MATH3007 Assignment 7

Due in class (12pm), Nov 21st

Problem 1 (20pts). Either prove or find a counterexample for each of the following statement (you can assume all the functions are second order continuously differentiable):

1. If $f(x)$ is convex, $g(x)$ is convex, then $f(g(x))$ is convex.
2. If $f(x)$ is convex and nondecreasing, $g(x)$ is convex, then $f(g(x))$ is convex.
3. If $f(x)$ is concave and nonincreasing, $g(x)$ is convex, then $f(g(x))$ is convex.
4. If $f(x)$ is increasing and non-negative, then $xf(x)$ is convex on $x \geq 0$.

Problem 2 (20pts). Consider the following function:

$$f(x_1, x_2) = \log\left(e^{x_1} + e^{x_2}\right)$$

1. Show that f is a convex function in (x_1, x_2).
2. Convert the following optimization problem into a convex optimization problem (hint: use result in part (1)):

$$\begin{array}{rc} \text{minimize} & x/y \\ \text{s.t.} & e^{-10} \leq x \leq e^3 \\ & x^2 + y/z \leq \sqrt{y} \\ & x/y = z^2 \\ & x, y, z \geq 0 \end{array}$$

3. Use CVX to solve the problem.

Problem 3 (20pts). Verify the Problem 1 in Assignment 6 is a convex optimization and use CVX to solve it. (Note that you may need to convert it in a convex form when inputting into CVX.)

Problem 4 (20pts). Show the entropy maximization problem (Problem 2 in Assignment 6) is a convex optimization problem.

Problem 5 (20pts). To model the influence of price on customer purchase probability, the following logit model is often used (p is the price, $\lambda(p)$ is the purchase probability):

$$\lambda(p) = \frac{e^{-p}}{1 + e^{-p}}$$

Assume the variable cost of the product is 0 (e.g., iPhone Apps). As the seller, you want to maximize the expected revenue by choosing the optimal price. That is, you want to solve:

$$\text{maximize}_p \quad p\lambda(p)$$

1. Draw a picture of $r(p) = p\lambda(p)$ (for p from 0 to 10) and use the picture to show that $r(p)$ is not concave (thus maximize $r(p)$ is not a convex optimization problem)

2. Write down p as a function of λ (the inverse function of $\lambda(p)$). Show that you can write the objective function as a function of λ: $\tilde{r}(\lambda)$, where $\tilde{r}(\lambda)$ is concave in λ.

3. From part 2, write the KKT condition for the optimal λ. Then transform it back to an optimal condition in p.