MAT 3007
Assignment 3 Solution

Problem 1
First, write the canonical form of the linear program:

$$\begin{array}{ll} \text{minimize} & -500x_1 - 250x_2 - 600x_3 \\ \text{s.t.} & 2x_1 + x_2 + x_3 + s_1 = 240 \\ & 3x_1 + x_2 + 2x_3 + s_2 = 150 \\ & x_1 + 2x_2 + 4x_3 + s_3 = 240 \\ & x_i, s_i \geq 0 \end{array}$$

B	-500	-250	-600	0	0	0	0
4	2	1	1	1	0	0	240
5	3	1	2	0	1	0	150
6	1	2	4	0	0	1	180
	0	-250/3	-800/3	0	500/3	0	25000
4	0	1/3	-1/3	1	-2/3	0	140
1	1	1/3	2/3	0	1/3	0	50
6	0	5/3	10/3	0	-1/3	1	130
	0	0	-100	0	150	50	31500
4	0	0	-1/2	1/2	-3/10	-1/10	57
1	1	0	0	0	14/15	-1/5	24
2	0	1	2	0	-1/5	3/5	78
	0	50	0	0	140	80	35400
4	0	1/2	0	1	-7/10	-1/10	153
1	1	0	0	0	14/15	-1/5	24
3	0	1/2	1	0	-1/10	3/10	39

In the initial tableau, the basic set is$\{4, 5, 6\}$, and the basic solution is $(0, 0, 0, 240, 150, 180)$. The objective function value is 0.
In the second tableau, the basic set is$\{4, 1, 6\}$, and the basic solution is $(50, 0, 0, 140, 0, 130)$. The objective function value is -25000.
In the third tableau, the basic set is$\{4, 1, 2\}$, and the basic solution is $(24, 78, 0, 57, 0, 0)$. The objective function value is -31500.
In the last tableau, the basic set is$\{4, 1, 3\}$, and the basic solution is $(24, 0, 39, 153, 0, 0)$. The objective function value is -35400.

Problem 2
Using Simplex method, after one iteration, we can see that the current basic solution is feasible, but the variable with negative reduced cost has a column with all negative entries. Therefore, the problem is unbounded.

B	-2	-3	1	12	0	0	0
5	-2	-9	1	9	1	0	0
6	1/3	1	-1/3	-2	0	1	0
	0	3	-1	0	0	6	0
5	0	-3	-1	-3	1	6	0
1	1	3	-1	-6	0	3	0

Problem 3
We first solve an auxiliary problem.

$$\begin{array}{ll} \text{minimize} & x_6 + x_7 + x_8 \\ \text{s.t.} & x_1 + 3x_2 + 4x_4 + x_5 + x_6 = 2 \\ & x_1 + 2x_2 - 3x_4 + x_5 + x_7 = 2 \\ & -x_1 - 4x_2 + 3x_3 + x_8 = 1 \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \geq 0 \end{array}$$

Use simplex tableau to solve the auxiliary problem.

B	0	0	0	0	0	1	1	1	0
c	-1	-1	-3	-1	-2	0	0	0	-5
6	1	3	0	4	1	1	0	0	2
7	1	2	0	-3	1	0	1	0	2
8	-1	-4	3	0	0	0	0	1	1
	0	2	-3	3	-1	1	0	0	-3
1	1	3	0	4	1	1	0	0	2
7	0	-1	0	-7	0	-1	1	0	0
8	0	-1	3	4	1	1	0	1	3
	0	1	0	7	0	2	0	1	0
1	1	3	0	4	1	1	0	0	2
7	0	-1	0	-7	0	-1	1	0	0
3	0	-1/3	1	4/3	1/3	1/3	0	1/3	1

The optimal value to the auxiliary problem is zero, which means the original problem has a feasible solution. Since one of the slack variables is still in the basis, and its optimal solution equals 0, we can just choose one variable from the rest to be the basic variable. Here, we choose x_2 to replace x_7. Next, use simplex method solve the original problem.

B	0	0	0	3	-5	-7
1	1	0	0	-17	1	2
2	0	1	0	7	0	0
3	0	0	1	11/3	1/3	1
	5	0	0	-82	0	3
5	1	0	0	-17	1	2
2	0	1	0	7	0	0
3	-1/3	0	1	28/3	0	1/3
	5	17/7	0	0	0	3
5	1	17/7	0	0	1	2
4	7/4	17/4	0	1	0	0
3	-1/3	-4/3	1	0	0	1/3

The optimal solution is $(0, 0, \frac{1}{3}, 0, 2)$, and the optimal objective function value is -3.

Problem 4

1. $\delta < 0$, $\alpha \leq 0$, $\gamma \leq 0$, $\beta \geq 0$
2. $\beta > 0$.
3. $\beta = 0$, $\delta \geq -\frac{2}{3}\gamma$.

Problem 5

1. $\beta \geq 0$.
2. $\alpha \geq 0$, $\beta < 0$.
3. $\beta > 0$, at least one of $\delta, \gamma, \xi < 0$. The reduced cost of one of the non-basic variables is negative.
4. $\beta \geq 0$, $\alpha \leq 0$, $\delta < 0$. The fourth column has all entries negative or zero.
5. $\beta \geq 0$, $\gamma < 0$, $\frac{2}{\eta} < \frac{3}{2}, \eta > 0 \rightarrow \eta > \frac{4}{3}$.