# MATH3007 Assignment 10

Due in class (12pm), Dec 19th

**Problem 1. (40pts)** Use branch-and-bound method to solve the following integer program. You are allowed to use LP solver to solve each linear program (the relaxed problems). Please specify the branch-and-bound tree and what you did at each node (similar as to what we did in the lecture slides).

$$\begin{aligned} \text{maximize} \quad & 17x + 12y \\ \text{subject to} \quad & 10x + 7y \leq 40 \\ & x + y \leq 5 \\ & x, y \geq 0 \\ & x, y \in \mathbb{Z} \end{aligned}$$

**Problem 2. (40pts)** You have $n$ items with sizes $a_1, ..., a_n$ to be packed into some bins. Each bin has capacity $V$. Formulate an integer program to find the packing method that uses the least number of bins. Hint: Use $I_j$ to denote whether bin $j$ is used; use $x_{ij}$ to denote whether item $i$ should be placed in bin $j$. (This is called bin packing problem.)

- Now consider a real case where you have items with size 4, 4, 5, 7 and the capacity of each bin is 10. Formulate the IP in that case. What are the optimal solutions to the IP and its LP relaxation? Is there an integrality gap in this case?

**Problem 3. (20pts)** Consider a seller who sells $m$ different products. For product $j$, there are $B_j$ units in inventory. There are $n$ customers, each customer $i$ is interested in buying a bundle of the product $S_i$, where $S_i \subseteq \{1, ..., m\}$ and is willing to pay a price $v_i$ for it. For each customer, the seller can only decide to accept his entire request $S_i$ or reject him. The objective of the seller is to maximize the revenue.

- Formulate this problem as an integer program.
- Consider the following example $B_1 = 1$, $B_2 = 2$, $B_3 = 3$, $S_1 = \{1, 2\}$, $v_1 = 2$, $S_2 = \{3\}$, $v_2 = 1$, $S_3 = \{1, 3\}$, $v_3 = 3$, $S_4 = \{2, 3\}$, $v_4 = 2$, $S_5 = \{2\}$, $v_5 = 2$. What is the optimal solution to the LP and IP respectively? What is the integrality gap?