Probability Model For A Multi-Logit Regression Model

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1 Multi-Logit Regression Model

The multi-logit regression model is given as

$$\begin{aligned} y_n &\sim \text{Categorical}\left(\mathbf{p}\right) \\ \mathbf{p} &= \left(p_1, \ldots, p_K\right) \\ p_k &= \frac{\exp\left(\sum\limits_{p=0}^{P} X_{n,p}\beta_{p,k}\right)}{\sum\limits_{k=1}^{K}\exp\left(\sum\limits_{p=0}^{P} X_{n,p}\beta_{p,k}\right)} \end{aligned} \tag{1}$$

where y_n has K discrete outcomes, say $y_n \in \{1, \ldots, K\}$, $X \in \mathbb{R}^{N\times(P+1)}$ is the predictor matrix with the entries in the first column all being one, and $\beta \in \mathbb{R}^{(P+1)\times K}$ is the coefficient matrix.

2 Probability Model

Stan allows us to use improper priors if we don't have any prior knowledge about the parameters. As an example, we could assume the following priors for β:

$$\beta_{p,k} \sim \text{Normal}\left(0, 5\right)$$

Putting it all together, the probability model for the multi-predictor logistic regression model is:

$$\begin{aligned} y_n &\sim \text{Categorical}\left(\mathbf{p}_n\right) \\ \mathbf{p}_n &= \left(p_{n1}, \ldots, p_{nK}\right) \\ p_{nk} &= \frac{\exp\left(\sum\limits_{p=0}^{P} X_{n,p}\beta_{p,k}\right)}{\sum\limits_{k=1}^{K}\exp\left(\sum\limits_{p=0}^{P} X_{n,p}\beta_{p,k}\right)} \\ \beta_{p,k} &\sim \text{Normal}\left(0, 5\right) \end{aligned}$$