Probability Model For A One Predictor Logistic Regression Model

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1 Logistic Regression Model

The logistic regression model is given as

$$\begin{aligned} y &\sim \text{Bernoulli}\,(p) \\ logit\,(p) &= \alpha + x\beta \end{aligned} \tag{1}$$

where $y \in \{0, 1\}$, $x \in \mathbb{R}^N$, $\alpha \in \mathbb{R}$, and $\beta \in \mathbb{R}$.
In this model, outcome y is binary, x is the predictor, α is the intercept, β is the slope coefficient.

2 Probability Model

Stan allows us to use improper priors if we don't have any prior knowledge about the parameters. We can therefore start with a simple model by assuming improper priors for α and β.

$$\begin{aligned} \alpha &\sim \text{Uniform}\,(-\infty, \infty) \\ \beta &\sim \text{Uniform}\,(-\infty, \infty) \end{aligned}$$

Putting it all together, the probability model for the single predictor logistic regression model is:

$$\begin{aligned} y_n &\sim \text{Bernoulli}\,(p_n) \\ p_n &= logit^{-1}\,(\alpha + x_n\beta) = \frac{e^{\alpha + x_n\beta}}{1 + e^{\alpha + x_n\beta}} \\ \alpha &\sim \text{Uniform}\,(-\infty, \infty) \\ \beta &\sim \text{Uniform}\,(-\infty, \infty) \end{aligned}$$