CSE 569 Homework #2

Total 3 points. Due Tuesday, Oct. 20 **by 11:59pm**.

Problem 1. A random variable X can take values 0, 1, 2, …., and it has the following distribution: $P(X = n \mid \lambda) = \dfrac{\lambda^n}{n!} e^{-\lambda}$

where $\lambda > 0$ is the parameter of the distribution. We are also given a training set $D = \{X_1\}$, i.e., only one sample drawn from the above distribution.

We further assume λ has the following prior distribution,

$$f(\lambda) = \begin{cases} e^{-\lambda}, & \text{if } \lambda > 0 \\ 0, & \text{otherwise} \end{cases}$$

Use the Bayesian Estimation technique to find an estimate for λ by finding the mean of the posterior density $f(\lambda|D)$. Discuss how this is different from the MLE.

Problem 2. On Bayesian Estimation. Consider the estimation of $p(\mathbf{x}|D)$ by revisiting the following expression,

$$p(\mathbf{x} \mid D) = \int p(\mathbf{x} \mid \boldsymbol{\theta}) p(\boldsymbol{\theta} \mid D) d\boldsymbol{\theta}$$

Discuss what role a *uniform* prior on $\boldsymbol{\theta}$ may play in the final estimation of $p(\mathbf{x}|D)$.

Problem 3. Discuss for what types of datasets PCA may be or may not be effective at all for the purpose of dimensionality reduction. Help your discussion by drawing 2-D examples.

Problem 4. Review the EM algorithm, and answer the following questions: (1) What does the E-step do? (2) If the E-step employs any distribution, what is that? (3) If the E-step employs any distribution, do the given training data have any impact on that distribution?

Problem 5. Assume that we have N observation sequences drawn from a hidden Markov model (HMM). Further assume that we have somehow obtained the corresponding state sequences that have generated these observation sequences. Outline a way to estimate all the model parameters for the underlying HMM.

Problem 6. Consider an HMM with a special emission probability matrix, in which each row has all entries being 1/M, where M is the number of possible observations (the cardinality of the set Ω). How does this special emission probability impact the HMM? In particular, revisit the Viterbi Algorithm (Slide 68 in Notes 04), and discuss how this special emission probability matrix impacts the specific steps of the algorithm (and thus impact the HMM).