MAT3007: Optimization - Assignment 1

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Problem 1

(a) Let x and y be the number of the first type product and the second type product produced by the company, respectively. The optimization problem can be written as:

$$\begin{aligned} \text{maximize}_{\text{x,y}} \quad & 7.8\text{x} + 7.1\text{y} \\ \text{subject to} \quad & \frac{1}{4}\text{x} + \frac{1}{3}\text{y} \le 90 \\ & \frac{1}{8}x + \frac{1}{3}y \le 80 \\ & x,\ y \ge 0 \end{aligned}$$

(b) The standard form of the LP is:

$$\begin{aligned} \text{minimize}_{\text{x,y}} \quad & -7.8\text{x} - 7.1\text{y} \\ \text{subject to} \quad & \frac{1}{4}\text{x} + \frac{1}{3}\text{y} + \text{s}_1 = 90 \\ & \frac{1}{8}x + \frac{1}{3}y + s_2 = 80 \\ & x,\ y,\ s_1,\ s_2 \ge 0 \end{aligned}$$

(c) The problem can be easily incorporated into the linear optimization formulation as follows:

$$\begin{aligned} \text{maximize}_{\text{x,y,z}} \quad & 7.8\text{x} + 7.1\text{y} - 7\text{z} \\ \text{subject to} \quad & \frac{1}{4}\text{x} + \frac{1}{3}\text{y} - \text{z} \le 90 \\ & \frac{1}{8}x + \frac{1}{3}y \le 80 \\ & z \le 50 \\ & x,\ y,\ z \ge 0 \end{aligned}$$

(d) The optimal solution is $x = 360$, $y = 0$. The optimal value is 2808.

Attached is the MATLAB code:

```
%% Problem 1(d)

cvx_begin quiet
variables x y;
    maximize 7.8*x + 7.1*y
    subject to
        1/4*x + 1/3*y <= 90;
        1/8*x + 1/3*y <= 80;
        [x, y] >= 0;
cvx_end

[x y]
cvx_optval
```

Problem 2

This can be equivalently written as:

$$\text{minimize} \quad 2x_2 + a$$

$$\text{subject to} \quad b + c \leq 5$$

$$a \geq x_1 - x_3$$

$$a \geq -x_1 + x_3$$

$$b \geq x_1 + 2$$

$$b \geq -x_1 - 2$$

$$c \geq x_2$$

$$c \geq -x_2$$

$$x_3 \geq -1$$

$$x_3 \leq 1$$

Its standard form is:

$$\text{minimize} \quad 2x_2 + a$$

$$\text{subject to} \quad b + c + s_1 = 5$$

$$a - s_2 = x_1 - x_3$$

$$a - s_3 = -x_1 + x_3$$

$$b - s_4 = x_1 + 2$$

$$b - s_5 = -x_1 - 2$$

$$c - s_6 = x_2$$

$$c - s_7 = -x_2$$

$$x_3 - s_8 = -1$$

$$x_3 + s_9 = 1$$

$$s_1,\ s_2,\ s_3,\ s_4,\ s_5,\ s_6,\ s_7,\ s_8,\ s_9 \geq 0$$

Problem 3

Let x_{ijg} denote the number of students of grade g who live in neighborhood i and go to school j. The optimization problem can be written as:

$$\text{minimize} \sum_{\text{i}\in\text{I}}\sum_{\text{j}\in\text{J}}\sum_{\text{g}\in\text{G}} \text{d}_{\text{ij}}\text{x}_{\text{ijg}}$$

$$\text{subject to} \sum_{\text{j}\in\text{J}} \text{x}_{\text{ijg}} = \text{S}_{\text{ig}} \qquad \forall \text{i} \in \text{I}, \text{g} \in \text{G}$$

$$\sum_{i\in I} x_{ijg} \leq C_{ig} \qquad \forall j \in J, g \in G$$

$$x_{ijg} \geq 0$$

Problem 4

I use w_{ij} to denote the cost of moving a car between region i and region j, x_{ij} is the number of cars moved from region i to region j. i, j = 1, 2, 3, 4, 5. The optimization problem can be written as:

$$\text{minimize} \quad \sum_{\text{i,j}} \text{w}_{\text{ij}}\text{x}_{\text{ij}}, \quad \forall \text{i} \neq \text{j}$$

$$\text{subject to} \quad 115 - \sum_{\text{j}\neq 1} \text{x}_{1\text{j}} + \sum_{\text{i}\neq 1} \text{x}_{\text{i}1} = 200$$

$$385 - \sum_{j\neq 2} x_{2j} + \sum_{i\neq 2} x_{i2} = 500$$

$$410 - \sum_{j\neq 3} x_{3j} + \sum_{i\neq 3} x_{i3} = 800$$

$$480 - \sum_{j\neq 4} x_{4j} + \sum_{i\neq 4} x_{i4} = 200$$

$$610 - \sum_{j\neq 5} x_{5j} + \sum_{i\neq 5} x_{i5} = 300$$

$$x_{ij} \geq 0, \quad \forall i \neq j$$

The optimal cost is 11370. The optimal solution is moving 115 cars from region 4 to region 2, moving 165 cars from region 4 to region 3, moving 85 cars from region 5 to region 1, and moving 225 cars from region 5 to region 3. Attached is the MATLAB code:

```
%% Problem 4

M = 100;
W = [M, 10, 12, 17, 34;
    10, M, 18, 8, 46;
    12, 18, M, 9, 27;
    17, 8, 9, M, 20;
    34, 46, 27, 20, M];

[n, ~] = size(W);

cvx_begin
variables x(n,n)
    minimize sum(sum(W .* x))
    subject to
        115 - sum(x(1,:)) + sum(x(:,1)) == 200;
        385 - sum(x(2,:)) + sum(x(:,2)) == 500;
        410 - sum(x(3,:)) + sum(x(:,3)) == 800;
        480 - sum(x(4,:)) + sum(x(:,4)) == 200;
        610 - sum(x(5,:)) + sum(x(:,5)) == 300;
        x >= 0
cvx_end
x
optval = cvx_optval
```

Problem 5

The optimization problem can be formed as follows:

$$\text{maximize} \quad \frac{1}{4}\sum_{\mathrm{i,j}} \mathrm{w_{ij}}(1 - \mathrm{x_i x_j})$$

$$\text{subject to} \quad \mathrm{x_i, x_j} \in \{1, \ -1\} \quad \mathrm{i, j} = 1, ..., \mathrm{n}$$