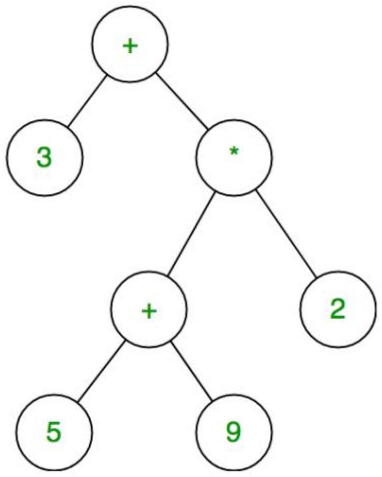
CS 260
Binary Parse Trees

Parse Trees

Parse Trees are binary trees, where each node has a parent and up to two children. They are used for parsing language and arithmetic expressions. This document focuses on parsing of arithmetic expressions.

Example Tree

In this example, the root contains the operator for addition, its left child is the number 3, its right child is the operator for multiplication, and so forth.



Expression Representation

Most people have been taught the use of infix notation. This is where the operator is written between the two operands and parentheses are used to indicate operations that are done out of the normal sequence. Other notations are postfix, where the operator follows its operands, and prefix, where the operator precedes its operands.

The expression shown in the above tree can be written in infix notation as **3 + (5 + 9) * 2**, or with full parentheses: (3 + ((5 + 9) * 2)). The same expression is written in postfix as **3 5 9 + 2 * +** and in prefix as: **+ 3 * + 5 9 2**.

Expression Evaluation

Most people have learned to evaluate an infix expression using the following rules.

1. Parentheses
2. Exponents
3. Multiplication/Division
4. Addition/Subtraction

and then evaluate the result left to right.

Computers normally use postfix notation because it does not require any knowledge of precedence of operators, use of parentheses, and is simply to evaluate using a stack. Postfix is also known as

Reverse Polish Notation or RPN. Learning to read and understand it is a good skill for people who work with computers.

The rules are simple:

1. Read the expression, one token at a time
    a. If operand, push on stack
    b. If operator, pop right side, pop left side, perform operation, push result on stack
2. When done reading expression, pop the result from stack

The evaluation of a postfix expression (3 5 9 + 2 * +) is illustrated in the following steps:

	Expression	Action	Stack
1	3 5 9 + 2 * +	Read 3 and push on stack	3
2	5 9 + 2 * +	Read 5 and push on stack	3, 5
3	9 + 2 * +	Read 9 and push on stack	3, 5, 9
4	+ 2 * +	Read +, pop 9, pop 5, push 5 + 9 on stack	3, 14
5	2 * +	Read 2 and push on stack	3, 14, 2
6	* +	Read *, pop 2, pop 14, push 14 * 2 on stack	3, 28
7	+	Read +, pop 28, pop 3, push 3 + 28 on stack	31
8		End of expression, pop 31 and return	

Displaying a Parse Tree

The same traversal methods are used to display the contents of a Binary Parse Tree as for any other binary tree. A prefix traversal yields the tree's expression in prefix notation, a postfix traversal yields it in postfix notation, and an infix traversal yields it in infix. For infix, it is necessary to modify the traversal code to add parentheses around each operator and its operands.

Parsing a postfix expression to build a Parse Tree

Since RPN (postfix) is so easy to use when evaluating an expression, it is also easy to use when creating a parse tree. The advantage of creating such a tree is that it provides an easy way to store an expression and then retrieve it in any of the three forms (postfix, prefix, or infix).

Instead of having a stack that holds operands and intermediate results, use a stack that holds the root node of a tree. Also realize that adding a node to such a stack is equivalent to adding the node and its entire subtree (since the subtree is attached via the left and right variables of the root node).

The basic steps for creating such a tree from a postfix expression are shown below.

```
Look at each character in the string in order
•  If a space
      o  do nothing
•  If an operand
      o  create a new node for the operand
      o  push it on the stack
•  If an operator
      o  create a new node for the operator
      o  pop the top of the stack and attach it on the right of the new node
```

        - pop the top of the stack and attach it on the left of the new node
        - push new node and its subtree on the stack
    - If end of expression
        - pop the top of the stack and save the node as root of the tree

Pseudo Code for creating a ParseTree from a postfix expression

```
doParse(expression)

  // Get next char
  // if char is letter, push
  // if char is operator, pop to right, pop to left, push
  stack theStack
  for i = 0; i < expression.length(); i++

     letter = expression[i]

     if isOperand(letter)
        theStack.push(new ParseNode(letter))

     else
        temp = new ParseNode(letter)

        temp.right = theStack.top()
        theStack.pop()

        temp.left = theStack.top()
        theStack.pop()

        theStack.push(temp)

  return theStack.top()
```

Parsing an infix expression to build a tree

The simplest way to parse an infix expression to build a tree is to convert it to postfix and then use the above algorithm. The algorithm for this conversion (assuming a string input and a string output) also uses a stack and is as shown below.

- Read the next character
    - if space
        - skip
    - If operand
        - add to output string
    - if L paren
        - push on the stack
    - if R paren
        - pop top item (continue until get L paren)
            - if not L paren, add to output string
            - else, discard and end popping
    - if operator
        - while stack not empty
            - pop top item
                - if L paren, push back on stack, stop popping

        - if not operator with higher precedence, push back on stack, stop popping
        - else add to output string
    - push operator

- At end of expression
    - while the stack is not empty
        - pop and add to buffer

Consider this algorithm on the expression (A + B) / (C - D)

	Input	Action	Stack	Output String
1	(A+B)/(C-D)	L paren, push on stack	(	
2	A+B)/(C-D)	A, operand, output	(	A
3	+B)/(C-D)	+, operator, pop stack, (, push on stack	(, +	A
4	B)/(C-D)	B, operand, output	(,+	AB
5	)/(C-D)	R paren, pop adding to output until L paren		AB+
6	/(C-D)	/, operator, push on stack	/	AB+
7	(C-D)	L paren, push on stack	/,(	AB+
9	C-D)	C, operand, output	/,(	AB+C
10	-D)	-, operator, pop stack, (, push on stack	/,(,-	AB+C
11	D)	D, operand, output	/,(,-	AB+CD
12	)	R paren, pop adding to output until L paren	/	AB+CD-
13		End of expression, pop adding to output		AB+CD-/

Note that this algorithm assumes a fully parenthesized expression.

Pseudo code for converting infix expression to postfix

```
inOrder2PostOrder(input)
   stack theStack
   string buffer = ""

   for i = 0; i < input.length()); i++
      next = input[i]

      // directly output all operands
      if isOperand(next)
         buffer += next

      // push opening parens onto stack
      else if next == LPAREN
         theStack.push(next)

      // for a closing paren
      //  pop stack and output
      //  until matching opening paren
      else if next == RPAREN
         isPopping = true
         while not theStack.empty() and isPopping

            value = theStack.top()
            theStack.pop()

            if value == LPAREN
                isPopping = false
            else
                buffer += value


    // for an operator
    // pop and output anything on stack
    //   until opening paren or higher precedence
    // then push this operator on stack
    else if isOperator(next)
        isPopping = true
        while not theStack.empty() and isPopping

            value = theStack.top()
            theStack.pop()

            if value == LPAREN
                theStack.push(value)
                isPopping = false

            else
                if not higherPrec(value, next)
                    theStack.push(value)
                    isPopping = false

                else
                    buffer += value

        theStack.push(next)

// get any remaining operators on stack
while not theStack.empty()
    buffer += theStack.top()
    theStack.pop()

// return the result
return buffer
```