MAT3007 Assignment 1

Due on Sep 19, 12pm

For those questions that ask you to write MATLAB codes to solve the problem. Please attach the code to the homework. You also need to clear state (write or type) what is the optimal solution and the optimal value you obtained. However, you do not need to attach the outputs in the command window of MATLAB.

Problem 1 (25pts). A company produces two kinds of products. A product of the first type requires 1/4 hours of assembly labor, 1/8 hours of testing, and \$1.2 worth of raw materials. A product of the second type requires 1/3 hours of assembly, 1/3 hours of testing, and \$0.9 worth of raw materials. Given the current personnel of the company, there can be at most 90 hours of assembly labor and 80 hours of testing each day. Products of the first and second type have a market value of \$9 and \$8 respectively.

(a) Formulate a linear optimization that maximizes the daily profit of the company.

(b) Write the standard form of the LP you formulated in part (a)

(c) Consider the following modification to the original problem: Suppose that up to 50 hours of overtime assembly labor can be scheduled, at a cost of \$7 per hour. Can it be easily incorporated into the linear optimization formulation and how?

(d) Solve the LP using MATLAB (for the original problem).

Problem 2 (25pts). Reformulate the problem:

$$\begin{aligned} \text{minimize} \quad & 2x_2 + |x_1 - x_3| \\ \text{subject to} \quad & |x_1 + 2| + |x_2| \leq 5 \\ & x_3^2 \leq 1 \end{aligned}$$

as a linear optimization problem. Also write down its standard form.

Problem 3 (25pts). Consider a school district with I neighborhoods, J schools, and G grades at each school. Each school j has a capacity of C_{jg} for grade g. In each neighborhood i, the student population of grade g is S_{ig}. Finally, the distance of school j from neighborhood i is d_{ij}. Formulate a linear optimization problem whose objective is to assign all students to schools, while minimizing the total distance traveled by all students.

Problem 4 (25pts). The China Railroad Ministry is in the process of planning relocations of freight cars among 5 regions of the country to get ready for the fall harvest. Table 1 shows the cost of moving a car between each pair of regions. Table 2 shows the current number of cars in each region and the number needed for harvest shipping.

From/To	1	2	3	4	5
1	-	10	12	17	34
2	10	-	18	8	46
3	12	18	-	9	27
4	17	8	9	-	20
5	34	46	27	20	-

Table 1: Costs of moving a car

	1	2	3	4	5
Present	115	385	410	480	610
Need	200	500	800	200	300

Table 2: Number of current and needed cars

Write down a linear optimization to compute the least costly way to move the cars such us the need is met. Solve the problem using MATLAB.

Problem 5 (bonus 20pts)[1] Consider a graph with n nodes. We denote the set of nodes by $V = \{1, ..., n\}$. For each pair of nodes (i, j), there is a edge connecting them with a weight $w_{ij} \geq 0$. Now we want to separate the nodes into two disjoint set S and T such that $S \bigcap T = \phi$ (empty set) and $S \bigcup T = V$. And we want to maximize the weights that are in the cut, i.e.,

$$\sum_{i \in S} \sum_{j \in T} w_{ij}.$$

This is called the *maximum cut problem* and has wide applications in problems such as circuit design (See Figure 1 for an illustration of a cut). Now using decision variables

$$x_i = \begin{cases} 1 & \text{if } x_i \in S \\ -1 & \text{if } x_i \in T \end{cases}$$

write an optimization problem (not necessarily a linear optimization) for max-cut problem.

[1]The total points of a homework is bounded by 100.

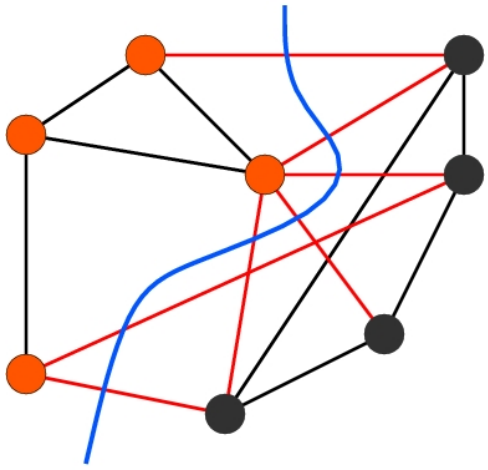

Figure 1: Illustration of a cut: The red edges are cut (therefore, there weights are counted)