MAT3007 2018 Fall
Assignment 1 Solution

1.(a) Let x_1 be the number of type 1 product, and x_2 be the number of type 2 product.

$$
\begin{array}{ll}
\text{maximize} & (9-1.2)x_1 + (8-0.9)x_2 \\
\text{s.t.} & \frac{1}{4}x_1 + \frac{1}{3}x_2 \leq 90 \\
 & \frac{1}{8}x_1 + \frac{1}{3}x_2 \leq 80 \\
 & x_1, x_2 \geq 0
\end{array}
$$

(b) The standard form is as follows.

$$
\begin{array}{ll}
\text{minimize} & -(9-1.2)x_1 - (8-0.9)x_2 \\
\text{s.t.} & \frac{1}{4}x_1 + \frac{1}{3}x_2 + s_1 = 90 \\
 & \frac{1}{8}x_1 + \frac{1}{3}x_2 + s_2 = 80 \\
 & x_1, x_2, s_1, s_2 \geq 0
\end{array}
$$

(c) Let x_3 be the number of overtime hours. We can form the following LP.

$$
\begin{array}{ll}
\text{maximize} & (9-1.2)x_1 + (8-0.9)x_2 - 7x_3 \\
\text{s.t.} & \frac{1}{4}x_1 + \frac{1}{3}x_2 \leq 90 + x_3 \\
 & \frac{1}{8}x_1 + \frac{1}{3}x_2 \leq 80 \\
 & x_3 \leq 50 \\
 & x_1, x_2, x_3 \geq 0
\end{array}
$$

(d) The optimal objective value is 2808. The optimal solutions are $x_1 = 360, x_2 = 0$.

```
cvx_begin
variables x1 x2

minimize -(9-1.2)*x1 - (8-0.9)*x2
subject to
        1/4*x1 + 1/3*x2 <= 90;
        1/8*x1 + 1/3*x2 <= 80;
        x1 >= 0;
        x2 >= 0;
cvx_end
```

2. Let $y_1 = |x_1 - x_3|$, $y_2 = |x_1 + 2|$, $y_3 = |x_2|$.

$$
\begin{array}{ll}
\text{minimize} & 2x_2 + y_1 \\
\text{s.t.} & y_1 \geq x_1 - x_3 \\
& y_1 \geq -x_1 + x_3 \\
& y_2 + y_3 \leq 5 \\
& y_2 \geq x_1 + 2 \\
& y_2 \geq -x_1 - 2 \\
& y_3 \geq x_2 \\
& y_3 \geq -x_2 \\
& x_3 \leq 1 \\
& x_3 \geq -1
\end{array}
$$

The standard form

$$
\begin{array}{ll}
\text{minimize} & 2x_2 + y_1 \\
\text{s.t.} & y_1 - (x_1^+ - x_1^-) + (x_3^+ - x_3^-) - s_1 = 0 \\
& y_1 + (x_1^+ - x_1^-) - (x_3^+ - x_3^-) - s_2 = 0 \\
& y_2 + y_3 - 5 + s_3 = 0 \\
& y_2 - (x_1^+ - x_1^-) - 2 - s_4 = 0 \\
& y_2 + (x_1^+ - x_1^-) + 2 - s_5 = 0 \\
& y_3 - (x_2^+ - x_2^-) - s_6 = 0 \\
& y_3 + (x_2^+ - x_2^-) - s_7 = 0 \\
& (x_3^+ - x_3^-) + 1 - s_8 = 0 \\
& (x_3^+ - x_3^-) - 1 + s_9 = 0 \\
& y_1, y_2, y_3, x_1^+, x_1^-, x_2^+, x_2^-, x_3^+, x_3^- \geq 0 \\
& s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9 \geq 0
\end{array}
$$

3. Let x_{ijg} be the number of students from neighborhood i going to school j in grade g.

$$
\begin{array}{lll}
\text{minimize} & \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{g=1}^{G} x_{ijg} d_{ij} & \\
\text{s.t.} & \sum_{j=1}^{J} x_{ijg} = S_{ig} & \forall i, g \\
& \sum_{i=1}^{I} x_{ijg} \leq C_{jg} & \forall j, g \\
& x_{ijg} \geq 0 & \forall i, j, g
\end{array}
$$

4. Let x_{ij} denote the number of cars moving from place i to j.

$$
\begin{array}{ll}
\text{minimize} & \sum_{i=1}^{5} \sum_{j=1}^{5} c_{ij} x_{ij} \\
\text{s.t.} & \sum_{i \neq 1} x_{i1} - \sum_{j \neq 1} x_{1j} \geq 200 - 115 \\
& \sum_{i \neq 2} x_{i2} - \sum_{j \neq 2} x_{2j} \geq 500 - 385 \\
& \sum_{i \neq 3} x_{i3} - \sum_{j \neq 3} x_{3j} \geq 800 - 410 \\
& \sum_{i \neq 4} x_{i4} - \sum_{j \neq 4} x_{4j} \geq 200 - 480 \\
& \sum_{i \neq 5} x_{i5} - \sum_{j \neq 5} x_{5j} \geq 300 - 610 \\
& x_{ij} \geq 0 \qquad \forall i, j
\end{array}
$$

The optimal solutions are $x_{41} = 0, x_{42} = 115, x_{43} = 165, x_{51} = 85, x_{53} = 225$, with the rest $x_{ij} = 0$. The optimal objective value is 11370.

```
c = [0 10 12 17 34;
    10 0 18 8 46;
    12 18 0 9 27;
    17 8 9 0 20;
    34 46 27 20 0];

cvx_begin
variables x(5,5)


minimize sum(sum(c.*x))
subject to
        x(2,1) + x(3,1) + x(4,1) + x(5,1) - x(1,2) - x(1,3) - x(1,4) - x(1,5) >= 200-115 ;
        x(1,2) + x(3,2) + x(4,2) + x(5,2) - x(2,1) - x(2,3) - x(2,4) - x(2,5) >= 500-385 ;
        x(1,3) + x(2,3) + x(4,3) + x(5,3) - x(3,1) - x(3,2) - x(3,4) - x(3,5) >= 800-410 ;
        x(1,4) + x(2,4) + x(3,4) + x(5,4) - x(4,1) - x(4,2) - x(4,3) - x(4,5) >= 200-480 ;
        x(1,5) + x(2,5) + x(3,5) + x(4,5) - x(5,1) - x(5,2) - x(5,3) - x(5,4) >= 300-610 ;
        x >= zeros(5,5);
cvx_end
```

5. In Problem 5, we want to sum on weight of the edges that have nodes from two different sets. So, we need to come up with a function of x_i and x_j such that it equals to 1 if i and j are in different sets, and 0 otherwise.

$$
\begin{array}{ll}
\text{maximize} & \sum_{i \in S} \sum_{j \in T} \frac{1}{2}(-x_i x_j + 1) w_{ij} \\
\text{s.t.} & x_i = \begin{cases} 1 & \text{if } x_i \in S \\ -1 & \text{if } x_i \in T \end{cases}
\end{array}
$$

This problem can also be formulated using another approach.

$$
\begin{array}{ll}
\text{maximize} & \sum_{i \in S} \sum_{j \in T} \frac{1}{2} |x_i - x_j| w_{ij} \\
\text{s.t.} & x_i = \begin{cases} 1 & \text{if } x_i \in S \\ -1 & \text{if } x_i \in T \end{cases}
\end{array}
$$