Introduction to Macroeconomic Theory II
The Graduate Center, CUNY, Spring 2023
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Due: April 3 (upload on Blackboard, by 11:59 pm)

Project

Consider the standard neoclassical growth model with an additive momentary utility function:

$$\max_{\{c_t, n_t, k_{t+1}\}_{t\in T}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\gamma}-1}{1-\gamma} + B\frac{(1-n_t)^{1-\mu}-1}{1-\mu} \right\}$$

$$\text{s.t. } c_t + k_{t+1} = (1-\delta)\,k_t + \theta_t k_t^{\alpha}\,(X_t n_t)^{1-\alpha},$$

$$\log\theta_t = \rho\log\theta_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}\left(0,\sigma^2\right),$$

with initial condition (k_0, θ_0) given. Here, $\beta \in (0,1)$ is the discount factor, $\delta \in (0,1)$ is the depreciation rate, c_t, n_t and $l_t = 1 - n_t$ are consumption, labor and leisure, respectively; γ, μ, $B > 0$ are the parameters of the utility function; k_t and n_t are aggregate capital and labor; X_t is labor-augmenting technological progress, growing at the rate g_x, i.e., $X_t = X_0 g_x^t$ (assume $X_0 = 1$).

1. Each of you need to introduce an additional exogenous shock to the model
$$\log z_t = \rho_z \log z_{t-1} + \varepsilon_t^z, \qquad \varepsilon_t^z \sim \mathcal{N}\left(0,\sigma_z^2\right), \tag{1}$$
however, each of you needs to do it in a different way. I will email each of you his/her own shock.

2. Because of labor-augmenting technological progress, the economy grows. Detrend the model by dividing the objective function and the constraint by X_t (for details see "Premier", pages 72-73).

3. Derive the FOCs for the model.

4. Calibrate the parameters of the model in the steady state to reproduce the observations on the US economy: the investment-to-output ratio $\left(\frac{i}{y}\right) = 0.2133$, the labor $n = 1/3$, and capital-output ratio $\left(\frac{k}{y}\right) = 10$. In the benchmark case, assume $\gamma = 1$, $\mu = 5$, $\sigma = 0.016$, $\rho = 0.95$, $g_x = 1.0029$. Since you do not know how to calibrate σ and ρ, try different values to achieve a better fit of the model to the data.

5. Modify the code of the article Maliar and Maliar (CE, 2005) (both the code and the paper are attached) to compute a solution to the model, namely, the decision functions for the model's variables $\{c_t, n_t, k_{t+1}\}$. Plot the constructed consumption, labor and capital functions.

6. Solve the model using the second-order perturbation implemented in Dynare.

7. Compute the accuracy of the solutions obtained using the PEA and Dynare. As a set of points for evaluating accuracy, $i = 1, ..., I$, take a set of simulated points, $t = 1, ..., 10,000$. In each simulated point, compute residuals $\mathcal{R}^{BC}(k_i, \theta_i)$, $\mathcal{R}^{EE}(k_i, \theta_i)$ and $\mathcal{R}^{MUL}(k_i, \theta_i)$. For the benchmark model, such residuals are defined as

$$\mathcal{R}^{BC}(k_i, \theta_i) = \frac{(1-\delta) k_i + \theta_i f(k_i, n_i)}{c_i + k_i'} - 1, \tag{2}$$

$$\mathcal{R}^{EE}(k_i, \theta_i) = \beta E\left\{\frac{u_1(c_i', n_i')}{u_1(c_i, n_i)}\left[1 - \delta + \theta_i' f_1(k_i', n_i')\right]\right\} - 1, \tag{3}$$

$$\mathcal{R}^{MUL}(k_i, \theta_i) = \frac{u_1(c_i, n_i)\,\theta_i f_2(k_i, n_i)}{u_2(c_i, n_i)} - 1. \tag{4}$$

8. To see whether the model can explain the regularities in the data, compare the predictions of the model with the statistics for the U.S. economy provided in the table below.

σ_c	σ_n	σ_i	σ_y
0.0085	0.0138	0.0426	0.0117
$corr(y, c)$	$corr(y, n)$	$corr(y, i)$	$corr(y, w)$
0.8581	0.8333	0.9124	0.3483

To do so, write a program that uses the constructed decision functions to run a large number of simulations and to compute the averages across simulated data sets. Take the length of one simulation 72 (this is the length of time-series on the U.S. economy used to compute the numbers in the table above).[1] The appropriate number of simulations will be 5000 or more (this is needed for the law of large numbers to work).

In every simulation $j = 1, ..., 5000$ start from the same initial condition (k_0, θ_0, z_0); as an initial state, assume the corresponding steady-state values. Draw a series of error terms $\{\varepsilon_t, \varepsilon_t^z\}_{t=1}^{72}$ by using a random number generator and generate the series $\{\theta_t, z_t\}_{t=1}^{72}$. Using the decision functions, compute recursively the *growing* series $\{c_t, n_t, k_{t+1}, i_t, y_t, w_t\}_{t=1}^{72}$. Compute the time series average of the following ratios:

$$\left(\frac{i}{y}\right)^{(j)} = mean\left(\frac{i_t}{y_t}\right), \quad \left(\frac{k}{y}\right)^{(j)} = mean\left(\frac{k_t}{y_t}\right), \quad n^{(j)} = mean\left(n_t\right).$$

Next, compute the logarithms of $\{c_t, n_t, k_{t+1}, i_t, y_t, w_t, \}_{t=1}^{72}$, detrend them by using the Hodrick Prescott filter (assume $\lambda = 1600$) and then compute the standard deviations (σ) and the correlation of each variable with output ($corr$). Therefore, for every simulation $j = 1, ..., 5000$, you will have the following set of statistics:

$$\left(\frac{i}{y}\right)^{(j)}, \quad \left(\frac{k}{y}\right)^{(j)}, \quad \sigma_c^{(j)}, \quad \sigma_n^{(j)}, \quad \sigma_k^{(j)}, \quad \sigma_i^{(j)}, \quad \sigma_y^{(j)}, \quad \sigma_w^{(j)},$$

$$corr^{(j)}\left(y_t, c_t\right), \quad corr^{(j)}\left(y_t, n_t\right), \quad corr^{(j)}\left(y_t, k_t\right), \quad corr^{(j)}\left(y_t, i_t\right), \quad corr^{(j)}\left(y_t, w_t\right).$$

After you finish all simulations, compute the averages and the standard deviations of each of the above statistics across the simulated data sets $j = 1, ..., 5000$. Report the mean and ± 2 standard deviations interval.

Compare the statistics from your model with the statistics from the US data, as well as with the benchmark model in which there is just shock θ_t. In particular, say if the empirical counterparts belong to the constructed confidence interval of the ± 2 standard deviations. Discuss how well the model reproduces the ratios $\left(\frac{i}{y}\right)$ and $\left(\frac{k}{y}\right)$ and the level for n targeted by the calibration procedure.

[1] Note that the number of periods is 72 because I used a time series of length 72 for computing the business cycle statistics on the US economy.

9. Write down the solution to this exam in the form of a research paper using Scientific Word (TEX) or LaTEX. The paper must be called: "Can additional shocks help to explain the US business cycles?" and must include an abstract (about 100 words) and have the following parts:

    - Section 1: Introduction.
    - Section 2: Evidence on the *relevant* US business cycles.
    - Section 3: The model (show FOCs here as well). Can you think of how to justify / interpret the shocks that are assumed in your case.
    - Section 4: Calibration and solution methodology (brief).
    - Section 5: Numerical results (include sensitivity experiments). Here, explain the intuition behind the effect of your shock.
    - Section 6: Conclusion.
    - References.
    - Appendix A: Derivation of FOCs.
    - Appendix B: Details on calibration and solution methodology and numerical experiments.

10. Please, make an effort to write the paper carefully and make it look nice - like a real research paper!

11. Submit your work (codes, paper, any supplementary results) on Blackboard.