

Probability Model For A Hierarchical Logistic Regression Model

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We consider a two level hierarchical logistic regression model in which the data are grouped into L distinct categories. Each category l gets its own coefficient vector β_l , and these coefficients are generated from the single set of hyperparameters.

1 Hierarchical Logistic Regression Model

The two level hierarchical logistic regression model is described as below:

$$y_n \sim \text{Bernoulli}(p_n)$$
$$p_n = \text{logit}^{-1} \left(\sum_{d=1}^D X_{n,d} \beta_{l_n,d} \right)$$

where $y_n \in \{0, 1\}$, $n = 1, \dots, N$; $X \in \mathbb{R}^{N \times D}$ is the predictor matrix, where N is the number of observations, and D is the number of predictors; $l_n \in \{1, \dots, L\}$ is the category associated with each binary outcome y_n ; $\beta_l \in \mathbb{R}^D$ is the coefficient vector for category l .

2 Probability Model

$\beta_{l,d}$ are the parameters for this hierarchical logistic regression model, where $l \in \{1, \dots, L\}$, $d = 1, \dots, D$. Assume $\beta_{l,d}$ are drawn from a single set of hyperparameters for all categories l as below:

$$\beta_{l,d} \sim \text{Normal}(\mu_d, \sigma_d)$$
$$d = 1, \dots, D$$

Then we need to further specify the priors for the hyperparameters μ_d and σ_d . For simulation purpose, we use the following reasonable weakly informative priors for the location and scale

parameters.

$$\begin{aligned}\mu &\sim \text{Normal}(0, 100) \\ \sigma &\sim \text{Gamma}(2, 0.1)\end{aligned}$$

where 0.1 is the rate parameter for gamma distribution.

Putting it all together, we have the probability model for this two level hierarchical logistic regression model as:

$$\begin{aligned}y_n &\sim \text{Bernoulli}(p_n) \\ p_n &= \text{logit}^{-1}\left(\sum_{d=1}^D X_{n,d}\beta_{l_n,d}\right) \\ \beta_{l,d} &\sim \text{Normal}(\mu_d, \sigma_d) \\ \mu_d &\sim \text{Normal}(0, 100) \\ \sigma_d &\sim \text{Gamma}(2, 0.1)\end{aligned}$$

where $l_n \in (1, \dots, L)$, and $d = 1, \dots, D$.