Probability Model For A One Predictor Linear Regression Model

Nan Wu
nanw@udel.edu

1 Linear Regression Model

The linear regression model is given as

$$y = \alpha + x\beta + \epsilon \tag{1}$$

where $y \in \mathbb{R}^N$, $x \in \mathbb{R}^N$, $\alpha \in \mathbb{R}$, $\beta \in \mathbb{R}$, $\epsilon \in \mathbb{R}^N$, and $\epsilon \sim \text{Normal}(0, \sigma)$.
In this model, x is the predictor, y is the outcome, α is the intercept, β is the slope coefficient, ϵ is the residual.

2 Probability Model

Stan allows us to use improper priors if we don't have any prior knowledge about the parameters. We can therefore start with a simple model by assuming improper priors for α, β and σ:

$$\begin{aligned}
\alpha &\sim \text{Uniform}(-\infty, \infty) \\
\beta &\sim \text{Uniform}(-\infty, \infty) \\
\sigma &\sim \text{Uniform}(0, \infty)
\end{aligned}$$

Putting it all together, the probability model for the single predictor linear regression model is:

$$\begin{aligned}
y_n &\sim \text{Normal}(\mu_n, \sigma) \\
\mu_n &= \alpha + x_n\beta \\
\alpha &\sim \text{Uniform}(-\infty, \infty) \\
\beta &\sim \text{Uniform}(-\infty, \infty) \\
\sigma &\sim \text{Uniform}(0, \infty)
\end{aligned}$$