MATH3007 Assignment 6

Due in class (12pm), Nov 14th

Problem 1 (20pts). Consider the function

$$f(x, y, z) = 2x^2 + xy + y^2 + yz + z^2 - 6x - 7y - 8z - 9$$

1. Use the first-order necessary conditions to find the candidate minimizer of $f(x, y, z)$.
2. Verify using the second-order sufficient conditions whether those candidates are indeed local minimizers.
3. Argue that the local minimizer you find in part 2 is also a global minimizer.

Problem 2 (20pts). The CUHK-SZ decides to build a circular fountain on campus. This time, it is required that this fountain contains the points $(0, 0)$, $(1, 5)$, $(2, 3)$ and $(3, 1)$. And one wants to build the smallest fountain as possible (the smallest radius).

1. Formulate this problem as a nonlinear optimization problem.
2. Write down its KKT conditions.

Problem 3 (20pts). Entropy maximization is a problem that is commonly encountered in information theory. The general form is given by

$$\begin{aligned} \text{minimize} \quad & \textstyle\sum_{i=1}^n x_i \log x_i \\ \text{subject to} \quad & \textstyle\sum_{i=1}^n a_i x_i = 1 \\ & x_i \geq 0 \end{aligned}$$

Write down the KKT conditions for this problem.

Problem 4 (20pts). A small computer company forecasts the demand over the next 12 months in the coming year to be d_i, $i = 1, 2, ..., 12$. In any month it can produce at most r units. The cost of producing x_i units is x_i^2 in month i. The firm can store units from month to month at a cost of s dollars per unit per month. Finally, the opportunity costs associated with unmet demand is k dollars per unit per month (any unmet demand is lost and does not roll over to the next month). Formulate the problem of determining the production schedule that minimizes cost (please explain your answer). Derive its KKT conditions.

Problem 5 (20pts). Construct the KKT condition for the following linear program:

$$\begin{aligned} \text{minimize} \quad & 5x_1 + 2x_2 + 5x_3 \\ \text{subject to} \quad & 2x_1 + 3x_2 + x_3 \geq 4 \\ & x_1 + 2x_2 + 3x_3 \geq 7 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$