FK1

Summary: Derive the Expressions for Linear strain rate, shear strain rate, & volumetric strain rate, as well as an expression for vorticity

Assumptions: steady state, incompressible

Analysis:

1) Linear Strain Rate:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \quad \varepsilon_{yy} = \frac{\partial v}{\partial y} \quad \varepsilon_{zz} = \frac{\partial w}{\partial z}$$

where,

$$\frac{d}{dt}\left(\frac{\left(u_\alpha + \frac{\partial u_\alpha}{\partial x_\alpha} dx_\alpha\right)dt + dx_\alpha - u_\alpha - u_\alpha dt - dx_\alpha}{dx_\alpha}\right) = \frac{\partial u_\alpha}{\partial x_\alpha}$$

Shear strain rate: $\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$ $\varepsilon_{zx} = \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)\frac{1}{2}$

$$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) & \frac{\partial w}{\partial z} \end{bmatrix}$$

Volumetric Strain Rate:

$$\frac{1}{V}\frac{DV}{Dt} = \frac{1}{V}\frac{dV}{dt} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

2) $w = \nabla \times U = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ u & v & w \end{vmatrix}$ $\omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}$ $\omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}$

$$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

Results: Vorticity is a measure of local spinning within a fluid usually inside the boundary layer and is defined as the curl of the velocity field: w = ∇ x U

FK2

Summary: Explain the units of vorticity and how it is measured, find and explain the behavior of vorticity on a hydrofoil. Derive an equation for freestream vorticity.

Assumptions: Incompressible Flow, non-rotational outside of boundary layer

Analysis:

Analysis:

1) units of vorticity are just cycles / hz, and the vorticity is negligable in freestream

Assumptions

- Everything inside the boundry is rotational
- Everything outside is not
-

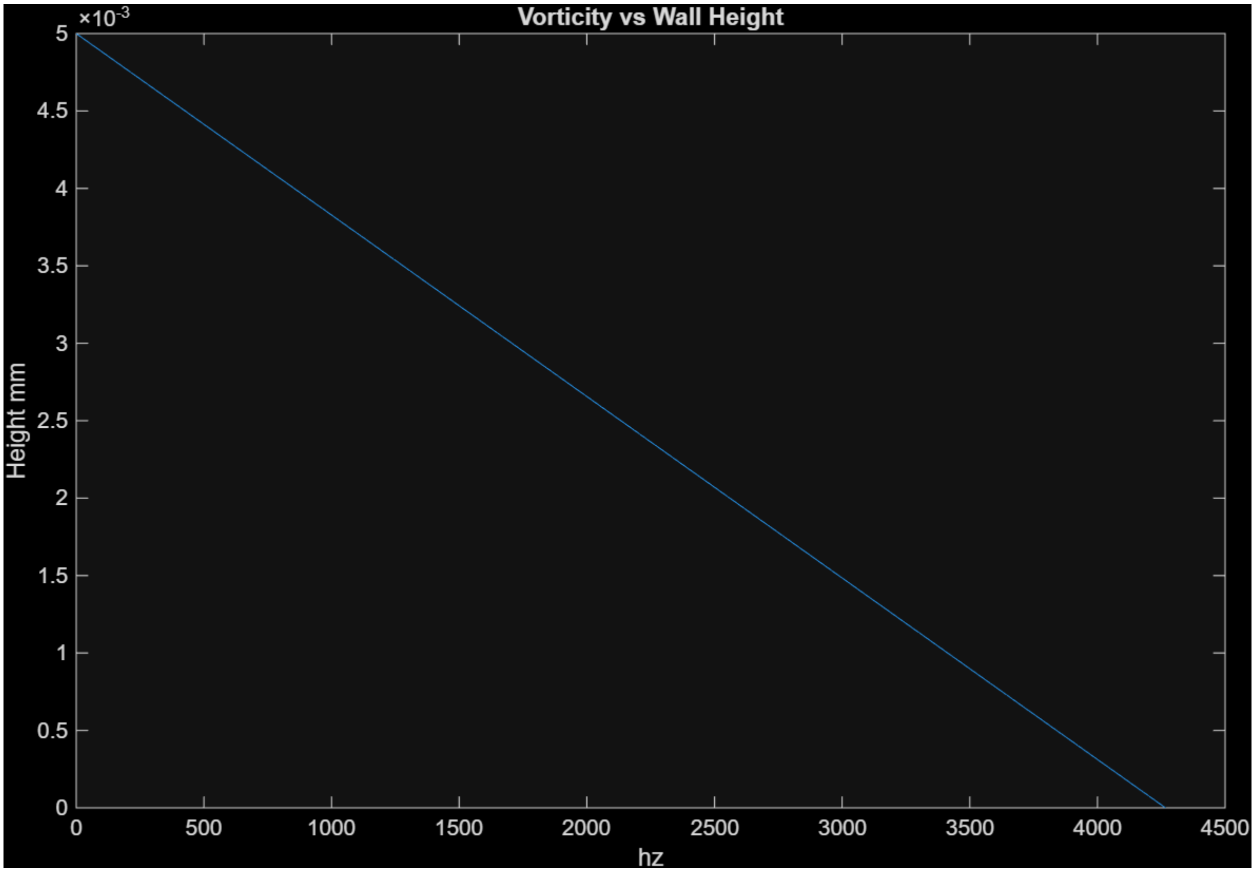
2) $Z = \vec{\nabla} \times \vec{U} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ u & v & w \end{vmatrix}$, $\omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}$, $\omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}$, $\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$

$w = 0$, $\frac{\partial}{\partial z} = 0 \rightarrow \omega_x = \omega_y = 0$, $\frac{\partial v}{\partial x} = 0 \rightarrow \omega = -\frac{\partial u}{\partial y} = -\frac{\rho g \sin(\theta)}{\mu}(h - y)$

Plot in MATLAB.

Vorticity at 5mm is the lowest at 0 hz on the graph, this value should be approaching zero at the top boundry as there is a boundry condition there.

Results/Plot:



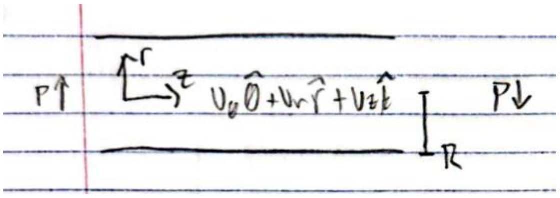
You can see the vorticity is zero at height zero and most frequent at highest height, confirming our predictions.

NSE2

Summary: Solve the Navier Stokes equations in cylindrical coordinates for fully developed flow moving in the z direction of pipe radius R and infinite length. Moving because of a pressure drop.

Assumptions: 1D Flow, P.E./Gravity negligible

Analysis:



$$\rho\left(\cancelto{0}{\frac{\partial v_z}{\partial t}} + \cancelto{0}{v_r\frac{\partial v_z}{\partial r}} + \cancelto{0}{\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}} + \cancelto{0}{v_z\frac{\partial v_z}{\partial z}}\right)$$

$$= -\frac{\partial P}{\partial z}\left(\frac{1}{r}\frac{\partial(r\tau_{rz})}{\partial r} + \cancelto{0}{\frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta}} + \cancelto{0}{\frac{\partial \tau_{zz}}{\partial z}}\right) + \cancelto{0}{\rho g_z}$$

$$\frac{\partial P}{\partial z} = \frac{1}{r}\frac{\partial(r\tau_{rz})}{\partial r} \longrightarrow \frac{\partial P}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\mu r\frac{\partial v}{\partial r}$$

$\rightarrow \frac{\partial^2 v}{\partial r^2} = \frac{r}{\mu}\frac{\partial P}{\partial z}$, Boundry conditions:
At center $\frac{\partial v}{\partial r} = 0$ (max)
At perimeter $V = 0$

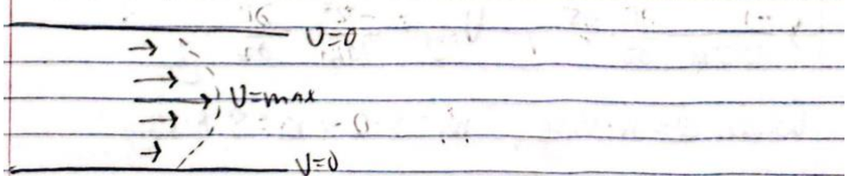
So,

$$\frac{\partial v}{\partial r} = \frac{1}{2}r^2\cdot\frac{1}{\mu}\frac{\partial P}{\partial z} + C_1 \qquad \text{Where, } C_1 = 0$$

$$V(r) = \frac{1}{6}r^3\cdot\frac{1}{\mu}\frac{\partial P}{\partial z} + C_1 + C_2 \qquad C_2 = -\frac{R^3}{6\mu}\frac{\partial P}{\partial z}$$

$$\rightarrow V(r) = \frac{r^3}{6\mu}\frac{\partial P}{\partial z} - \frac{R^3}{6\mu}\frac{\partial P}{\partial z}$$

Sketch:



From old HW: $\dot{m} = \rho A V_{Avg}$ $\quad$ $U(r)$

Where, $V_{Avg} = \frac{1}{A}\int_0^R \left(\frac{r^3}{6\mu}\frac{\partial P}{\partial z} - \frac{R^3}{6\mu}\frac{\partial P}{\partial z}\right) 2\pi r dr$

$\rightarrow \frac{1}{\pi r^2}\int_0^R \left(\frac{r^3}{6\mu}\frac{\partial P}{\partial z} - \frac{R^3}{6\mu}\frac{\partial P}{\partial z}\right) 2\cancel{\pi} r dr$

$\rightarrow \frac{2}{r^2}\int \left(\frac{r^3}{6\mu}\frac{\partial P}{\partial z} - \frac{R^3}{6\mu}\frac{\partial P}{\partial z}\right) r dr$

$\rightarrow \frac{2}{r^2}\int \left(\frac{r^4}{6\mu}\frac{\partial P}{\partial z} - \frac{R^3 r}{6\mu}\frac{\partial P}{\partial z}\right) dr$

$\rightarrow V_{Avg} = \frac{2\partial P}{R^2 6\mu \partial z}\int_0^R r^4 - R^3 r \, dr$

$\rightarrow \frac{2\partial P}{R^2 6\mu \partial z}\left(\frac{1}{5}r^5 - \frac{1}{2}R^3 r^2\right)\Big|_0^R$

$\rightarrow \frac{2\partial P}{R^2 6\mu \partial z}\left(\frac{1}{5}R^5 - \frac{1}{2}R^5\right) \rightarrow \frac{1}{R^2 6\mu}\frac{\partial P}{\partial z}\left(\frac{-3}{10}R^5\right)$

$\rightarrow \frac{-1}{10 R^2 \mu}\frac{\partial P}{\partial z}\cdot R^5$, $\quad V_{Avg} = \frac{-R^3}{10\mu}\cdot\frac{\partial P}{\partial z}$

Where $Q = A\cdot V_{Avg}$, $\quad \dot{m} = \rho\cdot Q \rightarrow \dot{m} = \rho\cdot A\cdot V_{Avg}$

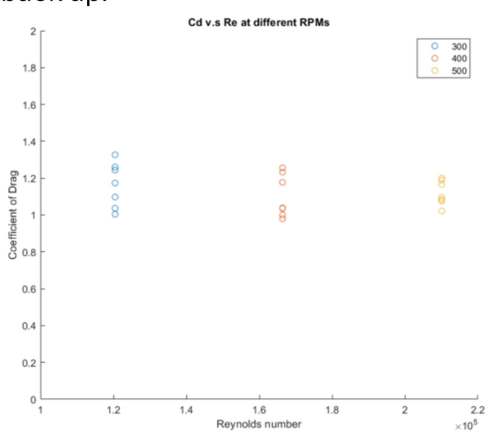
<u>CNM2</u>

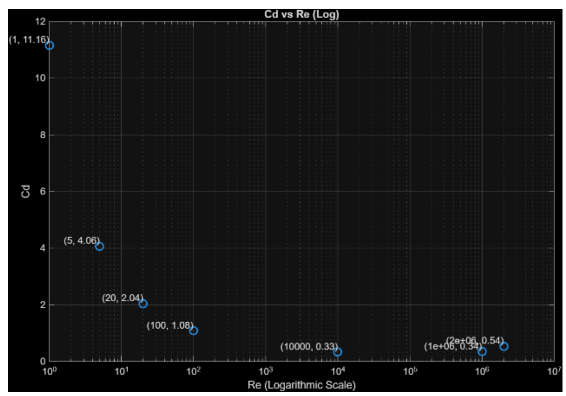
Summary: Use Ansys Fluent to solve for both Steady and Unsteady flows around a cylinder. Compare results of Cd vs Theta and Strouhal vs Re

Assumptions: 1D Flow, P.E./Gravity negligible

Analysis:

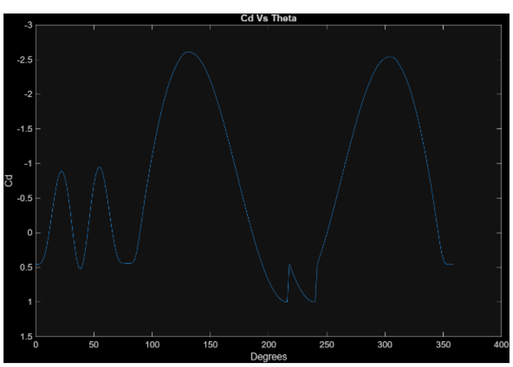
1. **Steady Flow Past a Cylinder**
    a. (Certificate was $15, but course is complete)
    b. Results at bottom before code
    c. These Reynolds numbers were tested/simulated and plotted on a log scale (bottom): 1,5,20,100,10000,1000000,2000000 their corresponding Cd: 11.1628,4.0640,2.0403,1.0834,0.3286,0.34426,0.5357. A clear pattern is seen emerging, where Cd is dropping / behaving inversely to the size of the Reynolds number. This matches theory and follows the trend witnessed in lab 2 of decreasing Cd with higher Re. Although at very high Reynolds Numbers the Cd seemed to pick back up.

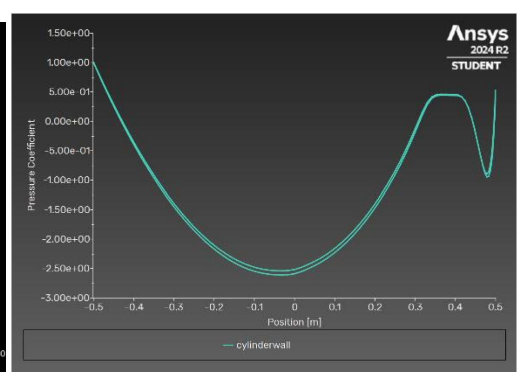




(Lab 2 Values) Ansys Simulated Values

    d. Solving for Cd at different parts of the cylinder provides two graphs that show a similar pattern seen in lab 2:

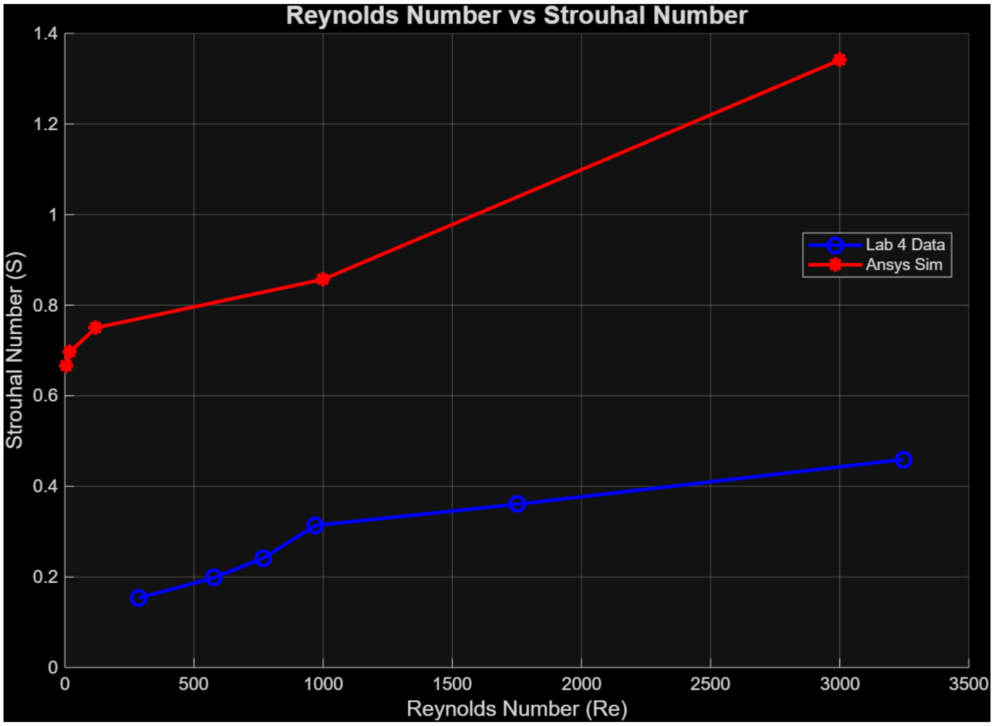




Cd vs Theta Cd vs Position

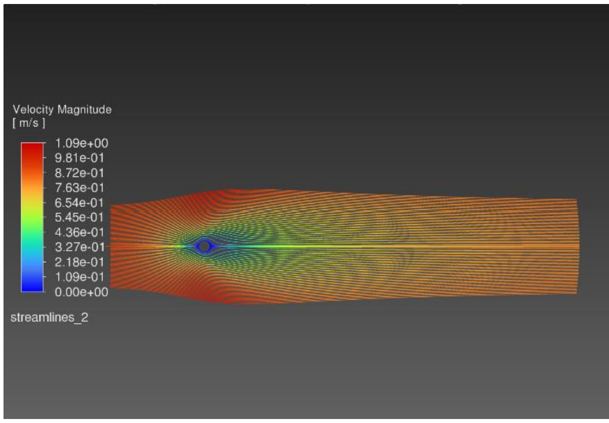
You can see in the Cd vs Position graphs there are two lines each one representing one half of the cylinder (top & bottom)

2. **Unsteady Flow Past a Cylinder**
    a. (Certificate was $15, but course is complete)
    b. Will be shown below
    c. Analyzing the change in the Strouhal number revealed that at least for lower Reynolds numbers the Strouhal number increases with Re:

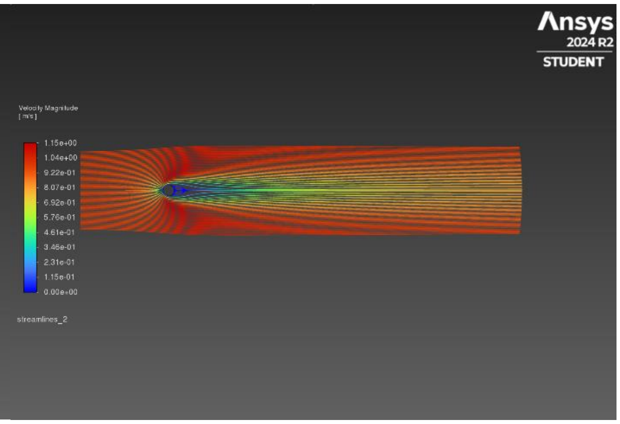


Here you can see that trend, although with a slight offset. You can still see the numbers rising.

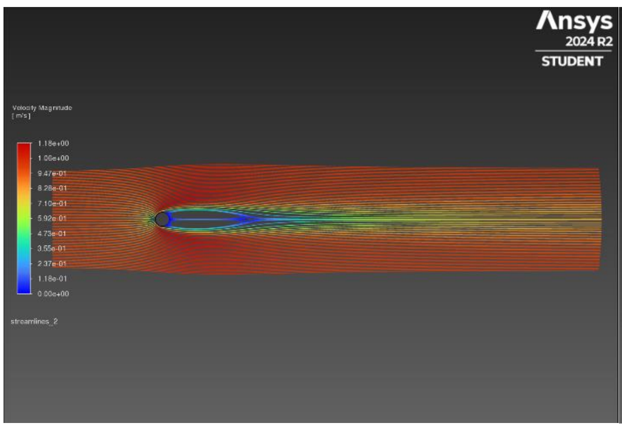
Steady Flow Past a Cylinder: Flow Fields:



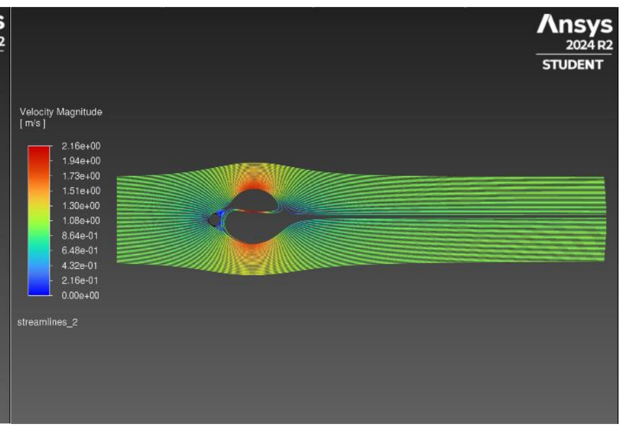
Re = 1



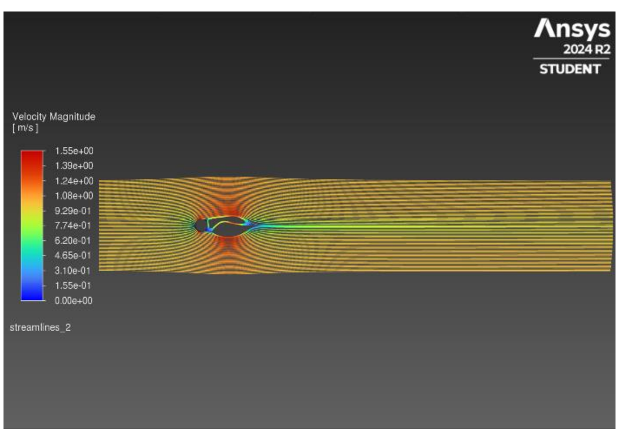
Re = 20



Re = 100

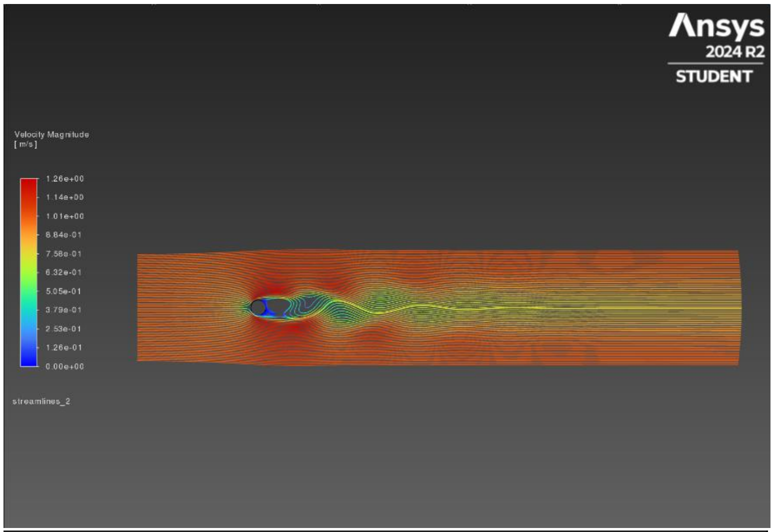


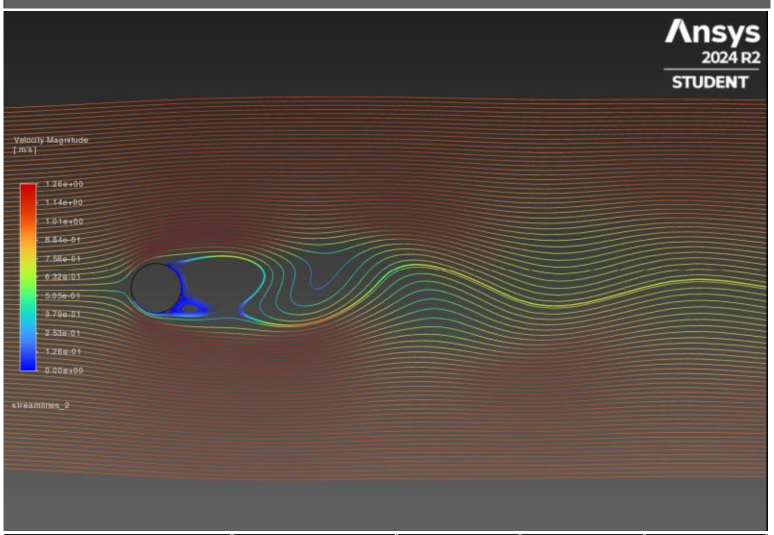
Re = 10,000



Re = 1,000,000

Unsteady Flow Past a Cylinder: Flow Fields:





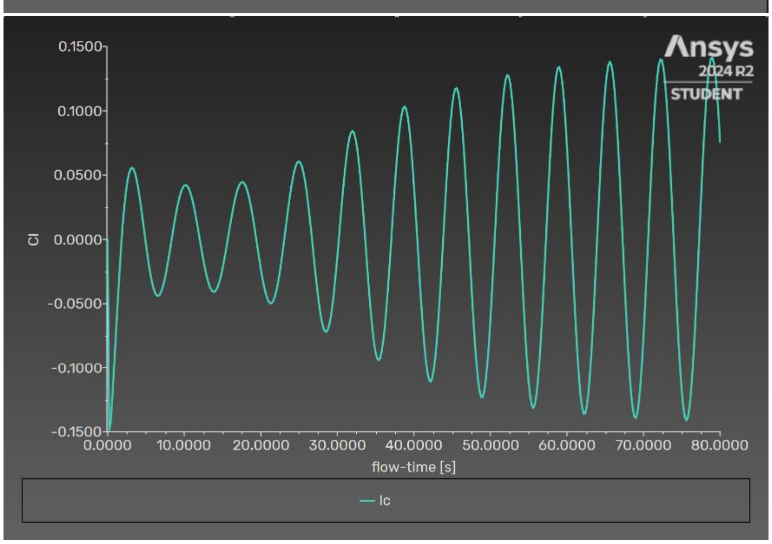


Table of Contents

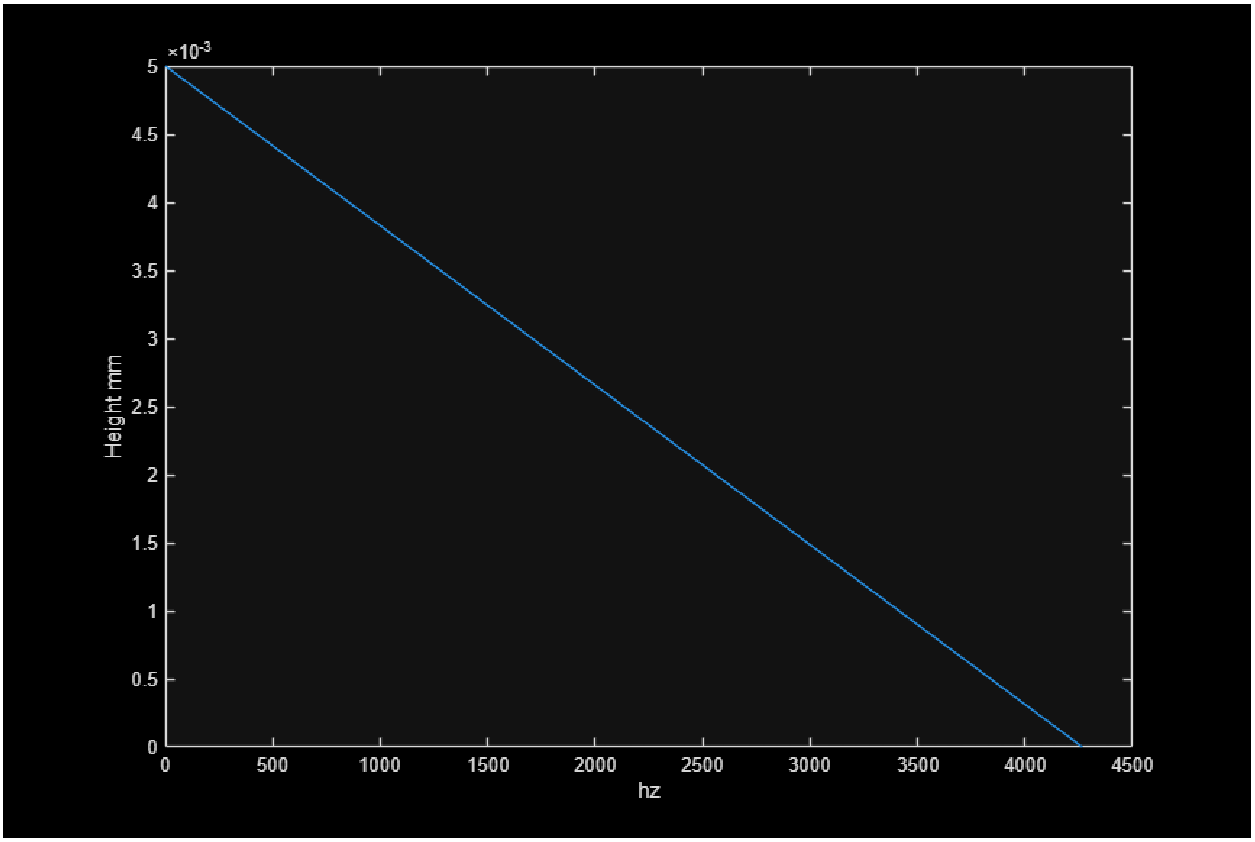

```
%Roshan Jaiswal-Ferri
%Section - 01
%AERO 302 Homework 3 - 11/20/24
```

Workspace Prep

```
format long      %Allows for more accurate decimals
close all;       %Clears all
clear all;       %Clears Workspace
clc;             %Clears Command Window
```

PART 1: FK2

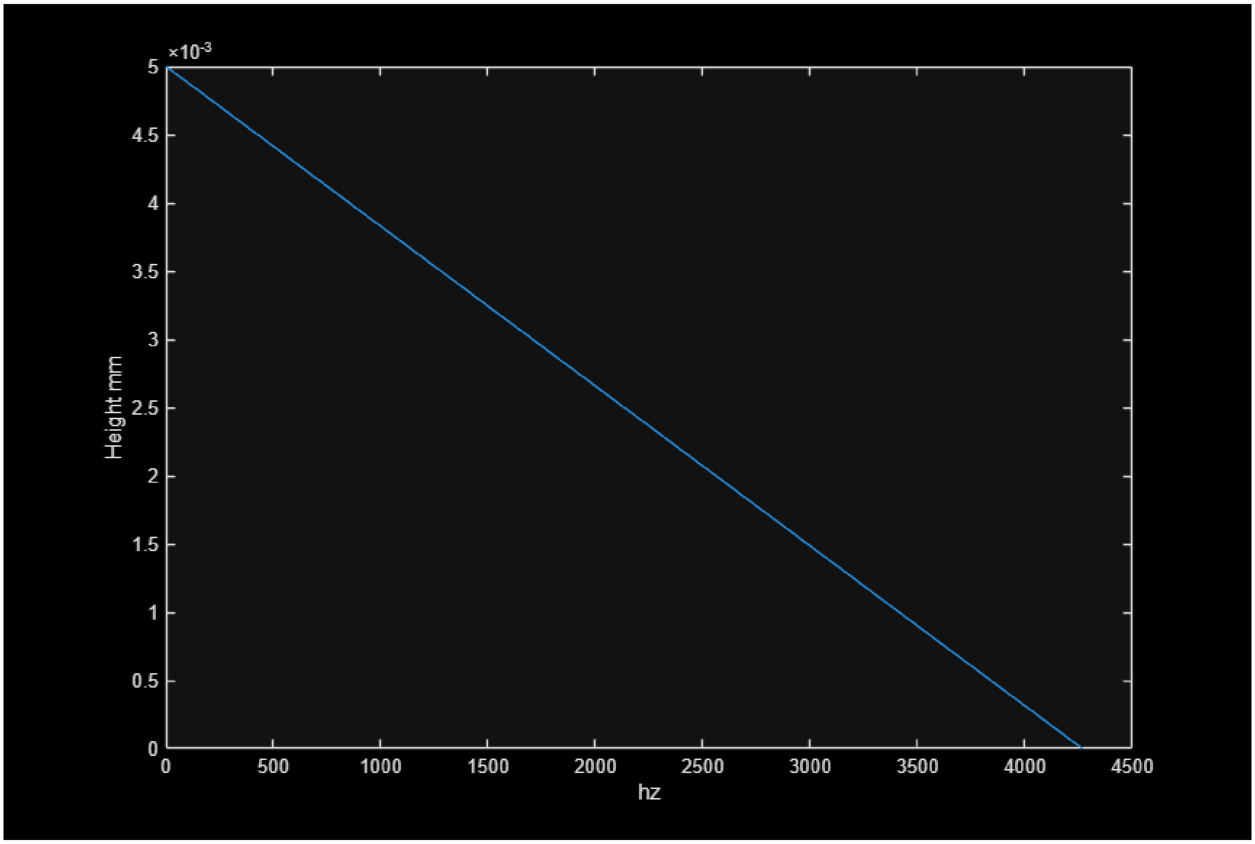
```
T = 293;
theta = 5;
g = -9.81;
y = linspace(0,(5/1000),200);
mu = 1.002e-3; %pa*s
rho = 1000; %kg/m^3
h = 5/1000;

zeta = -((rho*g*sind(theta))/mu)*(h-y);

plot(zeta, y);
xlabel('hz')
ylabel('Height mm')
```

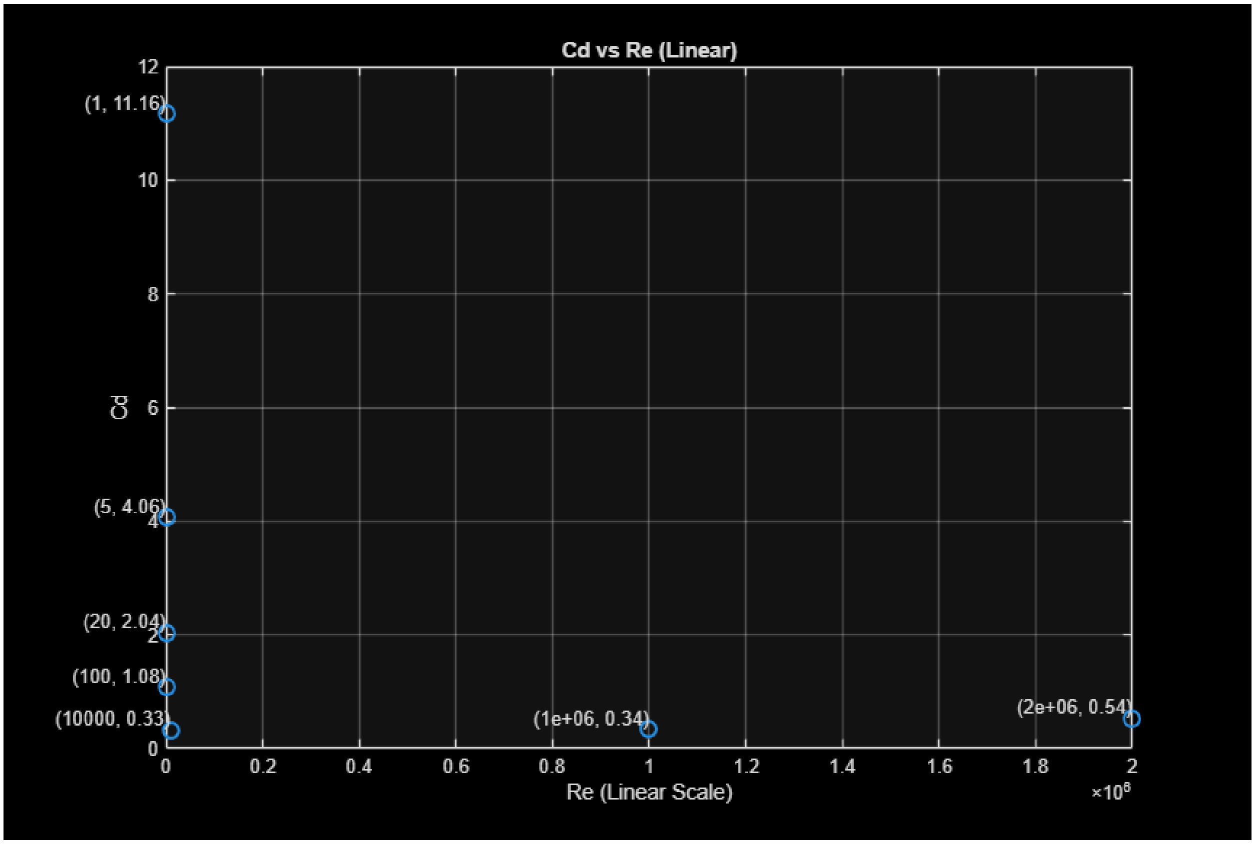


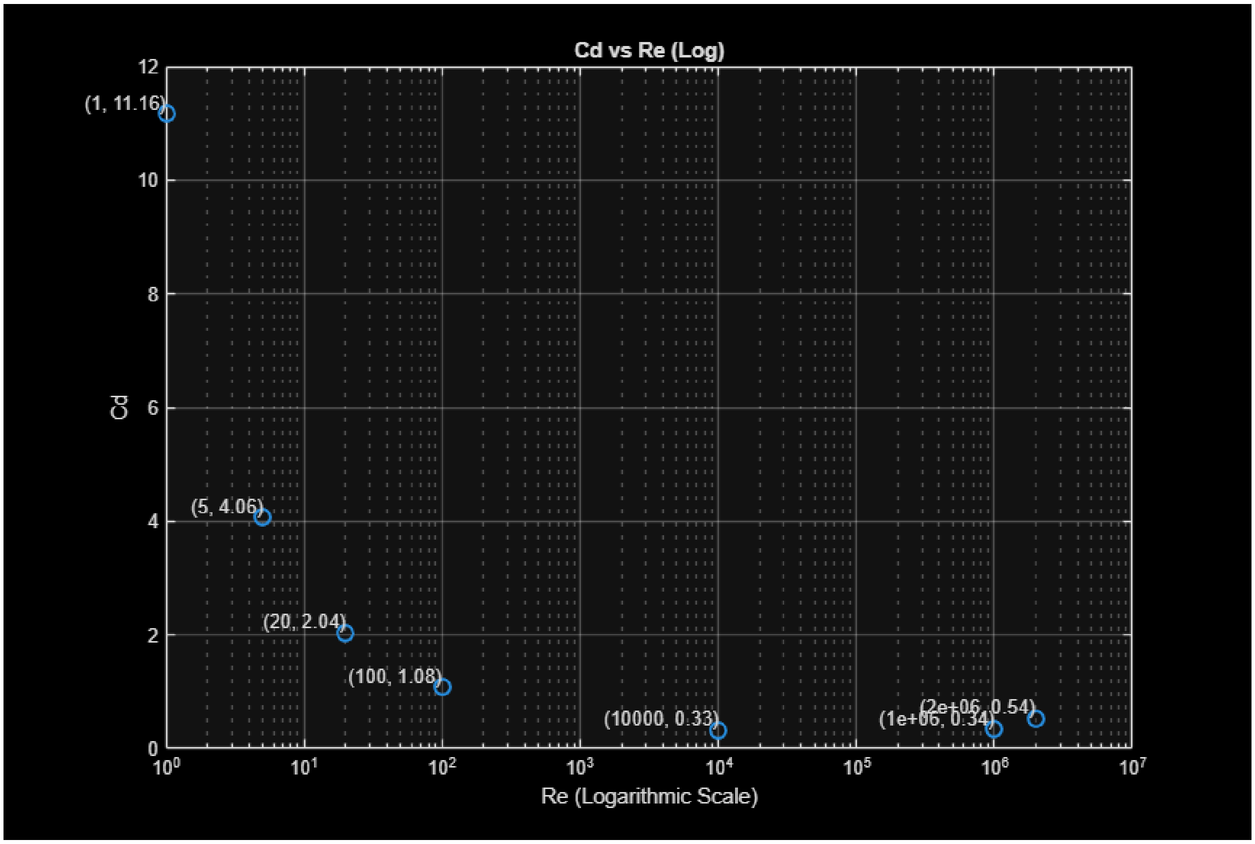
Cd vs Re

```
Re = [1,5,20,100,10000,1000000,2000000];
Cd = [11.1628,4.0640,2.0403,1.0834,0.3286,0.34426,0.5357];

figure('Name','Cd vs Re (Linear)')
plot(Re, Cd, 'o', 'MarkerSize', 8, 'LineWidth', 1.5);
xlabel('Re (Linear Scale)');
ylabel('Cd');
title('Cd vs. Re (Linear Scale)');
grid on;
% Add labels to points
for i = 1:length(Re)
    text(Re(i), Cd(i), sprintf('(%g, %.2f)', Re(i), Cd(i)), ...
        'VerticalAlignment', 'bottom', 'HorizontalAlignment', 'right');
end
title('Cd vs Re (Linear)');

figure('Name','Cd vs Re (Log)')
plot(Re, Cd, 'o', 'MarkerSize', 8, 'LineWidth', 1.5);
set(gca, 'XScale', 'log');
xlabel('Re (Logarithmic Scale)');
ylabel('Cd');
title('Cd vs. Re (Logarithmic Scale)');
grid on;
% Add labels to points
```

```
for i = 1:length(Re)
    text(Re(i), Cd(i), sprintf('(%g, %.2f)', Re(i), Cd(i)), ...
        'VerticalAlignment', 'bottom', 'HorizontalAlignment', 'right');
end
title('Cd vs Re (Log)');
```




Cd vs Re (Linear)
(1, 11.16)
(5, 4.06)
(20, 2.04)
(100, 1.08)
(10000, 0.33)
(1e+06, 0.34)
(2e+06, 0.54)
Cd
Re (Linear Scale)
×10^6


Cd vs Re (Log)
(1, 11.16)
(5, 4.06)
(20, 2.04)
(100, 1.08)
(10000, 0.33)
(1e+06, 0.34)
(2e+06, 0.54)
Cd
Re (Logarithmic Scale)

Workspace Prep

```
clear all;      %Clears Workspace
```

Strouhal #s

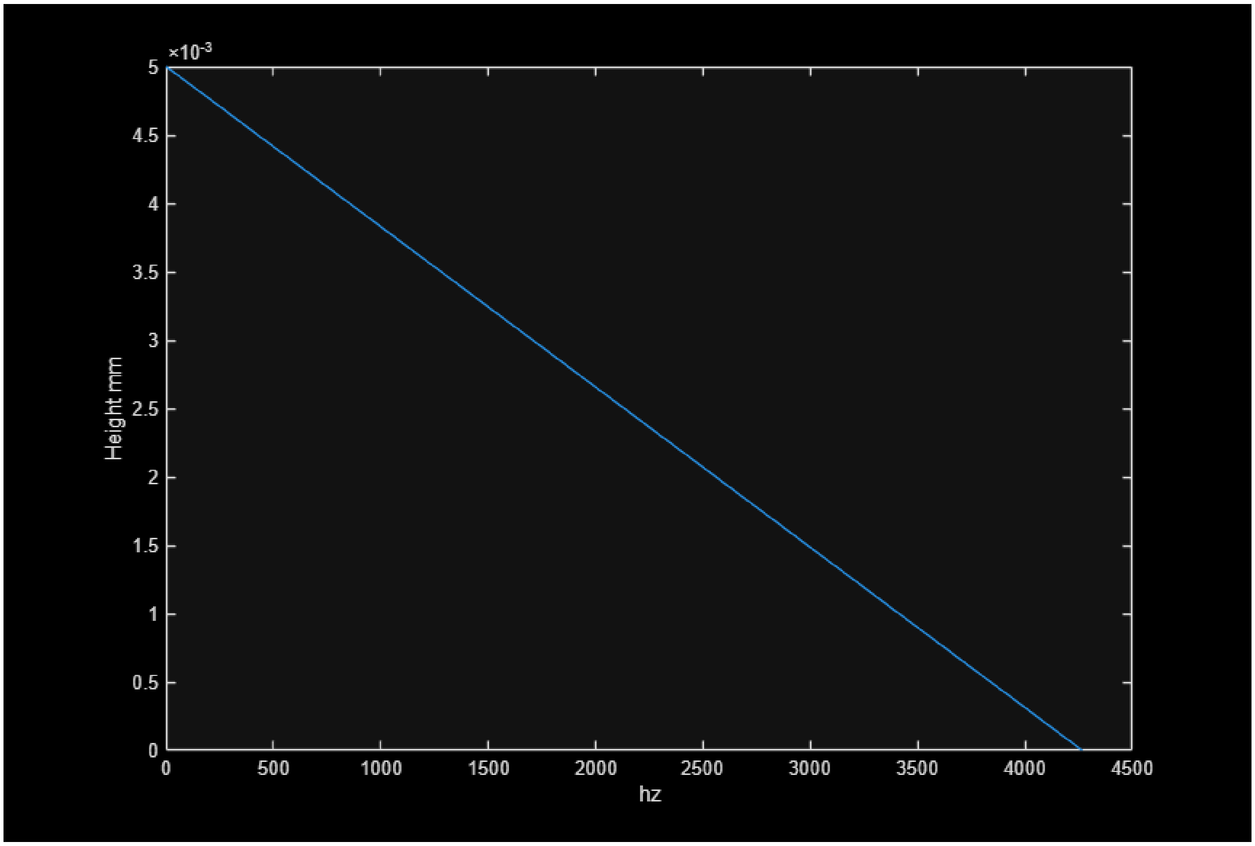
```
hz = 15;
U1 = 2.429*hz; %speed in mm/s
U = U1/1000; %speed in m/s

t = 54-47; %Time for 5 vorticies
vD6 = 5/t; %vorticies / second

t = 11;
vD5 = 5/t;

t = 12;
vD4 = 5/t;

t = 7;
vD3 = 3/7;

t = 11;
vD2 = 3/11;

t = 16;
vD1 = 3/16;

Di = [3.515,1.897,1.05,0.832,0.626,0.308]; % in inches
D = Di.*0.0254; %diam in meters

f = [vD1,vD2,vD3,vD4,vD5,vD6];

for i = 1:6
    S(i) = (f(i)*D(i))/U; %strouhal number
end

%Re Calc:

rho = 1000;
u = 0.0010016; %dyn visc of water at 20C

for i = 1:6
    Re(i) = (rho*U*D(i))/u;
end
```

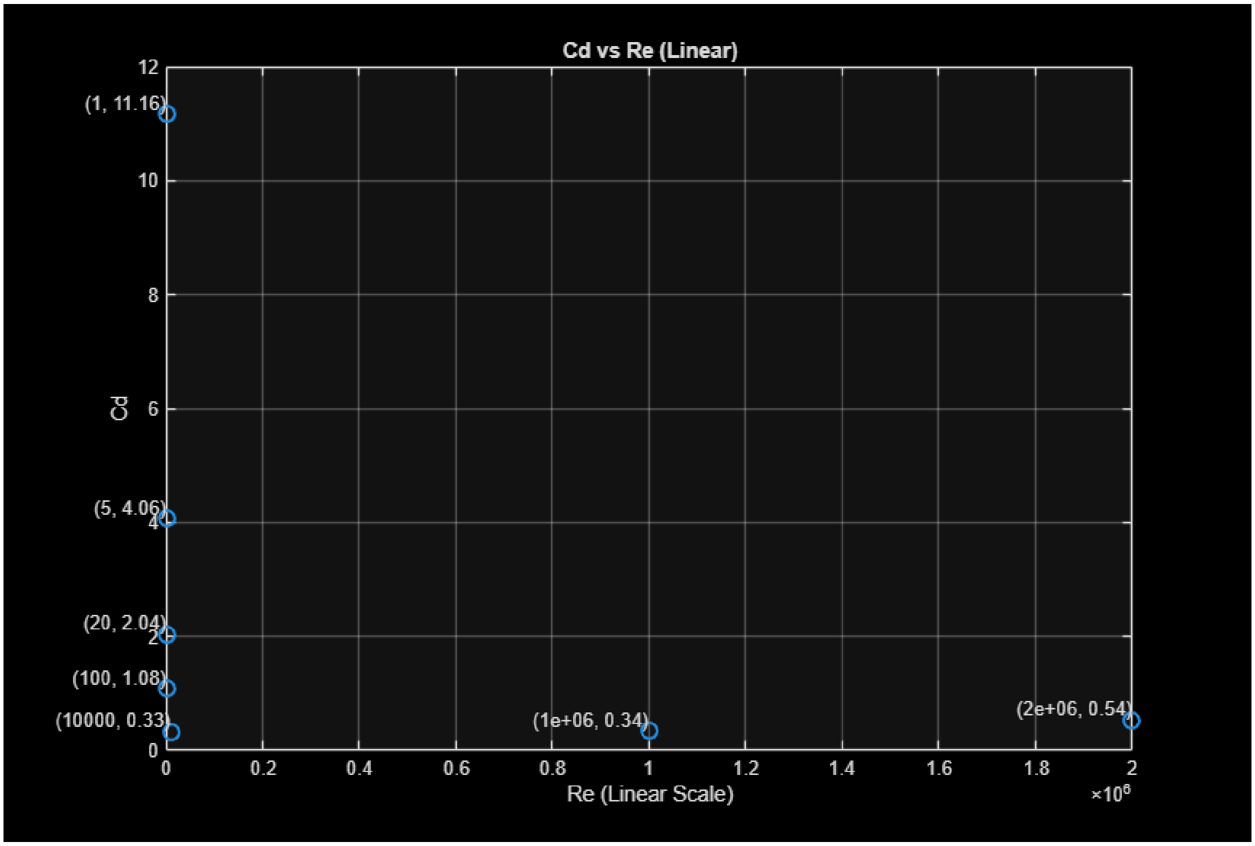
HW Calcs

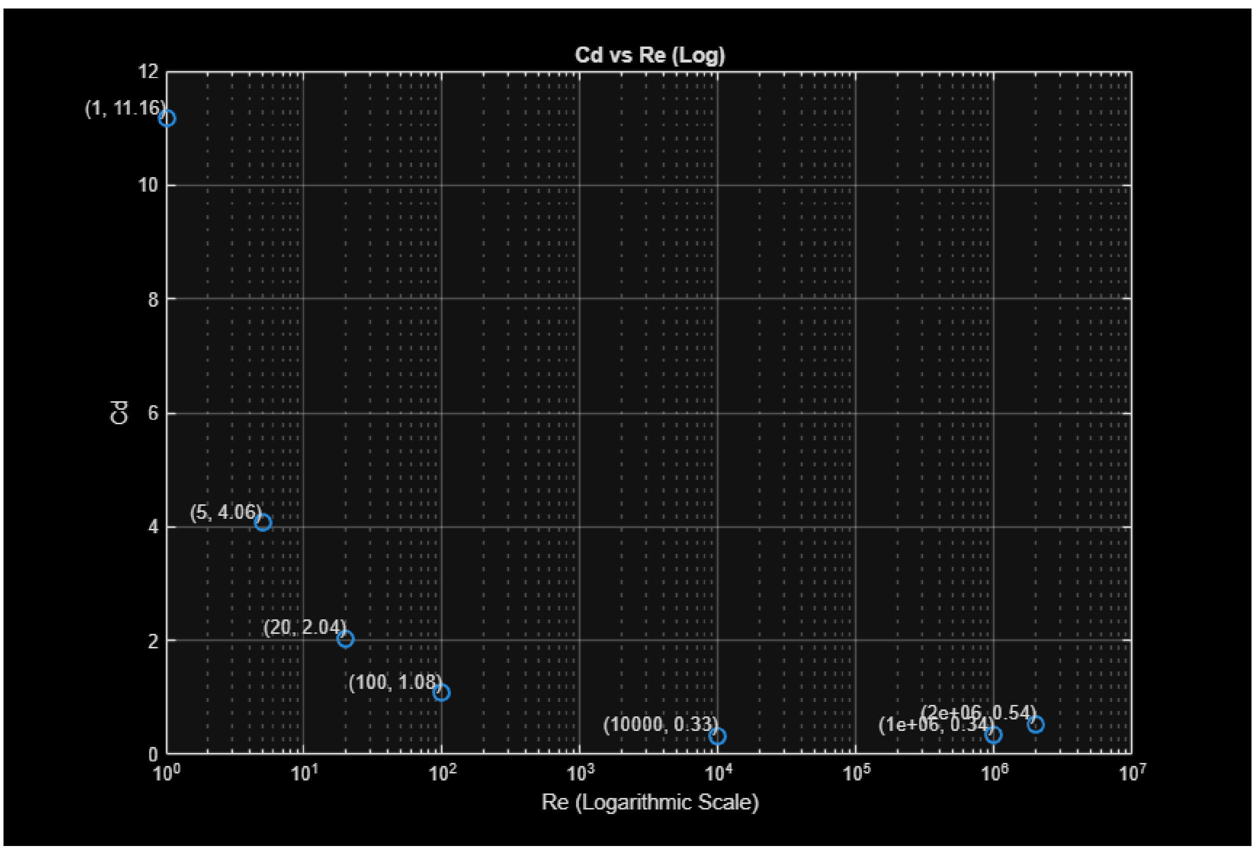
```
Re2 = [5,20,120,1000,3000]; %Calculated form # of vorticies per time in
animation
S2 = [0.666,0.697,0.75,0.857,1.341];
```

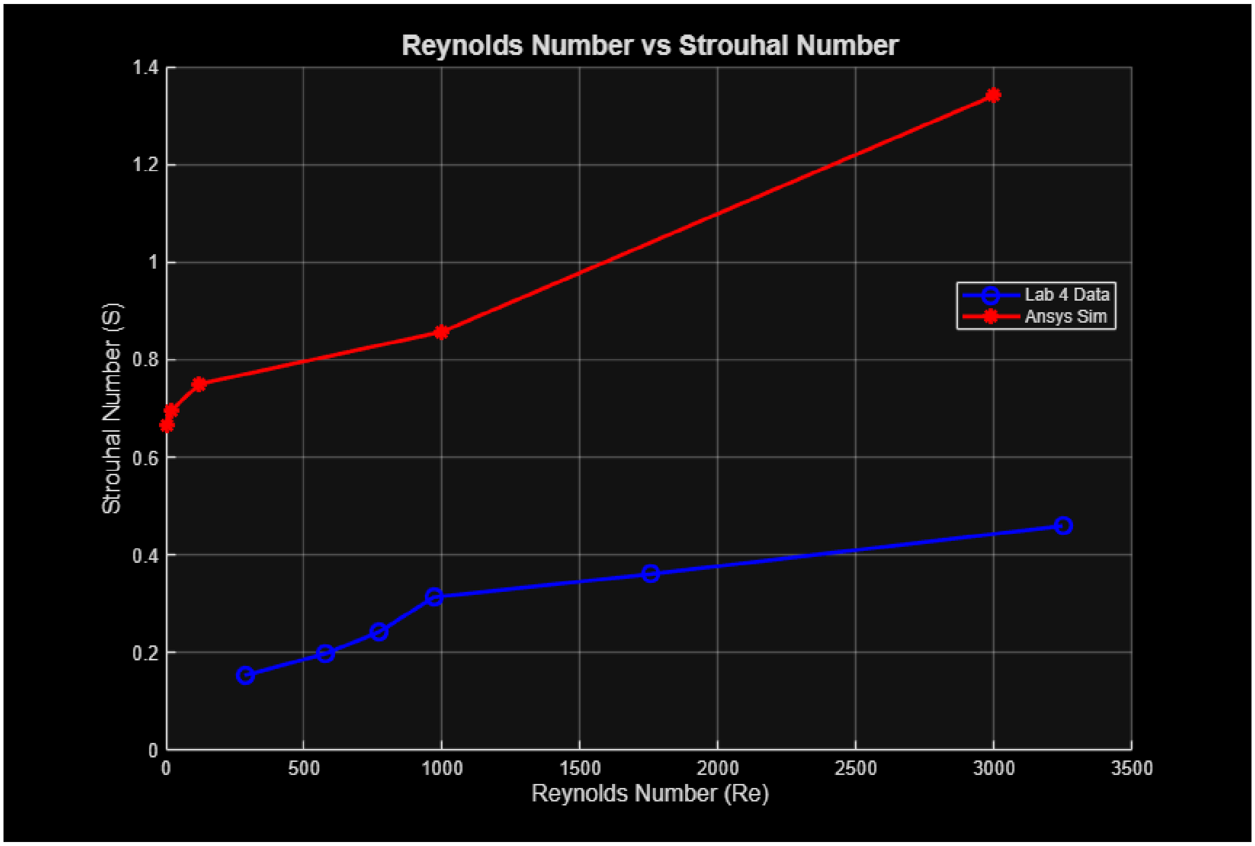
Plotting

```
figure;
hold on;
grid on;

% Plot S vs Re
plot(Re, S, 'bo-', 'LineWidth', 2, 'MarkerSize', 8);

% Plot S2 vs Re2
plot(Re2, S2, 'r*-', 'LineWidth', 2, 'MarkerSize', 8);

% Add labels, title, and legend
xlabel('Reynolds Number (Re)', 'FontSize', 12);
ylabel('Strouhal Number (S)', 'FontSize', 12);
title('Reynolds Number vs Strouhal Number', 'FontSize', 14);
legend('Lab 4 Data', 'Ansys Sim', 'Location', 'Best');
```

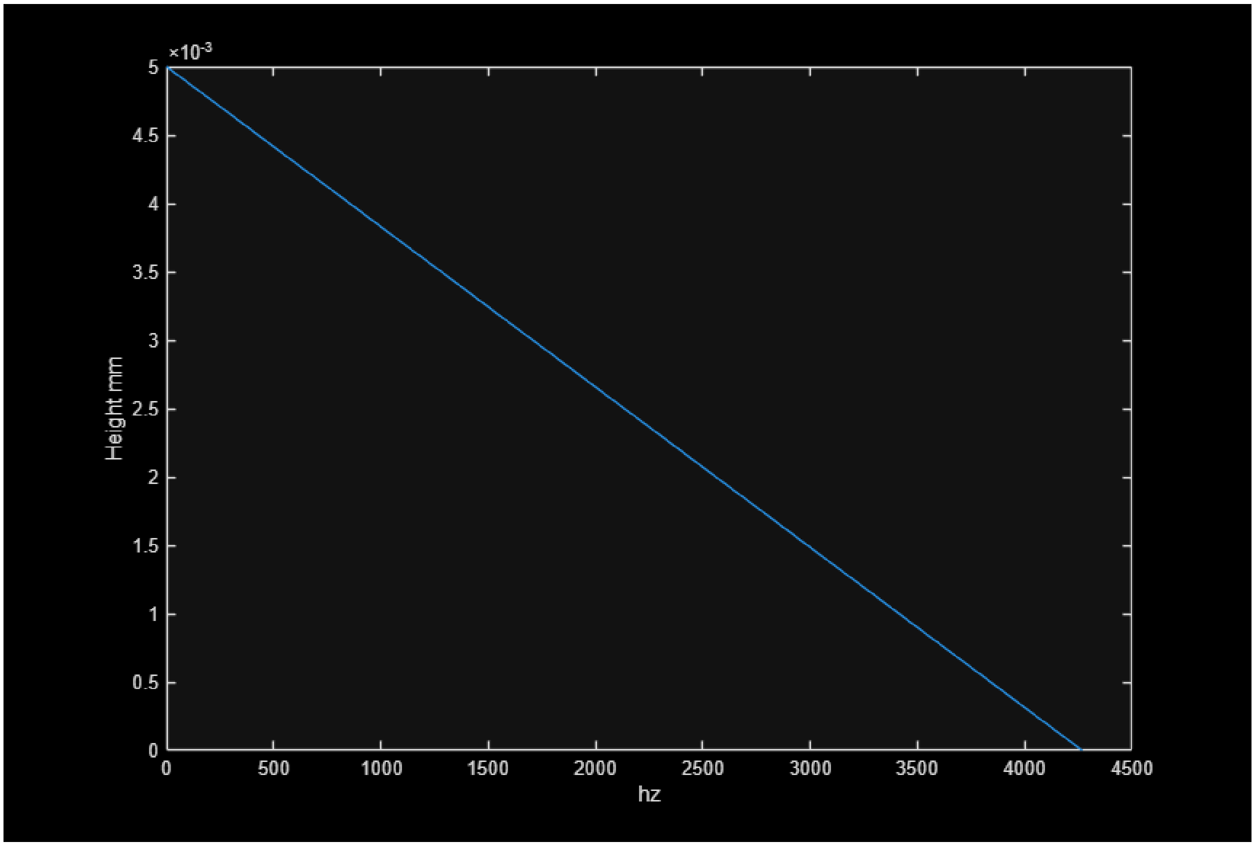


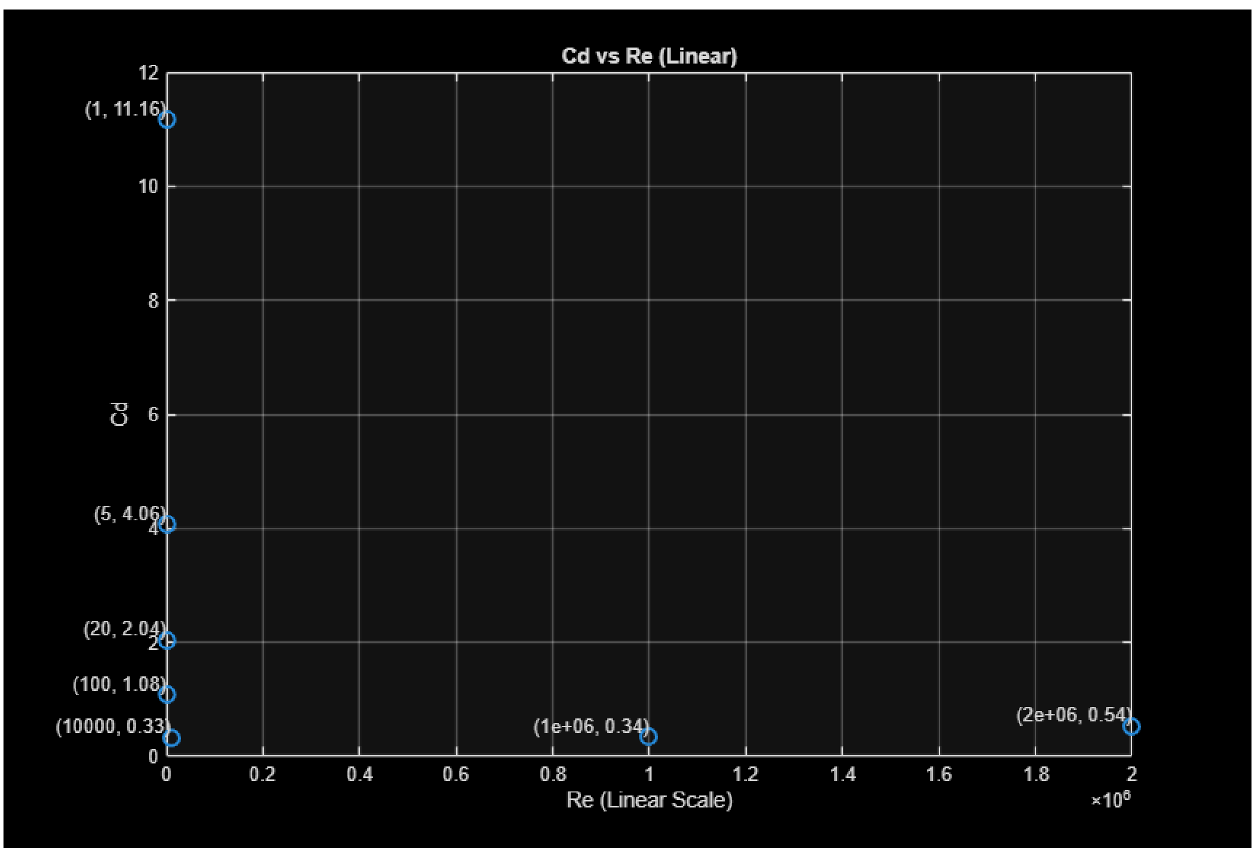

Cd vs Re (Linear)
Cd
Re (Linear Scale)
×10^6
(1, 11.16)
(5, 4.06)
(20, 2.04)
(100, 1.08)
(10000, 0.33)
(1e+06, 0.34)
(2e+06, 0.54)
12
10
8
6
4
2
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

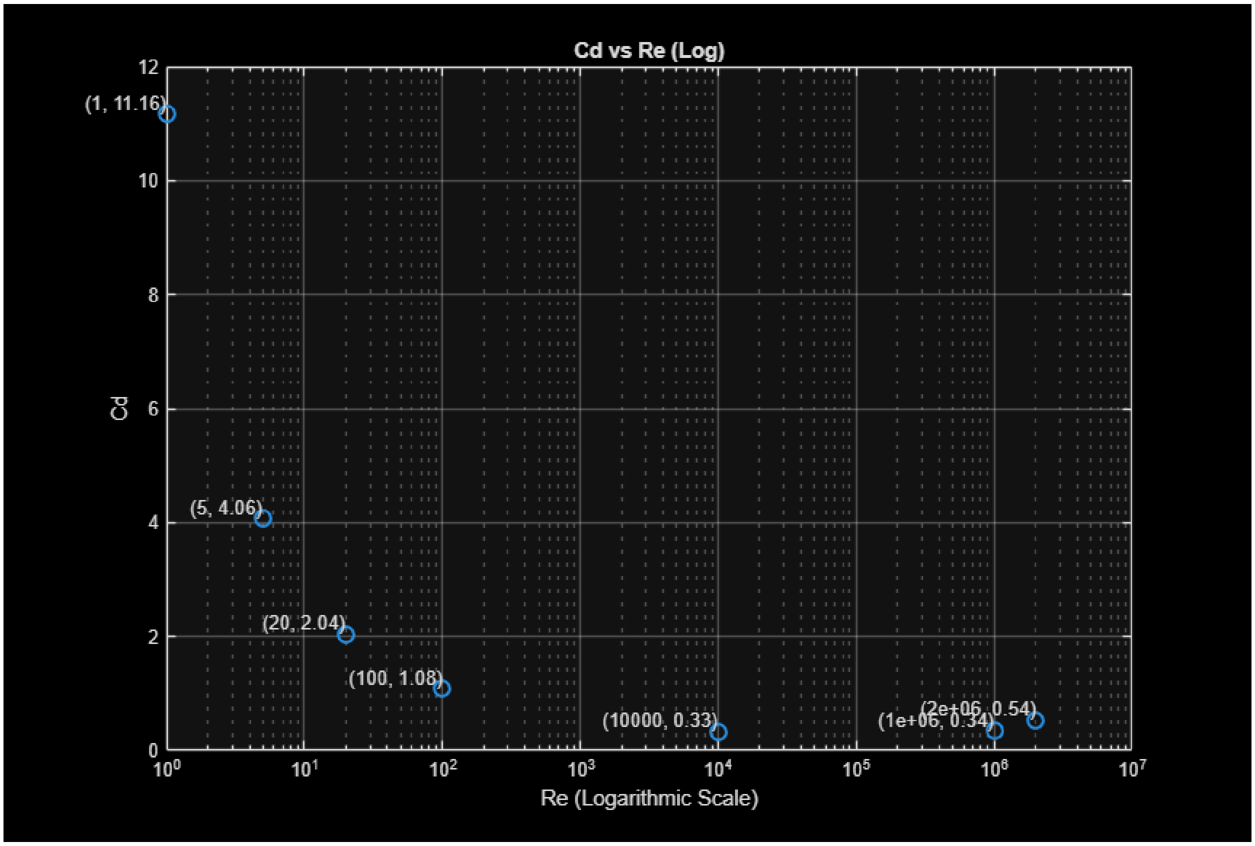

Cd vs Re (Log)
Cd
Re (Logarithmic Scale)
(1, 11.16)
(5, 4.06)
(20, 2.04)
(100, 1.08)
(10000, 0.33)
(1e+06, 0.34)
(2e+06, 0.54)
12
10
8
6
4
2
0
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7

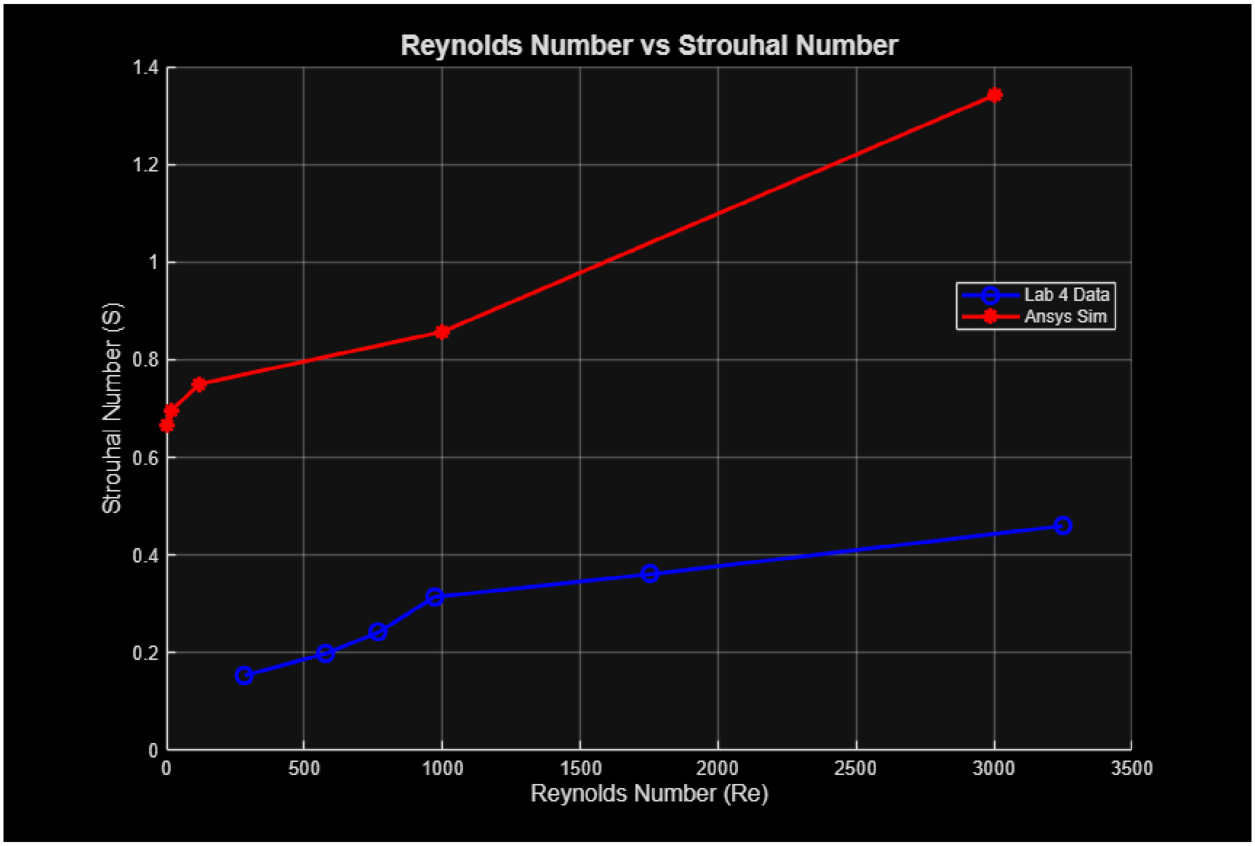


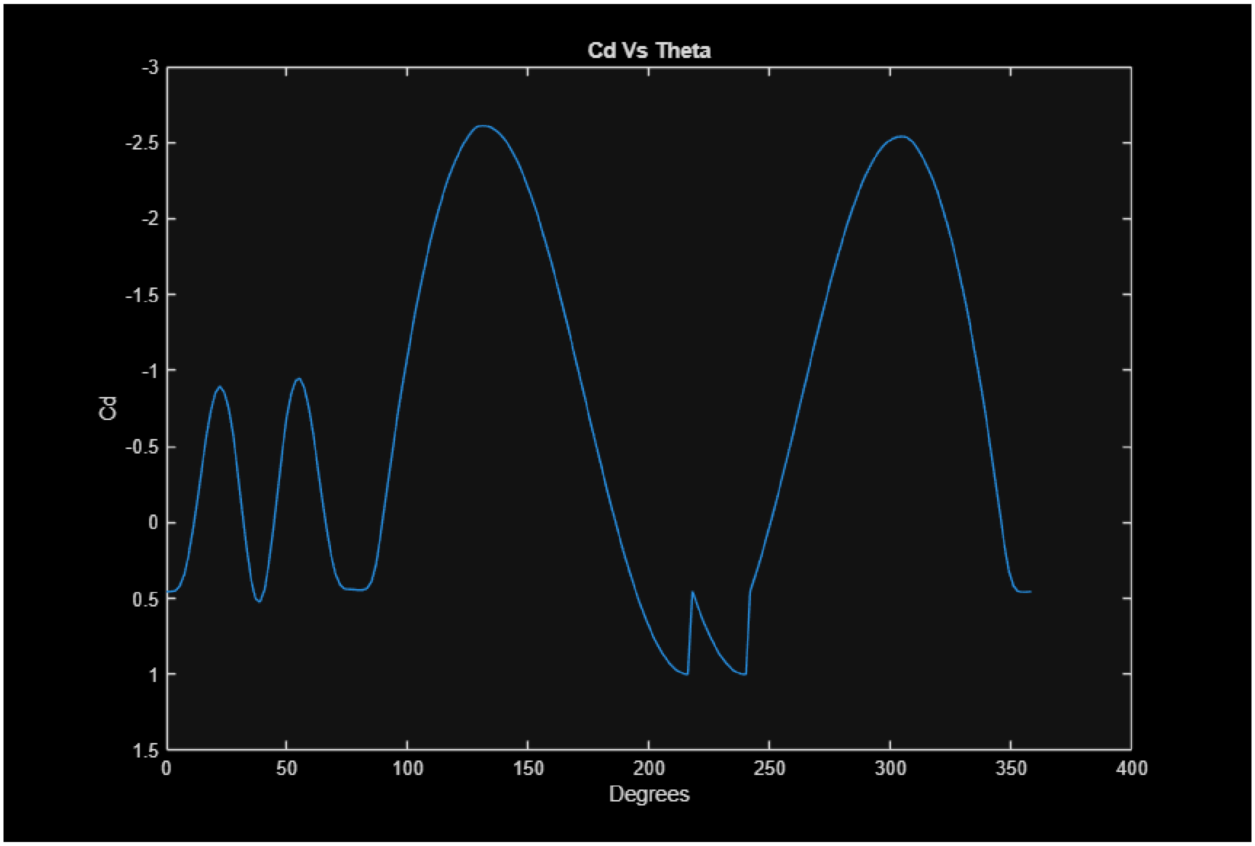

```
xy = readmatrix("CdPos");
theta = linspace(0,360,196);

figure
plot(theta,xy(:,2))
set(gca, 'YDir', 'reverse')
xlabel("Degrees")
ylabel('Cd')
title('Cd Vs Theta')
```


×10^-3
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Height mm
0
500
1000
1500
2000
2500
3000
3500
4000
4500
hz


Cd vs Re (Linear)
12
10
8
6
4
2
0
Cd
(1, 11.16)
(5, 4.06)
(20, 2.04)
(100, 1.08)
(10000, 0.33)
(1e+06, 0.34)
(2e+06, 0.54)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
×10^6
Re (Linear Scale)


Cd vs Re (Log)
Cd
Re (Logarithmic Scale)
(1, 11.16)
(5, 4.06)
(20, 2.04)
(100, 1.08)
(10000, 0.33)
(1e+06, 0.34)
(2e+06, 0.54)


Reynolds Number vs Strouhal Number
Strouhal Number (S)
Reynolds Number (Re)
Lab 4 Data
Ansys Sim



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