MAT3007 Assignment 2

Due by noon (12pm), Sep 26th

Problem 1 (20pts). Consider an LP in its standard form and the corresponding constraint set $P = \{\boldsymbol{x} | A\boldsymbol{x} = \boldsymbol{b}, \boldsymbol{x} \geq 0\}$. Suppose that the matrix A has dimensions $m \times n$ and that its rows are linearly independent. For each of the following statements, state whether it is true of false. Please explain your answers (if not true, please show a counterexample).

1. The set of all optimal solutions (assuming existence) must be bounded;
2. At every optimal solution, no more than m variables can be positive;
3. If there is more than one optimal solution, then there are uncountably many optimal solutions.

Problem 2 (20pts). Solve the following 2-dimensional linear optimization problem using the graphical method as in the lecture.

$$\begin{aligned} \text{maximize} \quad & x_1 + x_2 \\ \text{s.t.} \quad & -x_1 + x_2 \leq 2.5 \\ & x_1 + 2x_2 \leq 8 \\ & 0 \leq x_1 \leq 4 \\ & 0 \leq x_2 \leq 3 \end{aligned}$$

Which constraints are active at optimal solution? Also list all the vertices of the feasible region.

Problem 3 (20pts). Consider the following linear optimization problem:

$$\begin{aligned} \text{maximize} \quad & x_1 + 4x_2 + x_3 \\ \text{s.t.} \quad & 2x_1 + 2x_2 + x_3 \leq 4 \\ & x_1 - x_3 \geq 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

- Transform it into standard form;
- Argue without solving this LP that there must exist an optimal solution with no more than 2 positive variables;

- List all the basic solutions and basic feasible solutions (of the standard form);
- Find the optimal solution by using the results in step 3.

Problem 4 (20pts). (World Cup Market) Suppose there are 5 types of securities available in the 2018 World Cup Assets market for sale. The price of each share of the security is fixed and the payoff of it will be contingent on the outcome of the outcome of the world cup. The information of the 5 securities are shown in Table 1. Here, for example, Security 1's payoff is 1 if either Argentina, Brazil, or England wins the world cup, and the payoff is 0 if Germany or Spain wins. The Share Limit is the maximum number of shares one can purchase, and Price is the current purchasing price per share of each security.

Security	Price π	Share Limit q	Argentina	Brazil	England	Germany	Spain
1	0.75	10	\$1	\$1	\$1		
2	0.35	5				\$1	\$1
3	0.40	10	\$1		\$1		\$1
4	0.75	10	\$1	\$1	\$1	\$1	
5	0.65	5		\$1		\$1	\$1

Table 1: Payoff matrix

1. Assume that short is not allowed, that is, one can only buy shares but not sell. Formulate the problem as a *linear program* to decide how many shares of each security to purchase so as to maximize the worst-case (minimum) payoff when the game is finally realized. You can denote $\pi = (0.75, 0.35, 0.40, 0.75, 0.65)^T$, $\boldsymbol{q} = (10, 5, 10, 10, 5)^T$ and

$$A = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

2. Use any software/method to find the optimal solution of the program you set up in (a). Report the optimal solution and the optimal value (and explain what it means).

Problem 5 (20pts). The Chinese University of Hong Kong, Shenzhen decides to build a circular fountain on the campus. The school wants the fountain to be round and as large as possible but it must be restricted in a polygonal construction field, which is given by the following points: $(0, 1)$, $(0, 6)$, $(4, 10)$, $(8, 10)$, $(11, 7)$, $(11, 4)$, $(7, 0)$, and $(1, 0)$. Give a linear optimization formulation of this problem and then solve it using MATLAB or any other software. (Hint: First plot the points, and represent the polygon by using these points in the form of $\{\boldsymbol{x} \in \mathbb{R}^2 | a_i^T \boldsymbol{x} \leq b_i\}$. Use decision variable $\boldsymbol{y}$ (center of the fountain) and the radius r.)