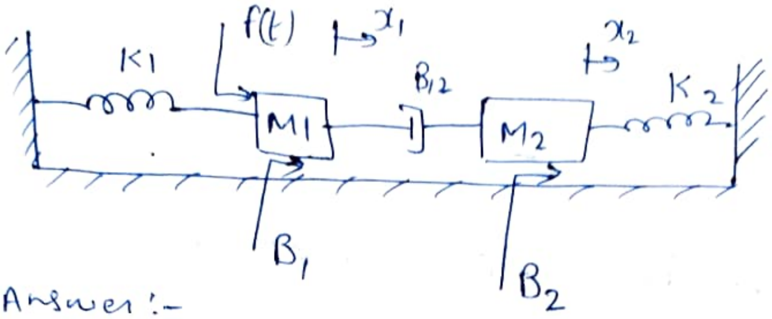
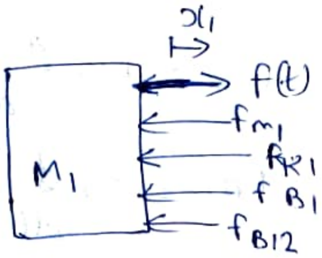
Q) Determine the transfer function, $\frac{X_1(s)}{F(s)}$ and $\frac{X_2(s)}{F(s)}$ for the system shown below



Answer:-

Free body diagram of mass M1:



opposing forces: $f_{m1} = M_1 \frac{d^2(x_1)}{dt^2}$

$f_{K1} = K_1 x_1$

$f_{B1} = B_1 \frac{d x_1}{dt}$

$f_{B12} = B_{12} \frac{d(x_1 - x_2)}{dt}$

applied force : f(t)

By Newton's 2^{nd} law

$$f_{m1} + f_{B1} + f_{B12} + f_{K1} = f(t)$$

$$M_1 \frac{d^2(x_1)}{dt^2} + B_1 \frac{d}{dt} x_1 + B_{12} \frac{d}{dt}(x_1 - x_2) + K_1 x_1 = f(t)$$

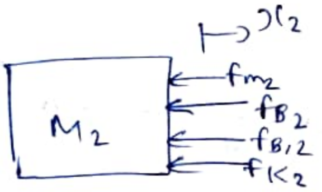
—①—

Taking laplace of ① ⇒

$$M_1 s^2 X_1(s) + B_1 s X_1(s) + B_{12} s \left(X_1(s) - X_2(s)\right) + K_1 X_1(s) = F(s)$$

-(2)-

Free body diagram of mass M_2



opposing forces: $f_{m2} = M_2 \frac{d^2}{dt^2}(x_2)$

$$f_{B2} = B_2 \frac{d}{dt}(x_2)$$

$$f_{B12} = B_{12} \frac{d}{dt}(x_2 - x_1)$$

$$f_{K2} = K_2 x_2$$

By Newton's 2nd law,

$$f_{m2} + f_{B2} + f_{B12} + f_{K2} = 0$$

$$M_2 \frac{d^2}{dt^2} x_2 + B_2 \frac{d}{dt} x_2 + B_{12} \frac{d}{dt}(x_2 - x_1) + K_2 x_2 = 0$$

-③-

Taking Laplace of ③

$$M_2 s^2 X_2(s) + B_2 s X_2(s) + B_{12} s \left(X_2(s) - X_1(s)\right) + K_2 X_2(s) = 0$$

$$X_2(s)\left[M_2 s^2 + B_2 s + B_{12} s + K_2\right] - B_{12} s\, X_1(s) = 0$$

$$X_2(s)\left[M_2 s^2 + (B_2 + B_{12}) s + K_2\right] = B_{12} s\, X_1(s)$$

$$X_2(s) = \frac{B_{12}\, s\, X_1(s)}{M_2 s^2 + (B_2 + B_{12}) s + K_2} \quad -(4)-$$

$$X_1(s) = \frac{X_2(s)\left[M_2 s^2 + (B_2 + B_{12}) s + K_2\right]}{B_{12}\, s} \quad -(5)-$$

(2) ⇒

$$X_1(s)\left[M_1 s^2 + (B + B_{12}) s + K_1\right] - B_{12}\, s\, X_2(s) = F(s)$$

$-(6)-$

substituting (4) in (6)

$$X_1(s)\left[M_1 s^2 + (B + B_{12}) s + K_1\right] - \frac{(B_{12} s)^2 X_1(s)}{\left[M_2 s^2 + (B_2 + B_{12}) s + K_2\right]} = F(s)$$

$$X_1(s)\left\{\frac{\left[M_1 s^2 + (B + B_{12}) s + K_1\right]\left[M_2 s^2 + (B_2 + B_{12}) s + K_2\right] - (B_{12} s)^2}{M_2 s^2 + (B_2 + B_{12}) s + K_2}\right. = F(s)$$

$$\frac{X_1(s)}{F(s)} = \frac{M_2 s^2 + (B_2 + B_{12}) s + K_2}{\left(M_1 s^2 + (B + B_{12}) s + K_1\right)\left(M_2 s^2 + (B_2 + B_{12}) s + K_2\right) - (B_{12}^2}$$

substituting (5) in (6)

$$X_2(s)\left\{\frac{\left(M_2 s^2+(B_2+B_{12})s+K_2\right)}{B_{12}s}\left(M_1 s^2+(B+B_{12})s+K_f\right)\right\}$$

$$- B_{12}s\,X_2(s) = F(s)$$

$$\frac{X_2(s)\left\{\left[M_2 s^2+(B_2+B_{12})s+K_2\right]\left[M_1 s^2+(B+B_{12})s+K_1\right] - (B_{12}s)^2\right\}}{B_{12}s}$$

$$= F(s)$$

$$\frac{X_2(s)}{F(s)} = \frac{B_{12}s}{\left[M_2 s^2+(B_2+B_{12})s+K_2\right]\left[M_1 s^2+(B+B_{12})s+K_1\right]-(B}$$