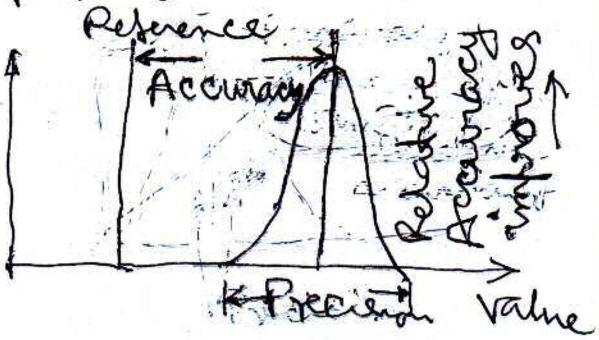
Question - Answer on FCEE 0106 Fundamentals of Electrical Engineering for Mid-Semester Examination held on 20 September 2024 (Friday)

Q. No. 1a Define accuracy, precision, relative accuracy of a measurement. 2

Ans: Accuracy and Precision are both ways and means to measure results. Accuracy measures how close results are to the true or known value. Precision, however, measures how close results are to each other. In fact both are useful for the quality of measurements. Relative accuracy focuses more on precision rather than accuracy. It emphasizes the consistency of measurements relative to each other within the measurements. Absolute accuracy prioritizes both accuracy and precision. It ensures that the measurements closely align with true or known value.

Side figure depicts about Accuracy as well as Precision (for comparison).



Q No. 1b Suppose you are measuring the area of a rectangle, where:

The length (L) is measured as 10.0 ± 0.2 cm

The width (W) is measured as 5.0 ± 0.1 cm

i. Calculate the Nominal value of the Area

ii. Determine the Absolute Error in the Area

iii. Determine the Relative Error in the Area 2

Ans. i) Normal value of Area (A)

$$A = L \times W = 10 \times 5 = 50\ cm^2$$

ii) Absolute Error in the area

In the case of positive extreme case,

$$A_{+ve} = (10.0+0.2) \times (5.0+0.1) = 10.2 \times 5.1 = 52.02\ cm^2$$

Associated error $= 52.02 - 50.0 = 2.02\ cm^2$

In the case of negative extreme case,

$$A_{-ve} = (10.0-0.2) \times (5.0-0.1) = 9.8 \times 4.9 = 48.02\ cm^2$$

Associated error $= 48.02 - 50.0 = -1.98\ cm^2$

Relative error in case of higher $\Rightarrow \frac{2.02}{50} \times 100\% = 4.04\%$

& the same in case of lower $\Rightarrow -\frac{1.98}{50} \times 100\%$
$= -3.96\%$

Q. No. 2a Describe the following in case of measuring instruments

i. Deflecting torque
ii. Controlling torque
iii. Damping torque

2

Ans. In electrical measuring instruments, the Deflecting torque is used for deflection by way of operation due to current, voltage, power, etc., while Controlling torque acts opposite to the Deflecting torque to restrain the deflection. Before coming to rest the pointer always oscillates due to inertia. To bring the pointer to rest within a short time Damping torque is used without effecting Controlling torque or inertia.

Q. No. 2b The change of inductance for a moving iron ammeter is $2\,\mu H$/degree. The control spring constant is 5×10^{-7} N-m/degree. The maximum deflection of the pointer is $100°$. What is the current corresponding to maximum deflection? 2

Ans. Please refer to the derivation against Q No. 5b.

Given $\frac{dL}{d\theta} = 2\,\mu H/\text{degree}$, $K = 5\times10^{-7}$ N-m/degree

$\theta = 100°$

Deflecting torque $T_d = \frac{1}{2} i^2 \frac{dL}{d\theta}$, while

Controlling torque $T_c = K\theta$

At equilibrium when $T_d = T_c$

$$\frac{1}{2} i^2 \frac{dL}{d\theta} = K\theta \text{; or, } i^2 = 2K\theta / (dL/d\theta)$$

$$\text{or, } i^2 = 2\times5\times10^{-7}\times100/2 = 50\times10^{-6}$$

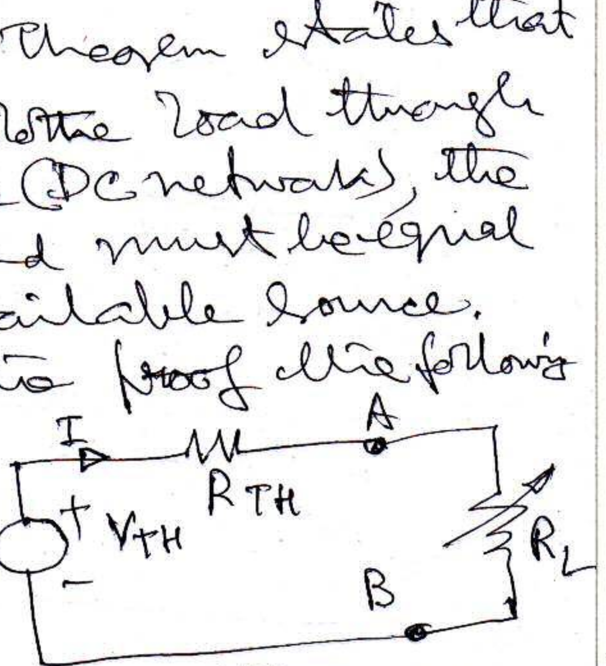
$$\text{or, } i = \sqrt{50}\times10^{-3}\,A = 7.07\,mA$$

Q No 3a State maximum power transfer theorem and also prove the condition when the maximum power is transferred in a circuit. 2

Ans.

Maximum Power Transfer Theorem states that to transfer maximum power to the load through a finite internal resistance (DC networks), the resistance of the given load must be equal to the resistance of the available source.

Consider for establishing the proof the following circuit with source voltage V_{TH}, internal resistance R_{TH} delivering power to a variable load represented through the varying resistance R_L.



Circuit Diagram

Current $I = \frac{V_{TH}}{R_{TH}+R_L}$, Power delivered to load P_L is given by $P_L = I^2 R_L = \frac{V_{TH}^2}{(R_{TH}+R_L)^2} \cdot R_L$

P_L can be maximized by varying R_L when

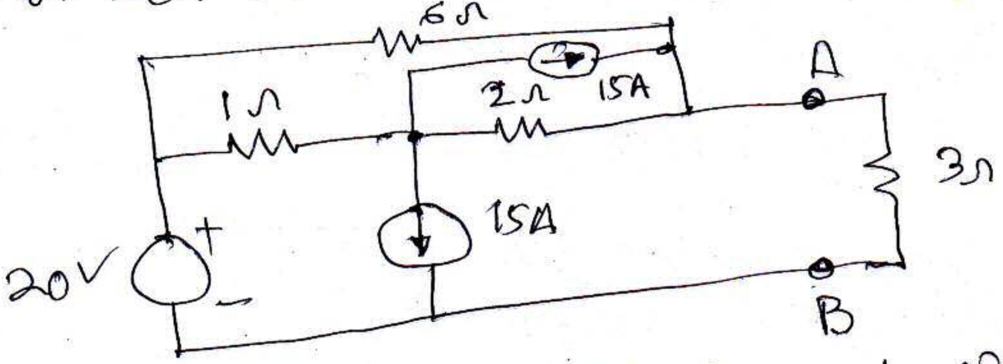
$$\frac{dP_L}{dR_L} = 0 \Rightarrow \frac{dP_L}{dR_L} = \frac{V_{TH}^2}{(R_{TH}+R_L)^4}\left[(R_{TH}+R_L)^2 - R_L \cdot 2(R_{TH}+R_L)\right]$$

$$\Rightarrow (R_{TH}+R_L)^2 = 2R_L(R_{TH}+R_L) \Rightarrow (R_{TH}+R_L) = 2R_L$$

or, $R_L = R_{TH}$

Accordingly $P_{L\,max} = \frac{V_{TH}^2}{(R_{TH}+R_{TH})^2} \cdot R_{TH} = \frac{V_{TH}^2}{4R_{TH}^2} \cdot R_{TH} = \frac{V_{TH}^2}{4R_{TH}}$

Q.No.36 Using Thevenin's Theorem, find the voltage across 3Ω resistor as shown in fig below.



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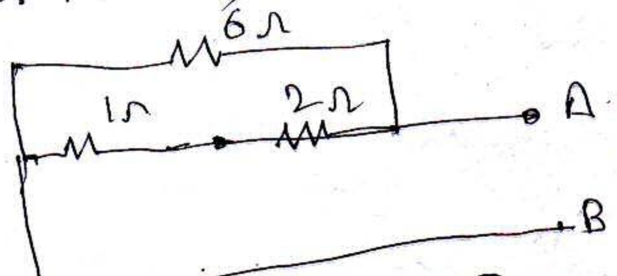
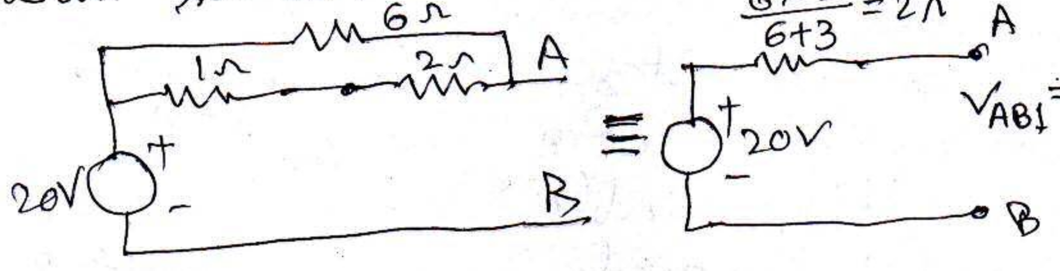
Ans. For finding R_{TH}, the Thevenin equivalent resistance, circuit reduces to as follows:



or $R_{TH} = \frac{6 \times 3}{6+3} = 2\Omega$

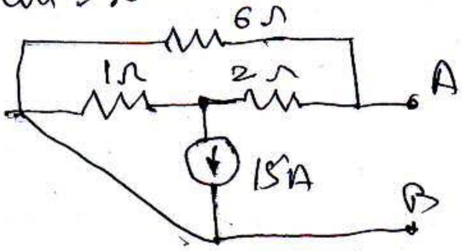
For finding V_{TH1}, let Superposition Theorem be used with 3 sources one by one.

- with source 1 -



$V_{AB1} = V_{TH1} = 20V$

– with source 2 –



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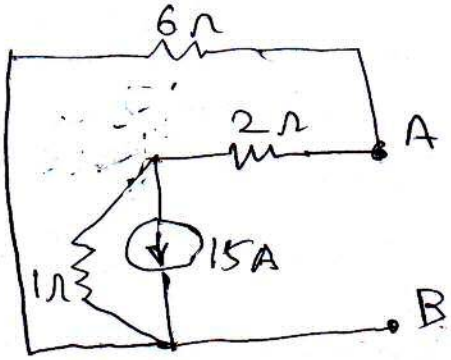

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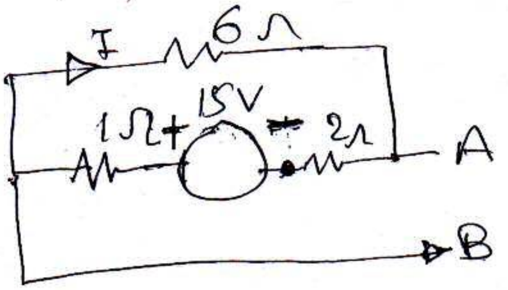
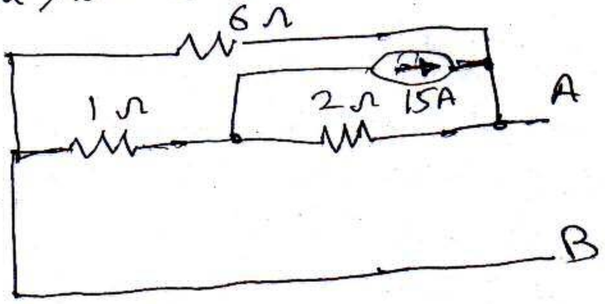


$I = \frac{15}{9} = 5/3\ A$ | $V_{AB2} = V_{TH2} = -6 \times 5/3 = -10V$

– with source 3 –



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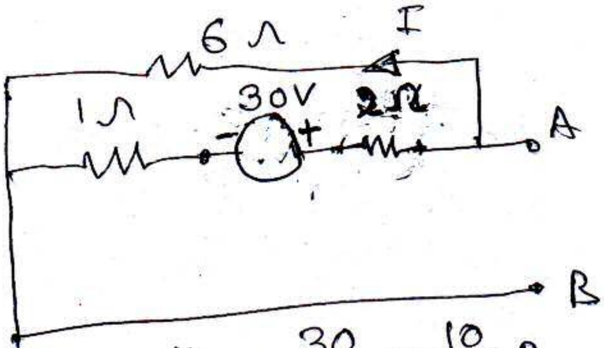


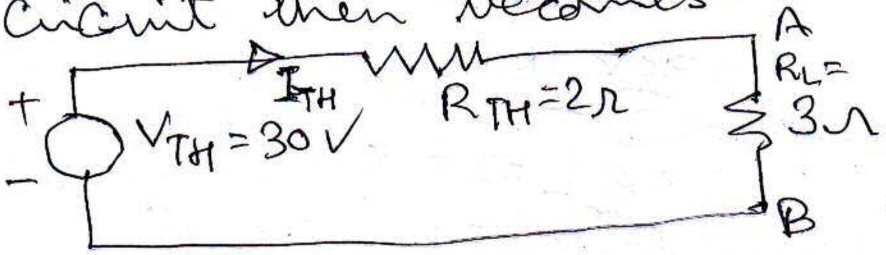
$I = \frac{30}{9} = \frac{10}{3}\ A$

$V_{AB3} = V_{TH3} = 6 \times 10/3 = 20V$

Hence in totality

$V_{TH} = V_{TH1} + V_{TH2} + V_{TH3}$
$= 20 - 10 + 20 = 30\ V$

Eqv. Circuit then becomes –

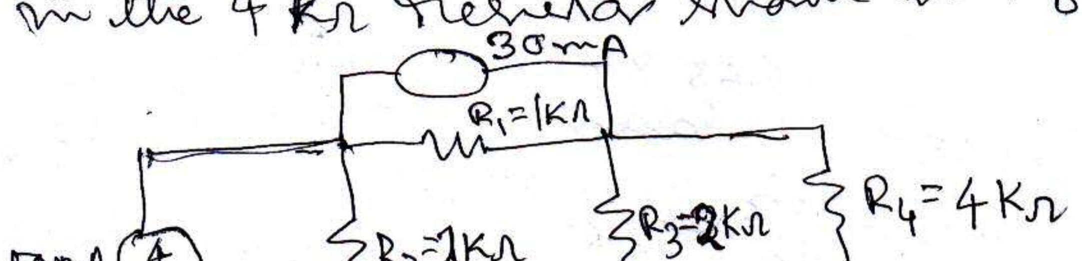


Now the ckt. is

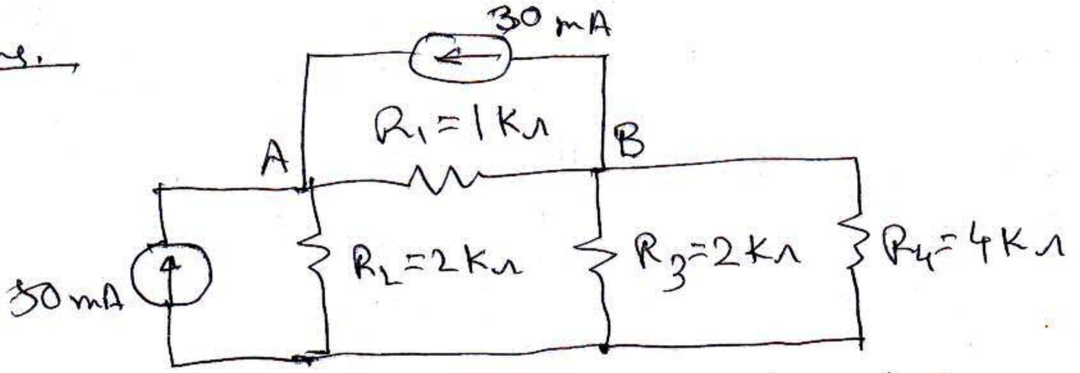
$I_{TH} = \frac{V_{TH}}{R_{TH} + R_L} = \frac{30}{2+3} = 6A$

$V_{AB} = R_L I_{TH} = 3 \times 6 = 18V$

Q No. 4a Use nodal analysis to find current in the 4 KΩ resistor shown in Fig below. 2



Ans.



At A $\rightarrow$ $50 + 30 = \frac{V_A}{2} + \frac{V_A - V_B}{1}$

or, $80 = 1.5V_A - V_B$

or, $3V_A - 2V_B = 160$

At B $\rightarrow$

$-30 = \frac{V_B - V_A}{1} + \frac{V_B}{2} + \frac{V_B}{4}$

or, $1.75V_B - V_A + 30 = 0$

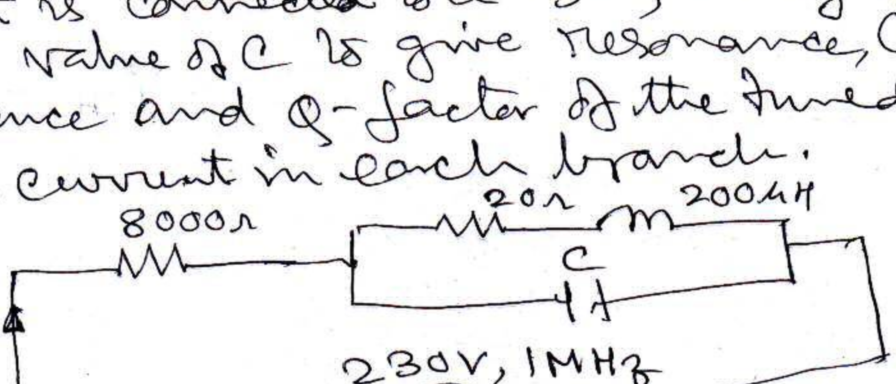
or, $4V_A - 7V_B = 120$

upon solving $V_B = 21.54$ V

Current in 4 KΩ resistor is given by $\frac{V_B}{4} = \frac{21.54}{4} = 5.385$ mA

Q No. 4b. A tuned circuit consisting of a coil having an inductance of 200 μH and a resistance of 20Ω is in parallel with a variable capacitor. This combination is in series with a resistor of 8000Ω. The entire circuit is connected to a 230V, 1 MHz supply. Calculate (i) the value of C to give resonance, (ii) the dynamic impedance and Q-factor of the tuned circuit and (iii) the current in each branch. 2

Ans.



Reactance of Coil, $X_L = 2\pi f_r L = 2\pi \times 1 \times 10^6 \times 200 \times 10^{-6} = 1,256\,\Omega$

As $R << X_L$, resonant frequency $f_r = \frac{1}{2\pi\sqrt{LC}}$

or, $f_r^2 = \frac{1}{4\pi^2 LC} \Rightarrow C = \frac{1}{4\pi^2 L f_r^2} = \frac{1}{4\pi^2 \times 200 \times 10^{-6} \times (1 \times 10^6)^2}$

$= 126.65 \times 10^{-12}\,F = 126.65\,pF$

Dynamic or Effective impedance $Z_r = \frac{L}{CR} = \frac{200 \times 10^{-6}}{126.65 \times 10^{-12} \times 20}$

$= 78,957\,\Omega$

Quality Factor $Q = \frac{\omega L}{R} = \frac{1,256}{20} = 62.8$

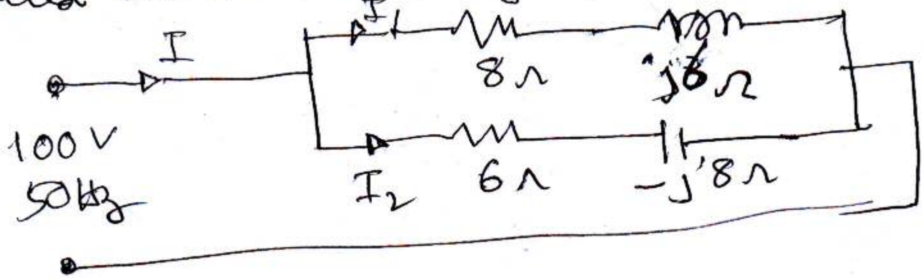
Total Circuit Impedance $= 8,000 + 78,957$ (almost fully resistive)

Total Current $= 230/86,957 = 2.64$ mA; Drop across tuned circuit $= 2.64 \times 78,957 = 208.4$V

Current in coil $= 208.4/\sqrt{20^2 + 1256^2} \simeq 166$ mA

Current through capacitor $= 208.4/X_c \simeq 166$ mA where $X_c = \frac{1}{2\pi \times 1 \times 10^6 \times 126.65 \times 10^{-12}}$

Q. No 5a — The parallel circuit shown in diagram is connected across a single phase 100 V, 50 Hz ac supply



Calculate
i. The total current
ii. The supply power factor
iii. Active and reactive power supplied 2

Ans.

$$I_1 = \frac{100}{8+j6} = \frac{100(8-j6)}{8^2+6^2} = 8-j6 = 10\angle -36.87^\circ \text{ A}$$

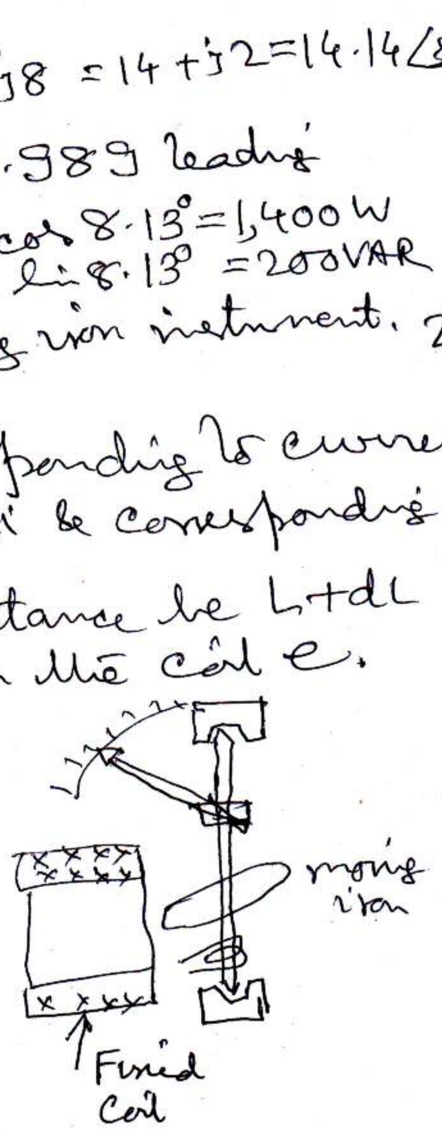
$$I_2 = \frac{100}{6-j8} = \frac{100(6+j8)}{36+64} = 6+j8 = 10\angle 53.13^\circ \text{ A}$$

- Total current $I = I_1 + I_2 = 8-j6+6+j8 = 14+j2 = 14.14\angle 8.13^\circ$ A
- Power factor, $pf = \cos\theta = \cos 8.13^\circ = 0.989$ leading
- Active Power $P = VI\cos\theta = 100 \times 14.14 \cos 8.13^\circ = 1,400$ W
& Reactive power $Q = VI\sin\theta = 100 \times 14.14 \sin 8.13^\circ = 200$ VAR

Q.No 5b Derive the torque equation for moving iron instrument. 2

Ans. Let θ be the deflection corresponding to current i Amp. Let current increases by di & corresponding deflection by $d\theta$. Let the new inductance be $L+dL$.
As a result let the induced emf in the coil e. from L with induced emf in the coil e.

Hence $e = \frac{d}{dt}(Li) = L\frac{di}{dt} + i\frac{dL}{dt}$



Multiplying both side by $i\,dt$

$e \times i\,dt = Li\,di + i^2 dL$

Therefore, change in energy stored

$= \frac{1}{2}(L+dL)(i+di)^2 - \frac{1}{2}Li^2$

$= \frac{1}{2}[(L+dL)(i+di)^2 - Li^2]$

neglecting product of smaller values

$= Li\,di + \frac{1}{2}i^2 dL$

- Mechanical work to move pointer by $d\theta$ is given by $T_d\, d\theta$

By law of conservation of energy,

Electrical Energy Supplied = Increased in stored energy + Mechanical energy

or, $Li\,di + i^2 dL = \left(Li\,di + \frac{1}{2} i^2 dL\right) + T_d \cdot d\theta$

or, $\frac{1}{2} i^2 dL = T_d\, d\theta \Rightarrow T_d = \frac{1}{2} i^2 \frac{dL}{d\theta}$

This is balanced in steady-state by the Controlling Torque, $T_c = K\theta$, where K is the spring Constant.

Then for $T_c = T_d$, $K\theta = \frac{1}{2} i^2 \frac{dL}{d\theta}$

or, $\theta = \frac{1}{2K} i^2 \frac{dL}{d\theta} \Rightarrow \theta \propto i^2$

[In AC measurement $\theta \propto I^2$ (where I is the current in rms)]