Measure Theory

Exam

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E20

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2. Brownian Motions
3. Martingales & Quadratic Variation
4. Itôs Formula
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Prerequisties

Define these things

- Brownian Motion
- Martingale
- Natural Filtration

If B is a one-dimensional $(\mathcal{F}_t)_{t\geq 0}$-Brownian motion, then B is a $(\mathcal{F}_t)_{t\geq 0}$ martingale.
In particular, every one-dimensional standard Brownian motion B is a martingale with respect to its natural filtration $(\mathcal{B}_t^B)_{t\geq 0}$.

Repeat Remark 6.7

By virtue of Remark 6.7 we can further conclude that every one-dimensional standard Brownian motion B is a martingale on the standard filtered probability space $(\Omega, \tilde{\mathcal{F}}, (\tilde{\mathcal{F}_t^B})_{t\geq 0}, \tilde{\mathbb{P}})$ obtained by completing $(\Omega, \mathcal{F}, (\mathcal{F}_t^B)_{t\geq 0}, \mathbb{P})$.

Let us first recall that

$$\int_{\Omega} |B_t| \, d\mathbb{P} = \frac{1}{(2\pi t)^{1/2}} \int_{\mathbb{R}} |x| e^{-x^2/2t} \, dx < \infty, \mathbb{E}[B_t] = \frac{1}{(2\pi t)^{1/2}} \int_{\mathbb{R}} x e^{-x^2/} \tag{3.1}$$

for every $t > 0$.

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