q_{\text{Cu}} = 0$$

$$0.25\text{ kg}\ 4\,186\frac{\text{J}}{\text{kg °C}}\left(T_f - 20\text{°C}\right) + 0.4\text{ kg}\ 900\frac{\text{J}}{\text{kg °C}}\left(T_f - 26\text{°C}\right) + 0.1\text{ kg}\ 387\frac{\text{J}}{\text{kg °C}}\left(T_f - 100\text{°C}\right) = 0$$

$$1\,046.5T_f - 20\,930\text{°C} + 360T_f - 9\,360\text{°C} + 38.7T_f - 3\,870\text{°C} = 0$$

$$1\,445.2T_f = 34\,160\text{°C}$$

$$T_f = \boxed{23.6\text{°C}}$$

P20.12 Vessel one contains oxygen described by $PV = nRT$:

$$n_c = \frac{PV}{RT} = \frac{1.75\left(1.013\times10^5\text{ Pa}\right)16.8\times10^{-3}\text{ m}^3}{8.314\text{ Nm/mol}\cdot\text{K}\ \ 300\text{ K}} = 1.194\text{ mol}$$

Vessel two contains this much oxygen:

$$n_h = \frac{2.25\left(1.013\times10^5\right)22.4\times10^{-3}}{8.314(450)}\text{ mol} = 1.365\text{ mol}$$

(a) The gas comes to an equilibrium temperature according to

$$(mc\Delta T)_{\text{cold}} = -(mc\Delta T)_{\text{hot}}$$

$$n_cMc\left(T_f - 300\text{ K}\right) + n_hMc\left(T_f - 450\text{ K}\right) = 0$$

The molar mass M and specific heat divide out:

$$1.194T_f - 358.2\text{ K} + 1.365T_f - 614.1\text{ K} = 0$$

$$T_f = \frac{972.3\text{ K}}{2.559} = \boxed{380\text{ K}}$$

(b) The pressure of the whole sample in its final state is

$$P = \frac{nRT}{V} = \frac{2.559\text{ mol}\ 8.314\text{ J}\ 380\text{ K}}{\text{mol K}(22.4+16.8)\times10^{-3}\text{ m}^3} = \boxed{2.06\times10^5\text{ Pa}} = 2.04\text{ atm}$$

Section 20.3 **Latent Heat**

P20.13 The energy input needed is the sum of the following terms:

$$Q_{\text{needed}} = (\text{heat to reach melting point}) + (\text{heat to melt})$$
$$+ (\text{heat to reach boiling point}) + (\text{heat to vaporize}) + (\text{heat to reach } 110°\text{C})$$

Thus, we have

$$Q_{\text{needed}} = 0.040\,0\text{ kg}\left[(2\,090\text{ J/kg}\cdot°\text{C})(10.0°\text{C}) + (3.33\times10^5\text{ J/kg})\right.$$
$$\left. + (4\,186\text{ J/kg}\cdot°\text{C})(100°\text{C}) + (2.26\times10^6\text{ J/kg}) + (2\,010\text{ J/kg}\cdot°\text{C})(10.0°\text{C})\right]$$

$$Q_{\text{needed}} = \boxed{1.22\times10^5\text{ J}}$$

P20.14 $Q_{\text{cold}} = -Q_{\text{hot}}$

$$(m_w c_w + m_c c_c)(T_f - T_i) = -m_s\left[-L_v + c_w(T_f - 100)\right]$$

$$\left[0.250\text{ kg}(4\,186\text{ J/kg}\cdot°\text{C}) + 0.050\,0\text{ kg}(387\text{ J/kg}\cdot°\text{C})\right](50.0°\text{C} - 20.0°\text{C})$$
$$= -m_s\left[-2.26\times10^6\text{ J/kg} + (4\,186\text{ J/kg}\cdot°\text{C})(50.0°\text{C} - 100°\text{C})\right]$$

$$m_s = \frac{3.20\times10^4\text{ J}}{2.47\times10^6\text{ J/kg}} = 0.012\,9\text{ kg} = \boxed{12.9\text{ g steam}}$$

P20.15 The bullet will not melt all the ice, so its final temperature is 0°C.

Then $\left(\frac{1}{2}mv^2 + mc|\Delta T|\right)_{\text{bullet}} = m_w L_f$

where m_w is the melt water mass

$$m_w = \frac{0.500(3.00\times10^{-3}\text{ kg})(240\text{ m/s})^2 + 3.00\times10^{-3}\text{ kg}(128\text{ J/kg}\cdot°\text{C})(30.0°\text{C})}{3.33\times10^5\text{ J/kg}}$$

$$m_w = \frac{86.4\text{ J} + 11.5\text{ J}}{333\,000\text{ J/kg}} = \boxed{0.294\text{ g}}$$

P20.16 (a) Q_1 = heat to melt all the ice

$$= (50.0\times10^{-3}\text{ kg})(3.33\times10^5\text{ J/kg}) = 1.67\times10^4\text{ J}$$

Q_2 = (heat to raise temp of ice to 100°C)

$$= (50.0\times10^{-3}\text{ kg})(4\,186\text{ J/kg}\cdot°\text{C})(100°\text{C}) = 2.09\times10^4\text{ J}$$

Thus, the total heat to melt ice and raise temp to 100°C $= 3.76\times10^4$ J

$$Q_3 = \begin{array}{c}\text{heat available}\\ \text{as steam condenses}\end{array} = (10.0\times10^{-3}\text{ kg})(2.26\times10^6\text{ J/kg}) = 2.26\times10^4\text{ J}$$

Thus, we see that $Q_3 > Q_1$, but $Q_3 < Q_1 + Q_2$.

Therefore, $\boxed{\text{all the ice melts}}$ but $T_f < 100°\text{C}$. Let us now find T_f

$Q_{\text{cold}} = -Q_{\text{hot}}$

$$(50.0\times10^{-3}\text{ kg})(3.33\times10^5\text{ J/kg}) + (50.0\times10^{-3}\text{ kg})(4\,186\text{ J/kg}\cdot°\text{C})(T_f - 0°\text{C})$$
$$= -(10.0\times10^{-3}\text{ kg})(-2.26\times10^6\text{ J/kg}) - (10.0\times10^{-3}\text{ kg})(4\,186\text{ J/kg}\cdot°\text{C})(T_f - 100°\text{C})$$

From which,

$$\boxed{T_f = 40.4°\text{C}}$$

continued on next page

(b) Q_1 = heat to melt all ice = 1.67×10^4 J [See part (a)]

$$Q_2 = \begin{matrix}\text{heat given up}\\ \text{as steam condenses}\end{matrix} = \left(10^{-3}\text{ kg}\right)\left(2.26\times10^{6}\text{ J/kg}\right) = 2.26\times10^{3}\text{ J}$$

$$Q_3 = \begin{matrix}\text{heat given up as condensed}\\ \text{steam cools to 0°C}\end{matrix} = \left(10^{-3}\text{ kg}\right)\left(4\,186\text{ J/kg}\cdot\text{°C}\right)(100\text{°C}) = 419\text{ J}$$

Note that $Q_2 + Q_3 < Q_1$. Therefore, $\boxed{\text{the final temperature will be 0°C}}$ with some ice remaining. Let us find the mass of ice which must melt to condense the steam and cool the condensate to 0°C.

$$mL_f = Q_2 + Q_3 = 2.68\times10^{3}\text{ J}$$

Thus,

$$m = \frac{2.68\times10^{3}\text{ J}}{3.33\times10^{5}\text{ J/kg}} = 8.04\times10^{-3}\text{ kg} = \boxed{8.04\text{ g of ice melts}}$$

Therefore, there is 42.0 g of ice left over, also at 0°C.

P20.17 $Q = m_{\text{Cu}}c_{\text{Cu}}\Delta T = m_{\text{N}_2}\left(L_{\text{vap}}\right)_{\text{N}_2}$

$$1.00\text{ kg}\left(0.092\,0\text{ cal/g}\cdot\text{°C}\right)(293-77.3)\text{°C} = m\left(48.0\text{ cal/g}\right)$$

$m = \boxed{0.414\text{ kg}}$

***P20.18** (a) Let n represent the number of stops. Follow the energy:

$$n\tfrac{1}{2}(1500\text{ kg})(25\text{ m/s})^2 = 6\text{ kg}(900\text{ J/kg}\cdot\text{°C})(660-20)\text{°}C$$

$$n = \frac{3.46\times10^{6}\text{ J}}{4.69\times10^{5}\text{ J}} = 7.37$$

Thus $\boxed{\text{seven}}$ stops can happen before melting begins.

(b) As the car is moving or stopping it transfers part of its kinetic energy into the air and into its rubber tires. As soon as the brakes rise above the air temperature they lose energy by heat, and lost it very fast if they attain a high temperature.

P20.19 (a) Since the heat required to melt 250 g of ice at 0°C *exceeds* the heat required to cool 600 g of water from 18°C to 0°C, the final temperature of the system (water + ice) must be $\boxed{0\text{°C}}$.

(b) Let m represent the mass of ice that melts before the system reaches equilibrium at 0°C.

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

$$mL_f = -m_wc_w\left(0\text{°C}-T_i\right)$$

$$m\left(3.33\times10^{5}\text{ J/kg}\right) = -\left(0.600\text{ kg}\right)\left(4\,186\text{ J/kg}\cdot\text{°C}\right)\left(0\text{°C}-18.0\text{°C}\right)$$

$m = 136$ g, so the ice remaining $= 250\text{ g} - 136\text{ g} = \boxed{114\text{ g}}$

***P20.20** The left-hand side of the equation is the kinetic energy of a 12-g object moving at 300 m/s together with an 8-g object moving at 400 m/s. If they are moving in opposite directions, collide head-on, and stick together, momentum conservation implies that we have a 20-g object moving with speed given by $8(400) - 12(300) = 20v \quad |v| = 20$ m/s, and the kinetic energy of a 20-g object moving at 20 m/s appears on the right-hand side. Thus we state

(a) Two speeding lead bullets, one of mass 12.0 g moving to the right at 300 m/s and one of mass 8.00 g moving to the left at 400 m/s, collide head-on and all of the material sticks together. Both bullets are originally at temperature 30.0°C. Describe the state of the system immediately thereafter.

(b) We find $\qquad 540 \text{ J} + 640 \text{ J} = 4 \text{ J} + 761 \text{ J} + m_\ell (24500 \text{ J/kg})$

So the mass of lead melted is $\qquad m_\ell = 415 \text{ J}/(24500 \text{ J/kg}) = 0.0169$ kg.

After the completely inelastic collision, a glob comprising 3.10 g of solid lead and 16.9 g of liquid lead is moving to the right at 20.0 m/s. Its temperature is 327.3°C.

Section 20.4 **Work and Heat in Thermodynamic Processes**

P20.21 $W_{if} = -\int_i^f P\,dV$

The work done on the gas is the negative of the area under the curve $P = \alpha V^2$ between V_i and V_f.

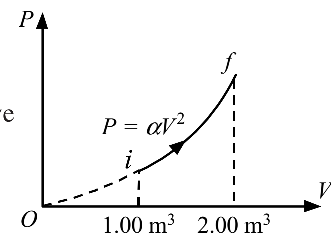


FIG. P20.21

$$W_{if} = -\int_i^f \alpha V^2 dV = -\frac{1}{3}\alpha\left(V_f^3 - V_i^3\right)$$

$$V_f = 2V_i = 2\left(1.00 \text{ m}^3\right) = 2.00 \text{ m}^3$$

$$W_{if} = -\frac{1}{3}\left[\left(5.00 \text{ atm/m}^6\right)\left(1.013\times10^5 \text{ Pa/atm}\right)\right]\left[\left(2.00 \text{ m}^3\right)^3 - \left(1.00 \text{ m}^3\right)^3\right] = \boxed{-1.18 \text{ MJ}}$$

P20.22 (a) $W = -\int P\,dV$

$$W = -\left(6.00\times10^6 \text{ Pa}\right)(2.00-1.00) \text{ m}^3 + -\left(4.00\times10^6 \text{ Pa}\right)(3.00-2.00) \text{ m}^3 + -\left(2.00\times10^6 \text{ Pa}\right)(4.00-3.00) \text{ m}^3$$

$$W_{i\to f} = \boxed{-12.0 \text{ MJ}}$$

(b) $W_{f\to i} = \boxed{+12.0 \text{ MJ}}$

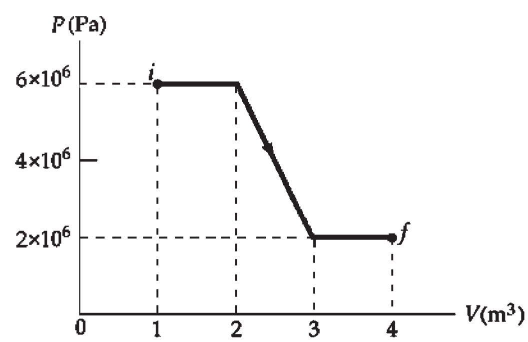


FIG. P20.22

P20.23 $W = -P\Delta V = -P\left(\frac{nR}{P}\right)\left(T_f - T_i\right) = -nR\Delta T = -(0.200)(8.314)(280) = \boxed{-466 \text{ J}}$

P20.24 $W = -\int_i^f PdV = -P\int_i^f dV = -P\Delta V = -nR\Delta T = \boxed{-nR(T_2 - T_1)}$ The negative sign for work *on* the sample indicates that the expanding gas *does* positive work. The quantity of work is directly proportional to the quantity of gas and to the temperature change.

P20.25 During the heating process $P = \left(\frac{P_i}{V_i}\right)V$

(a) $$W = -\int_i^f PdV = -\int_{V_i}^{3V_i}\left(\frac{P_i}{V_i}\right)VdV$$

$$W = -\left(\frac{P_i}{V_i}\right)\frac{V^2}{2}\Big|_{V_i}^{3V_i} = -\frac{P_i}{2V_i}\left(9V_i^2 - V_i^2\right) = \boxed{-4P_iV_i}$$

(b) $$PV = nRT$$

$$\left[\left(\frac{P_i}{V_i}\right)V\right]V = nRT$$

$$\boxed{T = \left(\frac{P_i}{nRV_i}\right)V^2}$$

Temperature must be proportional to the square of volume, rising to nine times its original value.

Section 20.5 The First Law of Thermodynamics

P20.26 (a) $Q = -W =$ Area of triangle

$$Q = \frac{1}{2}\left(4.00\ \text{m}^3\right)\left(6.00\ \text{kPa}\right) = \boxed{12.0\ \text{kJ}}$$

(b) $Q = -W = \boxed{-12.0\ \text{kJ}}$

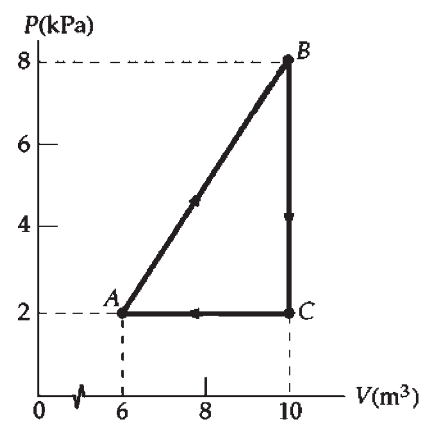


FIG. P20.26

P20.27 $\Delta E_{\text{int}} = Q + W$

$$Q = \Delta E_{\text{int}} - W = -500\ \text{J} - 220\ \text{J} = \boxed{-720\ \text{J}}$$

The negative sign indicates that positive energy is transferred *from* the system by heat.

P20.28 $W_{BC} = -P_B(V_C - V_B)$
$= -3.00 \text{ atm}(0.400 - 0.090\,0) \text{ m}^3$
$= -94.2 \text{ kJ}$

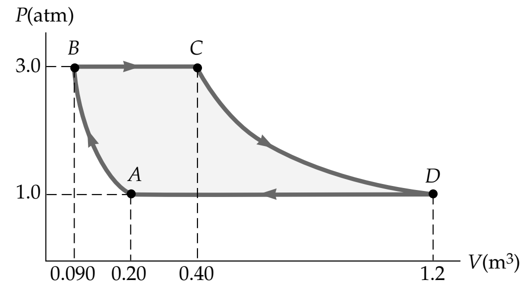


FIG. P20.28

$$\Delta E_{\text{int}} = Q + W$$
$$E_{\text{int, C}} - E_{\text{int, B}} = (100 - 94.2) \text{ kJ}$$
$$E_{\text{int, C}} - E_{\text{int, B}} = 5.79 \text{ kJ}$$

Since T is constant,

$$E_{\text{int, D}} - E_{\text{int, C}} = 0$$
$$W_{DA} = -P_D(V_A - V_D) = -1.00 \text{ atm}(0.200 - 1.20) \text{ m}^3$$
$$= +101 \text{ kJ}$$
$$E_{\text{int, A}} - E_{\text{int, D}} = -150 \text{ kJ} + (+101 \text{ kJ}) = -48.7 \text{ kJ}$$

Now, $E_{\text{int, B}} - E_{\text{int, A}} = -\left[\left(E_{\text{int, C}} - E_{\text{int, B}}\right) + \left(E_{\text{int, D}} - E_{\text{int, C}}\right) + \left(E_{\text{int, A}} - E_{\text{int, D}}\right)\right]$

$$E_{\text{int, B}} - E_{\text{int, A}} = -[5.79 \text{ kJ} + 0 - 48.7 \text{ kJ}] = \boxed{42.9 \text{ kJ}}$$

P20.29

	Q	W	ΔE_{int}	
BC	−	0	−	$\left(Q = \Delta E_{\text{int}} \text{ since } W_{BC} = 0\right)$
CA	−	+	−	$\left(\Delta E_{\text{int}} < 0 \text{ and } W > 0, \text{ so } Q < 0\right)$
AB	+	−	+	$(W < 0,\ \Delta E_{\text{int}} > 0 \text{ since } \Delta E_{\text{int}} < 0$ for $B \to C \to A;$ so $Q > 0)$

Section 20.6 **Some Applications of the First Law of Thermodynamics**

P20.30 (a) $W = -nRT \ln\left(\dfrac{V_f}{V_i}\right) = -P_f V_f \ln\left(\dfrac{V_f}{V_i}\right)$

so

$$`V_i = V_f \exp\left(+\frac{W}{P_f V_f}\right) = (0.025\,0)\exp\left[\frac{-3\,000}{0.025\,0\left(1.013\times10^5\right)}\right] = \boxed{0.007\,65 \text{ m}^3}$$

(b) $T_f = \dfrac{P_f V_f}{nR} = \dfrac{1.013\times10^5 \text{ Pa}\left(0.025\,0 \text{ m}^3\right)}{1.00 \text{ mol}(8.314 \text{ J/K}\cdot\text{mol})} = \boxed{305 \text{ K}}$

P20.31 (a) $\Delta E_{\text{int}} = Q - P\Delta V = 12.5 \text{ kJ} - 2.50 \text{ kPa}(3.00 - 1.00) \text{ m}^3 = \boxed{7.50 \text{ kJ}}$

(b) $\dfrac{V_1}{T_1} = \dfrac{V_2}{T_2}$

$$T_2 = \frac{V_2}{V_1}T_1 = \frac{3.00}{1.00}(300 \text{ K}) = \boxed{900 \text{ K}}$$

P20.32 (a) $W = -P\Delta V = -P[3\alpha V \Delta T]$

$$= -\left(1.013\times10^5\ \text{N/m}^2\right)\left[3\left(24.0\times10^{-6}\,{}^\circ\text{C}^{-1}\right)\left(\frac{1.00\ \text{kg}}{2.70\times10^3\ \text{kg/m}^3}\right)(18.0^\circ\text{C})\right]$$

$$W = \boxed{-48.6\ \text{mJ}}$$

(b) $Q = cm\Delta T = (900\ \text{J/kg}\cdot{}^\circ\text{C})(1.00\ \text{kg})(18.0^\circ\text{C}) = \boxed{16.2\ \text{kJ}}$

(c) $\Delta E_{\text{int}} = Q + W = 16.2\ \text{kJ} - 48.6\ \text{mJ} = \boxed{16.2\ \text{kJ}}$

P20.33 $W = -P\Delta V = -P\left(V_s - V_w\right) = -\dfrac{P(nRT)}{P} + P\left[\dfrac{18.0\ \text{g}}{\left(1.00\ \text{g/cm}^3\right)\left(10^6\ \text{cm}^3/\text{m}^3\right)}\right]$

$$W = -(1.00\ \text{mol})(8.314\ \text{J/K}\cdot\text{mol})(373\ \text{K}) + \left(1.013\times10^5\ \text{N/m}^2\right)\left(\frac{18.0\ \text{g}}{10^6\ \text{g/m}^3}\right) = \boxed{-3.10\ \text{kJ}}$$

$$Q = mL_v = 0.018\,0\ \text{kg}\left(2.26\times10^6\ \text{J/kg}\right) = 40.7\ \text{kJ}$$

$$\Delta E_{\text{int}} = Q + W = \boxed{37.6\ \text{kJ}}$$

P20.34 (a) The work done during each step of the cycle equals the negative of the area under that segment of the *PV* curve.

$$W = W_{DA} + W_{AB} + W_{BC} + W_{CD}$$

$$W = -P_i\left(V_i - 3V_i\right) + 0 - 3P_i\left(3V_i - V_i\right) + 0 = \boxed{-4P_iV_i}$$

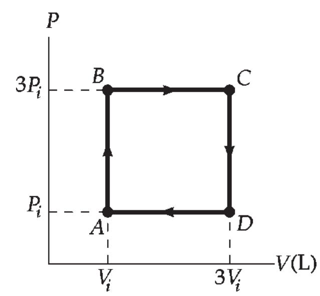


FIG. P20.34

(b) The initial and final values of *T* for the system are equal. Therefore,

$$\Delta E_{\text{int}} = 0 \qquad \text{and} \qquad Q = -W = \boxed{4P_iV_i}$$

(c) $W = -4P_iV_i = -4nRT_i = -4(1.00)(8.314)(273) = \boxed{-9.08\ \text{kJ}}$

P20.35 (a) $P_iV_i = P_fV_f = nRT = 2.00\ \text{mol}\left(8.314\ \text{J/K}\cdot\text{mol}\right)(300\ \text{K}) = 4.99\times10^3\ \text{J}$

$$V_i = \frac{nRT}{P_i} = \frac{4.99\times10^3\ \text{J}}{0.400\ \text{atm}}$$

$$V_f = \frac{nRT}{P_f} = \frac{4.99\times10^3\ \text{J}}{1.20\ \text{atm}} = \frac{1}{3}V_i = \boxed{0.041\,0\ \text{m}^3}$$

(b) $W = -\int P\,dV = -nRT\ln\left(\dfrac{V_f}{V_i}\right) = -\left(4.99\times10^3\right)\ln\left(\dfrac{1}{3}\right) = \boxed{+5.48\ \text{kJ}}$

(c) $\Delta E_{\text{int}} = 0 = Q + W$

$$Q = \boxed{-5.48\ \text{kJ}}$$

P20.36 $\Delta E_{\text{int, }ABC} = \Delta E_{\text{int, }AC}$ (conservation of energy)

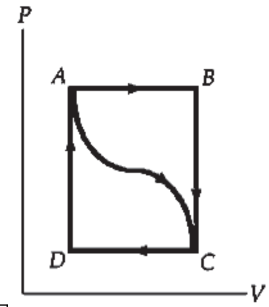


FIG. P20.36

(a) $\Delta E_{\text{int, }ABC} = Q_{ABC} + W_{ABC}$ (First Law)

$Q_{ABC} = 800\text{ J} + 500\text{ J} = \boxed{1\,300\text{ J}}$

(b) $W_{CD} = -P_C \Delta V_{CD}$, $\Delta V_{AB} = -\Delta V_{CD}$, and $P_A = 5P_C$

Then, $W_{CD} = \dfrac{1}{5} P_A \Delta V_{AB} = -\dfrac{1}{5} W_{AB} = \boxed{100\text{ J}}$

(+ means that work is done on the system)

(c) $W_{CDA} = W_{CD}$ so that $Q_{CA} = \Delta E_{\text{int, }CA} - W_{CDA} = -800\text{ J} - 100\text{ J} = \boxed{-900\text{ J}}$

(− means that energy must be removed from the system by heat)

(d) $\Delta E_{\text{int, }CD} = \Delta E_{\text{int, }CDA} - \Delta E_{\text{int, }DA} = -800\text{ J} - 500\text{ J} = -1\,300\text{ J}$

and $Q_{CD} = \Delta E_{\text{int, }CD} - W_{CD} = -1\,300\text{ J} - 100\text{ J} = \boxed{-1\,400\text{ J}}$

Section 20.7 **Energy Transfer Mechanisms**

P20.37 $\mathcal{P} = \dfrac{kA\Delta T}{L} = \dfrac{(0.800\text{ W/m}\cdot{}^\circ\text{C})(3.00\text{ m}^2)(25.0^\circ\text{C})}{6.00\times10^{-3}\text{ m}} = 1.00\times10^4\text{ W} = \boxed{10.0\text{ kW}}$

P20.38 $\mathcal{P} = \dfrac{A\Delta T}{\sum_i L_i/k_i} = \dfrac{(6.00\text{ m}^2)(50.0^\circ\text{C})}{2(4.00\times10^{-3}\text{ m})/[0.800\text{ W/m}\cdot{}^\circ\text{C}] + [5.00\times10^{-3}\text{ m}]/[0.023\,4\text{ W/m}\cdot{}^\circ\text{C}]}$

$= \boxed{1.34\text{ kW}}$

P20.39 In the steady state condition, $\mathcal{P}_{\text{Au}} = \mathcal{P}_{\text{Ag}}$

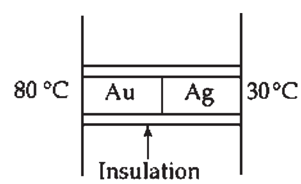


FIG. P20.39

so that $k_{\text{Au}} A_{\text{Au}} \left(\dfrac{\Delta T}{\Delta x}\right)_{\text{Au}} = k_{\text{Ag}} A_{\text{Ag}} \left(\dfrac{\Delta T}{\Delta x}\right)_{\text{Ag}}$

In this case $A_{\text{Au}} = A_{\text{Ag}}$

$\Delta x_{\text{Au}} = \Delta x_{\text{Ag}}$

$\Delta T_{\text{Au}} = (80.0 - T)$

and $\Delta T_{\text{Ag}} = (T - 30.0)$

where T is the temperature of the junction.

Therefore, $k_{\text{Au}}(80.0 - T) = k_{\text{Ag}}(T - 30.0)$

And $\boxed{T = 51.2^\circ\text{C}}$

P20.40 From the table of thermal resistances in the chapter text,

(a) $R = \boxed{0.890\ \text{ft}^2\cdot{}^\circ\text{F}\cdot\text{h/Btu}}$

(b) The insulating glass in the table must have sheets of glass less than $\frac{1}{8}$ inch thick. So we estimate the R-value of a 0.250-inch air space as $\frac{0.250}{3.50}$ times that of the thicker air space. Then for the double glazing

$$R_b = \left[0.890 + \left(\frac{0.250}{3.50}\right)1.01 + 0.890\right]\frac{\text{ft}^2\cdot{}^\circ\text{F}\cdot\text{h}}{\text{Btu}} = \boxed{1.85\frac{\text{ft}^2\cdot{}^\circ\text{F}\cdot\text{h}}{\text{Btu}}}$$

(c) Since A and $(T_2 - T_1)$ are constants, heat flow is reduced by a factor of $\frac{1.85}{0.890} = \boxed{2.08}$.

***P20.41** The net rate of energy loss from his skin is

$$\mathcal{P}_{\text{net}} = \sigma Ae\left(T^4 - T_0^4\right) = \left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left(1.50\ \text{m}^2\right)(0.900)\left[(308\ \text{K})^4 - (293\ \text{K})^4\right]$$
$$= 125\ \text{W}$$

Note that the temperatures must be in kelvins. The energy loss in ten minutes is

$$Q = \mathcal{P}_{\text{net}}\Delta t = (125\ \text{J/s})(600\ \text{s}) = \boxed{7.48\times10^4\ \text{J}}$$

In the infrared, the person shines brighter than a hundred-watt light bulb.

P20.42 $\mathcal{P} = \sigma AeT^4 = \left(5.669\,6\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left[4\pi\left(6.96\times10^8\ \text{m}\right)^2\right](0.986)(5\,800\ \text{K})^4$

$\mathcal{P} = \boxed{3.85\times10^{26}\ \text{W}}$

***P20.43** (a) The heat leaving the box during the day is given by $\mathcal{P} = kA\frac{(T_H - T_c)}{L} = \frac{Q}{\Delta t}$

$$Q = 0.012\frac{\text{W}}{\text{m}^\circ\text{C}}0.49\ \text{m}^2\frac{37^\circ\text{C} - 23^\circ\text{C}}{0.045\ \text{m}}12\ \text{h}\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right) = 7.90\times10^4\ \text{J}$$

The heat lost at night is

$$Q = 0.012\frac{\text{W}}{\text{m}^\circ\text{C}}0.49\ \text{m}^2\frac{37^\circ\text{C} - 16^\circ\text{C}}{0.045\ \text{m}}12\ \text{h}\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right) = 1.19\times10^5\ \text{J}$$

The total heat is $1.19\times10^5\ \text{J} + 7.90\times10^4\ \text{J} = 1.98\times10^5\ \text{J}$. It must be supplied by the solidifying wax: $Q = mL$

$$m = \frac{Q}{L} = \frac{1.98\times10^5\ \text{J}}{205\times10^3\ \text{J/kg}} = \boxed{0.964\ \text{kg or more}}$$

(b) The test samples and the inner surface of the insulation can be prewarmed to 37°C as the box is assembled. If this is done, nothing changes in temperature during the test period. Then the masses of the test samples and the insulation make no difference.

P20.44 Suppose the pizza is 70 cm in diameter and $\ell = 2.0$ cm thick, sizzling at 100°C. It cannot lose heat by conduction or convection. It radiates according to $\mathcal{P} = \sigma AeT^4$. Here, A is its surface area,

$$A = 2\pi r^2 + 2\pi r\ell = 2\pi(0.35\text{ m})^2 + 2\pi(0.35\text{ m})(0.02\text{ m}) = 0.81\text{ m}^2$$

Suppose it is dark in the infrared, with emissivity about 0.8. Then

$$\mathcal{P} = \left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)\left(0.81\text{ m}^2\right)(0.80)(373\text{ K})^4 = 710\text{ W}\ \boxed{\sim10^3\text{ W}}$$

If the density of the pizza is half that of water, its mass is

$$m = \rho V = \rho\pi r^2\ell = \left(500\text{ kg/m}^3\right)\pi(0.35\text{ m})^2(0.02\text{ m}) = 4\text{ kg}$$

Suppose its specific heat is $c = 0.6\text{ cal/g}\cdot°\text{C}$. The drop in temperature of the pizza is described by

$$Q = mc\left(T_f - T_i\right)$$

$$\mathcal{P} = \frac{dQ}{dt} = mc\frac{dT_f}{dt} - 0$$

$$\frac{dT_f}{dt} = \frac{\mathcal{P}}{mc} = \frac{710\text{ J/s}}{(4\text{ kg})(0.6\cdot4\,186\text{ J/kg}\cdot°\text{C})} = 0.07\ °\text{C/s}\ \boxed{\sim10^{-1}\text{ K/s}}$$

P20.45 $\mathcal{P} = \sigma AeT^4$

$$2.00\text{ W} = \left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)\left(0.250\times10^{-6}\text{ m}^2\right)(0.950)T^4$$

$$T = \left(1.49\times10^{14}\text{ K}^4\right)^{1/4} = \boxed{3.49\times10^3\text{ K}}$$

P20.46 We suppose the earth below is an insulator. The square meter must radiate in the infrared as much energy as it absorbs, $\mathcal{P} = \sigma AeT^4$. Assuming that $e = 1.00$ for blackbody blacktop:

$$1\,000\text{ W} = \left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)\left(1.00\text{ m}^2\right)(1.00)T^4$$

$$T = \left(1.76\times10^{10}\text{ K}^4\right)^{1/4} = \boxed{364\text{ K}}\ \text{(You can cook an egg on it.)}$$

***P20.47** Intensity is defined as power per area perpendicular to the direction of energy flow. The direction of sunlight is along the line from the sun to the object. The perpendicular area is the projected flat circular area enclosed by the *terminator*—the line that separates day from night on the object. The object radiates infrared light outward in all directions. The area perpendicular to this energy flow is its spherical surface area.

The sphere of radius R absorbs sunlight over area πR^2. It radiates over area $4\pi R^2$. Then, in steady state,

$$\mathcal{P}_{\text{in}} = \mathcal{P}_{\text{out}}$$

$$e\left(1\,370\text{ W/m}^2\right)\pi R^2 = e\sigma\left(4\pi R^2\right)T^4$$

The emissivity e, the radius R, and π all cancel.

Therefore,

$$T = \left[\frac{1\,370\text{ W/m}^2}{4\left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)}\right]^{1/4} = \boxed{279\text{ K}} = 6°\text{C}$$

It is chilly, well below room temperatures we find comfortable.

***P20.48** (a) Because the bulb is evacuated, the filament loses energy by radiation but not by convection; we ignore energy loss by conduction. From Stefan's law, the power ratio is

$$e\sigma AT_h^4/e\sigma AT_c^4 = (2373/2273)^4 = \boxed{1.19}$$

(b) The radiating area is the lateral surface area of the cylindrical filament, $2\pi r\ell$. Now we want

$$e\sigma 2\pi r_h \ell T_h^4 = e\sigma 2\pi r_c \ell T_c^4 \qquad \text{so} \qquad r_c/r_h = \boxed{1.19}$$

Additional Problems

P20.49 The increase in internal energy required to melt 1.00 kg of snow is

$$\Delta E_{\text{int}} = (1.00 \text{ kg})(3.33\times 10^5 \text{ J/kg}) = 3.33\times 10^5 \text{ J}$$

The force of friction is

$$f = \mu n = \mu mg = 0.200(75.0 \text{ kg})(9.80 \text{ m/s}^2) = 147 \text{ N}$$

According to the problem statement, the loss of mechanical energy of the skier is assumed to be equal to the increase in internal energy of the snow. This increase in internal energy is

$$\Delta E_{\text{int}} = f\Delta r = (147 \text{ N})\Delta r = 3.33\times 10^5 \text{ J}$$

and

$$\Delta r = \boxed{2.27\times 10^3 \text{ m}}$$

P20.50 (a) The energy thus far gained by the copper equals the energy loss by the silver. Your down parka is an excellent insulator.

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

or $$m_{\text{Cu}}c_{\text{Cu}}\left(T_f - T_i\right)_{\text{Cu}} = -m_{\text{Ag}}c_{\text{Ag}}\left(T_f - T_i\right)_{\text{Ag}}$$

$$(9.00 \text{ g})(387 \text{ J/kg}\cdot{}^\circ\text{C})(16.0^\circ\text{C}) = -(14.0 \text{ g})(234 \text{ J/kg}\cdot{}^\circ\text{C})\left(T_f - 30.0^\circ\text{C}\right)_{\text{Ag}}$$

$$\left(T_f - 30.0^\circ\text{C}\right)_{\text{Ag}} = -17.0^\circ\text{C}$$

so $$T_{f,\text{Ag}} = \boxed{13.0^\circ\text{C}}$$

(b) Differentiating the energy gain-and-loss equation gives: $m_{\text{Ag}}c_{\text{Ag}}\left(\frac{dT}{dt}\right)_{\text{Ag}} = -m_{\text{Cu}}c_{\text{Cu}}\left(\frac{dT}{dt}\right)_{\text{Cu}}$

$$\left(\frac{dT}{dt}\right)_{\text{Ag}} = -\frac{m_{\text{Cu}}c_{\text{Cu}}}{m_{\text{Ag}}c_{\text{Ag}}}\left(\frac{dT}{dt}\right)_{\text{Cu}} = -\frac{9.00 \text{ g}(387 \text{ J/kg}\cdot{}^\circ\text{C})}{14.0 \text{ g}(234 \text{ J/kg}\cdot{}^\circ\text{C})}(+0.500\,^\circ\text{C/s})$$

$$\left(\frac{dT}{dt}\right)_{\text{Ag}} = \boxed{-0.532\,^\circ\text{C/s}} \quad (\text{negative sign} \Rightarrow \text{decreasing temperature})$$

P20.51 (a) Before conduction has time to become important, the energy lost by the rod equals the energy gained by the helium. Therefore,

$$(mL_v)_{\text{He}} = (mc|\Delta T|)_{\text{Al}}$$

or $$(\rho V L_v)_{\text{He}} = (\rho V c|\Delta T|)_{\text{Al}}$$

so $$V_{\text{He}} = \frac{(\rho V c|\Delta T|)_{\text{Al}}}{(\rho L_v)_{\text{He}}}$$

$$V_{\text{He}} = \frac{(2.70\ \text{g/cm}^3)(62.5\ \text{cm}^3)(0.210\ \text{cal/g}\cdot{}^\circ\text{C})(295.8^\circ\text{C})}{(0.125\ \text{g/cm}^3)(2.09\times10^4\ \text{J/kg})(1.00\ \text{cal/4.186 J})(1.00\ \text{kg/1 000 g})}$$

$$V_{\text{He}} = 1.68\times10^4\ \text{cm}^3 = \boxed{16.8\ \text{liters}}$$

(b) The rate at which energy is supplied to the rod in order to maintain constant temperatures is given by

$$\mathcal{P} = kA\left(\frac{dT}{dx}\right) = (31.0\ \text{J/s}\cdot\text{cm}\cdot\text{K})(2.50\ \text{cm}^2)\left(\frac{295.8\ \text{K}}{25.0\ \text{cm}}\right) = 917\ \text{W}$$

This power supplied to the helium will produce a "boil-off" rate of

$$\frac{\mathcal{P}}{\rho L_v} = \frac{(917\ \text{W})(10^3\ \text{g/kg})}{(0.125\ \text{g/cm}^3)(2.09\times10^4\ \text{J/kg})} = 351\ \text{cm}^3/\text{s} = \boxed{0.351\ \text{L/s}}$$

***P20.52** (a) Work done by the gas is the negative of the area under the PV curve $W = -P_i\left(\frac{V_i}{2} - V_i\right) = \boxed{+\frac{P_iV_i}{2}}$

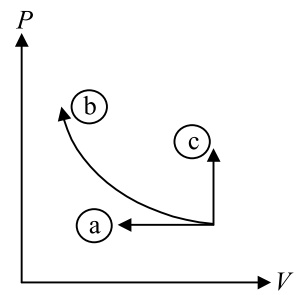


FIG. P20.52

Put the cylinder into a refrigerator at absolute temperature $T_i/2$. Let the piston move freely as the gas cools.

(b) In this case the area under the curve is $W = -\int P\,dV$. Since the process is isothermal,

$$PV = P_iV_i = 4P_i\left(\frac{V_i}{4}\right) = nRT_i$$

and

$$W = -\int_{V_i}^{V_i/4}\left(\frac{dV}{V}\right)(P_iV_i) = -P_iV_i\ln\left(\frac{V_i/4}{V_i}\right) = P_iV_i\ln 4$$

$$= \boxed{+1.39P_iV_i}$$

With the gas in a constant-temperature bath at T_i, slowly push the piston in.

(c) The area under the curve is 0 and $\boxed{W = 0}$.

Lock the piston in place and hold the cylinder over a hotplate at $3T_i$.

The student may be confused that the integral in part (c) is not explicitly covered in calculus class. Mathematicians ordinarily study integrals of functions, but the pressure is not a single-valued function of volume in a isovolumetric process. Our physics idea of an integral is more general. It still corresponds to the idea of area under the graph line.

P20.53 $Q = mc\Delta T = (\rho V)c\Delta T$ so that when a constant temperature difference ΔT is maintained, the rate of adding energy to the liquid is $\mathcal{P} = \frac{dQ}{dt} = \rho\left(\frac{dV}{dt}\right)c\Delta T = \rho Rc\Delta T$ and the specific heat of the liquid is $c = \boxed{\frac{\mathcal{P}}{\rho R\Delta T}}$

P20.54 The initial moment of inertia of the disk is

$$\frac{1}{2}MR^2 = \frac{1}{2}\rho VR^2 = \frac{1}{2}\rho\pi R^2 tR^2 = \frac{1}{2}\left(8\,920\ \text{kg/m}^3\right)\pi(28\ \text{m})^4\,1.2\ \text{m} = 1.033\times10^{10}\ \text{kg}\cdot\text{m}^2$$

The rotation speeds up as the disk cools off, according to

$$I_i\omega_i = I_f\omega_f$$

$$\frac{1}{2}MR_i^2\omega_i = \frac{1}{2}MR_f^2\omega_f = \frac{1}{2}MR_i^2\left(1-\alpha|\Delta T|\right)^2\omega_f$$

$$\omega_f = \omega_i\frac{1}{\left(1-\alpha|\Delta T|\right)^2} = 25\ \text{rad/s}\frac{1}{\left[1-\left(17\times10^{-6}\ 1/°\text{C}\right)830°\text{C}\right]^2} = 25.720\,7\ \text{rad/s}$$

(a) The kinetic energy increases by

$$\frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}I_i\omega_i\omega_f - \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}I_i\omega_i\left(\omega_f-\omega_i\right)$$

$$= \frac{1}{2}1.033\times10^{10}\ \text{kg}\cdot\text{m}^2(25\ \text{rad/s})0.720\,7\ \text{rad/s} = \boxed{9.31\times10^{10}\ \text{J}}$$

(b) $\Delta E_{\text{int}} = mc\Delta T = 2.64\times10^7\ \text{kg}(387\ \text{J/kg}\cdot°\text{C})(20°\text{C}-850°\text{C}) = \boxed{-8.47\times10^{12}\ \text{J}}$

(c) As 8.47×10^{12} J leaves the fund of internal energy, 9.31×10^{10} J changes into extra kinetic energy, and the rest, $\boxed{8.38\times10^{12}\ \text{J}}$ is radiated.

P20.55 The loss of mechanical energy is

$$\frac{1}{2}mv_i^2 + \frac{GM_Em}{R_E} = \frac{1}{2}670\ \text{kg}\left(1.4\times10^4\ \text{m/s}\right)^2 + \frac{6.67\times10^{-11}\ \text{Nm}^2\ 5.98\times10^{24}\ \text{kg}\ 670\ \text{kg}}{\text{kg}^2\ 6.37\times10^6\ \text{m}}$$

$$= 6.57\times10^{10}\ \text{J} + 4.20\times10^{10}\ \text{J} = 1.08\times10^{11}\ \text{J}$$

One half becomes extra internal energy in the aluminum: $\Delta E_{\text{int}} = 5.38\times10^{10}$ J. To raise its temperature to the melting point requires energy

$$mc\Delta T = 670\ \text{kg}\ 900\frac{\text{J}}{\text{kg°C}}\left(660-(-15°\text{C})\right) = 4.07\times10^8\ \text{J}$$

To melt it, $mL = 670\ \text{kg}\ 3.97\times10^5\ \text{J/kg} = 2.66\times10^8\ \text{J}$

To raise it to the boiling point, $mc\Delta T = 670(1\,170)(2\,450-660)\text{J} = 1.40\times10^9\ \text{J}$

To boil it, $mL = 670\ \text{kg}\ 1.14\times10^7\ \text{J/kg} = 7.64\times10^9\ \text{J}$

Then

$$5.38\times10^{10}\ \text{J} = 9.71\times10^9\ \text{J} + 670(1\,170)\left(T_f - 2\,450°\text{C}\right)\text{J/°C}$$

$$T_f = \boxed{5.87\times10^4\ °\text{C}}$$

P20.56 From $Q = mL_V$ the rate of boiling is described by

$$\mathcal{P} = \frac{Q}{\Delta t} = \frac{L_V m}{\Delta t} \qquad \therefore \frac{m}{\Delta t} = \frac{\mathcal{P}}{L_V}$$

Model the water vapor as an ideal gas

$$P_0 V_0 = nRT = \left(\frac{m}{M}\right) RT$$

$$\frac{P_0 V}{\Delta t} = \frac{m}{\Delta t}\left(\frac{RT}{M}\right)$$

$$P_0 A v = \frac{\mathcal{P}}{L_V}\left(\frac{RT}{M}\right)$$

$$v = \frac{\mathcal{P} RT}{M L_V P_0 A} = \frac{1\,000\text{ W}(8.314\text{ J/mol}\cdot\text{K})(373\text{ K})}{(0.018\,0\text{ kg/mol})(2.26\times10^{6}\text{ J/kg})(1.013\times10^{5}\text{ N/m}^2)(2.00\times10^{-4}\text{ m}^2)}$$

$$v = \boxed{3.76\text{ m/s}}$$

P20.57 The power incident on the solar collector is

$$\mathcal{P}_i = IA = (600\text{ W/m}^2)\left[\pi(0.300\text{ m})^2\right] = 170\text{ W}$$

For a 40.0% reflector, the collected power is $\mathcal{P}_c = 67.9$ W. The total energy required to increase the temperature of the water to the boiling point and to evaporate it is $Q = cm\Delta T + mL_V$:

$$Q = 0.500\text{ kg}\left[(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(80.0^\circ\text{C}) + 2.26\times10^{6}\text{ J/kg}\right]$$
$$= 1.30\times10^{6}\text{ J}$$

The time interval required is

$$\Delta t = \frac{Q}{\mathcal{P}_c} = \frac{1.30\times10^{6}\text{ J}}{67.9\text{ W}} = \boxed{5.31\text{ h}}$$

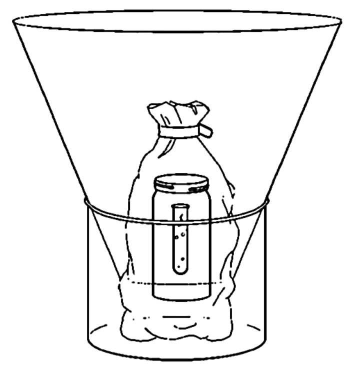


FIG. P20.57

P20.58 (a) The block starts with $K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(1.60\text{ kg})(2.50\text{ m/s})^2 = 5.00\text{ J}$

All this becomes extra internal energy in ice, melting some according to "Q" $= m_{\text{ice}} L_f$

Thus, the mass of ice that melts is

$$m_{\text{ice}} = \frac{\text{“}Q\text{”}}{L_f} = \frac{K_i}{L_f} = \frac{5.00\text{ J}}{3.33\times10^{5}\text{ J/kg}} = 1.50\times10^{-5}\text{ kg} = \boxed{15.0\text{ mg}}$$

For the block:	$Q = 0$ (no energy flows by heat since there is no temperature difference)
	$W = -5.00$ J
	$\Delta E_{\text{int}} = 0$ (no temperature change)
and	$\Delta K = -5.00$ J
For the ice,	$Q = 0$
	$W = +5.00$ J
	$\Delta E_{\text{int}} = +5.00$ J
and	$\Delta K = 0$

continued on next page

(b) Again, $K_i = 5.00\text{ J}$ and $m_{\text{ice}} = \boxed{15.0\text{ mg}}$

For the block of ice:	$Q = 0;\ \Delta E_{\text{int}} = +5.00\text{ J};\ \Delta K = -5.00\text{ J}$
so	$W = 0$
For the copper, nothing happens:	$Q = \Delta E_{\text{int}} = \Delta K = W = 0$

(c) Again, $K_i = 5.00\text{ J}$. Both blocks must rise equally in temperature.

"Q" $= mc\Delta T$: $\quad \Delta T = \dfrac{\text{“}Q\text{”}}{mc} = \dfrac{5.00\text{ J}}{2(1.60\text{ kg})(387\text{ J/kg}\cdot{}^\circ\text{C})} = \boxed{4.04\times10^{-3}\,{}^\circ\text{C}}$

At any instant, the two blocks are at the same temperature, so for both $Q = 0$.

For the moving block:	$\Delta K = -5.00\text{ J}$
and	$\Delta E_{\text{int}} = +2.50\text{ J}$
so	$W = -2.50\text{ J}$
For the stationary block:	$\Delta K = 0$
and	$\Delta E_{\text{int}} = +2.50\text{ J}$
so	$W = +2.50\text{ J}$

For each object in each situation, the general continuity equation for energy, in the form $\Delta K + \Delta E_{\text{int}} = W + Q$, correctly describes the relationship between energy transfers and changes in the object's energy content.

P20.59 Energy goes in at a constant rate $\mathcal{P}$. For the period from

50.0 min to 60.0 min, $Q = mc\Delta T$

$$\mathcal{P}(10.0\text{ min}) = (10\text{ kg} + m_i)(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(2.00^\circ\text{C} - 0^\circ\text{C})$$

$$\mathcal{P}(10.0\text{ min}) = 83.7\text{ kJ} + (8.37\text{ kJ/kg})m_i \qquad (1)$$

For the period from 0 to 50.0 min, $Q = m_i L_f$

$$\mathcal{P}(50.0\text{ min}) = m_i(3.33\times10^5\text{ J/kg})$$

Substitute $\mathcal{P} = \dfrac{m_i(3.33\times10^5\text{ J/kg})}{50.0\text{ min}}$ into Equation (1) to find

$$\frac{m_i(3.33\times10^5\text{ J/kg})}{5.00} = 83.7\text{ kJ} + (8.37\text{ kJ/kg})m_i$$

$$m_i = \frac{83.7\text{ kJ}}{(66.6 - 8.37)\text{ kJ/kg}} = \boxed{1.44\text{ kg}}$$

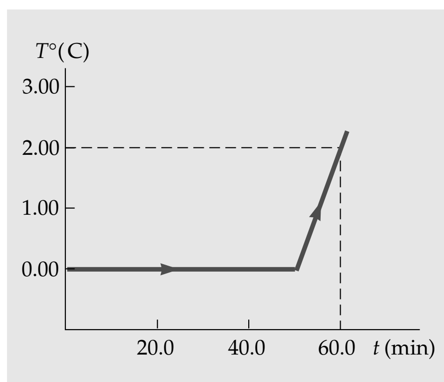


FIG. P20.59

P20.60 $\frac{L\rho A dx}{dt} = kA\left(\frac{\Delta T}{x}\right)$

$$L\rho \int_{4.00}^{8.00} x dx = k\Delta T \int_{0}^{\Delta t} dt$$

$$L\rho \left.\frac{x^2}{2}\right|_{4.00}^{8.00} = k\Delta T \Delta t$$

$$(3.33\times 10^5 \text{ J/kg})(917 \text{ kg/m}^3)\left(\frac{(0.080\,0 \text{ m})^2-(0.040\,0 \text{ m})^2}{2}\right) = (2.00 \text{ W/m}\cdot{}^\circ\text{C})(10.0^\circ\text{C})\Delta t$$

$$\Delta t = 3.66\times 10^4 \text{ s} = \boxed{10.2 \text{ h}}$$

P20.61 $A = A_{\text{end walls}} + A_{\text{ends of attic}} + A_{\text{side walls}} + A_{\text{roof}}$

$$A = 2(8.00 \text{ m}\times 5.00 \text{ m}) + 2\left[2\times\frac{1}{2}\times 4.00 \text{ m}\times(4.00 \text{ m})\tan 37.0^\circ\right]$$

$$+ 2(10.0 \text{ m}\times 5.00 \text{ m}) + 2(10.0 \text{ m})\left(\frac{4.00 \text{ m}}{\cos 37.0^\circ}\right)$$

$$A = 304 \text{ m}^2$$

$$\mathcal{P} = \frac{kA\Delta T}{L} = \frac{(4.80\times 10^{-4} \text{ kW/m}\cdot{}^\circ\text{C})(304 \text{ m}^2)(25.0^\circ\text{C})}{0.210 \text{ m}} = 17.4 \text{ kW} = 4.15 \text{ kcal/s}$$

Thus, the energy lost per day by heat is $(4.15 \text{ kcal/s})(86\,400 \text{ s}) = 3.59\times 10^5 \text{ kcal/day}$.

The gas needed to replace this loss is $\frac{3.59\times 10^5 \text{ kcal/day}}{9\,300 \text{ kcal/m}^3} = \boxed{38.6 \text{ m}^3\text{/day}}$.

P20.62 See the diagram appearing with the next problem. For a cylindrical shell of radius r, height L, and thickness dr, the equation for thermal conduction,

$$\frac{dQ}{dt} = -kA\frac{dT}{dx} \qquad \text{becomes} \qquad \frac{dQ}{dt} = -k(2\pi rL)\frac{dT}{dr}$$

Under equilibrium conditions, $\frac{dQ}{dt}$ is constant; therefore,

$$dT = -\frac{dQ}{dt}\left(\frac{1}{2\pi kL}\right)\left(\frac{dr}{r}\right) \qquad \text{and} \qquad \int_{T_a}^{T_b} dT = -\frac{dQ}{dt}\left(\frac{1}{2\pi kL}\right)\int_a^b \frac{dr}{r}$$

$$T_b - T_a = -\frac{dQ}{dt}\left(\frac{1}{2\pi kL}\right)\ln\left(\frac{b}{a}\right)$$

But $T_a > T_b$, so $\boxed{\frac{dQ}{dt} = \frac{2\pi kL(T_a - T_b)}{\ln(b/a)}}$

P20.63 From the previous problem, the rate of energy flow through the wall is

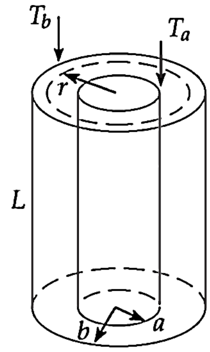


FIG. P20.63

$$\frac{dQ}{dt}=\frac{2\pi kL\left(T_a-T_b\right)}{\ln(b/a)}$$

$$\frac{dQ}{dt}=\frac{2\pi\left(4.00\times10^{-5}\ \text{cal/s}\cdot\text{cm}\cdot{}^\circ\text{C}\right)(3\,500\ \text{cm})(60.0^\circ\text{C})}{\ln(256\ \text{cm}/250\ \text{cm})}$$

$$\frac{dQ}{dt}=2.23\times10^3\ \text{cal/s}=\boxed{9.32\ \text{kW}}$$

This is the rate of energy loss from the plane by heat, and consequently is the rate at which energy must be supplied in order to maintain a constant temperature.

***P20.64** $Q_{\text{cold}}=-Q_{\text{hot}}$

or $\quad Q_{\text{Al}}=-\left(Q_{\text{water}}+Q_{\text{calo}}\right)$

$$m_{\text{Al}}c_{\text{Al}}\left(T_f-T_i\right)_{\text{Al}}=-\left(m_wc_w+m_cc_c\right)\left(T_f-T_i\right)_w$$

$$(0.200\ \text{kg})c_{\text{Al}}(+39.3^\circ\text{C})=-\big[0.400\ \text{kg}\left(4\,186\ \text{J/kg}\cdot{}^\circ\text{C}\right)$$
$$+0.040\,0\ \text{kg}\left(630\ \text{J/kg}\cdot{}^\circ\text{C}\right)\big]\ (-3.70^\circ\text{C})$$

$$c_{\text{Al}}=\frac{6.29\times10^3\ \text{J}}{7.86\ \text{kg}\cdot{}^\circ\text{C}}=\boxed{800\ \text{J/kg}\cdot{}^\circ\text{C}}$$

This differs from the tabulated value by (900–800)/900 =11%, so the values agree within 15%.

***P20.65** (a) If the energy flowing by heat through one spherical surface within the shell were different from the energy flowing through another sphere, the temperature would be changing at a radius between the layers, so the steady state would not yet be established.

For a spherical shell of radius r and thickness dr, the equation for thermal conduction,

$$\frac{dQ}{dt}=-kA\frac{dT}{dx},\qquad\text{becomes}\qquad\left|\frac{dQ}{dt}\right|=\mathcal{P}=k\left(4\pi r^2\right)\frac{dT}{dr}\qquad\text{so}\qquad\frac{dT}{dr}=\frac{\mathcal{P}}{k4\pi r^2}$$

(b) We separate the variables T and r and integrate from the interior to the exterior of the shell:

$$\int_{5^\circ\text{C}}^{40^\circ\text{C}}dT=\frac{\mathcal{P}}{4\pi k}\int_{3\text{cm}}^{7\text{cm}}\frac{dr}{r^2}$$

(c) $$T\Big|_{5^\circ\text{C}}^{40^\circ\text{C}}=\frac{\mathcal{P}}{4\pi k}\frac{r^{-1}}{-1}\Bigg|_{3\text{cm}}^{7\text{cm}}\qquad 40^\circ\text{C}-5^\circ\text{C}=\frac{\mathcal{P}}{4\pi(0.8\ \text{W/m}^\circ\text{C})}\left(-\frac{1}{7\ \text{cm}}+\frac{1}{3\ \text{cm}}\right)$$

$\mathcal{P}$= 35°C(4π)(0.8 W/100 cm·°C)/(0.190/cm) = $\boxed{18.5\ \text{W}}$

(d) With $\mathcal{P}$ now known, we take the equation from part (a), separate the variables again and integrate between a point on the interior surface and any point within the shell.

$$\int_{5^\circ\text{C}}^{T}dT=\frac{\mathcal{P}}{4\pi k}\int_{3\text{cm}}^{r}\frac{dr}{r^2}$$

(e) $$T\Big|_{5^\circ\text{C}}^{T}=\frac{\mathcal{P}}{4\pi k}\frac{r^{-1}}{-1}\Bigg|_{3\text{cm}}^{r}\qquad T-5^\circ\text{C}=\frac{18.5\ \text{W}}{4\pi(0.8\ \text{W/m}^\circ\text{C})}\left(-\frac{1}{r}+\frac{1}{3\ \text{cm}}\right)$$

$\boxed{T = 5^\circ\text{C} + 184\ \text{cm}\cdot{}^\circ\text{C}\ (1/3\ \text{cm} - 1/r)}$

(f) T = 5°C + 184 cm · °C (1/3 cm − 1/5 cm) = $\boxed{29.5^\circ\text{C}}$

***P20.66** (a) $\mathcal{P}=\sigma AeT^4=\left(5.67\times10^{-8}\ \text{W/m}^2\text{K}^4\right)5.1\times10^{14}\ \text{m}^2\ (0.965)\left(5\ 800\ \text{K}\right)^4=\boxed{3.16\times10^{22}\ \text{W}}$

(b) $T_{\text{avg}}=0.1\left(4\ 800\ \text{K}\right)+0.9\left(5\ 890\ \text{K}\right)=\boxed{5.78\times10^3\ \text{K}}$

$\boxed{\text{This is cooler than 5 800 K by } \frac{5\ 800-5\ 781}{5\ 800}=0.328\%}$

(c) $\mathcal{P}=\left(5.67\times10^{-8}\ \text{W/m}^2\text{K}^4\right)0.1\left(5.1\times10^{14}\ \text{m}^2\right)0.965\left(4\ 800\ \text{K}\right)^4$

$+\ 5.67\times10^{-8}\ \text{W}\ 0.9\left(5.1\times10^{14}\right)0.965\left(5\ 890\right)^4=\boxed{3.17\times10^{22}\ \text{W}}$

$\boxed{\text{This is larger than } 3.158\times10^{22}\text{ W by } \frac{1.29\times10^{20}\text{ W}}{3.16\times10^{22}\text{ W}}=0.408\%}$

ANSWERS TO EVEN PROBLEMS

P20.2 0.105°C

P20.4 The energy input to the water is 6.70 times larger than the laser output of 40.0 kJ.

P20.6 88.2 W

P20.8 (a) 16.1°C (b) 16.1°C (c) It makes no difference whether the drill bit is dull or sharp, or how far into the block it cuts. The answers to (a) and (b) are the same because work cannot be a final form of energy: all of the work done by the bit constitutes energy being transferred into the internal energy of the steel.

P20.10 (a) 25.8°C (b) The final temperature does not depend on the mass. Both the change in potential energy and the heat that would be required (from a stove to produce the change in temperature) are proportional to the mass; hence, the mass cancels in the energy relation.

P20.12 (a) 380 K (b) 2.04 atm

P20.14 12.9 g

P20.16 (a) all the ice melts; 40.4°C (b) 8.04 g melts; 0°C

P20.18 (a) 7 (b) As the car stops it transfers part of its kinetic energy into the air. As soon as the brakes rise above the air temperature they lose energy by heat, and lose it very fast if they attain a high temperature.

P20.20 (a) Two speeding lead bullets, one of mass 12.0 g moving to the right at 300 m/s and one of mass 8.00 g moving to the left at 400 m/s, collide head-on and all of the material sticks together. Both bullets are originally at temperature 30.0°C. Describe the state of the system immediately thereafter. (b) After the completely inelastic collision, a glob comprising 3.10 g of solid lead and 16.9 g is liquid lead is moving to the right at 20.0 m/s. Its temperature is 327.3°C.

P20.22 (a) −12.0 MJ (b) +12.0 MJ

P20.24 $-nR\left(T_2-T_1\right)$

P20.26 (a) 12.0 kJ (b) −12.0 kJ

P20.28 42.9 kJ

P20.30 (a) 7.65 L (b) 305 K

P20.32 (a) –48.6 mJ (b) 16.2 kJ (c) 16.2 kJ

P20.34 (a) $-4P_iV_i$ (b) $+4P_iV_i$ (c) –9.08 kJ

P20.36 (a) 1 300 J (b) 100 J (c) –900 J (d) –1 400 J

P20.38 1.34 kW

P20.40 (a) $0.890\ \text{ft}^2\cdot{}^\circ\text{F}\cdot\text{h/Btu}$ (b) $1.85\dfrac{\text{ft}^2\cdot{}^\circ\text{F}\cdot\text{h}}{\text{Btu}}$ (c) 2.08

P20.42 3.85×10^{26} W

P20.44 (a) $\sim10^3$ W (b) decreasing at $\sim10^{-1}$ K/s

P20.46 364 K

P20.48 (a) 1.19 (b) 1.19

P20.50 (a) 13.0°C (b) –0.532°C/s

P20.52 (a) $P_iV_i/2$. Put the cylinder into a refrigerator at absolute temperature $T_i/2$. Let the piston move freely as the gas cools. (b) $1.39P_iV_i$. With the gas in a constant-temperature bath at T_i, slowly push the piston in. (c) 0. Lock the piston in place and hold the cylinder over a hotplate at $3T_i$. See the solution.

P20.54 (a) 9.31×10^{10} J (b) -8.47×10^{12} J (c) 8.38×10^{12} J

P20.56 3.76 m/s

P20.58 (a) 15.0 mg. Block: $Q=0$, $W=-5.00$ J, $\Delta E_{\text{int}}=0$, $\Delta K=-5.00$ J. Ice: $Q=0$, $W=5.00$ J, $\Delta E_{\text{int}}=5.00$ J, $\Delta K=0$. (b) 15.0 mg. Block: $Q=0$, $W=0$, $\Delta E_{\text{int}}=5.00$ J, $\Delta K=-5.00$ J. Metal: $Q=0$, $W=0$, $\Delta E_{\text{int}}=0$, $\Delta K=0$. (c) 0.004 04°C. Moving block: $Q=0$, $W=-2.50$ J, $\Delta E_{\text{int}}=2.50$ J, $\Delta K=-5.00$ J. Stationary block: $Q=0$, $W=2.50$ J, $\Delta E_{\text{int}}=2.50$ J, $\Delta K=0$.

P20.60 10.2 h

P20.62 see the solution

P20.64 800 J/kg·°C. This differs from the tabulated value by 11%, so they agree within 15%.

P20.66 (a) 3.16×10^{22} W (b) 5.78×10^3 K, 0.328% less than 5 800 K (c) 3.17×10^{22} W, 0.408% larger

21

The Kinetic Theory of Gases

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q21.1 The molecules of all different kinds collide with the walls of the container, so molecules of all different kinds exert partial pressures that contribute to the total pressure. The molecules can be so small that they collide with one another relatively rarely and each kind exerts partial pressure as if the other kinds of molecules were absent. If the molecules collide with one another often, the collisions exactly conserve momentum and so do not affect the net force on the walls.

Q21.2 The helium must have the higher rms speed. According to Equation (21.4), the gas with the smaller mass per atom must have the higher average speed-squared and thus the higher rms speed.

Q21.3 The alcohol evaporates, absorbing energy from the skin to lower the skin temperature.

***Q21.4** (i) Statements a, d, and e are correct statements that describe the temperature increase of a gas.

(ii) Statement b is true if the molecules have any size at all, but molecular collisions with other molecules have nothing to do with temperature.

(iii) Statement c is incorrect. The molecular collisions are perfectly elastic. Temperature is determined by how fast molecules are moving through space, not by anything going on inside a molecule.

***Q21.5** (i) b. The volume of the balloon will decrease.

(ii) c. The pressure inside the balloon is nearly equal to the constant exterior atmospheric pressure. Snap the mouth of the balloon over an absolute pressure gauge to demonstrate this fact. Then from $PV = nRT$, volume must decrease in proportion to the absolute temperature. Call the process isobaric contraction.

***Q21.6** At 200 K, $\frac{1}{2}m_0 v_{\text{rms}0}^2 = \frac{3}{2}k_B T_0$. At the higher temperature, $\frac{1}{2}m_0\left(2v_{\text{rms}0}\right)^2 = \frac{3}{2}k_B T$
Then $T = 4T_0 = 4(200\text{ K}) = 800\text{ K}$. Answer (d).

***Q21.7** Answer c > a > b > e > d. The average vector velocity is zero in a sample macroscopically at rest. As adjacent equations in the text note, the asymmetric distribution of molecular speeds makes the average speed greater than the most probable speed, and the rms speed greater still. The most probable speed is $(2RT/M)^{1/2}$ and the speed of sound is $(\gamma RT/M)^{1/2}$, necessarily smaller. Sound represents an organized disturbance superposed on the disorganized thermal motion of molecules, and moving at a lower speed.

***Q21.8** Answer (b). The two samples have the same temperature and molecular mass, and so the same rms molecular speed. These are all intrinsic quantities. The volume, number of moles, and sample mass are extrinsic quantities that vary independently, depending on the sample size.

Q21.9 The dry air is more dense. Since the air and the water vapor are at the same temperature, they have the same kinetic energy per molecule. For a controlled experiment, the humid and dry air are at the same pressure, so the number of molecules per unit volume must be the same for both. The water molecule has a smaller molecular mass (18.0 u) than any of the gases that make up the air, so the humid air must have the smaller mass per unit volume.

Q21.10 Suppose the balloon rises into air uniform in temperature. The air cannot be uniform in pressure because the lower layers support the weight of all the air above them. The rubber in a typical balloon is easy to stretch and stretches or contracts until interior and exterior pressures are nearly equal. So as the balloon rises it expands. This is an isothermal expansion, with P decreasing as V increases by the same factor in $PV = nRT$. If the rubber wall is very strong it will eventually contain the helium at higher pressure than the air outside but at the same density, so that the balloon will stop rising. More likely, the rubber will stretch and break, releasing the helium to keep rising and "boil out" of the Earth's atmosphere.

Q21.11 A diatomic gas has more degrees of freedom—those of molecular vibration and rotation—than a monatomic gas. The energy content per mole is proportional to the number of degrees of freedom.

***Q21.12** (i) Answer (b). Average molecular kinetic energy increases by a factor of 3.

(ii) Answer (c). The rms speed increases by a factor of $\sqrt{3}$.

(iii) Answer (c). Average momentum change increases by $\sqrt{3}$.

(iv) Answer (c). Rate of collisions increases by a factor of $\sqrt{3}$ since the mean free path remains unchanged.

(v) Answer (b). Pressure increases by a factor of 3. This is the product of the answers to iii and iv.

Q21.13 As a parcel of air is pushed upward, it moves into a region of lower pressure, so it expands and does work on its surroundings. Its fund of internal energy drops, and so does its temperature. As mentioned in the question, the low thermal conductivity of air means that very little energy will be conducted by heat into the now-cool parcel from the denser but warmer air below it.

***Q21.14** Answer (a), temperature 900 K. The area under the curve represents the number of molecules in the sample, which must be 100 000 as labeled. With a molecular mass larger than that of nitrogen by a factor of 3, and the same speed distribution, krypton will have $(1/2)m_o v^2 = (3/2)k_B T$ average molecular kinetic energy larger by a factor of 3. Then its temperature must be higher by a factor of 3 than that of the sample of nitrogen at 300 K.

SOLUTIONS TO PROBLEMS

Section 21.1 **Molecular Model of an Ideal Gas**

P21.1 Use $1\text{ u} = 1.66\times10^{-24}\text{ g}$

(a) For He, $m_0 = 4.00\text{ u}\left(\dfrac{1.66\times10^{-24}\text{ g}}{1\text{ u}}\right) = \boxed{6.64\times10^{-24}\text{ g}}$

(b) For Fe, $m_0 = 55.9\text{ u}\left(\dfrac{1.66\times10^{-24}\text{ g}}{1\text{ u}}\right) = \boxed{9.29\times10^{-23}\text{ g}}$

(c) For Pb, $m_0 = 207\text{ u}\left(\dfrac{1.66\times10^{-24}\text{ g}}{1\text{ u}}\right) = \boxed{3.44\times10^{-22}\text{ g}}$

***P21.2** Because each mole of a chemical compound contains Avogadro's number of molecules, the number of molecules in a sample is N_A times the number of moles, as described by $N = nN_A$, and the molar mass is N_A times the molecular mass, as described by $M = m_0N_A$. The definition of the molar mass implies that the sample mass is the number of moles times the molar mass, as described by $m = nM$. Then the sample mass must also be the number of molecules times the molecular mass, according to $m = nM = nN_Am_0 = Nm_0$. The equations are true for chemical compounds in solid, liquid, and gaseous phases—this includes elements. We apply the equations also to air by interpreting M as the mass of Avogadro's number of the various molecules in the mixture.

P21.3 $\bar{F} = Nm\dfrac{\Delta v}{\Delta t} = 500\left(5.00\times10^{-3}\text{ kg}\right)\dfrac{\left[8.00\sin45.0° - (-8.00\sin45.0°)\right]\text{ m/s}}{30.0\text{ s}} = \boxed{0.943\text{ N}}$

$P = \dfrac{\bar{F}}{A} = 1.57\text{ N/m}^2 = \boxed{1.57\text{ Pa}}$

P21.4 $\bar{F} = Nm_0\dfrac{\Delta v}{\Delta t} = \dfrac{\left(5.00\times10^{23}\right)\left[\left(4.68\times10^{-26}\text{ kg}\right)2\left(300\text{ m/s}\right)\right]}{1.00\text{ s}} = 14.0\text{ N}$ and

$P = \dfrac{\bar{F}}{A} = \dfrac{14.0\text{ N}}{8.00\times10^{-4}\text{ m}^2} = \boxed{17.6\text{ kPa}}$

***P21.5** $PV = \left(\dfrac{N}{N_A}\right)RT$ and $N = \dfrac{PVN_A}{RT}$ so that

$$N = \frac{\left(1.00\times10^{-10}\right)(133)(1.00)\left(6.02\times10^{23}\right)}{(8.314)(300)} = \boxed{3.21\times10^{12}\text{ molecules}}$$

P21.6 Use the equation describing the kinetic-theory account for pressure: $P = \dfrac{2N}{3V}\left(\dfrac{m_0v^2}{2}\right)$. Then

$$K_{\text{av}} = \frac{m_0v^2}{2} = \frac{3PV}{2N} \text{ where } N = nN_A = 2N_A$$

$$K_{\text{av}} = \frac{3PV}{2\left(2N_A\right)} = \frac{3(8.00\text{ atm})\left(1.013\times10^5\text{ Pa/atm}\right)\left(5.00\times10^{-3}\text{ m}^3\right)}{2(2\text{ mol})\left(6.02\times10^{23}\text{ molecules/mol}\right)}$$

$$K_{\text{av}} = \boxed{5.05\times10^{-21}\text{ J/molecule}}$$

P21.7 $P=\frac{2}{3}\frac{N}{V}\left(\overline{KE}\right)$ from the kinetic-theory account for pressure.

$$N=\frac{3}{2}\frac{PV}{\left(\overline{KE}\right)}=\frac{3}{2}\frac{\left(1.20\times10^{5}\right)\left(4.00\times10^{-3}\right)}{\left(3.60\times10^{-22}\right)}=2.00\times10^{24}\text{ molecules}$$

$$n=\frac{N}{N_A}=\frac{2.00\times10^{24}\text{ molecules}}{6.02\times10^{23}\text{ molecules/mol}}=\boxed{3.32\text{ mol}}$$

P21.8 (a) $PV=nRT=\frac{Nm_0v^2}{3}$

The total translational kinetic energy is $\frac{Nm_0v^2}{2}=E_{\text{trans}}$:

$$E_{\text{trans}}=\frac{3}{2}PV=\frac{3}{2}\left(3.00\times1.013\times10^{5}\right)\left(5.00\times10^{-3}\right)=\boxed{2.28\text{ kJ}}$$

(b) $\frac{m_0v^2}{2}=\frac{3k_BT}{2}=\frac{3RT}{2N_A}=\frac{3(8.314)(300)}{2\left(6.02\times10^{23}\right)}=\boxed{6.21\times10^{-21}\text{ J}}$

P21.9 (a) $PV=Nk_BT$: $\quad N=\frac{PV}{k_BT}=\frac{1.013\times10^{5}\text{ Pa}\left[\frac{4}{3}\pi(0.150\text{ m})^3\right]}{\left(1.38\times10^{-23}\text{ J/K}\right)(293\text{ K})}=\boxed{3.54\times10^{23}\text{ atoms}}$

(b) $\bar{K}=\frac{3}{2}k_BT=\frac{3}{2}\left(1.38\times10^{-23}\right)(293)\text{ J}=\boxed{6.07\times10^{-21}\text{ J}}$

(c) For helium, the atomic mass is $m_0=\frac{4.00\text{ g/mol}}{6.02\times10^{23}\text{ molecules/mol}}=6.64\times10^{-24}\text{ g/molecule}$

$$m_0=6.64\times10^{-27}\text{ kg/molecule}$$

$\frac{1}{2}m_0\overline{v^2}=\frac{3}{2}k_BT$: $\quad\therefore v_{\text{rms}}=\sqrt{\frac{3k_BT}{m_0}}=\boxed{1.35\text{ km/s}}$

P21.10 (a) $1\text{ Pa}=(1\text{ Pa})\left(\frac{1\text{ N/m}^2}{1\text{ Pa}}\right)\left(\frac{1\text{ J}}{1\text{ N}\cdot\text{m}}\right)=\boxed{1\text{ J/m}^3}$

(b) For a monatomic ideal gas, $E_{\text{int}}=\frac{3}{2}nRT$

For any ideal gas, the energy of molecular translation is the same, $E_{\text{trans}}=\frac{3}{2}nRT=\frac{3}{2}PV$

Thus, the energy per volume is $\frac{E_{\text{trans}}}{V}=\boxed{\frac{3}{2}P}$

P21.11 (a) $\bar{K} = \frac{3}{2}k_B T = \frac{3}{2}\left(1.38\times10^{-23}\ \text{J/K}\right)\left(423\ \text{K}\right) = \boxed{8.76\times10^{-21}\ \text{J}}$

(b) $\bar{K} = \frac{1}{2}m_0 v_{rms}^2 = 8.76\times10^{-21}\ \text{J}$

so $$v_{rms} = \sqrt{\frac{1.75\times10^{-20}\ \text{J}}{m}} \quad (1)$$

For helium, $$m_0 = \frac{4.00\ \text{g/mol}}{6.02\times10^{23}\ \text{molecules/mol}} = 6.64\times10^{-24}\ \text{g/molecule}$$

$$m_0 = 6.64\times10^{-27}\ \text{kg/molecule}$$

Similarly for argon, $$m_0 = \frac{39.9\ \text{g/mol}}{6.02\times10^{23}\ \text{molecules/mol}} = 6.63\times10^{-23}\ \text{g/molecule}$$

$$m_0 = 6.63\times10^{-26}\ \text{kg/molecule}$$

Substituting in (1) above, we find for helium, $\boxed{v_{rms} = 1.62\ \text{km/s}}$

and for argon, $\boxed{v_{rms} = 514\ \text{m/s}}$

Section 21.2 **Molar Specific Heat of an Ideal Gas**

P21.12 $n = 1.00$ mol, $T_i = 300$ K

(b) Since $V =$ constant, $W = \boxed{0}$

(a) $\Delta E_{\text{int}} = Q + W = 209\ \text{J} + 0 = \boxed{209\ \text{J}}$

(c) $\Delta E_{\text{int}} = nC_V\Delta T = n\left(\frac{3}{2}R\right)\Delta T$

so $$\Delta T = \frac{2\Delta E_{\text{int}}}{3nR} = \frac{2(209\ \text{J})}{3(1.00\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})} = 16.8\ \text{K}$$

$$T = T_i + \Delta T = 300\ \text{K} + 16.8\ \text{K} = \boxed{317\ \text{K}}$$

P21.13 We use the tabulated values for C_P and C_V

(a) $Q = nC_P\Delta T = 1.00\ \text{mol}\left(28.8\ \text{J/mol}\cdot\text{K}\right)(420-300)\ \text{K} = \boxed{3.46\ \text{kJ}}$

(b) $\Delta E_{\text{int}} = nC_V\Delta T = 1.00\ \text{mol}\left(20.4\ \text{J/mol}\cdot\text{K}\right)(120\ \text{K}) = \boxed{2.45\ \text{kJ}}$

(c) $W = -Q + \Delta E_{\text{int}} = -3.46\ \text{kJ} + 2.45\ \text{kJ} = \boxed{-1.01\ \text{kJ}}$

P21.14 (a) Consider warming it at constant pressure. Oxygen and nitrogen are diatomic, so $C_P = \frac{7R}{2}$

$$Q = nC_P\Delta T = \frac{7}{2}nR\Delta T = \frac{7}{2}\left(\frac{PV}{T}\right)\Delta T$$

$$Q = \frac{7}{2}\frac{\left(1.013\times10^5\ \text{N/m}^2\right)\left(100\ \text{m}^3\right)}{300\ \text{K}}(1.00\ \text{K}) = \boxed{118\ \text{kJ}}$$

(b) $U_g = mgy$

$$m = \frac{U_g}{gy} = \frac{1.18\times10^5\ \text{J}}{\left(9.80\ \text{m/s}^2\right)2.00\ \text{m}} = \boxed{6.03\times10^3\ \text{kg}}$$

P21.15 Consider 800 cm^3 of (flavored) water at 90.0°C mixing with 200 cm^3 of diatomic ideal gas at 20.0°C:

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

or

$$m_{\text{air}}c_{P,\text{ air}}\left(T_f - T_{i,\text{ air}}\right) = -m_w c_w (\Delta T)_w$$

$$(\Delta T)_w = \frac{-m_{\text{air}}c_{P,\text{ air}}\left(T_f - T_{i,\text{ air}}\right)}{m_w c_w} = \frac{-(\rho V)_{\text{air}}\, c_{P,\text{ air}}\,(90.0^\circ\text{C} - 20.0^\circ\text{C})}{(\rho_w V_w)c_w}$$

where we have anticipated that the final temperature of the mixture will be close to 90.0°C.

The molar specific heat of air is $C_{P,\text{ air}} = \frac{7}{2}R$

So the specific heat per gram is $c_{P,\text{ air}} = \frac{7}{2}\left(\frac{R}{M}\right) = \frac{7}{2}(8.314\ \text{J/mol}\cdot\text{K})\left(\frac{1.00\ \text{mol}}{28.9\ \text{g}}\right) = 1.01\ \text{J/g}\cdot{}^\circ\text{C}$

$$(\Delta T)_w = -\frac{\left[\left(1.20\times10^{-3}\ \text{g/cm}^3\right)\left(200\ \text{cm}^3\right)\right](1.01\ \text{J/g}\cdot{}^\circ\text{C})(70.0^\circ\text{C})}{\left[\left(1.00\ \text{g/cm}^3\right)\left(800\ \text{cm}^3\right)\right](4.186\ \text{J/kg}\cdot{}^\circ\text{C})}$$

or

$$(\Delta T)_w \approx -5.05\times10^{-3}\,{}^\circ\text{C}$$

The change of temperature for the water is $\boxed{\text{between } 10^{-3}\,{}^\circ\text{C and } 10^{-2}\,{}^\circ\text{C}}$.

P21.16 (a) $C_V = \frac{5}{2}R = \frac{5}{2}(8.314\ \text{J/mol}\cdot\text{K})\left(\frac{1.00\ \text{mol}}{0.028\,9\ \text{kg}}\right) = 719\ \text{J/kg}\cdot\text{K} = \boxed{0.719\ \text{kJ/kg}\cdot\text{K}}$

(b) $m = Mn = M\left(\frac{PV}{RT}\right)$

$$m = (0.028\,9\ \text{kg/mol})\left(\frac{200\times10^3\ \text{Pa}\left(0.350\ \text{m}^3\right)}{(8.314\ \text{J/mol}\cdot\text{K})(300\ \text{K})}\right) = \boxed{0.811\ \text{kg}}$$

(c) We consider a constant volume process where no work is done.

$$Q = mC_V\Delta T = 0.811\ \text{kg}(0.719\ \text{kJ/kg}\cdot\text{K})(700\ \text{K} - 300\ \text{K}) = \boxed{233\ \text{kJ}}$$

(d) We now consider a constant pressure process where the internal energy of the gas is increased and work is done.

$$Q = mC_P\Delta T = m(C_V + R)\Delta T = m\left(\frac{7R}{2}\right)\Delta T = m\left(\frac{7C_V}{5}\right)\Delta T$$

$$Q = 0.811\ \text{kg}\left[\frac{7}{5}(0.719\ \text{kJ/kg}\cdot\text{K})\right](400\ \text{K}) = \boxed{327\ \text{kJ}}$$

P21.17 $Q = (nC_P\Delta T)_{\text{isobaric}} + (nC_V\Delta T)_{\text{isovolumetric}}$

In the isobaric process, V doubles so T must double, to $2T_i$.

In the isovolumetric process, P triples so T changes from $2T_i$ to $6T_i$.

$$Q = n\left(\frac{7}{2}R\right)(2T_i - T_i) + n\left(\frac{5}{2}R\right)(6T_i - 2T_i) = 13.5nRT_i = \boxed{13.5PV}$$

Section 21.3 **Adiabatic Processes for an Ideal Gas**

P21.18 (a) $P_iV_i^\gamma = P_fV_f^\gamma$ so $\dfrac{V_f}{V_i} = \left(\dfrac{P_i}{P_f}\right)^{1/\gamma} = \left(\dfrac{1.00}{20.0}\right)^{5/7} = \boxed{0.118}$

(b) $\dfrac{T_f}{T_i} = \dfrac{P_fV_f}{P_iV_i} = \left(\dfrac{P_f}{P_i}\right)\left(\dfrac{V_f}{V_i}\right) = (20.0)(0.118)$ $\qquad \dfrac{T_f}{T_i} = \boxed{2.35}$

(c) Since the process is adiabatic, $\boxed{Q = 0}$

Since $\gamma = 1.40 = \dfrac{C_P}{C_V} = \dfrac{R + C_V}{C_V}$, $\qquad C_V = \dfrac{5}{2}R$ and $\Delta T = 2.35T_i - T_i = 1.35T_i$

$$\Delta E_{\text{int}} = nC_V\Delta T = (0.016\,0\text{ mol})\left(\frac{5}{2}\right)(8.314\text{ J/mol}\cdot\text{K})[1.35(300\text{ K})] = \boxed{135\text{ J}}$$

and $\qquad W = -Q + \Delta E_{\text{int}} = 0 + 135\text{ J} = \boxed{+135\text{ J}}$.

P21.19 (a) $P_iV_i^\gamma = P_fV_f^\gamma$

$$P_f = P_i\left(\frac{V_i}{V_f}\right)^\gamma = 5.00\text{ atm}\left(\frac{12.0}{30.0}\right)^{1.40} = \boxed{1.39\text{ atm}}$$

(b) $$T_i = \frac{P_iV_i}{nR} = \frac{5.00(1.013\times10^5\text{ Pa})(12.0\times10^{-3}\text{ m}^3)}{2.00\text{ mol}(8.314\text{ J/mol}\cdot\text{K})} = \boxed{366\text{ K}}$$

$$T_f = \frac{P_fV_f}{nR} = \frac{1.39(1.013\times10^5\text{ Pa})(30.0\times10^{-3}\text{ m}^3)}{2.00\text{ mol}(8.314\text{ J/mol}\cdot\text{K})} = \boxed{253\text{ K}}$$

(c) The process is adiabatic: $\boxed{Q = 0}$

$$\gamma = 1.40 = \frac{C_P}{C_V} = \frac{R + C_V}{C_V},\ C_V = \frac{5}{2}R$$

$$\Delta E_{\text{int}} = nC_V\Delta T = 2.00\text{ mol}\left(\frac{5}{2}(8.314\text{ J/mol}\cdot\text{K})\right)(253\text{ K} - 366\text{ K}) = \boxed{-4.66\text{ kJ}}$$

$$W = \Delta E_{\text{int}} - Q = -4.66\text{ kJ} - 0 = \boxed{-4.66\text{ kJ}}$$

P21.20 $V_i = \pi\left(\frac{2.50\times10^{-2}\ \text{m}}{2}\right)^2 0.500\ \text{m} = 2.45\times10^{-4}\ \text{m}^3$

The quantity of air we find from $P_iV_i = nRT_i$

$$n = \frac{P_iV_i}{RT_i} = \frac{\left(1.013\times10^5\ \text{Pa}\right)\left(2.45\times10^{-4}\ \text{m}^3\right)}{(8.314\ \text{J/mol}\cdot\text{K})(300\ \text{K})}$$

$$n = 9.97\times10^{-3}\ \text{mol}$$

Adiabatic compression: $P_f = 101.3\ \text{kPa} + 800\ \text{kPa} = 901.3\ \text{kPa}$

(a) $P_iV_i^\gamma = P_fV_f^\gamma$

$$V_f = V_i\left(\frac{P_i}{P_f}\right)^{1/\gamma} = 2.45\times10^{-4}\ \text{m}^3\left(\frac{101.3}{901.3}\right)^{5/7}$$

$$V_f = \boxed{5.15\times10^{-5}\ \text{m}^3}$$

(b) $P_fV_f = nRT_f$

$$T_f = T_i\frac{P_fV_f}{P_iV_i} = T_i\frac{P_f}{P_i}\left(\frac{P_i}{P_f}\right)^{1/\gamma} = T_i\left(\frac{P_i}{P_f}\right)^{(1/\gamma-1)}$$

$$T_f = 300\ \text{K}\left(\frac{101.3}{901.3}\right)^{(5/7-1)} = \boxed{560\ \text{K}}$$

(c) The work put into the gas in compressing it is $\Delta E_{\text{int}} = nC_V\Delta T$

$$W = \left(9.97\times10^{-3}\ \text{mol}\right)\frac{5}{2}(8.314\ \text{J/mol}\cdot\text{K})(560-300)\ \text{K}$$

$$W = 53.9\ \text{J}$$

Now imagine this energy being shared with the inner wall as the gas is held at constant volume. The pump wall has outer diameter $25.0\ \text{mm} + 2.00\ \text{mm} + 2.00\ \text{mm} = 29.0\ \text{mm}$, and volume

$\left[\pi\left(14.5\times10^{-3}\ \text{m}\right)^2 - \pi\left(12.5\times10^{-3}\ \text{m}\right)^2\right]4.00\times10^{-2}\ \text{m} = 6.79\times10^{-6}\ \text{m}^3$ and mass

$\rho V = \left(7.86\times10^3\ \text{kg/m}^3\right)\left(6.79\times10^{-6}\ \text{m}^3\right) = 53.3\ \text{g}$

The overall warming process is described by

$$53.9\ \text{J} = nC_V\Delta T + mc\Delta T$$

$$53.9\ \text{J} = \left(9.97\times10^{-3}\ \text{mol}\right)\frac{5}{2}(8.314\ \text{J/mol}\cdot\text{K})\left(T_{ff}-300\ \text{K}\right) + \left(53.3\times10^{-3}\ \text{kg}\right)(448\ \text{J/kg}\cdot\text{K})\left(T_{ff}-300\ \text{K}\right)$$

$$53.9\ \text{J} = \left(0.207\ \text{J/K} + 23.9\ \text{J/K}\right)\left(T_{ff}-300\ \text{K}\right)$$

$$T_{ff} - 300\ \text{K} = \boxed{2.24\ \text{K}}$$

P21.21 $\frac{T_f}{T_i} = \left(\frac{V_i}{V_f}\right)^{\gamma-1} = \left(\frac{1}{2}\right)^{0.400}$

If $T_i = 300\ \text{K}$, then $T_f = \boxed{227\ \text{K}}$.

P21.22 We suppose the air plus burnt gasoline behaves like a diatomic ideal gas. We find its final absolute pressure:

$$21.0 \text{ atm}\left(50.0 \text{ cm}^3\right)^{7/5} = P_f\left(400 \text{ cm}^3\right)^{7/5}$$

$$P_f = 21.0 \text{ atm}\left(\frac{1}{8}\right)^{7/5} = 1.14 \text{ atm}$$

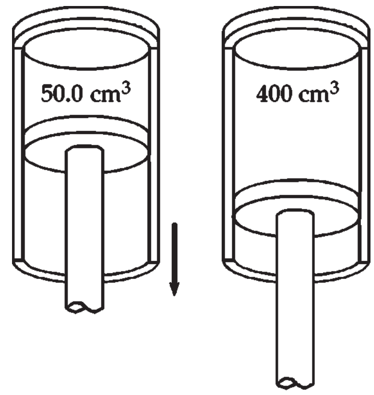


FIG. P21.22

Now $Q = 0$

and $W = \Delta E_{\text{int}} = nC_V\left(T_f - T_i\right)$

$$\therefore W = \frac{5}{2}nRT_f - \frac{5}{2}nRT_i = \frac{5}{2}\left(P_fV_f - P_iV_i\right)$$

$$W = \frac{5}{2}\left[1.14 \text{ atm}\left(400 \text{ cm}^3\right) - 21.0 \text{ atm}\left(50.0 \text{ cm}^3\right)\right]\left(\frac{1.013\times10^5 \text{ N/m}^2}{1 \text{ atm}}\right)\left(10^{-6} \text{ m}^3/\text{cm}^3\right)$$

$$W = -150 \text{ J}$$

The output work is $-W = +150$ J

The time for this stroke is $\frac{1}{4}\left(\frac{1 \text{ min}}{2\,500}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 6.00\times10^{-3}$ s

$$\mathcal{P} = \frac{-W}{\Delta t} = \frac{150 \text{ J}}{6.00\times10^{-3} \text{ s}} = \boxed{25.0 \text{ kW}}$$

P21.23 (a) See the diagram at the right.

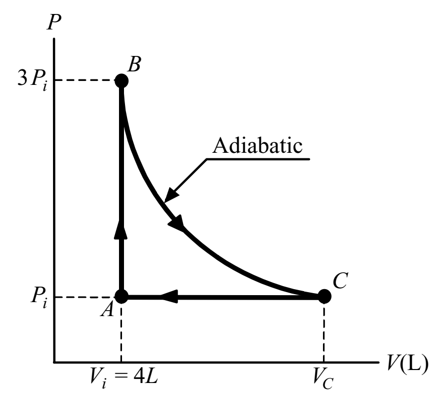


FIG. P21.23

(b) $P_BV_B^\gamma = P_CV_C^\gamma$

$3P_iV_i^\gamma = P_iV_C^\gamma$

$V_C = \left(3^{1/\gamma}\right)V_i = \left(3^{5/7}\right)V_i = 2.19V_i$

$V_C = 2.19(4.00 \text{ L}) = \boxed{8.77 \text{ L}}$

(c) $P_BV_B = nRT_B = 3P_iV_i = 3nRT_i$

$T_B = 3T_i = 3(300 \text{ K}) = \boxed{900 \text{ K}}$

(d) After one whole cycle, $T_A = T_i = \boxed{300 \text{ K}}$.

(e) In AB, $Q_{AB} = nC_V\Delta V = n\left(\frac{5}{2}R\right)\left(3T_i - T_i\right) = (5.00)nRT_i$

$Q_{BC} = 0$ as this process is adiabatic

$$P_CV_C = nRT_C = P_i\left(2.19V_i\right) = (2.19)nRT_i \qquad \text{so} \qquad T_C = 2.19T_i$$

$$Q_{CA} = nC_P\Delta T = n\left(\frac{7}{2}R\right)\left(T_i - 2.19T_i\right) = (-4.17)nRT_i$$

For the whole cycle,

$$Q_{ABCA} = Q_{AB} + Q_{BC} + Q_{CA} = (5.00 - 4.17)nRT_i = (0.829)nRT_i$$

$$\left(\Delta E_{\text{int}}\right)_{ABCA} = 0 = Q_{ABCA} + W_{ABCA}$$

$$W_{ABCA} = -Q_{ABCA} = -(0.829)nRT_i = -(0.829)P_iV_i$$

$$W_{ABCA} = -(0.829)\left(1.013\times10^5 \text{ Pa}\right)\left(4.00\times10^{-3} \text{ m}^3\right) = \boxed{-336 \text{ J}}$$

P21.24 (a) See the diagram at the right.

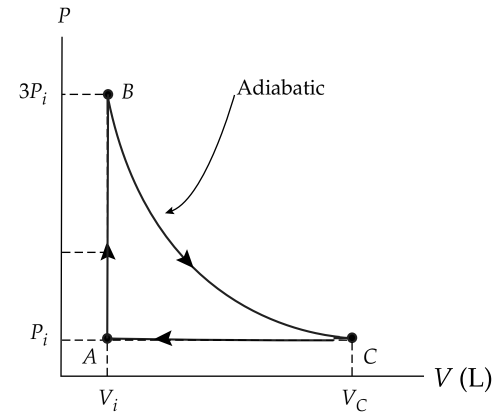


FIG. P21.24

(b) $P_B V_B^\gamma = P_C V_C^\gamma$

$3P_i V_i^\gamma = P_i V_C^\gamma$

$V_C = 3^{1/\gamma} V_i = 3^{5/7} V_i = \boxed{2.19 V_i}$

(c) $P_B V_B = nRT_B = 3P_i V_i = 3nRT_i$

$T_B = \boxed{3T_i}$

(d) After one whole cycle, $T_A = \boxed{T_i}$

(e) In AB, $Q_{AB} = nC_V \Delta T = n\left(\frac{5}{2}R\right)(3T_i - T_i) = (5.00)nRT_i$

$Q_{BC} = 0$ as this process is adiabatic

$P_C V_C = nRT_C = P_i(2.19V_i) = 2.19nRT_i \quad \text{so} \quad T_C = 2.19T_i$

$$Q_{CA} = nC_P \Delta T = n\left(\frac{7}{2}R\right)(T_i - 2.19T_i) = -4.17nRT_i$$

For the whole cycle,

$$Q_{ABCA} = Q_{AB} + Q_{BC} + Q_{CA} = (5.00 - 4.17)nRT_i = 0.830nRT_i$$

$$(\Delta E_{\text{int}})_{ABCA} = 0 = Q_{ABCA} + W_{ABCA}$$

$$W_{ABCA} = -Q_{ABCA} = -0.830nRT_i = \boxed{-0.830P_i V_i}$$

P21.25 (a) The work done *on* the gas is

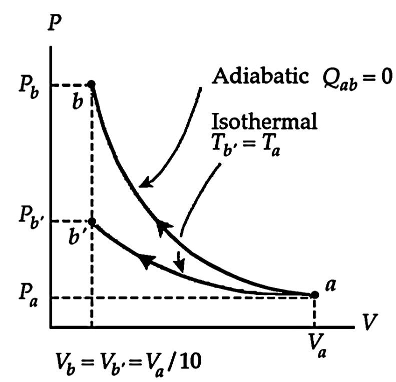


FIG. P21.25

$$W_{ab} = -\int_{V_a}^{V_b} P\,dV$$

For the isothermal process,

$$W_{ab'} = -nRT_a \int_{V_a}^{V_{b'}} \left(\frac{1}{V}\right) dV$$

$$W_{ab'} = -nRT_a \ln\left(\frac{V_{b'}}{V_a}\right) = nRT \ln\left(\frac{V_a}{V_{b'}}\right)$$

Thus,

$$W_{ab'} = 5.00 \text{ mol}(8.314 \text{ J/mol}\cdot\text{K})(293 \text{ K})\ln(10.0)$$

$$W_{ab'} = \boxed{28.0 \text{ kJ}}$$

continued on next page

(b) For the adiabatic process, we must first find the final temperature, T_b. Since air consists primarily of diatomic molecules, we shall use

$$\gamma_{\text{air}} = 1.40 \qquad \text{and} \qquad C_{V,\,\text{air}} = \frac{5R}{2} = \frac{5(8.314)}{2} = 20.8\ \text{J/mol}\cdot\text{K}$$

Then, for the adiabatic process

$$T_b = T_a\left(\frac{V_a}{V_b}\right)^{\gamma-1} = 293\ \text{K}(10.0)^{0.400} = 736\ \text{K}$$

Thus, the work done on the gas during the adiabatic process is

$$W_{ab}\left(-Q+\Delta E_{\text{int}}\right)_{ab} = \left(-0+nC_V\Delta T\right)_{ab} = nC_V\left(T_b-T_a\right)$$

or $$W_{ab} = 5.00\ \text{mol}\left(20.8\ \text{J/mol}\cdot\text{K}\right)(736-293)\ \text{K} = \boxed{46.0\ \text{kJ}}$$

(c) For the isothermal process, we have

$$P_{b'}V_{b'} = P_aV_a$$

Thus

$$P_{b'} = P_a\left(\frac{V_a}{V_{b'}}\right) = 1.00\ \text{atm}(10.0) = \boxed{10.0\ \text{atm}}$$

For the adiabatic process, we have $P_bV_b^\gamma = P_aV_a^\gamma$

Thus

$$P_b = P_a\left(\frac{V_a}{V_b}\right)^{\gamma} = 1.00\ \text{atm}(10.0)^{1.40} = \boxed{25.1\ \text{atm}}$$

Section 21.4 **The Equipartition of Energy**

P21.26 (1) $$E_{\text{int}} = Nf\left(\frac{k_BT}{2}\right) = f\left(\frac{nRT}{2}\right)$$

(2) $$C_V = \frac{1}{n}\left(\frac{dE_{\text{int}}}{dT}\right) = \frac{1}{2}fR$$

(3) $$C_P = C_V + R = \frac{1}{2}(f+2)R$$

(4) $$\gamma = \frac{C_P}{C_V} = \frac{f+2}{f}$$

continued on next page

P21.27 The sample's total heat capacity at constant volume is nC_V. An ideal gas of diatomic molecules has three degrees of freedom for translation in the x, y, and z directions. If we take the y axis along the axis of a molecule, then outside forces cannot excite rotation about this axis, since they have no lever arms. Collisions will set the molecule spinning only about the x and z axes.

(a) If the molecules do not vibrate, they have five degrees of freedom. Random collisions put equal amounts of energy $\frac{1}{2}k_BT$ into all five kinds of motion. The average energy of one molecule is $\frac{5}{2}k_BT$. The internal energy of the two-mole sample is

$$N\left(\frac{5}{2}k_BT\right)=nN_A\left(\frac{5}{2}k_BT\right)=n\left(\frac{5}{2}R\right)T=nC_VT$$

The molar heat capacity is $C_V=\frac{5}{2}R$ and the sample's heat capacity is

$$nC_V=n\left(\frac{5}{2}R\right)=2\text{ mol}\left(\frac{5}{2}(8.314\text{ J/mol}\cdot\text{K})\right)$$

$$\boxed{nC_V=41.6\text{ J/K}}$$

For the heat capacity at constant pressure we have

$$nC_P=n(C_V+R)=n\left(\frac{5}{2}R+R\right)=\frac{7}{2}nR=2\text{ mol}\left(\frac{7}{2}(8.314\text{ J/mol}\cdot\text{K})\right)$$

$$\boxed{nC_P=58.2\text{ J/K}}$$

(b) In vibration with the center of mass fixed, both atoms are always moving in opposite directions with equal speeds. Vibration adds two more degrees of freedom for two more terms in the molecular energy, for kinetic and for elastic potential energy. We have

$$nC_V=n\left(\frac{7}{2}R\right)=\boxed{58.2\text{ J/K}}\qquad\text{and}\qquad nC_P=n\left(\frac{9}{2}R\right)=\boxed{74.8\text{ J/K}}$$

P21.28 Rotational Kinetic Energy $=\frac{1}{2}I\omega^2$

$I=2m_0r^2$, $m_0=35.0\times1.67\times10^{-27}$ kg, $r=10^{-10}$ m

$I=1.17\times10^{-45}\text{ kg}\cdot\text{m}^2\qquad\omega=2.00\times10^{12}\text{ s}^{-1}$

$\therefore K_{\text{rot}}=\frac{1}{2}I\omega^2=\boxed{2.33\times10^{-21}\text{ J}}$

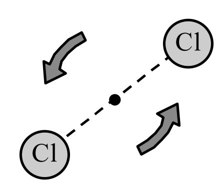


FIG. P21.28

***P21.29** Sulfur dioxide is the gas with the greatest molecular mass of those listed. If the effective spring constants for various chemical bonds are comparable, SO_2 can then be expected to have low frequencies of atomic vibration. Vibration can be excited at lower temperature than for the other gases. Some vibration may be going on at 300 K. With more degrees of freedom for molecular motion, the material has higher specific heat.

***P21.30** (a) The energy of one molecule can be represented as

$(1/2)m_0 v_x^2 + (1/2)m_0 v_y^2 + (1/2)m_0 v_z^2 + (1/2)I\omega_x^2 + (1/2)I\omega_z^2$

Its average value is $(1/2)k_BT + (1/2)k_BT + (1/2)k_BT + (1/2)k_BT + (1/2)k_BT = (5/2)k_BT$

The energy of one mole is obtained by multiplying by Avogadro's number, $E_{int}/n = (5/2)RT$

And the molar heat capacity at constant volume is $E_{int}/nT = \boxed{(5/2)R}$

(b) The energy of one molecule can be represented as

$(1/2)m_0 v_x^2 + (1/2)m_0 v_y^2 + (1/2)m_0 v_z^2 + (1/2)I\omega_x^2 + (1/2)I\omega_z^2 + (1/2)I\omega_y^2$

Its average value is $(1/2)k_BT + (1/2)k_BT + (1/2)k_BT + (1/2)k_BT + (1/2)k_BT + (1/2)k_BT = 3k_BT$

The energy of one mole is obtained by multiplying by Avogadro's number, $E_{int}/n = 3RT$

And the molar heat capacity at constant volume is $E_{int}/nT = \boxed{3R}$

(c) Let the modes of vibration be denoted by 1 and 2. The energy of one molecule can be represented as

$0.5m_0[v_x^2 + v_y^2 + v_z^2] + 0.5I\omega_x^2 + 0.5I\omega_z^2 + [0.5\mu v_{rel}^2 + 0.5kx^2]_1 + [0.5\mu v_{rel}^2 + 0.5kx^2]_2$

Its average value is

$(3/2)k_BT + (1/2)k_BT + (1/2)k_BT + (1/2)k_BT + (1/2)k_BT + (1/2)k_BT + (1/2)k_BT = (9/2)k_BT$

The energy of one mole is obtained by multiplying by Avogadro's number, $E_{int}/n = (9/2)RT$

And the molar heat capacity at constant volume is $E_{int}/nT = \boxed{(9/2)R}$

(d) The energy of one molecule can be represented as

$0.5m_0[v_x^2 + v_y^2 + v_z^2] + 0.5I\omega_x^2 + 0.5I\omega_z^2 + 0.5I\omega_y^2 + [0.5\mu v_{rel}^2 + 0.5kx^2]_1 + [0.5\mu v_{rel}^2 + 0.5kx^2]_2$

Its average value is $(3/2)k_BT + (3/2)k_BT + (1/2)k_BT + (1/2)k_BT + (1/2)k_BT + (1/2)k_BT = (5)k_BT$

The energy of one mole is obtained by multiplying by Avogadro's number, $E_{int}/n = 5RT$

And the molar heat capacity at constant volume is $E_{int}/nT = \boxed{5R}$

(e) Measure the constant-volume specific heat of the gas as a function of temperature and look for plateaus on the graph, as shown in Figure 21.7. If the first jump goes from $\frac{3}{2}R$ to $\frac{5}{2}R$, the molecules can be diagnosed as linear. If the first jump goes from $\frac{3}{2}R$ to $3R$, the molecules must be nonlinear. The tabulated data at one temperature are insufficient for the determination. At room temperature some of the heavier molecules appear to be vibrating.

Section 21.5 **Distribution of Molecular Speeds**

P21.31 (a) $v_{av} = \dfrac{\sum n_i v_i}{N} = \dfrac{1}{15}[1(2)+2(3)+3(5)+4(7)+3(9)+2(12)] = \boxed{6.80 \text{ m/s}}$

(b) $(v^2)_{av} = \dfrac{\sum n_i v_i^2}{N} = 54.9 \text{ m}^2/\text{s}^2$

so $v_{rms} = \sqrt{(v^2)_{av}} = \sqrt{54.9} = \boxed{7.41 \text{ m/s}}$

(c) $v_{mp} = \boxed{7.00 \text{ m/s}}$

P21.32 (a) The ratio of the number at higher energy to the number at lower energy is $e^{-\Delta E/k_BT}$ where ΔE is the energy difference. Here,

$$\Delta E = (10.2 \text{ eV})\left(\frac{1.60\times10^{-19} \text{ J}}{1 \text{ eV}}\right) = 1.63\times10^{-18} \text{ J}$$

and at 0°C,

$$k_BT = \left(1.38\times10^{-23} \text{ J/K}\right)(273 \text{ K}) = 3.77\times10^{-21} \text{ J}$$

Since this is much less than the excitation energy, nearly all the atoms will be in the ground state and the number excited is

$$\left(2.70\times10^{25}\right)\exp\left(\frac{-1.63\times10^{-18} \text{ J}}{3.77\times10^{-21} \text{ J}}\right) = \left(2.70\times10^{25}\right)e^{-433}$$

This number is much less than one, so $\boxed{\text{almost all of the time no atom is excited}}$.

(b) At 10 000°C,

$$k_BT = \left(1.38\times10^{-23} \text{ J/K}\right)10\,273 \text{ K} = 1.42\times10^{-19} \text{ J}$$

The number excited is

$$\left(2.70\times10^{25}\right)\exp\left(\frac{-1.63\times10^{-18} \text{ J}}{1.42\times10^{-19} \text{ J}}\right) = \left(2.70\times10^{25}\right)e^{-11.5} = \boxed{2.70\times10^{20}}$$

P21.33 In the Maxwell Boltzmann speed distribution function take $\dfrac{dN_v}{dv} = 0$ to find

$$4\pi N\left(\frac{m_0}{2\pi k_BT}\right)^{3/2}\exp\left(-\frac{m_0v^2}{2k_BT}\right)\left(2v - \frac{2m_0v^3}{2k_BT}\right) = 0$$

and solve for v to find the most probable speed.

Reject as solutions $v = 0$ and $v = \infty$ They describe minimally probable speeds.

Retain only $2 - \dfrac{m_0v^2}{k_BT} = 0$

Then $v_{\text{mp}} = \sqrt{\dfrac{2k_BT}{m_0}}$

P21.34 (a) $$\frac{V_{\text{rms, 35}}}{V_{\text{rms, 37}}} = \frac{\sqrt{3RT/M_{35}}}{\sqrt{3RT/M_{37}}} = \left(\frac{37.0 \text{ g/mol}}{35.0 \text{ g/mol}}\right)^{1/2} = \boxed{1.03}$$

(b) The lighter atom, $\boxed{^{35}\text{Cl}}$, moves faster.

P21.35 (a) From $v_{\text{av}} = \sqrt{\dfrac{8k_BT}{\pi m_0}}$

we find the temperature as $$T = \frac{\pi\left(6.64\times10^{-27} \text{ kg}\right)\left(1.12\times10^{4} \text{ m/s}\right)^2}{8\left(1.38\times10^{-23} \text{ J/mol}\cdot\text{K}\right)} = \boxed{2.37\times10^{4} \text{ K}}$$

(b) $$T = \frac{\pi\left(6.64\times10^{-27} \text{ kg}\right)\left(2.37\times10^{3} \text{ m/s}\right)^2}{8\left(1.38\times10^{-23} \text{ J/mol}\cdot\text{K}\right)} = \boxed{1.06\times10^{3} \text{ K}}$$

***P21.36** For a molecule of diatomic nitrogen the mass is
$m_0 = M/N_A = (28.0\times10^{-3}\text{ kg/mol})/(6.02\times10^{23}\text{ molecules/mol}) = 4.65\times10^{-26}\text{ kg/molecule}$

(a) $$v_{mp} = \sqrt{\frac{2k_BT}{m_0}} = \sqrt{\frac{2(1.38\times10^{-23}\text{ J/molecule}\cdot\text{K})(900\text{ K})}{4.65\times10^{-26}\text{ kg/molecule}}} = \boxed{731\text{ m/s}}$$

(b) $$v_{avg} = \sqrt{\frac{8k_BT}{\pi m_0}} = \sqrt{\frac{8(1.38\times10^{-23}\text{ J/molecule}\cdot\text{K})(900\text{ K})}{\pi\cdot4.65\times10^{-26}\text{ kg/molecule}}} = \boxed{825\text{ m/s}}$$

(c) $$v_{rms} = \sqrt{\frac{3k_BT}{m_0}} = \sqrt{\frac{3(1.38\times10^{-23}\text{ J/molecule}\cdot\text{K})(900\text{ K})}{4.65\times10^{-26}\text{ kg/molecule}}} = \boxed{895\text{ m/s}}$$

(d) $\boxed{\text{The graph appears to be drawn correctly within about 10 m/s.}}$

P21.37 (a) From the Boltzmann distribution law, the number density of molecules with gravitational energy m_0gy is $n_0e^{-m_0gy/k_BT}$. These are the molecules with height y, so this is the number per volume at height y as a function of y.

(b) $$\frac{n(y)}{n_0} = e^{-m_0gy/k_BT} = e^{-Mgy/N_Ak_BT} = e^{-Mgy/RT}$$
$$= e^{-(28.9\times10^{-3}\text{ kg/mol})(9.8\text{ m/s}^2)(11\times10^3\text{ m})/(8.314\text{ J/mol}\cdot\text{K})(293\text{ K})}$$
$$= e^{-1.279} = \boxed{0.278}$$

P21.38 (a) We calculate

$$\int_0^\infty e^{-m_0gy/k_BT}\,dy = \int_{y=0}^\infty e^{-m_0gy/k_BT}\left(-\frac{m_0g\,dy}{k_BT}\right)\left(-\frac{k_BT}{m_0g}\right)$$
$$= -\frac{k_BT}{m_0g}e^{-m_0gy/k_BT}\Big|_0^\infty = -\frac{k_BT}{m_0g}(0-1) = \frac{k_BT}{m_0g}$$

Using Table B.6 in the appendix,

$$\int_0^\infty ye^{-m_0gy/k_BT}\,dy = \frac{1!}{(m_0g/k_BT)^2} = \left(\frac{k_BT}{m_0g}\right)^2$$

Then

$$\bar{y} = \frac{\int_0^\infty ye^{-m_0gy/k_BT}\,dy}{\int_0^\infty e^{-m_0gy/k_BT}\,dy} = \frac{(k_BT/m_0g)^2}{k_BT/m_0g} = \frac{k_BT}{m_0g}$$

(b) $$\bar{y} = \frac{k_BT}{(M/N_A)g} = \frac{RT}{Mg} = \frac{8.314\text{ J }283\text{ K s}^2}{\text{mol}\cdot\text{K }28.9\times10^{-3}\text{ kg }9.8\text{ m}} = \boxed{8.31\times10^3\text{ m}}$$

Additional Problems

P21.39 (a) $P_f = \boxed{100 \text{ kPa}}$ $T_f = \boxed{400 \text{ K}}$

$$V_f = \frac{nRT_f}{P_f} = \frac{2.00 \text{ mol}(8.314 \text{ J/mol}\cdot\text{K})(400 \text{ K})}{100\times 10^3 \text{ Pa}} = 0.066\,5 \text{ m}^3 = \boxed{66.5 \text{ L}}$$

$$\Delta E_{\text{int}} = (3.50)nR\Delta T = 3.50(2.00 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(100 \text{ K}) = \boxed{5.82 \text{ kJ}}$$

$$W = -P\Delta V = -nR\Delta T = -(2.00 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(100 \text{ K}) = \boxed{-1.66 \text{ kJ}}$$

$$Q = \Delta E_{\text{int}} - W = 5.82 \text{ kJ} + 1.66 \text{ kJ} = \boxed{7.48 \text{ kJ}}$$

(b) $T_f = \boxed{400 \text{ K}}$

$$V_f = V_i = \frac{nRT_i}{P_i} = \frac{2.00 \text{ mol}(8.314 \text{ J/mol}\cdot\text{K})(300 \text{ K})}{100\times 10^3 \text{ Pa}} = 0.049\,9 \text{ m}^3 = \boxed{49.9 \text{ L}}$$

$$P_f = P_i\left(\frac{T_f}{T_i}\right) = 100 \text{ kPa}\left(\frac{400 \text{ K}}{300 \text{ K}}\right) = \boxed{133 \text{ kPa}} \qquad W = -\int P\,dV = \boxed{0} \text{ since } V = \text{constant}$$

$$\Delta E_{\text{int}} = \boxed{5.82 \text{ kJ}} \text{ as in part (a)} \qquad Q = \Delta E_{\text{int}} - W = 5.82 \text{ kJ} - 0 = \boxed{5.82 \text{ kJ}}$$

(c) $P_f = \boxed{120 \text{ kPa}}$ $T_f = \boxed{300 \text{ K}}$

$$V_f = V_i\left(\frac{P_i}{P_f}\right) = 49.9 \text{ L}\left(\frac{100 \text{ kPa}}{120 \text{ kPa}}\right) = \boxed{41.6 \text{ L}} \qquad \Delta E_{\text{int}} = (3.50)nR\Delta T = \boxed{0} \text{ since } T = \text{constant}$$

$$W = -\int P\,dV = -nRT_i\int_{V_i}^{V_f}\frac{dV}{V} = -nRT_i\ln\left(\frac{V_f}{V_i}\right) = -nRT_i\ln\left(\frac{P_i}{P_f}\right)$$

$$W = -(2.00 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(300 \text{ K})\ln\left(\frac{100 \text{ kPa}}{120 \text{ kPa}}\right) = \boxed{+909 \text{ J}}$$

$$Q = \Delta E_{\text{int}} - W = 0 - 909 \text{ J} = \boxed{-909 \text{ J}}$$

(d) $P_f = \boxed{120 \text{ kPa}}$ $\gamma = \dfrac{C_P}{C_V} = \dfrac{C_V + R}{C_V} = \dfrac{3.50R + R}{3.50R} = \dfrac{4.50}{3.50} = \dfrac{9}{7}$

$$P_fV_f^{\gamma} = P_iV_i^{\gamma}: \quad \text{so} \quad V_f = V_i\left(\frac{P_i}{P_f}\right)^{1/\gamma} = 49.9 \text{ L}\left(\frac{100 \text{ kPa}}{120 \text{ kPa}}\right)^{7/9} = \boxed{43.3 \text{ L}}$$

$$T_f = T_i\left(\frac{P_fV_f}{P_iV_i}\right) = 300 \text{ K}\left(\frac{120 \text{ kPa}}{100 \text{ kPa}}\right)\left(\frac{43.3 \text{ L}}{49.9 \text{ L}}\right) = \boxed{312 \text{ K}}$$

$$\Delta E_{\text{int}} = (3.50)nR\Delta T = 3.50(2.00 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(12.4 \text{ K}) = \boxed{722 \text{ J}}$$

$$Q = \boxed{0} \quad (\text{adiabatic process})$$

$$W = -Q + \Delta E_{\text{int}} = 0 + 722 \text{ J} = \boxed{+722 \text{ J}}$$

***P21.40** (a) $n = \frac{PV}{RT} = \frac{(1.013\times10^5\ \text{Pa})(4.20\ \text{m}\times3.00\ \text{m}\times2.50\ \text{m})}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})} = 1.31\times10^3\ \text{mol}$

$N = nN_A = (1.31\times10^3\ \text{mol})(6.02\times10^{23}\ \text{molecules/mol})$

$N = \boxed{7.89\times10^{26}\ \text{molecules}}$

(b) $m = nM = (1.31\times10^3\ \text{mol})(0.028\,9\ \text{kg/mol}) = \boxed{37.9\ \text{kg}}$

(c) $\frac{1}{2}m_0v^2 = \frac{3}{2}k_BT = \frac{3}{2}(1.38\times10^{-23}\ \text{J/k})(293\ \text{K}) = \boxed{6.07\times10^{-21}\ \text{J/molecule}}$

(d) For one molecule,

$m_0 = \frac{M}{N_A} = \frac{0.028\,9\ \text{kg/mol}}{6.02\times10^{23}\ \text{molecules/mol}} = 4.80\times10^{-26}\ \text{kg/molecule}$

$v_{\text{rms}} = \sqrt{\frac{2(6.07\times10^{-21}\ \text{J/molecule})}{4.80\times10^{-26}\ \text{kg/molecule}}} = \boxed{503\ \text{m/s}}$

(e), (f) $E_{\text{int}} = nC_VT = n\left(\frac{5}{2}R\right)T = \frac{5}{2}PV$

$E_{\text{int}} = \frac{5}{2}(1.013\times10^5\ \text{Pa})(31.5\ \text{m}^3) = \boxed{7.98\ \text{MJ}}$

The smaller mass of warmer air at 25°C contains the same internal energy as the cooler air. When the furnace operates, air expands and leaves the room.

***P21.41** For a pure metallic element, one atom is one molecule. Its energy can be represented as $(1/2)m_0v_x^2 + (1/2)m_0v_y^2 + (1/2)m_0v_z^2 + (1/2)k_xx^2 + (1/2)k_yy^2 + (1/2)k_zz^2$

Its average value is $(1/2)k_BT + (1/2)k_BT + (1/2)k_BT + (1/2)k_BT + (1/2)k_BT + (1/2)k_BT = 3k_BT$

The energy of one mole is obtained by multiplying by Avogadro's number, $E_{int}/n = 3RT$

And the molar heat capacity at constant volume is $E_{int}/nT = \boxed{3R}$

(b) $3(8.314\ \text{J/mole}\cdot\text{K}) = 3\times8.314\ \text{J}/[55.845\times10^{-3}\ \text{kg}]\cdot\text{K} = 447\ \text{J/kg}\cdot\text{K} =$

$\boxed{447\ \text{J/kg}\cdot{}^\circ\text{C. This agrees with the tabulated value of 448 J/kg}\cdot{}^\circ\text{C within 0.3\%.}}$

(c) $3(8.314\ \text{J/mole}\cdot\text{K}) = 3\times8.314\ \text{J}/[197\times10^{-3}\ \text{kg}]\cdot\text{K} = 127\ \text{J/kg}\cdot\text{K} =$

$\boxed{127\ \text{J/kg}\cdot{}^\circ\text{C. This agrees with the tabulated value of 129 J/kg}\cdot{}^\circ\text{C within 2\%.}}$

P21.42 (a) The average speed v_{avg} is just the weighted average of all the speeds.

$$v_{\text{avg}} = \frac{[2(v)+3(2v)+5(3v)+4(4v)+3(5v)+2(6v)+1(7v)]}{(2+3+5+4+3+2+1)} = \boxed{3.65v}$$

(b) First find the average of the square of the speeds,

$$(v^2)_{avg} = \frac{\left[2(v)^2+3(2v)^2+5(3v)^2+4(4v)^2+3(5v)^2+2(6v)^2+1(7v)^2\right]}{2+3+5+4+3+2+1} = 15.95v^2$$

The root-mean square speed is then $v_{\text{rms}} = \sqrt{(v_{avg})^2} = \boxed{3.99v}$

(c) The most probable speed is the one that most of the particles have;

i.e., five particles have speed $\boxed{3.00v}$.

continued on next page

(d) $PV = \frac{1}{3}Nm_0v_{\text{av}}^2$

Therefore,

$$P = \frac{20}{3}\frac{\left[m_0(15.95)v^2\right]}{V} = \boxed{106\left(\frac{m_0v^2}{V}\right)}$$

(e) The average kinetic energy for each particle is

$$\bar{K} = \frac{1}{2}m_0v_{\text{av}}^2 = \frac{1}{2}m_0\left(15.95v^2\right) = \boxed{7.98m_0v^2}$$

***P21.43** (a) $PV^\gamma = k$. So $\quad W = -\int_i^f PdV = -k\int_{V_i}^{V_f}\frac{dV}{V^\gamma} = \left.\frac{-kV^{1-\gamma}}{1-\gamma}\right|_{V_i}^{V_f}$

For k we can substitute $P_iV_i^\gamma$ and also $P_fV_f^\gamma$ to have

$$W = -\frac{P_fV_f^\gamma V_f^{1-\gamma} - P_iV_i^\gamma V_i^{1-\gamma}}{1-\gamma} = \frac{P_fV_f - P_iV_i}{\gamma-1}$$

(b) $dE_{\text{int}} = dQ + dW$ and $dQ = 0$ for an adiabatic process.

Therefore,

$$W = +\Delta E_{\text{int}} = nC_V\left(T_f - T_i\right)$$

To show consistency between these two equations, consider that $\gamma = \frac{C_P}{C_V}$ and $C_P - C_V = R$.
Therefore, $\frac{1}{\gamma-1} = \frac{C_V}{R}$
Using this, the result found in part (a) becomes $\quad W = \left(P_fV_f - P_iV_i\right)\frac{C_V}{R}$
Also, for an ideal gas $\frac{PV}{R} = nT$ so that $W = nC_V\left(T_f - T_i\right)$, as found in part (b).

P21.44 (a) $W = nC_V\left(T_f - T_i\right)$

$-2\,500\text{ J} = 1\text{ mol}\frac{3}{2}8.314\text{ J/mol}\cdot\text{K}\left(T_f - 500\text{ K}\right)$

$T_f = \boxed{300\text{ K}}$

(b) $P_iV_i^\gamma = P_fV_f^\gamma$

$P_i\left(\frac{nRT_i}{P_i}\right)^\gamma = P_f\left(\frac{nRT_f}{P_f}\right)^\gamma \qquad T_i^\gamma P_i^{1-\gamma} = T_f^\gamma P_f^{1-\gamma}$

$\frac{T_i^{\gamma/(\gamma-1)}}{P_i} = \frac{T_f^{\gamma/(\gamma-1)}}{P_f} \qquad P_f = P_i\left(\frac{T_f}{T_i}\right)^{\gamma/(\gamma-1)}$

$P_f = P_i\left(\frac{T_f}{T_i}\right)^{(5/3)(3/2)} = 3.60\text{ atm}\left(\frac{300}{500}\right)^{5/2} = \boxed{1.00\text{ atm}}$

P21.45 Let the subscripts '1' and '2' refer to the hot and cold compartments, respectively. The pressure is higher in the hot compartment, therefore the hot compartment expands and the cold compartment contracts. The work done by the adiabatically expanding gas is equal and opposite to the work done by the adiabatically compressed gas.

$$\frac{nR}{\gamma-1}\left(T_{1i}-T_{1f}\right)=-\frac{nR}{\gamma-1}\left(T_{2i}-T_{2f}\right)$$

Therefore

$$T_{1f}+T_{2f}=T_{1i}+T_{2i}=800\text{ K} \tag{1}$$

Consider the adiabatic changes of the gases.

$P_{1i}V_{1i}^{\gamma}=P_{1f}V_{1f}^{\gamma}$ and $P_{2i}V_{2i}^{\gamma}=P_{2f}V_{2f}^{\gamma}$

$\therefore \dfrac{P_{1i}V_{1i}^{\gamma}}{P_{2i}V_{2i}^{\gamma}}=\dfrac{P_{1f}V_{1f}^{\gamma}}{P_{2f}V_{2f}^{\gamma}}$

$\therefore \dfrac{P_{1i}}{P_{2i}}=\left(\dfrac{V_{1f}}{V_{2f}}\right)^{\gamma}$, since $V_{1i}=V_{2i}$ and $P_{1f}=P_{2f}$

$\therefore \dfrac{nRT_{1i}/V_{1i}}{nRT_{2i}/V_{2i}}=\left(\dfrac{nRT_{1f}/P_{1f}}{nRT_{2f}/P_{2f}}\right)^{\gamma}$, using the ideal gas law

$\therefore \dfrac{T_{1i}}{T_{2i}}=\left(\dfrac{T_{1f}}{T_{2f}}\right)^{\gamma}$, since $V_{1i}=V_{2i}$ and $P_{1f}=P_{2f}$

$$\therefore \frac{T_{1f}}{T_{2f}}=\left(\frac{T_{1i}}{T_{2i}}\right)^{1/\gamma}=\left(\frac{550\text{ K}}{250\text{ K}}\right)^{1/1.4}=1.756 \tag{2}$$

Solving equations (1) and (2) simultaneously gives $\boxed{T_{1f}=510\text{ K},\ T_{2f}=290\text{ K}}$

P21.46 The net work done *by* the gas on the bullet becomes the bullet's kinetic energy:

$$\frac{1}{2}mv^2=\frac{1}{2}1.1\times10^{-3}\text{ kg}\left(120\text{ m/s}\right)^2=7.92\text{ J}$$

The air in front of the bullet does work

$$P\Delta V=(1.013\times10^5\text{ N/m}^2)(-0.5\text{ m})(0.03\times10^{-4}\text{ m}^2)=-0.152\text{ J}$$

The hot gas behind the bullet then must do output work $+W$ in $+W-0.152\text{ J}=7.92\text{ J}$
$W=8.07$ J. The input work *on* the hot gas is -8.07 J

$$\frac{1}{\gamma-1}\left(P_fV_f-P_iV_i\right)=-8.07\text{ J}$$

Also $\quad P_fV_f^{\gamma}=P_iV_i^{\gamma} \qquad P_f=P_i\left(\dfrac{V_i}{V_f}\right)^{\gamma}$

So $\quad -8.07\text{ J}=\dfrac{1}{0.40}P_i\left[V_f\left(\dfrac{V_i}{V_f}\right)^{\gamma}-V_i\right]$

And $\quad V_f=12\text{ cm}^3+50\text{ cm }0.03\text{ cm}^2=13.5\text{ cm}^3$

Then $\quad P_i=\dfrac{-8.07\text{ J}(0.40)10^6\text{ cm}^3/\text{m}^3}{\left[13.5\text{ cm}^3(12/13.5)^{1.40}-12\text{ cm}^3\right]}=\boxed{5.85\times10^6\text{ Pa}}=57.7\text{ atm}$

P21.47 The pressure of the gas in the lungs of the diver must be the same as the absolute pressure of the water at this depth of 50.0 meters. This is:

$$P = P_0 + \rho g h = 1.00 \text{ atm} + \left(1.03\times10^3 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)(50.0 \text{ m})$$

or $$P = 1.00 \text{ atm} + 5.05\times10^5 \text{ Pa}\left(\frac{1.00 \text{ atm}}{1.013\times10^5 \text{ Pa}}\right) = 5.98 \text{ atm}$$

If the partial pressure due to the oxygen in the gas mixture is to be 1.00 atmosphere (or the fraction $\frac{1}{5.98}$ of the total pressure) oxygen molecules should make up only $\frac{1}{5.98}$ of the total number of molecules. This will be true if 1.00 mole of oxygen is used for every 4.98 mole of helium. The ratio by weight is then

$$\frac{(4.98 \text{ mol He})(4.003 \text{ g/mol He})g}{(1.00 \text{ mol } O_2)(2\times15.999 \text{ g/mol } O_2)g} = \boxed{0.623}$$

P21.48 (a) Maxwell's speed distribution function is

$$N_v = 4\pi N\left(\frac{m_0}{2\pi k_B T}\right)^{3/2} v^2 e^{-m_0 v^2/2k_B T}$$

With $N = 1.00\times10^4$,

$$m_0 = \frac{M}{N_A} = \frac{0.032 \text{ kg}}{6.02\times10^{23}} = 5.32\times10^{-26} \text{ kg}$$

$T = 500$ K

and $k_B = 1.38\times10^{-23}$ J/molecule·K

this becomes $N_v = \left(1.71\times10^{-4}\right)v^2 e^{-\left(3.85\times10^{-6}\right)v^2}$

To the right is a plot of this function for the range $0 \le v \le 1\,500$ m/s.

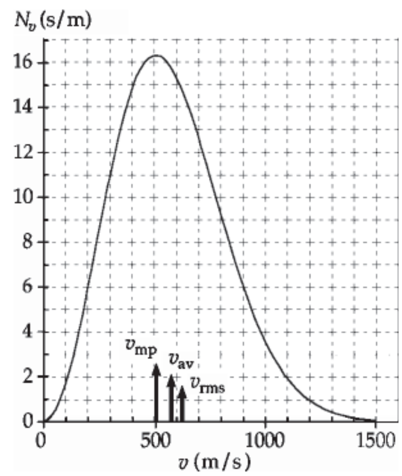


FIG. P21.48(a)

(b) The most probable speed occurs where N_v is a maximum.

From the graph, $\boxed{v_{mp} \approx 510 \text{ m/s}}$

(c) $$v_{av} = \sqrt{\frac{8k_B T}{\pi m_0}} = \sqrt{\frac{8\left(1.38\times10^{-23}\right)(500)}{\pi\left(5.32\times10^{-26}\right)}} = \boxed{575 \text{ m/s}}$$

Also,

$$v_{rms} = \sqrt{\frac{3k_B T}{m_0}} = \sqrt{\frac{3\left(1.38\times10^{-23}\right)(500)}{5.32\times10^{-26}}} = \boxed{624 \text{ m/s}}$$

(d) The fraction of particles in the range $300 \text{ m/s} \le v \le 600 \text{ m/s}$

is $$\frac{\int_{300}^{600} N_v dv}{N}$$

where $N = 10^4$

and the integral of N_v is read from the graph as the area under the curve. This is approximately (11 + 16.5 + 16.5 + 15)(1/4)(300) = 4 400 and the fraction is 0.44 or $\boxed{44\%}$.

P21.49 (a) Since $\boxed{\text{pressure increases as volume decreases}}$ (and vice versa),

$$\frac{dV}{dP} < 0 \qquad \text{and} \qquad -\frac{1}{V}\left[\frac{dV}{dP}\right] > 0$$

(b) For an ideal gas, $V = \dfrac{nRT}{P}$ and $\kappa_1 = -\dfrac{1}{V}\dfrac{d}{dP}\left(\dfrac{nRT}{P}\right)$

If the compression is isothermal, T is constant and

$$\kappa_1 = -\frac{nRT}{V}\left(-\frac{1}{P^2}\right) = \frac{1}{P}$$

(c) For an adiabatic compression, $PV^{\gamma} = C$ (where C is a constant) and

$$\kappa_2 = -\frac{1}{V}\frac{d}{dP}\left(\frac{C}{P}\right)^{1/\gamma} = \frac{1}{V}\left(\frac{1}{\gamma}\right)\frac{C^{1/\gamma}}{P^{(1/\gamma)+1}} = \frac{P^{1/\gamma}}{\gamma P^{1/\gamma+1}} = \frac{1}{\gamma P}$$

(d) $\kappa_1 = \dfrac{1}{P} = \dfrac{1}{(2.00\text{ atm})} = \boxed{0.500\text{ atm}^{-1}}$

$\gamma = \dfrac{C_P}{C_V}$ and for a monatomic ideal gas, $\gamma = \dfrac{5}{3}$, so that

$\kappa_2 = \dfrac{1}{\gamma P} = \dfrac{1}{\frac{5}{3}(2.00\text{ atm})} = \boxed{0.300\text{ atm}^{-1}}$

***P21.50** (a) The speed of sound is $v = \sqrt{\dfrac{B}{\rho}}$ where $B = -V\dfrac{dP}{dV}$

According to Problem 49, in an adiabatic process, this is $B = \dfrac{1}{\kappa_2} = \gamma P$

Also,

$$\rho = \frac{m_s}{V} = \frac{nM}{V} = \frac{(nRT)M}{V(RT)} = \frac{PM}{RT}$$

where m_s is the sample mass. Then, the speed of sound in the ideal gas is

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\gamma P\left(\frac{RT}{PM}\right)} = \boxed{\sqrt{\frac{\gamma RT}{M}}}$$

(b) $v = \sqrt{\dfrac{1.40(8.314\text{ J/mol}\cdot\text{K})(293\text{ K})}{0.028\,9\text{ kg/mol}}} = \boxed{344\text{ m/s}}$

$\boxed{\text{This agrees within 0.2\% with the 343 m/s listed in Table 17.1.}}$

(c) We use $k_B = \dfrac{R}{N_A}$ and $M = m_0 N_A$: $v = \sqrt{\dfrac{\gamma RT}{M}} = \sqrt{\dfrac{\gamma k_B N_A T}{m_0 N_A}} = \sqrt{\dfrac{\gamma k_B T}{m_0}}$

The most probable molecular speed is $\sqrt{\dfrac{2k_BT}{m_0}}$, the average speed is $\sqrt{\dfrac{8k_BT}{\pi m_0}}$,

and the rms speed is $\sqrt{\dfrac{3k_BT}{m_0}}$.

The speed of sound is somewhat less than each measure of molecular speed. Sound propagation is orderly motion overlaid on the disorder of molecular motion.

***P21.51** (a) The latent heat of evaporation per molecule is

$$2430\,\frac{\text{J}}{\text{g}}=2430\,\frac{\text{J}}{\text{g}}\left(\frac{18.0\text{ g}}{1\text{ mol}}\right)\left(\frac{1\text{ mol}}{6.02\times10^{23}\text{ molecule}}\right)=\boxed{7.27\times10^{-20}\text{ J/molecule}}$$

If the molecule has just broken free, we assume that it possesses the energy as translational kinetic energy.

(b) Consider one gram of these molecules: $K=(1/2)mv^2$

$2430\text{ J}=(1/2)(10^{-3}\text{ kg})\,v^2$ $\quad v=(4\,860\,000\text{ m}^2/\text{s}^2)^{1/2}=\boxed{2.20\times10^3\text{ m/s}}$

(c) The total translational kinetic energy of an ideal gas is $(3/2)nRT$, so we have

$(2430\text{ J/g})(18.0\text{ g/mol})=(3/2)(1\text{ mol})(8.314\text{ J/mol}\cdot\text{K})T$ $\quad T=\boxed{3.51\times10^3\text{ K}}$

The evaporating molecules are exceptional, at the high-speed tail of the distribution of molecular speeds. The average speed of molecules in the liquid and in the vapor is appropriate just to room temperature.

***P21.52** (a) Let $d=2r$ represent the diameter of the particle. Its mass is

$$m=\rho V=\rho\frac{4}{3}\pi r^3=\rho\frac{4}{3}\pi\left(\frac{d}{2}\right)^3=\frac{\rho\pi d^3}{6}.\text{ Then }\frac{1}{2}mv_{\text{rms}}^2=\frac{3}{2}kT\text{ gives }\frac{\rho\pi d^3}{6}v_{\text{rms}}^2=3kT\text{ so}$$

$$v_{\text{rms}}=\left(\frac{18kT}{\rho\pi d^3}\right)^{1/2}=\left(\frac{18\times1.38\times10^{-23}\text{ J/K }\times293\text{ K}}{1000\text{ kg/m}^3\ \pi}\right)^{1/2}d^{-3/2}=\boxed{4.81\times10^{-12}\text{ m}^{5/2}\text{s}^{-1}d^{-3/2}}$$

(b) $v=d/t$ $\quad [4.81\times10^{-12}\text{ m}^{5/2}/\text{s}]d^{-3/2}=d/t$

$$t=\frac{d}{[4.81\times10^{-12}\text{ m}^{5/2}/\text{s}]d^{-3/2}}=\boxed{2.08\times10^{11}\text{ s}\cdot\text{m}^{-5/2}d^{5/2}}$$

(c) $$v_{\text{rms}}=\left(\frac{18kT}{\rho\pi d^3}\right)^{1/2}=\left(\frac{18\left(1.38\times10^{-23}\text{ J/K}\right)(293\text{ K})}{\left(1\,000\text{ kg/m}^3\right)\pi\left(3\times10^{-6}\text{ m}\right)^3}\right)^{1/2}=\boxed{9.26\times10^{-4}\text{ m/s}}$$

$$v=\frac{x}{t}\qquad t=\frac{x}{v}=\frac{3\times10^{-6}\text{ m}}{9.26\times10^{-4}\text{ m/s}}=\boxed{3.24\text{ ms}}$$

(d) $70\text{ kg}=1\,000\text{ kg/m}^3\,\dfrac{\pi d^3}{6}$ $\qquad d=0.511\text{ m}$

$$v_{\text{rms}}=\left(\frac{18kT}{\rho\pi d^3}\right)^{1/2}=\left(\frac{18\left(1.38\times10^{-23}\text{ J/K}\right)(293\text{ K})}{\left(1\,000\text{ kg/m}^3\right)\pi(0.511\text{ m})^3}\right)^{1/2}=\boxed{1.32\times10^{-11}\text{ m/s}}$$

$t=\dfrac{0.511\text{ m}}{1.32\times10^{-11}\text{ m/s}}=\boxed{3.88\times10^{10}\text{ s}}=1\,230\text{ yr}$ $\quad$ This motion is too slow to observe.

(e) $\left(\dfrac{18kT}{\rho\pi d^3}\right)^{1/2}=\dfrac{d}{1\text{ s}}$ $\qquad \dfrac{18kT}{\rho\pi}=\dfrac{d^5}{1\text{ s}^2}$

$$d=\left(\frac{18\left(1.38\times10^{-23}\text{ J/K}\right)(293\text{ K})\left(1\text{ s}^2\right)}{\left(1\,000\text{ kg/m}^3\right)\pi}\right)^{1/5}=\boxed{2.97\times10^{-5}\text{ m}}$$

(f) Brownian motion is best observed with pollen grains, smoke particles, or latex spheres smaller than this 29.7-μm size. Then they can jitter about convincingly, showing relatively large accelerations several times per second. A simple rule is to use the smallest particles that you can clearly see with some particular microscopic technique.

P21.53 $n = \frac{m}{M} = \frac{1.20 \text{ kg}}{0.028\,9 \text{ kg/mol}} = 41.5 \text{ mol}$

(a) $V_i = \frac{nRT_i}{P_i} = \frac{(41.5 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(298 \text{ K})}{200\times10^3 \text{ Pa}} = \boxed{0.514 \text{ m}^3}$

(b) $\frac{P_f}{P_i} = \frac{\sqrt{V_f}}{\sqrt{V_i}}$ so $V_f = V_i\left(\frac{P_f}{P_i}\right)^2 = \left(0.514 \text{ m}^3\right)\left(\frac{400}{200}\right)^2 = \boxed{2.06 \text{ m}^3}$

(c) $T_f = \frac{P_fV_f}{nR} = \frac{\left(400\times10^3 \text{ Pa}\right)\left(2.06 \text{ m}^3\right)}{(41.5 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})} = \boxed{2.38\times10^3 \text{ K}}$

(d) $W = -\int_{V_i}^{V_f} PdV = -C\int_{V_i}^{V_f} V^{1/2}dV = -\left(\frac{P_i}{V_i^{1/2}}\right)\frac{2V^{3/2}}{3}\Bigg|_{V_i}^{V_f} = -\frac{2}{3}\left(\frac{P_i}{V_i^{1/2}}\right)\left(V_f^{3/2} - V_i^{3/2}\right)$

$W = -\frac{2}{3}\left(\frac{200\times10^3 \text{ Pa}}{\sqrt{0.514 \text{ m}}}\right)\left[\left(2.06 \text{ m}^3\right)^{3/2} - (0.514 \text{ m})^{3/2}\right] = \boxed{-4.80\times10^5 \text{ J}}$

(e) $\Delta E_{\text{int}} = nC_V\Delta T = (41.5 \text{ mol})\left[\frac{5}{2}(8.314 \text{ J/mol}\cdot\text{K})\right]\left(2.38\times10^3 - 298\right) \text{ K}$

$\Delta E_{\text{int}} = 1.80\times10^6 \text{ J}$

$Q = \Delta E_{\text{int}} - W = 1.80\times10^6 \text{ J} + 4.80\times10^5 \text{ J} = 2.28\times10^6 \text{ J} = \boxed{2.28 \text{ MJ}}$

P21.54 The ball loses energy $\frac{1}{2}mv_i^2 - \frac{1}{2}mv_f^2 = \frac{1}{2}(0.142 \text{ kg})\left[(47.2)^2 - (42.5)^2\right] \text{ m}^2/\text{s}^2 = 29.9 \text{ J}$

The air volume is $V = \pi(0.037\,0 \text{ m})^2(19.4 \text{ m}) = 0.083\,4 \text{ m}^3$

and its quantity is $n = \frac{PV}{RT} = \frac{1.013\times10^5 \text{ Pa}\left(0.083\,4 \text{ m}^3\right)}{(8.314 \text{ J/mol}\cdot\text{K})(293 \text{ K})} = 3.47 \text{ mol}$

The air absorbs energy as if it were warmed over a stove according to $Q = nC_P\Delta T$

So

$$\Delta T = \frac{Q}{nC_P} = \frac{29.9 \text{ J}}{3.47 \text{ mol}\left(\frac{7}{2}\right)(8.314 \text{ J/mol}\cdot\text{K})} = \boxed{0.296°\text{C}}$$

P21.55 $N_v(v) = 4\pi N\left(\dfrac{m_0}{2\pi k_B T}\right)^{3/2} v^2 \exp\left(\dfrac{-m_0 v^2}{2k_B T}\right)$ where exp(x) represents e^x

Note that $v_{\text{mp}} = \left(\dfrac{2k_B T}{m_0}\right)^{1/2}$

Thus, $N_v(v) = 4\pi N\left(\dfrac{m_0}{2\pi k_B T}\right)^{3/2} v^2 e^{\left(-v^2/v_{\text{mp}}^2\right)}$

And $\dfrac{N_v(v)}{N_v(v_{\text{mp}})} = \left(\dfrac{v}{v_{\text{mp}}}\right)^2 e^{\left(1-v^2/v_{\text{mp}}^2\right)}$

For $v = \dfrac{v_{\text{mp}}}{50}$

$$\frac{N_v(v)}{N_v(v_{\text{mp}})} = \left(\frac{1}{50}\right)^2 e^{\left[1-(1/50)^2\right]} = 1.09\times 10^{-3}$$

The other values are computed similarly, with the following results:

$\dfrac{v}{v_{\text{mp}}}$	$\dfrac{N_v(v)}{N_v(v_{\text{mp}})}$
$\frac{1}{50}$	1.09×10^{-3}
$\frac{1}{10}$	2.69×10^{-2}
$\frac{1}{2}$	0.529
1	1.00
2	0.199
10	1.01×10^{-41}
50	$1.25\times 10^{-1\,082}$

To find the last value, we note:

$(50)^2\, e^{1-2\,500} = 2\,500 e^{-2\,499}$

$10^{\log 2\,500} e^{(\ln 10)(-2\,499/\ln 10)} = 10^{\log 2\,500} 10^{-2\,499/\ln 10} = 10^{\log 2\,500 - 2\,499/\ln 10} = 10^{-1\,081.904} = 10^{0.096}\times 10^{-1082}$

P21.56 (a) The effect of high angular speed is like the effect of a very high gravitational field on an atmosphere. The result is: $\boxed{\text{The larger-mass molecules settle to the outside}}$ while the region at smaller r has a higher concentration of low-mass molecules.

(b) Consider a single kind of molecules, all of mass m_0. To cause the centripetal acceleration of the molecules between r and $r + dr$, the pressure must increase outward according to $\sum F_r = m_0 a_r$. Thus,

$$PA - (P + dP)A = -\left(nm_0 A\, dr\right)\left(r\omega^2\right)$$

where n is the number of molecules per unit volume and A is the area of any cylindrical surface. This reduces to $dP = nm_0\omega^2 r dr$.

But also $P = nk_B T$, so $dP = k_B T dn$. Therefore, the equation becomes

$$\frac{dn}{n} = \frac{m_0\omega^2}{k_B T} r dr \quad \text{giving} \quad \int_{n_0}^{n} \frac{dn}{n} = \frac{m_0\omega^2}{k_B T}\int_0^r r dr \quad \text{or} \quad \ln(n)\Big|_{n_0}^{n} = \frac{m_0\omega^2}{k_B T}\left(\frac{r^2}{2}\right)\Bigg|_0^r$$

$$\ln\left(\frac{n}{n_0}\right) = \frac{m_0\omega^2}{2k_B T} r^2 \quad \text{and solving for } n \text{ gives} \quad \boxed{n = n_0 e^{m_0 r^2\omega^2/2k_B T}}$$

P21.57 First find v_{av}^2 as $v_{\text{av}}^2 = \frac{1}{N}\int_0^\infty v^2 N_v dv$. Let $a = \frac{m_0}{2k_B T}$

Then,

$$v_{\text{av}}^2 = \frac{\left[4N\pi^{-1/2}a^{3/2}\right]}{N}\int_0^\infty v^4 e^{-av^2 dv} = \left[4a^{3/2}\pi^{-1/2}\right]\frac{3}{8a^2}\sqrt{\frac{\pi}{a}} = \frac{3k_B T}{m}$$

The root-mean square speed is then $v_{\text{rms}} = \sqrt{v_{\text{av}}^2} = \boxed{\sqrt{\frac{3k_B T}{m_0}}}$

To find the average speed, we have

$$v_{\text{av}} = \frac{1}{N}\int_0^\infty v N_v dv = \frac{\left(4Na^{3/2}\pi^{-1/2}\right)}{N}\int_0^\infty v^3 e^{-av^2} dv = \frac{4a^{3/2}\pi^{-1/2}}{2a^2} = \boxed{\sqrt{\frac{8k_B T}{\pi m_0}}}$$

P21.58 We want to evaluate $\frac{dP}{dV}$ for the function implied by $PV = nRT =$ constant, and also for the different function implied by $PV^\gamma =$ constant. We can use implicit differentiation:

From $PV =$ constant $\qquad P\frac{dV}{dV} + V\frac{dP}{dV} = 0 \qquad \left(\frac{dP}{dV}\right)_{\text{isotherm}} = -\frac{P}{V}$

From $PV^\gamma =$ constant $\qquad P\gamma V^{\gamma-1} + V^\gamma\frac{dP}{dV} = 0 \qquad \left(\frac{dP}{dV}\right)_{\text{adiabat}} = -\frac{\gamma P}{V}$

Therefore, $\qquad \left(\frac{dP}{dV}\right)_{\text{adiabat}} = \gamma\left(\frac{dP}{dV}\right)_{\text{isotherm}}$

The theorem is proved.

***P21.59** (a) $n = \frac{PV}{RT} = \frac{\left(1.013\times10^5\ \text{Pa}\right)\left(5.00\times10^{-3}\ \text{m}^3\right)}{(8.314\ \text{J/mol}\cdot\text{K})(300\ \text{K})}$

$= \boxed{0.203\ \text{mol}}$

(b) $T_B = T_A\left(\frac{P_B}{P_A}\right) = 300\ \text{K}\left(\frac{3.00}{1.00}\right) = \boxed{900\ \text{K}}$

$T_C = T_B = \boxed{900\ \text{K}}$

$V_C = V_A\left(\frac{T_C}{T_A}\right) = 5.00\ \text{L}\left(\frac{900}{300}\right) = \boxed{15.0\ \text{L}}$

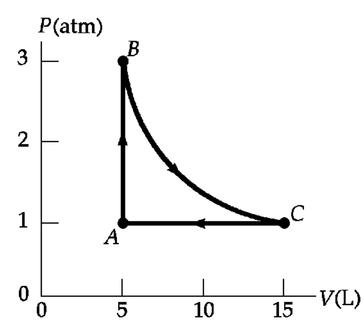


FIG. P21.59

(c) $E_{\text{int},A} = \frac{3}{2}nRT_A = \frac{3}{2}(0.203\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(300\ \text{K}) = \boxed{760\ \text{J}}$

$E_{\text{int},B} = E_{\text{int},C} = \frac{3}{2}nRT_B = \frac{3}{2}(0.203\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(900\ \text{K}) = \boxed{2.28\ \text{kJ}}$

(d)

	P (atm)	V(L)	T(K)	E_{int}(kJ)
A	1.00	5.00	300	0.760
B	3.00	5.00	900	2.28
C	1.00	15.00	900	2.28

continued on next page

(e) For the process *AB*, lock the piston in place and put the cylinder into an oven at 900 K. For *BC*, keep the sample in the oven while gradually letting the gas expand to lift a load on the piston as far as it can. For *CA*, carry the cylinder back into the room at 300 K and let the gas cool without touching the piston.

(f) For *AB*: $W = \boxed{0}$ $\Delta E_{\text{int}} = E_{\text{int},B} - E_{\text{int},A} = (2.28 - 0.760)\text{ kJ} = \boxed{1.52\text{ kJ}}$

$$Q = \Delta E_{\text{int}} - W = \boxed{1.52\text{ kJ}}$$

For *BC*: $\Delta E_{\text{int}} = \boxed{0}$, $W = -nRT_B \ln\left(\frac{V_C}{V_B}\right)$

$$W = -(0.203\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(900\text{ K})\ln(3.00) = \boxed{-1.67\text{ kJ}}$$

$$Q = \Delta E_{\text{int}} - W = \boxed{1.67\text{ kJ}}$$

For *CA*: $\Delta E_{\text{int}} = E_{\text{int},A} - E_{\text{int},C} = (0.760 - 2.28)\text{ kJ} = \boxed{-1.52\text{ kJ}}$

$$W = -P\Delta V = -nR\Delta T = -(0.203\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(-600\text{ K}) = \boxed{1.01\text{ kJ}}$$

$$Q = \Delta E_{\text{int}} - W = -1.52\text{ kJ} - 1.01\text{ kJ} = \boxed{-2.53\text{ kJ}}$$

(g) We add the amounts of energy for each process to find them for the whole cycle.

$$Q_{ABCA} = +1.52\text{ kJ} + 1.67\text{ kJ} - 2.53\text{ kJ} = \boxed{0.656\text{ kJ}}$$

$$W_{ABCA} = 0 - 1.67\text{ kJ} + 1.01\text{ kJ} = \boxed{-0.656\text{ kJ}}$$

$$(\Delta E_{\text{int}})_{ABCA} = +1.52\text{ kJ} + 0 - 1.52\text{ kJ} = \boxed{0}$$

P21.60 (a) $(10\,000\text{ g})\left(\frac{1.00\text{ mol}}{18.0\text{ g}}\right)\left(\frac{6.02\times10^{23}\text{ molecules}}{1.00\text{ mol}}\right) = \boxed{3.34\times10^{26}\text{ molecules}}$

(b) After one day, 10^{-1} of the original molecules would remain. After two days, the fraction would be 10^{-2}, and so on. After 26 days, only 3 of the original molecules would likely remain, and after $\boxed{27\text{ days}}$, likely none.

(c) The soup is this fraction of the hydrosphere: $\left(\frac{10.0\text{ kg}}{1.32\times10^{21}\text{ kg}}\right)$

Therefore, today's soup likely contains this fraction of the original molecules. The number of original molecules likely in the pot again today is:

$$\left(\frac{10.0\text{ kg}}{1.32\times10^{21}\text{ kg}}\right)(3.34\times10^{26}\text{ molecules}) = \boxed{2.53\times10^{6}\text{ molecules}}$$

P21.61 (a) For escape, $\frac{1}{2}m_0v^2 = \frac{Gm_0M}{R_E}$. Since the free-fall acceleration at the surface is $g = \frac{GM}{R_E^2}$, this can also be written as:

$$\frac{1}{2}m_0v^2 = \frac{Gm_0M}{R_E} = \boxed{m_0gR_E}$$

continued on next page

(b) For O_2, the mass of one molecule is

$$m_0 = \frac{0.032\,0 \text{ kg/mol}}{6.02\times10^{23} \text{ molecules/mol}} = 5.32\times10^{-26} \text{ kg/molecule}$$

Then, if $m_0 g R_E = 10\left(\frac{3k_B T}{2}\right)$, the temperature is

$$T = \frac{m_0 g R_E}{15k_B} = \frac{(5.32\times10^{-26} \text{ kg})(9.80 \text{ m/s}^2)(6.37\times10^6 \text{ m})}{15(1.38\times10^{-23} \text{ J/mol}\cdot\text{K})} = \boxed{1.60\times10^4 \text{ K}}$$

P21.62 (a) For sodium atoms (with a molar mass $M = 32.0$ g/mol)

$$\frac{1}{2}m_0 v^2 = \frac{3}{2}k_B T$$

$$\frac{1}{2}\left(\frac{M}{N_A}\right)v^2 = \frac{3}{2}k_B T$$

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.314 \text{ J/mol}\cdot\text{K})(2.40\times10^{-4} \text{ K})}{23.0\times10^{-3} \text{ kg}}} = \boxed{0.510 \text{ m/s}}$$

(b) $t = \dfrac{d}{v_{\text{rms}}} = \dfrac{0.010 \text{ m}}{0.510 \text{ m/s}} = \boxed{20 \text{ ms}}$

ANSWERS TO EVEN PROBLEMS

P21.2 Because each mole of a chemical compound contains Avogadro's number of molecules, the number of molecules in a sample is N_A times the number of moles, as described by $N = nN_A$, and the molar mass is N_A times the molecular mass, as described by $M = m_0 N_A$. The definition of the molar mass implies that the sample mass is the number of moles times the molar mass, as described by $m = nM$. Then the sample mass must also be the number of molecules times the molecular mass, according to $m = nM = nN_A m_0 = Nm_0$. The equations are true for chemical compounds in solid, liquid, and gaseous phases—this includes elements. We apply the equations also to air by interpreting M as the mass of Avogadro's number of the various molecules in the mixture.

P21.4 17.6 kPa

P21.6 5.05×10^{-21} J/molecule

P21.8 (a) 2.28 kJ (b) 6.21×10^{-21} J

P21.10 see the solution

P21.12 (a) 209 J (b) 0 (c) 317 K

P21.14 (a) 118 kJ (b) 6.03×10^3 kg

P21.16 (a) 719 J/kg·K (b) 0.811 kg (c) 233 kJ (d) 327 kJ

P21.18 (a) 0.118 (b) 2.35 (c) 0; +135 J; +135 J

P21.20 (a) 5.15×10^{-5} m^3 (b) 560 K (c) 2.24 K

P21.22 25.0 kW

P21.24 (a) see the solution (b) 2.19 V_i (c) $3T_i$ (d) T_i (e) $-0.830P_iV_i$

P21.26 see the solution

P21.28 2.33×10^{-21} J

P21.30 (a) $5R/2$ (b) $3R$ (c) $9R/2$ (d) $5R$ (e) Measure the constant-volume specific heat of the gas as a function of temperature and look for plateaus on the graph, as shown in Figure 21.7. If the first jump goes from $\frac{3}{2}R$ to $\frac{5}{2}R$, the molecules can be diagnosed as linear. If the first jump goes from $\frac{3}{2}R$ to $3R$, the molecules must be nonlinear. The tabulated data at one temperature are insufficient for the determination. At room temperature some of the heavier molecules appear to be vibrating.

P21.32 (a) No atom, almost all the time (b) 2.70×10^{20}

P21.34 (a) 1.03 (b) ^{35}Cl

P21.36 (a) 731 m/s (b) 825 m/s (c) 895 m/s (d) The graph appears to be drawn correctly within about 10 m/s.

P21.38 (a) see the solution (b) 8.31 km

P21.40 (a) 7.89×10^{26} molecules (b) 37.9 kg (c) 6.07×10^{-21} J/molecule (d) 503 m/s (e) 7.98 MJ (f) 7.98 MJ The smaller mass of warmer air contains the same internal energy as the cooler air. When the furnace operates, air expands and leaves the room.

P21.42 (a) $3.65v$ (b) $3.99v$ (c) $3.00v$ (d) $106\left(\dfrac{m_0 v^2}{V}\right)$ (e) $7.98 m_0 v^2$

P21.44 (a) 300 K (b) 1.00 atm

P21.46 5.85 MPa

P21.48 (a) see the solution (b) 5.1×10^2 m/s (c) $v_{av} = 575$ m/s; $v_{rms} = 624$ m/s (d) 44%

P21.50 (a) see the solution (b) 344 m/s, in good agreement with Table 17.1 (c) The speed of sound is somewhat less than each measure of molecular speed. Sound propagation is orderly motion overlaid on the disorder of molecular motion.

P21.52 (a) $[18\, k_B T/\pi\rho d^3]^{1/2} = [4.81 \times 10^{-12}\ \text{m}^{5/2}/\text{s}]d^{-3/2}$ (b) $[2.08 \times 10^{11}\ \text{s} \cdot \text{m}^{-5/2}]d^{5/2}$ (c) 0.926 mm/s and 3.24 ms (d) 1.32×10^{-11} m/s and 3.88×10^{10} s (e) 29.7 μm (f) It is good to use the smallest particles that you can clearly see with some particular microscopic technique.

P21.54 0.296°C

P21.56 (a) The effect of high angular speed is like the effect of a very high gravitational field on an atmosphere. The result is that the larger-mass molecules settle to the outside while the region at smaller r has a higher concentration of low-mass molecules. (b) see the solution

P21.58 see the solution

P21.60 (a) 3.34×10^{26} molecules (b) during the 27th day (c) 2.53×10^{6} molecules

P21.62 (a) 0.510 m/s (b) 20 ms

22

Heat Engines, Entropy, and the Second Law of Thermodynamics

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q22.1 First, the efficiency of the automobile engine cannot exceed the Carnot efficiency: it is limited by the temperature of burning fuel and the temperature of the environment into which the exhaust is dumped. Second, the engine block cannot be allowed to go over a certain temperature. Third, any practical engine has friction, incomplete burning of fuel, and limits set by timing and energy transfer by heat.

***Q22.2** For any cyclic process the total input energy must be equal to the total output energy. This is a consequence of the first law of thermodynamics. It is satisfied by processes ii, iv, v, vi, vii but not by processes i, iii, viii. The second law says that a cyclic process that takes in energy by heat must put out some of the energy by heat. This is not satisfied for processes v, vii, and viii. Thus the answers are (i) b (ii) a (iii) b (iv) a (v) c (vi) a (vii) c (viii) d.

Q22.3 A higher steam temperature means that more energy can be extracted from the steam. For a constant temperature heat sink at T_c, and steam at T_h, the efficiency of the power plant goes as $\dfrac{T_h - T_c}{T_h} = 1 - \dfrac{T_c}{T_h}$ and is maximized for a high T_h.

Q22.4 No. The first law of thermodynamics is a statement about energy conservation, while the second is a statement about stable thermal equilibrium. They are by no means mutually exclusive. For the particular case of a cycling heat engine, the first law implies $|Q_h| = W_{eng} + |Q_c|$, and the second law implies $|Q_c| > 0$.

Q22.5 Take an automobile as an example. According to the first law or the idea of energy conservation, it must take in all the energy it puts out. Its energy source is chemical energy in gasoline. During the combustion process, some of that energy goes into moving the pistons and eventually into the mechanical motion of the car. The chemical energy turning into internal energy can be modeled as energy input by heat. The second law says that not all of the energy input can become output mechanical energy. Much of the input energy must and does become energy output by heat, which, through the cooling system, is dissipated into the atmosphere. Moreover, there are numerous places where friction, both mechanical and fluid, turns mechanical energy into heat. In even the most efficient internal combustion engine cars, less than 30% of the energy from the fuel actually goes into moving the car. The rest ends up as useless heat in the atmosphere.

***Q22.6** Answer (b). In the reversible adiabatic expansion *OA*, the gas does work against a piston, takes in no energy by heat, and so drops in internal energy and in temperature. In the free adiabatic expansion *OB*, there is no piston, no work output, constant internal energy, and constant temperature for the ideal gas. The points *O* and *B* are on a hyperbolic isotherm. The points *O* and *A* are on an adiabat, steeper than an isotherm by the factor γ.

Q22.7 A slice of hot pizza cools off. Road friction brings a skidding car to a stop. A cup falls to the floor and shatters. Your cat dies. Any process is irreversible if it looks funny or frightening when shown in a videotape running backwards. The free flight of a projectile is nearly reversible.

Q22.8 (a) When the two sides of the semiconductor are at different temperatures, an electric potential (voltage) is generated across the material, which can drive electric current through an external circuit. The two cups at 50°C contain the same amount of internal energy as the pair of hot and cold cups. But no energy flows by heat through the converter bridging between them and no voltage is generated across the semiconductors.

(b) A heat engine must put out exhaust energy by heat. The cold cup provides a sink to absorb output or wasted energy by heat, which has nowhere to go between two cups of equally warm water.

***Q22.9** (i) Answer (a). The air conditioner operating in a closed room takes in energy by electric transmission and turns it all into energy put out by heat. That is its whole net effect.

(ii) Answer (b). The frozen stuff absorbs energy by heat from the air. But if you fill the ice trays with tap water and put them back into the freezer, the refrigerator will pump more heat into the air than it extracts from the water to make it freeze.

***Q22.10** (i) Answer (d). (ii) Answer (d). The second law says that you must put in some work to pump heat from a lower-temperature to a higher-temperature location. But it can be very little work if the two temperatures are very nearly equal.

Q22.11 One: Energy flows by heat from a hot bowl of chili into the cooler surrounding air. Heat lost by the hot stuff is equal to heat gained by the cold stuff, but the entropy decrease of the hot stuff is less than the entropy increase of the cold stuff.

Two: As you inflate a soft car tire at a service station, air from a tank at high pressure expands to fill a larger volume. That air increases in entropy and the surrounding atmosphere undergoes no significant entropy change.

Three: The brakes of your car get warm as you come to a stop. The shoes and drums increase in entropy and nothing loses energy by heat, so nothing decreases in entropy.

Q22.12 (a) For an expanding ideal gas at constant temperature, the internal energy stays constant. The gas must absorb by heat the same amount of energy that it puts out by work. Then its entropy change is $\Delta S = \dfrac{\Delta Q}{T} = nR\ln\left(\dfrac{V_2}{V_1}\right)$

(b) For a reversible adiabatic expansion $\Delta Q = 0$, and $\Delta S = 0$. An ideal gas undergoing an irreversible adiabatic expansion can have any positive value for ΔS up to the value given in part (a).

***Q22.13** Answer (f). The whole Universe must have an entropy change of zero or more. The environment around the system comprises the rest of the Universe, and must have an entropy change of +8.0 J/K, or more.

***Q22.14** (i) Consider the area that fits under each of the arrows, between its line segment and the horizontal axis. Count it as positive for arrows to the right, zero for vertical arrows, and negative for arrows tending left. Then $E > F > G > H = D > A > B > C$.

(ii) The thin blue hyperbolic lines are isotherms. Each is a set of points representing states with the same internal energy for the ideal gas simple. An arrow tending farther from the origin than the *BE* hyperbola represents a process for which internal energy increases. So we have $D = E > C > B = F > G > A = H$.

(iii) The arrows *C* and *G* are along an adiabat. Visualize or sketch in a set of these curves, uniformly steeper than the blue isotherms. The energy input by heat is determined by how far above the starting adiabat the process arrow ends. We have $E > D > F > C = G > B > H > A$.

***Q22.15** Processes *C* and *G* are adiabatic. They can be carried out reversibly. Along these arrows entropy does not change. Visualize or sketch in a set of these adiabatic curves, uniformly steeper than the blue isotherms. The entropy change is determined by how far above the starting adiabat the process arrow ends. We have $E > D > F > C = G > B > H > A$.

***Q22.16** (a) The reduced flow rate of 'cooling water' reduces the amount of heat exhaust Q_c that the plant can put out each second. Even with constant efficiency, the rate at which the turbines can take in heat is reduced and so is the rate at which they can put out work to the generators. If anything, the efficiency will drop, because the smaller amount of water carrying the heat exhaust will tend to run hotter. The steam going through the turbines will undergo a smaller temperature change. Thus there are two reasons for the work output to drop.

(b) The engineer's version of events, as seen from inside the plant, is complete and correct. Hot steam pushes hard on the front of a turbine blade. Still-warm steam pushes less hard on the back of the blade, which turns in response to the pressure difference. Higher temperature at the heat exhaust port in the lake works its way back to a corresponding higher temperature of the steam leaving a turbine blade, a smaller temperature drop across the blade, and a lower work output.

***Q22.17** Answer (d). Heat input will not *necessarily* produce an entropy increase, because a heat input could go on simultaneously with a larger work output, to carry the gas to a lower-temperature, lower-entropy final state. Work input will not *necessarily* produce an entropy increase, because work input could go on simultaneously with heat output to carry the gas to a lower-volume, lower-entropy final state. Either temperature increase at constant volume, or volume increase at constant temperature, or simultaneous increases in both temperature and volume, will necessarily end in a more disordered, higher-entropy final state.

Q22.18 An analogy used by Carnot is instructive: A waterfall continuously converts mechanical energy into internal energy. It continuously creates entropy as the organized motion of the falling water turns into disorganized molecular motion. We humans put turbines into the waterfall, diverting some of the energy stream to our use. Water flows spontaneously from high to low elevation and energy spontaneously flows by heat from high to low temperature. Into the great flow of solar radiation from Sun to Earth, living things put themselves. They live on energy flow, more than just on energy. A basking snake diverts energy from a high-temperature source (the Sun) through itself temporarily, before the energy inevitably is radiated from the body of the snake to a low-temperature sink (outer space). A tree builds organized cellulose molecules and we build libraries and babies who look like their grandmothers, all out of a thin diverted stream in the universal flow of energy crashing down to disorder. We do not violate the second law, for we build local reductions in the entropy of one thing within the inexorable increase in the total entropy of the Universe. Your roommate's exercise puts energy into the room by heat.

Q22.19 Either statement can be considered an instructive analogy. We choose to take the first view. All processes require energy, either as energy content or as energy input. The kinetic energy which it possessed at its formation continues to make the Earth go around. Energy released by nuclear reactions in the core of the Sun drives weather on the Earth and essentially all processes in the biosphere. The energy intensity of sunlight controls how lush a forest or jungle can be and how warm a planet is. Continuous energy input is not required for the motion of the planet. Continuous energy input is required for life because energy tends to be continuously degraded, as heat flows into lower-temperature sinks. The continuously increasing entropy of the Universe is the index to energy-transfers completed. Arnold Sommerfeld suggested the idea for this question.

Q22.20 Shaking opens up spaces between jellybeans. The smaller ones more often can fall down into spaces below them. The accumulation of larger candies on top and smaller ones on the bottom implies a small increase in order, a small decrease in one contribution to the total entropy, but the second law is not violated. The total entropy increases as the system warms up, its increase in internal energy coming from the work put into shaking the box and also from a bit of gravitational energy loss as the beans settle compactly together.

SOLUTIONS TO PROBLEMS

Section 22.1 Heat Engines and the Second Law of Thermodynamics

P22.1 (a) $e=\dfrac{W_{\text{eng}}}{|Q_h|}=\dfrac{25.0\text{ J}}{360\text{ J}}=\boxed{0.069\,4}$ or $\boxed{6.94\%}$

(b) $|Q_c|=|Q_h|-W_{\text{eng}}=360\text{ J}-25.0\text{ J}=\boxed{335\text{ J}}$

***P22.2** The engine's output work we identify with the kinetic energy of the bullet:

$$W_{\text{eng}}=K=\frac{1}{2}mv^2=\frac{1}{2}0.002\,4\text{ kg}(320\text{ m/s})^2=123\text{ J}$$

$$e=\frac{W_{\text{eng}}}{Q_h}$$

$$Q_h=\frac{W_{\text{eng}}}{e}=\frac{123\text{ J}}{0.011}=1.12\times10^4\text{ J}$$

$$Q_h=W_{\text{eng}}+|Q_c|$$

The energy exhaust is

$$|Q_c|=Q_h-W_{\text{eng}}=1.12\times10^4\text{ J}-123\text{ J}=1.10\times10^4\text{ J}$$

$$Q=mc\Delta T$$

$$\Delta T=\frac{Q}{mc}=\frac{1.10\times10^4\text{ J kg°C}}{1.80\text{ kg }448\text{ J}}=\boxed{13.7°\text{C}}$$

P22.3 (a) We have $e=\dfrac{W_{\text{eng}}}{|Q_h|}=\dfrac{|Q_h|-|Q_c|}{|Q_h|}=1-\dfrac{|Q_c|}{|Q_h|}=0.250$

with $|Q_c|=8\,000$ J, we have $|Q_h|=\boxed{10.7\text{ kJ}}$

(b) $W_{\text{eng}}=|Q_h|-|Q_c|=2\,667$ J

and from $\mathcal{P}=\dfrac{W_{\text{eng}}}{\Delta t}$, we have $\Delta t=\dfrac{W_{\text{eng}}}{\mathcal{P}}=\dfrac{2\,667\text{ J}}{5\,000\text{ J/s}}=\boxed{0.533\text{ s}}$

P22.4 (a) The input energy each hour is

$$\left(7.89\times10^{3}\ \text{J/revolution}\right)\left(2\,500\ \text{rev/min}\right)\frac{60\ \text{min}}{1\ \text{h}}=1.18\times10^{9}\ \text{J/h}$$

implying fuel input $\left(1.18\times10^{9}\ \text{J/h}\right)\left(\frac{1\ \text{L}}{4.03\times10^{7}\ \text{J}}\right)=\boxed{29.4\ \text{L/h}}$

(b) $Q_h=W_{\text{eng}}+|Q_c|$. For a continuous-transfer process we may divide by time to have

$$\frac{Q_h}{\Delta t}=\frac{W_{\text{eng}}}{\Delta t}+\frac{|Q_c|}{\Delta t}$$

$$\text{Useful power output}=\frac{W_{\text{eng}}}{\Delta t}=\frac{Q_h}{\Delta t}-\frac{|Q_c|}{\Delta t}$$

$$=\left(\frac{7.89\times10^{3}\ \text{J}}{\text{revolution}}-\frac{4.58\times10^{3}\ \text{J}}{\text{revolution}}\right)\frac{2\,500\ \text{rev}}{1\ \text{min}}\frac{1\ \text{min}}{60\ \text{s}}=1.38\times10^{5}\ \text{W}$$

$$\mathcal{P}_{\text{eng}}=1.38\times10^{5}\ \text{W}\left(\frac{1\ \text{hp}}{746\ \text{W}}\right)=\boxed{185\ \text{hp}}$$

(c) $\mathcal{P}_{\text{eng}}=\tau\omega\Rightarrow\tau=\frac{\mathcal{P}_{\text{eng}}}{\omega}=\frac{1.38\times10^{5}\ \text{J/s}}{\left(2\,500\ \text{rev/60 s}\right)}\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right)=\boxed{527\ \text{N}\cdot\text{m}}$

(d) $\frac{|Q_c|}{\Delta t}=\frac{4.58\times10^{3}\ \text{J}}{\text{revolution}}\left(\frac{2\,500\ \text{rev}}{60\ \text{s}}\right)=\boxed{1.91\times10^{5}\ \text{W}}$

P22.5 The heat to melt 15.0 g of Hg is $|Q_c|=mL_f=\left(15\times10^{-3}\ \text{kg}\right)\left(1.18\times10^{4}\ \text{J/kg}\right)=177\ \text{J}$

The energy absorbed to freeze 1.00 g of aluminum is

$$|Q_h|=mL_f=\left(10^{-3}\ \text{kg}\right)\left(3.97\times10^{5}\ \text{J/kg}\right)=397\ \text{J}$$

and the work output is $W_{\text{eng}}=|Q_h|-|Q_c|=220\ \text{J}$

$$e=\frac{W_{\text{eng}}}{|Q_h|}=\frac{220\ \text{J}}{397\ \text{J}}=0.554,\ \text{or}\ \boxed{55.4\%}$$

The theoretical (Carnot) efficiency is $\frac{T_h-T_c}{T_h}=\frac{933\ \text{K}-243.1\ \text{K}}{933\ \text{K}}=0.749=74.9\%$

Section 22.2 **Heat Pumps and Refrigerators**

P22.6 $\text{COP}(\text{refrigerator})=\frac{Q_c}{W}$

(a) If $Q_c=120$ J and COP $=5.00$, then $\boxed{W=24.0\ \text{J}}$

(b) Heat expelled = Heat removed + Work done.

$$Q_h=Q_c+W=120\ \text{J}+24\ \text{J}=\boxed{144\ \text{J}}$$

P22.7 $\text{COP} = 3.00 = \frac{Q_c}{W}$. Therefore, $W = \frac{Q_c}{3.00}$.

The heat removed each minute is

$$\frac{Q_C}{t} = (0.030\,0\text{ kg})(4\,186\text{ J/kg°C})(22.0\text{°C}) + (0.030\,0\text{ kg})(3.33\times10^5\text{ J/kg})$$
$$+(0.030\,0\text{ kg})(2\,090\text{ J/kg°C})(20.0\text{°C}) = 1.40\times10^4\text{ J/min}$$

or, $$\frac{Q_c}{t} = 233\text{ J/s}$$

Thus, the work done per second is

$$\mathcal{P} = \frac{233\text{ J/s}}{3.00} = \boxed{77.8\text{ W}}$$

P22.8 (a) $\left(10.0\ \frac{\text{Btu}}{\text{h}\cdot\text{W}}\right)\left(\frac{1055\text{ J}}{1\text{ Btu}}\right)\left(\frac{1\text{ h}}{3\,600\text{ s}}\right)\left(\frac{1\text{ W}}{1\text{ J/s}}\right) = \boxed{2.93}$

(b) The energy extracted by heat from the cold side divided by required work input is by definition the coefficient of performance for a refrigerator: $\boxed{(\text{COP})_{\text{refrigerator}}}$

(c) With EER 5, $5\ \frac{\text{Btu}}{\text{h}\cdot\text{W}} = \frac{10\,000\text{ Btu/h}}{\mathcal{P}}$: $\quad \mathcal{P} = \frac{10\,000\text{ Btu/h}}{5\text{ Btu/h}\cdot\text{W}} = 2\,000\text{ W} = 2.00\text{ kW}$

Energy purchased is $\mathcal{P}\Delta t = (2.00\text{ kW})(1\,500\text{ h}) = 3.00\times10^3\text{ kWh}$

$\text{Cost} = (3.00\times10^3\text{ kWh})(0.100\text{ \$/kWh}) = \$300$

With EER 10, $10\ \frac{\text{Btu}}{\text{h}\cdot\text{W}} = \frac{10\,000\text{ Btu/h}}{\mathcal{P}}$: $\quad \mathcal{P} = \frac{10\,000\text{ Btu/h}}{10\text{ Btu/h}\cdot\text{W}} = 1\,000\text{ W} = 1.00\text{ kW}$

Energy purchased is $\mathcal{P}\Delta t = (1.00\text{ kW})(1\,500\text{ h}) = 1.50\times10^3\text{ kWh}$

$\text{Cost} = (1.50\times10^3\text{ kWh})(0.100\text{ \$/kWh}) = \$150$

Thus, the cost for air conditioning is half as much for an air conditioner with EER 10 compared with an air conditioner with EER 5.

Section 22.3 **Reversible and Irreversible Processes**

Section 22.4 **The Carnot Engine**

P22.9 $T_c = 703\text{ K} \qquad T_h = 2\,143\text{ K}$

(a) $e_c = \frac{\Delta T}{T_h} = \frac{1\,440}{2\,143} = \boxed{67.2\%}$

(b) $|Q_h| = 1.40\times10^5\text{ J}$, $W_{\text{eng}} = 0.420|Q_h|$

$$\mathcal{P} = \frac{W_{\text{eng}}}{\Delta t} = \frac{5.88\times10^4\text{ J}}{1\text{ s}} = \boxed{58.8\text{ kW}}$$

P22.10 When $e = e_c$, $1 - \frac{T_c}{T_h} = \frac{W_{\text{eng}}}{|Q_h|}$ and $\frac{W_{\text{eng}}/\Delta t}{|Q_h|/\Delta t} = 1 - \frac{T_c}{T_h}$

(a) $|Q_h| = \frac{(W_{\text{eng}}/\Delta t)\Delta t}{1-(T_c/T_h)} = \frac{(1.50\times10^5\ \text{W})(3\,600\ \text{s})}{1-293/773}$

$|Q_h| = 8.70\times10^8\ \text{J} = \boxed{870\ \text{MJ}}$

(b) $|Q_c| = |Q_h| - \left(\frac{W_{\text{eng}}}{\Delta t}\right)\Delta t = 8.70\times10^8 - (1.50\times10^5)(3\,600) = 3.30\times10^8\ \text{J} = \boxed{330\ \text{MJ}}$

***P22.11** We use amounts of energy to find the actual efficiency. $Q_h = Q_c + W_{\text{eng}} = 20$ kJ + 1.5 kJ = 21.5 kJ
$e = W_{\text{eng}}/Q_h = 1.5$ kJ/21.5 kJ = 0.0698

We use temperatures to find the Carnot efficiency of a reversible engine $e_C = 1 - T_c/T_h =$
$1 - 373$ K/453 K = 0.177

$\boxed{\text{The actual efficiency of 0.0698 is less than four-tenths of the Carnot efficiency of 0.177.}}$

***P22.12** (a) $e_C = 1 - T_c/T_h = 1 - 350/500 = \boxed{0.300}$

(b) In $e_C = 1 - T_c/T_h$ we differentiate to find $de_C/dT_h = 0 - T_c(-1)T_h^{-2} = T_c/T_h^{\,2} = 350/500^2 =$ $\boxed{1.40\times10^{-3}}$ This is the increase of efficiency per degree of increase in the temperature of the hot reservoir.

(c) In $e_C = 1 - T_c/T_h$ we differentiate to find $de_C/dT_c = 0 - 1/T_h = -1/500 = -2.00\times10^{-3}$

Then $de_C/(-dT_c) = \boxed{+2.00\times10^{-3}}$ This is the increase of efficiency per degree of decrease in the temperature of the cold reservoir. Note that it is a better deal to cool the exhaust than to supercharge the firebox.

P22.13 Isothermal expansion at $T_h = 523$ K

Isothermal compression at $T_c = 323$ K

Gas absorbs 1 200 J during expansion.

(a) $|Q_c| = |Q_h|\left(\frac{T_c}{T_h}\right) = 1\,200\ \text{J}\left(\frac{323}{523}\right) = \boxed{741\ \text{J}}$

(b) $W_{\text{eng}} = |Q_h| - |Q_c| = (1\,200 - 741)\ \text{J} = \boxed{459\ \text{J}}$

P22.14 The Carnot summer efficiency is $e_{c,s} = 1 - \frac{T_c}{T_h} = 1 - \frac{(273+20)\ \text{K}}{(273+350)\ \text{K}} = 0.530$

And in winter, $e_{c,w} = 1 - \frac{283}{623} = 0.546$

Then the actual winter efficiency is $0.320\left(\frac{0.546}{0.530}\right) = \boxed{0.330}$ or $\boxed{33.0\%}$

P22.15 (a) In an adiabatic process, $P_f V_f^{\gamma} = P_i V_i^{\gamma}$. Also, $\left(\dfrac{P_f V_f}{T_f}\right)^{\gamma} = \left(\dfrac{P_i V_i}{T_i}\right)^{\gamma}$

Dividing the second equation by the first yields $T_f = T_i \left(\dfrac{P_f}{P_i}\right)^{(\gamma-1)/\gamma}$

Since $\gamma = \dfrac{5}{3}$ for Argon, $\dfrac{\gamma - 1}{\gamma} = \dfrac{2}{5} = 0.400$ and we have

$$T_f = (1\,073\text{ K})\left(\frac{300\times 10^3\text{ Pa}}{1.50\times 10^6\text{ Pa}}\right)^{0.400} = \boxed{564\text{ K}}$$

(b) $\Delta E_{\text{int}} = nC_V\Delta T = Q - W_{\text{eng}} = 0 - W_{\text{eng}}$, so $W_{\text{eng}} = -nC_V\Delta T$,
and the power output is

$$\mathcal{P} = \frac{W_{\text{eng}}}{t} = \frac{-nC_V\Delta T}{t} \text{ or}$$

$$= \frac{(-80.0\text{ kg})(1\text{ mol}/0.0399\text{ kg})\left(\frac{3}{2}\right)(8.314\text{ J/mol}\cdot\text{K})(564 - 1\,073)\text{K}}{60.0\text{ s}}$$

$$\mathcal{P} = 2.12\times 10^5\text{ W} = \boxed{212\text{ kW}}$$

(c) $$e_C = 1 - \frac{T_c}{T_h} = 1 - \frac{564\text{ K}}{1\,073\text{ K}} = 0.475 \text{ or } \boxed{47.5\%}$$

P22.16 (a) $$e_{\max} = 1 - \frac{T_c}{T_h} = 1 - \frac{278}{293} = 5.12\times 10^{-2} = \boxed{5.12\%}$$

(b) $$\mathcal{P} = \frac{W_{\text{eng}}}{\Delta t} = 75.0\times 10^6\text{ J/s}$$

Therefore, $W_{\text{eng}} = (75.0\times 10^6\text{ J/s})(3\,600\text{ s/h}) = 2.70\times 10^{11}\text{ J/h}$

From $e = \dfrac{W_{\text{eng}}}{|Q_h|}$ we find $|Q_h| = \dfrac{W_{\text{eng}}}{e} = \dfrac{2.70\times 10^{11}\text{ J/h}}{5.12\times 10^{-2}} = 5.27\times 10^{12}\text{ J/h} = \boxed{5.27\text{ TJ/h}}$

(c) As fossil-fuel prices rise, this way to use solar energy will become a good buy.

***P22.17** (a) $$e = \frac{W_{\text{eng1}} + W_{\text{eng2}}}{Q_{1h}} = \frac{e_1 Q_{1h} + e_2 Q_{2h}}{Q_{1h}}$$

Now

$$Q_{2h} = Q_{1c} = Q_{1h} - W_{\text{eng1}} = Q_{1h} - e_1 Q_{1h}$$

So

$$e = \frac{e_1 Q_{1h} + e_2(Q_{1h} - e_1 Q_{1h})}{Q_{1h}} = \boxed{e_1 + e_2 - e_1 e_2}$$

continued on next page

(b) $e = e_1 + e_2 - e_1 e_2 = 1 - \frac{T_i}{T_h} + 1 - \frac{T_c}{T_i} - \left(1 - \frac{T_i}{T_h}\right)\left(1 - \frac{T_c}{T_i}\right)$

$$= 2 - \frac{T_i}{T_h} - \frac{T_c}{T_i} - 1 + \frac{T_i}{T_h} + \frac{T_c}{T_i} - \frac{T_c}{T_h} = \boxed{1 - \frac{T_c}{T_h}}$$

The combination of reversible engines is itself a reversible engine so it has the Carnot efficiency. No improvement in net efficiency has resulted.

(c) With $W_{\text{eng2}} = W_{\text{eng1}}$, $e = \frac{W_{\text{eng1}} + W_{\text{eng2}}}{Q_{1h}} = \frac{2W_{\text{eng1}}}{Q_{1h}} = 2e_1$

$$1 - \frac{T_c}{T_h} = 2\left(1 - \frac{T_i}{T_h}\right)$$

$$0 - \frac{T_c}{T_h} = 1 - \frac{2T_i}{T_h}$$

$$2T_i = T_h + T_c$$

$$\boxed{T_i = \frac{1}{2}\left(T_h + T_c\right)}$$

(d) $e_1 = e_2 = 1 - \frac{T_i}{T_h} = 1 - \frac{T_c}{T_i}$

$$T_i^2 = T_c T_h$$

$$\boxed{T_i = \left(T_h T_c\right)^{1/2}}$$

***P22.18** (a) "The actual efficiency is two thirds the Carnot efficiency" reads as an equation

$$\frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}}}{|Q_c| + W_{\text{eng}}} = \frac{2}{3}\left(1 - \frac{T_c}{T_h}\right) = \frac{2}{3}\frac{T_h - T_c}{T_h}.$$

All the T's represent absolute temperatures. Then

$$\frac{|Q_c| + W_{\text{eng}}}{W_{\text{eng}}} = \frac{1.5\,T_h}{T_h - T_c} \qquad \frac{|Q_c|}{W_{\text{eng}}} = \frac{1.5\,T_h}{T_h - T_c} - 1 = \frac{1.5\,T_h - T_h + T_c}{T_h - T_c}$$

$$|Q_c| = W_{\text{eng}}\frac{0.5\,T_h + T_c}{T_h - T_c} \qquad \frac{|Q_c|}{\Delta t} = \frac{W_{\text{eng}}}{\Delta t}\frac{0.5\,T_h + T_c}{T_h - T_c} = \boxed{1.40\text{ MW}\frac{0.5\,T_h + 383\text{ K}}{T_h - 383\text{ K}}}$$

The dominating T_h in the bottom of this fraction means that the exhaust power decreases as the firebox temperature increases.

(b) $\frac{|Q_c|}{\Delta t} = 1.40\text{ MW}\frac{0.5\,T_h + 383\text{ K}}{T_h - 383\text{ K}} = 1.40\text{ MW}\frac{0.5(1073\text{ K}) + 383\text{ K}}{(1073 - 383)\text{ K}} = \boxed{1.87\text{ MW}}$

(c) We require $\frac{|Q_c|}{\Delta t} = \frac{1}{2}1.87\text{ MW} = 1.40\text{ MW}\frac{0.5\,T_h + 383\text{ K}}{T_h - 383\text{ K}} \qquad \frac{0.5\,T_h + 383\text{ K}}{T_h - 383\text{ K}} = 0.666$

$$0.5\,T_h + 383\text{ K} = 0.666T_h - 255\text{ K} \qquad T_h = 638\text{ K}/0.166 = \boxed{3.84 \times 10^3\text{ K}}$$

(d) The minimum possible heat exhaust power is approached as the firebox temperature goes to infinity, and it is $|Q_c|/\Delta t$ = 1.40 MW(0.5/1) = 0.7 MW. The heat exhaust power cannot be as small as (1/4)(1.87 MW) = 0.466 MW. So

no answer exists. The energy exhaust cannot be that small.

P22.19 $(\text{COP})_{\text{refrig}} = \dfrac{T_c}{\Delta T} = \dfrac{270}{30.0} = \boxed{9.00}$

P22.20 (a) First, consider the adiabatic process $D \to A$:

$P_D V_D^\gamma = P_A V_A^\gamma$ so $P_D = P_A\left(\dfrac{V_A}{V_D}\right)^\gamma = 1\,400 \text{ kPa}\left(\dfrac{10.0 \text{ L}}{15.0 \text{ L}}\right)^{5/3} = \boxed{712 \text{ kPa}}$

Also $\left(\dfrac{nRT_D}{V_D}\right)V_D^\gamma = \left(\dfrac{nRT_A}{V_A}\right)V_A^\gamma$

or $T_D = T_A\left(\dfrac{V_A}{V_D}\right)^{\gamma-1} = 720 \text{ K}\left(\dfrac{10.0}{15.0}\right)^{2/3} = \boxed{549 \text{ K}}$

Now, consider the isothermal process $C \to D$: $T_C = T_D = \boxed{549 \text{ K}}$

$$P_C = P_D\left(\frac{V_D}{V_C}\right) = \left[P_A\left(\frac{V_A}{V_D}\right)^\gamma\right]\left(\frac{V_D}{V_C}\right) = \frac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}}$$

$$P_C = \frac{1\,400 \text{ kPa}(10.0 \text{ L})^{5/3}}{24.0 \text{ L}(15.0 \text{ L})^{2/3}} = \boxed{445 \text{ kPa}}$$

Next, consider the adiabatic process $B \to C$: $P_B V_B^\gamma = P_C V_C^\gamma$

But, $P_C = \dfrac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}}$ from above. Also considering the isothermal process, $P_B = P_A\left(\dfrac{V_A}{V_B}\right)$

Hence, $P_A\left(\dfrac{V_A}{V_B}\right)V_B^\gamma = \left(\dfrac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}}\right)V_C^\gamma$ which reduces to $V_B = \dfrac{V_A V_C}{V_D} = \dfrac{10.0 \text{ L}(24.0 \text{ L})}{15.0 \text{ L}} = \boxed{16.0 \text{ L}}$

Finally, $P_B = P_A\left(\dfrac{V_A}{V_B}\right) = 1\,400 \text{ kPa}\left(\dfrac{10.0 \text{ L}}{16.0 \text{ L}}\right) = \boxed{875 \text{ kPa}}$

State	P (kPa)	V (L)	T (K)
A	1 400	10.0	720
B	875	16.0	720
C	445	24.0	549
D	712	15.0	549

continued on next page

(b) For the isothermal process $A \to B$: $\Delta E_{\text{int}} = nC_V \Delta T = \boxed{0}$

so $Q = -W = nRT \ln\left(\frac{V_B}{V_A}\right) = 2.34 \text{ mol}(8.314 \text{ J/mol}\cdot\text{K})(720 \text{ K})\ln\left(\frac{16.0}{10.0}\right) = \boxed{+6.58 \text{ kJ}}$

For the adiabatic process $B \to C$: $Q = \boxed{0}$

$$\Delta E_{\text{int}} = nC_V(T_C - T_B) = 2.34 \text{ mol}\left[\frac{3}{2}(8.314 \text{ J/mol}\cdot\text{K})\right](549 - 720) \text{ K} = \boxed{-4.98 \text{ kJ}}$$

and $W = -Q + \Delta E_{\text{int}} = 0 + (-4.98 \text{ kJ}) = \boxed{-4.98 \text{ kJ}}$

For the isothermal process $C \to D$: $\Delta E_{\text{int}} = nC_V \Delta T = \boxed{0}$

and $Q = -W = nRT \ln\left(\frac{V_D}{V_C}\right) = 2.34 \text{ mol}(8.314 \text{ J/mol}\cdot\text{K})(549 \text{ K})\ln\left(\frac{15.0}{24.0}\right) = \boxed{-5.02 \text{ kJ}}$

Finally, for the adiabatic process $D \to A$: $Q = \boxed{0}$

$$\Delta E_{\text{int}} = nC_V(T_A - T_D) = 2.34 \text{ mol}\left[\frac{3}{2}(8.314 \text{ J/mol}\cdot\text{K})\right](720 - 549) \text{ K} = \boxed{+4.98 \text{ kJ}}$$

and $W = -Q + \Delta E_{\text{int}} = 0 + 4.98 \text{ kJ} = \boxed{+4.98 \text{ kJ}}$

Process	Q (kJ)	W (kJ)	ΔE_{int} (kJ)
$A \to B$	+6.58	−6.58	0
$B \to C$	0	−4.98	−4.98
$C \to D$	−5.02	+5.02	0
$D \to A$	0	+4.98	+4.98
ABCDA	+1.56	−1.56	0

The work done *by* the engine is the negative of the work input. The output work W_{eng} is given by the work column in the table with all signs reversed.

(c) $e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{-W_{ABCD}}{Q_{A\to B}} = \frac{1.56 \text{ kJ}}{6.58 \text{ kJ}} = 0.237$ or $\boxed{23.7\%}$

$e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{549}{720} = 0.237$ or $\boxed{23.7\%}$

P22.21 (a) For a complete cycle, $\Delta E_{\text{int}} = 0$ and $W = |Q_h| - |Q_c| = |Q_c|\left[\frac{|(Q_h)|}{|Q_c|} - 1\right]$

The text shows that for a Carnot cycle (and only for a reversible cycle) $\frac{|Q_h|}{|Q_c|} = \frac{T_h}{T_c}$

Therefore,

$$W = \boxed{|Q_c|\left[\frac{T_h - T_c}{T_c}\right]}$$

(b) We have the definition of the coefficient of performance for a refrigerator, $\text{COP} = \frac{|Q_c|}{W}$

Using the result from part (a), this becomes $\text{COP} = \boxed{\frac{T_c}{T_h - T_c}}$

P22.22 $(\text{COP})_{\text{heat pump}} = \dfrac{|Q_c|+W}{W} = \dfrac{T_h}{\Delta T} = \dfrac{295}{25} = \boxed{11.8}$

P22.23 $(\text{COP})_{\text{Carnot refrig}} = \dfrac{T_c}{\Delta T} = \dfrac{4.00}{289} = 0.013\,8 = \dfrac{|Q_c|}{W}$

$\therefore W = \boxed{72.2\text{ J}}$ per 1 J energy removed by heat.

P22.24 $\text{COP} = 0.100\text{COP}_{\text{Carnot cycle}}$

or

$$\frac{|Q_h|}{W} = 0.100\left(\frac{|Q_h|}{W}\right)_{\text{Carnot cycle}} = 0.100\left(\frac{1}{\text{Carnot efficiency}}\right)$$

$$\frac{|Q_h|}{W} = 0.100\left(\frac{T_h}{T_h - T_c}\right) = 0.100\left(\frac{293\text{ K}}{293\text{ K} - 268\text{ K}}\right) = 1.17$$

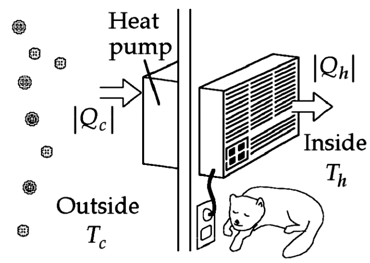


FIG. P22.24

Thus, $\boxed{\text{1.17 joules of energy enter the room by heat for each joule of work done.}}$

***P22.25** $\dfrac{|Q_c|}{W} = \text{COP}_\text{C}\ (\text{refrigerator}) = \dfrac{T_c}{T_h - T_c} = \dfrac{|Q_c|/\Delta t}{W/\Delta t}$

$$\frac{0.150\text{ W}}{W/\Delta t} = \frac{260\text{ K}}{40.0\text{ K}}$$

$$\mathcal{P} = \frac{W}{\Delta t} = 0.150\text{ W}\left(\frac{40.0\text{ K}}{260\text{ K}}\right) = \boxed{23.1\text{ mW}}$$

P22.26 $e = \dfrac{W}{Q_h} = 0.350 \qquad W = 0.350Q_h$

$Q_h = W + Q_c \qquad Q_c = 0.650Q_h$

$$\text{COP}(\text{refrigerator}) = \frac{Q_c}{W} = \frac{0.650Q_h}{0.350Q_h} = \boxed{1.86}$$

Section 22.5 **Gasoline and Diesel Engines**

P22.27 (a) $P_iV_i^\gamma = P_fV_f^\gamma$

$$P_f = P_i\left(\frac{V_i}{V_f}\right)^\gamma = \left(3.00\times10^6\text{ Pa}\right)\left(\frac{50.0\text{ cm}^3}{300\text{ cm}^3}\right)^{1.40} = \boxed{244\text{ kPa}}$$

(b) $W = \displaystyle\int_{V_i}^{V_f} P\,dV \qquad P = P_i\left(\frac{V_i}{V}\right)^\gamma$

Integrating,

$$W = \left(\frac{1}{\gamma-1}\right)P_iV_i\left[1-\left(\frac{V_i}{V_f}\right)^{\gamma-1}\right]$$

$$= (2.50)\left(3.00\times10^6\text{ Pa}\right)\left(5.00\times10^{-5}\text{ m}^3\right)\left[1-\left(\frac{50.0\text{ cm}^3}{300\text{ cm}^3}\right)^{0.400}\right]$$

$$= \boxed{192\text{ J}}$$

P22.28 (a), (b) The quantity of gas is

$$n = \frac{P_A V_A}{RT_A} = \frac{(100\times 10^3\ \text{Pa})(500\times 10^{-6}\ \text{m}^3)}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})} = 0.020\,5\ \text{mol}$$

$$E_{\text{int},A} = \frac{5}{2}nRT_A = \frac{5}{2}P_A V_A = \frac{5}{2}(100\times 10^3\ \text{Pa})(500\times 10^{-6}\ \text{m}^3) = \boxed{125\ \text{J}}$$

In process AB, $P_B = P_A\left(\frac{V_A}{V_B}\right)^{\gamma} = (100\times 10^3\ \text{Pa})(8.00)^{1.40} = \boxed{1.84\times 10^6\ \text{Pa}}$

$$T_B = \frac{P_B V_B}{nR} = \frac{(1.84\times 10^6\ \text{Pa})(500\times 10^{-6}\ \text{m}^3/8.00)}{(0.020\,5\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})} = \boxed{673\ \text{K}}$$

$$E_{\text{int},B} = \frac{5}{2}nRT_B = \frac{5}{2}(0.020\,5\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(673\ \text{K}) = \boxed{287\ \text{J}}$$

so $\Delta E_{\text{int},AB} = 287\ \text{J} - 125\ \text{J} = \boxed{162\ \text{J}} = Q - W_{\text{out}} = 0 - W_{\text{out}}$ $\quad W_{AB} = \boxed{-162\ \text{J}}$

Process BC takes us to:

$$P_C = \frac{nRT_C}{V_C} = \frac{(0.020\,5\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(1\,023\ \text{K})}{62.5\times 10^{-6}\ \text{m}^3} = \boxed{2.79\times 10^6\ \text{Pa}}$$

$$E_{\text{int},C} = \frac{5}{2}nRT_C = \frac{5}{2}(0.020\,5\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(1\,023\ \text{K}) = \boxed{436\ \text{J}}$$

$$E_{\text{int},BC} = 436\ \text{J} - 287\ \text{J} = \boxed{149\ \text{J}} = Q - W_{\text{out}} = Q - 0$$

$$Q_{BC} = \boxed{149\ \text{J}}$$

In process CD:

$$P_D = P_C\left(\frac{V_C}{V_D}\right)^{\gamma} = (2.79\times 10^6\ \text{Pa})\left(\frac{1}{8.00}\right)^{1.40} = \boxed{1.52\times 10^5\ \text{Pa}}$$

$$T_D = \frac{P_D V_D}{nR} = \frac{(1.52\times 10^5\ \text{Pa})(500\times 10^{-6}\ \text{m}^3)}{(0.020\,5\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})} = \boxed{445\ \text{K}}$$

$$E_{\text{int},D} = \frac{5}{2}nRT_D = \frac{5}{2}(0.020\,5\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(445\ \text{K}) = \boxed{190\ \text{J}}$$

$$\Delta E_{\text{int},CD} = 190\ \text{J} - 436\ \text{J} = \boxed{-246\ \text{J}} = Q - W_{\text{out}} = 0 - W_{\text{out}}$$

$$W_{CD} = \boxed{246\ \text{J}}$$

and $\Delta E_{\text{int},DA} = E_{\text{int},A} - E_{\text{int},D} = 125\ \text{J} - 190\ \text{J} = \boxed{-65.0\ \text{J}} = Q - W_{\text{out}} = Q - 0$

$$Q_{DA} = \boxed{-65.0\ \text{J}}$$

For the entire cycle, $\Delta E_{\text{int,net}} = 162\ \text{J} + 149 - 246 - 65.0 = \boxed{0}$. The net work is

$$W_{\text{eng}} = -162\ \text{J} + 0 + 246\ \text{J} + 0 = \boxed{84.3\ \text{J}}$$

$$Q_{\text{net}} = 0 + 149\ \text{J} + 0 - 65.0\ \text{J} = \boxed{84.3\ \text{J}}$$

continued on next page

The tables look like:

State	T (K)	P (kPa)	V (cm^3)	E_{int} (J)
A	293	100	500	125
B	673	1 840	62.5	287
C	1 023	2 790	62.5	436
D	445	152	500	190
A	293	100	500	125

Process	Q (J)	output W (J)	ΔE_{int} (J)
AB	0	−162	162
BC	149	0	149
CD	0	246	−246
DA	−65.0	0	−65.0
$ABCDA$	84.3	84.3	0

(c) The input energy is $Q_h = \boxed{149 \text{ J}}$, the waste is $|Q_c| = \boxed{65.0 \text{ J}}$, and $W_{\text{eng}} = \boxed{84.3 \text{ J}}$.

(d) The efficiency is: $e = \dfrac{W_{\text{eng}}}{Q_h} = \dfrac{84.3 \text{ J}}{149 \text{ J}} = \boxed{0.565}$

(e) Let f represent the angular speed of the crankshaft. Then $\dfrac{f}{2}$ is the frequency at which we obtain work in the amount of 84.3 J/cycle:

$$1\,000 \text{ J/s} = \left(\frac{f}{2}\right)(84.3 \text{ J/cycle})$$

$$f = \frac{2\,000 \text{ J/s}}{84.3 \text{ J/cycle}} = 23.7 \text{ rev/s} = \boxed{1.42 \times 10^3 \text{ rev/min}}$$

P22.29 Compression ratio = 6.00, $\gamma = 1.40$

(a) Efficiency of an Otto-engine $e = 1 - \left(\dfrac{V_2}{V_1}\right)^{\gamma - 1}$

$$e = 1 - \left(\frac{1}{6.00}\right)^{0.400} = \boxed{51.2\%}$$

(b) If actual efficiency $e' = 15.0\%$ losses in system are $e - e' = \boxed{36.2\%}$

Section 22.6 **Entropy**

P22.30 For a freezing process,

$$\Delta S = \frac{\Delta Q}{T} = \frac{-(0.500 \text{ kg})(3.33 \times 10^5 \text{ J/kg})}{273 \text{ K}} = \boxed{-610 \text{ J/K}}$$

***P22.31** The process of raising the temperature of the sample in this way is reversible, because an infinitesimal change would make δ negative, and energy would flow out instead of in. Then we may find the entropy change of the sample as

$$\Delta S = \int_{T_i}^{T_f} dS = \int_{T_i}^{T_f} \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{mc\,dT}{T} = mc \ln T \Big|_{T_i}^{T_f} = mc\left[\ln T_f - \ln T_i\right] = mc \ln(T_f/T_i)$$

***P22.32** (a) The process is isobaric because it takes place under constant atmospheric pressure. As described by Newton's third law, the stewing syrup must exert the same force on the air as the air exerts on it. The heating process is not adiabatic (because energy goes in by heat), isothermal (T goes up), isovolumetric (it likely expands a bit), cyclic (it is different at the end), or isentropic (entropy increases). It could be made as nearly reversible as you wish, by not using a kitchen stove but a heater kept always just incrementally higher in temperature than the syrup. The process would then also be eternal, and impractical for food production.

(b) The final temperature is

$$220°\text{F} = 212°\text{F} + 8°\text{F} = 100°\text{C} + 8°\text{F}\left(\frac{100 - 0°\text{C}}{212 - 32°\text{F}}\right) = 104°\text{C}$$

For the mixture,

$$\begin{aligned} Q &= m_1c_1\Delta T + m_2c_2\Delta T \\ &= (900\ \text{g}\ 1\ \text{cal/g}\cdot°\text{C} + 930\ \text{g}\ 0.299\ \text{cal/g}\cdot°\text{C})(104.4°\text{C} - 23°\text{C}) \\ &= 9.59\times10^4\ \text{cal} = \boxed{4.02\times10^5\ \text{J}} \end{aligned}$$

(c) Consider the reversible heating process described in part (a):

$$\begin{aligned} \Delta S &= \int_i^f \frac{dQ}{T} = \int_i^f \frac{(m_1c_1 + m_2c_2)dT}{T} = (m_1c_1 + m_2c_2)\ln\frac{T_f}{T_i} \\ &= [900(1) + 930(0.299)](\text{cal/°C})\left(\frac{4.186\ \text{J}}{1\ \text{cal}}\right)\left(\frac{1°\text{C}}{1\ \text{K}}\right)\ln\left(\frac{273+104}{273+23}\right) \\ &= (4\,930\ \text{J/K})0.243 = \boxed{1.20\times10^3\ \text{J/K}} \end{aligned}$$

P22.33 $$\Delta S = \int_i^f \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{mcdT}{T} = mc\ln\left(\frac{T_f}{T_i}\right)$$

$$\Delta S = 250\ \text{g}(1.00\ \text{cal/g}\cdot°\text{C})\ln\left(\frac{353}{293}\right) = 46.6\ \text{cal/K} = \boxed{195\ \text{J/K}}$$

Section 22.7 **Entropy Changes in Irreversible Processes**

P22.34 $$\Delta S = \frac{Q_2}{T_2} - \frac{Q_1}{T_1} = \left(\frac{1\,000}{290} - \frac{1\,000}{5\,700}\right)\ \text{J/K} = \boxed{3.27\ \text{J/K}}$$

P22.35 The car ends up in the same thermodynamic state as it started, so it undergoes zero changes in entropy. The original kinetic energy of the car is transferred by heat to the surrounding air, adding to the internal energy of the air. Its change in entropy is

$$\Delta S = \frac{\frac{1}{2}mv^2}{T} = \frac{750(20.0)^2}{293}\ \text{J/K} = \boxed{1.02\ \text{kJ/K}}$$

***P22.36** Define T_1 = Temp Cream = 5.00°C = 278 K. Define T_2 = Temp Coffee = 60.0°C = 333 K

The final temperature of the mixture is: $T_f = \dfrac{(20.0\ \text{g})T_1 + (200\ \text{g})T_2}{220\ \text{g}} = 55.0°\text{C} = 328\ \text{K}$

The entropy change due to this mixing is $\Delta S = (20.0\ \text{g})\int_{T_1}^{T_f} \dfrac{c_V dT}{T} + (200\ \text{g})\int_{T_2}^{T_f} \dfrac{c_V dT}{T}$

$$\Delta S = (84.0\ \text{J/K})\ln\left(\frac{T_f}{T_1}\right) + (840\ \text{J/K})\ln\left(\frac{T_f}{T_2}\right) = (84.0\ \text{K/J})\ln\left(\frac{328}{278}\right) + (840\ \text{J/K})\ln\left(\frac{328}{333}\right)$$

$$\Delta S = \boxed{+1.18\ \text{J/K}}$$

P22.37 Sitting here writing, I convert chemical energy, in ordered molecules in food, into internal energy that leaves my body by heat into the room-temperature surroundings. My rate of energy output is equal to my metabolic rate,

$$2\,500\ \text{kcal/d} = \frac{2\,500 \times 10^3\ \text{cal}}{86\,400\ \text{s}}\left(\frac{4.186\ \text{J}}{1\ \text{cal}}\right) = 120\ \text{W}$$

My body is in steady state, changing little in entropy, as the environment increases in entropy at the rate

$$\frac{\Delta S}{\Delta t} = \frac{Q/T}{\Delta t} = \frac{Q/\Delta t}{T} = \frac{120\ \text{W}}{293\ \text{K}} = 0.4\ \text{W/K} \sim \boxed{1\ \text{W/K}}$$

When using powerful appliances or an automobile, my personal contribution to entropy production is much greater than the above estimate, based only on metabolism.

P22.38 $c_{\text{iron}} = 448\ \text{J/kg}\cdot°\text{C}$; $c_{\text{water}} = 4\,186\ \text{J/kg}\cdot°\text{C}$

$Q_{\text{cold}} = -Q_{\text{hot}}$: $4.00\ \text{kg}(4\,186\ \text{J/kg}\cdot°\text{C})(T_f - 10.0°\text{C}) = -(1.00\ \text{kg})(448\ \text{J/kg}\cdot°\text{C})(T_f - 900°\text{C})$

which yields $T_f = 33.2°\text{C} = 306.2\ \text{K}$

$$\Delta S = \int_{283\ \text{K}}^{306.2\ \text{K}} \frac{c_{\text{water}} m_{\text{water}} dT}{T} + \int_{1173\ \text{K}}^{306.2\ \text{K}} \frac{c_{\text{iron}} m_{\text{iron}} dT}{T}$$

$$\Delta S = c_{\text{water}} m_{\text{water}} \ln\left(\frac{306.2}{283}\right) + c_{\text{iron}} m_{\text{iron}} \ln\left(\frac{306.2}{1173}\right)$$

$$\Delta S = (4\,186\ \text{J/kg}\cdot\text{K})(4.00\ \text{kg})(0.078\,8) + (448\ \text{J/kg}\cdot\text{K})(1.00\ \text{kg})(-1.34)$$

$$\Delta S = \boxed{718\ \text{J/K}}$$

P22.39 $\Delta S = nR\ln\left(\dfrac{V_f}{V_i}\right) = R\ln 2 = \boxed{5.76\ \text{J/K}}$

There is $\boxed{\text{no change in temperature}}$ for an ideal gas.

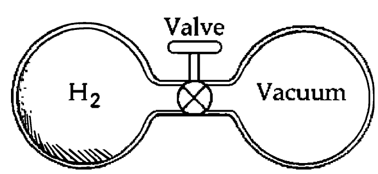


FIG. P22.39

P22.40 $\Delta S = nR\ln\left(\frac{V_f}{V_i}\right) = (0.044\,0)(2)R\ln 2$

$\Delta S = 0.088\,0(8.314)\ln 2 = \boxed{0.507\ \text{J/K}}$

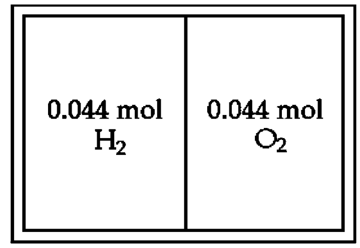


FIG. P22.40

Section 22.8 **Entropy on a Microscopic Scale**

P22.41 (a) A 12 can only be obtained $\boxed{\text{one}}$ way, as 6 + 6

(b) A 7 can be obtained $\boxed{\text{six}}$ ways: 6 + 1, 5 + 2, 4 + 3, 3 + 4, 2 + 5, 1 + 6

P22.42 (a) The table is shown below. On the basis of the table, the most probable recorded result of a toss is $\boxed{\text{2 heads and 2 tails}}$.

(b) The most ordered state is the least likely macrostate. Thus, on the basis of the table this is $\boxed{\text{either all heads or all tails}}$.

(c) The most disordered is the most likely macrostate. Thus, this is $\boxed{\text{2 heads and 2 tails}}$.

Result	Possible Combinations	Total
All heads	HHHH	1
3H, 1T	THHH, HTHH, HHTH, HHHT	4
2H, 2T	TTHH, THTH, THHT, HTTH, HTHT, HHTT	6
1H, 3T	HTTT, THTT, TTHT, TTTH	4
All tails	TTTT	1

P22.43 (a)

Result	Possible Combinations	Total
All red	RRR	1
2R, 1G	RRG, RGR, GRR	3
1R, 2G	RGG, GRG, GGR	3
All green	GGG	1

(b)

Result	Possible Combinations	Total
All red	RRRRR	1
4R, 1G	RRRRG, RRRGR, RRGRR, RGRRR, GRRRR	5
3R, 2G	RRRGG, RRGRG, RGRRG, GRRRG, RRGGR, RGRGR, GRRGR, RGGRR, GRGRR, GGRRR	10
2R, 3G	GGGRR, GGRGR, GRGGR, RGGGR, GGRRG, GRGRG, RGGRG, GRRGG, RGRGG, RRGGG	10
1R, 4G	RGGGG, GRGGG, GGRGG, GGGRG, GGGGR	5
All green	GGGGG	1

Additional Problems

P22.44 The conversion of gravitational potential energy into kinetic energy as the water falls is reversible. But the subsequent conversion into internal energy is not. We imagine arriving at the same final state by adding energy by heat, in amount mgy, to the water from a stove at a temperature infinitesimally above 20.0°C. Then,

$$\Delta S = \int \frac{dQ}{T} = \frac{Q}{T} = \frac{mgy}{T} = \frac{5\,000\text{ m}^3\left(1\,000\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)(50.0\text{ m})}{293\text{ K}} = \boxed{8.36\times 10^6\text{ J/K}}$$

***P22.45** For the Carnot engine, $e_c = 1 - \dfrac{T_c}{T_h} = 1 - \dfrac{300\text{ K}}{750\text{ K}} = 0.600$

Also, $\quad e_c = \dfrac{W_{\text{eng}}}{|Q_h|}$

so $\quad |Q_h| = \dfrac{W_{\text{eng}}}{e_c} = \dfrac{150\text{ J}}{0.600} = 250\text{ J}$

and $\quad |Q_c| = |Q_h| - W_{\text{eng}} = 250\text{ J} - 150\text{ J} = 100\text{ J}$

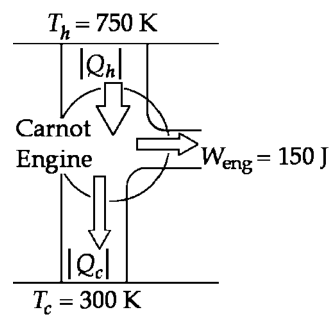


FIG. P22.45

(a) $|Q_h| = \dfrac{W_{\text{eng}}}{e_S} = \dfrac{150\text{ J}}{0.700} = \boxed{214\text{ J}}$

$|Q_c| = |Q_h| - W_{\text{eng}} = 214\text{ J} - 150\text{ J} = \boxed{64.3\text{ J}}$

(b) $|Q_{h,\text{net}}| = 214\text{ J} - 250\text{ J} = \boxed{-35.7\text{ J}}$

$|Q_{c,\text{net}}| = 64.3\text{ J} - 100\text{ J} = \boxed{-35.7\text{ J}}$

The net flow of energy by heat from the cold to the hot reservoir without work input, is impossible.

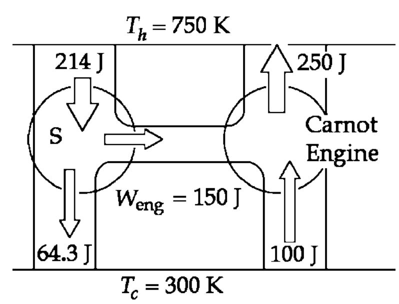


FIG. P22.45(b)

(c) For engine S: $\quad |Q_c| = |Q_h| - W_{\text{eng}} = \dfrac{W_{\text{eng}}}{e_S} - W_{\text{eng}}$

so $\quad W_{\text{eng}} = \dfrac{|Q_c|}{\frac{1}{e_S} - 1} = \dfrac{100\text{ J}}{\frac{1}{0.700} - 1} = \boxed{233\text{ J}}$

and $\quad |Q_h| = |Q_c| + W_{\text{eng}} = 233\text{ J} + 100\text{ J} = \boxed{333\text{ J}}$

(d) $|Q_{h,\text{net}}| = 333\text{ J} - 250\text{ J} = \boxed{83.3\text{ J}}$

$W_{\text{net}} = 233\text{ J} - 150\text{ J} = \boxed{83.3\text{ J}}$

$|Q_{c,\text{net}}| = \boxed{0}$

The output of 83.3 J of energy from the heat engine by work in a cyclic process without any exhaust by heat is impossible.

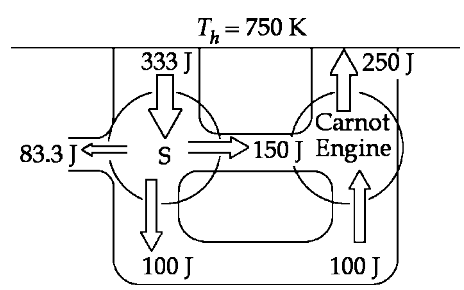


FIG. P22.45(d)

continued on next page

(e) Both engines operate in cycles, so $\Delta S_S = \Delta S_{\text{Carnot}} = 0$

For the reservoirs, $\Delta S_h = -\frac{|Q_h|}{T_h}$ and $\Delta S_c = +\frac{|Q_c|}{T_c}$

Thus,

$$\Delta S_{\text{total}} = \Delta S_S + \Delta S_{\text{Carnot}} + \Delta S_h + \Delta S_c = 0 + 0 - \frac{83.3 \text{ J}}{750 \text{ K}} + \frac{0}{300 \text{ K}} = \boxed{-0.111 \text{ J/K}}$$

$\boxed{\text{A decrease in total entropy is impossible.}}$

***P22.46** (a) Let state i represent the gas before its compression and state f afterwards, $V_f = \frac{V_i}{8}$. For a diatomic ideal gas, $C_v = \frac{5}{2}R$, $C_p = \frac{7}{2}R$, and $\gamma = \frac{C_p}{C_V} = 1.40$. Next,

$$P_i V_i^\gamma = P_f V_f^\gamma$$

$$P_f = P_i\left(\frac{V_i}{V_f}\right)^\gamma = P_i 8^{1.40} = 18.4P_i$$

$$P_i V_i = nRT_i$$

$$P_f V_f = \frac{18.4 P_i V_i}{8} = 2.30 P_i V_i = 2.30nRT_i = nRT_f$$

so $T_f = 2.30T_i$

$$\Delta E_{\text{int}} = nC_V\Delta T = n\frac{5}{2}R\left(T_f - T_i\right) = \frac{5}{2}nR1.30T_i = \frac{5}{2}1.30P_iV_i$$

$$= \frac{5}{2}1.30\left(1.013\times10^5 \text{ N/m}^2\right)0.12\times10^{-3} \text{ m}^3 = 39.4 \text{ J}$$

Since the process is adiabatic, $Q = 0$ and $\Delta E_{\text{int}} = Q + W$ gives $W = \boxed{39.4 \text{ J}}$

(b) The moment of inertia of the wheel is $I = \frac{1}{2}MR^2 = \frac{1}{2}5.1 \text{ kg}(0.085 \text{ m})^2 = 0.018\ 4 \text{ kg}\cdot\text{m}^2$

We want the flywheel to do work 39.4 J, so the work on the flywheel should be −39.4 J:

$$K_{\text{rot }i} + W = K_{\text{rot }f}$$

$$\frac{1}{2}I\omega_i^2 - 39.4 \text{ J} = 0$$

$$\omega_i = \left(\frac{2(39.4 \text{ J})}{0.018\ 4 \text{ kg}\cdot\text{m}^2}\right)^{1/2} = \boxed{65.4 \text{ rad/s}}$$

(c) Now we want $W = 0.05K_{\text{rot }i}$

$$39.4J = 0.05\frac{1}{2}0.018\ 4 \text{ kg}\cdot\text{m}^2\omega_i^2$$

$$\omega = \left(\frac{2(789 \text{ J})}{0.018\ 4 \text{ kg}\cdot\text{m}^2}\right)^{1/2} = \boxed{293 \text{ rad/s}}$$

P22.47 (a) $\mathcal{P}_{\text{electric}} = \dfrac{H_{ET}}{\Delta t}$ so if all the electric energy is converted into internal energy, the steady-state condition of the house is described by $H_{ET} = |Q|$.

Therefore,

$$\mathcal{P}_{\text{electric}} = \frac{Q}{\Delta t} = \boxed{5\,000\text{ W}}$$

(b) For a heat pump, $(\text{COP})_{\text{Carnot}} = \dfrac{T_h}{\Delta T} = \dfrac{295\text{ K}}{27\text{ K}} = 10.92$

$$\text{Actual COP} = 0.6(10.92) = 6.55 = \frac{|Q_h|}{W} = \frac{|Q_h|/\Delta t}{W/\Delta t}$$

Therefore, to bring 5 000 W of energy into the house only requires input power

$$\mathcal{P}_{\text{heat pump}} = \frac{W}{\Delta t} = \frac{|Q_h|/\Delta t}{\text{COP}} = \frac{5\,000\text{ W}}{6.56} = \boxed{763\text{ W}}$$

P22.48 $\Delta S_{\text{hot}} = \dfrac{-1\,000\text{ J}}{600\text{ K}}$

$\Delta S_{\text{cold}} = \dfrac{+750\text{ J}}{350\text{ K}}$

(a) $\Delta S_U = \Delta S_{\text{hot}} + \Delta S_{\text{cold}} = \boxed{0.476\text{ J/K}}$

(b) $e_c = 1 - \dfrac{T_1}{T_2} = 0.417$

$W_{\text{eng}} = e_c|Q_h| = 0.417(1\,000\text{ J}) = \boxed{417\text{ J}}$

(c) $W_{\text{net}} = 417\text{ J} - 250\text{ J} = 167\text{ J}$

$T_1\Delta S_U = 350\text{ K}(0.476\text{ J/K}) = \boxed{167\text{ J}}$

P22.49 (a) For an isothermal process, $Q = nRT\ln\left(\dfrac{V_2}{V_1}\right)$

Therefore, $Q_1 = nR(3T_i)\ln 2$

and $Q_3 = nR(T_i)\ln\left(\dfrac{1}{2}\right)$

For the constant volume processes, $Q_2 = \Delta E_{\text{int},\,2} = \dfrac{3}{2}nR(T_i - 3T_i)$

and $Q_4 = \Delta E_{\text{int},\,4} = \dfrac{3}{2}nR(3T_i - T_i)$

The net energy by heat transferred is then

$$Q = Q_1 + Q_2 + Q_3 + Q_4$$

or $Q = \boxed{2nRT_i\ln 2}$

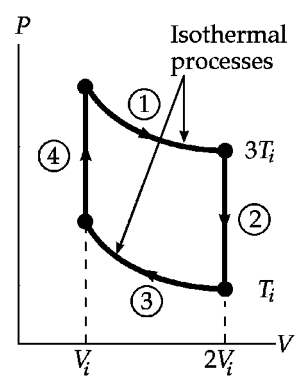


FIG. P22.49

(b) A positive value for heat represents energy transferred into the system.

Therefore, $|Q_h| = Q_1 + Q_4 = 3nRT_i(1 + \ln 2)$

Since the change in temperature for the complete cycle is zero,

$$\Delta E_{\text{int}} = 0 \text{ and } W_{\text{eng}} = Q$$

Therefore, the efficiency is $e_c = \dfrac{W_{\text{eng}}}{|Q_h|} = \dfrac{Q}{|Q_h|} = \dfrac{2\ln 2}{3(1+\ln 2)} = \boxed{0.273}$

P22.50 (a) $35.0°\text{F} = \frac{5}{9}(35.0 - 32.0)°\text{C} = (1.67 + 273.15)\ \text{K} = 274.82\ \text{K}$

$$98.6°\text{F} = \frac{5}{9}(98.6 - 32.0)°\text{C} = (37.0 + 273.15)\ \text{K} = 310.15\ \text{K}$$

$$\Delta S_{\text{ice water}} = \int \frac{dQ}{T} = (453.6\ \text{g})(1.00\ \ \text{cal/g}\cdot\text{K}) \times \int_{274.82}^{310.15} \frac{dT}{T} = 453.6 \ln\left(\frac{310.15}{274.82}\right) = 54.86\ \ \text{cal/K}$$

$$\Delta S_{\text{body}} = -\frac{|Q|}{T_{\text{body}}} = -(453.6)(1.00)\frac{(310.15 - 274.82)}{310.15} = -51.67\ \ \text{cal/K}$$

$$\Delta S_{\text{system}} = 54.86 - 51.67 = \boxed{3.19\ \ \text{cal/K}}$$

(b) $(453.6)(1)(T_F - 274.82) = (70.0 \times 10^3)(1)(310.15 - T_F)$

Thus,

$$(70.0 + 0.453\,6) \times 10^3 T_F = \left[(70.0)(310.15) + (0.453\,6)(274.82)\right] \times 10^3$$

and $T_F = 309.92\ \text{K} = 36.77°\text{C} = \boxed{98.19°\text{F}}$

$$\Delta S'_{\text{ice water}} = 453.6 \ln\left(\frac{309.92}{274.82}\right) = 54.52\ \ \text{cal/K}$$

$$\Delta S'_{\text{body}} = -(70.0 \times 10^3)\ln\left(\frac{310.15}{309.92}\right) = -51.93\ \ \text{cal/K}$$

$$\Delta S'_{\text{sys}} = 54.52 - 51.93 = \boxed{2.59\ \ \text{cal/K}}$$

$\boxed{\text{This is significantly less than the estimate in part (a).}}$

P22.51 $e_c = 1 - \frac{T_c}{T_h} = \frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}}/\Delta t}{|Q_h|/\Delta t}$: $\quad \frac{|Q_h|}{\Delta t} = \frac{\mathcal{P}}{(1 - T_c/T_h)} = \frac{\mathcal{P}T_h}{T_h - T_c}$

$|Q_h| = W_{\text{eng}} + |Q_c|$: $\quad \frac{|Q_c|}{\Delta t} = \frac{|Q_h|}{\Delta t} - \frac{W_{\text{eng}}}{\Delta t}$

$$\frac{|Q_c|}{\Delta t} = \frac{\mathcal{P}T_h}{T_h - T_c} - \mathcal{P} = \frac{\mathcal{P}T_c}{T_h - T_c}$$

$|Q_c| = mc\Delta T$: $\quad \frac{|Q_c|}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right)c\Delta T = \frac{\mathcal{P}T_c}{T_h - T_c}$

$$\frac{\Delta m}{\Delta t} = \frac{\mathcal{P}T_c}{(T_h - T_c)c\Delta T}$$

$$\frac{\Delta m}{\Delta t} = \frac{(1.00 \times 10^9\ \text{W})(300\ \text{K})}{200\ \text{K}(4\,186\ \ \text{J/kg}\cdot°\text{C})(6.00°\text{C})} = \boxed{5.97 \times 10^4\ \ \text{kg/s}}$$

P22.52 $e_c = 1 - \frac{T_c}{T_h} = \frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}} / \Delta t}{|Q_h| / \Delta t}$ $\qquad \frac{|Q_h|}{\Delta t} = \frac{\mathcal{P}}{1-(T_c / T_h)} = \frac{\mathcal{P} T_h}{T_h - T_c}$

$$\frac{|Q_c|}{\Delta t} = \left(\frac{|Q_h|}{\Delta t}\right) - \mathcal{P} = \frac{\mathcal{P} T_c}{T_h - T_c}$$

$|Q_c| = mc\Delta T$, where c is the specific heat of water.

Therefore, $$\frac{|Q_c|}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right) c\Delta T = \frac{\mathcal{P} T_c}{T_h - T_c}$$

and $$\frac{\Delta m}{\Delta t} = \boxed{\frac{\mathcal{P} T_c}{(T_h - T_c) c\Delta T}}$$

We test for dimensional correctness by identifying the units of the right-hand side:

$\frac{\text{W} \cdot {}^\circ\text{C}}{{}^\circ\text{C}(\text{J/kg} \cdot {}^\circ\text{C}){}^\circ\text{C}} = \frac{(\text{J/s})\text{kg}}{\text{J}} = \text{kg/s}$, as on the left hand side. Think of yourself as a power-company engineer arranging to have enough cooling water to carry off your thermal pollution. If the plant power $\mathcal{P}$ increases, the required flow rate increases in direct proportion. If environmental regulations require a smaller temperature change ΔT, then the required flow rate increases again, now in inverse proportion. Next note that T_h is in the bottom of the fraction. This means that if you can run the reactor core or firebox hotter, the required coolant flow rate decreases! If the turbines take in steam at higher temperature, they can be made more efficient to reduce waste heat output.

P22.53 Like a refrigerator, an air conditioner has as its purpose the removal of energy by heat from the cold reservoir.

Its ideal COP is $\text{COP}_{\text{Carnot}} = \dfrac{T_c}{T_h - T_c} = \dfrac{280\text{ K}}{20\text{ K}} = 14.0$

(a) Its actual COP is $0.400(14.0) = 5.60 = \dfrac{|Q_c|}{|Q_h| - |Q_c|} = \dfrac{|Q_c/\Delta t|}{|Q_h/\Delta t| - |Q_c/\Delta t|}$

$$5.60\left|\frac{Q_h}{\Delta t}\right| - 5.60\left|\frac{Q_c}{\Delta t}\right| = \left|\frac{Q_c}{\Delta t}\right|$$

$$5.60(10.0\text{ kW}) = 6.60\left|\frac{Q_c}{\Delta t}\right| \quad \text{and} \quad \left|\frac{Q_c}{\Delta t}\right| = \boxed{8.48\text{ kW}}$$

(b) $|Q_h| = W_{\text{eng}} + |Q_c|$: $\dfrac{W_{\text{eng}}}{\Delta t} = \left|\dfrac{Q_h}{\Delta t}\right| - \left|\dfrac{Q_c}{\Delta t}\right| = 10.0\text{ kW} - 8.48\text{ kW} = \boxed{1.52\text{ kW}}$

(c) The air conditioner operates in a cycle, so the entropy of the working fluid does not change. The hot reservoir increases in entropy by

$$\frac{|Q_h|}{T_h} = \frac{(10.0\times10^3\text{ J/s})(3\,600\text{ s})}{300\text{ K}} = 1.20\times10^5\text{ J/K}$$

The cold room decreases in entropy by

$$\Delta S = -\frac{|Q_c|}{T_c} = -\frac{(8.48\times10^3\text{ J/s})(3\,600\text{ s})}{280\text{ K}} = -1.09\times10^5\text{ J/K}$$

The net entropy change is positive, as it must be:

$$+1.20\times10^5\text{ J/K} - 1.09\times10^5\text{ J/K} = \boxed{1.09\times10^4\text{ J/K}}$$

(d) The new ideal COP is $\text{COP}_{\text{Carnot}} = \dfrac{T_c}{T_h - T_c} = \dfrac{280\text{ K}}{25\text{ K}} = 11.2$

We suppose the actual COP is $0.400(11.2) = 4.48$

As a fraction of the original 5.60, this is $\dfrac{4.48}{5.60} = 0.800$, so the fractional change is to $\boxed{\text{drop by }20.0\%}$.

***P22.54** (a) For the isothermal process AB, the work on the gas is

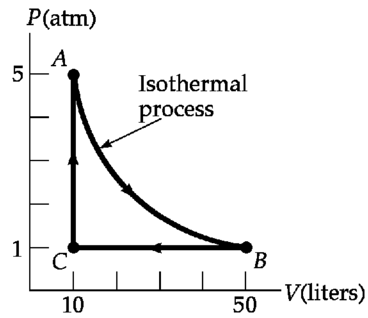


FIG. P22.54

$$W_{AB} = -P_A V_A \ln\left(\frac{V_B}{V_A}\right)$$

$$W_{AB} = -5\left(1.013\times 10^5 \text{ Pa}\right)\left(10.0\times 10^{-3} \text{ m}^3\right)\ln\left(\frac{50.0}{10.0}\right)$$

$$W_{AB} = -8.15\times 10^3 \text{ J}$$

where we have used $1.00 \text{ atm} = 1.013\times 10^5 \text{ Pa}$

and $1.00 \text{ L} = 1.00\times 10^{-3} \text{ m}^3$

$$W_{BC} = -P_B\Delta V = -\left(1.013\times 10^5 \text{ Pa}\right)\left[(10.0-50.0)\times 10^{-3}\right]\text{m}^3 = +4.05\times 10^3 \text{ J}$$

$W_{CA} = 0$ and $W_{\text{eng}} = -W_{AB} - W_{BC} = 4.10\times 10^3 \text{ J} = \boxed{4.10 \text{ kJ}}$

(b) Since AB is an isothermal process, $\Delta E_{\text{int}, AB} = 0$

and $Q_{AB} = -W_{AB} = 8.15\times 10^3 \text{ J}$

For an ideal monatomic gas, $C_V = \frac{3R}{2}$ and $C_P = \frac{5R}{2}$

$$T_B = T_A = \frac{P_B V_B}{nR} = \frac{\left(1.013\times 10^5\right)\left(50.0\times 10^{-3}\right)}{R} = \frac{5.06\times 10^3}{R}$$

Also, $$T_C = \frac{P_C V_C}{nR} = \frac{\left(1.013\times 10^5\right)\left(10.0\times 10^{-3}\right)}{R} = \frac{1.01\times 10^3}{R}$$

$$Q_{CA} = nC_V\Delta T = 1.00\left(\frac{3}{2}R\right)\left(\frac{5.06\times 10^3 - 1.01\times 10^3}{R}\right)$$

$$= 6.08 \text{ kJ}$$

so the total energy absorbed by heat is $Q_{AB} + Q_{CA} = 8.15 \text{ kJ} + 6.08 \text{ kJ} = \boxed{14.2 \text{ kJ}}$

(c) $Q_{BC} = nC_P\Delta T = \frac{5}{2}(nR\Delta T) = \frac{5}{2}P_B\Delta V_{BC}$

$$Q_{BC} = \frac{5}{2}\left(1.013\times 10^5\right)\left[(10.0-50.0)\times 10^{-3}\right] = -1.01\times 10^4 \text{ J} = \boxed{-10.1 \text{ kJ}}$$

(d) $e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}}}{Q_{AB}+Q_{CA}} = \frac{4.10\times 10^3 \text{ J}}{1.42\times 10^4 \text{ J}} = 0.288$ or $\boxed{28.8\%}$

(e) A Carnot engine operating between $T_{\text{hot}} = T_A = 5060/R$ and $T_{\text{cold}} = T_C = 1010/R$ has efficiency $1 - T_c/T_h = 1 - 1/5 = 80.0\%$.

$\boxed{\text{The three-process engine considered in this problem has much lower efficiency.}}$

***P22.55** At point A, $P_iV_i = nRT_i$ and $n = 1.00$ mol

At point B, $3P_iV_i = nRT_B$ so $T_B = 3T_i$

At point C, $(3P_i)(2V_i) = nRT_C$ and $T_C = 6T_i$

At point D, $P_i(2V_i) = nRT_D$ so $T_D = 2T_i$

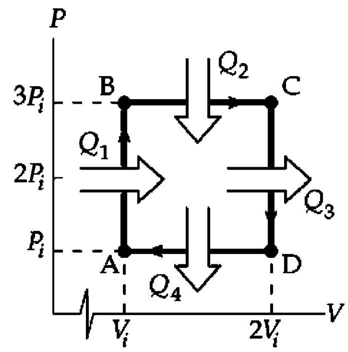


FIG. P22.55

The heat for each step in the cycle is found using $C_V = \dfrac{3R}{2}$ and $C_P = \dfrac{5R}{2}$:

$$Q_{AB} = nC_V(3T_i - T_i) = 3nRT_i$$
$$Q_{BC} = nC_P(6T_i - 3T_i) = 7.50nRT_i$$
$$Q_{CD} = nC_V(2T_i - 6T_i) = -6nRT_i$$
$$Q_{DA} = nC_P(T_i - 2T_i) = -2.50nRT_i$$

(a) Therefore, $Q_{\text{entering}} = |Q_h| = Q_{AB} + Q_{BC} = \boxed{10.5nRT_i}$

(b) $Q_{\text{leaving}} = |Q_c| = |Q_{CD} + Q_{DA}| = \boxed{8.50nRT_i}$

(c) Actual efficiency, $e = \dfrac{|Q_h| - |Q_c|}{|Q_h|} = \boxed{0.190}$

(d) Carnot efficiency, $e_c = 1 - \dfrac{T_c}{T_h} = 1 - \dfrac{T_i}{6T_i} = \boxed{0.833}$

$\boxed{\text{The Carnot efficiency is much higher.}}$

P22.56 $\Delta S = \displaystyle\int_i^f \frac{dQ}{T} = \int_i^f \frac{nC_P dT}{T} = nC_P \int_i^f T^{-1} dT = nC_P \ln T\Big|_{T_i}^{T_f} = nC_P(\ln T_f - \ln T_i) = nC_P \ln\left(\frac{T_f}{T_i}\right)$

$$\Delta S = nC_P \ln\left(\frac{PV_f}{nR}\frac{nR}{PV_i}\right) = \boxed{nC_P \ln 3}$$

P22.57 (a) The ideal gas at constant temperature keeps constant internal energy. As it puts out energy by work in expanding it must take in an equal amount of energy by heat. Thus its entropy increases. Let P_i, V_i, T_i represent the state of the gas before the isothermal expansion. Let P_C, V_C, T_i represent the state after this process, so that $P_iV_i = P_CV_C$. Let P_i, $3V_i$, T_f represent the state after the adiabatic compression.

Then $P_CV_C^\gamma = P_i(3V_i)^\gamma$

Substituting $P_C = \dfrac{P_iV_i}{V_C}$

gives $P_iV_iV_C^{\gamma-1} = P_i(3^\gamma V_i^\gamma)$

Then $V_C^{\gamma-1} = 3^\gamma V_i^{\gamma-1}$ and $\dfrac{V_C}{V_i} = 3^{\gamma/(\gamma-1)}$

The work output in the isothermal expansion is

$$W = \int_i^C P\,dV = nRT_i\int_i^C V^{-1}dV = nRT_i\ln\left(\frac{V_C}{V_i}\right) = nRT_i\ln\left(3^{\gamma/(\gamma-1)}\right) = nRT_i\left(\frac{\gamma}{\gamma-1}\right)\ln 3$$

This is also the input heat, so the entropy change is

$$\Delta S = \frac{Q}{T} = nR\left(\frac{\gamma}{\gamma-1}\right)\ln 3$$

Since $C_P = \gamma C_V = C_V + R$

we have $(\gamma-1)C_V = R$, $C_V = \dfrac{R}{\gamma-1}$

and $C_P = \dfrac{\gamma R}{\gamma-1}$

Then the result is $\boxed{\Delta S = nC_P\ln 3}$

(b) The pair of processes considered here carries the gas from the initial state in Problem 56 to the final state there. Entropy is a function of state. Entropy change does not depend on path. Therefore the entropy change in Problem 56 equals $\Delta S_{\text{isothermal}} + \Delta S_{\text{adiabatic}}$ in this problem. Since $\Delta S_{\text{adiabatic}} = 0$, the answers to Problems 56 and 57(a) must be the same.

***P22.58** (a) $W = \displaystyle\int_{V_i}^{V_f} P\,dV = nRT\int_{V_i}^{2V_i}\frac{dV}{V} = (1.00)RT\ln\left(\frac{2V_i}{V_i}\right) = \boxed{RT\ln 2}$

(b) While it lasts, this process does convert all of the energy input into work output. But the gas sample is in a different state at the end than it was at the beginning. The process cannot be done over unless the gas is recompressed by a work input. To be practical, a heat engine must operate in a cycle. The second law refers to a heat engine operating in a cycle, so this process is consistent with the second law of thermodynamics.

P22.59 The heat transfer over the paths CD and BA is zero since they are adiabatic.

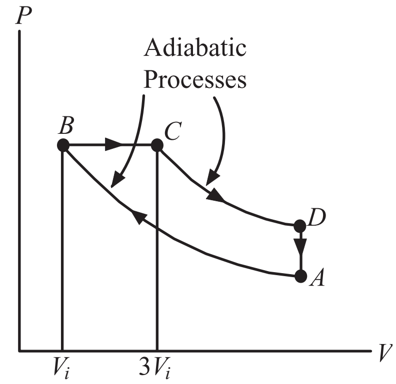


FIG. P22.59

Over path BC: $Q_{BC} = nC_P(T_C - T_B) > 0$

Over path DA: $Q_{DA} = nC_V(T_A - T_D) < 0$

Therefore, $|Q_c| = |Q_{DA}|$ and $Q_h = Q_{BC}$

The efficiency is then

$$e = 1 - \frac{|Q_c|}{Q_h} = 1 - \frac{(T_D - T_A)C_V}{(T_C - T_B)C_P}$$

$$e = 1 - \frac{1}{\gamma}\left[\frac{T_D - T_A}{T_C - T_B}\right]$$

P22.60 Simply evaluate the maximum (Carnot) efficiency.

$$e_C = \frac{\Delta T}{T_h} = \frac{4.00\text{ K}}{277\text{ K}} = 0.014\,4$$

The proposal does not merit serious consideration. Operating between these temperatures, this device could not attain so high an efficiency.

***P22.61** (a) $\boxed{20.0°\text{C}}$

(b) $\Delta S = mc\ln\frac{T_f}{T_1} + mc\ln\frac{T_f}{T_2} = 1.00\text{ kg}(4.19\text{ kJ/kg}\cdot\text{K})\left[\ln\frac{T_f}{T_1} + \ln\frac{T_f}{T_2}\right]$

$= (4.19\text{ kJ/K})\ln\left(\frac{293}{283}\cdot\frac{293}{303}\right)$

(c) $\Delta S = \boxed{+4.88\text{ J/K}}$

(d) $\boxed{\text{Yes, the mixing is irreversible. Entropy has increased.}}$

P22.62 (a) Use the equation of state for an ideal gas

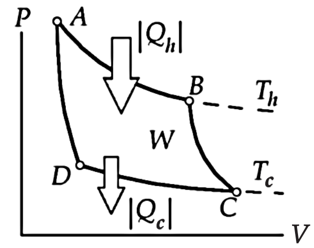


FIG. P22.62

$$V=\frac{nRT}{P}$$

$$V_A=\frac{1.00(8.314)(600)}{25.0\left(1.013\times10^5\right)}=\boxed{1.97\times10^{-3}\ \text{m}^3}$$

$$V_C=\frac{1.00(8.314)(400)}{1.013\times10^5}=\boxed{32.8\times10^{-3}\ \text{m}^3}$$

Since AB is isothermal, $P_AV_A=P_BV_B$

and since BC is adiabatic, $P_BV_B^{\gamma}=P_CV_C^{\gamma}$

Combining these expressions, $V_B=\left[\left(\frac{P_C}{P_A}\right)\frac{V_C^{\gamma}}{V_A}\right]^{1/(\gamma-1)}=\left[\left(\frac{1.00}{25.0}\right)\frac{\left(32.8\times10^{-3}\ \text{m}^3\right)^{1.40}}{1.97\times10^{-3}\ \text{m}^3}\right]^{(1/0.400)}$

$$V_B=\boxed{11.9\times10^{-3}\ \text{m}^3}$$

Similarly, $V_D=\left[\left(\frac{P_A}{P_C}\right)\frac{V_A^{\gamma}}{V_C}\right]^{1/(\gamma-1)}=\left[\left(\frac{25.0}{1.00}\right)\frac{\left(1.97\times10^{-3}\ \text{m}^3\right)^{1.40}}{32.8\times10^{-3}\ \text{m}^3}\right]^{(1/0.400)}$

or $V_D=\boxed{5.44\times10^{-3}\ \text{m}^3}$

Since AB is isothermal, $P_AV_A=P_BV_B$

and $P_B=P_A\left(\frac{V_A}{V_B}\right)=25.0\ \text{atm}\left(\frac{1.97\times10^{-3}\ \text{m}^3}{11.9\times10^{-3}\ \text{m}^3}\right)=\boxed{4.14\ \text{atm}}$

Also, CD is an isothermal and $P_D=P_C\left(\frac{V_C}{V_D}\right)=1.00\ \text{atm}\left(\frac{32.8\times10^{-3}\ \text{m}^3}{5.44\times10^{-3}\ \text{m}^3}\right)=\boxed{6.03\ \text{atm}}$

Solving part (c) before part (b):

(c) For this Carnot cycle, $e_c=1-\frac{T_c}{T_h}=1-\frac{400\ \text{K}}{600\ \text{K}}=\boxed{0.333}$

(b) Energy is added by heat to the gas during the process AB. For the isothermal process, $\Delta E_{\text{int}}=0$.

and the first law gives $Q_{AB}=-W_{AB}=nRT_h\ln\left(\frac{V_B}{V_A}\right)$

or $|Q_h|=Q_{AB}=1.00\ \text{mol}(8.314\ \text{J/mol}\cdot\text{K})(600\ \text{K})\ln\left(\frac{11.9}{1.97}\right)$

$$=8.97\ \text{kJ}$$

Then, from $e=\frac{W_{\text{eng}}}{|Q_h|}$

the net work done per cycle is $W_{\text{eng}}=e_c|Q_h|=0.333(8.97\ \text{kJ})=\boxed{2.99\ \text{kJ}}$

ANSWERS TO EVEN PROBLEMS

P22.2 13.7°C

P22.4 (a) 29.4 L/h (b) 185 hp (c) 527 N·m (d) 1.91×10^5 W

P22.6 (a) 24.0 J (b) 144 J

P22.8 (a) 2.93 (b) coefficient of performance for a refrigerator (c) The cost for air conditioning is half as much for an air conditioner with EER 10 compared with an air conditioner with EER 5.

P22.10 (a) 870 MJ (b) 330 MJ

P22.12 (a) 0.300 (b) 1.40×10^{-3} (c) 2.00×10^{-3}

P22.14 33.0%

P22.16 (a) 5.12% (b) 5.27 TJ/h (c) As fossil-fuel prices rise, this way to use solar energy will become a good buy.

P22.18 (a) $|Q_c|/\Delta t = 700\text{ kW}(T_h + 766\text{ K})/(T_h - 383\text{ K})$ The exhaust power decreases as the firebox temperature increases. (b) 1.87 MW (c) 3.84×10^3 K (d) No answer exists. The energy exhaust cannot be that small.

P22.20 (a)

State	P (kPa)	V (L)	T (K)
A	1 400	10.0	720
B	875	16.0	720
C	445	24.0	549
D	712	15.0	549

(b)

Process	Q (kJ)	W (kJ)	ΔE_{int} (kJ)
$A \rightarrow B$	6.58	−6.58	0
$B \rightarrow C$	0	−4.98	−4.98
$C \rightarrow D$	−5.02	5.02	0
$D \rightarrow A$	0	4.98	4.98
$ABCDA$	1.56	−1.56	0

(c) 23.7%; see the solution

P22.22 11.8

P22.24 1.17 J

P22.26 1.86

P22.28 (a), (b) see the solution (c) $Q_h = 149$ J; $|Q_c| = 65.0$ J; $W_{eng} = 84.3$ J (d) 56.5% (e) 1.42×10^3 rev/min

P22.30 −610 J/K

P22.32 (a) The process is isobaric because it takes place under constant atmospheric pressure. The heating process is not adiabatic (because energy goes in by heat), isothermal (T goes up), isovolumetric (it likely expands a bit), cyclic (it is different at the end), or isentropic (entropy increases). It could be made as nearly reversible as you wish, by not using a kitchen stove but a heater kept always just incrementally higher in temperature than the syrup. (b) 402 kJ (c) 1.20 kJ/K

P22.34 3.27 J/K

P22.36 +1.18 J/K

P22.38 718 J/K

P22.40 0.507 J/K

P22.42 (a) 2 heads and 2 tails (b) All heads or all tails (c) 2 heads and 2 tails

P22.44 8.36 MJ/K

P22.46 (a) 39.4 J (b) 65.4 rad/s = 625 rev/min (c) 293 rad/s = 2 790 rev/min

P22.48 (a) 0.476 J/K (b) 417 J (c) $W_{\text{net}} = T_1 \Delta S_U = 167$ J

P22.50 (a) 3.19 cal/K (b) 98.19ºF, 2.59 cal/K This is significantly less than the estimate in part (a).

P22.52 $\dfrac{\mathcal{P} T_c}{(T_h - T_c) c \Delta T}$

P22.54 (a) 4.10 kJ (b) 14.2 kJ (c) 10.1 kJ (d) 28.8% (e) The three-process engine considered in this problem has much lower efficiency than the Carnot efficiency.

P22.56 $nC_p \ln 3$

P22.58 (a) see the solution (b) While it lasts, this process does convert all of the energy input into work output. But the gas sample is in a different state at the end than it was at the beginning. The process cannot be done over unless the gas is recompressed by a work input. To be practical, a heat engine must operate in a cycle. The second law refers to a heat engine operating in a cycle, so this process is consistent with the second law of thermodynamics.

P22.60 The proposal does not merit serious consideration. Operating between these temperatures, this device could not attain so high an efficiency.

P22.62 (a)

	P, atm	V, L
A	25.0	1.97
B	4.14	11.9
C	1.00	32.8
D	6.03	5.44

(b) 2.99kJ (c) 33.3%

23

Electric Fields

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q23.1 A neutral atom is one that has no net charge. This means that it has the same number of electrons orbiting the nucleus as it has protons in the nucleus. A negatively charged atom has one or more excess electrons.

***Q23.2** (i) Suppose the positive charge has the large value 1 μC. The object has lost some of its conduction electrons, in number 10^{-6} C (1 e/1.60×10^{-19} C) = 6.25×10^{12} and in mass 6.25×10^{12} (9.11×10^{-31} kg) = 5.69×10^{-18} kg. This is on the order of 10^{14} times smaller than the ~1 g mass of the coin, so it is an immeasurably small change. Answer (d).

(ii) The coin gains extra electrons, gaining mass on the order of 10^{-14} times its original mass for the charge -1 μC. Answer (b).

Q23.3 All of the constituents of air are nonpolar except for water. The polar water molecules in the air quite readily "steal" charge from a charged object, as any physics teacher trying to perform electrostatics demonstrations in the summer well knows. As a result—it is difficult to accumulate large amounts of excess charge on an object in a humid climate. During a North American winter, the cold, dry air allows accumulation of significant excess charge, giving the potential (pun intended) for a shocking (pun also intended) introduction to static electricity sparks.

Q23.4 Similarities: A force of gravity is proportional to the product of the intrinsic properties (masses) of two particles, and inversely proportional to the square of the separation distance. An electrical force exhibits the same proportionalities, with charge as the intrinsic property.

Differences: The electrical force can either attract or repel, while the gravitational force as described by Newton's law can only attract. The electrical force between elementary particles is vastly stronger than the gravitational force.

Q23.5 No. The balloon induces polarization of the molecules in the wall, so that a layer of positive charge exists near the balloon. This is just like the situation in Figure 23.4a, except that the signs of the charges are reversed. The attraction between these charges and the negative charges on the balloon is stronger than the repulsion between the negative charges on the balloon and the negative charges in the polarized molecules (because they are farther from the balloon), so that there is a net attractive force toward the wall. Ionization processes in the air surrounding the balloon provide ions to which excess electrons in the balloon can transfer, reducing the charge on the balloon and eventually causing the attractive force to be insufficient to support the weight of the balloon.