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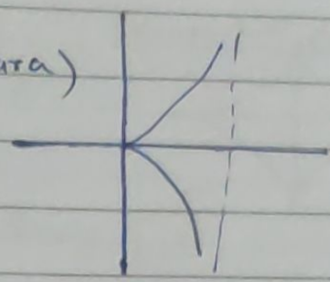
(1) $y^2(a-x) = x^3$ (Cissoid)(Cartesian form)

$x = a\sin^2 t$; $y = \dfrac{a\sin^3 t}{\cos t}$ (Para)

$r = \dfrac{a\sin^2\theta}{\cos\theta}$ (Polar)



$\Rightarrow r = \dfrac{a\sin^2\theta}{\cos\theta}$ Converting to Cartesian, by using $x = r\cos\theta$; $y = r\sin\theta$.

$\therefore r = a\left(\dfrac{y}{r}\right)^2\left(\dfrac{r}{x}\right)$ $\therefore r^2 x = a y^2$

$(x^2+y^2)x = ay^2$

$y^2(a-x) = x^3$.

$\Rightarrow$ • $x = a\sin^2 t$; $y = \dfrac{a\sin^3 t}{\cos t}$

Converting to Cartesian.

$\sin^2 t = \dfrac{x}{a}$; $\cos t = \cancel{a\sin} \sqrt{1-\sin^2 t}$

$\cos t = \sqrt{1-\dfrac{x}{a}} = \sqrt{\dfrac{a-x}{a}}$

$\therefore y = a\sin^2 t . \sin t . \dfrac{1}{\cos t}$

$y = \cancel{a} . \dfrac{x}{\cancel{a}}\sqrt{\dfrac{x}{a}} . \dfrac{\sqrt{a}}{\sqrt{a-x}}$

$y = \dfrac{x\sqrt{x}\,\cancel{\sqrt{a}}}{\cancel{\sqrt{a}}\sqrt{a-x}}$

$(a-x)y^2 = x^2 x$

$\cancel{a^2y^2 = x^3(a-x)}$

$(a-x)y^2 = x^3$

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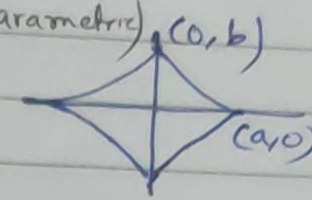
2) $\dfrac{x^{2/3}}{a^{2/3}} + \dfrac{y^{2/3}}{b^{2/3}} = 1$ (Hypocycloid.)

$x = a\cos^3 t$; $y = b\sin^3 t$. (Parametric)

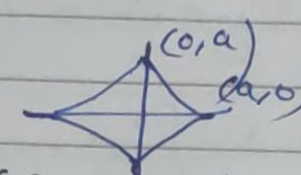
$\dfrac{x^2}{a^2} = \cos^6 t$; $\dfrac{y^2}{b^2} = \sin^6 t$.



Taking cube root & adding.

$$\frac{x^{2/3}}{a^{2/3}} + \frac{y^{2/3}}{b^{2/3}} = \cos^2 t + \sin^2 t = 1$$

(3) $x^{2/3} + y^{2/3} = a^{2/3}$



$x = a\cos^3 t$; $y = a\sin^3 t$ (Parametric)

$\dfrac{x^2}{a^2} = \cos^6 t$; $\dfrac{y^2}{a^2} = \sin^6 t$

Taking cube root & adding.

$$x^{2/3} + y^{2/3} = a^{2/3}$$

(4) $x^2 + y^2 = a^2$ circle

$\Rightarrow x = a\cos t$; $y = a\sin t$

$\dfrac{x^2}{a^2} + \dfrac{y^2}{a^2} = 1$ i.e $x^2 + y^2 = a^2$

$\Rightarrow x = \dfrac{1-t^2}{1+t^2}$; $y = \dfrac{2t}{1+t^2}$

$$x^2 + y^2 = \frac{(1-t^2)^2 + 4t^2}{(1+t^2)^2} = \frac{1+t^4-2t^2+4t^2}{(1+t^2)^2}$$

$$= \frac{(1+t^2)^2}{(1+t^2)^2} = 1$$

$\therefore x^2 + y^2 = 1$

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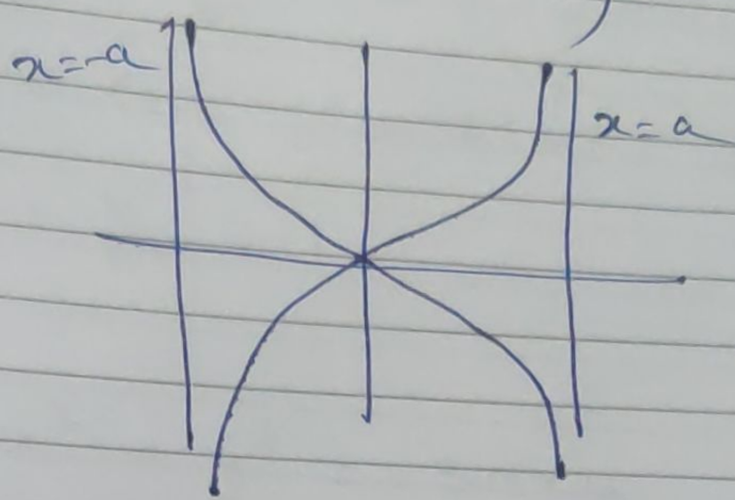
(8) $x^2 y^2 = a^2 (y^2 - x^2)$



(9) $(x^2+y^2)(3ay - x^2 - y^2) = 4ay^3$

$r^2(\cos^2\theta + \sin^2\theta)\left[3ar\sin\theta - r^2\right] = 4ar^3\sin^3\theta$

$\cancel{r^2} \cdot \cancel{r}(3a\sin\theta - r) = 4a\cancel{r^3}\sin^3\theta$

$r = 3a\sin\theta - 4a\sin^3\theta$

$r = a(3\sin\theta - 4\sin^3\theta)$

$r = a\sin 3\theta$.

(10) $x = t^2$; $y = t - \frac{1}{3}t^3$

$y = t\left(1 - \frac{1}{3}t^2\right)$

$y^2 = t^2\left(1 - \frac{1}{3}t^2\right)^2$

$y^2 = x \cdot \left(1 - \frac{1}{3}x\right)^2$

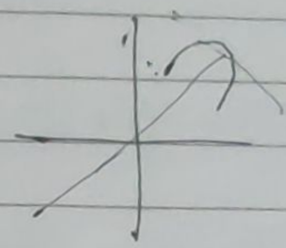
y is 0 at $x=0$ and $x=3$.
$\therefore$ This curve will form a loop between $(0,0)$ & $(3,0)$

(5) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellipse

$x = a\cos t$; $y = b\sin t$. (Parametric)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(6) $x^3 + y^3 = 3axy$.



$x = \frac{3at}{1+t^3}$; $y = \frac{3at^2}{1+t^3}$

$x^3 + y^3 = \frac{27a^3t^3}{(1+t^3)^3} + \frac{27a^3t^6}{(1+t^3)^3}$

$x^3 + y^3 = \frac{27a^3t^3}{(1+t^3)^3}(1+t^3)$

$x^3 + y^3 = \frac{27a^3t^3}{(1+t^3)^2}$

$x^3 + y^3 = 3a \times \frac{3at}{1+t^3} \times \frac{3at^2}{1+t^3}$

$x^3 + y^3 = 3axy$.

(7) $y^2(a+x) = x^2(a-x)$

or $(x^2+y^2)x - a(x^2-y^2) = 0$.