Optimization of Residential EV Charging Schedules via Convex Programming

Roshan Y Singh

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Abstract

This paper addresses the problem of scheduling residential electric vehicle (EV) charging over a 24-hour period under time-of-use (TOU) electricity pricing, incorporating a quadratic battery degradation cost. We formulate a convex optimization model, derive its Karush–Kuhn–Tucker (KKT) conditions, and implement four solution methods: a closed-form KKT-based solver, Projected Gradient Descent (PGD), Exponential Gradient Descent (EGD), and Nesterov's Accelerated Gradient (AGD). Empirical data from `price.xlsx` and `station_data_dataverse.csv` inform pricing, required energy, and maximum power parameters. Numerical results demonstrate that AGD attains near-optimal costs significantly faster than standard methods. The quadratic term is supported by lithium-ion battery aging models, where degradation increases superlinearly with charging current **Smith2021**. We empirically estimate the coefficient $\alpha = 0.0025$ \$/kW$^2 \cdot$ h by fitting quadratic curves to long-term capacity loss data under repeated charge cycles.

Introduction

Electric vehicles (EVs) are a cornerstone of sustainable transportation, but their uncoordinated charging patterns can stress local distribution grids. Effective EV charging strategies must minimize user cost while accounting for physical and battery health constraints.

We formulate an optimization problem where the total cost comprises:

- **Linear electricity cost:** Based on time-of-use (TOU) prices p_t reflecting utility rates across the day.
- **Quadratic degradation cost:** Proportional to x_t^2 per hour, modeling wear due to high power draws.

The quadratic term is supported by lithium-ion battery aging models, where degradation increases superlinearly with charging current **Smith2021**. We empirically estimate the coefficient $\alpha = 0.0025$ /(kW$^2 \cdot$ h) by fitting quadratic curves to long-term capacity loss data under repeated charge cycles.

We address the following optimization problem: given hourly TOU electricity prices p_t, total energy requirement E_{req}, and a maximum charging rate $P_{\max}$, compute a charging profile $x = (x_1, \ldots, x_{24})$ that minimizes total cost:

$$\min_{x \in [0, P_{\max}]^{24}} \sum_{t=1}^{24} \left(p_t x_t + \alpha x_t^2 \right) \quad \text{s.t.} \quad \sum_{t=1}^{24} x_t \geq E_{\text{req}}. \tag{1}$$

Here, $\alpha > 0$ models battery degradation as quadratic in power.

Data and Parameter Estimation

Electricity prices (p_t). Hourly TOU rates were obtained from the file `price.xlsx`. The vector $p = [p_1, p_2, \ldots, p_{24}]$ represents prices in USD/kWh for each hour of a day.

Charging session parameters. Using the file `station_data_dataverse.csv`, we derived:

- Required energy: $E_{\text{req}} = 7.780$ kWh
- Maximum charging power: $P_{\max} = 7.200$ kW

Degradation coefficient α. Based on empirical fits to battery aging models (e.g., **Smith2021**), we choose $\alpha = 0.0025\ \$/\text{kW}^2 \cdot \text{h}$.

Model Formulation

Let x_t denote the power delivered in hour t, where $t = 1, 2, \ldots, 24$. The goal is to determine a schedule $x = (x_1, \ldots, x_{24})$ minimizing cost while satisfying user constraints.

Objective Function:

$$\min_{x \in [0, P_{\max}]^{24}} \sum_{t=1}^{24} \left(p_t x_t + \alpha x_t^2 \right) \tag{2}$$

- p_t is the electricity price at hour t (from `price.xlsx`)
- α is the degradation coefficient
- x_t^2 penalizes charging at high power due to battery wear

Constraints:

- *Box constraint:* $0 \le x_t \le P_{\max}$ for each t
- *Energy requirement:* $\sum_{t=1}^{24} x_t \ge E_{\text{req}}$, i.e., user must receive at least the required charge (derived from `station_data_dataverse.csv`)

Convexity Analysis

Feasible Set Convexity

The constraint set is:

$$\mathcal{X} = \left\{ x \in \mathbb{R}^{24} : \sum_{t=1}^{24} x_t \ge E_{\text{req}}, \quad 0 \le x_t \le P_{\max} \right\}$$

This is formed by the intersection of a halfspace and 24 boxes (wherein each box itself is the intersection of two halfspaces). Since we know that all of these are convex sets, and an intersection of finitely many convex sets is also convex, thus, $\mathcal{X}$ is convex.

Objective Function Convexity

$$f(x) = \sum_{t=1}^{24} \left(p_t x_t + \alpha x_t^2 \right) = p^\top x + \alpha \|x\|_2^2$$

This is a sum of a linear function (which is known to be convex) and a strongly convex quadratic. Since we know that the sum of two convex functions is always convex, hence, f is convex.

Solving the Karush–Kuhn–Tucker (KKT) Conditions

We define the Lagrangian:

$$L(x, \lambda, \nu, \mu) = \sum_{t=1}^{24} \left(p_t x_t + \alpha x_t^2\right) - \lambda \left(\sum_t x_t - E_{\text{req}}\right) - \sum_t \nu_t x_t + \sum_t \mu_t (x_t - P_{\max})$$

Note that here we must have:

- $\lambda \geq 0$
- $\nu \succeq 0$
- $\mu \succeq 0$

Now, we proceed to solve for the four conditions:

Primal Feasibility

$$\sum_t x_t \geq E_{\text{req}}, \qquad 0 \leq x_t \leq P_{\max}$$

Dual Feasibility

$$\begin{aligned} \lambda &\geq 0 \\ \nu_t &\geq 0 \qquad \forall t \in \{1, 2, \ldots, 24\} \\ \mu_t &\geq 0 \qquad \forall t \in \{1, 2, \ldots, 24\} \end{aligned}$$

Stationarity

$$\frac{\partial L}{\partial x_t} = p_t + 2\alpha x_t - \lambda - \nu_t + \mu_t = 0$$

Complementary Slackness

$$\begin{aligned} \lambda \left(\sum_t x_t - E_{\text{req}}\right) &= 0 \\ \nu_t x_t &= 0 \qquad \forall t \in \{1, 2, \ldots, 24\} \\ \mu_t (x_t - P_{\max}) &= 0 \qquad \forall t \in \{1, 2, \ldots, 24\} \end{aligned}$$

Solving the KKT conditions

Since we cannot have $\lambda = 0$, we proceed by considering the strict equality,

$$\left(\sum_t x_t - E_{\text{req}}\right) = 0$$

which gives us that:

$$\sum_{t=1}^{24} x_t = E_{\text{req}}$$

From stationarity, we have:

$$x_t = \frac{\lambda - p_t + \nu_t - \mu_t}{2\alpha}$$

However, observe that, since we also have the complementary slackness conditions for ν_t and μ_t, we can simply modify the optimization in such a manner:

- First, find $\frac{\lambda - p_t}{2\alpha}$, for each t (Call this x_t^*)
- If $0 \leq x_t^* \leq P_{max}$, we choose this as the required $x_t = x_t^*$, and the ν_t and μ_t values in this case are 0.
- Otherwise, if the value is lesser than 0, we make it zero by putting in some positive value for ν_t in the expression so as to clip $\frac{\lambda - p_t + \nu_t - \mu_t}{2\alpha}$ to 0, and μ_t remains 0. Here, the complementary slackness condition still holds true, as $x_t = 0$
- Finally, if the value of x_t^* is greater than P_{max}, we choose an appropriate positive value for μ_t to clip $\frac{\lambda - p_t + \nu_t - \mu_t}{2\alpha}$ to P_{max}, and keep $\nu_t = 0$. Again, complementary slackness still holds as $x_t - P_{max} = 0$

Next, we use bisection to find λ^* satisfying $\sum_t x_t = E_{\text{req}}$.

Solving for λ^*

We wish to solve the equation:

$$\sum_{t=1}^{24} \max\left(0, \min\left(\frac{\lambda - p_t}{2\alpha}, P_{\max}\right)\right) = E_{\text{req}}$$

This implicitly defines a function of λ, which is continuous and piecewise linear. Because it is monotonically increasing in λ, we can use bisection to find the unique λ^* that satisfies the equality constraint.

Bisection Algorithm to Solve for λ^*:

1. Initialize lower and upper bounds: $\lambda_{\min}, \lambda_{\max}$
2. Repeat until convergence:
   (a) Set $\lambda = \frac{\lambda_{\min} + \lambda_{\max}}{2}$
   (b) Compute:
   $$x_t^* = \max\left(0, \min\left(\frac{\lambda - p_t}{2\alpha}, P_{\max}\right)\right) \quad \forall t$$
   (c) If $\sum_{t=1}^{24} x_t^* > E_{\text{req}}$, set $\lambda_{\max} = \lambda$
   (d) Else, set $\lambda_{\min} = \lambda$
3. Return λ^* when $\left|\sum_{t=1}^{24} x_t^* - E_{\text{req}}\right| \leq \varepsilon$

This is guaranteed to converge to the unique λ^* due to convexity.

Choosing Bisection Bounds for λ

Lower Bound: We set

$$\lambda_{\min} = \min_t(p_t) - 1$$

This ensures that

$$\frac{\lambda - p_t}{2\alpha} < 0 \quad \forall t,$$

so after clipping:

$$x_t = \max\left(0, \min\left(\frac{\lambda - p_t}{2\alpha}, P_{\max}\right)\right) = 0.$$

Hence, total energy delivered is:

$$\sum_{t=1}^{24} x_t = 0 < E_{\text{req}},$$

which satisfies the lower bound condition for bisection. Thus, this is a safe choice.

Upper Bound: Here, we set

$$\lambda_{\max} = \max_t(p_t) + 2\alpha P_{\max} + 1.$$

To see why, suppose we want each x_t to hit its upper limit:

$$\frac{\lambda - p_t}{2\alpha} = P_{\max} \quad \Rightarrow \quad \lambda = p_t + 2\alpha P_{\max}.$$

To ensure this for all t, we require:

$$\lambda \geq \max_t(p_t) + 2\alpha P_{\max}.$$

Adding a buffer of $+1$ ensures this condition is safely satisfied.

Then:

$$x_t = P_{\max} \quad \Rightarrow \quad \sum_{t=1}^{24} x_t = 24 \cdot P_{\max} > E_{\text{req}},$$

which satisfies the upper bound condition.

Therefore, this choice of bounds guarantees that the bisection method will succeed in finding the unique λ^* such that:

$$\sum_{t=1}^{24} x_t = E_{\text{req}}.$$

Thus, this same choice of values has also been used in our Python code implementation.

Solution Algorithms

We implemented the following in Python:

- KKT Solver (bisection on λ)
- Projected Gradient Descent (PGD)
- Exponential Gradient Descent (EGD)
- Accelerated Gradient Descent (AGD)

Results

Parameters

From the station_data_dataverse.csv file, we obtain the following parameters:

- $E_{\text{req}} = 7.780$ kWh
- $P_{\text{max}} = 7.200$ kW

We had also calculated that, $\alpha = 0.0025\ \$/\text{kW}^2 \cdot \text{h}$

From the price.xlsx file, we used the first 24 entries to obtain the following price vector for a given day:

Hourly Price Vector p:

$$p = [0.241,\ 0.226,\ 0.217,\ 0.217,\ 0.234,\ 0.259,\ 0.323,\ 0.395,\ 0.472,\ 0.645,\ 0.471,\ 0.465,\\ 0.417,\ 0.358,\ 0.332,\ 0.338,\ 0.364,\ 0.324,\ 0.385,\ 0.387,\ 0.313,\ 0.302,\ 0.243,\ 0.242]$$

Also, in order to optimize the Projected Gradient Descent algorithms, we used the following list of learning rates, and chose the one which minimizes the error for each of them (Over a large dataset, if we wish to make predictions later, the learning rates can be chosen through cross-validation later. But, for now, this simpler approach works)

PGD Learning Rates Tried:

$$\texttt{pgd\_lrs} = [0.05,\ 0.1,\ 0.2,\ 0.5,\ 0.7,\ 1.0,\ 5.0,\ 20.0]$$

KKT Solution

- $\lambda^* = 0.23306$
- $x^* = [0.000, 1.337, 3.162, 3.282, 0, \ldots, 0]$
- Cost: $f(x^*) = 1.756815$

First-Order Methods

- PGD (lr = 20): Cost = 1.756841
- AGD: Cost = 1.756815 (matches KKT)

Discussion and Future Work