Solutions to the Given Problems

Problem 1: $\frac{d}{dx}(1x + 2x + 3x) = \frac{d}{dx}\left(\sum_{i=1}^{3} ix\right) = ?$

Solution:

First, simplify the expression:

$$1x + 2x + 3x = 6x$$

Now, differentiate using the **sum rule**:

$$\frac{d}{dx}(6x) = 6$$

Alternatively, using the summation notation:

$$\frac{d}{dx}\left(\sum_{i=1}^{3} ix\right) = \sum_{i=1}^{3} \frac{d}{dx}(ix) = \sum_{i=1}^{3} i = 1 + 2 + 3 = 6$$

Problem 2: $\frac{d}{dx}\left((2x+1)^2 + 3x - 2\right) = ?$

Solution:

Differentiate term by term:

$$\frac{d}{dx}\left((2x+1)^2\right) + \frac{d}{dx}(3x) + \frac{d}{dx}(-2)$$

For $(2x+1)^2$, use the **chain rule**:

$$\frac{d}{dx}\left((2x+1)^2\right) = 2(2x+1) \cdot 2 = 4(2x+1)$$

For $3x$ and -2, differentiate directly:

$$\frac{d}{dx}(3x) = 3, \quad \frac{d}{dx}(-2) = 0$$

Combine the results:

$$\frac{d}{dx}\left((2x+1)^2 + 3x - 2\right) = 4(2x+1) + 3 = 8x + 7$$

Problem 3: $\frac{d}{dx}\left((2x+1)(e^{-2x})\right) - \log_2(x^2) = ?$

Solution:

First, differentiate $(2x+1)(e^{-2x})$ using the **product rule**:

$$\frac{d}{dx}\left((2x+1)(e^{-2x})\right) = \frac{d}{dx}(2x+1) \cdot e^{-2x} + (2x+1) \cdot \frac{d}{dx}(e^{-2x})$$

Compute each derivative:

$$\frac{d}{dx}(2x+1) = 2, \quad \frac{d}{dx}(e^{-2x}) = -2e^{-2x}$$

Substitute back:

$$2 \cdot e^{-2x} + (2x+1) \cdot (-2e^{-2x}) = 2e^{-2x} - 4xe^{-2x} - 2e^{-2x} = -4xe^{-2x}$$

Next, differentiate $\log_2(x^2)$ using the **chain rule**:

$$\frac{d}{dx}\left(\log_2(x^2)\right) = \frac{1}{x^2 \ln 2} \cdot 2x = \frac{2}{x \ln 2}$$

Combine the results:

$$\frac{d}{dx}\left((2x+1)(e^{-2x})\right) - \log_2(x^2) = -4xe^{-2x} - \frac{2}{x \ln 2}$$

Problem 4: $\frac{d}{dx}\left((e^{2x}+1)^3+\ln(x^2)\right)=?$

Solution:

Differentiate term by term:

$$\frac{d}{dx}\left((e^{2x}+1)^3\right)+\frac{d}{dx}\left(\ln(x^2)\right)$$

For $(e^{2x}+1)^3$, use the **chain rule**:

$$\frac{d}{dx}\left((e^{2x}+1)^3\right)=3(e^{2x}+1)^2\cdot\frac{d}{dx}(e^{2x}+1)=3(e^{2x}+1)^2\cdot 2e^{2x}=6e^{2x}(e^{2x}+1)^2$$

For $\ln(x^2)$, use the **chain rule**:

$$\frac{d}{dx}\left(\ln(x^2)\right)=\frac{1}{x^2}\cdot 2x=\frac{2}{x}$$

Combine the results:

$$\frac{d}{dx}\left((e^{2x}+1)^3+\ln(x^2)\right)=6e^{2x}(e^{2x}+1)^2+\frac{2}{x}$$

Problem 5: $\int_0^1 x^2-x+1\,dx=?$

Solution:

Integrate term by term:

$$\int_0^1 x^2\,dx-\int_0^1 x\,dx+\int_0^1 1\,dx$$

Compute each integral:

$$\int x^2\,dx=\frac{x^3}{3},\quad \int x\,dx=\frac{x^2}{2},\quad \int 1\,dx=x$$

Evaluate from 0 to 1:

$$\left[\frac{x^3}{3}\right]_0^1=\frac{1}{3},\quad \left[\frac{x^2}{2}\right]_0^1=\frac{1}{2},\quad [x]_0^1=1$$

Combine the results:

$$\frac{1}{3}-\frac{1}{2}+1=\frac{2}{6}-\frac{3}{6}+\frac{6}{6}=\frac{5}{6}$$