Laser Scanning and Etalon

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Introduction:

The laser scanning and etalon experiment demonstrates the frequency of the laser in a periodic fashion where it allows us to controllably sweep the frequency of the laser over many Rubidium atomic transitions in a short period of time which will be viewed on the oscilloscope as a photodiode signal. The scan control module of the laser controllers is set for modulation frequency and modulation amplitude whereby calibration of frequency of modulation and amplitude of modulation would be determined.

Results:

4.4.1: Calibrating the Scan Controller

Index of refraction of air, $n_1 = 1.000$

Index of refraction of YAG, $n_2 = 1.833$

Angle of incidence, $\Theta_i = 0°$

Angle of transmission, $\Theta_t = 0°$

Using Fresnel's equation, reflectance, R

$$R = \left|\frac{n_1 - n_2}{n_1 + n_2}\right|^2 = \left|\frac{1.000 - 1.833}{1.000 + 1.833}\right|^2 = 0.09$$

$$Finesse = \frac{\pi\sqrt{R}}{1 - R} = \frac{\pi\sqrt{0.09}}{1 - 0.09} = 1.04$$

$$\Delta\lambda_{FSR} = \frac{\lambda^2}{n2D} = \frac{(780 \times 10^{-9}m)^2}{(1.833)2(27.00 \times 10^{-3}m)} = 6.14 \times 10^{-12}\text{m}$$

$$\Delta f_{FSR} = \frac{c}{n2D} = \frac{(3 \times 10^8)m/s}{(1.833)2(27.00 \times 10^{-3}m)} = 3.03 \times 10^9\text{s}^{-1}$$

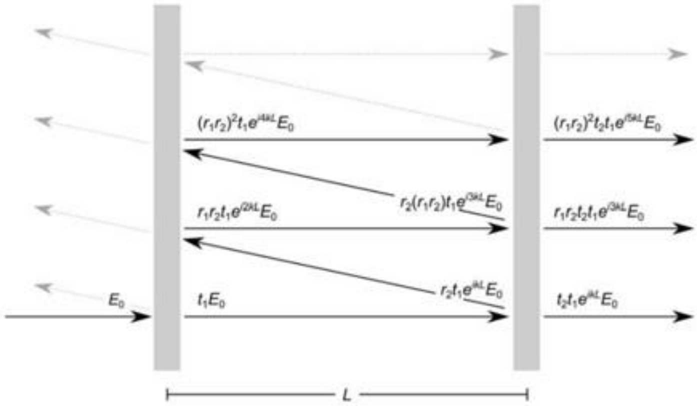


Figure1: Scheme of a Fabry-Perot resonator. *Massimo Inguscio &Leornado Fallani. (2013). Atomic Physic.*

Consider two mirrors separated by a distance L, which are characterized by a field of reflectivity, r and transmittivity, t. The reflectivity and transmittivity coefficients for the intensities are $R = |r|^2$ and $T = |t|^2$ respectively and by the conservation of energy, a condition is imposed where $R + T = 1$. An incident beam with electric field E_0 entering the cavity experiences an infinite sequence of partial reflections and transmissions from the two mirrors. Assuming the angle of incident of normal incidence whereby all beams propagate along the same direction and overlap with each other, the total transmitted electric field can be calculated by summing the infinite partial beam leaking from the cavity in the forward direction

$$E_T = t^2 e^{ikL} E_0 \sum_{n=0}^{\infty} (r^2 e^{i2kL})^2 = t^2 e^{ikL} E_0 \frac{1}{1 - r^2 e^{i2kL}}$$

The sum formula for the geometric series is used which is convergent since $r^2 e^{i2kL} < 1$. The transmitted intensity is taken to be the square modulus of the total electric field

$$I_T = I_0 \frac{|t|^2}{|1 - r^2 e^{i2kL}|^2} = I_0 \frac{(1 - R)^2}{|1 - Re^{i2kL}|^2}$$

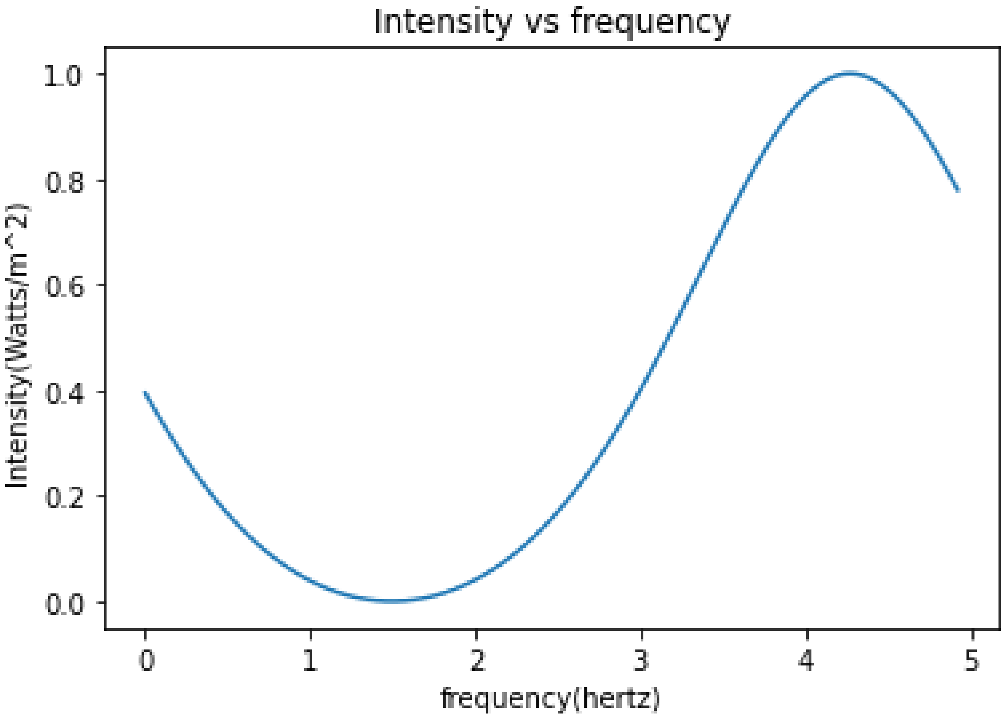


Figure 2: Graph of intensity vs frequency for a solid etalon with reflectance 0.09 and 27.00mm long.

4.5.3: Calibrate the Frequency of modulation

Frequency of modulation (inverse of the period of modulation) is recorded for ten allowable settings of the range frequency selector of 0-9.

The effect of the etalon on the transmission of the laser creates what looks like a sin wave due to the mirror's reflectivity where it affects the number of bounces and improves the quality of modulation. When the reflectivity of the mirror increases, the modulation peaks become sharper and decreases in width. Similarly, when the reflectivity of the mirror decreases, the modulation peak increases in width and becomes wider. The transmission spectrum of an etalon will have a series of peaks where constructive interference occurs spaced by the free spectral range.

Range Frequency	Period	Frequency, f (s^{-1})	
0	0	0	
1	4.00ms	250.0	
2	820ms	1.22	
3	250ms	4.00	
4	75ms	13.33	
5	35.6ms	28.09	
6	11.2ms	89.29	
7	3.72ms	268.8	
8	1.32ms	757.6	
9	410μs	2439.0	
		$\sum f$	3851.33

Table 1: Frequency of modulation (inverse of the period of modulation) for ten allowable settings of the range frequency selector of 0-9.

Mean of frequency:

$$\bar{f} = \frac{\sum f}{n} = \frac{3851.33}{10} = 385.1\text{s}^{-1}$$

Standard deviation of frequency:

Range frequency	f	$f - \bar{f}$	$(f - \bar{f})^2$	
0	0	-385.1	148302.1	
1	250	-135.1	18252.01	
2	1.22	-383.9	147379.21	
3	4.00	-381.1	145237.21	
4	13.33	-371.8	138235.24	
5	28.09	-357.0	127449	
6	89.29	-295.8	87497.64	
7	268.8	-116.3	13525.69	
8	757.6	372.5	138756.25	
9	2439.0	2053.9	4218505.21	
			$\sum(f - \bar{f})^2$	5183139.56

Table 2: Calculation of the standard deviation.

$$\sigma = \sqrt{\frac{\sum(X-\bar{X})^2}{n-1}} = \sqrt{\frac{5183139.56}{10-1}} = 7.588 \times 10^2 \mathrm{s}^{-1}$$

Uncertainty of the mean:

$$\bar{\sigma} = \frac{\sigma}{\sqrt{n}} = \frac{7.588 \times 10^2}{\sqrt{10}} = 2.4 \times 10^2 \mathrm{s}^{-1}$$

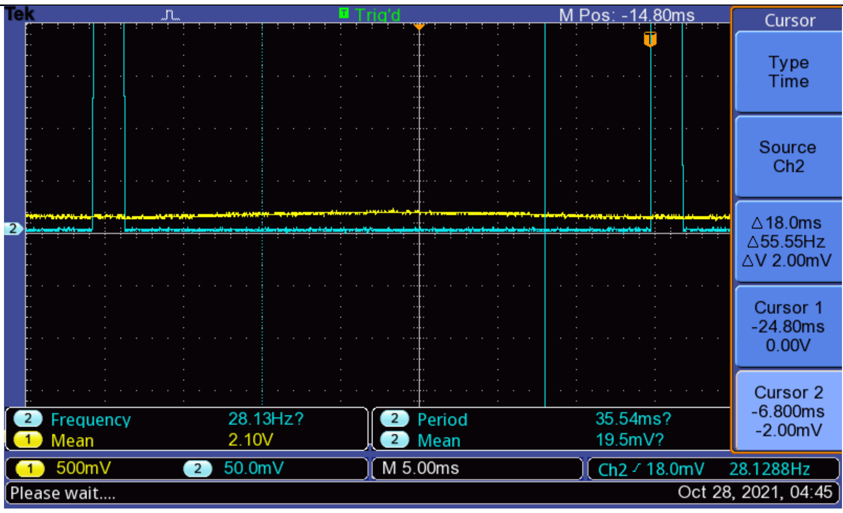
4.5.4: Calibrate the amplitude of Modulation

Magnitude of the frequency modulation for each data is recorded. Here, magnitude means the amplitude. The amplitude of modulation is related to the frequency change of the laser. Therefore, the total amount of hertz is determined over the scanning of the laser's frequency on each back and forth scan for each setting of the red amplitude dial on the laser controller. In order to proceed, the free spectral range of the etalon, that is the distance between the peaks in hertz is known. Then by looking at the photodiode data and counting how many of these etalon peaks the laser scanned over for each amplitude setting, the total amount of hertz of each scan can be generated. The number of peaks determined is half of the total of peaks counted within the blue trigger period.

On small amplitude where we may be scanning over fractions of a single etalon peak and would thus find it hard to count them, the high amplitude data with many peaks is used to extrapolate to lower amplitudes using a best fit where we fit a line to the log of the number of peaks, making sure to impose the y-intercept to be zero where at zero amplitude, it would be scanning over zero peaks.

$$\Delta f_{FSR} = \frac{c}{n2D} = \frac{(3 \times 10^8)m/s}{(1.833)2(27.00 \times 10^{-3}m)} = 3.03 \times 10^9 \mathrm{s}^{-1}$$

Magnitude of frequency modulation, $A = (n \times \Delta f_{FSR})$
where n: number of etalon peaks

<table>
<tr><th>Amplitude setting</th><th>Photodiode data</th><th>Number of peaks</th></tr>
<tr><td>0</td><td>
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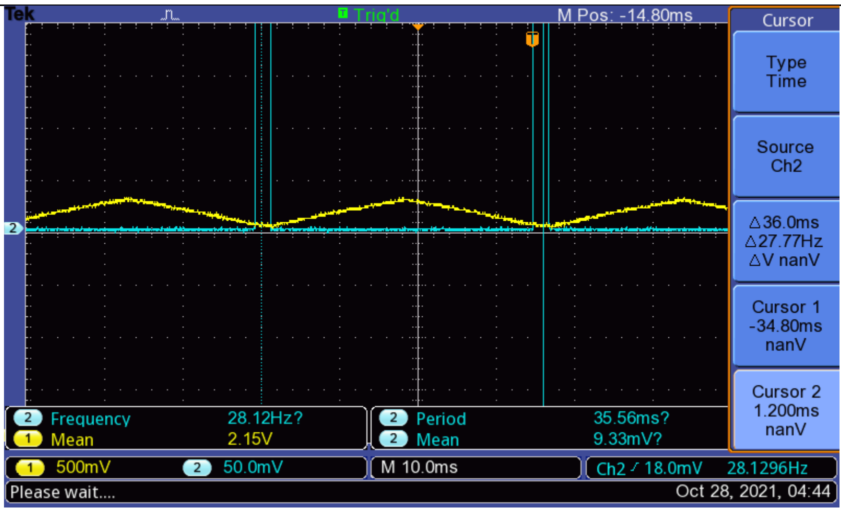
<tr><td>1</td><td>
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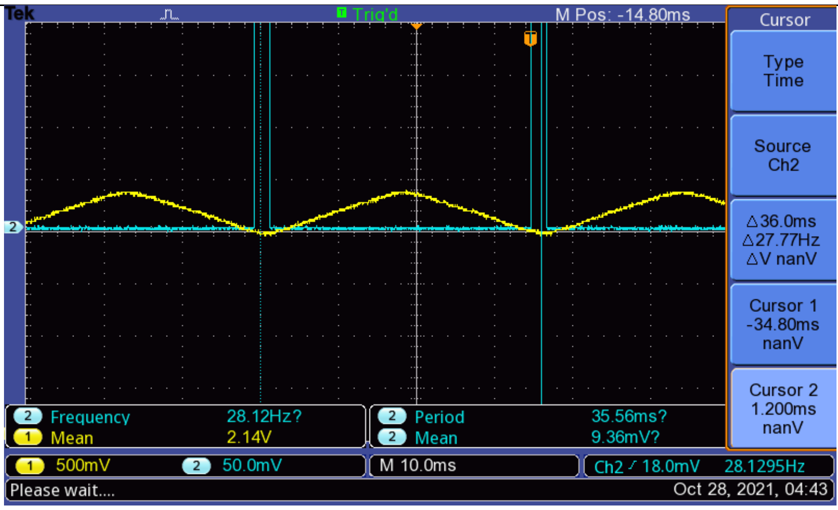
<tr><td>2</td><td>
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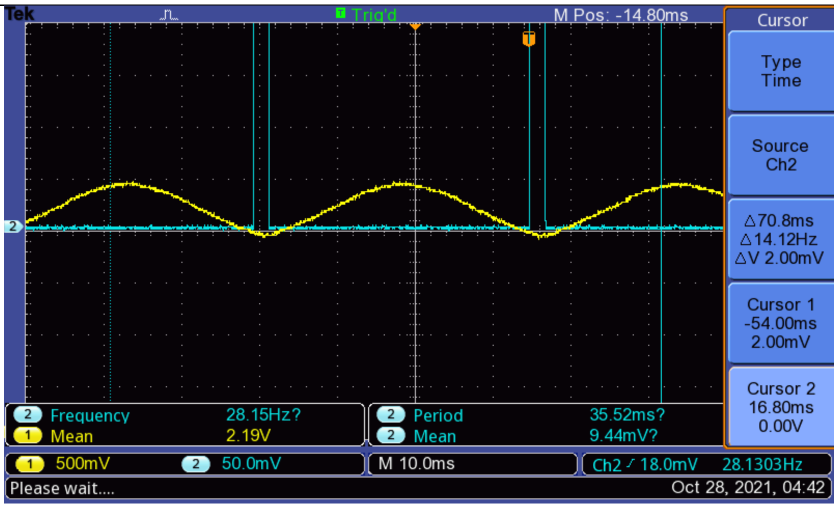
<tr><td>3</td><td>
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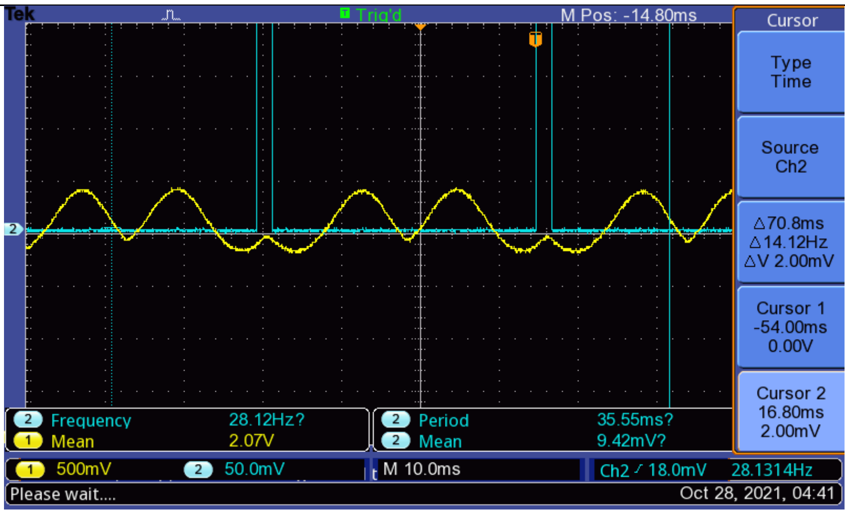
<tr><td>4</td><td>
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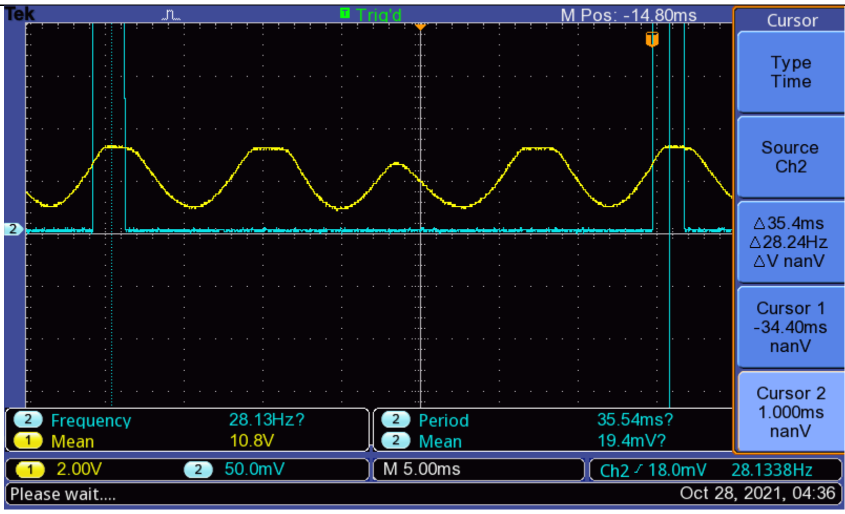
<tr><td>5</td><td>
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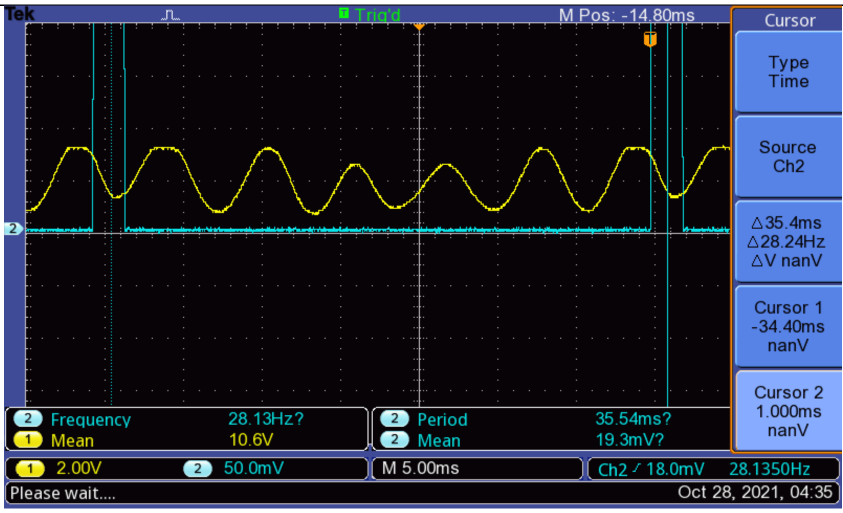
<tr><td>6</td><td>
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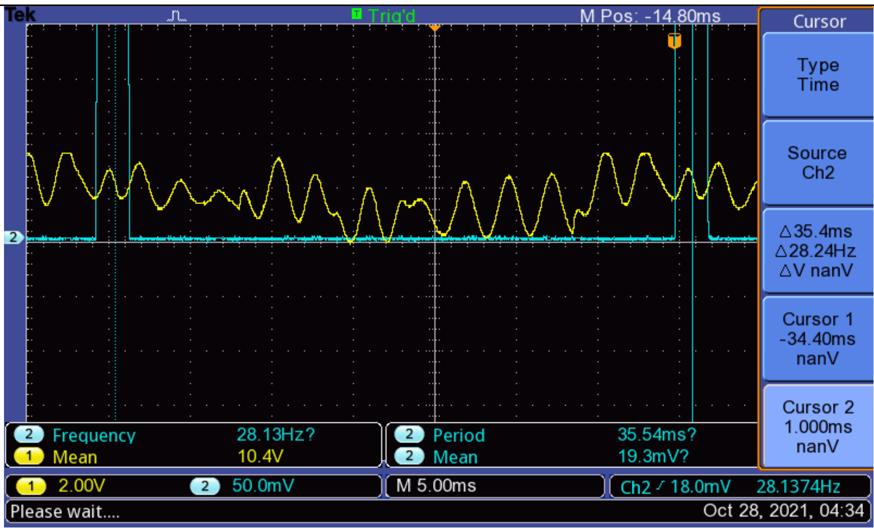
<tr><td>7</td><td>
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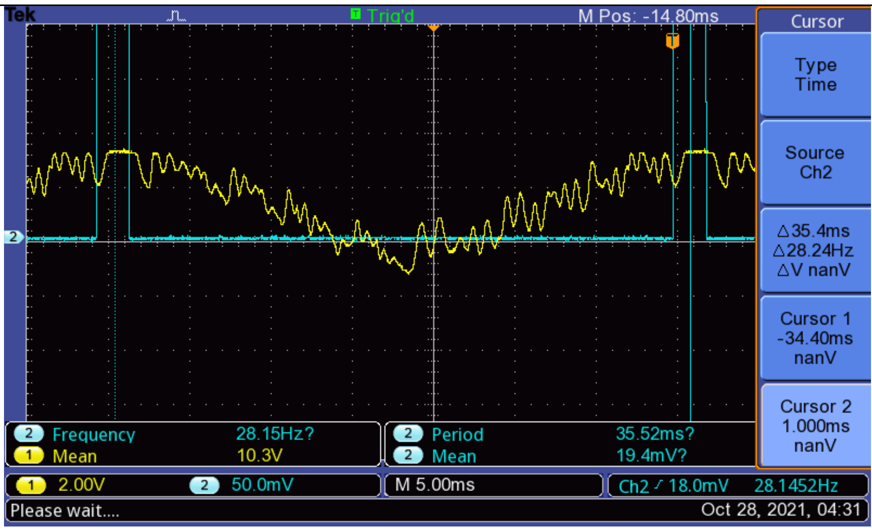
<tr><td>8</td><td>
</td><td>20.5</td></tr>
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Table 3: Photodiode signals at different amplitude settings. Number of peaks determined is half of the total of peaks counted within the blue trigger period.

Using amplitude 4 to amplitude 8 to extrapolate lower amplitude to find number of peaks for amplitude 0, amplitude 1 and amplitude3, fitting a line to the log of the number of peaks.

Amplitude	Number of peaks, n	Log(n)
4	2	0.30103
5	2.5	0.39794
6	3.0	0.47712
7	7.5	0.87506
8	20.5	1.3118

Table 4: Calculation of log of the number of peaks

$y = mx + c$ where y-intercept is set to zero where at zero amplitude, it scans over zero peaks.

Slope obtained is 0.11934926. Therefore, the number of peaks would be

$$log(number\ of\ peaks) = (0.11934926)(Amplitude)$$

$$\therefore number\ of\ peaks = (10)^{(0.11934926)(Amplitude)}$$

Amplitude	Number of peaks, n	Magnitude of frequency modulation, A (s^{-1})
0	0	0
1	1.3	3.939×10^9
2	1.7	5.151×10^9
3	2.3	6.969×10^9
4	2.0	6.060×10^9
5	2.5	7.575×10^9
6	3.0	9.090×10^9
7	7.5	22.725×10^9

8	20.5	62.115×10^9	
		$\sum A$	3.28149×10^{11}

Table 5: Magnitude of frequency modulation for settings 8 through 0 in integer steps of the amplitude dial.

Mean of magnitude of frequency modulation:

$$\bar{A} = \frac{\sum A}{n} = \frac{3.281449 \times 10^{11}}{9} = 3.646 \times 10^{10}\mathrm{s}^{-1}$$

Standard deviation of magnitude of frequency modulation:

Amplitude	A	$A - \bar{A}$	$(A - \bar{A})^2$	
0	0	-3.646$\times 10^{10}$	1.3293×10^{21}	
1	3.939×10^9	-3.252$\times 10^{10}$	1.0576×10^{21}	
2	5.151×10^9	-3.131$\times 10^{10}$	9.8032×10^{20}	
3	6.969×10^9	-2.949$\times 10^{10}$	8.6966×10^{20}	
4	6.060×10^9	-3.040$\times 10^{10}$	9.2416×10^{20}	
5	7.575×10^9	-2.889$\times 10^{10}$	8.3463×10^{20}	
6	9.090×10^9	-2.737$\times 10^{10}$	7.4912×10^{20}	
7	22.725×10^9	-1.374×10^{10}	1.8879×10^{20}	
8	62.115×10^9	2.566×10^{10}	6.5844×10^{20}	
			$\sum(A - \bar{A})^2$	7.59202×10^{21}

Table 6: Calculation of standard deviation.

$$\sigma = \sqrt{\frac{\sum(X - \bar{X})^2}{n-1}} = \sqrt{\frac{7.59202 \times 10^{21}}{9-1}} = 3.0816 \times 10^{10}\mathrm{s}^{-1}$$

Uncertainty of the mean:

$$\bar{\sigma} = \frac{\sigma}{\sqrt{n}} = \frac{3.0816 \times 10^{10}}{\sqrt{9}} = 1.0 \times 10^{10}\mathrm{s}^{-1}$$

Specification sheet for the scan control module

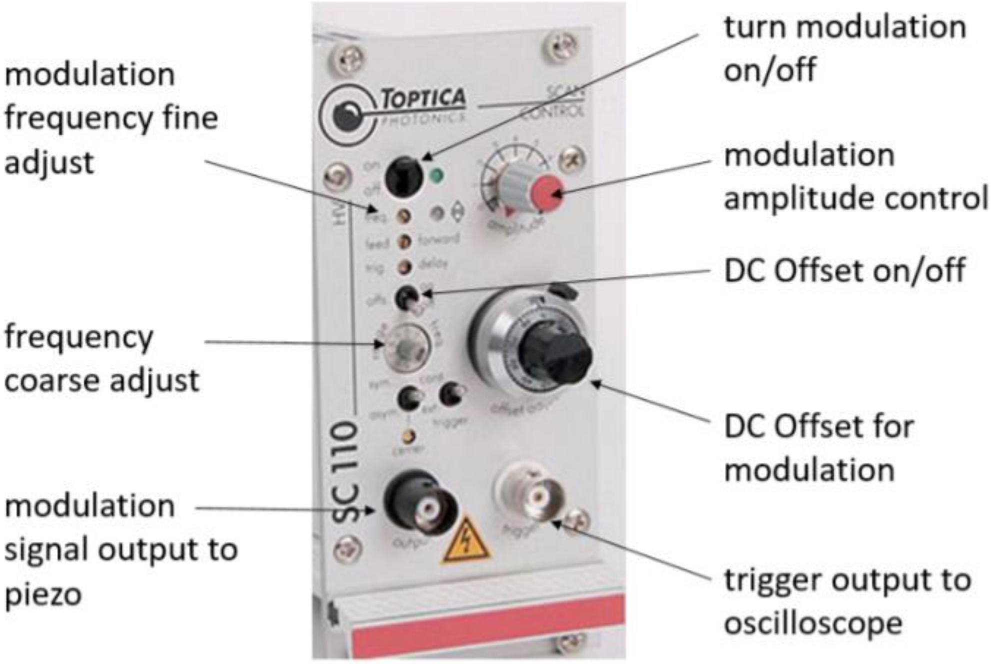


Figure 2: Scan control module of the laser controller.

On/Off Switch	Switch sets the Scan Control SC 110 into operation and an output voltage is available at BNC-connectors.
Amplitude tuning	Amplitude tuning sets the output amplitude continuously. Amplitude Magnitude of the frequency modulation (s^{-1}) 0 0 1 3.939×10^9 2 5.151×10^9 3 6.969×10^9 4 6.060×10^9 5 7.575×10^9 6 9.090×10^9 7 22.725×10^9 8 62.115×10^9
DC Offset	The offset offers variable settings of the output offset between the maximum output values (±20V). Output amplitude is 40V peak to peak as maximum when switch on.
Trigger output	Output trigger socket for synchronization of an oscilloscope with the internal ramp
Modulation signal output	Output voltage for connection
Frequency course range	Tuning of the output frequency can be set using the 10 positions of the frequency range. Position Frequency (Hz)

	0 0.3 1 1 2 3 3 10 4 30 5 100 6 300 7 1000 8 3000 9 10000
Frequency fine	Precise setting of the output frequency within the coarse frequency.
Cont./Ext.Trig. Switch	Determines whether TTL-Trigger BNC-connector is used as in or output trigger.
Sym./Asym switch	Option of asymmetrical ramps (sawtooth). The switch changes the normally symmetrical output of the scan control to an adjustable sawtooth form.

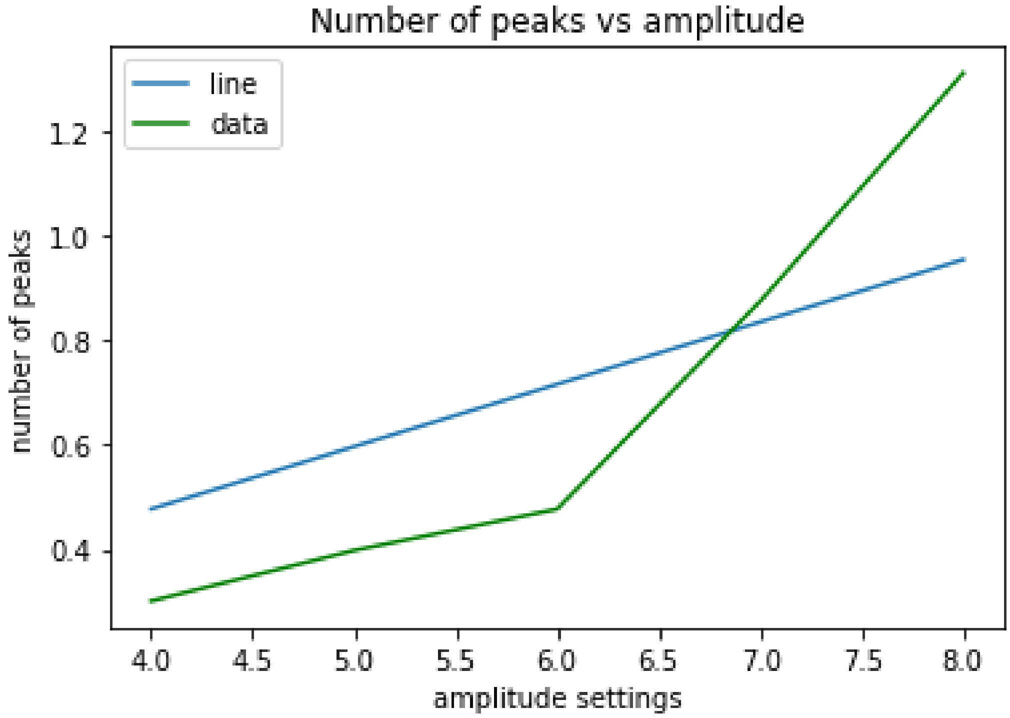
Table 7: Specification sheet for the scan control module

Conclusion

The magnitude of frequency modulation for settings 8 through 0 of the amplitude dials was found to be $(3.6 \pm 1.0) \times 10^{10}\mathrm{s}^{-1}$ whereas the frequency of modulation for each of the ten allowable settings of the range frequency selector is $(3.9 \pm 2.4) \times 10^{2}\mathrm{s}^{-1}$. On calibration of the amplitude of modulation, it is observed from table 3 that small amplitude such as amplitude 0, 1,2 and 3 was not usable due to the etalon peaks being too wide. This is due to the fact that on such small amplitude, we may be scanning over fractions of a single etalon peak and thus would be hard to count them accurately. The one solution that was imposed was taking higher amplitude data with many peaks and to make a fit to the log of the number of peaks whereby extrapolation could simply take place. Another thing to note from table 3 of amplitude 7 and 8 is that there appears to be double peak looking spot on the figure, this is due to the fact that scan goes up and down within the trigger period whereby the number of peaks is counted only up to the halfway point which explains the symmetry to it.

Appendix A:

Plotted graph for number of peaks versus the amplitude



Appendix B

<table>
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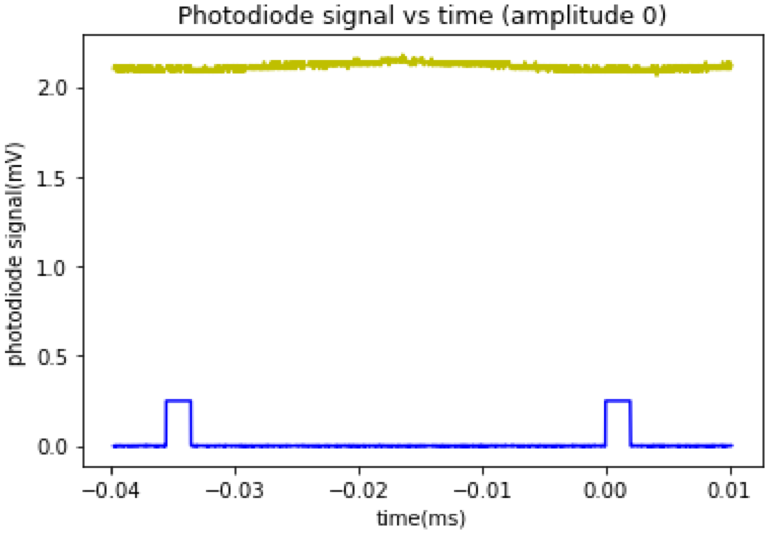
<tr><td>0</td><td>
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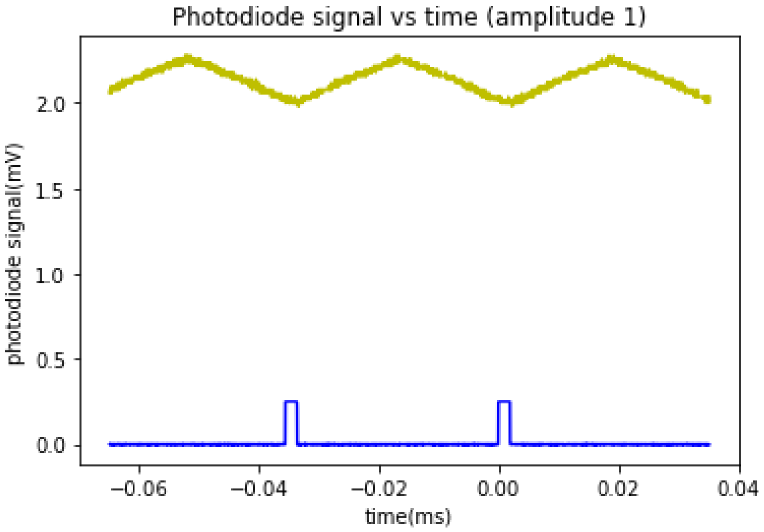
<tr><td>1</td><td>
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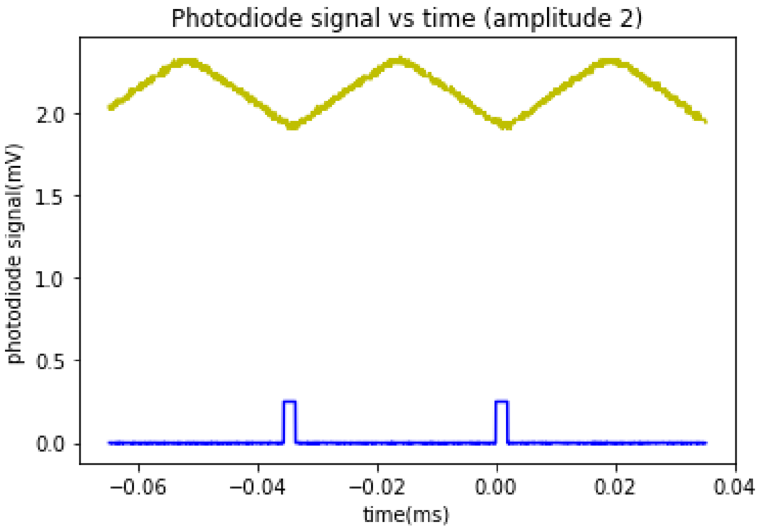
<tr><td>2</td><td>
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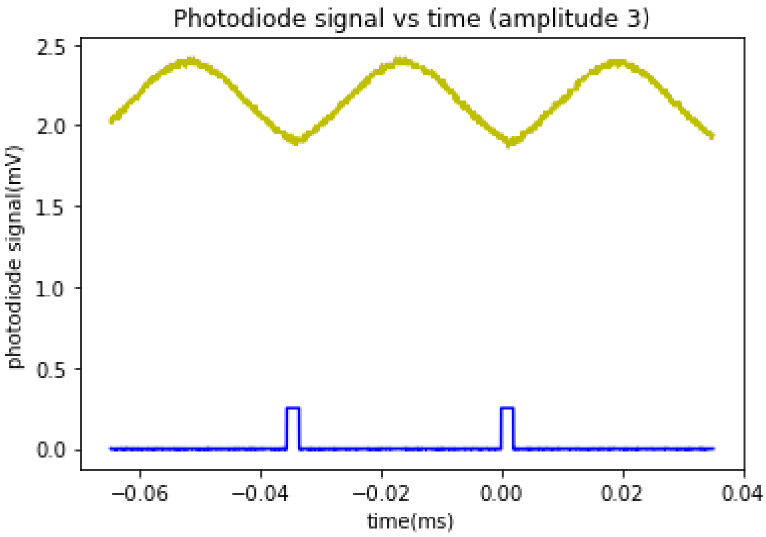
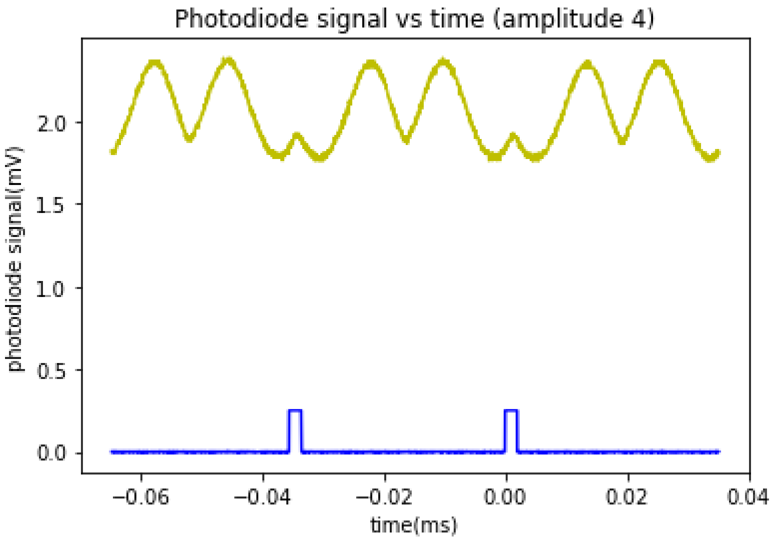
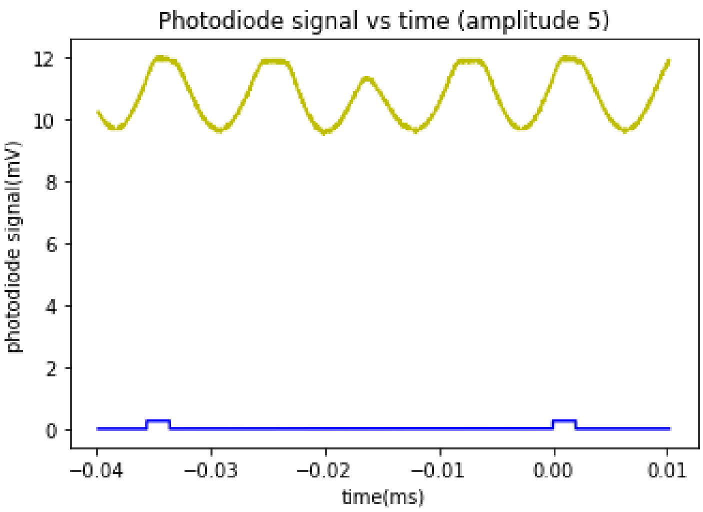
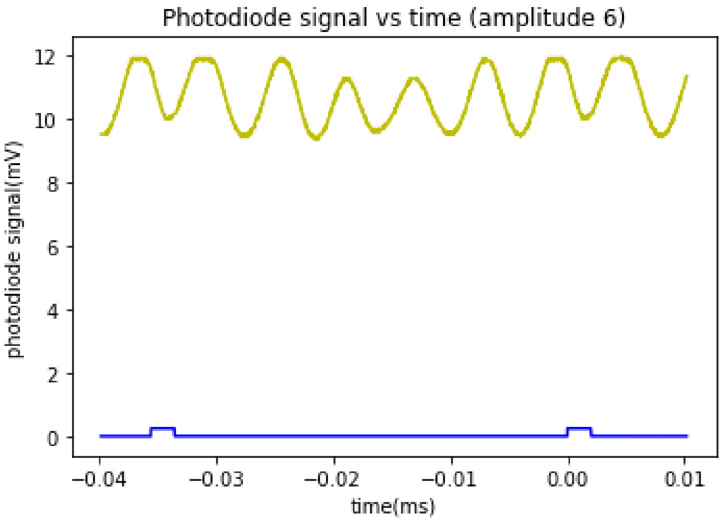
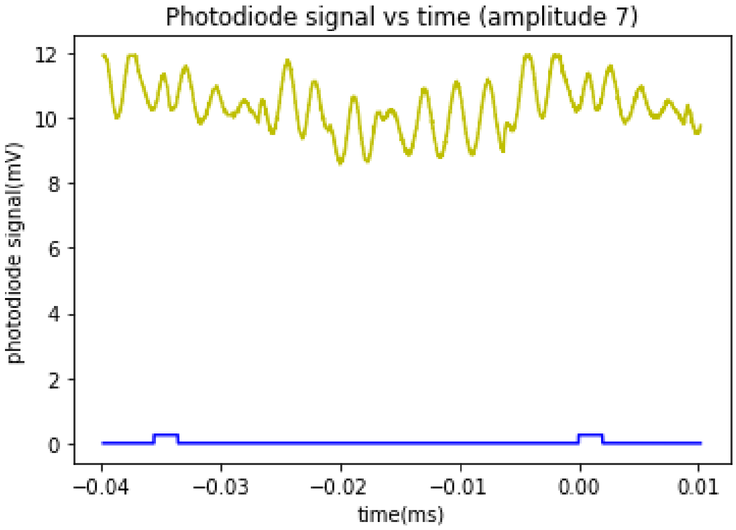
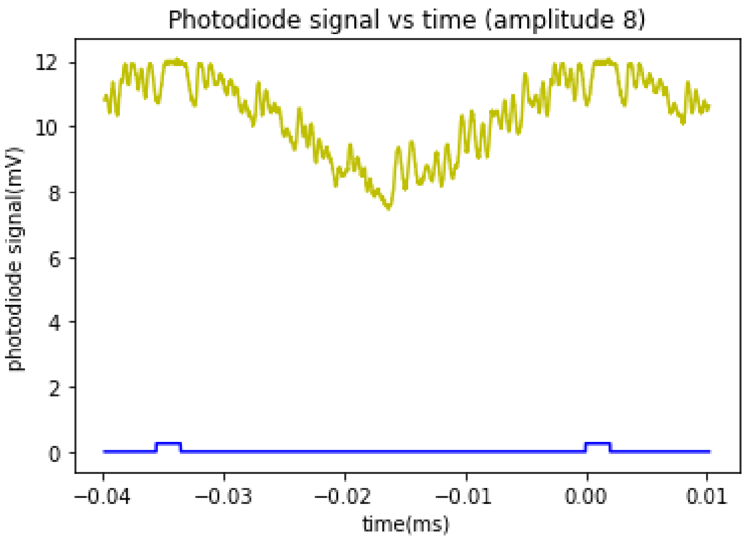
3	Photodiode signal vs time (amplitude 3) photodiode signal(mV) time(ms) 2.5 2.0 1.5 1.0 0.5 0.0 −0.06 −0.04 −0.02 0.00 0.02 0.04
4	Photodiode signal vs time (amplitude 4) photodiode signal(mV) time(ms) 2.0 1.5 1.0 0.5 0.0 −0.06 −0.04 −0.02 0.00 0.02 0.04
5	Photodiode signal vs time (amplitude 5) photodiode signal(mV) time(ms) 12 10 8 6 4 2 0 −0.04 −0.03 −0.02 −0.01 0.00 0.01

6	Photodiode signal vs time (amplitude 6) photodiode signal(mV) time(ms)
7	Photodiode signal vs time (amplitude 7) photodiode signal(mV) time(ms)
8	Photodiode signal vs time (amplitude 8) photodiode signal(mV) time(ms)