# Integration Theory

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MATØK6

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# 1 Lebesque Integration Theory

(Chapter 2-5 of Bartle's book, the most important results are: monotone convergence theorem, Fatou's lemma and Lebesgue dominated convergence theorem)

4.6 - Monotone Convergence Thereom

**Theorem 1.1** (Monotone Convergence Thereom)**.** *If $(f_n)$ is a monotone increasing sequence of functions in $M^+(X,m)$ which converges to $f$, then*

$$\int f\,d\mu = \lim \int f_n\,d\mu. \tag{1.1}$$

4.8 - Fatou's Lemma

**Theorem 1.2** (Fatou's Lemma)**.** *If $(f_n)$ belongs to $M^+(X,m)$, then*

$$\int (\liminf f_n)\,d\mu \leq \liminf \int f_n\,d\mu. \tag{1.2}$$

5.6 - Lebesgue Dominated Convergence Theorem

**Theorem 1.3** (Lebesgue Dominated Convergence Theorem)**.** *Let $(f_n)$ be a sequence of integrable function which converges almost everywhere to a real-valued measureable function $f$. If there exists an integrable function $g$ such that $|f_n| \leq g$ for all $n$, then $f$ is integrable and*

$$\int f\,d\mu = \lim \int f_n\,d\mu. \tag{1.3}$$

# 2 $L^p$ Spaces

(Chapter 6 of Bartle's book, the most important results are: Hölder's inequality, Minkowski's inequality and Riesz-Fischer Theorem)

6.9 - Hölder's Inequality

**Theorem 2.1** (Hölder's Inequality)**.** *Let $f \in L_p$ and $g \in L_q$ where $p > 1$ and $(1/p) + (1/q) = 1$. Then $fg \in L_1$ and $\|fg\|_1 \leq \|f\|_p \|g\|_q$.*

6.11 - Minkowski's Inequality

**Theorem 2.2** (Minkowski's Inequality)**.** *If $f$ and $h$ belong to $L_p$, $p \geq 1$, then $f + h$ belongs to $L_p$ and*

$$\|f + h\|_p \leq \|f\|_p + \|h\|_p. \tag{2.1}$$

6.14 - Completeness Theorem (Riesz-Fischer Theorem)

**Theorem 2.3** (Completeness Theorem (Riesz-Fischer Theorem))**.** *If $1 \leq p < \infty$, then the space $L_p$ is a complete normed linear space under the norm*

$$\|f\|_p = \left\{ \int |f|^p \, d\mu \right\}^{1/p}. \tag{2.2}$$

# 3 Decomposition of Measures

(Chapter 8 of Bartle's book, the most important results are: Radon-Nikodym theorem, Lebesgue decomposition theorem and Riesz representation theorem)

8.9 - Radon-Nikodým Theorem

8.11 - Lebesgue Decomposition Theorem

8.14 - Riesz Representation Theorem

# 4 Generation of Measures and Product Measures

(Chapter 9 and 10 of Bartle's book, the most important results are: Carathéodory extension theorem, Hahn extension theorem, Product measure theorem, Fubini's theorem and Tonelli's theorem)

9.7 - Carathéodory Extension Theorem
9.8 - Hahn Extension Theorem
10.4 - Product Measure Thereom
10.9 - Tonelli's Theorem
10.10 - Fubini's Theorem

## 5 TBA

## 6 TBA