MAT3007: Optimization - Assignment 3

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October 10, 2018

Problem 1

The optimization problem can be written as the following standard form:

$$\begin{aligned} \text{minimize} \quad & -500\mathrm{x}_1 - 250\mathrm{x}_2 - 600\mathrm{x}_3 \\ \text{subject to} \quad & 2\mathrm{x}_1 + \mathrm{x}_2 + \mathrm{x}_3 + \mathrm{s}_1 = 240 \\ & 3\mathrm{x}_1 + \mathrm{x}_2 + 2\mathrm{x}_3 + \mathrm{s}_2 = 150 \\ & \mathrm{x}_1 + 2\mathrm{x}_2 + 4\mathrm{x}_3 + \mathrm{s}_3 = 180 \\ & \mathrm{x}_1,\ \mathrm{x}_2,\ \mathrm{x}_3,\ \mathrm{s}_1,\ \mathrm{s}_2,\ \mathrm{s}_3 \geq 0 \end{aligned}$$

The canonical form is as follows:

B	- 500	- 250	- 600	0	0	0	0
4	2	1	1	1	0	0	240
5	3	1	2	0	1	0	150
6	1	2	4	0	0	1	180

The current basis is $s_1,\ s_2,\ s_3$, the current basic solution is (0, 0, 0, 240, 150, 180), and the corresponding objective value is 0.

Choose column 1 to be the incoming basis:

B	0	- 250/3	- 800/3	0	500/3	0	25000
4	0	1/3	-1/3	1	-2/3	0	140
1	1	1/3	2/3	0	1/3	0	50
6	0	5/3	10/3	0	-1/3	1	130

The current basis is $x_1,\ s_1,\ s_3$, the current basic solution is (50, 0, 0, 140, 0, 130), and the corresponding objective value is -25000.

Choose column 2 to be the incoming basis:

B	0	0	- 100	0	150	50	31500
4	0	0	-1	1	-3/5	-1/5	114
1	1	0	0	0	2/5	-1/5	24
2	0	1	2	0	-1/5	3/5	78

The current basis is $x_1,\ x_2,\ s_3$, the current basic solution is (24, 78, 0, 114, 0, 0), and the corresponding objective value is -31500.

Choose column 3 to be the incoming basis:

B	0	50	0	0	140	80	35400
4	0	1/2	0	1	-7/10	1/10	153
1	1	0	0	0	2/5	-1/5	24
3	0	1/2	1	0	-1/10	3/10	39

All the reduced costs are positive now.

Therefore, the current basis is $x_1,\ x_3,\ s_1$, the optimal solution is (24, 0, 39, 153, 0, 0), and the optimal value is -35400.

Problem 2

The optimization problem is in its standard form:

$$\begin{aligned} \text{minimize} \quad & -2\mathrm{x}_1 - 3\mathrm{x}_2 + \mathrm{x}_3 + 12\mathrm{x}_4 \\ \text{subject to} \quad & -2\mathrm{x}_1 - 9\mathrm{x}_2 + \mathrm{x}_3 + 9\mathrm{x}_4 + \mathrm{x}_5 = 0 \\ & 1/3\mathrm{x}_1 + \mathrm{x}_2 - 1/3\mathrm{x}_3 - 2\mathrm{x}_4 + \mathrm{x}_6 = 0 \\ & \mathrm{x1},\ \mathrm{x2},\ \mathrm{x3},\ \mathrm{x4},\ \mathrm{x5},\ \mathrm{x6} \geq 0 \end{aligned}$$

The canonical form is as follows:

B	-2	-3	1	12	0	0	0
5	2	9	-1	-9	-1	0	0
6	1/3	1	-1/3	-2	0	1	0

Choose column 1 to be the incoming basis:

B	0	6	0	3	-1	0	0
1	1	9/2	-1/2	-9/2	-1/2	0	0
6	0	-1/2	-1/6	-1/2	1/6	1	0

Choose column 5 to be the incoming basis:

B	0	3	-1	0	0	6	0
1	1	3	-1	-6	0	3	0
5	0	-3	-1	-3	1	6	0

Choose column 3 to be the incoming basis:

B	-1	0	6	6	0	3	0
3	-1	-3	1	6	0	-3	0
5	-1	-6	0	3	1	3	0

Notice that in the column 1, both rows are negative. Therefore, the problem is unbounded.

Problem 3

First, construct the auxiliary problem:

$$\begin{aligned} \text{minimize} \quad & x_6 + x_7 + x_8 \\ \text{subject to} \quad & x_1 + 3x_2 + 4x_4 + x_5 + x_6 = 2 \\ & x_1 + 2x_2 - 3x_4 + x_5 + x_7 = 2 \\ & -x_1 - 4x_2 + 3x_3 + x_8 = 1 \\ & x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6,\ x_7,\ x_8 \geq 0 \end{aligned}$$

The initial tableau for the auxiliary problem is:

B	-1	-1	-3	-1	-2	0	0	0	-5
6	1	3	0	4	1	1	0	0	2
7	1	2	0	-3	1	0	1	0	2
8	-1	-4	3	0	0	0	0	1	1

Then carry out the simplex method.

Step 1:

B	0	2	-3	3	-1	1	0	0	-3
1	1	3	0	4	1	1	0	0	2
7	0	1	0	7	0	1	-1	0	0
8	0	-1	3	4	1	1	0	1	3

Step 2:

B	0	1	0	7	0	2	0	1	0
1	1	3	0	4	1	1	0	0	2
7	0	1	0	7	0	1	-1	0	0
3	0	-1/3	1	4/3	1/3	1/3	0	1/3	1

It still contains basic indices x_7 in the auxiliary variables, I pick x_2 to make the BFS x = (2, 0, 1, 0, 0, 0), B = {1, 2, 3}.

B	0	0	0	0	0	1	1	1	0
1	1	0	0	-17	1	-2	3	0	2
2	0	1	0	7	0	1	-1	0	0
3	0	0	1	11/3	1/3	2/3	-1/3	1/3	1

$\bar{c} = c^T - c_B^T A_B^{-1} A$ = (0, 0, 0, 3, -5)

The current objective value is: 7.

Now the Simplex tableau becomes:

B	0	0	0	3	-5	-7
1	1	0	0	-17	1	2
2	0	1	0	7	0	0
3	0	0	1	11/3	1/3	1

The next pivot:

B	5	0	0	-82	0	3
5	1	0	0	-17	1	2
2	0	1	0	7	0	0
3	-1/3	0	1	28/3	0	1/3

The next pivot:

B	5	82/7	0	0	0	3
5	1	17/7	0	0	1	2
4	0	1/7	0	1	0	0
3	-1/3	-4/3	1	0	0	1/3

This is optimal. The optimal solution is x = (0, 0, 1/3, 0, 2). The optimal value is -3.

Problem 4

1. $\delta < 0,\ \alpha \leq 0,\ \gamma \leq 0,\ \beta \geq 0$. If we bring x_1 into the basis, the optimal value is $-\infty$.
2. $\beta > 0$. The current solution is feasible but not optimal, since we can still bring x_2 into the basis.
3. $\beta = 0,\ \delta = 0,\ \gamma = 0,\ \eta \leq 0$. If we let x_2 enter the basis, we obtain $\theta^* = \beta/3 = 0$. The reduced costs turn out to be nonnegative, which means that we get the optimal value. And this means that we find multiple optimal bases achieve the same objective value.

Problem 5

1. $\beta \geq 0$, the basic variables are feasible.
2. $\alpha \geq 0,\ \beta < 0$ indicates that the problem is infeasible.
3. $\beta \geq 0,\ \delta < 0,\ \alpha > 0$. This will make the basic solution feasible, but not reach an optimal basic set B.
4. $\beta > 0,\ \delta < 0,\ \alpha \leq 0$. If we bring x_4 into the basis, the optimal value is $-\infty$.
5. $\beta \geq 0,\ \gamma < 0,\ \eta > \frac{4}{3}$.