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```
%Section - 02
%Aero 431 HW3: 5/12/25
```

Workspace Prep

```
%warning off
format long     %Allows for more accurate decimals
close all;      %Clears all
clear all;      %Clears Workspace
clc;            %Clears Command
```

Rayleigh-Ritz

```
% Material and plate properties
E = 70e9;          % Youngs Modulus Pa
nu = 0.3;          % Poisson's Ratio
rho = 2710;        % kg/m^3
h = 0.01;          % thickness m
L = 1;            % side length m

% rigidity
D = E*h^3 / (12*(1-nu^2));
```

Part 1 — Rayleigh Quotient Method

```
% symbolic Rayleigh quotient
omega1_sq = (72*E*h^2) / (L^4*rho*(1 - nu^2));
freq1 = sqrt(omega1_sq) / (2*pi);  % Hz

omega2_sq = (1584*E*h^2*429) / (L^4*rho*6292*(1-nu^2));
freq2 = sqrt(omega2_sq) / (2*pi);  % Hz

fprintf('--- Fundamental Frequency Estimates (Rayleigh Method) ---\n');
fprintf('1st Ansatz (simple):      %.4f Hz\n', freq1);
fprintf('2nd Ansatz (refined, a=0): %.4f Hz\n\n', freq2);
```

```
--- Fundamental Frequency Estimates (Rayleigh Method) ---
1st Ansatz (simple):      71.9499 Hz
2nd Ansatz (refined, a=0): 88.1202 Hz
```

Part 2 — Generalized Eigenvalues

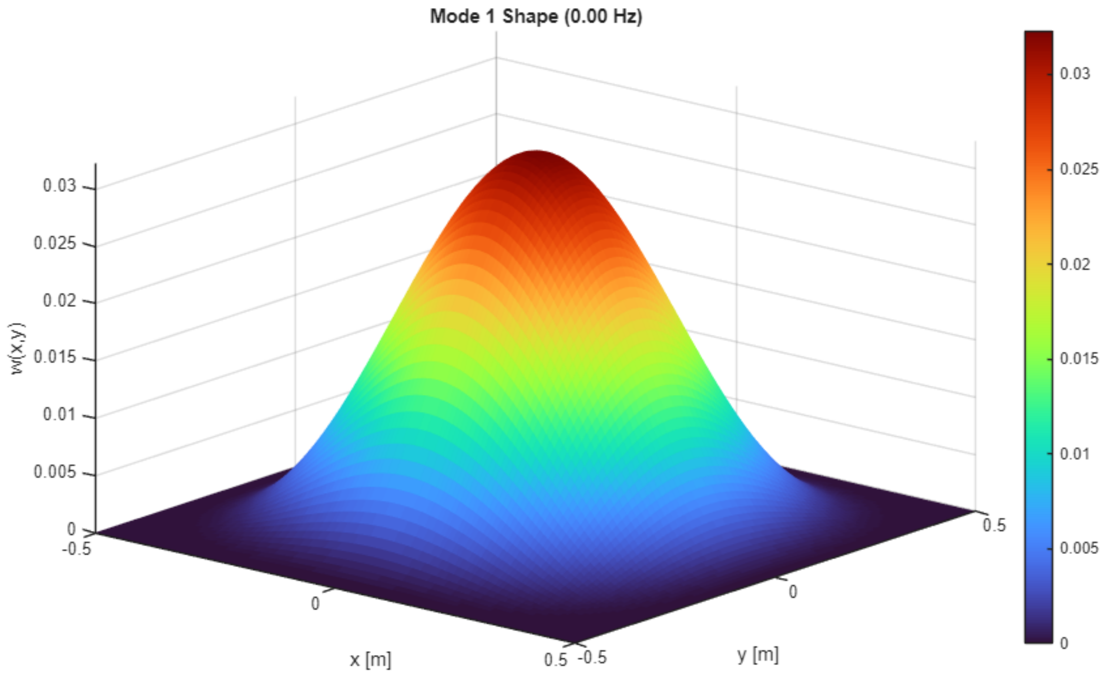
```
% Mass / area
rhoh = rho*h;

% mass and stiffness matrix
M = [L^10/900,     0,                 0,            L^12/12600;
     0,           L^12/25200,       0,            0;
     0,           0,                 L^12/25200,   0;
     L^12/12600,  0,                 0,            L^14/105840];

K = [0.0013990,     0,            0,            7.1737e-04;
     0,             3.7919e-04,  0,            0;
     0,             0,            3.7919e-04,  0;
     7.1737e-04,    0,            0,            2.3851e-04];

K_scaled = D * K;
M_scaled = rhoh * M;

% solve generalized eigenvalue problem
[mode_shapes, omega_sq_vals] = eig(K_scaled, M_scaled);
omega_vals = sqrt(diag(omega_sq_vals));
frequencies = omega_vals / (2 * pi);

% Sort frequencies
[frequencies_sorted, sort_idx] = sort(real(frequencies));

fprintf('--- Frequencies from Generalized Eigenvalue Problem ---\n');
for i = 1:4
    fprintf('Mode %d: %.4f Hz\n', i, frequencies_sorted(i));
end

% Comments:
% The simple mode found a frequency about 16hz lower than the refined
% mode. The inclusion of more terms for displacement shows that the refined
% mode is creating a better aproximation of the actual frequency.
```

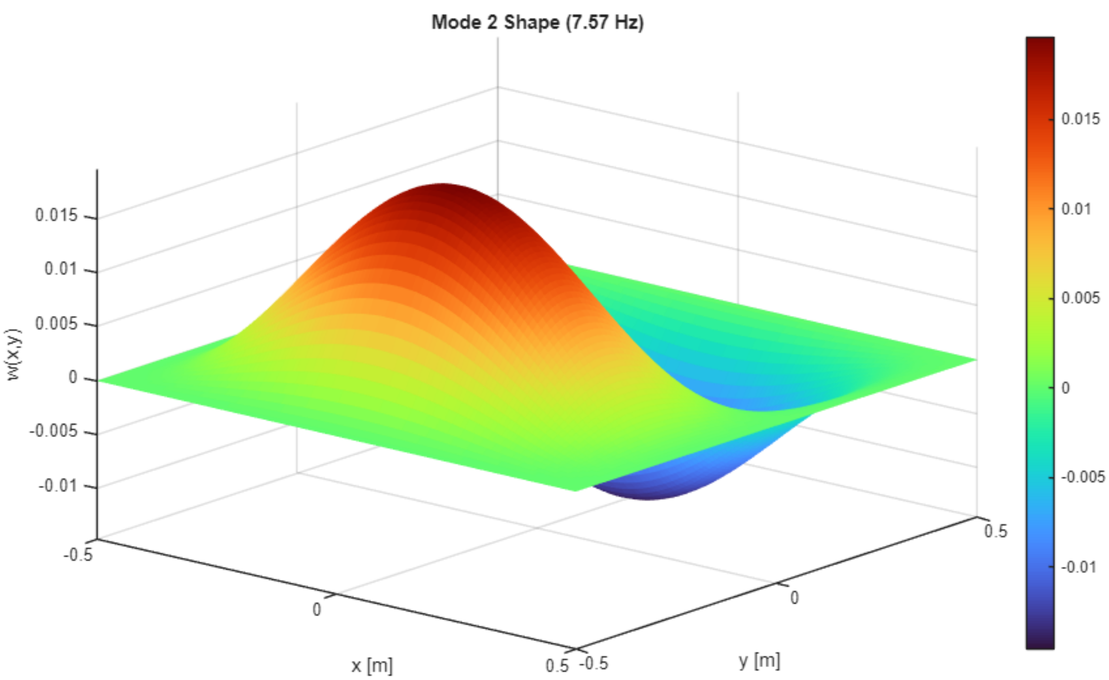
```
--- Frequencies from Generalized Eigenvalue Problem ---
Mode 1: 0.0000 Hz
Mode 2: 7.5666 Hz
Mode 3: 7.5666 Hz
Mode 4: 15.5222 Hz
```

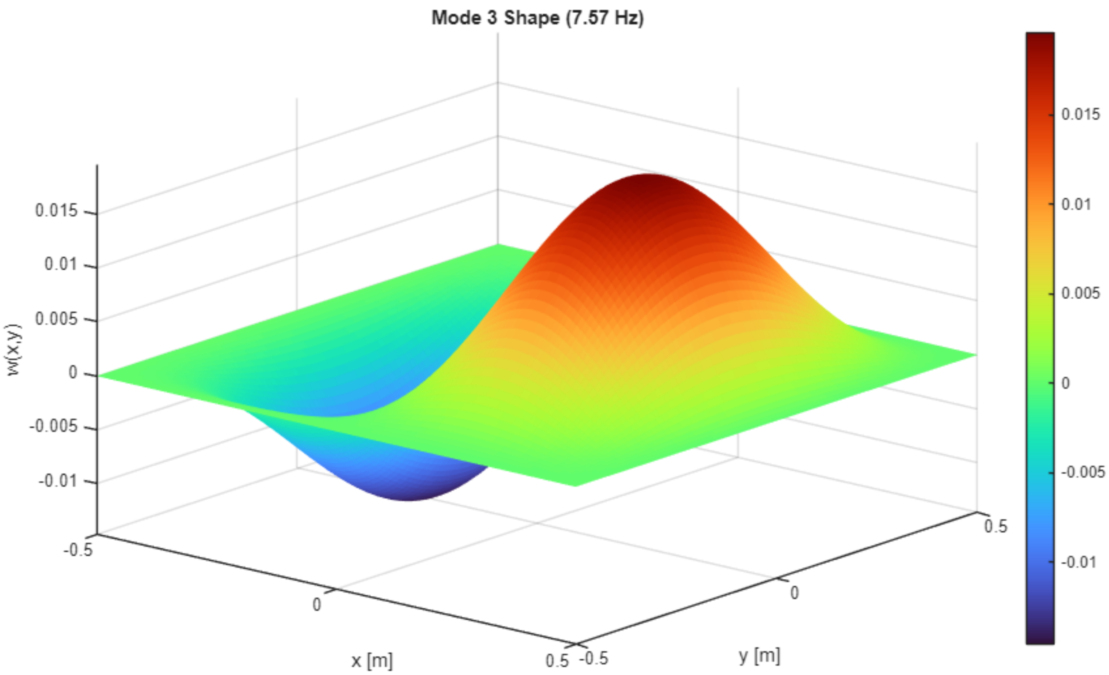
Plotting

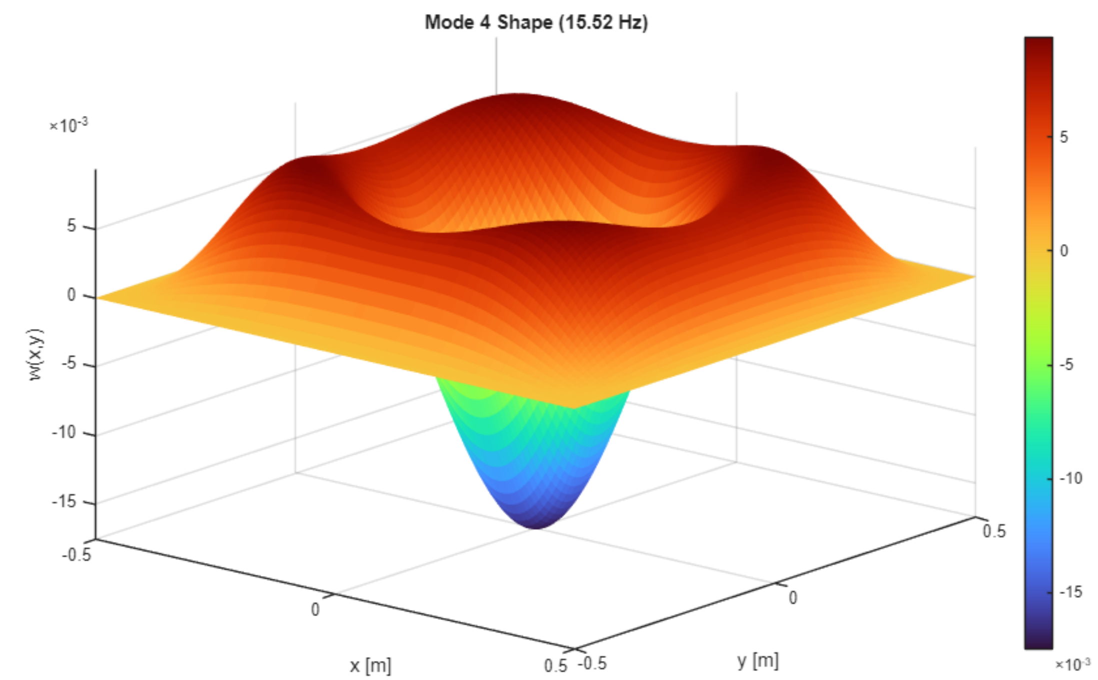
```
phi = {
    @(x, y) 1;
    @(x, y) x;
```

```
    @(x, y) y;
    @(x, y) x.^2 + y.^2
};

% weight function
wgt = @(x, y) (x.^2 - L^2/4).^2 .* (y.^2 - L^2/4).^2;

x_vals = linspace(-L/2, L/2, 120);
y_vals = linspace(-L/2, L/2, 80);
[Y, X] = ndgrid(y_vals, x_vals);

for i = 1:4
    coeffs = mode_shapes(:, sort_idx(i));

    W = wgt(X, Y);
    for j = 1:4
        W = W + coeffs(j) * wgt(X, Y) .* phi{j}(X, Y);
    end

    % Plot
    figure;
    surf(X, Y, W, 'EdgeColor', 'none');
    colormap turbo;
    colorbar;
    title(sprintf('Mode %d Shape (%.2f Hz)', i, frequencies_sorted(i)));
    xlabel('x [m]');
    ylabel('y [m]');
    zlabel('w(x,y)');
    view(40, 25);
    axis tight;
end
```


Mode 1 Shape (0.00 Hz)
w(x,y)
0.03
0.025
0.02
0.015
0.01
0.005
0
-0.5
0
0.5
x [m]
-0.5
0
0.5
y [m]
0.03
0.025
0.02
0.015
0.01
0.005
0


Mode 2 Shape (7.57 Hz)
w(x,y)
0.015
0.01
0.005
0
-0.005
-0.01
-0.5
0
0.5
x [m]
-0.5
0
0.5
y [m]
0.015
0.01
0.005
0
-0.005
-0.01





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