Orbital COE's Hand Calcs

→

eccentricity 1) $\vec{e} = \frac{1}{\mu}\left(|\vec{V}|^2 - \frac{\mu}{|\vec{R}|}\right)\vec{R} - (\vec{R}\cdot\vec{V})\vec{V}$

$\vec{R} = \langle -2315.9,\ 2169.6,\ 6314.5 \rangle$ km

$\vec{V} = \langle -3.0599,\ 6.0645,\ -3.0249 \rangle$ km/s

$\mu = 398600$ km^3/s^2

$\vec{e} = \frac{1}{398600}\left((7.5106)^2 - \frac{398600}{7066.8}\right)\vec{R}$

$R\cdot V = 3.7133$ km^2/s

$\vec{h} = \vec{R} \times \vec{V} = \langle -45243,\ -26743,\ 7409 \rangle$ km^2/s

$-(3.7133)\vec{V} = \vec{e}$

$|\vec{R}|_{mag} = 7066.8$ km

$|\vec{V}|_{mag} = 7.5106$ km/s

$1\cdot10^{-3}\langle .0028,\ -.0324,\ .1 \rangle = \vec{e}$

$|R \times V| = 53076$ km^2/s

$e = |\vec{e}| \quad \sqrt{(1\cdot10^{-3}\langle .0018^2,\ -.0324^2,\ .1^2} = \boxed{.00010519 \text{ unit(-)}}$

Semi-major axis 2) $\mathcal{E} = \frac{|\vec{V}|^2}{2} - \frac{\mu}{|\vec{R}|} \rightarrow \frac{(7.5106)^2}{2} - \frac{398600}{7066.8} = -28.2002$

$\text{Semi-major} = -\frac{\mu}{2\mathcal{E}} \rightarrow \frac{398600}{2(-28.2002)} = \boxed{7067.31 \text{ km}}$

inclination 3) $i = \cos^{-1}\left(\frac{\hat{k}\cdot\vec{h}}{|\hat{k}|\cdot|\vec{h}|}\right) \rightarrow \boxed{98.0244^\circ}$

RAAN 4) Quad Check node line $= \langle 0,0,1 \rangle \times \langle h \rangle = 1\cdot10^4\langle 2.6793,\ -4.5243,\ 0 \rangle = \vec{n}$

y component < 0 so correct with -2π

$(2\pi) - \cos\left(\frac{(\hat{I}\cdot\vec{n})}{|\hat{I}|\,|\vec{n}|}\right) = 300.5868^\circ$

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Arg of peri 5) For quad check, check k component of $\vec{e}$ to see if you are above or below reference plane for us $\vec{e}_k > 0$ so no correction needed

$$\text{Arg of peri} = \cos^{-1}\left(\frac{\vec{n}\cdot\vec{e}}{|\vec{n}||\vec{e}|}\right) = 73.8304^\circ$$

True anom 6) Quad check R·V if it is > or < than 0, R·V = 3.71 so > 0 no quad check needed.

$$\text{True anom} = \cos^{-1}\left(\frac{\vec{e}\cdot\vec{R}}{|\vec{e}||\vec{R}|}\right) = 41.6963^\circ$$

→ Define COE's in my words:

eccentricity: measure of how stretched an orbit is between circular and ellipsoid

RAAN: Angle from first point of Aries to node line / where the reference plane and orbit plane meet.

inclination: angle between orbit plane and reference plane

Semi-major axis: the length from the origin to the farthest point in the ellipse.

Argument of periapsis: angle of node line to eccentricity vector (which points to point of periapsis)

True anomaly: Angle between R vector and eccentricity Vector

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Orbital COE write up:

During this assignment I learned the basics of orbital mechanics and how to calculate their values (classical orbital elements). We learned to verify quadrants when doing calculations with inverse trig functions (arccos). I also gained a lot of experience working w/ Vectors and Matricies.

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