

Received October 4, 2016, accepted October 13, 2016, date of publication October 19, 2016, date of current version November 8, 2016.

Digital Object Identifier 10.1109/ACCESS.2016.2618772

## **Virtual Spatial Modulation**

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This work was supported by the National Nature Science Foundation of China under Grant 61501190, Grant 61201249, Grant 61622101, and Grant 61571020, and the Nature Science Foundation of Guangdong Province under Grant 2014A030310389.

**ABSTRACT** In this paper, we propose a virtual spatial modulation (VSM) scheme that performs index modulation on the virtual parallel channels resulting from the singular value decomposition of the multi-input-multi-output channels. The VSM scheme conveys information through both the indices of the virtual parallel channels and the *M*-ary modulated symbols. We derive a closed-form upper bound on the average bit error probability (ABEP), which considers the impact of imperfect channel estimation. Moreover, the asymptotic ABEP is also studied, which characterizes the error floor under imperfect channel estimation and the resulting diversity order as well as the coding gain under perfect channel estimation. Computer simulations verify the analysis and show that the VSM scheme can outperform the existing pre-coding aided spatial modulation schemes under the same spectral efficiency.

**INDEX TERMS** Singular value decomposition (SVD), pre-coding, spatial modulation, average bit error probability.

### I. INTRODUCTION

Multi-input-multi-output (MIMO) systems can greatly increase the channel capacity without sacrificing additional spectral resources in wireless communications [1], [2]. Various well-known categories of MIMO techniques, such as vertical-bell laboratories layered space-time (V-BLAST) [3] and space time block coding [4], have been proposed to exploit the benefits. However, several problems imply in MIMO systems due to the activation of multiple transmit antennas, such as the requirements of multiple radio frequency (RF) chains and inter-antenna synchronization, and the arising of the inter-channel interference (ICI).

To overcome above-mentioned problems, spatial modulation (SM) [5]–[8] has been proposed recently, in which only a single transmit antenna is activated for each transmission. Since a single RF chain is needed, SM leads to low transceiver complexity and low power consumption, which is beneficial for practical application [9], [10]. Note that the impact of channel estimation on the performance of SM or SSK has also been studied in more practical systems [11], [12]. Following the idea of SM, a great deal of works have been proposed. In [13], space shift keying (SSK), which only relies on the index of the active antenna to convey information, is proposed

to mitigate the implementation burden of SM. Generalized SSK (GSSK) [14] and SM (GSM) [15] extend SSK and SM by allowing activation of more than one transmit antenna, which enhance the spectral efficiencies of SSK and SM, respectively. The applications of SM and SSK into the relay systems are provided in [16] and [17]. In [18], quadrature SM (QSM), which expands the spatial domain to include both real and imaginary domains, is also proposed to increase the spectral efficiency of SM. Combining SM with trellis coding and space time block coding (STBC), more extensions, such as trellis coded SM (TCSM) and STBC-SM, have been proposed to improve the transmit diversity of SM [19]–[21]. To dispense with the channel estimation at the receiver, differential SM (DSM) is proposed in [22] and [23], which utilizes the antenna activation order to convey additional information.

More recently, the concept of SM is further extended to the receiver side. In [24], in favor of the zero-forcing (ZF) or minimum mean square error (MMSE) pre-coding, pre-coding SM (PSM) is proposed, which activates a single receive antenna at each time slot and conveys additional information through its index. At the cost of the requirement of channel state information (CSI) at the transmitter, PSM is shown to outperform most transmitter-side SM schemes and the

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conventional MIMO scheme. Attracted by its advantages, plenty of works related to PSM have also been proposed [25]–[29]. Generalised PSM (GPSM), which allows PSM to activate multiple receive antennas, is designed in [25]. The extension of GPSM into the multi-stream system is investigated in [26]. In [27], generalised pre-coding aided quadrature SM (GPQSM) is proposed as another extension of PSM, where the transmitted information is further embedded into the real and imaginary signals. By regarding the distributed relays as receive antennas, the idea of PSM is applied in a dual-hop relaying, where the source node of multiple antennas employs pre-coding operation to select a single relay, whose index carries information, to forward signals in an SM manner to the destination node [28]. Following [28] and [29] proposes the use of pre-coding aided DSM instead of PSM for the source node to avoid channel estimation at all receiving nodes.

All above-mentioned pre-coding schemes require the ZF or MMSE pre-coding to transfer the MIMO channel into multiple parallel channels. To apply, however, the number of transmit antennas,  $N_t$ , must not be less than that of receive antennas,  $N_r$ . On the other hand, the complete elimination of ICI by the ZF or MMSE pre-coding incurs high correlation between the resulting parallel channels. The correlated channels, however, will seriously deteriorate the performance of an index modulation system, which relies on the diversity of channels to convey information. The above observation motivates us to propose a novel pre-coding scheme, named virtual SM (VSM), which transfers the MIMO channels into virtual parallel channels with less channel correlation. The VSM scheme exploits the indices of the virtual parallel channels resulting from the singular value decomposition (SVD) of the MIMO channel to convey information. The right unitary matrix is chosen as the pre-coding matrix. For ease of implementation, a low-complexity near-optimal detector is also designed. Assuming Rayleigh fading channels, we study the average bit error probability (ABEP) of the VSM scheme, which takes into account the impact of imperfect channel estimation. Specifically, according to the union bounding technique, we derive a closed-from upper bound on the ABEP. In addition, inspired by [30] and [31], an asymptotic expression of ABEP in the high signal-to-noise ratio (SNR) region is also provided, which characterizes the ABEP error floor with channel estimation errors and the diversity order as well as coding gain without channel estimation errors. Computer simulations are conducted to investigate the performance of the VSM scheme. Results show that the VSM scheme significantly reduces the ABEP error floor under imperfect channel estimation and achieves the same diversity order of  $N_t - N_r + 1$ as the existing receiver-side SM schemes while obtaining a much higher coding gain.

The rest of this paper is organized as follows. In Section II, we introduce the system model and the GPQSM scheme. The principle and the implementation of the VSM scheme are clarified in Section III. Section IV presents the ABEP upper bound and asymptotic analysis for the VSM scheme.

In Section V, comparisons in terms of the ABEP are made. Finally, this paper is concluded in Section VI.

Notations: Upper and lower case boldface letters denote matrices and column vectors, respectively. The complex number field is represented by  $\mathbb{C}$ .  $(\cdot)^T$ ,  $(\cdot)^H$  and  $(\cdot)^{-1}$  represent the transpose, Hermitian transpose and inversion operations, respectively.  $\mathbf{I}_M$  is an identity matrix of size  $M \times M$ . Tr $\{\mathbf{X}\}$  returns the trace of the matrix  $\mathbf{X}$ .  $||\cdot||$  and  $\mathbf{C}(\cdot,\cdot)$  denote the Frobenius norm and binomial operations, respectively.  $Q(\cdot)$  and  $\Re\{\cdot\}$  represent the Gaussian Q-function [32] and the real component of the argument, respectively. diag $\{\mathbf{x}\}$  denotes a diagonal matrix whose diagonal elements are drawn from  $\mathbf{x}$ . The probability density function (PDF) and the probability of an event are denoted by  $f(\cdot)$  and  $\Pr(\cdot)$ , respectively.  $\Gamma(z)$  denotes the Gamma function, which is given by  $\Gamma(z) = x^z \int_0^{+\infty} e^{-xt} t^{z-1} dt$  [33].  $\operatorname{sort}(\cdot)$  denotes an ordering function that reorders the elements in an ascending order.

## II. SYSTEM MODEL AND OVERVIEW OF GPQSM

#### A. SYSTEM MODEL

We consider a MIMO system with an M-ary constellation set S,  $N_t$  transmit and  $N_r$  receive antennas. The MIMO channels are denoted by  $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ , where  $h_{i,j}$  with  $i \in \{1, \dots, N_r\}$  and  $j \in \{1, \dots, N_t\}$  denotes the channel between the j-th transmit antenna to the i-th receive antenna, which is assumed to be a complex Gaussian random variable (RV) with zero mean and unit variance. For simplicity, we do not take into account the channel correlation, which implies that the entries of  $\mathbf{H}$  are independent of each other. The received signal vector with pre-coding for all schemes can be expressed in a general form as

$$\mathbf{v} = \mathbf{H}\mathbf{P}\mathbf{x} + \mathbf{w},\tag{1}$$

where  $\mathbf{x} \in \mathbb{C}^{N_r \times 1}$  is the signal vector,  $\mathbf{P} \in \mathbb{C}^{N_t \times N_r}$  is the precoding matrix and  $\mathbf{w} \in \mathbb{C}^{N_r \times 1}$  is an additive white Gaussian noise vector with zero mean and covariance matrix  $N_0 \mathbf{I}_{N_r}$ .

## **B. GPQSM REVISITED**

In the sequel, we introduce the representative pre-coding aided SM scheme, GPQSM, which performs SM on the receive antennas and the in-phase and quadrature parts of the received signal independently [27].

In GPQSM, the limitation of  $N_t \ge N_r$  is a prerequisite to ensure the availability of the ZF or MMSE pre-coding. The signal vector  $\mathbf{x}$  has a length of  $N_r$  and the pre-coding matrix  $\mathbf{P}$  based on the ZF or MMSE criterion is given by

$$\mathbf{P}_{ZF} = \mathbf{\Omega}\mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1},\tag{2}$$

or

$$\mathbf{P}_{MMSE} = \mathbf{\Omega}\mathbf{H}^{H}(\mathbf{H}\mathbf{H}^{H} + N_{0}\mathbf{I}_{N_{r}})^{-1}, \tag{3}$$

respectively, where  $\Omega = \text{diag}\{\omega_1, \omega_2, \dots, \omega_{N_r}\}$  is the normalizing diagonal matrix. To ensure a constant instantaneous transmit power, the diagonal elements can be chosen as  $\omega_i = 1/\sqrt{||\mathbf{p}_i||^2}, i = 1, 2, \dots, N_r$ , where  $\mathbf{p}_i$  denotes the *i*-th



column of  $\mathbf{P}_{ZF}$  or  $\mathbf{P}_{MMSE}$  for ZF or MMSE pre-coding, respectively. Take ZF precoding as an example. The received signal vector can be expressed from (1) and (2) as

$$y = \Omega x + w, \tag{4}$$

which implies that the ICI is completely eliminated and  $N_r$  parallel channels with fading coefficients  $\{\omega_1, \omega_2, \ldots, \omega_{N_r}\}$  are resulted. On the other hand, we can also set the average transmit power to be constant, in which case the normalizing diagonal matrix becomes  $\Omega = \sqrt{N_t/\text{Tr}\{(\mathbf{H}^H\mathbf{H})^{-1}\}\mathbf{I}_{N_r}}$ . From above, it can be readily figured out that by applying either the instantaneous transmit power constraint or the average transmit power constraint, the resulting parallel channels are highly correlated with each other. The correlated channels significantly degrade the system performance, as will be verified in Section V.

#### III. PROPOSED VSM

To avoid the high channel correlation resulting from the ZF pre-coding or MMSE pre-coding, we propose a novel pre-coding scheme, i.e., the VSM scheme.

#### A. PRINCIPLE OF VSM

The well-known parallel decomposition of **H** using SVD is given by

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H,\tag{5}$$

where  $\mathbf{U} \in \mathbb{C}^{N_r \times N_r}$  and  $\mathbf{V} \in \mathbb{C}^{N_t \times N_t}$  are unitary matrices and  $\mathbf{\Sigma} \in \mathbb{C}^{N_r \times N_t}$  is the singular value matrix. Inspired by SVD, we aim to transmit the spatial bits through the virtual parallel channels in  $\mathbf{\Sigma}$ . If we have perfect channel estimation at the receiver and perfect feedback of the CSI from the receiver to the transmitter, the pre-coding matrix can be simply obtained from  $\mathbf{V}$ . However, the assumption never holds in practical systems. Therefore, to be practical we take into account the imperfect channel estimation and limited feedback in this paper. Note that the receiver uses the orthogonal pilots to estimate the CSI and the maximum-likelihood (ML) channel estimation is employed at the receiver. Therefore, the resulting estimated channel with errors at the receiver can be modeled as [34], [35]

$$\tilde{\mathbf{H}} = \mathbf{H} + \mathbf{H}_{\rho},\tag{6}$$

where  $\mathbf{H}_e$  denotes the channel estimation error matrix whose entries are independent and identically distributed (i.i.d.) complex Gaussian RVs with zero mean and variance  $\delta_e^2$ . Note that  $\mathbf{H}_e$  is independent of  $\mathbf{H}$  and  $\tilde{\mathbf{H}}$  depends on  $\mathbf{H}$  with the correlation coefficient  $\rho = 1/(1+\delta_e^2)$ . It can be seen that  $\delta_e^2 = 0$  indicates a special case of the channel estimation without errors. Based on the first-order autoregressive model, we have

$$\mathbf{H} = \rho \tilde{\mathbf{H}} + \mathbf{G},\tag{7}$$

where  $\mathbf{G} \in \mathbb{C}^{N_r \times N_t}$  is a complex Gaussian matrix independent of  $\tilde{\mathbf{H}}$ , whose entries have zero mean and variance  $(1-\rho)$ .

According to  $\hat{\mathbf{H}}$ , the limited feedback channel  $\hat{\mathbf{H}}$  is given by CSI quantization [36]

$$\tilde{\mathbf{H}} = \bar{\mathbf{H}} + \mathbf{E}_F,\tag{8}$$

where  $\mathbf{E}_F \in \mathbb{C}^{N_r \times N_t}$  denotes a noise matrix independent of  $\bar{\mathbf{H}}$ , whose entries are i.i.d. complex Gaussian RVs with zero mean and variance  $\delta_F^2$ . Here,  $\delta_F^2$  represents the average channel quantization distortion. Note that  $\delta_F^2 = 0$  indicates the channel quantization without distortion. Since  $\mathbf{G}$ ,  $\mathbf{E}_F$  and  $\bar{\mathbf{H}}$  are independent with each other, the true channel matrix can be expressed through the limited feedback channel by substituting (8) into (7) as

$$\mathbf{H} = \rho \bar{\mathbf{H}} + \mathbf{\Psi},\tag{9}$$

where  $\Psi \in \mathbb{C}^{N_r \times N_t}$  is the corresponding noise matrix whose entries are i.i.d. complex Gaussian RVs with zero mean and variance  $\rho^2 \delta_F^2 + (1-\rho)$ . From (9), we can see that the effects of the channel estimation and limited feedback channel are incorporated into  $\Psi$ . Therefore, for simplicity, it is advisable to attribute the effect of the limited feedback channel to the imperfect channel estimation and only consider the impact of the imperfect channel estimation ( $\tilde{\mathbf{H}} = \bar{\mathbf{H}}$  with  $\delta_F^2 = 0$ ). In this paper, the value of  $\rho$  is chosen according to two different criterions: (1) fixed  $\rho$ : the value of estimation error is fixed for all SNRs; (2) unfixed  $\rho$ : the estimation error changes with the given SNR as  $\delta_e^2 = 1/(l\gamma)$ , where  $\gamma = 1/N_0$  denotes the SNR and l denotes the number of pilot symbols used for channel estimation [37].

Due to the channel estimation, the receiver knows  $\tilde{\mathbf{H}}$  instead of  $\mathbf{H}$ . Performing SVD to  $\tilde{\mathbf{H}}$ , we have

$$\tilde{\mathbf{H}} = \tilde{\mathbf{U}}\tilde{\mathbf{\Sigma}}\tilde{\mathbf{V}}^H,\tag{10}$$

where  $\tilde{\mathbf{U}} \in \mathbb{C}^{N_r \times N_r}$  and  $\tilde{\mathbf{V}} \in \mathbb{C}^{N_t \times N_t}$  are unitary matrices and  $\tilde{\mathbf{\Sigma}} \in \mathbb{C}^{N_r \times N_t}$  is the singular value matrix. Under the assumptions of  $N_t \geq N_r$  and a rich scattering environment,  $\tilde{\mathbf{\Sigma}} \in \mathbb{C}^{N_r \times N_t}$  contains non-zero singular values  $\{\tilde{\sigma}_i\}_{i=1}^{N_r}$ , which provide  $N_r$  virtual parallel channels to carry spatial bits. Therefore, the SVD of  $\tilde{\mathbf{H}}$  in (10) can be rewritten as

$$\tilde{\mathbf{H}} = \tilde{\mathbf{U}}[\tilde{\mathbf{\Sigma}}_1, \mathbf{0}][\tilde{\mathbf{V}}_1, \tilde{\mathbf{V}}_0]^H = \tilde{\mathbf{U}}\tilde{\mathbf{\Sigma}}_1\tilde{\mathbf{V}}_1^H, \tag{11}$$

where  $\tilde{\mathbf{\Sigma}}_1 \in \mathbb{C}^{N_r \times N_r}$  is a diagonal matrix whose diagonal elements are  $\{\tilde{\sigma}_i\}_{i=1}^{N_r}, \tilde{\mathbf{V}}_1 \in \mathbb{C}^{N_t \times N_r}$  and  $\tilde{\mathbf{V}}_0 \in \mathbb{C}^{N_t \times (N_t - N_r)}$  are sub-matrices of  $\tilde{\mathbf{V}}$ , which contain  $N_r$  right-singular vectors corresponding to nonzero singular values and  $N_t - N_r$  right-singular vectors corresponding to zero singular values, respectively. From (11), the signal vector can be set as  $\mathbf{x} \in \mathbb{C}^{N_r \times 1}$ , and  $\tilde{\mathbf{V}}_1$  is fed back to the transmitter for pre-coding purpose.

## B. VSM TRANSMITTER

The transmitter structure of VSM is depicted in Fig. 1. In VSM,  $N_p$  out of  $N_r$  virtual parallel channels are activated to transmit the real (imaginary) parts of the modulated symbols. In order to modulate the information bits, only  $N = 2^{\lfloor \log_2(C(N_r,N_p)) \rfloor}$  combinations are selected as the legitimate

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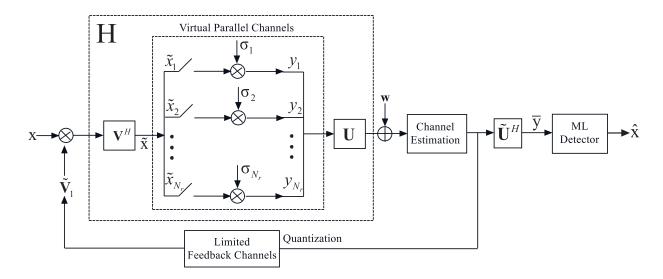


FIGURE 1. The schematic of VSM.

antenna combinations. Specifically, VSM works as follows. At each time slot,  $d=N_p\log_2(M)+2\lfloor\log_2(C(N_r,N_p))\rfloor$  information bits are divided into three parts. The first part of  $N_p\log_2(M)$  bits are mapped into a symbol vector  $\mathbf{s}=[s_1,\ldots,s_{N_p}]^T$ , where  $s_\varphi=s_\varphi^I+js_\varphi^Q, \varphi\in\{1,\ldots,N_p\}$ , with  $s_\varphi^I$  and  $s_\varphi^Q$  denoting the in-phase and quadrature parts of  $s_\varphi$ , respectively. Note that  $s_\varphi^I\in\mathcal{S}^I$  and  $s_\varphi^Q\in\mathcal{S}^Q$ , where  $\mathcal{S}^I$  and  $\mathcal{S}^Q$  denote the in-phase and quadrature sets of  $\mathcal{S}$ , respectively. The second part of  $\lfloor\log_2(C(N_r,N_p))\rfloor$  bits select an active channel combination  $I_v^I=\{i_v^I(1),\ldots,i_v^I(N_p)\}$ , where  $i_v^I(\alpha)\in\{1,\ldots,N_r\}$  with  $v\in\{1,\ldots,N\}$  and  $\alpha\in\{1,\ldots,N_p\}$ , to transmit the real part of  $\mathbf{s}$ , yielding  $\mathbf{x}^I=[\ldots,s_1^I,\ldots,s_{N_p}^I,\ldots]$ . Similarly, the third part of  $\lfloor\log_2(C(N_r,N_p))\rfloor$  bits select an active channel combination  $I_\varphi^Q=\{i_\mu^Q(1),\ldots,i_\mu^Q(N_p)\}$ , where  $i_\mu^Q(\varsigma)\in\{1,\ldots,N_r\}$  with  $\mu\in\{1,\ldots,N\}$  and  $\varsigma\in\{1,\ldots,N_p\}$ , to transmit the imaginary part of  $\mathbf{s}$ , yielding  $\mathbf{x}^Q=[\ldots,s_1^Q,\ldots,s_{N_p}^Q,\ldots]$ . Finally, the information vector is generated by combining the in-phase and quadrature parts:

$$\mathbf{x} = \mathbf{x}^I + j\mathbf{x}^Q. \tag{12}$$

As an example, we present a mapping table of VSM for  $N_t = 4$ ,  $N_r = 2$ ,  $N_p = 1$  and 4-QAM in Table I. In this example, we have d = 4. Suppose that the information bits  $q = [0 \ 1 \ 1 \ 0]$  are transmitted at a given time slot. The first  $N_p \log_2(M) = 2$  bits  $[0\ 1]$  are mapped into  $\mathbf{s} = [-1-j]$ . Then,  $\mathbf{s}$  is divided into in-phase and quadrature parts  $\mathbf{s}^I = [-1]$  and  $\mathbf{s}^Q = [-1]$ , respectively. The second  $\lfloor \log_2(\mathbf{C}(N_r, N_p)) \rfloor = 1$  bit [1] indicates that the active channel combination  $I_2^I = \{2\}$  is chosen to transmit  $\mathbf{s}^I$ , resulting in the vector  $\mathbf{x}^I = [0, -1]^T$ . The last  $\lfloor \log_2(\mathbf{C}(N_r, N_p)) \rfloor = 1$  bit [0] indicates to select the active channel combination  $I_1^Q = \{1\}$  to transmit  $\mathbf{s}^Q$ , which results in the vector  $\mathbf{x}^Q = [1, 0]^T$ . Finally, the information vector is obtained as  $\mathbf{x} = [-j, -1]^T$ .

Input bits	Signal vector	Input bits	Signal vector
0000	$[-1+j,0]^T$	1000	$[1+j,0]^T$
0001	$[-1, j]^T$	1001	$[1,j]^T$
0010	$[j, -1]^T$	1010	$[j,1]^T$
0011	$[0,-1+j]^T$	1011	$[0,1+j]^T$
0100	$[-1-j,0]^T$	1100	$[1-j,0]^T$
0101	$[-1, -j]^T$	1101	$[1, -j]^T$
0110	$[-j, -1]^T$	1110	$[-j, 1]^T$
0111	$[0,-1-j]^T$	1111	$[0, 1-j]^T$

**TABLE 1.** Mapping table for VSM with  $N_t = 4$ ,  $N_r = 2$ ,  $N_p = 1$  and 4-QAM.

Before transmitted into the wireless channel, the information vector is pre-coded by  $P = \tilde{V}_1$ .

## C. VSM RECEIVER

The receiver structure of VSM is depicted in Fig.1. The received signal vector after receiver shaping can be expressed as

$$\bar{\mathbf{y}} = \tilde{\mathbf{U}}^H \mathbf{y} = \tilde{\mathbf{U}}^H \mathbf{H} \tilde{\mathbf{V}}_1 \mathbf{x} + \tilde{\mathbf{U}}^H \mathbf{w}. \tag{13}$$

Based on (13), we propose two detection methods corresponding to whether or not the receiver has the knowledge of the correlation coefficient  $\rho$ . When  $\rho$  is known to the receiver, the maximum-likelihood (ML) detector can be derived as

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\bar{\mathbf{y}} - \rho \tilde{\mathbf{\Sigma}}_1 \mathbf{x}\|^2. \tag{14}$$

On the other hand, when  $\rho$  is unknown to the receiver, it is reasonable to assume perfect channel estimation for detection and thus the detector follows from (14) by setting  $\rho = 1$ :

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{arg\,min}} \|\bar{\mathbf{y}} - \tilde{\mathbf{\Sigma}}_1 \mathbf{x}\|^2. \tag{15}$$



However, both receivers in (14) and (15) impose great computational burden to the receiver when  $N_r$  or M goes to a very large value. To solve this problem, inspired by [27] we further propose a low-complexity detector in the following.

For brevity, we only consider the detection with  $\rho$  unknown to the receiver. In light of (12), we can rewrite (15) as

$$\hat{\mathbf{x}} = \underset{\mathbf{x}^{I}}{\arg\min} \|\bar{\mathbf{y}}^{I} - \tilde{\mathbf{\Sigma}}_{1}\mathbf{x}^{I}\|^{2} + j \underset{\mathbf{x}^{Q}}{\arg\min} \|\bar{\mathbf{y}}^{Q} - \tilde{\mathbf{\Sigma}}_{1}\mathbf{x}^{Q}\|^{2},$$
(16)

where  $\bar{\mathbf{y}}^I$  and  $\bar{\mathbf{y}}^Q$  denote the real and imaginary parts of  $\bar{\mathbf{y}}$ , respectively. Due to the independence between the real and imaginary parts of  $\mathbf{x}$ , the detections for the real and imaginary parts can be performed independently. Without loss of generality, we only consider the detection for the real part, which can be expressed as

$$\hat{\mathbf{x}}^{I} = \arg\min_{\mathbf{x}^{I}} \|\bar{\mathbf{y}}^{I} - \tilde{\mathbf{\Sigma}}_{1}\mathbf{x}^{I}\|^{2}$$

$$= \arg\min_{\{x_{i}^{I}\}_{i=1}^{N_{r}}} \sum_{i=1}^{N_{r}} |\bar{y}_{i}^{I} - \tilde{\sigma}_{i}x_{i}^{I}|^{2}$$

$$= \arg\min_{\{x_{i}^{I}\}_{i=1}^{N_{r}}} \sum_{i=1}^{N_{r}} \left[ \left( \bar{y}_{i}^{I} \right)^{2} - 2\tilde{\sigma}_{i}x_{i}^{I}\bar{y}_{i}^{I} + \left( \tilde{\sigma}_{i}x_{i}^{I} \right)^{2} \right]$$

$$= \arg\min_{I_{v}^{I}} \sum_{i \in I_{v}^{I}} \min_{s_{i}^{I}} \left[ \left( \tilde{\sigma}_{i}s_{i}^{I} \right)^{2} - 2\tilde{\sigma}_{i}s_{i}^{I}\bar{y}_{i}^{I} \right]. \tag{17}$$

Denote 
$$D_i = (\tilde{\sigma}_i \hat{s}_i^I)^2 - 2\tilde{\sigma}_i \hat{s}_i^I \bar{y}_i^I$$
 and 
$$\hat{s}_i^I = \underset{s \in S^I}{\arg \min} |\bar{y}_i^I - \tilde{\sigma}_i s|^2,$$
 (18)

which represents the hard decision on  $\bar{y}_i^I$ . The detection for the active channel combination (legitimate channel combination) in (17) is thus equivalent to

$$\hat{I}_{\nu}^{I} = \underset{I_{\nu}^{I}}{\operatorname{arg\,min}} \sum_{i \in I_{\nu}^{I}} D_{i}. \tag{19}$$

It is clear that the order of the search complexity in (19) is  $N_r M^I + N$ , where  $M^I$  denotes the cardinality of  $S^I$ . However, the search complexity is still high especially when  $N_r$  or  $N_p$  is large. To further reduce the search complexity, it is advisable to select active channel combination via  $[z_1, \ldots, z_{N_r}]$  $\operatorname{sort}(\{D_i\}_{i=1}^{N_r})$ , where  $z_1$  and  $z_{N_r}$  are the indices of the maximum and minimum values in  $\{D_i\}_{i=1}^{N_r}$ , respectively. Therefore, we can simply obtain the active channel combination by  $\hat{I}_{v}^{I} = \text{sort}([z_1, \dots, z_{N_n}])$ . In this manner, however, we may obtain an illegitimate channel combination, which does not exist in the mapping table. To relieve this adverse effect, when this event occurs, we select  $\hat{I}_{v}^{I} = \text{sort}([z_1, \dots, z_{N_{p-1}}, z_{N_{p+1}}])$ instead, which is the second most likely to happen. Note that if the alternative channel combination is still illegal, we simply set the estimated information bits to be all 0 bits. After getting  $\hat{I}_{\nu}^{I}$ , the estimation of  $\mathbf{x}^{I}$  is directly obtained by

$$\hat{x}_i^I = \begin{cases} \hat{s}_i^I, & i \in \hat{I}_v^I \\ 0, & \text{otherwise.} \end{cases}$$
 (20)

From above, we see that the equivalent detection only leads to a search complexity of order  $N_r M^I$ . Compared with (15), the search complexity is largely reduced.

#### IV. PERFORMANCE ANALYSIS

In this section, we analytically derive the ABEP of VSM for the ML detection in (14). To study the diversity order and coding gain, we further derive the asymptotic ABEP of VSM. The provided results, as will be verified in Section V, is also valid for the ML detection in (15).

## A. ANALYSIS OF ABEP

According to (7),  $\bar{\mathbf{y}}$  can be rewritten from (13) as

$$\bar{\mathbf{y}} = \rho \tilde{\mathbf{\Sigma}}_1 \mathbf{x} + \mathbf{q},\tag{21}$$

where  $\mathbf{q} = \tilde{\mathbf{U}}^H \mathbf{G} \tilde{\mathbf{V}}_1 \mathbf{x} + \tilde{\mathbf{U}}^H \mathbf{w}$ . Since multiplying a matrix by any unitary matrix does not change the probability distribution, we can conclude that  $\tilde{\mathbf{U}}^H \mathbf{G} \tilde{\mathbf{V}}_1$  and  $\tilde{\mathbf{U}}^H \mathbf{w}$  have the same distribution as  $\mathbf{G}$  and  $\mathbf{w}$ , respectively. In addition, since  $\mathbf{G}$  and  $\mathbf{w}$  are mutually independent, each entry of  $\mathbf{q}$  follows the complex Gaussian distribution with zero mean and variance

$$\psi = N_0 + (1 - \rho)||\mathbf{x}||^2. \tag{22}$$

At this point, it is straightforward to arrive at (14) or (15) from (21) for detection.

Denote  $\mathbf{c} \triangleq [c_1, \dots, c_{N_r}]^T = \mathbf{x} - \hat{\mathbf{x}}$ . In light of (15) and (21), the average PEP of detecting  $\hat{\mathbf{x}}$  when  $\mathbf{x}$  is transmitted on  $\tilde{\Sigma}_1$  can be calculated as

$$Pr(\mathbf{x} \to \hat{\mathbf{x}}|\tilde{\mathbf{\Sigma}}_{1}) = Pr(||\bar{\mathbf{y}} - \tilde{\mathbf{\Sigma}}_{1}\mathbf{x}||^{2} > ||\bar{\mathbf{y}} - \tilde{\mathbf{\Sigma}}_{1}\hat{\mathbf{x}}||^{2})$$

$$= Q\left(\sqrt{\frac{||\rho\tilde{\mathbf{\Sigma}}_{1}(\mathbf{x} - \hat{\mathbf{x}})||^{2}}{2\psi}} - (1 - \rho)\frac{||\tilde{\mathbf{\Sigma}}_{1}\mathbf{x}||^{2} - ||\tilde{\mathbf{\Sigma}}_{1}\hat{\mathbf{x}}||^{2}}{\sqrt{2\psi||\tilde{\mathbf{\Sigma}}_{1}(\mathbf{x} - \hat{\mathbf{x}})||^{2}}}\right).$$
(23)

However, averaging (23) is nearly unavailable due to the difficulty in obtaining the exact PDF of the second part in Q-function. Therefore, it may be difficult to obtain closed-form upper-bounded and asymptotic ABEPs of VSM that adopts the receiver in (15). Alternatively, we resorts to (14) for the derivation of PEP in this paper. In light of (14) and (21), the average PEP can be calculated as

$$Pr(\mathbf{x} \to \hat{\mathbf{x}}|\tilde{\mathbf{\Sigma}}_{1})$$

$$= Pr(||\bar{\mathbf{y}} - \rho\tilde{\mathbf{\Sigma}}_{1}\mathbf{x}||^{2} > ||\bar{\mathbf{y}} - \rho\tilde{\mathbf{\Sigma}}_{1}\hat{\mathbf{x}}||^{2})$$

$$= Q\left(\sqrt{\frac{||\rho\tilde{\mathbf{\Sigma}}_{1}(\mathbf{x} - \hat{\mathbf{x}})||^{2}}{2\psi}}\right)$$

$$= Q\left(\sqrt{\frac{\sum_{i=1}^{N_{r}} \lambda_{i}|c_{i}|^{2}}{2\psi}}\right), \tag{24}$$

where  $\lambda_i = \rho^2 \tilde{\sigma}_i^2$ ,  $i \in \{1, \dots, N_r\}$ .

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From (24), it can be seen that the derivation of PEP needs the joint PDF of  $\Lambda = [\lambda_1, \dots, \lambda_{N_r}]^T$ , which is given by [38]

$$f(\mathbf{\Lambda}) = \frac{1}{K} e^{-\sum_{i=1}^{N_r} \lambda_i} \underbrace{\prod_{i=1}^{N_r} \lambda_i^{N_t - N_r} \prod_{j=i+1}^{N_r} (\lambda_i - \lambda_j)^2}_{\Upsilon}$$
$$= \frac{1}{K} \sum_{r=1}^{L} a_{\kappa} e^{-\sum_{i=1}^{N_r} \lambda_i} \lambda_1^{b_1^1} \cdots \lambda_{N_r}^{b_{\kappa}^{N_r}}, \tag{25}$$

where K is a normalizing coefficient, L denotes the number of monomials in  $\Upsilon$ ,  $a_{\kappa}$  denotes the coefficient of the  $\kappa$ -th monomial in  $\Upsilon$  with  $\kappa \in \{1, \ldots L\}$  and  $\{b_{\kappa}^i\}_{i=1}^{N_r}$  denote the power coefficients of  $\{\lambda_i\}_{i=1}^{N_r}$  in the  $\kappa$ -th monomial in  $\Upsilon$ . An example for all values of  $(K, a_{\kappa}, b_{\kappa}^1, \ldots, b_{\kappa}^{N_r})$  with  $N_t = 4$  and  $N_r = 3$  is given in Table 2. By averaging (24) with (25), we have

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}) = E_{\mathbf{\Lambda}} \left[ Q \left( \sqrt{\frac{\sum_{i=1}^{N_r} \lambda_i |c_i|^2}{2\psi}} \right) \right]$$

$$\cong E_{\mathbf{\Lambda}} \left[ \frac{1}{12} e^{-\sum_{i=1}^{N_r} \frac{\lambda_i |c_i|^2}{4\psi}} + \frac{1}{4} e^{-\sum_{i=1}^{N_r} \frac{\lambda_i |c_i|^2}{3\psi}} \right] (26)$$

$$= \frac{1}{12} \int e^{-\sum_{i=1}^{N_r} \frac{\lambda_i |c_i|^2}{4\psi}} f(\mathbf{\Lambda}) d\mathbf{\Lambda}$$

$$+ \frac{1}{4} \int e^{-\sum_{i=1}^{N_r} \frac{\lambda_i |c_i|^2}{3\psi}} f(\mathbf{\Lambda}) d\mathbf{\Lambda}, \qquad (27)$$

where (26) is obtained by [39]

$$Q(x) \cong \frac{1}{12}e^{-\frac{x^2}{2}} + \frac{1}{4}e^{-\frac{2x^2}{3}}.$$
 (28)

According to [33, eq. (8.312.2)], the first integral of (27) can be calculated as

$$\frac{1}{12} \int e^{\frac{-\sum_{i=1}^{N_r} \lambda_i |c_i|^2}{4\psi}} f(\mathbf{\Lambda}) d\mathbf{\Lambda}$$

$$= \frac{1}{12K} \sum_{\kappa=1}^{L} a_{\kappa} \int e^{-\sum_{i=1}^{N_r} (\frac{|c_i|^2}{4\psi} + 1)\lambda_i} \lambda_1^{b_{\kappa}^1} \cdots \lambda_{N_r}^{b_{\kappa}^{N_r}} d\mathbf{\Lambda}$$

$$= \frac{1}{12K} \sum_{\kappa=1}^{L} a_{\kappa} \int e^{-(\frac{|c_1|^2}{4\psi} + 1)\lambda_1} \lambda_1^{b_{\kappa}^1} d\lambda_1$$

$$\times \cdots \times \int e^{-(\frac{|c_{N_r}|^2}{4\psi} + 1)\lambda_{N_r}} \lambda_{N_r}^{b_{\kappa}^{N_r}} d\lambda_{N_r}$$

$$= \frac{1}{12K} \sum_{\kappa=1}^{L} a_{\kappa} \prod_{i=1}^{N_r} \left(\frac{|c_i|^2}{4\psi} + 1\right)^{-(b_{\kappa}^i + 1)} \Gamma(b_{\kappa}^i + 1). \quad (29)$$

In the same manner, the second integral of (27) is given by

$$\frac{1}{4} \int e^{-\sum_{i=1}^{N_r} \frac{\lambda_i |c_i|^2}{3\psi}} f(\mathbf{\Lambda}) d\mathbf{\Lambda}$$

$$= \frac{1}{4K} \sum_{\kappa=1}^{L} a_{\kappa} \prod_{i=1}^{N_r} \left( \frac{|c_i|^2}{3\psi} + 1 \right)^{-(b_{\kappa}^i + 1)} \Gamma(b_{\kappa}^i + 1). \tag{30}$$

By substituting (29) and (30) into (27), the unconditional PEP can be expressed as

$$\Pr(\mathbf{x} \to \hat{\mathbf{x}}) = \frac{1}{4K} \sum_{\kappa=1}^{L} a_{\kappa} \left( \prod_{i=1}^{N_r} \Gamma(b_{\kappa}^i + 1) \right) \times \left[ \frac{1}{3} \prod_{i=1}^{N_r} \left( \frac{|c_i|^2}{4\psi} + 1 \right)^{-(b_{\kappa}^i + 1)} + \prod_{i=1}^{N_r} \left( \frac{|c_i|^2}{3\psi} + 1 \right)^{-(b_{\kappa}^i + 1)} \right]. \quad (31)$$

After obtaining the PEP, an upper bound on the ABEP can be readily derived according to the union bounding technique as [40]

$$P_e \le \frac{1}{d2^d} \sum_{\mathbf{x}} \sum_{\hat{\mathbf{x}} \ne \mathbf{x}} N(\mathbf{x} \to \hat{\mathbf{x}}) \Pr(\mathbf{x} \to \hat{\mathbf{x}}), \tag{32}$$

where  $N(\mathbf{x} \to \hat{\mathbf{x}})$  measures the number of bits in error between  $\mathbf{x}$  and  $\hat{\mathbf{x}}$ .

#### **B. ASYMPTOTIC ABEP ANALYSIS**

Recall  $\psi = N_0 + (1 - \rho)||\mathbf{x}||^2$ . If we have channel estimation error  $(\rho \neq 1)$ ,  $N_0 = 1/\gamma$  tends to be zero and  $\psi \to \psi^* = (1 - \rho)||\mathbf{x}||^2$  as  $\gamma \to +\infty$ . By substituting (31) into (32) with  $\psi = \psi^*$ , the asymptotic ABEP can be thus expressed as

$$P_{e} \to \frac{1}{d2^{d+2}K} \sum_{\mathbf{x}} \sum_{\hat{\mathbf{x}}} N(\mathbf{x} \to \hat{\mathbf{x}}) \sum_{\kappa=1}^{L} a_{\kappa} \left( \prod_{i=1}^{N_{r}} \Gamma(b_{\kappa}^{i} + 1) \right) \times \left[ \frac{1}{3} \prod_{i=1}^{N_{r}} \left( \frac{|c_{i}|^{2}}{4\psi^{*}} + 1 \right)^{-(b_{\kappa}^{i} + 1)} + \prod_{i=1}^{N_{r}} \left( \frac{|c_{i}|^{2}}{3\psi^{*}} + 1 \right)^{-(b_{\kappa}^{i} + 1)} \right].$$
(33)

The above equation reveals that the ABEP tends to be a constant irrelevant to the SNR in the high SNR region, which, as reflected in the ABEP performance, creates an error floor.

On the other hand, if no channel estimation error exists ( $\rho=1$ ), we have  $\psi=1/\gamma$  and the asymptotic ABEP can be derived as

$$P_{e} \to \frac{1}{d2^{d+2}K} \sum_{\mathbf{x}} \sum_{\hat{\mathbf{x}}} N(\mathbf{x} \to \hat{\mathbf{x}}) \sum_{\kappa=1}^{L} a_{\kappa} \left( \prod_{i=1}^{N_{r}} \Gamma(b_{\kappa}^{i} + 1) \right) \times \left[ \frac{1}{3} \prod_{i=1}^{N_{r}} \left( \frac{|c_{i}|^{2} \gamma}{4} \right)^{-(b_{\kappa}^{i} + 1)} + \prod_{i=1}^{N_{r}} \left( \frac{|c_{i}|^{2} \gamma}{3} \right)^{-(b_{\kappa}^{i} + 1)} \right].$$
(34)

From (34), we see that the high SNR approximation of  $P_e$  turns out to be a linear combination of PEPs, which are

$\kappa$	$a_{\kappa}$	$b_{\kappa}^{1}$	$b_{\kappa}^2$	$b_{\kappa}^3$	$\kappa$	$a_{\kappa}$	$b_{\kappa}^{1}$	$b_{\kappa}^2$	$b_{\kappa}^3$
1	1	1	3	5	11	2	3	2	4
2	-2	1	4	4	12	2	4	2	3
3	1	1	5	3	13	2	4	3	2
4	2	2	3	4	14	-2	4	4	1
5	2	2	4	3	15	1	3	1	5
6	-2	2	5	2	16	-2	4	1	4
7	-6	3	3	3	17	1	5	1	3
8	2	3	4	2	18	-2	5	2	2
9	1	3	5	1	19	1	5	3	1
10	-2	2	2	5					

**TABLE 2.** A look-up table for  $N_t = 4$ ,  $N_r = 3$  (K = 144).

dominated by the sum of the terms associated with the largest exponent (equivalently the smallest value among  $\{b_{\kappa}^i\}_{i=1}^{N_r}$  for all  $\kappa(s)$  of  $\gamma$ . For instance, from Table 2 with  $N_t=4$  and  $N_r=3$ , the smallest coefficient is 1. From the above analysis, the asymptotic ABEP in (34) can be further approximated as

$$P_{e} \rightarrow \frac{\gamma^{-d_{min}}}{d2^{d+2}K} \sum_{\kappa, d_{min}} \sum_{s, \hat{s} \in \mathcal{S}} a_{\kappa} \left( \prod_{i=1}^{N_{r}} \Gamma(b_{\kappa}^{i} + 1) \right) N(\mathbf{x} \rightarrow \hat{\mathbf{x}})$$

$$\times \left( \frac{|s - \hat{s}|^{-2d_{min}}}{3 \cdot 4^{-d_{min}}} + \frac{|s - \hat{s}|^{-2d_{min}}}{3^{-d_{min}}} \right), \tag{35}$$

where  $d_{min} = N_t - N_r + 1$  is the diversity order achieved by the VSM scheme and  $\sum_{\kappa,d_{min}}$  denotes the sum applying to all  $\kappa(s)$  for which there exists the coefficient  $d_{min}$ . From (35), we see that similar to the existing pre-coding schemes, the diversity order of VSM is determined by the configuration of the transceiver antennas only and is irreverent to  $N_p$ . By definition [41], the coding gain achieved by the VSM scheme can be derived from (35) as

$$G_{c} = \left[ \frac{1}{Kd2^{d+2}} \sum_{\kappa, d_{min}} \sum_{s, \hat{s} \in \mathcal{S}} a_{\kappa} \left( \prod_{i=1}^{N_{r}} \Gamma(b_{\kappa}^{i} + 1) \right) N(\mathbf{x} \to \hat{\mathbf{x}}) \right. \\ \times \left. \left( \frac{|s - \hat{s}|^{-2d_{min}}}{3 \cdot 4^{-d_{min}}} + \frac{|s - \hat{s}|^{-2d_{min}}}{3^{-d_{min}}} \right) \right]^{-\frac{1}{d_{min}}}, \tag{36}$$

which satisfies

$$P_e \cong (G_c \gamma)^{-d_{min}},\tag{37}$$

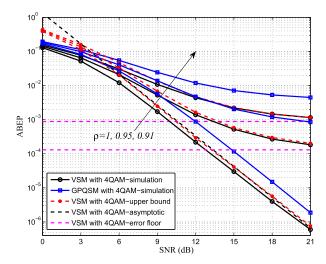
in the high SNR region.

## V. SIMULATION RESULTS AND ANALYSIS

We conduct computer simulations to evaluate the ABEP performance of VSM. The 4QAM constellation is assumed for all schemes.

## A. PERFORMANCE COMPARISON WITH GPQSM

We compare the ABEP performance of VSM with that of GPQSM in Figs. 2 and 3. The simulation parameters for Fig. 2



**FIGURE 2.** Comparison between VSM and GPQSM in terms of ABEP with  $\rho = 1$ , 0.95, 0.91 for  $N_t = 4$ ,  $N_T = 2$  and  $N_P = 1$ .

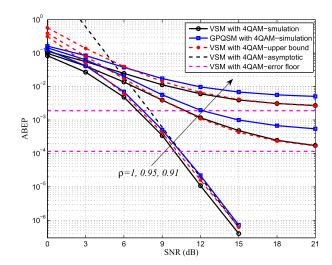
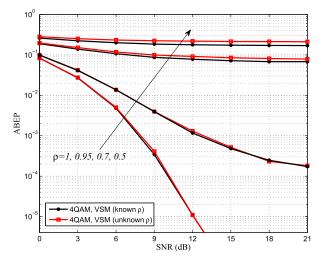


FIGURE 3. Comparison between VSM and GPQSM in terms of ABEP with  $\rho=1,0.95,0.91$  for  $N_t=8,N_r=4$  and  $N_p=2$ .

are chosen as  $N_t = 4$ ,  $N_r = 2$  and  $N_p = 1$ , and those for Fig. 3 are  $N_t = 8$ ,  $N_r = 4$  and  $N_p = 2$ . Since the proposed lowcomplexity detector achieves almost the same performance as the ML detector, we have removed its ABEP curves for figure clarity. To examine the effect of channel estimation, we consider  $\rho = 1, 0.95, \text{ and } 0.91, \text{ which correspond to}$  $\delta_e^2 = 0, 0.05, \text{ and } 0.1, \text{ respectively. To verify our analysis,}$ the analytical ABEP upper bounds and asymptotic results of VSM are also added in the figures. It is observed from Figs. 2 and 3 that, similar to GPQSM, under imperfect channel estimation, VSM results in error floors, whose levels are accurately characterized by the analytical results. In addition, the error floor increases with the channel estimation error, applying to both schemes. However, the floor level for VSM is much lower. The above observation can be explained by (21), which reveals that the power of the equivalent noise  $\bf{q}$ scales with the channel estimation error rather than the SNR.



On the other hand, under perfect channel estimation, VSM archives a diversity order of  $N_t - N_r + 1$  as GPQSM while obtaining a much larger coding gain. For example, as seen from Fig. 2, about 2dB SNR gain is obtained at an ABEP value of  $10^{-5}$ . This superiority can be accounted for the more diverse parallel channels resulted from the SVD.

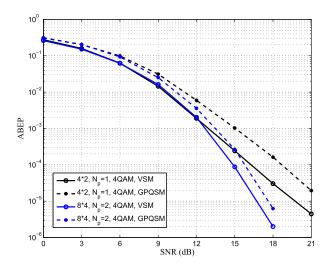


**FIGURE 4.** Comparison of the performances of VSM receivers (14) and (15) in terms of ABEP with  $\rho=$  1, 0.95, 0.7, 0.5 for  $N_t=$  8,  $N_r=$  4 and  $N_D=$  2.

#### B. IMPACT OF $\rho$ ON PERFORMANCE

We examine the impact of the knowledge of  $\rho$  to the receiver on the performance of VSM systems. Fig. 4 shows the comparison results between the ABEPs of the receivers (14) and (15) with the fixed  $\rho = 1, 0.95, 0.7, 0.5$  for  $N_t = 8$ ,  $N_r = 4$  and  $N_p = 2$ . From the figure, we see that for large correlation coefficients such as  $\rho = 1$  and 0.95, nearly the same performance is resulted no matter whether  $\rho$  is known to the receiver or not. For smaller correlation coefficients such as  $\rho = 0.7, 0.5$ , the performance with the knowledge of  $\rho$  to the receiver is superior to that without the knowledge of  $\rho$ . This can be easily understood since the influence of channel estimation becomes more significant as  $\rho$  becomes smaller and the knowledge of  $\rho$  helps the receiver tract the true channel more accurately. However, we note that as the performance gap becomes apparent, the ABEP value also goes very large, e.g., of the order of  $10^{-1}$  for  $\rho = 0.7, 0.5$ , which is intolerable for most applications of wireless communications. Therefore, in practice the receiver (15) is preferred for VSM systems in consideration of its simplicity in implementation since usually the true value of  $\rho$  is difficult to obtain.

Then, we consider the impact of a different channel estimation model on the ABEP performance of VSM. It is assumed that  $\rho$  changes with the SNR according to  $\rho = l\gamma/(l\gamma + 1)$ , where l is chosen to be 2. Fig. 5 shows the comparison results between performances of VSM and GPQSM with unfixed  $\rho$  for two different system configurations, which are 1)  $N_t = 4$ ,  $N_r = 2$  and  $N_p = 1$ ; 2)  $N_t = 8$ ,  $N_r = 4$  and  $N_p = 2$ . From



**FIGURE 5.** Comparison of the performances of VSM and GPQSM in terms of ABEP with unfixed  $\rho$  for two different system configurations: 1)  $N_t=4$ ,  $N_r=2$ , and  $N_p=1$ ; 2)  $N_t=8$ ,  $N_r=4$ , and  $N_p=2$ .

the figure, it can be seen that different from what we observe under the situation of fixed  $\rho$ , the error floors never appear for both VSM and GPQSM. This is because  $\rho$  approaches one, which happens to be the perfect channel estimation case, in the high SNR region. Therefore, the asymptotic results also apply to the situation of unfixed  $\rho$ , which are not shown for figure clarity. On the other hand, from Fig. 5, we see that similarly VSM is superior to GPQSM in the whole SNR region.

#### **VI. CONCLUSIONS AND FUTURE WORK**

In this paper, we have proposed the VSM scheme, which conveys the spatial bits via the indices of active virtual parallel channels. We have derived closed-form upper bounded and asymptotic ABEPs of VSM with practical channel estimation. The diversity order and coding gain of VSM have also been characterized. Finally, simulation results have shown that the proposed VSM scheme provides a significant performance gain compared with the conventional GPQSM scheme with and without channel estimation errors.

For its nature, VSM also applies to the case of  $N_t < N_r$ . In this case, VSM performs index modulation on  $N_t$  virtual parallel channels, which can be considered as a transmitter-side SM scheme. The performance analysis in this paper is also applicable to VSM for  $N_t < N_r$ . The potential of VSM as a transmitter-side SM scheme will be studied in the future.

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