The Rubidium Atom

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Introduction

The Rubidium Atom experiment demonstrates some general properties of Rubidium where it describes the general structure of the Rubidium atom with reference to quantum numbers n, L, S, J and F. Here, the application of the Beer-Lambert law is applied to determine the pressure of Rubidium by treating the Rubidium vapour as an ideal gas. Absorption features of four resonances will be observed on the oscilloscope where the Doppler-broadened width will be determined and to examine the absorption spectra of a resonant laser through a Rubidium vapour cell.

5.2: The Rubidium Atom

The total electron angular momentum, symbolized J is a combination of the electron orbital angular momentum L, and the electron spin angular momentum S. It is mathematically represented as:

$$\vec{J} = \vec{L} + \vec{S}$$

The magnitude of $\vec{J}$ can take values from $|L - S|$ to $|L + S|$ in steps of +1.

5S state of rubidium:

L=0 and $S = \frac{1}{2}$

$\vec{J} = |L - S| = \left|0 - \frac{1}{2}\right| = \frac{1}{2}$

$\vec{J} = |L + S| = \left|0 + \frac{1}{2}\right| = \frac{1}{2}$

The allowed value of $\vec{J}$ for the 5S state of rubidium is $\frac{1}{2}$.

5P state of Rubidium:

L= 1 and S = $\frac{1}{2}$

$\vec{J} = |L - S|$ =$\left|1 - \frac{1}{2}\right| = \frac{1}{2}$

$\vec{J} = |L + S|$ =$\left|1 + \frac{1}{2}\right| = \frac{3}{2}$

The allowed value of $\vec{J}$ for the 5P state of rubidium are $\frac{1}{2}$ and $\frac{3}{2}$.

One convert frequency to energy units by using the following expression:

$$E = hf$$

h: $planck\ constant\ of$ $6.626 \times 10^{-34} Js$

$\mathbf{5S_{\frac{1}{2}} \rightarrow 5P_{\frac{1}{2}}}$

$E = hf = (6.626 \times 10^{-34} Js)(380 \times 10^{12} s^{-1})$ =2.518×10^{-19}J

$\mathbf{5P_{\frac{1}{2}} \rightarrow 5P_{\frac{3}{2}}}$

$E = hf = (6.626 \times 10^{-34} Js)(7 \times 10^{12} s^{-1})$ =4.638×10^{-19}J

Wavelength corresponding to the 5S to 5P energy splitting is

$$\lambda = \frac{c}{f}$$

$$\lambda = \frac{c}{f} = \frac{(3 \times 10^{8})m}{s(380 \times\ 10^{12} s^{-1})} = 7.895 \times 10^{-7} = 789nm$$

Visible light corresponds to the 5S and 5P energy splitting.

In addition to the spin and orbital angular momentum of the electron, the nucleus, symbolized I. This angular momentum can couple with the total electron angular momentum to give the total angular momentum F.

$$\vec{F} = \vec{J} + \vec{I}$$

The allowed values of total angular momentum are $|J - I| \leq F \leq |J + I|$ where F increases in steps of 1.

$\mathbf{5S_{\frac{1}{2}}}$**:**

$J = \frac{1}{2}$ and $I = \frac{5}{2}$

$$|J - I| \leq F \leq |J + I| = \left|\frac{1}{2} - \frac{5}{2}\right| \leq F \leq \left|\frac{1}{2} + \frac{5}{2}\right| = 2 \leq F \leq 3$$

The allowed values of F are 2 and 3.

$\mathbf{5P_{\frac{1}{2}}}$**:**

$J = \frac{1}{2}$ and $I = \frac{5}{2}$

$$|J - I| \leq F \leq |J + I| = \left|\frac{1}{2} - \frac{5}{2}\right| \leq F \leq \left|\frac{1}{2} + \frac{5}{2}\right| = 2 \leq F \leq 3$$

The allowed values of F are 2 and 3.

$\mathbf{5P_{\frac{3}{2}}}$**:**

$J = \frac{3}{2}$ and $I = \frac{5}{2}$

$$|J - I| \leq F \leq |J + I| = \left|\frac{3}{2} - \frac{5}{2}\right| \leq F \leq \left|\frac{3}{2} + \frac{5}{2}\right| = 1 \leq F \leq 4$$

The allowed values of F are 1,2,3 and 4.

5.2.2: Selection Rules

The selection rules describe the necessary requirements for the electron to transition from one state to another whereby

- $\Delta L = 1$
- $\Delta F = -1,0,1$

The allowable electric-dipole transitions for ^{87}Rb are

$5P_{\frac{3}{2}} \rightarrow 5S_{\frac{1}{2}}$, $F = 1$ =1 to $F' = 0,1,2$ and $F = 2$ =1 to $F' = 1,2,3$

$5P_{\frac{1}{2}} \rightarrow 5S_{\frac{1}{2}}$, $F = 1$ to $F' = 1,2$ and $F = 2$ =1 to $F' = 1,2$

The probable speed for a room temperature distribution of Rubidium is determined from the following expression

$$V = \sqrt{\frac{2K_B T}{M}}$$

K_B:Boltzmann constant (1.38$\times 10^{-23} J/K$)

T: Temperature

M: mass of Rubidium atom

Unit conversion is given by:

1u=1.6605402$\times 10^{-27}$kg

Mass of ^{85}Rb: 85.4678u = 1.419$\times 10^{-25} kg$

Mass of ^{87}Rb: 86.9091835u = 1.443$\times 10^{-25} kg$

Probable speed of ^{85}Rb:

$$V_{85\mathrm{Rb}} = \sqrt{\frac{2K_BT}{M}} = \sqrt{\frac{2(1.38\times10^{-23}kgm^2)(300K)}{K(1.419\times10^{-25}kg)s^2}} = 241.6\,\frac{m}{s}$$

Probable speed of ^{87}Rb:

$$V_{87\mathrm{Rb}} = \sqrt{\frac{2K_BT}{M}} = \sqrt{\frac{2(1.38\times10^{-23}kgm^2)(300K)}{K(1.443\times10^{-25}kg)s^2}} = 239.5\,\frac{m}{s}$$

Parameter of doppler shift is given by:

$$\beta = \frac{v}{c}$$

Doppler shift for ^{85}Rb:

$$\beta = \frac{v}{c} = \frac{241.6}{3\times10^8} = 8.053\times10^{-7}$$

$$\therefore \beta \times f_0 = (8.053\times10^{-7})(380\ \times10^{12}Hz) = 306MHz$$

Doppler shift for ^{87}Rb:

$$\beta = \frac{v}{c} = \frac{239.5}{3\times10^8} = 7.983\times10^{-7}$$

$$\therefore \beta \times f_0 = (7.983\times10^{-7})(380\ \times10^{12}Hz) = 303MHz$$

5.6.2: Data Collection

Doppler- broadened absorption features of Rubidium is observed at scan amplitude setting of 6 and range frequency to 5.

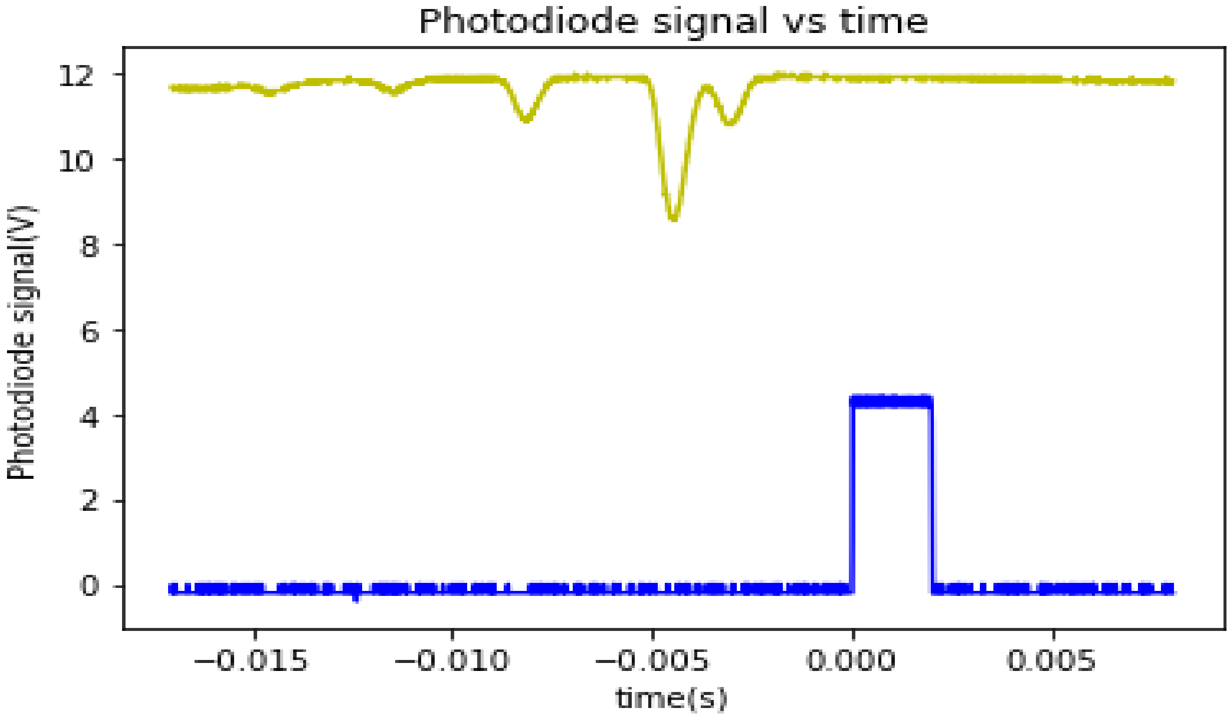


Figure 1: Oscilloscope trace across all four resonances at once.

5.6.3: Background Subtraction

The background data is scaled down to overlap the signal data as best as possible to have a better background subtraction as the background beam was at a different intensity level.

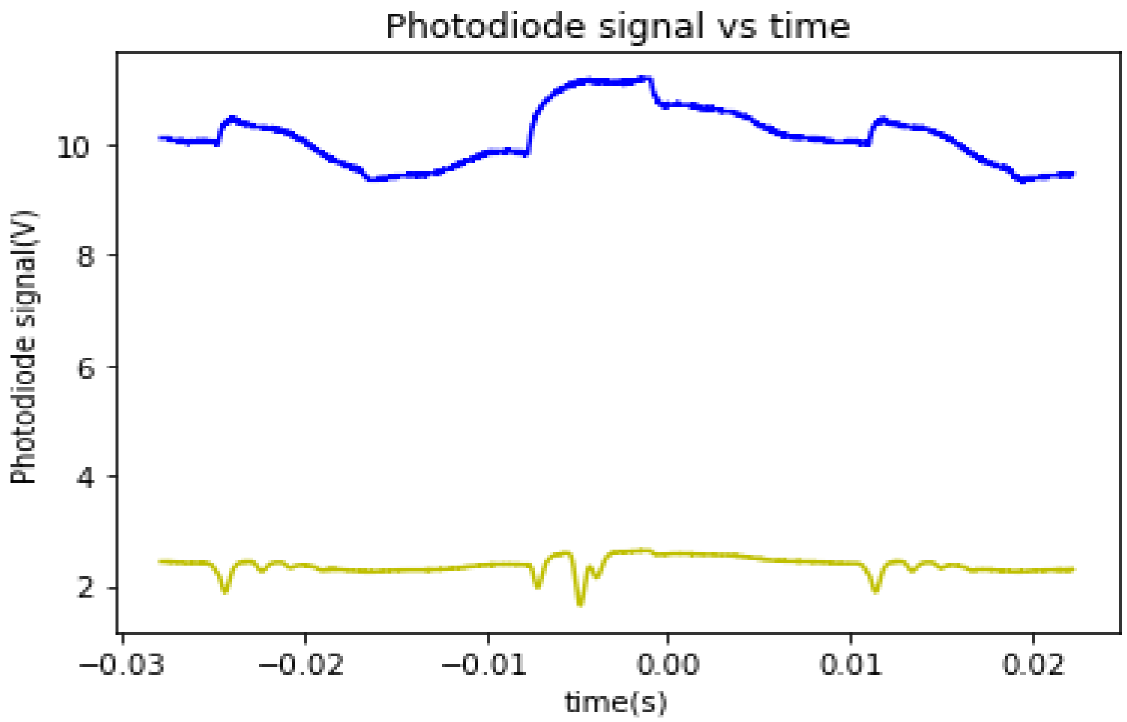


Figure 2: Traces of both photodiode signals on the oscilloscope before background data is scaled.

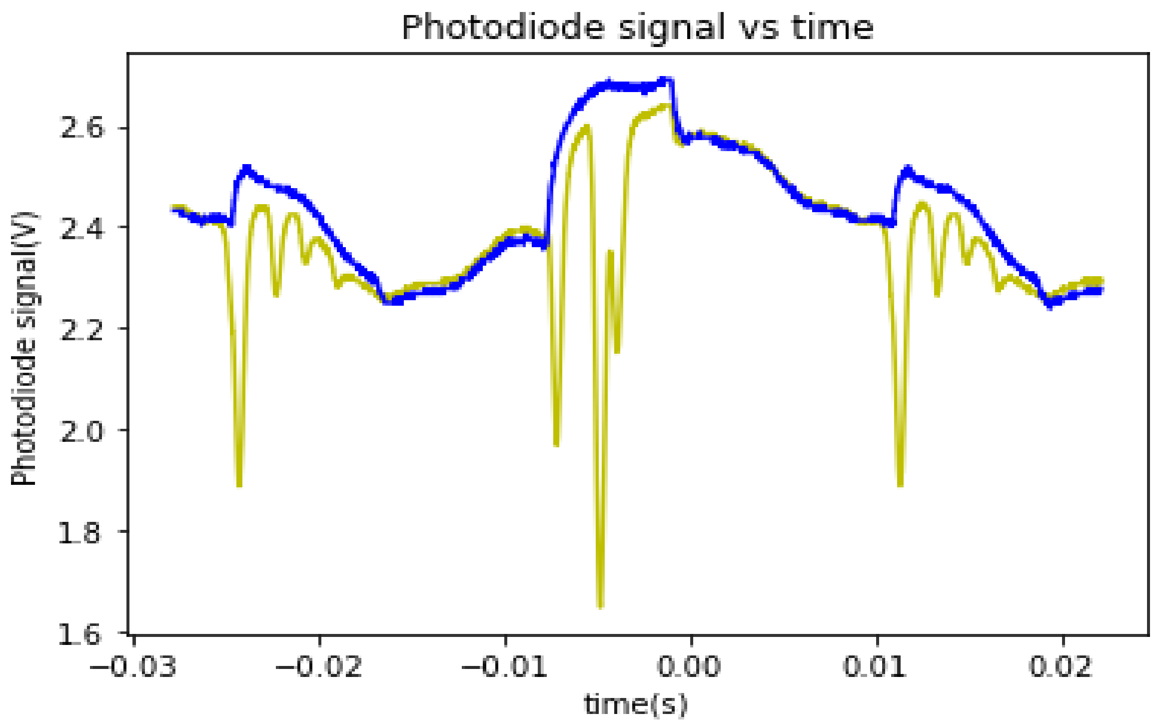


Figure 3: Overlap of traces of both photodiode signals after background data is scaled.

Part 1:

Four background absorption features are fitted to a gaussian where the Doppler broadened linewidth of these transitions is determined from the following equation

$$FWHM = 2\sqrt{2ln2}\,\sigma \approx 2.355\sigma$$

σ: standard deviation

The scan amplitude used is set to 6 whereby the calibration is 9.090×10^{9}Hz.

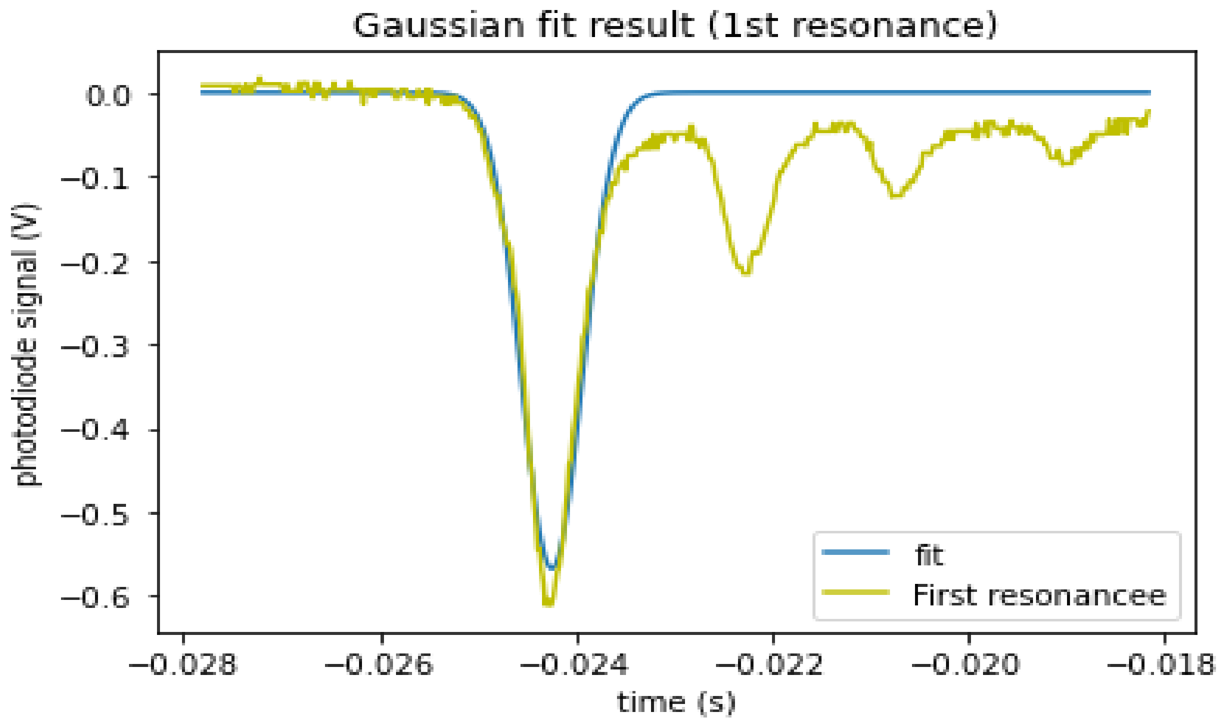


Figure 4: Gaussian fit to the first resonance

Standard deviation of the curve fit, σ : 0.000310345 ± $9.50939\times 10^{-6}s$

$$FWHM = 2.355\sigma = 2.355(0.000310345 \pm\ 9.50939 \times 10^{-6}) = 7.309 \times 10^{-4} \pm 2.239 \times 10^{-5}s$$

$$= (7.3 \pm 0.2) \times 10^{-4}s$$

Taking the inverse of FWHM to obtain in unit Hz

$$\therefore linewidth = \left(\frac{1}{7.309 \times 10^{-4}s}\right)(9.090 \times 10^{9}\text{Hz}) = 1.244 \times 10^{13}\text{Hz} = 12.4\text{THz}$$

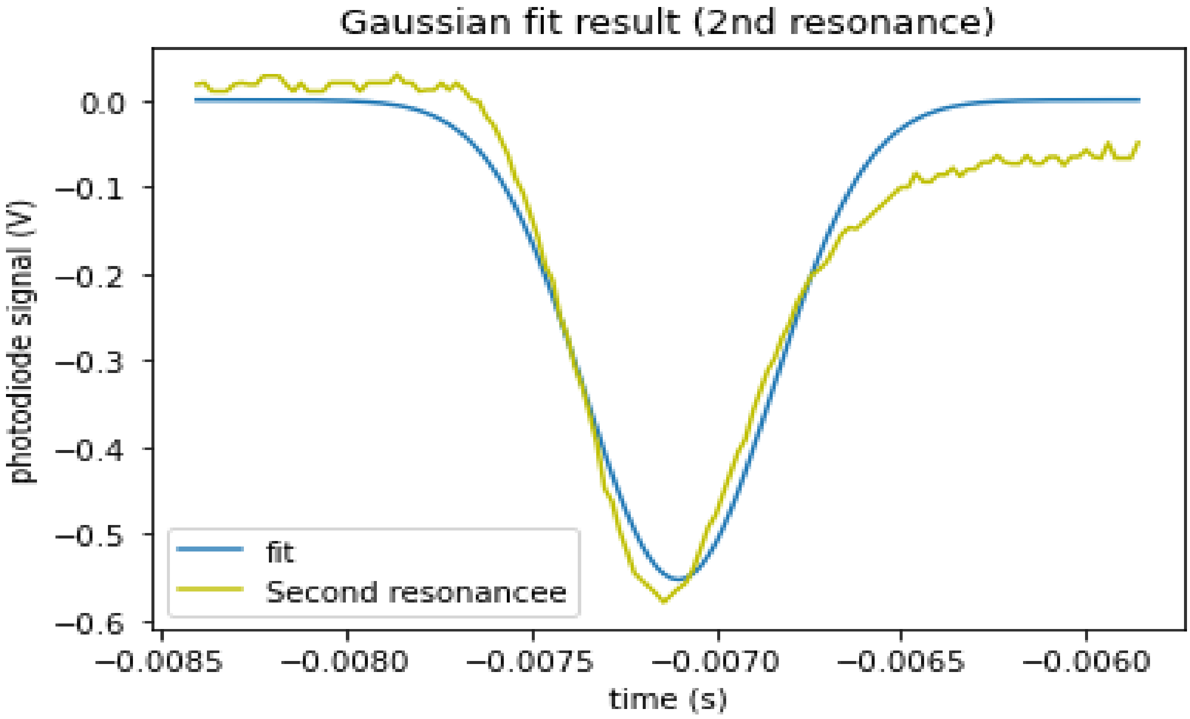


Figure 5: Gaussian fit to the second resonance

Standard deviation of the curve fit, σ : (2.53 ± 0.06)$\times 10^{-4}$

$$FWHM = 2.355\sigma = 2.355(0.000253723 \pm\ 6.31071 \times 10^{-6}) = 5.975 \times 10^{-4} \pm 1.486 \times 10^{-5}s$$

$$= (6.0 \pm 0.1) \times 10^{-4}s$$

Taking the inverse of FWHM to obtain in unit Hz:

$$\therefore linewidth = \left(\frac{1}{5.975 \times 10^{-4}s}\right)(9.090 \times 10^{9}\text{Hz}) = 1.521 \times 10^{13}\text{Hz} = 15.2\text{THz}$$

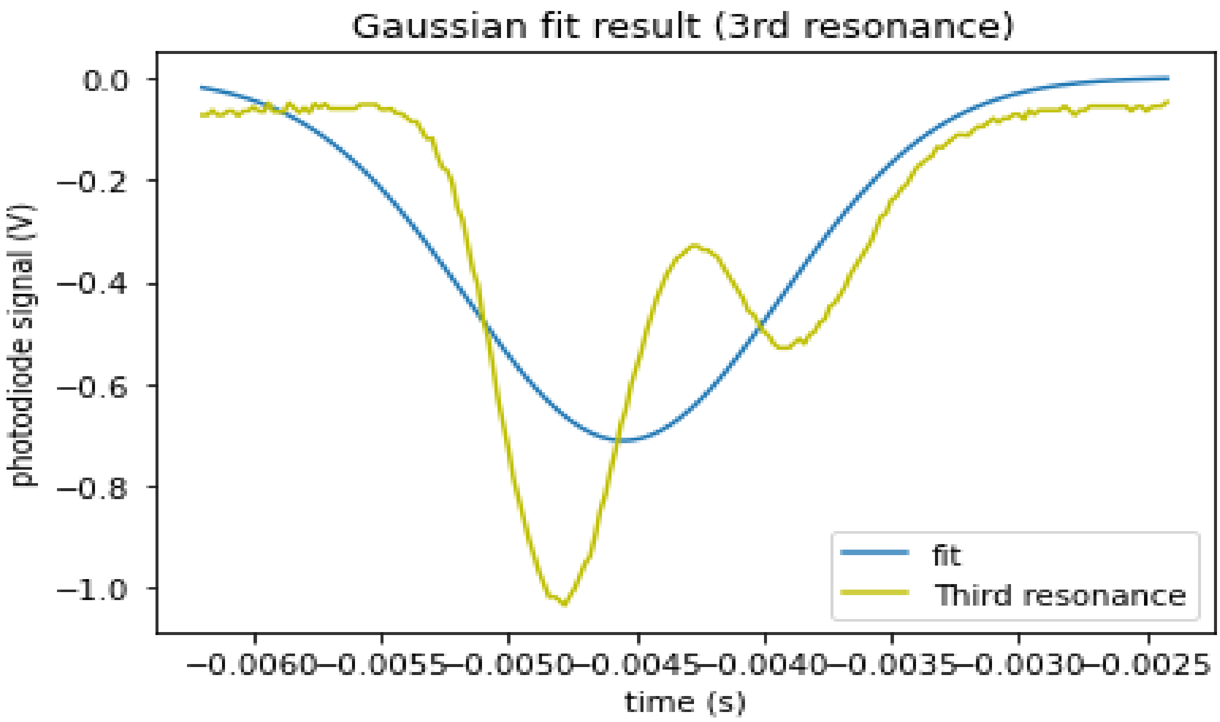


Figure 6: Gaussian fit to the third resonance

Standard deviation of the curve fit, σ : $(6.17 \pm 0.26)\times 10^{-4}$

$$FWHM = 2.355\sigma = 2.355(0.000617178 \ \pm 2.59278 \times 10^{-5}\ ) = 1.453 \times 10^{-3} \ \pm 6.106 \times 10^{-5}$$

$$= (1.45 \pm 0.06) \times 10^{-3}\text{s}$$

Taking the inverse of FWHM to obtain in unit Hz:

$$\therefore linewidth = \left(\frac{1}{1.453 \times 10^{-3} s}\right)(9.090 \times 10^{9}\text{Hz}) = 6.27 \times 10^{12}\text{Hz} = 6.27\text{THz}$$

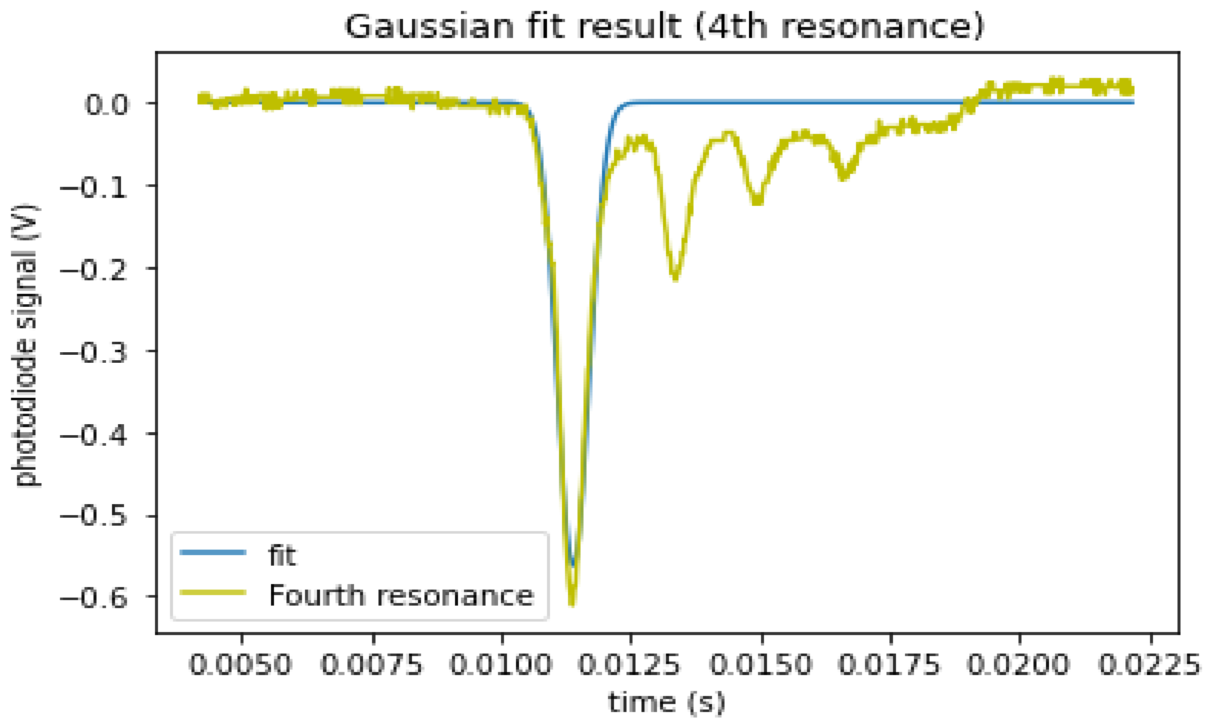


Figure 7: Gaussian fit to the fourth resonance

Standard deviation of the curve fit, σ : (3.07 ± 0.07)$\times\ 10^{-4}$

$$FWHM = 2.355\sigma = 2.355(0.000307325 \ \pm 7.14686 \times 10^{-6}) = 7.238 \times 10^{-4} \pm 1.683 \times 10^{-5}$$

$$= (7.2 \pm 0.2) \times 10^{-4} s$$

Taking the inverse of FWHM to obtain in unit Hz:

$$\therefore\ linewidth = \left(\frac{1}{7.238 \times 10^{-4} s}\right)(9.090 \times 10^{9}\text{Hz}) = 1.256 \times 10^{13}\text{Hz} = 12.6\text{THz}$$

Part 2:

The number density of Rubidium in the vapour cell is generated by the following Beer-Lambert equation

$$I(L) = I_o e^{-\sigma n L}$$

L: distance the light travels through the absorbing medium

I_0: initial intensity

$I(L)$: intensity of light after passing through the medium

σ: cross-section of absorption

Considering the largest absorption features, the x-axis of the maximum peak of -0.0048 is taken whereby intensity of light after passing through the medium correspond to 1.648 while the initial intensity of light is 2.672119. The relative intensity that is the ratio between the photodiode signals is found to be $\frac{I(L)}{I_o} = \frac{1.648}{2.672119}$. Thus, the number density of Rubidium is

$$\Rightarrow \ln\left(\frac{I(L)}{I_o}\right) = -\sigma n L$$

$$\therefore n = \frac{-\ln\left(\frac{1.648}{2.672110}\right)}{(1.4\times10^{-16}m^2)(75.0\times10^{-3})m} = 4.603 \times 10^{16}\mathrm{m}^{-3}$$

Treating the rubidium vapour as an ideal gas,

$$PV = nK_BT$$

$\frac{n}{V}$: the number density of Rubidium atoms

$$\therefore P = \frac{nK_BT}{V} = (4.603 \times 10^{16}m^{-3})\left(1.38 \times 10^{-23}\frac{J}{K}\right)(300K) = 1.906 \times 10^{-4}\ \mathrm{Pa}$$

Part 3:

We see two peaks for every isotope such that the $5S_{\frac{1}{2}}$ and $5S_{\frac{3}{2}}$ state have hyperfine splitting where only the $5S_{\frac{1}{2}}$ state has a large enough hyperfine splitting than the Doppler broadening. Distance in unit of frequency is determined between two peaks for ^{85}Rb and ^{87}Rb whereby the distance obtained would be the hyperfine splitting of the $5S_{\frac{1}{2}}$ state.

The scan amplitude used is set to 6 whereby the calibration is 9.090×10^{9}Hz.

^{85}Rb:

Distance of laser moving through one full cycle: 6.11×10^{-3}s

Distance of a laser going through a single ramp: 3.05×10^{-3}s

Distance between two peaks: 8.4×10^{-4}s

Ratio of distance between two peaks and distance of laser of a single ramp: $\frac{8.4\times10^{-4}}{3.05\times10^{-3}} = 0.275$

$\therefore hyperfine\ splitting = (0.275) \times (9.090\times 10^{9})\ \text{Hz} = 2.5 \times 10^{9}\text{Hz}.$

Accepted value: 3GHz

$$Percentage\ error, \delta = \left|\frac{v_A - v_e}{v_e}\right| \times 100\%$$

$v_A = measured\ value$
$v_e = expected\ value$

$$Percentage\ error, \delta = \left|\frac{v_A - v_e}{v_e}\right| \times 100\% = \left|\frac{2.5 \times 10^{9} - 3.0 \times 10^{9}}{3.0 \times 10^{9}}\right| \times 100\% = 16.6\%$$

^{87}Rb:

Distance of laser moving through one full cycle: 6.11$\times 10^{-3}$s

Distance of a laser going through a single ramp: 3.05×10^{-3}s

Distance between two peaks: 2.52$\times 10^{-3}$s

Ratio of distance between two peaks and distance of laser of a single ramp: $\frac{2.52\times10^{-3}}{3.05\times10^{-3}} = 0.826$

$\therefore \textit{hyperfine splitting} =$ (0.826) × (9.090$\times 10^9$) Hz $= 7.5 \times 10^9$Hz.

Accepted value: 6GHz

$$Percentage\ error, \delta = \left|\frac{v_A - v_e}{v_e}\right| \times 100\%$$

$v_A = measured\ value$
$v_e = expected\ value$

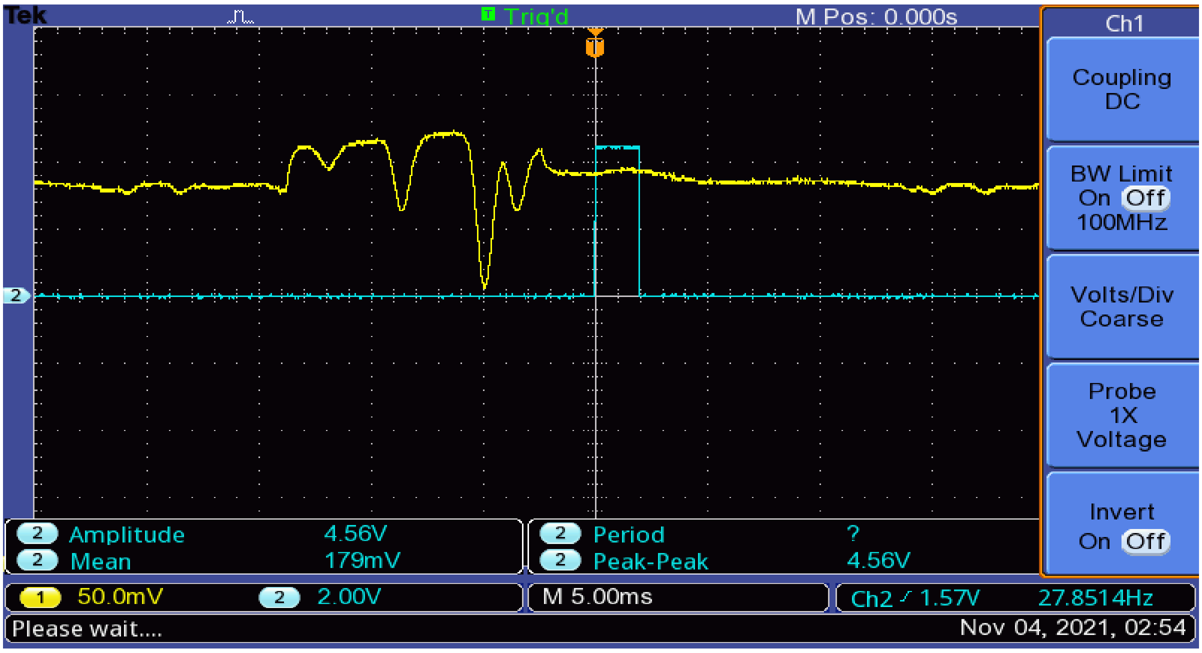
$$Percentage\ error, \delta = \left|\frac{v_A - v_e}{v_e}\right| \times 100\% = \left|\frac{7.5 \times 10^9 - 6.0 \times 10^9}{6.0 \times 10^9}\right| \times 100\% = 25.0\%$$

Conclusion:

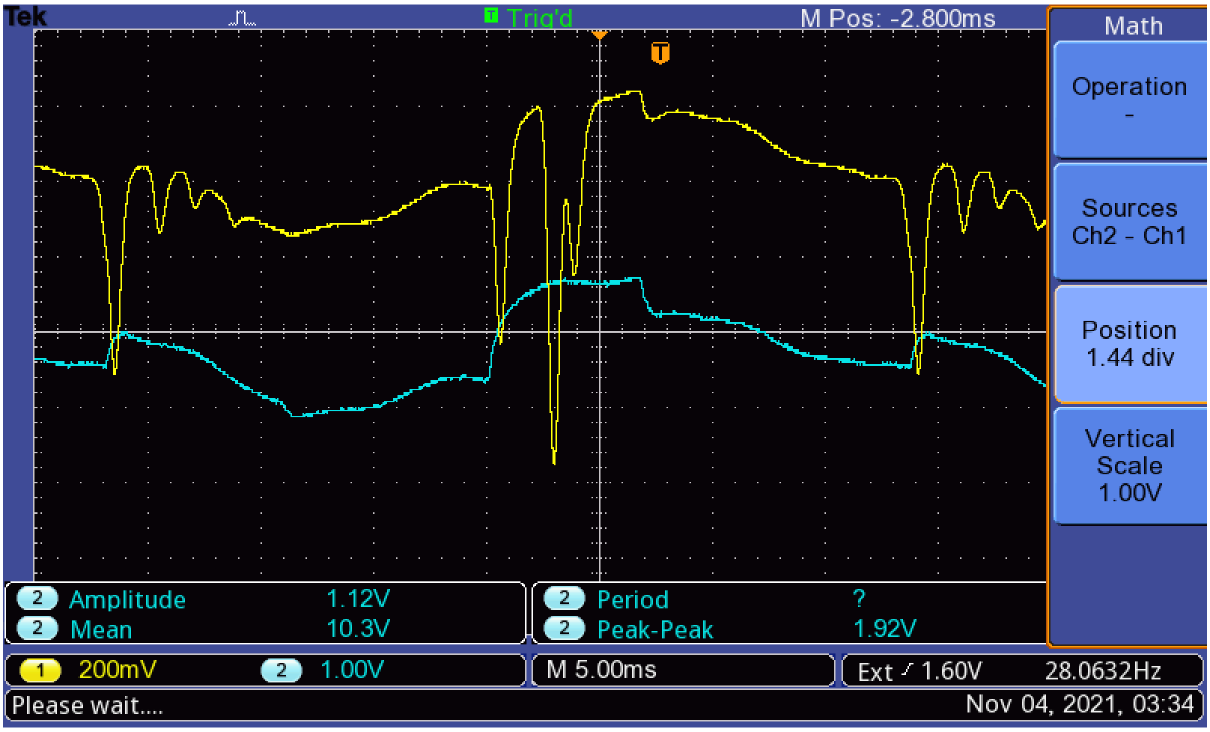
The Doppler broadened linewidth for each transition was found to be 12.4THz for the first resonance, 15.2THz for the second, 6.27THz for the third and 12.6THz for the fourth resonance. The number density of Rubidium atoms was found to be $4.603\times 10^{16} m^{-3}$which correspond to a pressure of 1.906×10^{-4}Pa. However, in comparison with the accepted value of the hyperfine splitting of the $5S_{\frac{1}{2}}$ state for ^{85}Rb and ^{87}Rb, the measured value was found to be 2.5×10^{9}Hz and 7.5×10^{9}Hz respectively. The measured values appeared to be close to the accepted values but with a relatively high percentage error of 16.6% for ^{85}Rb and 25% for ^{87}Rb. This could be due to the fact that the amplitude calibration used here was not done correctly so in the previous lab or the laser beam was being too saturated. Nevertheless, the data and results generated from this experiment is deemed reasonable.

Appendix A

Oscilloscope trace of four resonances:



Traces of both photodiode signal on the oscilloscope:



Appendix B

Standard deviation for four resonances with its uncertainty in array 2

Resonance	Standard deviation	Uncertainty
1	popt - NumPy object array 0: -0.567565 1: -0.024236 2: 0.000310345	popt3 - NumPy object array 0: -0.56254 1: 0.0113959 2: 0.000307325
2	popt1 - NumPy object array 0: -0.55323 1: -0.00710093 2: 0.000253723	perr1 - NumPy object array 0: 0.0119165 1: 6.31071e-06 2: 6.31071e-06
3	popt2 - NumPy object array 0: -0.710749 1: -0.00454898 2: 0.000617178	perr2 - NumPy object array 0: 0.0257755 1: 2.58309e-05 2: 2.59278e-05
4	popt3 - NumPy object array 0: -0.56254 1: 0.0113959 2: 0.000307325	perr3 - NumPy object array 0: 0.0113282 1: 7.14686e-06 2: 7.14686e-06

Appendix C

Excel file

subtract.xlsx