MAT3007
Assignment 5 Solution

Problem 1

1. Let x_i denote the number of time process i is being carried out. The LP formulation is as follows:

$$\begin{aligned} \text{maximize} \quad & 200x_1 + 60x_2 + 206x_3 \\ \text{s.t.} \quad & 3x_1 + x_2 + 5x_3 \leq 8000 \\ & 5x_1 + x_2 + 3x_3 \leq 5000 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Using simplex method to find an optimal solution. At optimal, we have $x_1 = 0, x_2 = 500, x_3 =$

	-200	-60	-206	0	0	0
4	3	1	5	1	0	8000
5	5	1	3	0	1	5000
	0	-20	-86	0	40	200000
4	0	0.4	3.2	1	-0.6	5000
1	1	0.2	0.6	0	0.2	1000
	100	0	-26	0	60	300000
4	-2	0	2	1	-1	3000
2	5	1	3	0	1	5000
	74	0	0	13	47	339000
3	-1	0	1	0.5	-0.5	1500
2	8	1	0	-1.5	2.5	500

$1,500$, and the optimal values is $339,000$.

2. Let Δc be the amount of increase in gasoline. So the new cost coefficient $\tilde{c} = c + \Delta c$. Let $N = \{1, 4, 5\}$, $B = \{2, 3\}$.

$$\tilde{c}_N = \begin{pmatrix} -200 - 4\Delta c \\ 0 \\ 0 \end{pmatrix}$$

$$\tilde{c}_B = \begin{pmatrix} -60 - \Delta c \\ -206 - 3\Delta c \end{pmatrix}$$

$$A_N^T (A_B^{-1})^T = \begin{pmatrix} 8 & -1 \\ -1.5 & 0.5 \\ 2.5 & -0.5 \end{pmatrix}$$

We need the following inequality to hold:

$$\tilde{c}_N - A_N^T (A_B^{-1})^T \tilde{c}_B \geq 0$$

which are

$$\begin{cases} -200 - 4\Delta c + 480 + 8\Delta c - 206 - 3\Delta c \geq 0 \\ 0 - 90 - 1.5\Delta c + 103 + 1.5\Delta c \geq 0 \\ 0 + 150 + 2.5\Delta c - 103 - 1.5\Delta c \geq 0 \end{cases}$$

Solving the inequality, we have $\Delta c \geq -47$. This means as long as the price doesn't decrease by 47, the optimal basis stays the same.

3. We introduce a new constraint:

$$4x_1 + 3x_2 + 5x_3 \leq 10000$$

We bring in the optimal solution $(0, 500, 1500)$ to check whether the constraint holds. Simple calculation shows that this constraint is not violated. Thus, the optimal solution will not change.

Problem 2

1. Let x_i denote the number of ith insurance product. The linear program is

$$\begin{aligned} \text{maximize} \quad & 500x_1 + 250x_2 + 600x_3 \\ \text{s.t.} \quad & 2x_1 + x_2 + x_3 \leq 240 \\ & 3x_1 + x_2 + 2x_3 \leq 150 \\ & x_1 + 2x_2 + 4x_3 \leq 180 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

2. From the final tableau, we see that the shadow prices are \$0, \$140, \$80 respectively. If the company sells one unit of mortgage, it requires 1 hour for underwriting, 1 hour for administration, and 2 hours for claims. So, the cost to sell 1 unit of mortgage is $140 + 2 \times 80 = 300$. However, the profit for one unit of mortgage is only \$250 per unit. Therefore, the company cannot make any net profit by selling mortgage.

3. We have $B = \{4, 1, 3\}$, and we need to determine the range of λ such that

$$\bar{b} + \lambda A_B^{-1} e_3 \geq 0$$

A_B^{-1} is the last three columns of the final tableau, so

$$A_B^{-1} = \begin{pmatrix} 1 & -0.7 & 0.1 \\ 0 & 0.4 & -0.2 \\ 0 & -0.1 & 0.3 \end{pmatrix}$$

It is then straightforward to verify that we must have $-130 \leq \lambda \leq 120$, and therefore $b_3 \in [50, 300]$.

4. The optimal basis remains the same if and only if $r_N - \lambda A_N^T (A_B^{-1})^T e_1 \geq 0$.

$$\begin{cases} 50 \geq 0 \\ 140 - 0.4\lambda \geq 0 \\ 80 + 0.2\lambda \geq 0 \end{cases}$$

We can find $A_B^{-1} A_N$ from the final tableau, and it is therefore straightforward to verify that $-400 \leq \lambda \leq 350$, and therefore that $c_1 \in [150, 900]$.